Resource Scheduling for Intelligent Reflecting Surface-assisted Full-duplex Wireless Powered Communication Networks with Phase Errors

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Abstract—Intelligent reflecting surface (IRS) is envisioned as a promising technique to improve the performance of full-duplex wireless powered communication networks (FD-WPCNs). This paper investigates the joint phase beamforming design and resource management for IRS-assisted FD-WPCNs, where multiple wireless devices (WDs) can harvest downlink radio-frequency energy and transmit uplink information to the hybrid access point (HAP) over the same band with the aid of IRS. We first formulate a total transmission time minimization problem subject to the minimum transmit rate and energy causality constraints of WDs. In particular, the random phase error of IRS is integrated into our optimization model. Furthermore, we develop an alternating optimization method to obtain the optimal solution of formulated non-convex problem by iteratively solving two subproblems. For the phase beamforming optimization subproblem, we first convert the random phase errors to a deterministic expression, and then utilize the successive convex approximation method to solve the phase beamforming optimization problem. For the transmit power and time-slot allocation subproblem, the optimal transmit power of WDs is derived in closed-form expressions, and the approximation method and variable substitution technique are adopted to obtain the optimal time-slot allocation and transmit power of HAP. Finally, numerical results are provided to evaluate the performance of our proposed method, and reveal the benefits introduced by the IRS technique as compared to benchmark methods.

Index Terms—Intelligent reflecting surface, wireless powered communication networks, full duplex, phase shift errors.

I. INTRODUCTION

Internet of things (IoT) has been proposed as a novel paradigm to accommodate massive connectivity for a large number of low-power wireless devices (WDs), e.g., sensors, wearable devices, and so on [1]–[3]. In general, the limited battery capability of WDs is the main bottleneck for the deployment of various IoT applications, such as smart healthcare, forest fire monitoring, automatic manufacturing, etc [4]. Therefore, it is crucial to extend the lifetime of WDs in a low-cost manner [5]. Wireless powered communication network (WPCN) is thus proposed as an efficient solution to overcome this bottleneck [6]–[8], where the WDs are able to harvest radio-frequency energy from the HAP, and further transmit the uplink information to the HAP by utilizing the harvested energy. Due to the high pathloss of wireless energy transfer, full-duplex technique has been integrated into WPCNs for extending the energy transfer time and increasing the amount of energy harvested by WDs [9].

Meanwhile, intelligent reflecting surface (IRS) has been proposed as an innovative technique to achieve smart and reconfigurable wireless communications in the sixth-generation (6G) era [10]–[15]. In general, IRS is composed by massive passive/active reflection elements, and their amplitudes and phase shifts can be adjusted to enhance the signal strength at intended receivers. Therefore, IRS has great potential to improve the efficiency of wireless energy/information transmissions in WPCNs. Motivated by these observations, this paper studies the optimal design for IRS-assisted full-duplex WPCNs.

A. Related Works

In this subsection, we will review the existing works from three aspects: (1) Resource Allocation for WPCNs; (2) Full-duplex enhanced WPCNs; (3) IRS-assisted WPCNs.
1) Resource Allocation for WPCNs: Wireless powered communication network is recognized as a key technique to support sustainable communications in 6G era [16]. Due to the coupled energy and information transmissions, resource allocation plays an important role in improving the performance of WPCNs [17]. In [18], the authors first proposed a "Harvest-then-Transmit" protocol to balance downlink energy transfer and uplink information reception at the HAP, and they maximized the system throughput by proposing a resource allocation strategy. Wu et al. in [19] investigated the joint time allocation and power control method to maximize the system energy efficiency of WPCNs. In [20], Lee et al. focused on maximizing the total uplink rate for WPCNs considering finite energy storage. In [21], Xiong et al. studied the user cooperation strategy for achieving "Win-Win" resource sharing in WPCNs. In [22], the authors presented the robust resource management strategy for WPCNs with non-linear energy harvesting model and imperfect channel state information. To improve the performance of WPCNs, Xie et al. in [23] proposed an unmanned aerial vehicle (UAV)-assisted WPCN, where the UAV serves as a flying access point to provide energy/information transmission services for ground wireless devices.

2) Full-duplex enhanced WPCNs: In-band full duplex technique has been envisioned as a promising solution to improve the spectrum efficiency of wireless networks [24]. In recent years, some research works proposed to utilize the full-duplex technique to improve the efficiency of energy/information transmissions in WPCNs. In [25], Ju et al. first investigated the resource allocation method for full-duplex WPCNs, where the HAP is able to transmit the downlink energy signals and receive uplink information signals simultaneously. In [26], Kang et al. presented resource management strategies for maximizing the throughput and minimize the transmission latency in full-duplex assisted WPCNs, considering the energy causality constraints. In [27], the authors concentrated on maximizing the secrecy rate for full-duplex-assisted WPCNs with multiple potential eavesdroppers. In [28], [29], Iqbal et al. formulated the minimum length scheduling problem via jointly optimizing the transmit power and time allocation for full-duplex WPCNs. The authors in [30] investigated the joint beamforming and resource allocation strategy for full-duplex WPCNs, in which the HAP is equipped with multiple antennas for improving the efficiency of energy transfer and information reception.

3) IRS-assisted WPCNs: In recent years, IRS has attracted extensive attention from academia due to its ability to reconstruct wireless propagation environments in a low-cost manner. Therefore, IRS has great potential to improve the performance of WPCNs [31], [32]. Wu et al. in [33] first introduced the IRS into WPCNs for improving the efficiency of wireless energy and information transmissions. In [34], the authors focused on maximizing the system throughput for IRS-assisted WPCNs. Due to the hardware restriction and finite resolution of IRS, the adjustable phase shift of IRS is generally discrete. Different from the existing works considering continuous phase shifts, Chu et al. in [35] investigated the joint wireless resource and discrete phase shift optimization problem for IRS-assisted WPCNs. Xu et al. in [36] focused on the energy efficiency maximization problem for an IRS-assisted multi-antenna WPCNs. In [37], Li et al. presented a robust beamforming and resource allocation method for IRS-assisted WPCNs by taking account of the imperfect channel state information. In [38], Lyu et al. investigated the optimal design for self-sustainable IRS-assisted WPCNs, where the IRS first harvests energy from downlink energy signals, and then utilizes its reflection link gain to improve the efficiency of energy/information transmissions between HAP and IoT devices. To improve the spectral efficiency, Hua et al. in [39] integrated the full-duplex technique into IRS-assisted WPCNs, and they further studied the joint phase beamforming and resource scheduling strategy to maximize the system throughput.

Above works generally considered an ideal scenario without hardware impairments at IRS, which is an unrealistic assumption due to harmful noises, quantization errors, etc. In recent year, some research literature investigated the robust optimization strategy for IRS-assisted communication systems [40], [41]. In [40], Li et al. analyzed the ergodic capacity for IRS-assisted communication systems with phase errors, and they modelled the phase error as a random variable that follows uniform distribution at the interval \((-\frac{\pi}{2}, \frac{\pi}{2})\). Chu et al. in [41] revealed the impact of transceiver hardware impairments and phase errors of IRS on the throughput performance for IRS-assisted WPCNs.

To sum up, the resource allocation problems for WPCNs have been extensively studied in existing literatures [18]–[23]. Besides, IRS and full-duplex technique were also exploited to improve the efficiency of information and energy transmissions [24]–[39]. Nevertheless, the following issues need to be further addressed:

- Existing works generally focused on the optimization of sum transmission rate or energy efficiency for IRS-assisted WPCNs. The total transmission time is another vital performance indicator, especially in the time-critical applications, such as emergency alert networks. In the literature, it lacks of study to minimize the total transmission time for IRS-assisted WPCNs.
- The full-duplex technique were seldom used to reduce the total transmission time for IRS-assisted WPCNs. Due to the simultaneous energy and information transmissions introduced by full-duplex technique, thus it is a promising solution to reduce the total transmission time by integrating the full-duplex technique into IRS-assisted WPCNs. However, it is difficult to tackle the residual self-interference signals and highly coupled optimization variables in the resource allocation problem for IRS-assisted full-duplex WPCNs.
- In practical IRS-assisted networks, the phase errors of IRS will make a negative impact on the network performance. However, existing works generally assume the perfect phase shift model. Hence, it is essential to optimize the performance of IRS-assisted full-duplex WPCNs considering the practical phase errors at the IRS.
B. Novelty and Contribution

To fill the gap in existing works, this paper investigates the joint resource allocation and reflection optimization method for IRS-aided full-duplex WPCNs with phase errors. The contribution of this paper is outlined as follows.

- We formulate a total transmission time minimization problem subject to the transmission rate and energy causality constraints of WDs, to jointly optimize time-slot allocation, transmit power control of wireless devices and HAP, and phase beamforming of IRS. Different from the existing works, the phase error of IRS is considered in the formulated problem, and it is modeled as a random variable that follows the uniform distribution.

- To address the formulated non-convex and non-linear optimization problem, we further develop an alternating optimization method to iteratively solve two subproblems. For the phase beamforming optimization subproblem, we first transform the stochastic phase errors to a deterministic form, and then utilize the success convex approximation technique to design an iterative optimization algorithm for obtaining the optimal phase shift matrix of IRS. For the time-slot allocation and transmission power control subproblem, we first derive the closed-form solution for the optimal transmission power of WDs, and further design an iterative algorithm to obtain the optimal time-slot allocation and transmit power of HAP by exploiting the variable substitution technique and approximation method.

- In simulations, we show that the proposed method can realize significantly lower total completion time than the Without IRS scheme and Random phase shift scheme, which verifies the necessity to deploy IRS with the optimal phase shift design in full-duplex WPCNs. Moreover, we also reveal that the proposed iterative method will converge to the optimal solution within several iterations, and the proposed method with phase errors and energy causality constraints only requires slightly longer transmission time than the Without energy causality scheme and Without phase errors scheme, which demonstrates the effectiveness of our proposed method.

The remainder of this paper is organized as follows. Section II introduces the system model and transmission protocol for IRS-assisted FD-WPCNs with phase errors. Section III presents the total transmission time minimization problem and the corresponding solution. Section IV illustrates the benchmark methods. Section V provides the simulation results to validate the performance of the proposed method. We conclude this paper in Section VI.

Notations: Hereinafter, lower-case letter, boldface lower-case letter, and boldface upper-case letter stand for scalar, vector and matrix, respectively. For a vector $x$, $|x|$, $x^T$, $\text{conj}(x)$, $x^H$ and $\text{diag}(x)$ denote its module, transpose, conjugate, hermitian transpose and diagonalization, respectively. For a matrix $X$, $\text{Tr}(X)$ and $\text{Rank}(X)$ indicate its trace and rank, respectively. $E_x(X)$ represents the expectation of $X$ with respect to the random variable $x$. $1_{M \times 1}$ denotes the identity column vector with all elements of 1.

II. SYSTEM MODEL

As illustrated in Fig. 1(a), this paper considers an IRS-assisted full-duplex WPCN, consisting of a single-antenna hybrid access point (HAP), an IRS with $M$ reflection elements, and $K$ single-antenna wireless devices (WDs). The sets of reflection units and WDs are indicated by $M = \{1, 2, \cdots, M\}$ and $K = \{1, 2, \cdots, K\}$, respectively. In addition, it is assumed that the WDs have no stable energy source and have to harvest radio-frequency (RF) energy from HAP. To improve the spectral efficiency, the HAP works in full-duplex mode for supporting simultaneous uplink information reception and downlink energy transfer over the same band. Moreover, considering the limited size and signal processing capability, the WDs operate in time division duplex mode to avoid harmful co-channel interference. We denote $g_{d,k}$, $g_{r}$ and $g_{l,k}$ as the uplink channels from the $k$-th WD to the HAP, from the IRS to the HAP, and from the $k$-th WD to the IRS, respectively. Besides, the counterpart downlink channels are represented as $h_{d,k}$, $h_{r}$ and $h_{l,k}$. It is assumed that all the considered channels follow block-based flat fading, i.e., the channel coefficient keeps fixed during the current time block but may change at the boundaries of time blocks. To reveal the lower bound of total transmission time, the perfect channel state information (CSI) can be obtained at the HAP. In general, the direct channels between the HAP and WDs can be estimated via traditional methods by setting the IRS into the absorbing state. Besides, we can measure the CSI of cascade links between the IRS and HAP/WDs by equipping a small number of low-power radio-frequency chains [42]. The main symbols used in this paper are illustrated in Table I for reference.

The transmission protocol is illustrated in Fig. 1(b), where the first time slot is dedicated for downlink RF energy transfer, while the other time slots are used for both uplink information transmission and downlink energy transfer at the same band simultaneously. In particular, this paper considers the energy causality constraints of WDs, i.e., the $k$-th WD can only utilize the harvested energy before $\tau_k$ to transmit its information to the HAP. Therefore, the available energy of the $k$-th WD is expressed as

$$E_{H,k} = \eta_k \sum_{i=0}^{k-1} \tau_i P_{A,i} \| h_{i,k}^H \tilde{\Gamma}_i h_r + h_{d,k} \|^2, \forall k \in K,$$

where $\eta_k \in (0, 1)$ denotes the linear energy conversion efficiency, $P_{A,i}$ stands for the downlink transmission power of the HAP during $\tau_i$, and $\tilde{\Gamma}_i = \text{diag}\{e^{j(\theta_{i,1} + \theta_{E,1})}, e^{j(\theta_{i,2} + \theta_{E,2})}, \cdots, e^{j(\theta_{i,M} + \theta_{E,1})}\}$ represents the actual phase beamforming matrix of the IRS during $\tau_i$, where $\theta_{i,m}$ and $\theta_{E,i,m}$ denote the phase shift

\[1\]This paper assumes that the phase shift of IRS can be arbitrarily chosen from $\{0, 2\pi\}$, in order to reveal the lower bound of total transmission time for the considered system. In fact, the phase shift of IRS can only be chosen from a given discrete set due to the limited resolution of phases. This paper will provide a guidance for the future work considering discrete phase shifts of IRS.
and the additive random phase errors, respectively. Similar to [41], [43], it is assumed that the phase errors follows the uniform distribution, i.e., $\theta_{E,i,m} \sim \mathcal{U}(-\frac{\pi}{2}, \frac{\pi}{2})$. Moreover, this paper considers a linear energy harvesting model, and the method proposed in this work can be directly applied in the scenario with non-linear energy harvesting model. Utilizing the harvested RF energy, the $k$-th WD will transmit its own information to the HAP during $\tau_k$. Due to the existence of residual self-interference at the HAP, the achievable transmission rate of $k$-th WD is given by

$$R_k = \tau_k \log_2 \left(1 + \frac{P_k |\hat{g}_k^H \hat{e}_k + g_{d,k}|^2}{\delta^2 + \gamma P_{A,k}}\right), \forall k \in \mathcal{K},$$  

where $P_k$ denotes the transmission power of the $k$-th WD, $\gamma$ is a factor to measure the residual self interference at the HAP, and $\delta^2$ stands for the Gaussian noise power. It should be noted that the interference item generated by the downlink energy signal reflected by IRS is ignored in this paper, since its power is significantly smaller than that of the residual self-interference at the HAP due to the double pathloss [39].

### III. Problem Formulation

This work focuses on minimizing the total completion time for downlink energy transfer and uplink information transmissions of all WDs, through joint optimization of time-slot allocation \(\{\tau_k, \forall k \in \{0, \mathcal{K}\}\}\), phase beamforming \(\{\hat{\Gamma}_k, \forall k \in \{0, \mathcal{K}\}\}\) of the IRS, downlink transmission power \(\{P_{A,k}, \forall k \in \{0, \mathcal{K}\}\}\) of HAP, and uplink transmission power \(\{P_k, \forall k \in \mathcal{K}\}\) of wireless devices. The total transmission time minimization problem can be formulated as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{\mathcal{K}} \tau_k \\
\text{s.t.} & \quad R_{k, \min} \leq \tau_k, \forall k \in \mathcal{K}, \\
& \quad P_k \tau_k \leq \eta_k \sum_{i=0}^{k-1} P_{A,i} \tau_i |\hat{h}_{f,k}^H \hat{r}_i + h_{d,k}|^2, \\
& \quad \forall k \in \mathcal{K}, \\
& \quad 0 \leq \theta_{k,m} \leq 2\pi, \forall k \in \{0, \mathcal{K}\}, \forall m \in \mathcal{M}, \\
& \quad 0 \leq P_{A,k} \leq P_{A,\max}, \forall k \in \{0, \mathcal{K}\}, \\
& \quad P_k \geq 0, \tau_k \geq 0, \tau_0 \geq 0, \forall k \in \mathcal{K},
\end{align*}
\]

where C1 implies that the achievable transmission rate of WDs should be larger than their minimum rate requirements, C2 restricts that the communication energy consumption of each WD cannot exceed the harvested RF energy before its transmission time slot, C3 represents the phase shift constraint of reflection elements integrated on the IRS, and C4 restricts the transmit power at the HAP.

As observed, (3) is a non-linear and non-convex optimization problem due to the coupled variables among transmit power, time, and phase beamforming matrix. Moreover, the randomness of phase errors make the problem more difficult to tackle. To deal with this problem, we will design an alternating optimization method to decouple the original problem into two subproblems, i.e., phase beamforming optimization subproblem, and time-slot allocation and transmit power control subproblem. For the phase beamforming subproblem, the random phase error is first transformed to a deterministic form, and then we utilize the successive convex approximation method to obtain the optimal phase shifts of IRS. For the time-slot allocation and transmit power control subproblem, we first derive the optimal transmit power of WDs in closed-form expressions. Then, we rewrite the non-convex transmit rate expression to a more tractable convex form, and design
an iterative method to obtain the optimal time allocation and transmit power control of HAP. The detailed flowchart for solving (3) is illustrated in Fig. 2.

**A. Phase Beamforming Optimization Subproblem**

Given \(\{\tau^*_k, P_{A,k}, P^*_k\}\), (3) is simplified to the following phase beamforming optimization subproblem

\[
\text{find } \{\Gamma_k\} \\
\tau^*_k \text{ log}_2 \left( 1 + \frac{P^*_k |\mathbf{g}^H \Gamma_k \mathbf{g}_k + g_{d,k}|^2}{\delta^2 + \gamma P^*_k} \right) \geq R_{k,\min}, \quad \forall k \in \mathcal{K}, \\
P^*_k \tau^*_k \leq \eta_k \sum_{i=0}^{k-1} P^*_{A,i} \tau^*_{i} |\mathbf{h}^H \Gamma_k \mathbf{h}_i + h_{d,k}|^2, \quad \forall k \in \mathcal{K}, \quad \text{C3.} 
\]

Since \(\hat{\Gamma}_k = \Gamma_k \Gamma_{E,k}\) with \(\Gamma_k = \text{diag}\{e^{j\theta_{k,1}}, e^{j\theta_{k,2}}, \ldots, e^{j\theta_{k,M}}\}\) and \(\Gamma_{E,k} = \text{diag}\{e^{j\Theta_{E,k,1}}, e^{j\Theta_{E,k,2}}, \ldots, e^{j\Theta_{E,k,M}}\}\), we can derive that

\[
ak_k = |\mathbf{g}^H \Gamma_k \mathbf{g}_k + g_{d,k}|^2 \\
= |\mathbf{g}^H \Gamma_{E,k} \Gamma_{E,k} \mathbf{g}_k + g_{d,k}|^2 \\
= \mathbf{v}^H_k \mathbf{A}_k \text{conj}(\mathbf{v}_{E,i}) \mathbf{v}^T_{E,i} \mathbf{A}_k^H \mathbf{v}_k + \mathbf{v}^H_k \mathbf{A}_k \text{conj}(\mathbf{v}_{E,i}) g_{d,k} + g_{d,k}^2 \mathbf{v}^H_k \mathbf{A}_k \mathbf{v}_k + g_{d,k}^2 g_{d,k},
\]

and

\[
b_{k,i} = |\mathbf{h}^H \Gamma_k \mathbf{h}_i + h_{d,k}|^2 \\
= |\mathbf{h}^H \Gamma_{E,k} \Gamma_{E,k} \mathbf{h}_i + h_{d,k}|^2 \\
= \mathbf{v}^H_k \mathbf{B}_k \text{conj}(\mathbf{v}_{E,i}) \mathbf{v}^T_{E,i} \mathbf{B}_k^H \mathbf{v}_i + \mathbf{v}^H_k \mathbf{B}_k \text{conj}(\mathbf{v}_{E,i}) h_{d,k} + h_{d,k}^2 \mathbf{v}^H_k \mathbf{B}_k \mathbf{v}_i + h_{d,k}^2 h_{d,k},
\]

where \(\mathbf{v}_k = [e^{j\Theta_{k,1}}, \ldots, e^{j\Theta_{k,M}}]^T\), \(\mathbf{v}_{E,k} = [e^{j\Theta_{E,k,1}}, \ldots, e^{j\Theta_{E,k,M}}]^T\), \(\mathbf{A}_k = \text{diag}(\mathbf{g}^H_k \mathbf{g}_k)\text{diag}(\mathbf{g}_k)\) and \(\mathbf{B}_k = \text{diag}(\mathbf{h}^H_k)\text{diag}(\mathbf{h}_k)\). Introducing (5)-(6) into (4), the phase beamforming optimization subproblem will be converted as

\[
\text{find } \{\mathbf{v}_k\} \\
\tau^*_k \text{ log}_2 \left( 1 + \frac{P^*_k \mathbb{E}_{\mathbf{v}_{E,k}}(a_k)}{\delta^2 + \gamma P^*_k} \right) \geq R_{k,\min}, \quad \forall k \in \mathcal{K}, \\
P^*_k \tau^*_k \leq \eta_k \sum_{i=0}^{k-1} P^*_{A,i} \tau^*_{i} \mathbb{E}_{\mathbf{v}_{E,i}}(b_{k,i}), \forall k \in \mathcal{K}, 
\]

where \(\mathbb{E}_{\mathbf{v}_{E,k}}(a_k)\) and \(\mathbb{E}_{\mathbf{v}_{E,i}}(b_{k,i})\) can be rewritten as

\[
a_k = \mathbb{E}_{\mathbf{v}_{E,k}}(a_k) \\
= g_{d,k}^H g_{d,k} + \mathbf{v}_k^H \mathbf{A}_k \mathbb{E}_{\mathbf{v}_{E,k}}(\text{conj}(\mathbf{v}_{E,k})) \mathbf{A}_k^H \mathbf{v}_k + \mathbf{v}_k^H \mathbf{A}_k \mathbb{E}_{\mathbf{v}_{E,k}}(\text{conj}(\mathbf{v}_{E,k})) g_{d,k} + g_{d,k}^2 \mathbf{v}_k^H \mathbf{A}_k \mathbf{v}_k \\
= g_{d,k}^H g_{d,k} + \mathbf{v}_k^H \mathbf{A}_k \mathbf{R} \mathbf{A}_k^H \mathbf{v}_k + 2 \mathbf{v}_k^H \mathbf{A}_k g_{d,k} + 2 \mathbf{v}_k^H \mathbf{A}_k \mathbf{v}_k \\
\]

(8)

and

\[
b_{k,i} = \mathbb{E}_{\mathbf{v}_{E,i}}(b_{k,i}) \\
= h_{d,k}^H h_{d,k} + \mathbf{v}_i^H \mathbf{B}_k \mathbb{E}_{\mathbf{v}_{E,i}}(\text{conj}(\mathbf{v}_{E,i})) \mathbf{B}_k^H \mathbf{v}_i + \mathbf{v}_i^H \mathbf{B}_k \mathbb{E}_{\mathbf{v}_{E,i}}(\text{conj}(\mathbf{v}_{E,i})) h_{d,k} + h_{d,k}^2 \mathbf{v}_i^H \mathbf{B}_k \mathbf{v}_i \\
= h_{d,k}^H h_{d,k} + \mathbf{v}_i^H \mathbf{B}_k \mathbf{R} \mathbf{B}_k^H \mathbf{v}_i + 2 \mathbf{v}_i^H \mathbf{B}_k h_{d,k} + 2 \mathbf{v}_i^H \mathbf{B}_k \mathbf{v}_i \\
\]

(9)

respectively, where

\[
\mathbf{R} = \begin{bmatrix} 1 & \frac{1}{\pi^2} & \ldots & \frac{1}{\pi^2} \\ \frac{1}{\pi^2} & 1 & \ldots & \frac{1}{\pi^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\pi^2} & \frac{1}{\pi^2} & \ldots & 1 \end{bmatrix}_{M \times M} 
\]

(10)

**Proof:** See Appendix A.

According to Lemma 1, (7) will be rewritten as

\[
\text{find } \{\mathbf{v}_k\} 
\]

(11a)
After some matrix transformations, (11) is equivalent to the following problem

\[
\begin{align*}
\text{minimize} & \quad \{ \tilde{v}_k \} \\
\text{s.t.} & \quad \tau_k^* \log_2 \left( 1 + \frac{P_k^* \hat{a}_k}{\delta^2 + \gamma P_{A,k}} \right) \geq R_{k,\min}, \quad \forall k \in \mathcal{K}, \\
& \quad P_k^* \tau_k^* \leq \eta k \sum_{i=0}^{k-1} P_{A,i}^* \tau_i^* \hat{b}_{k,i}, \quad \forall k \in \mathcal{K}, \\
& \quad \text{C3}.
\end{align*}
\]

By rewriting the convex eigenvalue function \( \lambda_{\max}(\tilde{V}_k) \) as its first Taylor approximation, (16) is transformed as

\[
\begin{align*}
\text{minimize} & \quad \{ \tilde{v}_k \} \\
\text{s.t.} & \quad (15b)-(15e), (16b), \quad \forall k \in \mathcal{K}.
\end{align*}
\]

Then, the non-convex rank-one constraint (15f) will be removed. In general, the trace of semi-definite matrix \( \tilde{V}_k \) should be no less than its maximum eigenvalue, namely \( \text{Tr}(\tilde{V}_k) \geq \lambda_{\max}(\tilde{V}_k) \), and the equality holds when \( \text{Rank}(\tilde{V}_k) = 1 \). Hence, we can minimize \( \sum_{k=0}^{K} \text{Tr}(\tilde{V}_k) - \lambda_{\max}(\tilde{V}_k) \) in the objective function of (15) and drop the rank-one constraint (15f), to reformulate (15) as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{K} \text{Tr}(\tilde{V}_k) \quad \text{subject to} \quad \tau_k \log_2 \left( 1 + \frac{P_k^* \hat{a}_k}{\delta^2 + \gamma P_{A,k}} \right) \geq R_{k,\min}, \quad \forall k \in \mathcal{K}, \quad (16a)
\end{align*}
\]

\[
\begin{align*}
\text{s.t.} & \quad (15b)-(15e), \quad (16b), \quad \forall k \in \mathcal{K}.
\end{align*}
\]

We summarize the whole procedure for solving (4) in Algorithm 1.

**Algorithm 1: SCA-based method for solving the phase beamforming optimization subproblem (4)**

1. **Initialize:** Set \( \{ \tilde{V}_0 \} = \{ \tilde{V}_0^{(0)}, \tilde{V}_1^{(0)}, \cdots, \tilde{V}_K^{(0)} \} \), \( \{ \tilde{v}_{0,\max}^{(0)}, \tilde{v}_{1,\max}^{(0)}, \cdots, \tilde{v}_{K,\max}^{(0)} \} \), and \( i = 1 \).
2. **Repeat:**
   3. Solving (18) by utilizing the convex toolbox, such as CVX or Yalmip, to obtain optimal \( \{ \tilde{V}_k \} \);
   4. Setting \( \tilde{V}_k^{(i)} = \tilde{V}_k^{(i-1)} \);
   5. Calculating the maximum eigenvalue \( \lambda_{\max}(\tilde{V}_k^{(i)}) \) and the corresponding eigenvector \( \tilde{v}_{k,\max}^{(i)} \);
   6. Updating the iterative number \( i = i + 1 \);
3. **Until Convergence.**

8. **Return** optimal phase beamforming matrix \( \{ \tilde{V}_k \} \).

### B. Time-Slot Allocation and Transmission Power Control

Under given \( \{ \tilde{g}_i^k \} \), (3) will be reduced to the following problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{K} \tau_k \\
\text{s.t.} & \quad \tau_k \log_2 \left( 1 + \frac{P_k^* g_{i,k} \tilde{g}_i^k + g_{d,k}}{\delta^2 + \gamma P_{A,k}} \right) \geq R_{k,\min}, \quad \forall k \in \mathcal{K}, \quad (20a)
\end{align*}
\]
where $\alpha > 0$ and $\beta > 0$, we can construct another solution \{\(\tau_0^+, \tau_j^+, P_{A,k}^+, P_{d,k}^+\)\} satisfying $\tau_0^+ = \tau_0^-$, $\tau_j^+ = \beta \tau_j^-$, $\tau_k^+ = \tau_k^-$, $P_{A,k}^+ = P_{A,k}^-$, $P_{d,k}^+ = \alpha P_d^+$, and $P_{d,k}^+ = P_d^+$, where $\alpha > 1$ and $\beta < 1$, which satisfies $\beta \tau_j^+ \log_2 \left( 1 + \frac{\alpha P_d^+ |h_{f,j}^H \hat{f}_j^* h_r + g_{d,j}^2|^2}{\delta^2 + \gamma A,k} \right) \geq R_{t,\text{min}}$ and $\alpha \beta P_d^+ \tau_j^+ \leq \eta_j \sum_{i=0}^{k-1} P_{A,i}^+ \tau_i^+ |h_{f,j}^H \hat{f}_j^* h_r + g_{d,j}^2|^2$. First, it is easy to verify that \{\(\tau_0^+, \tau_j^+, P_{A,k}^+, P_{d,k}^+\)\} still satisfies constraints (20b)-(20d), and we have $\sum_{k=0}^{K} \tau_k^+ = \beta \tau_j^+ + \sum_{k=0}^{K} \tau_k^+ < \sum_{k=0}^{K} \tau_k^+$, which contradicts with the assumption. Theorem 1 is thus proved.

Based on Theorem 1, (20) will be simplified as the following problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{K} \tau_k \\
\text{s.t.} & \quad \tau_k \log_2 \left( 1 + \frac{\sum_{i=0}^{k-1} r_{i,k} P_{A,i} \tau_i}{\tau_k (\delta^2 + \gamma A,k)} \right) \geq R_{t,\text{min}}, \forall k \in \mathcal{K}, \\
& \quad 0 \leq P_{A,k} \leq P_{A,\text{max}}, \forall k \in \{0, \mathcal{K}\}, \\
& \quad \tau_k \geq 0, \tau_0 \geq 0, \forall k \in \mathcal{K},
\end{align*}
\]  

where $r_{i,k} = \eta_k |h_{f,i}^H \hat{f}_i^* h_r + h_{d,k}^2|^2 |g_{f,i}^H \hat{f}_i^* g_{d,k} + g_{d,k}^2|^2$. To decouple the time and power variables, we introduce a set of auxiliary variables $\epsilon_{A,k} = P_{A,k} \tau_k$ to reformulate (22) as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{K} \tau_k \\
\text{s.t.} & \quad \tau_k \log_2 \left( 1 + \frac{\sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i} \tau_i}{\tau_k (\delta^2 + \gamma A,k)} \right) \geq R_{t,\text{min}}, \forall k \in \mathcal{K}, \\
& \quad 0 \leq \epsilon_{A,k} \leq \tau_k P_{A,\text{max}}, \forall k \in \{0, \mathcal{K}\}, \\
& \quad \tau_k \geq 0, \tau_0 \geq 0, \forall k \in \mathcal{K},
\end{align*}
\]

As observed, (23) is nonconvex due to the constraint (23b). The following Lemma 2 will be proposed to tackle it.

**Lemma 2:** Defining $f(y) = -y x + \ln(y) + 1$, we have

\[
-\ln x = \max_{y>0} f(y).
\]

The equality holds when $y = 1/x$. Defining $x = \tau_k \delta^2 + \gamma \epsilon_{A,k}$ and $y = y_k^{(i)}$, $R_{k}(\hat{\Gamma}_k^*)/\tau_k$ can be rewritten as

\[
\begin{align*}
& \quad \frac{R_k(\hat{\Gamma}_k^*)}{\tau_k} = \log_2 \left( 1 + \frac{\sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i} \tau_i}{\tau_k (\delta^2 + \gamma A,k)} \right) \\
& \quad = \frac{1}{\ln 2} \left( \ln(\tau_k \delta^2 + \gamma \epsilon_{A,k} + \sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i}) - \ln(\tau_k \delta^2 + \gamma \epsilon_{A,k}) \right) \\
& \quad = \frac{1}{\ln 2} \left( \ln(\tau_k \delta^2 + \gamma \epsilon_{A,k} + \sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i}) - y_k^{(i)} (\tau_k \delta^2 + \gamma \epsilon_{A,k}) \right) \\
& \quad + \ln(y_k^{(i)} + 1).
\end{align*}
\]

Therefore, (23) will be converted as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=0}^{K} \tau_k \\
\text{s.t.} & \quad \frac{1}{\ln 2} \left( \ln(\tau_k \delta^2 + \gamma \epsilon_{A,k} + \sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i}) - y_k^{(i)} (\tau_k \delta^2 + \gamma \epsilon_{A,k}) \right) \\
& \quad \geq \frac{R_{k,\text{min}}}{\tau_k}, \forall k \in \mathcal{K},
\end{align*}
\]

**Theorem 2:** The time allocation and transmit power control subproblem (26) is a convex optimization problem.

**Proof:** $\ln(\tau_k \delta^2 + \gamma \epsilon_{A,k} + \sum_{i=0}^{k-1} r_{i,k} \epsilon_{A,i})$ is a concave function with respect to $\{\tau_k, \epsilon_{A,k}\}$. In addition $\frac{R_{k,\text{min}}}{\tau_k}$ is a convex function of $\tau_k$. Therefore, (26b) is the typical convex constraint. Integrated with the line objective function and constraints, we will prove that (26) is convex.

Therefore, we can adopt some convex optimization toolboxes to solve (26). The detailed procedure for solving (20) is illustrated in the following Algorithm 2.

**C. Algorithm Procedure and Complexity Analysis**

In summary, the alternating optimization method for solving the total completion time minimization problem (3) is presented in Algorithm 3.
Algorithm 2: Proposed method for solving time-slot allocation and power control problem (20)

1. Initialize: Set \( \{ r_k^{(i)}, P_{A,k}^{(i)}, P_k^{(i)} \} \), and \( i = 1 \).
2. Repeat:
   3. Computing \( P_k^{(i)} \) according to (21);
   4. Calculating \( y_k^{(i)} = \frac{1}{\tau_k^{(i)} + 1} \tau_k^{(i)} P_{A,k}^{(i)} \);
   5. Acquiring the optimal \( \{ r_k^{(i)}, e_k^{(i)} \} \) by solving (26);
   6. Computing \( P_k^{(i)} = \frac{y_k^{(i)}}{2} \);
   7. Updating the iteration factor \( i = i + 1 \);
8. Until convergence.
9. Obtaining optimal solution \( \{ r_k^*, P_{A,k}^*, P_k^* \} \).

Algorithm 3: Alternating optimization method for solving the total completion time minimization problem (3)

1. Initialize: Set \( \{ r_k^{(0)}, P_{A,k}^{(0)}, P_k^{(0)}, V_k^{(0)} \} \), and \( n = 1 \).
2. Repeat:
   3. Adopting Algorithm 1 to obtain the optimal phase beamforming matrix \( V_k^{(n)} \).
   4. Computing the optimal time-slot allocation and power control strategy \( \{ r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)} \} \) by executing Algorithm 2;
   5. Updating the iteration factor \( n = n + 1 \);
6. Until convergence.
7. Return: \( (r_k^*, P_{A,k}^*, P_k^*, V_k^*) = (r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)}, V_k^{(n)}) \).

Theorem 3: Algorithm 3 can converge to the optimal solution after several iterations.

Proof: Denoting \( T_{tot}(r_k, P_{A,k}, P_k, V_k) \) as the objective function of (3) and \( n \) as the iteration index, we will prove the convergence of Algorithm 3 as follows. In the step 3 of Algorithm 3, since \( \{ V_k^{(n+1)} \} \) is the suboptimal phase beamforming strategy under given \( \{ r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)} \} \), we have

\[
T_{tot}(r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)}, V_k^{(n)}) \geq T_{tot}(r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)}, V_k^{(n+1)}).
\]

In the step 4 of Algorithm 3, given \( \{ V_k^{(n+1)} \} \), \( \{ r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n+1)} \} \) is the suboptimal time-slot allocation and transmit power control scheme of (20), which satisfies

\[
T_{tot}(r_k^{(n)}, P_{A,k}^{(n)}, P_k^{(n)}, V_k^{(n+1)}) \geq T_{tot}(r_k^{(n+1)}, P_{A,k}^{(n+1)}, P_k^{(n+1)}, V_k^{(n+1)}).
\]

Hence, \( T_{tot}(r_k, P_{A,k}, P_k, V_k) \) is non-increasing versus the iteration index. Besides, the total transmission time is limited. According to Cauchy theory, we derive that Algorithm 3 will converge to an optimal value after a finite number of iterations.

In each iteration, the computational complexity of Algorithm 3 mainly includes the complexity for solving two subproblems.

- For the phase beamforming optimization subproblem, the complexity for solving convex problem (18) will be \( O((3K+1+K(M+1)^2+(K+1)(M+1))(K(M+1)^2)^2) \) with \( K(M+1)^2 \) variables and \( 3K+1+(K+1)(M+1) \) constraints, where \( \epsilon_1 \) denotes the tolerance factor for solving (18). In addition, the complexity for calculating eigenvalues and eigenvectors of \( \{ V_k \} \) will be \( O((K+1)(M+1)^3) \). Therefore, the total complexity of Algorithm 1 is given by \( O((3K+1+K(M+1)^2+(K+1)(M+1))(K(M+1)^2)^2) \) with \( 2(K+1) \) constraints and \( 3K+2 \) constraints, where \( \epsilon_2 \) stands for the tolerance factor for solving (26). The total complexity for Algorithm 2 is given by \( O(4(5K+4)(K+1)^2\sqrt{3K+2}+2\log(1/\epsilon_2)) \).

In summary, the complexity of Algorithm 3 is \( O(4(5K+4)(K+1)^2\sqrt{3K+2}+2\log(1/\epsilon_2)) \).

IV. SIMULATION RESULTS

This section illustrates the simulation results to validate the superiority of the proposed approach in terms of the total transmission time for IRS-assisted full-duplex WPCNs, as compared to several benchmark methods, i.e., Without phase errors scheme, Without IRS scheme, Without energy causality scheme, and Random phase shift scheme. It should be noted that the benchmark methods are illustrated in Appendix B. The simulation parameters are set as follows: \( M = 60 \), \( K = 2 \), \( \delta^2 = 10^{-9} \) W, \( u_k = 0.8 \), \( \gamma = 10^{-12} \), \( P_{A,k} = 5 \) W, and \( R_{k,\text{min}} = 0.1 \) bits/Hz. The coordinates of HAP, IRS, and WDs are \( [0, 0] \), \( [0, 3] \), and \( [2, 3] \), respectively.

In simulations, the channel coefficient is modeled as

\[
C_h = \sqrt{P_u d^{-\alpha}}[\sqrt{\frac{M_r}{M_r+1}C_r^{\text{LoS}}} + \sqrt{\frac{1}{M_r+1}C_r^{\text{NLoS}}}],
\]

where \( P_u = 10^{-2} \) is the path-loss of unit distance, \( d \) stands for the distance among communication nodes, \( \alpha = 2 \) represents the path-loss factor, \( M_r = 3 \) denotes the Ricean factor, and \( C_r^{\text{LoS}} \) and \( C_r^{\text{NLoS}} \) indicate the Line-of-Sight (LoS) and Rayleigh fading components, respectively.

In Fig. 3, we illustrate the total completion time against the transmission rate requirements of WDs. As observed, the total transmission time yields an increasing trend with \( R_{k,\text{min}} \) for all schemes. This is because that the increase of \( R_{k,\text{min}} \) will inevitably lead to extend the information/energy transmission time for satisfying strict transmission rate requirements. Moreover, the proposed method outperforms the Without IRS scheme and the Random phase shift scheme. This observation
confirms that the optimization design of IRS exhibits an important role in reducing the total transmission time for IRS-assisted full-duplex WPCNs. Besides, the perfect case without phase errors provides a lower total completion time compared to the proposed method with phase errors, which means that the phase errors have a negative impact on the system performance.

Fig. 4 shows the relationship between the total completion time and path-loss factor of direct links. From this figure, we find that the total completion time of all schemes exhibits a declining trend with the path-loss factor of direct links. This is due to the fact that more severe channel fading will lead to a lower energy/information transmission efficiency, and it will further result in the increase of total completion time. Meanwhile, the time reduction achieved by the proposed method increases with the path-loss factor of direct links, as compared with the Without IRS scheme. Because that the cascade links aided by the IRS become a good choice to improve the efficiency of information/energy transmissions, when the direct links undergo deep fading. In addition, the proposed method with energy causality constraints have a slightly degraded performance than the Without energy causality scheme, because the latter method allows the WD to utilize the harvested energy after its uplink transmit slot, and in turn, decreases the information transmission duration.

In Fig. 5, we reveal the impact of the number of reflection elements on the total completion time. From this figure, the total completion time achieved by IRS-assisted schemes has a decreasing trend with the increase of $M$. Specifically, the time reduction achieved by our proposed method increases with $M$ than both the Without IRS scheme and Random phase shift scheme. This validates the necessity to optimize the phase beamforming method for IRS-assisted full-duplex WPCNs, especially when the IRS is equipped with a large number of reflection elements.

In Fig. 6, we plot the total completion time against the maximum transmit power of HAP. As desired, the total completion time achieved by all schemes decreases with the increase of maximum transmit power at the HAP. Because the energy transmission time will be reduced with a larger transmit power of HAP. Moreover, we also see that the proposed method can realize at least 83% and 29% time reduction than the Without IRS scheme and Random phase shift scheme, respectively. Meanwhile, it is also shown that the proposed method consumes 20% and 26% more transmission time than the Without energy causality scheme and Without phase
errors scheme, respectively. Fig. 7 reveal the convergence rate of Algorithm 3, where one can see that the proposed Algorithm 3 will converge to the optimal solution after several iterations under different parameter settings. It demonstrates the good convergence property of our proposed alternating optimization method.

V. CONCLUSION

This paper studied the impact of phase errors on the performance of IRS-assisted full-duplex WPCNs in terms of total transmission time. First, we formulated a total completion time minimization problem under the transmit rate and energy causality constraints of WDs, via jointly optimizing the time-slot allocation, phase beamforming of IRS, and transmit power control of WDs and HAP. To solve this problem, we decoupled the original problem into two subproblems by exploiting the block coordinate descent method. For the phase beamforming optimization subproblem, we transformed the random phase errors to a deterministic form, and further designed an iterative method to obtain the optimal phase beamforming strategy by exploiting the successive convex approximation technique. For the time allocation and transmit power control subproblem, we derive the optimal transmit power of WDs in closed-form expressions, and utilized the variable substitution and approximation method to obtain the optimal time allocation and transmit power of HAP. Simulation results demonstrated that the completion time can be greatly reduced by our proposed method, as compared with the Without energy causality scheme and Without phase errors scheme.

In the future, we can extend the current work to several interesting research directions. First, active IRS can be considered to improve the performance of full-duplex WPCNs via jointly optimizing phase shifts and amplitudes of reflection elements on IRS. Second, the robust resource management and reflection optimization strategy is able to be designed considering the imperfect CSI and transceiver hardware impairments. Finally, it is meaningful to investigate the optimal design problem for the multi-cell scenario, and meanwhile multiple IRS can be deployed to improve the performance of full-duplex WPCNs.

APPENDIX A: PROOF OF Lemma 1

We first derive $E_{\nu_{E,k}}(\text{conj}(v_{E,k})v_{E,k}^T)$, $E_{\nu_{E,k}}(\text{conj}(v_{E,k}))$ and $E_{\nu_{E,k}}(v_{E,k}^T)$. The expression of $\text{conj}(v_{E,k})v_{E,k}^T$ is given by

$$
\begin{bmatrix}
1 & e^{j(\theta_{E,k,1}-\theta_{E,k,2})} & \cdots & e^{j(\theta_{E,k,1}-\theta_{E,k,M})} \\
& 1 & \cdots & \cdots \\
& \vdots & \ddots & \vdots \\
& e^{j(\theta_{E,k,1}-\theta_{E,k,M})} & \cdots & 1
\end{bmatrix}
$$

The domain of integration for distribution function.

Since $\theta_{E,k,m} \sim U(-\frac{\pi}{2}, \frac{\pi}{2})$, we will derive the probability density function (PDF) of $\Delta \theta_{i,j} = \theta_{E,k,i} - \theta_{E,k,j}$, $\forall i \neq j$. It exists the following several cases:

- $|\Delta \theta_{i,j}| > \pi$: it is out of the scope of $\Delta \theta_{i,j}$, so its PDF will be $f_{\Delta \theta_{i,j}}(\Delta \theta) = 0$.

- $-\pi < \Delta \theta_{i,j} < 0$: According to Fig. 8 (a), the distribution function $F_{\Delta \theta_{i,j}}(\Delta \theta)$ will be

$$
F_{\Delta \theta_{i,j}}(\Delta \theta) = \frac{\Delta \theta^2}{2\pi^2} + \frac{\Delta \theta}{\pi} + \frac{1}{2}.
$$

Therefore, the PDF of $\Delta \theta_{i,j}$ at the interval $[-\pi, 0]$ is given by

$$
f_{\Delta \theta_{i,j}}(\Delta \theta) = \frac{\partial F_{\Delta \theta_{i,j}}(\Delta \theta)}{\partial \Delta \theta} = \frac{\Delta \theta}{\pi^2} + \frac{1}{\pi}.
$$

- $0 < \Delta \theta_{i,j} < \pi$: By observing Fig. 8(b), the distribution
Therefore, the PDF of $\Delta \theta_{i,j}$ can be expressed as
$$f_{\Delta \theta_{i,j}}(\Delta \theta) = \frac{\partial F_{\Delta \theta_{i,j}}(\Delta \theta)}{\partial \Delta \theta} = -\frac{\Delta \theta}{\pi^2} + \frac{1}{\pi},$$
(34)

According to above descriptions, the PDF of $\Delta \theta_{i,j}$ will be
$$f_{\Delta \theta_{i,j}}(\Delta \theta) = \begin{cases} \frac{\Delta \theta}{\pi^2} + \frac{1}{\pi}, & -\pi \leq \Delta \theta \leq 0 \\ 0, & \text{otherwise} \end{cases}$$
(35)

Then, $E_{\Delta \theta_{i,j}}(e^{j\Delta \theta_{i,j}})$ is derived as
$$E_{\Delta \theta_{i,j}}(e^{j\Delta \theta_{i,j}}) = \int_{-\pi}^{0} \left( \frac{\Delta \theta}{\pi^2} + \frac{1}{\pi} \right) e^{j\Delta \theta} d\Delta \theta$$
$$= \frac{4}{\pi^2}.$$  
(36)

Therefore, $E_{v_{E,k}}(\text{conj}(v_{E,k})v_{E,k}^T)$ will be
$$E_{v_{E,k}}(\text{conj}(v_{E,k})v_{E,k}^T) = \begin{bmatrix} E_{\Delta \theta_{E,k}}(e^{j\Delta \theta_{E,k}}) & \ldots & E_{\Delta \theta_{E,k}}(e^{j\Delta \theta_{E,k}}) \\ \vdots & \ddots & \vdots \\ E_{\Delta \theta_{E,k}}(e^{j\Delta \theta_{E,k}}) & \ldots & E_{\Delta \theta_{E,k}}(e^{j\Delta \theta_{E,k}}) \end{bmatrix}$$
$$= \begin{bmatrix} \frac{4}{\pi^2} & \ldots & \frac{4}{\pi^2} \\ \vdots & \ddots & \vdots \\ \frac{4}{\pi^2} & \ldots & \frac{4}{\pi^2} \end{bmatrix} = R.$$  
(37)

Next, we will derive $E_{\theta_{E,k}}(e^{j\theta_{E,k}})$ and $E_{\theta_{E,k}}(e^{-j\theta_{E,k}})$ as follows.
$$E_{\theta_{E,k}}(e^{j\theta_{E,k}}) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{j\theta_{E,k}} d\theta_{E,k}$$
$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{\pi} \left( \cos(\theta_{E,k}) + i \sin(\theta_{E,k}) \right) d\theta_{E,k}$$
$$= \frac{2}{\pi}.$$  
(38)

Thus, we have the following results

$E_{v_{E,k}}(\text{conj}(v_{E,k})v_{E,k}^T) = [E_{\theta_{E,k}}(e^{j\theta_{E,k}}), E_{\theta_{E,k}}(e^{-j\theta_{E,k}}), \ldots, E_{\theta_{E,k}}(e^{j\theta_{E,k}}), E_{\theta_{E,k}}(e^{-j\theta_{E,k}}), \ldots, E_{\theta_{E,k}}(e^{j\theta_{E,k}}), E_{\theta_{E,k}}(e^{-j\theta_{E,k}})]^T = 2 \cdot \frac{4}{\pi^2} T.$

Therefore, we can derive $\hat{a}_k$ and $\hat{b}_k$ described in (8)-(9) according to (37), (40) and (41).

**APPENDIX B**

**BENCHMARK METHODS**

1) Without phase errors: In this scheme, it considers an ideal scenario without phase errors at the IRS. Therefore, the corresponding optimization problem will be
$$\text{minimize} \sum_{k=0}^{K} \tau_k$$
$$\text{s.t.} \quad \tau_k \log_2 \left( 1 + \frac{P_k |g_{h,k}|^2}{\delta^2 + \gamma^2 P_{A,k}} \right) \geq R_{k,\min}, \forall k \in \mathcal{K},$$
$$P_k \tau_k \leq \eta_k \sum_{i=0}^{k-1} P_{A,i} \tau_i |h_{f,k}^H \Gamma_i h_f + h_{d,k}|^2, \forall k \in \mathcal{K},$$

2) Without IRS: In this scheme, the HAP transmits the downlink energy signals to WDs and receives the uplink information from WDs without the aid of IRS. Hence, the corresponding optimization problem is given by
$$\text{minimize} \sum_{k=0}^{K} \tau_k$$
$$\text{s.t.} \quad \tau_k \log_2 \left( 1 + \frac{P_k |g_{d,k}|^2}{\delta^2 + \gamma^2 P_{A,k}} \right) \geq R_{k,\min}, \forall k \in \mathcal{K},$$
$$P_k \tau_k \leq \eta_k \sum_{i=0}^{k-1} P_{A,i} \tau_i |h_{d,k}|^2, \forall k \in \mathcal{K},$$

C4-C5.
3) Without energy causality: In this scheme, the $k$-th wireless device can utilize the harvested energy after \(\tau_k\) for transmitting its uplink information, i.e., \(E_{H,k} = \eta_k \sum_{i \neq k} P_{A,i} \tau_i |h_{H,i}^T \Gamma_k \hat{h}_r + h_{d,k}|^2\). Therefore, the corresponding optimization problem can be formulated as

\[
\text{minimize} \quad \sum_{k=0}^{K} \tau_k \quad (44a)
\]

subject to

\[
P_k \tau_k \leq \eta_k \sum_{i \neq k} P_{A,i} \tau_i |h_{H,i}^T \Gamma_k \hat{h}_r + h_{d,k}|^2, \quad \forall k \in K, \quad (44b)
\]

\[
\tau_k \log_2 \left(1 + \frac{P_k|E_{H,k}\Gamma_k \hat{h}_r + g_{d,k}|^2}{\sigma^2 + \gamma P_{A,k}}\right) \geq R_{k,\text{min}}, \quad \forall k \in K, \quad (45a)
\]

\[
P_k \tau_k \leq \eta_k \sum_{i=0}^{k-1} P_{A,i} \tau_i |h_{H,i}^T \Gamma_k \hat{h}_r + h_{d,k}|^2, \quad \forall k \in K, \quad (45b)
\]

\[
C1, C3-C5. \quad (44c)
\]

\[
C4-C5. \quad (45c)
\]

\[
(45d)
\]

A. Random phase shift

In this scheme, the phase shift \(\{\Gamma_k\}\) of IRS is randomly generated from \((0, 2\pi]\), and the transmit power and time-slot variables are optimized by solving the following problem

\[
\text{minimize} \quad \sum_{k=0}^{K} \tau_k \quad (45a)
\]

subject to

\[
P_k \tau_k \leq \eta_k \sum_{i \neq k} P_{A,i} \tau_i |h_{H,i}^T \Gamma_k \hat{h}_r + h_{d,k}|^2, \quad \forall k \in K, \quad (45b)
\]

\[
C1, C3-C5. \quad (44c)
\]

\[
(45d)
\]

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