Designing Public Support for SMEs during Crises: The Role of Private Bank Lending*

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November 26, 2024

Abstract

When designing crisis support schemes for SMEs, governments grapple with adverse selection problems. We show how governments can improve policy targeting by partnering with private banks that have no superior information on SME types. The mechanism utilises SMEs' loan demand information and external financing being more costly than internal financing. If policy aims at employment protection, optimal partnership fully subsidises funding costs of *only* those SMEs with sufficiently low loan per worker. If policy aims at efficiency, the partnership targets SMEs with sufficiently high loan demand. We estimate our design could save at least 16% more jobs than the UK Bounce-Back Loan scheme while significantly reducing costs to taxpayers.

Keywords: public crisis intervention, adverse selection, small and medium-sized enterprises, employment protection, public-private partnership

JEL Codes: D82, G21, G38, H81

*We thank Cláudia Custódio, Albert Menkveld, Samuel Rosen, Oren Sussman, Alex Vardoulakis, Remco Zwinkels, and participants at the seminar of CPB Netherlands Bureau for Economic Policy Analysis, Ministry of Economic Affairs and Climate Policy, 2024 EEA Summer Meeting, 2023 Asia Meeting of the Econometric Society in NTU Singapore, 2023 Econometric Society Meeting in China, 2023 FMA European Meeting, 2023 Royal Economic Society and Scottish Economic Society Annual Conference, 2023 IFABS Meeting, and VU Amsterdam finance seminar series for helpful comments and discussions. We thank Sijmen Rijks for excellent research assistance.

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1 Introduction

The Small and Medium-sized Enterprises (SME) sector is a powerhouse in many economies, accounting for 60-70% jobs and representing over 99% of all businesses in most OECD countries.[1] In response to crises such as the Global Financial Crisis and the COVID-19 pandemic, governments have unleashed substantial public support for SMEs (see, e.g., Gourinchas, Kalemli-Ozcan, Penciakova and Sander, 2020 and Bonfim, Custódio and Raposo, 2023). However, a significant challenge arises in designing these schemes: governments often lack the necessary information to effectively target the most deserving firms.

For an SME to be a *right* firm that qualifies for public funding, two basic requirements must be met. First, the cost of survival should not surpass the firm's present value conditional on survival. That is, a right SME should have a positive net present value (NPV) of survival. Second, SMEs should exhaust their own funding before seeking public assistance. That is, a right SME should be *in deficit*: its own funding is inadequate to cover the cost of survival. Moreover, among those SMEs that meet the two basic requirements, some deserve public funding more than others, depending on the government's objective. In practice, governments are unlikely to observe the NPVs and the self-funding of SMEs. As a result, lack of targeting is a serious issue that hampers the effectiveness of governments' support schemes. For instance, Gourinchas et al. (2020) simulate that without targeting SMEs that meet the two basic requirements, the untargeted government COVID-19 support scheme was over three times as costly as a targeted approach to achieve the same outcomes. Similarly, Autor et al. (2022) argue that the *Paycheck Protection Program* in the US was "essentially untargeted," which led to inefficient outcomes, with only a quarter of the funds protecting jobs that were truly at risk. This raises a critical question: How can governments best target SMEs in crisis support schemes, especially in the face of adverse selection problems?

Our idea is that the government can improve targeting by forming a public-private part-

[1] According to OECD report on "SME and entrepreneurship Outlook".

nership with banks. The mechanism we propose does not rely on banks having superior information about SME types than the government. Instead, it operates by utilising the information embedded in the SME loan demand and the fact that bank loans are more expensive than SMEs' self-funding. Our analysis highlights how various government support schemes implemented during the pandemic crisis could be modified to significantly improve their efficacy. Using the UK Bounce-Back Loan scheme as an estimation example, we find that, under our proposed design, the scheme could have protected at least 16% more jobs and cost substantially less of taxpayer money.

We model an economy in a crisis. A continuum of SMEs decide whether to pay a cost to survive the crisis. Each SME has a certain quantity of own funds. An SME's type is characterised by the surviving cost and the quantity of its own funds and is its private information, not observed by either the government or banks. If the surviving cost exceeds the present value conditional on survival, the SME's NPV of survival is negative, and the SME finds it not worthwhile to survive by using its own funding. If the surviving cost is larger than the SME's own funding, it can make up the deficit by borrowing from banks. The government has a limited budget to help SMEs survive. An effective deployment of public funds should at least meet the two basic requirements, as mentioned above: the recipient should have a positive NPV and be in deficit. Even meeting the two basic requirements is a challenge, because the government does not observe SME types.

One way to overcome this challenge is for the government to form the following Simple Partnership with private banks. In this partnership, the government subsidises banks instead of directly funding SMEs: for each dollar a bank lends to an SME, the government subsidises the bank with certain cents; the scale of the subsidy depends on the budget of the scheme. Essentially, banks pool the public funds with their own funds to lend to SMEs. Remarkably, this simple partnership can already meet the two basic requirements previously outlined. In equilibrium, bank funding is more expensive than SMEs' self-funding (as is known in the corporate finance literature). Hence, SMEs exhaust their own funding before borrowing bank loans, meaning only those in deficit will demand loans and thus use public funding. Moreover, if an SME finds it not worthwhile to survive by using its own funding, it will not find it worthwhile to survive by using bank funding. Hence, only those with a positive NPV will demand loans and use public funding. Note that the mechanism does not require banks to have more information than the government about SMEs' types but rather utilises only the information in loan demand, as with all the public-private partnerships analysed below.

This Simple Partnership meets the two basic requirements but is not the best way to deploy the public funds. The issue is that *all* types of SMEs with access to bank funding benefit from the public scheme under the Simple Partnership; this feature is shared by many COVID-19 support schemes in the developed economies.[2] The government can significantly enhance the effects of public funding by targeting *specific* types of SMEs among those that have access to bank funding. Which types to target depends on the government's objective. Government support schemes typically aim to protect employment, as illustrated in the COVID-19 pandemic crisis.[3] Thus, we primarily consider the case where the government aims to protect employment. Additionally, we also consider the case where the government aims to maximise economic efficiency.

We regard the public funding support as an investment. To achieve the maximum effect, the government should target SMEs where the investment delivers the highest return rates. The return rate of public funding for employment protection should be calculated as follows. Suppose the government funds the deficit for an SME to survive the crisis; the return is that the jobs created by the SME are saved. Hence, the return rate is the number

[2] See details in Section 6.3.

[3] This can be evidenced by the US Department of the Treasury's introductory article on the US Paycheck Protection Programme (https://home.treasury.gov/policy-issues/coronavirus/assistance-for-small-businesses/paycheck-protection-program), and the UK HM Treasury's article on government-backed financial support schemes (https://www.gov.uk/government/news/government-backed-loans-help-thousands-of-businesses-to-protect-jobs-during-pandemic). As the UK then Chancellor of the Exchequer, Rishi Sunak, put it: 'I said we would do whatever it takes to protect jobs and livelihoods and that (government-backed loan support) is exactly what we have done.'

of these jobs per unit of deficit. To allocate funds efficiently, the government should only support SMEs where this return rate exceeds a set threshold. However, neither the government nor banks observe SMEs' deficits and, hence, the return rates. The remedy lies in the information contained in the SME's loan demand. Since bank funding is more costly than self-funding, SMEs will only demand bank funding that exactly bridges their deficits. Their loan demands, therefore, mirror their deficits. Hence, the return rate of public funding for employment protection is equal to the job-to-loan rate – the number of jobs per unit of loan demand. Thus, the Employment Protection partnership takes the following form: the government subsidises an SME proportionately to its loan demand if and only if its job-to-loan rate is higher than a qualification threshold.

Given the public budget, a trade-off emerges between the qualification threshold and the subsidy size. If the subsidy per unit of loan demand increases, each qualified SME receives more subsidy, and therefore, fewer SMEs will be qualified, leading to a higher threshold. The subsidy size should not surpass SMEs' net funding cost – the spread between the cost of bank funding and self-funding – because exceeding this would encourage overborrowing by offering a net gain to SMEs. We show that the optimal partnership should *fully* subsidise the net funding cost of the qualified SMEs. Suppose the partnership only partially subsidises the net funding cost of qualified SMEs. For intuition, consider an increase in the subsidy size, which results in a higher qualification threshold. Those SMEs with job-to-loan rates lower than the new qualification threshold are disqualified and are thus selected out. Given the public budget, some SMEs are selected in due to the increase in the subsidy size. These SMEs were previously excluded because their NPVs of survival are too low to afford bank funding without the increased subsidy. Because these newly selected SMEs meet the new qualification threshold, their job-to-loan rates must be higher than the new threshold. Taken together, the job-to-loan rates of the newly selected SMEs are higher than the new threshold, whereas the rates of those selected out are lower. As the job-to-loan rate equals the return rate of using public funding for employment protection, the net effect of increasing the subsidy size is that more jobs are saved. Hence, the optimal employment-protection partnership *fully* subsidises the net funding cost of only those SMEs whose number of jobs per unit of loan demand is higher than a threshold.

We compare our optimal Employment Protection partnership with support schemes implemented during COVID-19 through a simple empirical estimation. In practice, these schemes typically include two features: 1) they provide guarantees on loan principals, and 2) SME borrowers are permitted to access additional loans not covered by the schemes. The first feature can lead some recipients to treat the schemes' funding as free cash, resulting in two potential forms of waste of taxpayer money. The first is a moral hazard problem, where scheme users may use the funding for unproductive expenditures with no intentions to repay (see UK National Audit Office, 2020; Beggs and Harvison, 2023 for evidence of fraud and abuse and US Department of Justice, 2022 for court cases); the second is that SMEs obtain public funding before exhausting their own funds. Our model addresses the moral hazard problem by leveraging private banks' expertise in arranging and monitoring loan contracts. Additionally, our design ensures that only SMEs in deficit access public funding. Consequently, our design avoids both forms of waste, potentially saving taxpayer money substantially. E.g., as estimated by Autor et al. (2022), 66-77% of the Paycheck Protection Program loans issued in 2020 "did *not* go to paychecks, however, but instead accrued to business owners and shareholders." For our estimation, we assume the existing schemes in practice are free from these two forms of waste and focus solely on the second feature. That is, we focus on the difference that in our design only SMEs whose job-to-loan rates are higher than a specified threshold receive public funding, whereas the existing schemes have no such qualification criteria. Using the UK Bounce Back Loan (BBL) Scheme as an example, we estimate that, due to this difference, our optimal Employment Protection partnership could rescue at least 16% more jobs than the BBL. Considering that our design avoids the two forms of waste associated with the first feature, our optimal Employment Protection partnership not only saves more jobs but also uses less taxpayer money.

Thus far, we have analysed the design of the public-private partnership if the government's objective is employment protection. We also consider the case where the government's objective is to maximise efficiency. Because our optimal Employment-Protection partnership targets SMEs whose job-to-loan ratios are above a threshold, it tends to prioritise the SMEs whose loan demand is low. In contrast, if the government's objective is to maximise efficiency, the partnership prioritises the SMEs whose loan demand is sufficiently large. The intuition is as follows. Since the government aims to maximise efficiency, it should target SMEs with sufficiently high NPVs. An SME's loan demand is positively correlated with its NPV: because the bank funding is costly, only SMEs with sufficiently high NPVs can afford large bank loans. Hence, the partnership should prioritise the SMEs with sufficiently large loan demand.

2 Related Literature

The existing body of literature on public support for non-financial firms in crises has expanded considerably, particularly since the Global Financial Crisis and the recent pandemic crisis. The effective deployment of public funding support depends on it targeting the "right" firms. Our study contributes to this literature by introducing a novel channel for targeting firms based on their loan demand.

Our paper aligns closely with the studies focusing on crisis intervention with information asymmetry (see, e.g., Bebchuk and Goldstein, 2011; Goodhart, Tsomocos and Wang, 2023; Lee and Neuhann, 2023; Li and Li, 2021; Kahn and Wagner, 2021; Vardoulakis, 2020); many of them address the role of banks in alleviating information asymmetry. Bebchuk and Goldstein (2011) study how the coordination failure between lenders leads to credit market freezes during crises. To mitigate the impact of credit market freezes, the government could directly lend to real sectors but it faces adverse selection problems. The authors show that this issue could be addressed by utilising the screening expertise of financial firms. Goodhart et al. (2023) focus on the distorting effect of loan guarantees on lenders' monitoring incentives, which we do not consider. The authors propose a screening device to deter certain unprofitable businesses from borrowing loans. The critical policy trade-off is between short-term employment stabilisation and long-run allocative efficiency. Lee and Neuhann (2023) study a dynamic model with collateralised lending whereby lenders do not observe the collateral quality. The authors show that a government without commitment power can fall into intervention traps in which ex-post efficient lending subsidies reduce long-run growth and welfare. Li and Li (2021) underscore a critical trade-off of public liquidity support between the preservation of production capacity and the cleansing effects of crises on firm quality. Kahn and Wagner (2021) consider both direct liquidity provision to businesses by the central bank and traditional liquidity provision by banks. Notably, the central bank has superior knowledge of the externalities during a pandemic, whereas the private banks have informational advantages over the businesses. Vardoulakis (2020) proposes that the central bank can design a multi-tier loan pricing facility to reduce its adverse selection problem, and that the welfare gain of the multi-tier scheme can be measured based on three sufficient statistics. In these studies, typically banks help with alleviating adverse selection problems because they hold information advantage over borrower types. In comparison, our framework does not rely on banks having superior information than the government on firm types. Banks are needed in our framework for two reasons: 1) bank funding is more costly than borrowers' self-funding, and this is used in our framework as a screening mechanism, and 2) banks have information on SMEs' loan demand, on which targeting in our proposed scheme is based.[4]

Our model uncovers a novel mechanism to improve policy targeting in providing financial support for SMEs to survive crises. The importance of policy targeting has been

[4]Other theory papers on government loan support are based on different types of frictions than adverse selection. Segura and Villacorta (2020) analyse different government interventions to support firms and establish a pecking order between direct transfers and indirect support. The critical friction in their paper is the moral hazard due to the borrower's unobserved effort cost. Philippon (2020) focuses on the wage rigidity friction and delves into how government interventions can improve efficiency when the decentralised economy amid COVID-19 is distorted by wage rigidity.

underscored in many empirical and quantitative studies. Autor et al. (2022) document that because the US *Paycheck Protection Program* lacks targeting, only around one-quarter of the programme funds protected jobs that would otherwise have been lost. Cororaton and Rosen (2021) also study the US Paycheck Protection Program and document that 13.5% of funding recipients, in particular those facing more public scrutiny, returned their loans after public backlash. Using a structural model, Elenev, Landvoigt and Van Nieuwerburgh (2020) suggest that the US Paycheck Protection Program "wastes resources on firms that do not need the aid", and that if firms that are truly in need can be targeted, "a much smaller-sized program is needed to prevent a lot more bankruptcies." Gourinchas, Kalemli-Ozcan, Penciakova and Sander (2020) estimate the impact of the COVID-19 crisis on SME failures in seventeen countries. They find that untargeted public funding support is more than three times as costly as targeted policy and may provide support to firms that would have failed regardless of COVID-19 or that do not need it. However, the authors emphasise that such targeting requires much more granular information than what governments typically possess. Our paper provides one way out of such a conundrum. On the other hand, a public support scheme can be fairly effective if it targets the right firms reasonably well. Bonfim, Custódio and Raposo (2023) exploit the *SME-Leader Program* adopted in Portugal after the Global Financial Crisis, which uses firm-level financial variables to target eligible firms. They find that during the crisis eligible firms borrow more, invest more in total assets, and increase their employment by more than non-eligible firms. The authors emphasise that "its targeted nature also significantly reduced the (absolute and relative) costs."

Since our message is that public policy should utilise private sectors' information and expertise, our paper therefore connects with the literature on public-private partnerships (PPP) (see, for example, Laffont and Tirole, 1991; Hart, 2003; Bennett and Iossa, 2006; Hoppe and Schmitz, 2010; Iossa and Martimort, 2012; Hoppe and Schmitz, 2013). In particular, Hart (2003) develops a model of public-private partnerships and provides the foundation for the determinants of the boundaries between public and private firms in an advanced capitalist economy. Cui, Liu, Hope and Wang (2018) review the existing public-private partnership research to explore the status quo, trends, and gaps in infrastructure projects. This group of literature typically focuses on the costs and benefits of PPP in terms of bundling tasks or ownership in contexts plagued with agency costs, transaction costs and contract incompleteness. We contribute to this literature by considering a different form of PPP whereby government subsidies join forces with bank lending operation, and we underscore utilising private sector's expertise and information. Furthermore, whereas this group of literature focuses on the agency issues between the government and the private partners, in our paper, the agency issue stems from a third party (i.e., the SMEs) which both the government and the private partners are subject to. In this sense, we see our work as complementary.

3 The Model

The economy is in a crisis, in which a continuum of SMEs are struggling to survive. Survival is costly, and SMEs have some funds of their own. In addition, SMEs may be able to obtain external financing from banks, and the government also puts aside a budget to help SMEs survive. The type of an SME is characterised by two variables, (a,x), where a is the amount of the SME's own funds and x is the cost of surviving the crisis. We assume (a,x) distributes on $[0,\overline{a}] \times [\underline{x},\overline{x}]$ with p.d.f. h. An SME's type is its private information, which represents the adverse selection issue discussed in the introduction.

Besides adverse selection, bank lending and public schemes in reality are often shrouded in moral hazard issues on the part of borrowers or scheme users. After an SME's owner obtains funds from a bank or the government, the question is whether she will use the funds for the enterprise or for her private benefit. Our model accommodates this type of issue in a canonical manner. We assume that funding the surviving cost is only a necessary condition for survival; if this condition is satisfied, the chance of survival depends on the SME owner's effort choice. If she works hard, which incurs a private cost of c, it survives with probability

p. If she shirks, the surviving probability is reduced to $p - \Delta > 0$. The effort choice is the owner's private information. All the surviving enterprises generate profits Π. Thus, the quality of the business projects is represented by the cost x. Making the profit variable will not qualitatively change the results. We assume that

$$p\Pi - c > (p - \Delta)\Pi. \tag{1}$$

That is, working hard is socially efficient. Let $V \equiv p\Pi$ be the present value of a surviving SME conditional on working hard. An SME is worth saving if and only if

$$V \geq x + c.$$

If an SME's own funding is left idle, it earns a gross interest rate normalised to 1. If $a < x$, the SME needs external funding to survive, and we call this SME *in deficit*. The source of external funds, excluding the public budget, is from banks, which have a total of K units of funding for SMEs. Both SMEs and banks are risk-neutral.

The government allocates a budget of G to help SMEs survive the crisis. Our modelling of government support captures a wide range of schemes implemented in practice aimed at helping SMEs survive crises, despite many of them taking the form of credit guarantees,[5] instead of an immediate expenditure out of the public purse. Regardless of the specific form the government scheme takes, it helps more SMEs survive only if it enlarges the funding supply for them. In the case of credit guarantee schemes, it may appear that the enlargement of funding comes from the private sector presently. However, since the enlargement would not realise should the government scheme be absent, the extra funding must be induced by the future payments of the support scheme; that is, it is equal to the present value of these payments. This present value is what we capture with G in the model.

[5]For more details see Section 6.3.

We start our analysis with the case in the absence of government funding and then investigate how the government funding G can be most effectively deployed.

4 The benchmark: Market Equilibrium without Government Support

In the absence of government funding, SMEs can only turn to banks for loans. In what follows, we determine the gross risk-free lending rate R that clears the loan market. Given we have normalised the gross return rate of SMEs' self-funding to 1, we interpret $R-1$ as SMEs' net funding cost. In this market, the aggregate supply of bank funding is K, and the aggregate demand depends on R. We assume the bank funding is such that in equilibrium it is more expensive than SMEs' self-funding, i.e., $R > 1$; the exact condition for this to hold will be characterised later. Under this assumption, SMEs demand bank funding and survive the crisis if and only if their types (a,x) satisfy the following three conditions. First, because $R > 1$, only those SMEs who are in deficit borrow from banks. That is, the types of borrowers (a,x) are in the set DF where

$$DF \equiv \{(a,x)|a \leq x\}. \tag{2}$$

Second, as in Holmstrom and Tirole (1997), the moral hazard of SME owners generates a wedge between the present value of the business and the value pledgeable to the bank. SME owners work hard only if their stake in the business is no less than c_Δ ($\equiv \frac{c}{\Delta}$) conditional on the project's survival; otherwise, they will shirk.[6] Hence, the expected pledgeable income to the lender is no larger than $p(\Pi - c_\Delta)$. Given the gross risk-free rate R, then the maximum

[6]Suppose the wedge is M, so the pledeable income is $\Pi - M$ conditional on surviving. If the project succeeds, banks get paid $\Pi - M$. If the SME owner works hard, her expected gain is $pM - c$; if she shirks, her expected gain is $(p-\Delta)M$. The banks ensure the expected gain by working hard is no smaller than that of shirking, and hence, we can solve for the wedge $M \geq \frac{c}{\Delta}$.

amount that an SME could borrow from a bank is

$$k_e = \frac{p(\Pi - c_\Delta)}{R} = \frac{V - pc_\Delta}{R}.$$

A type (a,x) can find sufficient bank funding to make up the deficit if and only if it is in the set IC where

$$IC \equiv \{(a,x)|x - a \leq k_e\}. \tag{3}$$

Third and lastly, not all the SMEs eligible for bank funding find it worthwhile to weather through the crisis by borrowing bank funds. The reason is that the cost of bank funding $R > 1$ so that each unit of bank funding incurs a loss of $R-1$ to the borrower. The total loss to an SME of type (a,x) is therefore $(R-1)(x-a)$. Its NPV is $V-c-x$. Therefore, the SME will borrow from the banks to survive if and only if its type satisfies the individual rationality (IR) constraint: $V-c-x \geq (R-1)(x-a)$, and hence, is in set IR, where

$$IR \equiv \{(a,x)|Rx - (R-1)a \leq V - c\}. \tag{4}$$

Therefore, the aggregate demand for bank loans $D(R)$ is

$$D(R) \equiv \iint_{DF \,\cap\, IC \,\cap\, IR} (x-a)h(a,x)dadx.$$

The cost of bank funding R is determined by the following market clearing condition

$$D(R) = K. \tag{5}$$

We show in Lemma 1 below that this demand is a decreasing function of R, the price of bank funding.

Lemma 1. $D'(R) < 0$ *for* $R \geq 1$ *and* $\lim_{R\to\infty} D(R) = 0$.

Proof: See Appendix A.

As a result, the market clearing price of bank funding $R > 1$ whenever (6) holds, which we assume.

$$K < D(1). \tag{6}$$

A type (a,x) that survives the crisis by borrowing from banks must be in the joint set of (2), (3), and (4), as illustrated in Figure 1 below.

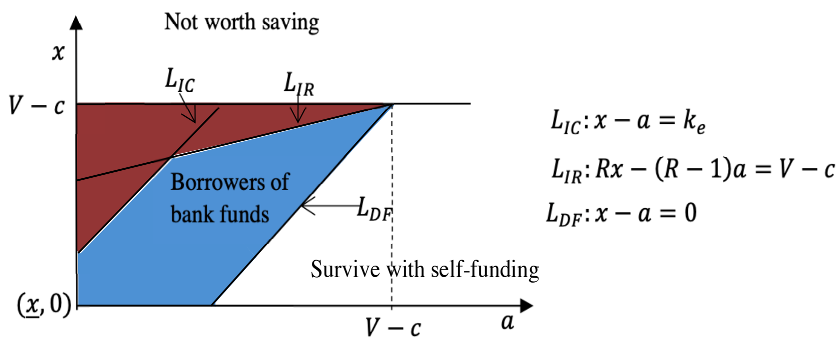


Figure 1: The types in the blue area survive in the absence of public support.

The SMEs in the dark red area of Figure 1 have a positive NPV but fail to survive the crisis. This is an efficiency loss. They fail to survive because of a failure to meet either the IC constraint (3) or the IR constraint (4). Intuitively, constraint (3) says that an SME must have an adequate stake to ensure the bank that there is no shirking; constraint (4) says that the SME must have a large enough NPV to afford borrowing expensive bank funds. These two constraints point to two sources of efficiency loss: the former is that SMEs cannot borrow sufficiently due to the IC constraint; the latter is that bank funds are too expensive relative to the NPV of the SME. Note that reducing the cost R of bank funding for SMEs only ameliorates the latter problem, but not so much the former.

Hereinafter we call *outsiders* those types (a,x) that are in deficit, with positive NPV projects, but outside the loan market, i.e., the types in the dark red area in Figure 1. These types have a positive NPV but need the support of public funding to survive the crisis.

The government has a limited quantity of G units of public funds for the outsiders. What is the best way to deploy them? Our main thesis is that a public-private partnership (PPP) with banks is necessary for the government to effectively use public funding. To appreciate that, let us for now imagine how much the government could achieve if it were to implement the scheme on its own. Since the government does not observe SMEs' types, there is no targeting at all. It would have to blindly distribute the public funds to SMEs. Given that each SME receives only a small amount of funding, few of them would be saved by the public support, among which even fewer have positive NPVs. A large quantity of public funds is therefore squandered on the "wrong" SMEs, i.e., SMEs that either have negative NPVs or can survive with their self-funding.

While the PPP is necessary for the effective use of public funding, different forms of PPP may deliver different levels of effects. To begin with, we consider a simple form of the partnership.

5 A Simple Form of PPP

Among all forms of PPP, the simplest one might be as follows.

The Simple Partnership: The government gives funding G to banks, which pool the public funds with their own funds K to lend to the SMEs.

The effect of the Simple Partnership is the enlargement of funding supply from K to $K+G$, thereby reducing its price R. We assume G is sufficiently small so that the net funding cost $R-1$ for SMEs is still positive. This is captured by the following equation.

$$G < D(1) - K. \tag{7}$$

The gross risk-free lending rate R is thus determined by the following market-clearing condition:

$$D(R) = K + G. \quad (8)$$

A reduction in R (due to the enlargement of funding from K to $K+G$) affects both the IC and the IR constraints. First, as R falls, external finance becomes cheaper and enterprises with smaller NPV can afford it. Hence, the IR constraint (4) is relaxed. This is represented by line L_{IR} In Figure (1) rotating clockwise around point O (coordinate $(x = V-c, a = V-c)$). Second, a reduced R also increases the present value of the pledgeable income k_e to the bank and thus relaxes the IC constraint (3).[7] This is represented by line L_{IC} in Figure (1) parallelly moving northwestward. These two effects are illustrated in Figure 2 below.

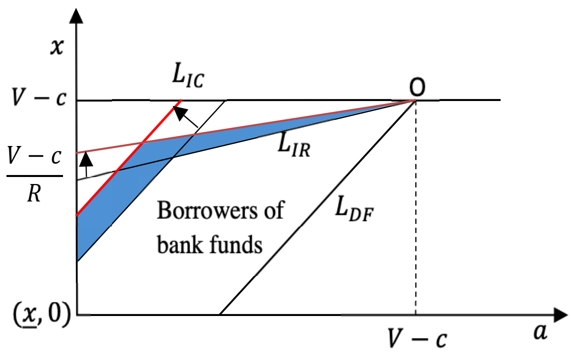


Figure 2: The effects of Simple Partnership: compared to Figure 1, with R reduced, line L_{IR} rotates clockwise around point O and line L_{IC} moves northwestward. Hence, the blue area marks the SMEs that the policy saves.

With the Simple Partnership, the SMEs in the blue area of Figure (2) are saved, in addition to those in the blue area of Figure (1). The funding shortfalls of these newly included SMEs must be covered by the public funding G. That is,

$$\int_{\text{blue area}} (x-a)h(a,x)dadx = G. \quad (9)$$

Despite being simple, the Simple Partnership already improves targeting compared to

[7]Since $k_e = (V - pc_{\Delta})/R$ decreases with R.

the scenario of the government distributing funds on its own. Because $R > 1$ and bank funding is still more expensive than SMEs' self-funding, it excludes the "wrong" SMEs which received funding in the previous scenario, i.e., those with a negative NPV and those not in deficit.

Albeit the Simple Partnership has these merits, we show the partnership between the government and banks can still do better. How it can do better depends on the government's objective. In crisis times, the main concern for the government typically is employment protection. For example, many of the existing COVID-19 support programmes aim to protect jobs and livelihoods. Therefore, we first suppose the government's object is to protect employment. Later, we also consider the other case where the government's objective is to maximise economic efficiency.

6 The Case of Protecting Employment

To demonstrate the efficacy of the PPP, we first consider a hypothetical case where the government has full information on firm types but operates on its own. Then we go back to our model where the government and banks cannot observe firm types but they form a partnership. And we show the optimal partnership for employment protection can do even better than the hypothetical full-information case. Finally, we compare our optimal Employment Protection partnership with some existing government support programmes and empirically estimate how much better our optimal partnership can perform. For our purposes, let n denote the number of jobs an SME provides. In our graphic illustrations below, we assume n is the same among SMEs; however, as we shall explain, our analytic results hold true regardless of this assumption.

6.1 The hypothetical full-information case

In this hypothetical case the government observes the type (a, x) but operates disjointly from banks. Hence, the government has no problems of adverse selection. It seems that in this full information case, public support achieves the maximum effect – i.e., saving the most jobs. However, as we will demonstrate, a well-designed PPP can perform even better, despite neither the government nor banks being able to observe the SME types.

Which outsiders (i.e., the SMEs that are in deficit but with positive NPV) should the public funding support in this full-information case? We approach the allocation of public funds as an investment decision. Suppose the government covers the deficit $x - a$ of an outsider (i.e., an SME that is in deficit, with a positive NPV, but outside the loan market), thereby saving the n jobs it provides. We define the number of jobs per unit of public funding, $n/(x - a)$, as the *job-saving return rate* of investing public funds. To maximise the effect of employment protection, the government should prioritise outsiders with higher job-saving return rates.

Full-Information Policy for Protecting Employment (FI-E): The government covers the shortfall of a type (a, x) outsider that provides n jobs if and only if its job-saving return rate is no lower than a threshold λ^*. That is,

$$\frac{n}{x - a} \geq \lambda^*. \tag{10}$$

For the purpose of illustration, we assume all the SMEs have the same number of workers. Under this assumption, (10) is equivalent to the funding shortfall $x - a$ being no larger than a threshold s, where $s = n/\lambda^*$. Therefore, the government finances the shortfall of those outsiders for whom $0 < x - a \leq s$. Obviously, the larger the scale G of public funding is, the higher the threshold s is. As we assume a limited budget G, we focus on the case where $s < k_e$. Hence, the types of outsiders that FI-E saves are illustrated in Figure 3 below, where line $x - a = s$ is to the right of line L_{IC} because of the assumption $s < k_e$.

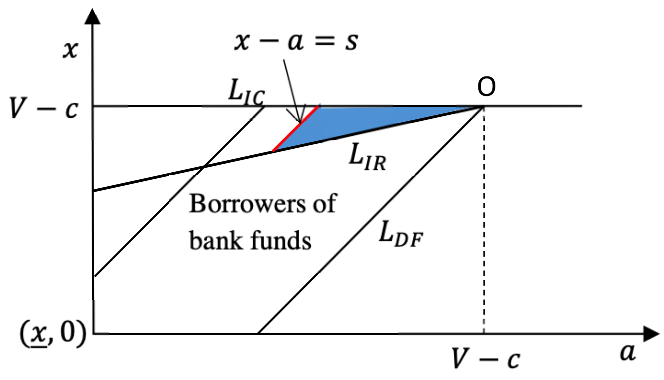


Figure 3: The blue area marks the SMEs saved by FI-E.

Because all types in the blue area of Figure 3 have their deficits $x-a$ covered by the public funding, the following budget constraint holds.

$$\int_{\text{blue area}} (x-a)h(a,x)dadx = G. \tag{11}$$

FI-E saves more jobs than the Simple Partnership. As we can observe from Eq (9) and Eq (11), the sums of the funding shortfalls of the SMEs selected in by the FI-E and by the Simple Partnership are both equal to G. However, from the perspective of investments, since the FI-E prioritises the SMEs with higher job-saving return rates, it delivers higher returns, i.e., saves more employment, than the Simple Partnership.

6.2 The optimal Employment Protection partnership

Now let us turn to the model economy where the government observes neither a nor x of any SME. The government cannot directly target SMEs with high job-saving return rates $n/(x-a)$ of using the funds for employment protection, but a partnership with private banks can help. Because bank funding is expensive (i.e., $R > 1$), SMEs will only borrow the quantity of bank funding that just suffices to cover their shortfalls of $x-a$. Put differently, SMEs' demand for loans b reflects their shortfalls. Therefore, to target SMEs with the highest return rates, the government should target those with the smallest loan demand per worker, b/n. A partnership to protect employment takes the following form:

Employment Protecting (EP) Partnership: Suppose an SME borrows b units of loans and provides n jobs. The SME obtains yb units of public funding if its loan per worker b/n is no larger than a threshold b_w^*, and obtains no public funds otherwise.

The subsidy rate y should satisfy condition (12):

$$y \leq R - 1. \tag{12}$$

If $y > R - 1$, this partnership causes borrowing to deliver a net gain to the borrower and all SMEs will use the public support, which then defeats its purpose of targeting. The two parameters b_w^* and y in such a partnership are negatively related. If b_w^* goes up, then more SMEs will be qualified for the support. Given the budget G, the subsidy rate y to each qualified SME has to go down. Hence, there is only one degree of freedom, and we can index the partnership with y.

Given y, let us examine which SMEs are added to the survival group by the partnership. First, with this partnership, an SME who borrows b from a bank will have $(1+y)b$ units of funds in hand. To cover the deficit, $b = (x-a)/(1+y)$. To qualify for the benefit of the public support, the borrowing scale $b/n \leq b_w^*$, which is equivalent to $n/(x-a) \geq 1/((1+y)b_w^*)$. Therefore, the qualification standard of the EP partnership for an SME type (a,x) is equivalent to the requirement that the job-saving return rates of using public funds be above a certain threshold:

$$\frac{n}{x-a} \geq \frac{1}{(1+y)b_w^*}. \tag{13}$$

Second, because the partnership has no impact on the pledgeable income, it does not affect the IC constraint, which is still given by (3). It affects the IR constraint with two effects. 1) It gives a subsidy $y \cdot b = y \cdot (x-a) \,/\, (1+y)$ to a recipient type (a,x). As a result, its NPV becomes $V - x - c + y \cdot (x-a)/(1+y)$. 2) with the partnership, the cost of bank funding amounts to $b \cdot (R-1) = (x-a)/(1+y) \cdot (R-1)$. With the aid of EP partnership, the NPV is no smaller than the cost of bank funding if and only if

$$V - c \geq \frac{R}{1+y}x - \frac{R-(1+y)}{1+y}a. \tag{14}$$

These two conditions characterise types (a,x) of the outsiders that the partnership saves, which, under the assumption that all SMEs provide an equal number of n jobs, are illustrated in the blue area of Figure (4) below.

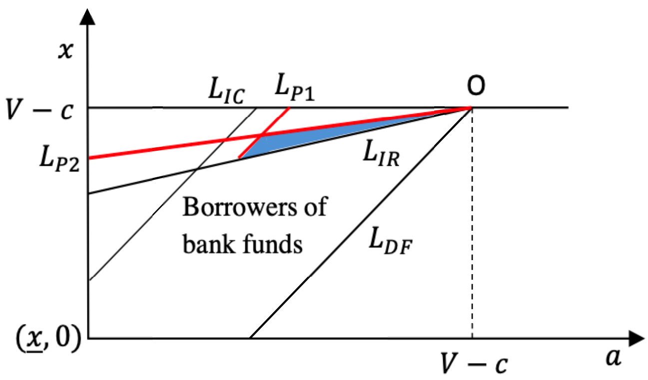


Figure 4: If $y < R-1$, the types that are saved by EP partnership compose the blue area, where Line L_{P1} is defined by the equality form of (13), and Line L_{P2} by the equality form of (14).

With the EP partnership, the SMEs in the blue area of Figure 4 are saved, in addition to those saved by bank funding in the absence of the partnership. When market clears, the total deficits must be equal to $G+K$. Hence, the deficits of those newly saved SMEs are covered by the public funding:

$$\int_{\text{blue area}} (x-a)h(a,x)dadx = G. \tag{15}$$

We have shown that the EP partnership is characterised by $y \in (0, R-1]$; b_w^* is a decreasing function of y via (15). For a given $y < R-1$, we show the net effect of a marginal increase in y is an increase in the number of jobs saved, and hence, the following proposition.

Proposition 1. *The optimal value of y is $R-1$. That is, in the optimal EP partnership, the public funds fully subsidise the funding cost of SMEs whose loan per worker is no larger than a threshold, which is determined by the scale G of the public funds.*

The proof of the proposition is as follows. We have seen the negative relationship between the subsidy y and the qualification threshold b_w^*. The marginal increase from y to y' reduces the qualification threshold from b_w^* to b'_w. Now consider which SMEs are selected out due to this change and which are selected in. This change selects in SMEs because it leads to a more generous subsidy and therefore relaxes the IR constraint (14). Given that the IR constraint is relaxed, if an SME is selected out, that must be because it is disqualified due to the change in the threshold; that is, the SME violates the qualification constraint (13) with the new threshold:

$$\frac{n}{x-a} < \frac{1}{(1+y')b'_w}. \tag{16}$$

On the other hand, those who are selected in must meet the new qualification constraint, i.e.,

$$\frac{n}{x-a} \geq \frac{1}{(1+y')b'_w}. \tag{17}$$

Comparing (16) and (17), for each SME selected in, the job-saving return rate of using public funding is larger than that for each SME selected out. Furthermore, from Equation (15), the total deficits $x-a$ of those selected in equal that of those selected out. Hence, the marginal increase in y ends up saving more jobs.

Our EP partnership has a qualification standard. Below we show the qualification standard is restrictive.

Lemma 2. *Not all SMEs that borrow from banks benefit from the optimal EP partnership.*

We prove Lemma 2 by contradiction. Suppose on the contrary, all borrowers benefit from the optimal EP partnership. Then, it is equivalent to the Simple Partnership, because each borrower gets a government fund in proportion to the bank funding that they demand. Moreover, the equivalence of the optimal EP partnership to the Simple Partnership implies the market-clearing rate with the Simple Partnership $R = 1$, because the optimal EP partnership fully subsidises the funding cost. It follows that the market clearing condition (8) holds at $R = 1$, which contradicts with assumption (7).

To illustrate the effects of the optimal EP partnership, we resort to the assumption that n is the same among all SMEs. Let $b^* \equiv b_w^* n$. Given the optimal $y = R - 1$, conditions (13) and (14) become:

$$Rb^* \geq x - a, \tag{18}$$

$$V - c \geq x. \tag{19}$$

By Lemma 2, $Rb^* < k_e$, and the optimal EP partnership saves the SMEs that are illustrated in the blue area of Figure 5.

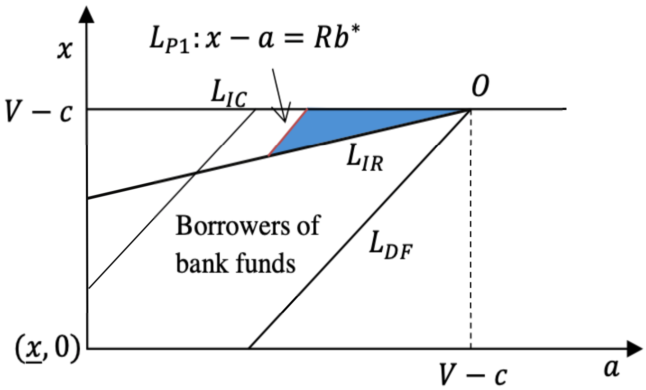


Figure 5: The types of SMEs saved by the optimal EP partnership compose the blue area.

Assuming that the equilibrium lending rate R is unaffected by this partnership, Figure 5 is exactly the same as Figure 3, which illustrates what the government could best achieve if it had full information and operated disjointly from banks. Hence,

Proposition 2. *If the equilibrium lending rate is unchanged, the optimal Employment Protecting partnership achieves the same effect as the full information case.*

Proposition 2 demonstrates the power of public-private partnerships in overcoming information asymmetry: the partnership can achieve under asymmetric information what the government could achieve on its own under full information, if the equilibrium lending rate is not changed. In practice, this condition should hold true, because, relative to the aggregate private funding K, the scale of the public funding G is probably not large enough to move the equilibrium lending rate R substantially.

Interestingly, if the EP partnership does move the lending rate, it can perform even better.

Proposition 3. *If the equilibrium lending rate increases due to the partnership, the optimal EP partnership under asymmetric information saves more jobs than the government on its own with full information.*

We first prove Proposition 3 under the assumption that all SMEs employ an equal number of workers, and we then explain that this proposition holds in the absence of this assumption.

We illustrate the proof using Figure 3 which illustrates what the government achieves on its own under full information. Suppose the EP partnership increases R. As R increases, Line L_{IC} shifts southeastward and Line L_{IR} rotates anti-clockwise around point O, as illustrated in Figure (6). The shifts of these two lines squeeze out types in the red area of Figure (6). Given that the aggregate deficits all SMEs saved are equal to $K+G$, squeezing out these types generates space for more types to be saved. That is, L_{P1} shifts leftward, selecting in the types in the grey area of Figure (6). Observe that the types squeezed out (i.e., those in the red area of Figure (6)) all have a greater shortfall $x-a$ than those selected in (i.e. those in the grey area). Therefore, the net effect is that the optimal EP partnership under asymmetric information saves more SMEs and hence more jobs than the government operating alone but with full information.

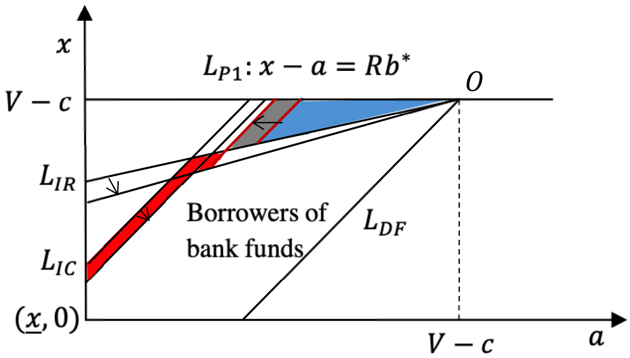


Figure 6: If the optimal EP partnership increases R, types in the red area are selected out, those in the grey selected in.

We now prove Proposition 3 in the absence of the assumption that SMEs employ an equal number of workers. For this purpose, we prove that the types selected out by the increase in R all have a lower job-saving return rate $n/(x-a)$ than those selected in. The rise in R increases the cost of bank funding. However, this increase has no impact on an SME that is qualified for the subsidy under the optimal EP partnership, which fully covers the cost of bank funding. Hence, if an SME is selected out, that must be because it is not qualified for the subsidy. This means its job-saving return rate $n/(x-a)$ of using public funding is lower than the qualification threshold. On the other hand, despite the increase in the cost of bank funding, if an SME is selected in, that must be because it is qualified for the subsidy. Therefore, its job-saving return rate is higher than the qualification threshold.

When does the partnership increase the equilibrium lending rate? Given the fixed supply of bank funding, the partnership affects the equilibrium lending rate via its impact on the aggregate demand for bank funding. Regarding this impact, this partnership has both a positive effect and a negative effect on the aggregate demand. First, the partnership saves a group of SMEs who would not survive without the public support, illustrated in Area A1 in Figure 7 (triangle BCO). That they join the demand side of bank funding raises the aggregate demand, which is the positive effect. Second, a group of SMEs who would survive the crisis by borrowing from the banks without the partnership are nevertheless qualified to receive public support according to the partnership. These are the SMEs who have a loan demand per worker no larger than b_w^*, illustrated in Area A2 in Figure 7 (polygon XBODE). Due to the partnership, their demand for loans decreases from $x-a$ to $(x-a)/R$. This is the negative effect on the aggregate demand. If the p.d.f. $h(a,x)$ is sufficiently large in Area A1 relative to A2, then the positive effect dominates the negative effect. The aggregate demand for bank funding, and therefore, the equilibrium lending rate, both increase.

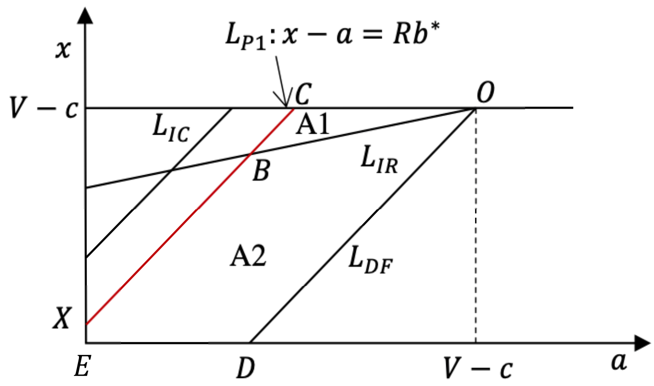


Figure 7: With the optimal EP partnership, types in Area A1 join the loan market, increasing the aggregate demand for bank funding, and types in Area A2 reduce their demand for bank funding.

If the partnership causes the equilibrium lending rate to decrease, we see opposite movements compared to Figure 6, and therefore, the optimal EP partnership under asymmetric information performs worse than the government operating on its own but with full information.

The optimal EP partnership always performs better than the Simple Partnership. As illustrated in Figure 5 the optimal EP partnership saves the SMEs with the highest job-saving return rates $n/(x-a)$ of using public funding. Relative to the Simple Partnership, those selected in by the optimal EP partnership have higher return rates than those selected out. Since for both schemes, the total shortfalls of SMEs saved because of public funding are equal to G, the optimal EP partnership saves more jobs than the Simple Partnership.

Let us finish the analysis of this section by considering the implementation of the optimal Employment Protection partnership. Note that $R-1$ is the funding cost for the SMEs, not the *ex ante* net interest rate observed in the loan contract. Given an SME survives with probability p, the gross loan rate is R/p, and the net loan rate that is observed in the loan contract is $r_o = R/p-1$. That subsidy for the funding cost $y = R-1$ means the government pays an interest expense of $(R-1)/p$, which is smaller than the whole interest expense r_o. To implement the optimal Employment Protection partnership precisely, the government needs to observe either the default risk $1-p$ or the funding cost $R-1$. In practice, the government will not be able to observe p. If it asks banks to report p, then banks may misreport its value and induce the government to shoulder more interest expense. However, there are reliable ways of measuring the funding cost $R-1$. In equilibrium, banks should obtain the same profit margin $R-1$ from extending loans of different risk profiles. We can assume the default risk of the safest loans is 0 (i.e., $p=1$), such as the mortgage lending to high-quality borrowers (e.g., those with stable income, low loan-to-value ratios). Then, the net loan rate r_{safe} of this type of safe loan is equal to the funding cost $R-1$. From $r_{safe} = R-1$ and $r_o = R/p-1$, it follows that $p = \left(r_{safe}+1\right)/\left(r_0+1\right)$. The interest subsidy $(R-1)/p$ should hence be $r_{safe}\left(r_o+1\right)/\left(r_{safe}+1\right)$, or a fraction ϕ of the interest cost r_o, where

$$\phi = \frac{N_g\left(r_{safe}\right)}{N_g\left(r_o\right)}, \tag{20}$$

with $N_g(r) := r/(1+r)$ denotes the ratio of net to gross lending rate.

During the COVID-19 pandemic, many governments implemented support programmes for SMEs to protect employment. We next formalise the key features of some of those support programmes in our framework and empirically estimate the magnitude of the improvement that our optimal Employment Protection partnership can achieve over the support programmes in practice.

6.3 Empirical magnitude of improvement over COVID-19 policies

Typically, the schemes governments implemented to support SMEs during the pandemic place a cap on the size of loans they subsidise. In addition to the subsidised loans, SMEs are free to borrow any additional amount from banks through unsubsidised loans. For example, the UK Bounce-Back Loan (BBL) scheme provides 100% guarantee for a loan, both the principal and interest included, up to 25% of the turnover and has an upper limit of £50,000 per claimant, and the borrowers are free to take out other loans uncovered by the scheme.[8] Schemes of similar features were also launched in many euro area countries, such as the *KfW-Schnellkredit* in Germany, and in the US, such as *Paycheck Protection Program*.[9]

To sum up, these employment protection schemes have the following three features: 1) they provide guarantees on the loan principals, besides subsidies on the interest expense, 2) there is a size cap on the guaranteed loans, and 3) the subsidised loan borrowers are free to access additional loans uncovered by the scheme.

Because these schemes provide guarantees on the loan principals, this feature may lead some recipients to treat the schemes' funding as free cash, leading to two potential forms of taxpayer money waste. The first is moral hazard: some scheme users simply spend the money on unproductive expenditures with no intentions to repay.[10] The second is that SMEs may obtain public funding before exhausting their own funds. Our model addresses the moral hazard issue by utilising private banks' expertise in arranging and monitoring loan contracts, ensuring that the SMEs using the funding meet their IC constraints. Moreover, our design ensures that only SMEs in deficit will use public funding, effectively avoiding

[8]This has been confirmed via the authors' freedom of information request from the British Business Bank, a state-owned economic development bank established by the UK government with the aim of increasing the supply of credit to SMEs and providing business advice. Also see British Business Bank https://www.british-business-bank.co.uk/.

[9]For details on Germany's KfW-Schnellkredit see `https://static.eurofound.europa.eu/covid19db/cases/DE-2020-13_349.html`. For other similar schemes in the euro area, see the ECB Financial Stability Review, May 2020 and ECB Economic Bulletin, Issue 6 2020. And for the US case see Autor et al. (2022).

[10]See evidence of fraud and abuse in UK National Audit Office (2020); Beggs and Harvison (2023) and court cases US Department of Justice (2022).

both forms of taxpayer money waste. In our estimation, we assume existing schemes are free from these two forms of waste, and we focus on a key difference: in our optimal Employment Protection partnership, only SMEs whose job-to-loan rates are above a specified threshold receive public funding, whereas under the existing schemes the subsidised SME borrowers are free to access further loans not covered by the schemes (Feature 3). Thus, we estimate the magnitude of improvement our partnership achieves over the existing schemes, assuming the latter were free from moral hazard and inefficient allocation to SMEs with adequate self-funding. Taking into account the taxpayer money wasted because of these two issues, our optimal EP partnership not only protects more jobs, but also significantly saves taxpayer money.

Specifically, we model the existing scheme as follows. In this scheme, borrowers can borrow a costless loan ι up to a cap δ, and these loans are financed by the public funding G. To ensure the scheme does not give free cash, the borrowers still need to fully repay ι. That is, they need to pay back ι/p in case their enterprises survive the crisis. Outside this scheme, borrowers are free to further borrow bank loans at the market rate R.

We now examine which types of SMEs are saved under this scheme. First, some types of SMEs can be saved by only borrowing the costless loan which gives them funding up to δ. The types of these SMEs are characterised by the condition

$$0 < x - a \leq \delta. \tag{21}$$

As we assume the scheme only pays the borrowing cost of the loans it covers, borrowers still need to pay back δ in value. To induce SME owners to work hard and not shirk, their stake *ex ante* shall not be smaller than pc_Δ. Hence, δ satisfies the following IC constraint:

$$V - \delta \geq pc_\Delta. \tag{22}$$

Second, if condition (21) is violated, types (a,x) need to borrow additional bank funding at the market rate R. We examine how the IC and IR constraints of these SMEs are affected. Regarding the IC constraint, the pledgeable income to the bank is now no larger than $V - \delta - pc_\Delta$. The maximum bank funding that the SME can borrow changes to

$$k_e' = \frac{V - \delta - pc_\Delta}{R} = k_e - \delta/R.$$

By assumption (22), $k_e' > 0$, so that the SMEs who use this scheme can still borrow normal bank loans, consistent with what happened in reality. An SME in deficit can survive the crisis if and only if the shortfall $x - (a + \delta)$ is no greater than this threshold k_e'. That is, the IC constraint now changes to

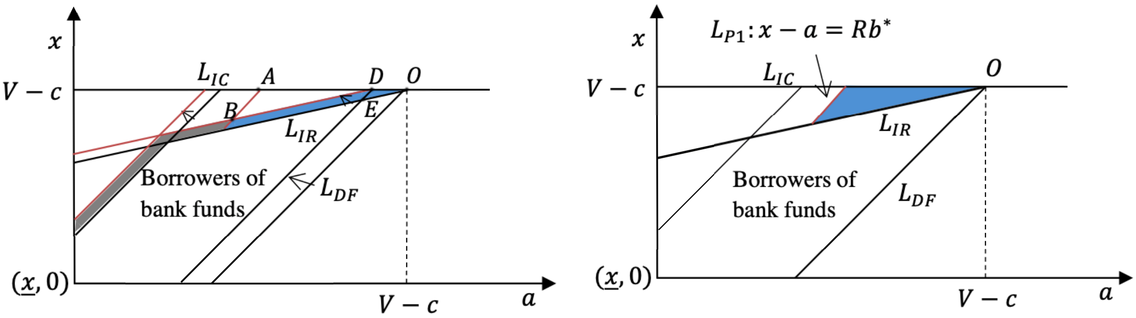
$$x - a \leq k_e + \delta\,(1 - 1/R)\,. \tag{23}$$

The binding case of this constraint defines the new IC line L_{IC}. Regarding the IR constraint, an SME gains no net value but the funding deficit is reduced to $x - (a + \delta)$. If it borrows bank funding to cover the deficit, it incurs a loss of $[x - (a + \delta)](R - 1)$. The SME is willing to borrow if and only if this loss is no greater than its NPV of $V - x - c$, which is equivalent to

$$Rx - (R - 1)\,a \leq V - c + (R - 1)\,\delta. \tag{24}$$

The binding case of this constraint defines the new IR line L_{IR}. Altogether, the types that are saved under the scheme are those that meet condition (21) or conditions (23) and (24). We have seen that the types that are saved without public funding are illustrated by the blue area in Figure 1. Hence, the types that are saved because of the scheme are illustrated by the blue and grey areas in Figure 8a.

The effect of the optimal Employment Protection partnership is illustrated in Figure 5, which is replicated in Figure 8b. From the two panels of Figure 8, compared with the optimal EP partnership, the scheme selects in SMEs in the grey stripes in Figure 8a, while selecting out the SMEs in the white triangle *ABD*. By the design of our EP partnership, any selected-out SME has a job-saving return rate $n/(x-a)$ lower than any selected-in SME. Thus, the scheme performs worse than our optimal Employment Protection partnership.



(a) Effect of the scheme (b) Effect of the optimal EP partnership

Figure 8: Compared with the optimal EP partnership, the scheme selects in the types in the grey stripes, while crowding out the types in the white triangle *ABD*.

Now we conduct a simple simulation exercise to broadly assess the magnitude of how many more jobs our optimal EP partnership can save relative to the scheme, based on the assumption all SMEs provide an equal number of jobs. To facilitate the empirical estimation, we normalise the SME's own funds a and the cost of surviving the crisis x by the NPV $V-c$. We use ˆ to denote the normalised variable. For example, $\hat{a} \equiv \frac{a}{V-c}$, and $\hat{x} \equiv \frac{x}{V-c}$. The normalised IC and IR constraints, from (3) and (4), are simply

$$\hat{k}_e \geq \hat{x} - \hat{a}; \tag{25}$$

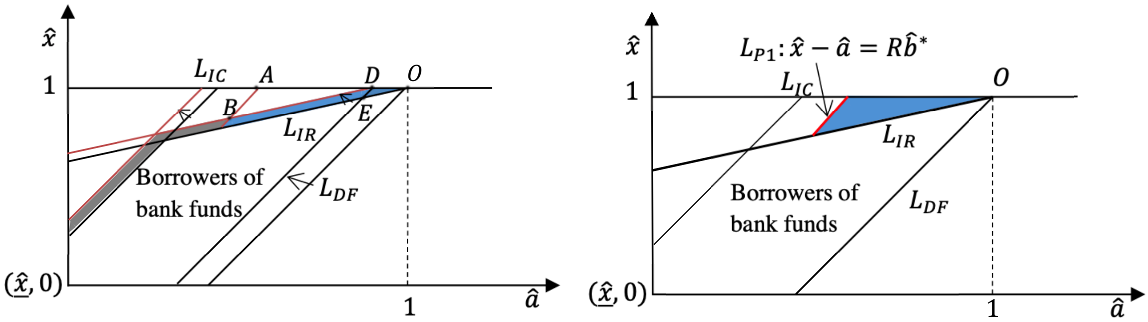
$$1 \geq R\hat{x} - (R-1)\hat{a}. \tag{26}$$

The normalised IC and IR constraints with the scheme, from (23) and (24), become

$$\hat{k}_e + \hat{\delta}(1 - 1/R) \geq \hat{x} - \hat{a}; \tag{27}$$

$$1 + (R-1)\hat{\delta} \geq R\hat{x} - (R-1)\hat{a}. \tag{28}$$

Figure 8 is thus modified as below.



(a) Scheme normalised

(b) Optimal EP partnership normalised

Figure 9: The effects of the scheme and optimal EP partnership normalised by $V - c$.

Our estimation strategy consists of the following four steps.

Step 1. We look for the empirical counterparts of $\hat{k}_e$, $\hat{\delta}$ and R, such that we fix the exact positions of all the lines in Figure 9, except $\hat{x} - \hat{a} = R\hat{b}^*$.

Step 2. Estimate the government budget $\hat{G}$ by using $\int_{\text{colour area}}(\hat{x} - \hat{a})h(\hat{a}, \hat{x})d\hat{a}d\hat{x} = \hat{G}$ in Figure 9a, assuming uniform distribution for (a, x).

Step 3. Calculate $\hat{b}^*$ in Figure 9b by noting that the $\hat{G}$ used in Figure 9a is the same as that in Figure 9b, so $\int_{\text{blue area}}(\hat{x} - \hat{a})h(\hat{a}, \hat{x})d\hat{a}d\hat{x} = \hat{G}$ in Figure 9b.

Step 4. Calculate $size_a$, the size of the coloured area in Figure 9a and $size_b$, the size of the blue area in Figure 9b. As the size represents the number of SMEs saved, which is proportional to the number of jobs saved, $\Delta size = (size_b - size_a)/size_a$ provides us with the empirical magnitude of the percentage increase in jobs saved by our Employment Protection partnership relative to existing government schemes.

In the existing schemes, the upper bound of the covered loan is typically a fraction α of turnover. We estimate $NPV \approx turnover \times profit\ margin \times PE$, where the PE represents the price-to-earning ratio. Thus,

$$\hat{\delta} = \alpha \times turnover/NPV \approx \alpha/(profit\ margin \times PE).$$

We take the PE ratio of UK firms to be 10 and we estimate the profit margin by using firm markups according to (29).[11]

$$profit\ margin = (markup - 1)/markup. \tag{29}$$

So far, we have not found data on firm markups in the euro area and the UK, but De Loecker et al. (2020) has estimated the markup for US firms. They find that on the markup distribution, the median is invariant over time, but markups for the higher percentiles sharply increase. For example, the 90th percentile has seen an increase from 1.5 in 1980 to 2.5 in 2016. As SMEs typically do not possess large market power, we use the median markup of 1.25 as a reasonable baseline and conduct sensitivity analysis for a range of markup values. To obtain the empirical counterpart of $\hat{k}_e$, the maximum loans an SME can obtain normalised by its value, we approximate it with SMEs' average debt-to-value ratio. We take a conservative take by assuming it to be around the debt-to-value ratio of 0.5, as reported by British Business Bank. Finally, to proxy for R, the risk-free lending rate of the banks, we use the gross interest rate paid by small unincorporated businesses on secured debt in the summer of 2019.

Incorporating these empirical estimates in our four-step calibration strategy, we obtain $\Delta size = 16\%$. This means in our baseline estimation, our employment-protection partnership saves 16% more jobs than the UK government loan support policy. Our estimation is sensitive to markups, and as we use the weighted average markup of 1.6, $\Delta size$ increases staggeringly to 39%. Considering that our optimal EP partnership avoids the above-mentioned two forms of taxpayer money waste due to moral hazard problems, our design not only saves

[11] Per unit price p is equal to markup times cost c, i.e., $p = markup \times c$. The profit per unit of product is $p - c = (markup - 1)c$. As $profit\ margin = (p - c)/p$, Equation (29) obtains.

more jobs but also uses less taxpayer money.

During crises, public support for SMEs typically aims to protect jobs. Thus far, we have considered how to best deploy public funding if the government's objective is to protect employment. Such deployment of public funding, however, does not prioritise economic efficiency: our optimal EP partnership, although giving no funding to negative NPV SMEs, excludes some SMEs with high NPVs while saving SMEs whose NPVs are lower. In the next section, we consider the case where the government's objective is to maximise economic efficiency.

7 The case of maximising efficiency

Suppose the government's objective is to maximise economic efficiency. Given the same shortfall, $x-a$, higher NPV projects should take priority. Again, we first consider the hypothetical scenario in which the government observes the type (a,x) of SMEs and operates disjointly from banks. In this scenario, as in the preceding section, we treat public funding as an investment, and therefore, the targeting of its recipients should be guided by the return rate of the investment. If the government invests funds of $x-a$ to help an enterprise survive, the social value of this investment is $V-c-x$. The return rate of using public funding to improve efficiency is thus $(V-c-x)/(x-a)$. The SMEs with higher return rates should be prioritised. This hypothetical full-information value-maximising policy is thus given as follows:

Full-information Policy for Value Maximisation (FI-V): The government finances the shortfall of those outsiders whose types (a,x) satisfy:

$$\frac{V-c-x}{x-a} \geq \lambda^* \text{ for some } \lambda^* > 0.$$

The types saved by FI-V are illustrated in Figure 10. Because $(V-c-x)/(x-a) = \lambda$ is equivalent to

$$x - \frac{\lambda}{1+\lambda}a = \frac{V-c}{1+\lambda},$$

all the isoquant-return curves are straight lines that pass point O $(V-c, V-c)$. In particular, L_{IR} is an isoquant-return line. Moreover, the greater the return rate λ is, the steeper the slope $\lambda/(1+\lambda)$ of the isoquant-return line is. The threshold λ^* depends on G. The larger the public funding G is, the more SMEs saved, and hence, the lower the threshold λ^* is. Therefore, if G is larger than a certain threshold, then λ^* is small enough that the slope $\lambda^*/(1+\lambda^*)$ of the boundary isoquant-return line is smaller than the slope of the IR line L_{IR}, in which case the types of SMEs saved by the policy are illustrated in the left panel of Figure 10. If G is smaller than the threshold, then the slope $\lambda^*/(1+\lambda^*)$ of the boundary isoquant-return line is larger than that of the IR line, in which case the types of SMEs saved by the policy are illustrated in the right panel of Figure 10. We focus on the right panel as we assume the public funding G is limited.

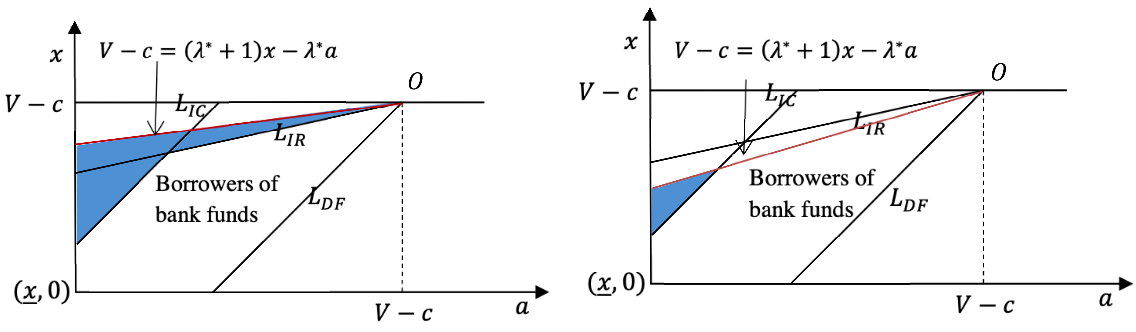


Figure 10: The types that FI-V saves are in the blue area of the left panel if G is larger than a threshold and are in the blue area of the right panel if G is smaller than the threshold.

We now turn back to our model setup and consider designing the partnership with banks. The position of the blue area in both Figures 10 suggests that the government should prioritise SMEs with a large enough shortfall $x-a$, and therefore, should target SMEs with a large enough loan demand. Intuitively, this is because the size of an SME's loan demand is positively correlated with its NPV. Only when the SME has a sufficiently high NPV can it afford the funding cost of a large loan. Therefore, for value maximisation, the government should subsidise the borrowers of large loans. This is the opposite to the optimal Employment Protection partnership, which tends to subsidise only small loans. If we restrict to the case that the subsidy rate should be a constant for the qualified loans, then the Value Maximising partnership should take the following form:

Value Maximising (VM) partnership: The government subsidy to loans of size b is as follows:

$$s(b) = \max\{y(b-b^*), 0\} \tag{30}$$

where $b^* < k_e$ because k_e is the maximum loan banks grant, and y satisfies Condition (12).

From Equation (30), the partnership only subsidises the part of loan that is larger than the threshold b^*, because this gives no incentives for SMEs to over-borrow. Suppose the subsidy is in the following form:

$$s(b) = \begin{cases} yb & \text{if } b > b^* \\ 0 & \text{if } b \leq b^*. \end{cases}$$

Then the SMEs whose shortfalls are below and close to b^* will over-borrow. For example, the SMEs whose shortfalls are b^* will borrow $b^* + \varepsilon$ whereby they obtain the subsidy $y(b^* + \varepsilon)$ but only incur the loss $\varepsilon(R-1)$. In our form of VM partnership, this incentive is ruled out: if a type with shortfall $x - a \leq b^*$ borrows $b > b^*$, then its cost is increased by $(R-1-y)(b-b^*) + (R-1)(b^* - (x-a))$.

Thus, only SMEs with a shortfall $x - a \geq b^*$ are affected by the partnership. Consider such an SME. If it borrows b, the government will offer her $y(b-b^*)$. Hence, the shortfall is filled if $b + y(b-b^*) = x - a$, or

$$(1+y)b - yb^* = x - a. \tag{31}$$

The maximum the SME can borrow is k_e, and hence, $b \leq k_e$. The incentive compatibility constraint, for this group of SMEs, is thus given as follows:

$$\widetilde{k} \geq x - a, \tag{32}$$

where $\widetilde{k} := (1+y)\,k_e - yb^*$. The binding case of (32) gives rise to a line L_{P1}; that is,

$$L_{P1} := \left\{(a,x)\,|x - a = \widetilde{k}\right\}. \tag{33}$$

As $b^* < k_e$, we have $\widetilde{k} > k_e$. Therefore, L_{P1} is to the left of the IC line ($x - a = k_e$) in the absence of the partnership.

As for the IR constraint, from (31), a qualified type (a,x) SME needs to borrow $b = (x - a + yb^*)\,/\,(1+y)$ to fill the shortfall. The SME is willing to borrow if and only if its NPV $V - x - c$ is no smaller than the funding cost $b(R-1)$, which is equivalent to the following condition:

$$V - c - \frac{Ry}{1+y}b^* \geq \frac{R}{1+y}x - \frac{R-(1+y)}{1+y}a. \tag{34}$$

The binding case of (34) gives rise to a line L_{P2} in the $a - x$ plane:

$$L_{P2} := \left\{(a,x)\,|\,\frac{(1+y)\,(V-c)}{R} - yb^* = x - \frac{R-(1+y)}{R}a\right\}. \tag{35}$$

This line is above L_{IR}, the IR line in the absence of the partnership.

In this section, we assume that the effect of the partnership on the funding cost R is negligible. The effect of the Value Maximising partnership is illustrated as follows.

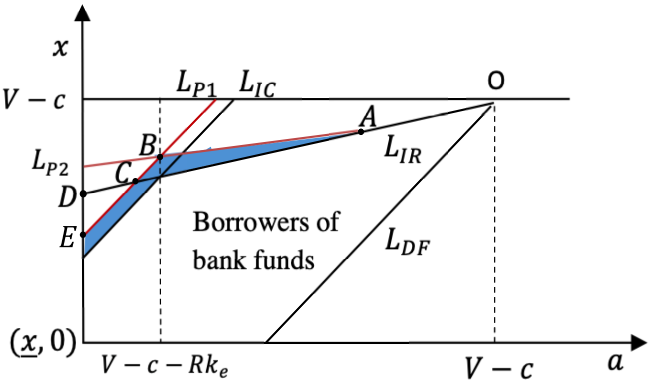


Figure 11: The effect of VM partnership if its effect on R is negligible: The types in the blue area are saved. Lines L_{IR} and L_{P1} cross at point A, and lines L_{P1} and L_{P2} at point B.

The parameters of the partnership (y, b^*) satisfy the usual market-clearing condition:

$$\int_{\text{blue area}} (x-a)h(a,x)dadx = G. \tag{36}$$

If the threshold b^* of the subsidy increases, there will be fewer SMEs who qualify for the subsidy, and given the budget, the subsidy rate y increases. That is, $\partial y/\partial b^* > 0$.

The optimal VM partnership is with (y, b^*) that solves the following problem

$$\max_{y,b^*} \int_{\text{blue area}} (V-c-x)h(a,x)dadx, \text{ s.t. } (36).$$

To begin with, we establish that any VM partnership performs strictly worse than the FI-V. Observe Figure 11. Because L_{IR} is an isoquant-return line, the SMEs in the blue triangle area ABC above L_{IR} all have return rates lower than the ones in the white triangle area CDE demarcated by L_{P1}, L_{IR}, and the vertical axis. The partnership saves the former but not the latter, which is an efficiency loss. In contrast, in the case of employment protection, we have shown that the optimal partnership achieves the full-information implementation if the effect on R is negligible. The reason for this difference is that as a signal, the loan demand is much more informative of the SME's shortfall than it is of their NPV. As a result, it does not offer as much help in overcoming the asymmetric information in the case of value maximisation as it does in the case of employment protection.

While any VM partnership is strictly dominated by the FI-V, we can still find VM partnerships that dominate the Simple Partnership. To prove this, first note that the Simple Partnership is a special case of the VM partnership at $b^* = 0$: the Simple Partnership works by reducing the lending rate from R to R', and this is equivalent to each loan receiving a subsidy $R - R'$, which is equivalent to the VM partnership with $b^* = 0$ and $y = R - R'$. We now show a marginal increase in b^* is an improvement upon the Simple Partnership. Observe that if $b^* = 0$, then point A in Figure 11 coincides with point O, whose coordinates are $(V - c, V - c)$, and hence, line L_{P2} is an isoquant-return line of the subsidy return function $(V - c - x) / (x - a)$. Now we consider a marginal rise in b^* from $b^* = 0$. To see how the rise in b^* moves lines L_{P1} and L_{P2}, we examine how it moves points A and B of Figure 11. Point A is the intersection of line L_{P2} and line L_{IR} and has coordinates $a = V - c - Rb^*$ and $x = V - c - (R - 1)b^*$. The rise in b^* therefore moves A down along line L_{IR}. Point B is the intersection of lines L_{P1} and L_{P2} and has a coordinate $a = V - c - Rk_e$, which is independent of (y, b^*), and hence it slides along the vertical line $a = V - c - Rk_e$ up or down. As A moves down along line L_{IR}, if point B is unmoved, the blue area in Figure 11 will diminish, which will violate condition (36). To keep the condition holding, therefore, point B should slide straight up along line $a = V - c - Rk_e$. Altogether, the effect of the increase in b^* from $b^* = 0$ is illustrated as follows:

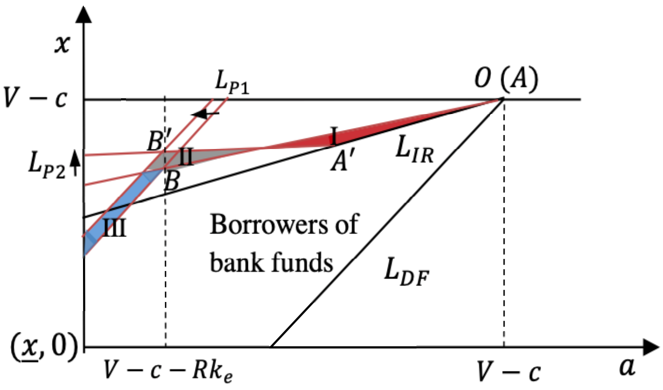


Figure 12: With a marginal increase in b^* from $b^* = 0$, point A moves from the position of point O to A', point B to B'.

We can see types in Area I are selected out, and types in Areas II and III selected in. Given that at $b^* = 0$, line L_{P2} is an isoquant-return line, the types in Area II have the same subsidy return rate $(V - c - x)/(x - a)$ as those in Area I to the first order effect; that is because the return rates in these two areas are infinitely close to the isoquant line that is line L_{P2} with $b^* = 0$. Given Area III is below this ISO-return line, types in this area all have a higher subsidy return rate than those in Area I. Therefore, the types selected in (i.e., those in Areas II and III) have a subsidy return rate larger than or almost equal to those squeezed out (i.e., types in Area I) do. Therefore, the net effect is an increase in value. That is, the marginal rise in b^* from $b^* = 0$ increases efficiency. Hence,

Proposition 4. *The optimal* $b^* > 0$*; thus, the optimal VM partnership does better than the Simple Partnership.*

From the argument above, a rise in b^* moves point A downwards along line L_{IR} and therefore moves point B upwards. Figure 13 shows the effect of VM partnership when the threshold $b^* > 0$. As b^* rises, which is equivalent to y increasing, point B moves upward on line $a = V - c - Rk_e$, so the area of strip CBFE is getting larger, and given condition (36), the area of triangle BAF is getting smaller. When the budget G is extremely small, we can prove that the return rates of subsidising types in the strip area are larger than those in the triangle area. As a result, the optimal VM partnership should maximise the length of BF, which is equivalent to maximising y. Due to Condition (12), the optimal y is equal to $R-1$, which implies that line AB is flat. That is, the following Proposition 5 holds.

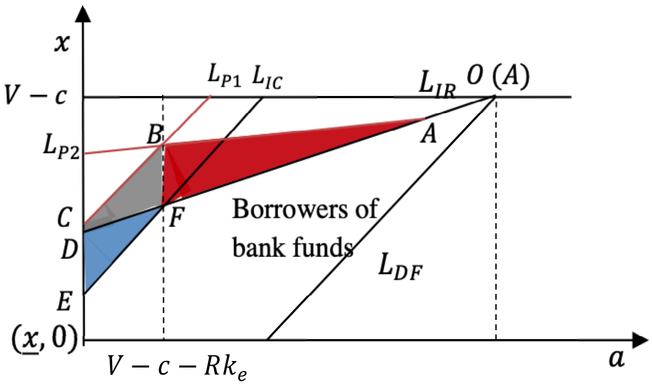


Figure 13: The effect of VM partnership with $b^* > 0$, where A is the intersection of L_{IR} and L_{P2}, B the intersection of L_{P1} and L_{P2}, C the intersection of L_{P1} and line $a = 0$, D be the intersection of L_{IR} and line $a = 0$, E the intersection of L_{IC} and line $a = 0$, and F the intersection of the IC and IR. Points B and F are always on line $a = V - c - Rk_e$ regardless of b^*.

Proposition 5. *If $G \approx 0$, then the optimal $y = R-1$; that is, the optimal VM partnership finances all the funding cost of the part of bank loans that is above the threshold.*

To prove this proposition, as in Figure 13, the SMEs saved by the partnership is are the strip $CBEF$ and triangle ABF. Suppose $G \approx 0$, then B moves infinitely close to F, and C approaches infinitely close to E. This means the strip $CBEF$ is almost 100% immersed in the triangle DEF. Note that the investment return of the SMEs in the triangle DEF is larger than that in the triangle ABF (because IR is an ISO-return line), and therefore, the investment return rate in the strip $CBEF$ is larger than that in the triangle ABF. Therefore, the optimal VM partnership should maximise the area of the strip, which leads to $y = R-1$.

8 Conclusion

When, in a crisis, the government is using a limited budget to save SMEs to survive the crisis, both the government and private banks encounter significant information constraints.

Although the government should target saving those SMEs with a positive NPV but inadequate financial resources to survive, it cannot observe either the NPV of an SME conditional on survival or the necessary surviving cost. Additionally, the government is unable to observe the extent of an SME's self-funding. This paper shows that the government can screen SMEs and enhance its targeting, by forming a partnership with private banks and exploiting information in the SME loan demand. If the government's objective is to protect employment, the optimal partnership is to set a cap and fully fund the funding cost of only those SMEs if and only if their loan demand per worker is no larger than the cap. Conversely, if the objective is to maximise economic efficiency, the government's partnership with private banks should target SMEs whose loan demand is above a threshold, subsidising the funding cost for the part of the loans above the threshold. In either case, the government utilises SMEs' loan demand to deduce the funding shortfall or NPV, which is the target of government support for employment protection or efficiency maximisation, respectively. In general, we believe a sophisticated design of a public support scheme should utilise the information and expertise dispersed within private sectors to improve its efficacy.

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Appendices

A Proof of Lemma 1

An increase in R has the following effects, referring back to Figure 1. Line L_{DF} stays unchanged. Line L_{IC} moves southeastward closer to line L_{DF}, since $k_e = (V - pc_\Delta)/R$ decreases with R. Finally, line L_{IR} rotates anti-clockwise around point O $(x = V - c, a = V - c)$, closer to line L_{DF}, since the intersection of the line with $x-$axis, $\frac{V-c}{R}$ decreases with R. Therefore, the area of borrowing SMEs is diminished and fewer SMEs borrow from banks, and thus, the total demand for loans, D, decreases.

When $R \to \infty$, both lines L_{IC} and L_{IR} converge to L_{DF} and the area of the borrowing SMEs goes to 0, i.e., $D \to 0$. □