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Fair immunization and network topology of complex financial ecosystems

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ABSTRACT

The aftermath of the recent financial crisis has shown how expensive and unfair the stabilization of financial ecosystems can be. The main cause is the level of complexity of financial interactions that poses a problem for regulators. We provide an analytical framework that decomposes complex ecosystems in both their overall level of instability and the contribution of institutions to instability. These ingredients are then used to study the pathways of the ecosystems towards stability by means of immunization schemes. The latter can be designed to penalize institutions proportionally to their contribution to instability, and therefore enhance fairness. We show that fair immunization schemes can also be cost-efficient when employed on ecosystems characterized by a tiered network structure of interactions concentrated among few core nodes that at the same time form many closed cycles that exacerbate instability. For less tiered network topologies we observe a trade-off between fairness and dollar-cost of immunization, allowing regulators to choose the combination that best meets their objectives. The implementation of immunization schemes on real cross-border financial networks of the Bank of International Settlement (BIS) reporting country banking systems is also provided.

1. Introduction

The resilience of an ecosystem is its ability to return to its original state following any shocks or external perturbations [1]. If the ecosystem is not able to self-inoculate against these disturbances, it will be exposed to a system-wise disruption and the potential extinction of its constituent species. Immunization strategies are therefore crucial to guide unstable ecosystems towards stability. The work of May [2,3] provides insights on how the topology of the network contributes to instability in an epidemiology model. Although initially based on random matrices, the May–Wigner stability theorem has been generalized [4] and tested on several ecosystems, including financial markets [5,6]. The recent financial crisis is an example of an unstable financial ecosystem, highly vulnerable to systemic risk. The latter is defined as a negative externality [5], a "disruption to the flow of financial services that is caused by the impairment of all or parts of the financial system and has the potential to have serious negative spillovers to the real economy" [7].

The derived financial stability condition from May [2,3] complements a large body of empirical studies based on shock propagation algorithms and distribution of financial losses (see, [8–12], for recent reviews). The moral hazard problem

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caused by one of the largest tax payer bailouts of financial intermediaries, costing over $14 trillion [13], promoted the debate of ex-ante solutions aimed at stabilizing the financial system and therefore avoid unfair social and economic consequences. This paper discusses recent developments of ex-ante solutions to immunize financial ecosystems in the pursuit of mitigating future disruptions.

Although several studies propose interesting metrics to assess the level on instability of both observed and reconstructed financial systems [14–16], there are still limited studies addressing the problem of internalizing instability via immunization strategies, i.e. by imposing taxes/restrictions to the level of interaction among participants [17–19]. More importantly, immunization strategies should also be designed to target those participants that pose the largest threat to the resilience of the whole ecosystem [20]. We design different immunization schemes, both homogeneous with a common rate for all players, and heterogeneous with rates tailored to the contribution of each financial institution to the level of instability of the entire system. The latter scheme aims at maximizing the fairness of the immunization by targeting systemically important players. We show an interesting trade-off between the dollar-cost of immunization and its level of fairness that is highly sensitive to the topological properties of the network. In highly tiered systems that well represent real financial networks, such as scale-free and core–periphery structures, core nodes tend to be both systemically important and responsible for the creation of many closed cycles that exacerbate instability [21]. Within these structures, heterogeneous schemes can be both fair and cost-efficient, the ideal choice for proper financial regulation. However, for less tiered/more random structures with low correlation between systemic importance and contribution to cyclical structures, pursuing a fairer scheme might come at an higher cost compared to the homogeneous case. This trade-off is also found when immunization schemes are tested on real cross-border financial networks of country banking systems using BIS consolidated banking statistics.

2. Methods

Metrics that attempt to measure the level of disruption in a systemic event can be classified on the basis of the information used to assess individual contribution to systemic risk, i.e. market based vs. balance-sheet network based metrics (see [22] for a detailed review of market-based systemic risk metrics). The latter are increasingly being used to better understand the stability of complex financial systems. Since the classic Eisenberg&Noe [23] and Furfine [24] stress test contagion algorithms, along with the recent Debt-Rank [25], a growing body of work using network analysis has been developed for systemic risk management that uses financial balance sheet interlinkages to analyse contagion and distribution of losses from the failure of a trigger institution. Examples of applications and extensions of the clearing payment vector of [23] can be found in [26] that includes default costs in their greatest clearing vector algorithm, and recent extension of [27] that shows issues in the existence of unique results when banks can use CDS to cover their losses. The inclusion of uncertainty on the value of the external assets is another important extension of the general Eisenberg&Noe model that was initially employed by [28] in their assessment of the stability of Austrian banks, and further developed by [29,30]. Analytical generalizations of network valuation for systemic risks assessments are also presented in [30,31].

Instead of performing numerical simulations of contagion conditional to a distress event [23–25], we exploit the ex-ante stability conditions of the ecosystem as a function of the topology of the network from which we derive and validate immunization strategies. We follow the insight from [2,3] that network stability depends on the size of the maximum eigenvalue of an appropriate dynamical characterization of the network and a common failure threshold [32–34]. This spectral analysis led to the so-called Eigen-Pair model that has already been employed on the global derivatives market [5], the US CDS market [6] and for risk assessment of CCPs [11,35] and global banking [32]. Recent work of [21] employs a similar spectral approach on banks leverage matrix to study the pathways of financial network topology towards instability. In contrast, this study looks at pathways towards stability by evaluating alternative immunization schemes for banks. While the seminal work of [5,6] first and recent generalization of stability conditions of [32] contributed to the estimation of the level of systemic risk of the financial system, our approach builds on this framework by proposing solutions to reduce the estimated level of systemic risk. Specifically, the [32] eigenpair solution of their systemic risk indicator, which provides both a macro index of systemic risk as well as an individual contribution to that level of risk both as importance and vulnerability rank, provides an elegant and analytically sound foundation to derive immunization solutions to the problem of systemic risk.

Our study is closely related to that of [17,19] as both proposed solutions to internalize systemic risk by means of taxation. Of course, tax rates require a proper assessment of systemic risk and the individual contribution to instability. The first study proposes two measures, Marginal Expected Shortfalls (MES) and Systemic Risk Contribution (SRISK) as proxy of the firm vulnerability in terms of capital loss. The second is based of Debt-Rank and implements an agent-based model to assess the restructuring of the financial network when a systemic risk tax is imposed to participants. Our model follows the same principle that network participants should be taxed according to their individual contribution to the overall level of systemic risk. Our model differs from these studies on the assessment model implemented, in our case the eigenpair model. The advantage of our approach is that the fixed point solution of the eigenpair model provides simultaneously both the overall level of instability as well as the individual contribution of participants to the level of instability. Therefore, our approach does not require individual aggregations of level of risk to assess the overall stability of the system as for [17,19].

2.1. Contagion model

When modelling network contagion, both the source and magnitude of pairwise connections must be accurately defined. With regards to the source of contagion within a balance sheet interlinkage network, financial institutions can face losses from both sides of their balance sheet, i.e. (i) as a lender when borrowers fail to repay their debt in the event of defaults, or more generally due to deterioration of their cross-border investments exacerbated by mark-to-market accounting (solvency shocks [6,23–25,36,37]), or (ii) as a borrower when credit lines from lenders are withdrawn (liquidity shortfalls [38–40]). We focus on the first channel (solvency) as a source of contagion in this study.

$$\mathbf{X} = \begin{bmatrix} 0 & x_{12} & \cdots & x_{1j} & \cdots & x_{1N} \\ x_{21} & 0 & \cdots & x_{2j} & \cdots & x_{2N} \\ \vdots & \vdots & 0 & \cdots & \cdots & \cdots \\ x_{i1} & \vdots & \cdots & 0 & \cdots & x_{iN} \\ \vdots & \vdots & \cdots & \cdots & 0 & \vdots \\ x_{N1} & x_{N2} & \cdots & x_{Nj} & \cdots & 0 \end{bmatrix}. \tag{1}$$

Let i and j be two nodes. When a direct link originates with i and ends with j, viz. a gross (nominal) liability for i, it is represented by the entry x_{ij}, denoting payments for which i is the obligor. The financial system is defined by a $N \times N$ directed weighted network represented by the matrix $\mathbf{X}$ in Eq. (1) such that the sum of an ith row $\sum_{j=1}^{N} x_{ij}$ represents the total liabilities of node i to the whole system. Likewise, the sum of the jth column $\sum_{i=1}^{N} x_{ij}$ represents the total receivable amount in the form of claims of node j. Note that $x_{ii} = 0$.

With regard to the magnitude of bilateral losses, popular models for risk assessments evaluate net bilateral exposures [24,41], especially when considering large institutions that are allowed to assess their exposure on a net basis via master agreements. In order to relate system failure from insolvency contagion arising from creditors having insufficient buffers set by regulatory capital requirements, we model the dynamics of i's capital at time $t+1$, $c_{i,t+1}$, as follows:

$$c_{i,t+1} = c_{i,0} - \sum_{j} \left(x_{ji} - x_{ij}\right)^{+} p_{j,t} - (1-\rho_i)\left(c_{i,0} - c_{i,t}\right). \tag{2}$$

where $c_{i,0}$ is i's the initial capital, $\left(x_{ij} - x_{ji}\right)^{+}$ equals to the net liability exposure of a node j viz-á-viz its debtor i, and the cumulative rate of capital depletion that is used to assess the propensity of default of node i at $t+1$, denoted by $p_{i,t+1}$, is given by subtracting the equivalent $c_{i,t+1}$ from $c_{i,0}$ and dividing through by $c_{i,0}$ in (2). Finally, the individual stability thresholds (or cure rates) ρ_i, are defined as the percentage of capital each node allocates to loss absorption, ranging from zero (no capital to mitigate losses) to one (the entire capital of the node is allocated to loss absorption). We also adopt the notion of stability matrix Θ whose elements $\theta_{ij} = \frac{(x_{ij}-x_{ji})^{+}}{c_{j,0}}$ are equal to the net liability exposure of a node j viz-á-viz its debtor i, $\left(x_{ij} - x_{ji}\right)^{+}$, as percentage of the initial capital of j, $c_{j,0}$ [32]. The individual propensity of default $p_{i,t+1}$ of i at time $t+1$ is described by the dynamical system

$$p_{i,t+1} = (1-\rho_i)\, p_{i,t} + \sum_{j} \theta_{ji} p_{j,t}. \tag{3}$$

The propensity of default at $t+1$ in (3), $p_{i,t+1}$, is determined by (i) i's own 'cure rate' ρ_i given its current propensity of default $p_{i,t}$ (typically defined by the ratio involving cumulative capital losses at time t given as $\frac{c_{i,0}-c_{i,t}}{c_{i,0}}$) and (ii) the sum of 'infection rates' defined by the sum of pairwise losses from its exposure to j weighted by the counterparty propensities of default. Note that the linear form of the contagion channel in (3) captures the losses of asset values due to the deterioration of the underline (still not defaulted) institution, as in [21]. This extends the standard view that losses can only be propagated as the result of a default, requiring step functions as in [23,24] to name few.

Converted into matrix notation, the dynamical system takes the form

$$\mathbf{P}_t = \left[(\mathbf{I} - \mathbf{R}) + \Theta'\right] \mathbf{P}_{t-1} = \mathbf{Q}\mathbf{P}_{t-1} \tag{4}$$

with $\mathbf{P}_t = \left(p_{1,t}, p_{2,t}, \ldots, p_{n,t}\right)$, $\mathbf{R} = diag[\rho_1, \ldots, \rho_n]$ is a diagonal matrix of individual ρ_i and $\mathbf{I}$ is the identity matrix. The stability condition is given by $\lambda < 1$, where λ is the maximum eigenvalue of the matrix $\mathbf{Q}$. Financial ecosystems characterized by $\lambda > 1$ are therefore unstable to perturbations and would require immunization strategies to avoid systemic disruptions (see Supplementary Methods for the proof).

The spectral decomposition of the instability matrix does not only provide a stability metric for the system as a whole (the eigenvalue analysis), but also captures the individual contributions of financial nodes to the level of instability via the right eigenvector associated with the largest eigenvalue (see Supplementary Methods). The latter is usually ignored in the field of network stability, and only employed in network analysis as a node centrality index. The eigen-pair method of [5,32] is a notable exception, and their findings have inspired the creation of the immunization schemes proposed here.

2.2. Immunization

With an unstable financial system, $\lambda > 1$, our objective is to stabilize the ecosystem by bringing the λ below one. Remember that the matrix $\mathbf{Q}$ is a function of the individual stability thresholds ρ_i (cure rate), the positive net liabilities $\left(x_{ij} - x_{ji}\right)^+$ and the individual level of capital c_i. Supplementary Methods provide a formula to derive individual thresholds ρ_i that satisfy the capital adequacy ratio of regulatory tier 1 capital over risk weighted assets set by recent banking regulation. This provides direct implementation of this systemic risk assessment to real bank data as shown later. By setting individual stability thresholds to satisfy the actual capital requirements imposed by financial regulators, we can effectively evaluate the contribution and vulnerability of each participant to the overall systemic risk.

The main literature agrees that there are two major ways to internalize systemic risk: by regulation or by taxation [42]. Since systemic risk is a negative externality [5,6], economists prefer the solution of taxing the externality [19] as taxation can be designed specifically to address the marginal contribution of systemic risk [42] as well as not requiring heavy-handed government intervention into the decision making of market participants. Specifically, a Pigovian tax, the 'Systemic Risk Tax', of [17] that can dynamically induce participant to reduce their contribution (in an agent-based simulated model), or a 'bail-in' escrow fund (super-spreader taxes) of [5] are among those proposed.

In our model, we follow the preferred taxation approach suggested above that would bring λ down by reducing the bilateral net liabilities. Alternatively, reducing ρ_i and/or increasing initial capital buffers $c_{i,0}$ would also internalize systemic risk by means of regulation. We discuss this alternative approach in the Supplementary Methods.

Since imposing a systemic risk tax on financial participants based on unrealized potential future losses can be very hard to implement, we therefore suggest that individual contributions to internalize systemic risk can be escrowed into a systemic risk fund, similar to what regulators are recommending to CCPs as part of the default fund dedicated to systemic risk events [11]. Note that regulators and/or market makers can also evaluate alternative assets to be eligible in the fund in order to facilitate transactions, including high quality liquid assets and/or guarantees or insurance policies to cover the marginal contribution of systemic risk in a systemic event as recommended by [19]. We leave the analysis on the impact of these alternative assets to systemic risk management to future studies.

A simple solution to the problem of quantifying individual contributions to the systemic risk fund would be to apply a common, i.e. homogeneous, rate $\bar{\tau}$ to all net liabilities such that the stability of the system is restored. For a given unstable financial system, the rate $\bar{\tau}$ that stabilizes the network is confined in the range $1 > \bar{\tau} > max\left[1 - \frac{\min_i(\rho_i)}{\lambda(\Theta)}, 0\right]$. However, the homogeneous rate applies the same % to all nodes, regardless of their actual contribution to λ. We generalize the above homogeneous scheme by taking into consideration the right eigenvector that is usually ignored in the field of network stability, and only employed in network analysis as a node centrality index.

Consider the following generalized immunization rate vector

$$\tau = k\bar{\tau}\mathbf{v}^{\alpha} \tag{5}$$

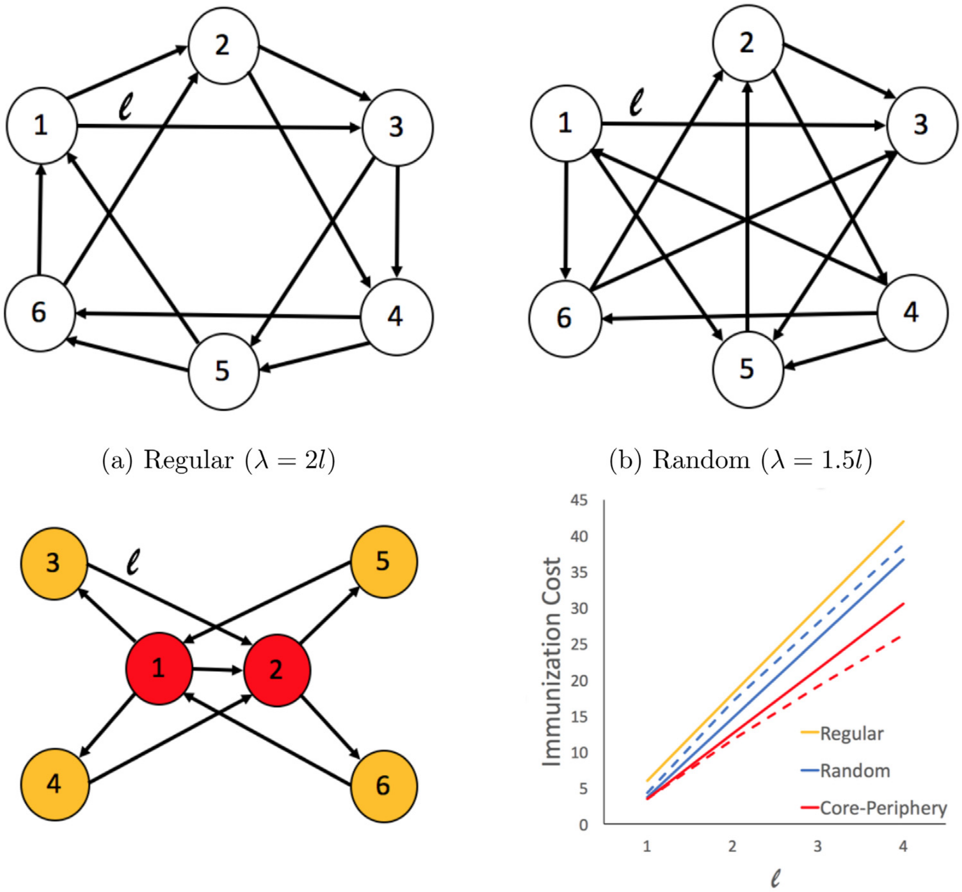
with $\mathbf{v}$ being the right eigenvector vector associated with λ. Note that α is a parameter used to tune τ towards the contribution $\mathbf{v}$ of each node to λ. The higher the α, the more heterogeneous the immunization will be towards a $\mathbf{v}$-oriented scheme. Once α is chosen, the scalar k is calibrated such that $\lambda = 1 - \epsilon$, with ϵ a very small positive number. With $\alpha = 0$, $\tau_i = \bar{\tau} \quad \forall i$, making the homogeneous immunization scheme a special case of the generalized regime vector τ. The other special case $\alpha = 1$ represents the scheme in which rates are proportional to $\mathbf{v}$, i.e. their contribution to the whole system instability. Note that elements of τ are bounded to 1, i.e. contributions to the systemic risk fund cannot exceed the total net liabilities of a node (see Supplementary Methods). We focus on these two special cases, the homogeneous with $\alpha = 0$ and the heterogeneous with $\alpha = 1$. Sensitivity analysis of immunization schemes under different levels of α is also reported.

Finally, we introduce performance measures to evaluate both the dollar-cost of immunization and its fairness, i.e. the ability to target nodes that contribute more to the instability of the system. The dollar-cost of immunization is simply the sum of all contributions, i.e. $\sum_i \sum_j \left[\left(x_{ij} - x_{ji}\right)^+ \tau_i\right]$, while the fairness of the scheme is evaluated by the level of inequality. The latter is defined as the deviation of rates imposed on individual nodes to their actual contribution to instability,

$$inequality = \frac{1}{2} \sum_i \left| \frac{\bar{m}_i^+ \tau_i}{\sum_i \bar{m}_i^+ \tau_i} - v_i^2 \right|. \tag{6}$$

where $\bar{m}_i^+ = \sum_j \left(x_{ij} - x_{ji}\right)^+$. Inequality takes min value of 0 in the case the immunization is exactly proportional to the right eigenvector and fairness is maximized. Conversely, the largest deviation from maximum fairness, meaning most unfair scheme based on each individual contribution to instability, is equal to 1. As $\sum_i \left| \frac{\bar{m}_i^+ \tau_i}{\sum_i \bar{m}_i^+ \tau_i} \right| = \sum_i v_i^2 = 1$, this implies that $\sum_i \left| \frac{\bar{m}_i^+ \tau_i}{\sum_i \bar{m}_i^+ \tau_i} - v_i^2 \right| \leq 2$. Note that the vector $\mathbf{v}$ is normalized using Euclidean norm, therefore the sum of the squares of v_i equals one by construction.

Ideally, we would opt for a cheap immunization scheme that maximizes fairness. However, there could be some trade-off between cost reduction and fairness depending on the topological properties of the financial network. Ultimately, the right balance between cost and fairness must be chosen by regulators on the basis of their policy priorities.



(a) Regular ($\lambda = 2l$)

(b) Random ($\lambda = 1.5l$)

(c) Core-Periphery ($\lambda = 1.6l$)

(d) Cost of Immunization

Fig. 1. Illustration of three network topologies with equal link weight l representing (a) regular, (b) random [43,44] and (c) core–periphery structures [45]. Panel (d) displays the cost of immunization for each of the three network topologies using both homogeneous (solid line) and heterogeneous (dashed line) immunization schemes. Note that both schemes are identical for the regular network case.

As discussed in [21], the presence of cycles in the network has a big, and positive, impact to the levels of instability λ. Building upon this findings, we can predict that schemes which indirectly penalize cycles can achieve faster (and cheaper) pathways towards stability.

3. Results

3.1. Numerical example

Consider as an illustration the example of Fig. 1. For simplicity, links are of equal weight l and there can be only one directional link between two nodes to be consistent with a net liability network. The regular layout (a) depicts a network where all nodes have the same in and out degrees, with $v_i = 0.41$ for all is. The second example (b) adds some randomness to the regular layout. It shows a more heterogeneous network with 1 and 4 being the most systemically important nodes with slightly larger eigenvector values and three outdegrees each, whereas 3 and 5 are the least systemically important with just one outdegree each. The last topology (c) is a core–periphery structure [45,46], very common in financial networks, with core nodes 1 and 2 being fully connected to each other and to the other peripheral nodes. The latter nodes only connect to the core and not to each other. Core nodes have by construction expected right eigenvector values way greater than the peripheral nodes. Fig. 1(d) compares the cost of the two main immunization schemes for each of the three configurations, such as the homogeneous (solid lines) and the heterogeneous case (dashed lines).

Due to the homogeneity of the regular layout, the eigenvector values of the nodes are about the same and therefore no discrimination can be achieved by heterogeneous schemes. Therefore, both homogeneous and heterogeneous schemes are identical as no one is posing more risk to the system than others. However, the regular case is more unstable than the

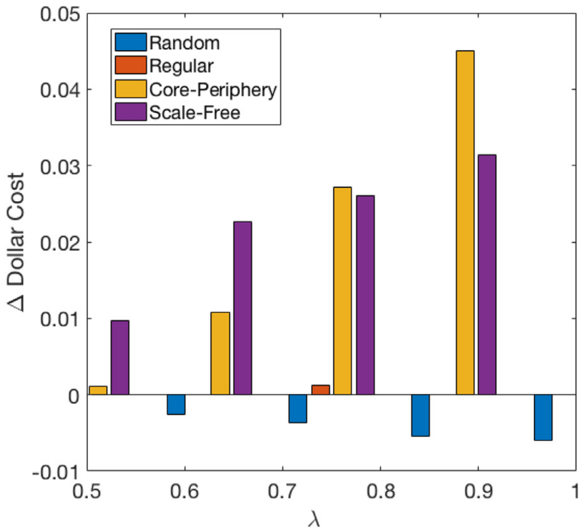


Fig. 2. The effect of heterogeneous schemes over the homogeneous case on dollar-cost. For each of the four topological network classes, i.e random (blue), regular (red), core–periphery (yellow) and scale-free (purple), we plot the average change of dollar-cost between the homogeneous ($\alpha = 0$) and the heterogeneous ($\alpha = 1$) regimes. See Supplementary Methods for the construction of the sample. Positive values, mainly for core–periphery and scale-free networks, shows the % cost saving of heterogeneous schemes compared to homogeneous ones. The opposite is true for negative values.

random case as a result of a larger number of cycles and therefore would require high immunization costs. For the random network, the homogeneous case provides a more cost-efficient immunization than the heterogeneous, although the latter minimizes inequality. This is due to the fact that the out-degrees of both nodes 3 and 5 directly contribute to closed cycles, although with low values of v_i, $i = 3, 5$, and therefore not well penalized by the heterogeneous immunization. The opposite case is observed for node 1, for which all its three out-degrees do not contribute to any closed cycle although v_1 is the highest value. The core–periphery network, on the contrary, is a clear example of central nodes in terms of v_i that are at the same time forming closed cycles (by construction all links are connected directly to the core, which in turn is itself fully connected). This well tiered network structure provides the optimal configuration for the heterogeneous scheme to outperform the homogeneous one in both cost-efficiency and inequality.

3.2. Network topologies

We investigate even further the relationship between network topology and the performance of the heterogeneous immunizations to provide evidence of a trade-off between dollar-cost and inequality. We numerically generate a large set of financial networks following four of the main topological classes, such as (i) *Erdös–Rényi random* [43], *regular*, *core–periphery* [45] and *scale-free* [47] (for a detailed literature survey, see [48]). Each network sampled from any of the four categories has $N = 100$ nodes, and both liabilities and external net assets are normalized to have same amount of total gross liabilities $L = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ij}$ and total capital $C = \sum_{i=1}^{N} c_i$. We also assume all failure thresholds $\rho_i = 0.25 \quad \forall i$ for simplicity. Therefore, the stability condition can be derived in terms of the matrix Θ, i.e. $\lambda(\Theta) < \rho = 0.25$ (see [32]). We sample 40,000 networks, 10,000 for each of the four network topologies whose properties are given in the Supplementary Methods.

Fig. 2 plots the change of the dollar-cost, defined as *Δ Dollar Cost* of immunization between homogeneous $\alpha = 0$ and heterogeneous regimes $\alpha = 1$, as average value for each network topology sample. Specifically,

$$\Delta\ Dollar\ Cost = Cost^{\alpha=0} - Cost^{\alpha=1} \tag{7}$$

Positive numbers of *Δ Dollar Cost* would indicate % dollar-cost saving of heterogeneous scheme over the homogeneous one. We notice a small increase in average dollar-cost ($<$1%) for the random network for highly unstable systems. Opposite evidence is depicted for core–periphery and scale-free samples where average dollar-cost reduction reaches values up to 4.5% and 3% respectively.

Table 1 confirms that the last two topologies are characterized by a specific network structure driven by higher level of correlations between **v** and cycle centrality. The cycle centrality of a node is the number of cycles each node is part of (weighted by the link weights), normalized using norm 1. A cycle is a path of edges such that the first and the last nodes are the same node [49]. We find that for network topologies characterized by a tiered structure, like scale-free and in particular core–periphery networks, the correlation between eigenvector centrality and cycle centrality is extremely high

Table 1
Descriptive statistic table of correlations between nodes right eigenvector and cycle centrality.

Topology	Mean	Median	std	min	max
Random	0.0063	0.0065	0.1028	−0.3875	0.3901
Regular	0.0081	0.0175	0.2196	−0.7405	0.6108
Core–periphery	0.8310	0.8337	0.0443	0.5781	0.9693
Scale-free	0.2737	0.2670	0.1176	−0.1138	0.7626

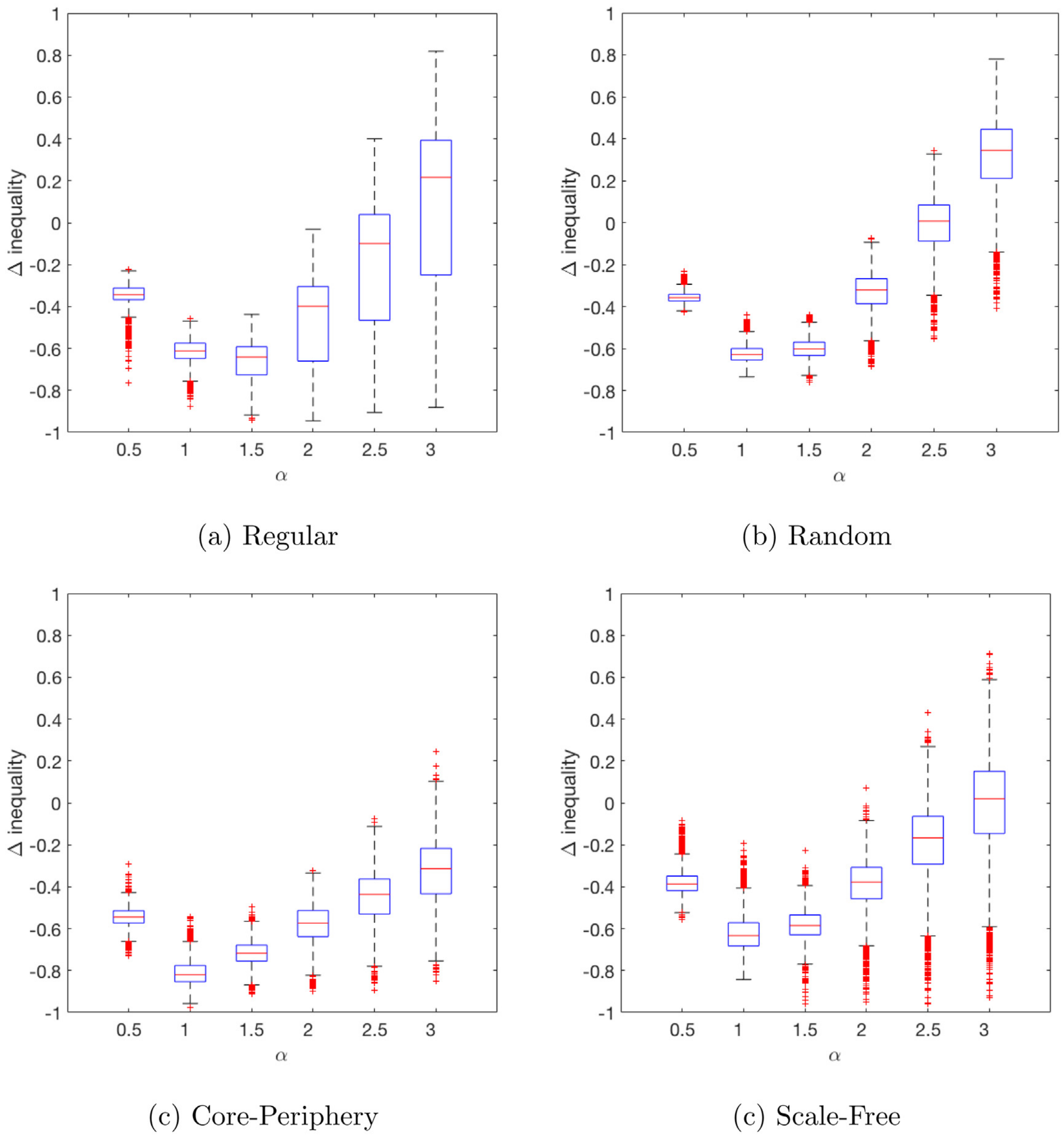


Fig. 3. Boxplot representing the effect of α to the reduction of inequality from the homogeneous for the four network topologies. We display the statistical properties of the difference in inequality values between the reference homogeneous regime ($\alpha = 0$) and heterogeneous ones varying the level of α as plotted in the horizontal axis.

compared with random and regular networks, supporting our claims. Recent extensive studies have assessed the relation between the topological structure of a network and the spectral properties of the associated graph (see e.g. [50,51] and references therein).

For completeness, Fig. 3 evaluates the sensitivity of the inequality with a wider range of α values for the four topologies considered. By construction, minimum inequality is achieved with $\alpha = 1$, representing exactly the magnitude of $\mathbf{v}$. For the core–periphery topology, inequality shows substantial reduction up to 80% on average, along with a 60%–65% reduction for the other topologies.

3.3. Application to global cross-border bank networks

This Section implements our immunization (taxation) model on real networks of countries banking systems that are connected via cross border claims as reported by the Bank of International Settlements (BIS) consolidated banking statistics. Since data on bilateral exposure of counties banking systems is publicly available, no estimation procedure is required. However, BIS data aggregation of the financial flows is done at the banking system level. Therefore, the application of our model will be generalized to country banking systems i rather than individual banks. This generalization might limit the accuracy of the assessment compared to one at the individual bank level. The Supplementary Methods document all our efforts in ensuring that the aggregation procedure to assess the global banking network is consistent across all variables.

Our financial network is characterized by a set of 18 Country Banking System (CBS) nodes connected via cross-border contractual flows of liquidity and/or obligations to make and receive payments in the period 2005 quarter 4 (2005Q4 for short) and 2014Q4 (see Supplementary Methods for details).

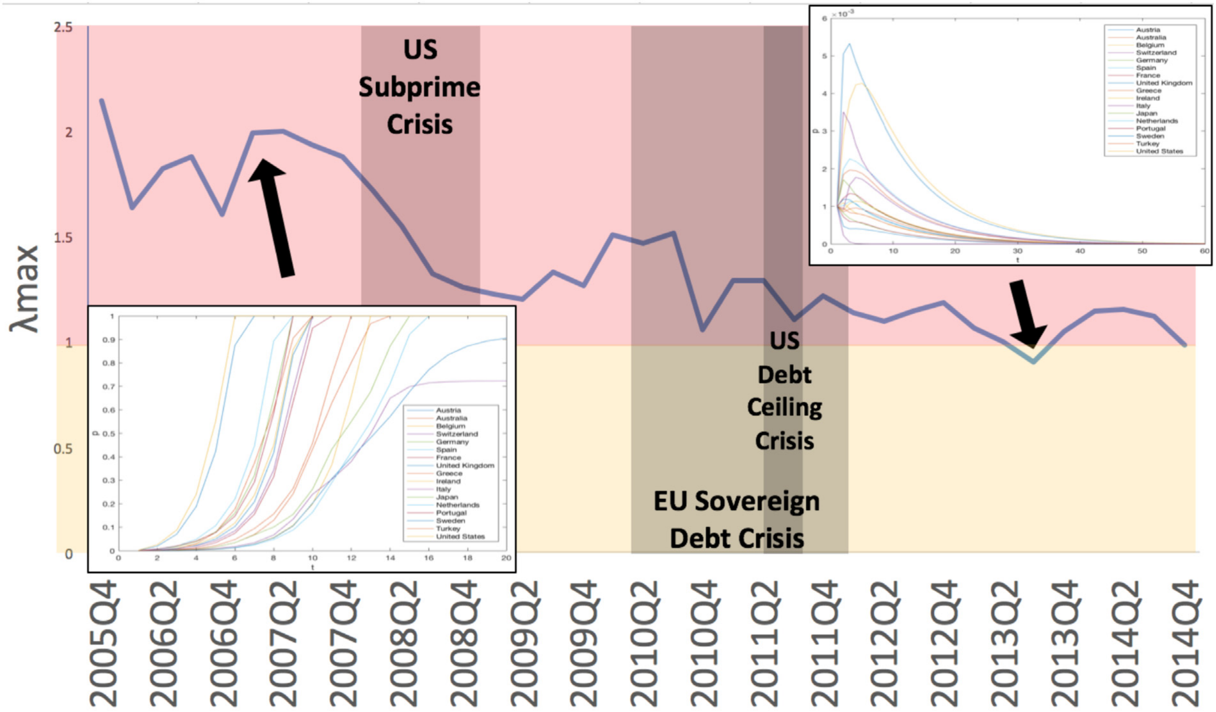
As discussed in [32], high instability prior to the crisis with many European banking systems at high risk of distress is signalled. Those propensities suggest high level of instability of the whole ecosystem, captured by λ and plotted in Fig. 4(a), since 2005Q4, with peaks up to 200% of expected capital loss, well in anticipation of the 2008 US subprime crisis. Due to the heavy recapitalization process started in early 2008, the level of instability went down in order to prevent the burst of the subprime bubble. Unfortunately, that capital injection came too little, too late. Instability peaked up again in 2009 in anticipation to the EU sovereign debt crisis. The LHS subplot within Fig. 4(a) simulates the asymptotic convergence of the propensities of defaults of the system in 2007Q1 towards a quick and complete systemic financial meltdown. The global cross-border banking network appears to be stable, $\lambda < 1$, only in 2013 Q3 and 2014 Q4, showing a slow asymptotic convergence of all p_i towards zero (Fig. 4(a) RHS subplot). For a comparison of this approach with other market-based systemic risk metrics, see [32].

We finally employ the above taxation on the cross-border networks. Fig. 5(a) plots the overall cost of heterogeneous taxation against our systemic risk index λ for all the 37 quarter networks from 2005Q4 to 2014Q4. As discussed above, we observe monotonic decline of instability with increased taxation, although the relation might not be linear. This is particularly true for very unstable ecosystems where every extra dollar-tax collected above $1tn produces a larger marginal reduction of λ than the homogeneous case (2007Q1 case in Fig. 5(a)). The trade-off between immunization cost and inequality is captured in Fig. 5(b). Throughout the 9 years of investigation, we observe cases where narrow trends in equality between the two immunizations schemes resulted in massive gaps of costs in favour of the homogeneous rates. These are the areas depicted in blue, mainly representing periods of high instability in which the upper bound limit on individual tax rates of 1 limits the effectiveness of a pure eigenvector-based tax scheme (see Supplementary Methods). There are however many other time periods highlighted in yellow where the large gain in fairness by the heterogeneous scheme is just marginally more expensive than the homogeneous case. As mentioned above, we also find two cases highlighted in green that show how a fair immunization scheme can also be cheaper that traditional homogeneous ones. Note that this visual comparison cannot directly support policy recommendations as lacking on a clear criterion to discriminate between immunization schemes. Regulators need to properly quantify the cost–benefit level of each scheme based on their policy targets and choose the scheme accordingly.

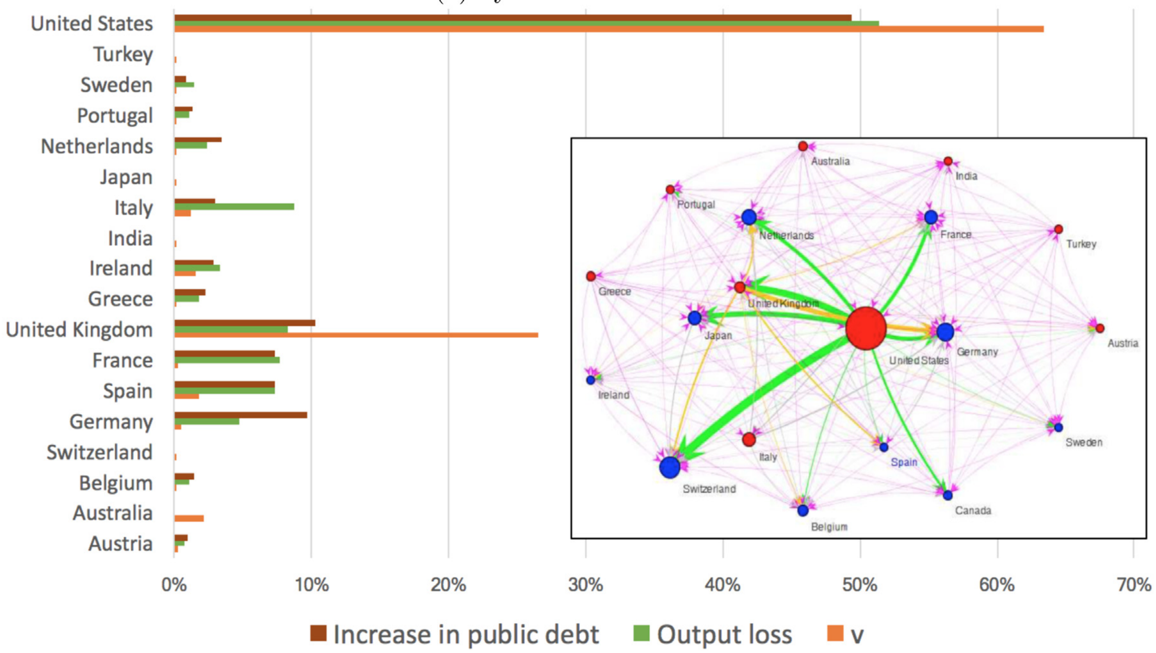
4. Discussion

We provide an elegant analytical framework to evaluate the level of instability of financial ecosystems and design immunization schemes tailored to the contribution of institutions to the instability of the system. Our numerical examples provide evidence of how the topology of the ecosystem network impacts the efficiency of the immunization scheme. Heterogeneous schemes are fairer than homogeneous ones by construction. They can also be more cost-effective depending on the topology of the network. Regulators can easily opt for a fairer scheme when dealing with an highly tiered structure with well defined core and peripheral nodes. In this scenario, tougher penalties on central nodes can also mitigate closed cycles in the network and therefore bringing the system towards stability more efficiently. However, on less tiered structures with higher degree of randomness in the allocation of links, heterogeneous schemes tend to be more expensive although fairer than homogeneous one, raising the issue of dealing with the trade-off between cost-efficiency and fairness of immunization. The latter is also discussed on real financial networks derived by cross-border financial exposures of country banking systems provided by BIS consolidated banking statistics. Our findings contribute to the predictive power of the systemic risk approach proposed by [6,32] and enhance its application beyond the initial assessment of systemic risk studied in this literature.

Our immunization function paired with the analytical framework of stability provides regulators with a parsimonious model to monitor complexity and select the right immunization scheme for the ecosystem. The framework can be easily extended to alternative risk assessments, such as liquidity shortfalls [39] and liquidity hoarding [53]. The recent Basel III regulatory framework provides individual liquidity thresholds that can be used to calibrate the dynamical system and assess liquidity contagion. As reported in [54], liquidity shocks can lead to solvency problems and eventually exacerbate defaults, at the expense of higher costs of immunization.

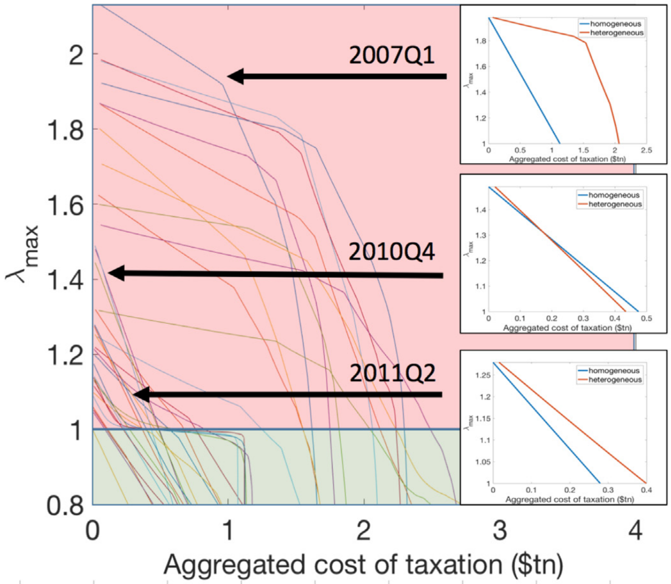


(a) Systemic Risk Index

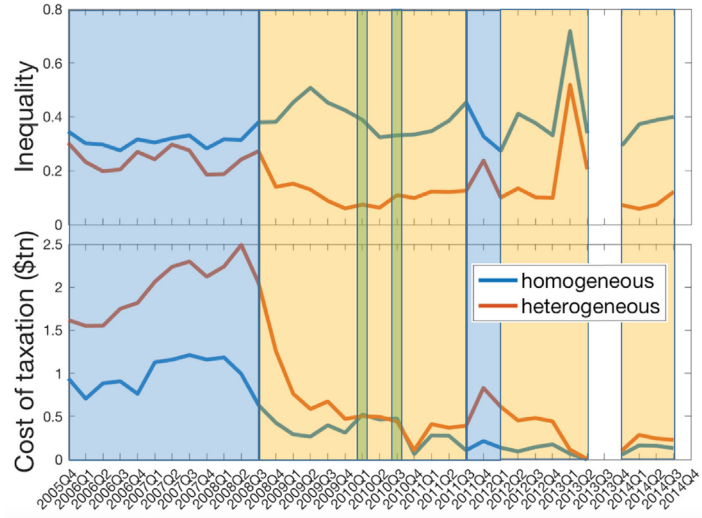


(b) Systemically Important Countries

Fig. 4. (a) Quarterly level of instability of the BIS cross-border banking system captured by λ over the period 2005–2014, along with snapshots of convergence of individual propensities of defaults for both unstable (2007Q1) and stable (2013Q4) quarters. (b) Contribution of countries banking systems to systemic risk captured by average right eigenvector values of the pre crisis period 2005–2007, and compared with actual output losses and increased public debt [52] as percentage of the overall capital of the system, along with a snapshot of the network plot. The latter depicts size of net liabilities of nodes (total liabilities greater than total assets) and net assets (total assets greater than total liabilities), colour coded in red and blue respectively. Links are also weighted and colour coded in green (links from core nodes), yellow (mid nodes) and purple (periphery nodes). See [5] for details of the network layout.



(a) Systemic Risk Index



(b) Systemically Important Countries

Fig. 5. (a) The impact of heterogeneous taxation $\alpha = 1$ in terms of aggregated dollar-cost compared to the level of instability λ max for all the unstable quarters in 2005–2014. Those are complemented by three snapshots of comparison between homogeneous and heterogeneous tax schemes. (b) Comparison between homogeneous and heterogeneous tax schemes in terms of both inequality (top) and dollar-cost (bottom) when systems are fully stabilized ($\lambda = 1 - \epsilon$). The blue areas depicts scenarios with the homogeneous scheme being way cheaper than heterogeneous although slightly more unequal, whereas yellow areas show the heterogeneous scheme being way more equal than homogeneous although slightly more expensive. The green areas captures scenarios in which both minimization of cost and inequality are achieved by the heterogeneous taxation.

Supplementary methods

Dynamical system of financial instability

Tipping point. The stability of the system is governed by the characterization of the instability matrix Θ and the cure rate vector ρ. The dynamical system describing the propensities of default can be expressed using unconditional propensities $\mathbf{P}_t = \mathbf{Q}\mathbf{P}_{t-1} = \mathbf{Q}^t\mathbf{P}_0$, with $\mathbf{Q} = \left[(\mathbf{I} - \mathbf{R}) + \Theta'\right]$. Using spectral decomposition as in [6], we can express $\mathbf{Q}$ in terms of its eigenvalues diagonal matrix $\mathbf{D}$ and relative right eigenvectors matrix $\mathbf{V}$ (spanning the N dimensional vector space of real numbers R^N) and a $N \times 1$ vector of weights $\mathbf{g}$, $\mathbf{Q}^t\mathbf{P}_0 = \mathbf{Q}^t\mathbf{V}\mathbf{g} = \mathbf{D}^t\mathbf{V}\mathbf{g} = \sum_i \left(\lambda_i^t g_i v_i\right)$. The stability of the system is achieved when the propensity of default for large t dies away. By arranging all eigenvalues in descending order $\lambda_{max} = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and factoring out λ_{max} we have $\mathbf{P}_t = \lambda_{max}^t \sum_i \left(\left(\frac{\lambda_i}{\lambda_{max}}\right)^t v_i g_i\right)$. For $i > 1$, $\frac{\lambda_i}{\lambda_{max}} < 1$ so that $\mathbf{P}_t \to \lambda_{max}^t v_1 g_1$ as $t \to 0$, $g_1 \neq 0$. The stability is therefore governed by the largest eigenvalue and its corresponding eigenvector, implying that for the 'infection' from other FIs to die off a necessary condition is $\lambda_{max} < 1$. This also leads to the importance of the magnitude of λ_{max} as it directly affects positive or negative growth of losses (as percentage of capital) in the system. If $\lambda_{max} - 1 > 0$, the system will lose capital at this rate at each t. If $\lambda_{max} - 1 < 0$, then as buffers exceed losses, the latter will be reduced at that rate, yielding negative growth of losses as percentage of capital.

Individual contribution to instability. As shown in [55,56], there is a close relationship between the stability condition of the dynamical network system and power iteration fixed point algorithm that yields the eigen-value equation with the maximum eigenvalue λ_{max} defined as the systemic risk index giving the near side (%) loss of capital for the system as a whole. The right eigenvector centralities (v_i) are a measure of systemic importance and given the contribution of each banking system to systemic risk. The left eigenvector centralities ($\tilde{v}_i$) are a measure of vulnerability and give the fixed point solution for each banking system's propensities to fail. The product of $\lambda_{max}\tilde{v}_i$ gives the expected loss of capital for i. Only under conditions of system instability when $\lambda_{max} > 1$, will all the banking systems' asymptotic (as t tends to infinity) propensities to fail converges to 1, some faster than others.

Banks capital requirements and cure rate. In financial contagion, the cure rate ρ_i represents the proportion of capital institution i has at its disposal to absorb external shocks. This capital threshold is calibrated to satisfy specific capital requirements put in place by banking regulation. Basel III criteria for capital adequacy, for example, states that the minimum ratio of Common Equity Tier 1 (CET1) capital c_i over risk weighted assets rwa_i of bank i is 4.5%, $\frac{c_i}{rwa_i} > 4.5\%$. This means that the capital surplus on top on the 4.5% ratio is a buffer that institution i has at its disposal to absorb losses, i.e. $\frac{c_i - \rho_i c_i}{rwa_i} = 4.5\%$. Therefore, the ρ_i consistent with Basel II capital requirement will be $\rho_i = \max\left(1 - 4.5\%\frac{rwa_i}{c_i}, 0\right)$.

Immunization of systemic risk by regulation. The calibration of capital requirements and cure rates can also be designed to internalize systemic risk. This would be consistent with the idea of immunizing an unstable financial system by regulation [19]. This contrasts with the taxation approach adopted in this study. There are two ways to internalize systemic risk by regulation. The first approach can be achieved by either a more lenient capital constraint below the current tier1 ratio defined by Basel III. In our case we can set the minimum value of $\frac{c_i}{rwa_i}$ to be below 4.5%. This approach would lower banks capital buffers needed by regulation and of course contradicting with the current trend of higher capital requirements. The second way to adjust ρ_i is by increasing the available capital c_i in Eq. (2). The increase of capital will also reduce vulnerability of lenders j's by lowering θ_{ij}. This approach has been preferred by regulators due to the problem of reaching a consensus on evaluating systemic risk. However, it penalizes vulnerable banks more that systemically important ones since it enforces higher capital requirements on the lenders rather than the borrower who actually contributes to systemic risk.

Derivation of homogeneous rate. Given a non-symmetric matrix $\mathbf{Q} = \left[(\mathbf{I} - \mathbf{R}) + \Theta'\right]$, $\mathbf{R} = diag[\rho_1, \ldots, \rho_n]$ is a diagonal matrix of individual ρ_i and $\mathbf{I}$ is the identity matrix, $\lambda_{max}(\mathbf{Q}) \leq (1 - \min_i \rho_i) + \lambda_{max}(\Theta)$. This decomposition allows us to derive a sufficient but not necessary condition of stability of the ecosystem in terms of Θ, i.e. $\lambda_{max}(\Theta) < \min_i \rho_i$. Consider a immunized system $\Phi \equiv (1 - \bar{\tau})\,\Theta$. We can express the maximum eigenvalue of Φ in terms of Θ, $\lambda_{max}(\Phi) = (1 - \bar{\tau})\,\lambda_{max}(\Theta)$. The stability condition $\lambda_{max}(\Phi) < 1$ holds if and only if $(1 - \bar{\tau}) < \frac{\min_i \rho_i}{\lambda_{max}(\Theta)}$. Note that immunization rates are bound in the range [0, 1] as the most severe immunization would be equal to the total net liability of an institution, therefore the final condition for the homogeneous rate is $1 > \bar{\tau} > max(1 - \frac{\min_i \rho_i}{\lambda_{max}(\Theta)}, 0)$. This restriction on the upper bound of rates can affect the performance of heterogeneous schemes in highly unstable systems.

Network sampling

We sample 40,000 networks, 10,000 for each of the four network topologies whose properties are given below.

- *Random* - $p \sim$ U in the range $\left[\frac{1}{N}, 1\right]$ is the probability of each potential link to be activated, which for the random network also corresponds to its theoretical connectivity. The minimum connectivity $\frac{1}{N}$ guarantees the irreducibility of the network in expectation (network is strongly connected) [43].
- *Regular* - $k \sim$ U in the range $[1, N - 1]$ is the number of links for each node that are activated in a regular fashion (node i is connected with node $i + 1, i + 2, \ldots, i + k$).
- *Core–Periphery* - $k \sim$ U in the range $[2, 0.3N]$ is the size of the core (from a minimum of 2 to a maximum of 30% of the total nodes). The core is fully connected, whereas $p \sim$ U in the range [0, 1] is the probability of each node in the core (periphery) to be connected to a node in the periphery (core). We guarantee that each peripheral node has at least one active link with the core [45,46].
- *Scale-free* - $k \sim$ U in the range $[1, 0.1N]$ is the number of links per node that are selected using a preferential attachment procedure (the higher the connectivity that a node already has, the higher the probability of receiving a link from other nodes) [47].

Cross-border BIS data

The BIS *Consolidated Banking Statistics* foreign claims quarterly dataset is used to build the cross global banking system network. The information of tier1 Capital of the cross-border banks in each of the 18 reporting countries is retrieved from Bankscope. The sample selected covers the period of 2005 Q4–2014 Q4 and represents the major 18 BIS reporting countries banking system, such as Australia, Austria, Belgium, France, Germany, Greece, India, Ireland, Italy, Japan, Netherlands, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States. Of these the so called GIIPS are Greece, Italy, Ireland, Portugal and Spain. The analysis is limited to foreign claims on ultimate base risk [57] among banking sectors only, which refers to the so-called sectoral breakdown data. The availability of the latter started in 2005 Q1 for 9 major BIS reporting banking systems, i.e. Australia, Belgium, France, Ireland, Japan, Portugal, Sweden, UK, US. The remaining countries started reporting a bit later [58]. In the absence of the sectoral breakdown, the aggregated values, i.e. the foreign claims of each reporting country to all the sectors of the foreign country, are rescaled by the proportion of banking sector liability of the foreign country over the total liabilities of all sectors vis-á-vis all the 25 BIS reporting countries, information still provided by BIS *Consolidated Banking Statistics*. Details on the data manipulations are given in [32].

CRediT authorship contribution statement

Simone Giansante: Conceptualization, Methodology, Study design, Software, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Project administration, Funding acquisition. **Sabato Manfredi:** Conceptualization, Methodology, Study design, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Funding acquisition. **Sheri Markose:** Conceptualization, Methodology, Study design, Validation, Formal analysis, Investigation, Resources, Data curation, Writing – original draft, Writing – review & editing, Visualization, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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