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Intelligent Reflecting Surface-assisted Over-the-Air Computation for Backscatter Sensor Networks

Sun Mao, Ning Zhang, Jie Hu, and Kun Yang

***Abstract*—For achieving efficient data aggregation and energy supply in internet of things, this paper investigates the optimal design for an intelligent reflecting surface (IRS)-aided backscatter sensor network with over-the-air computation (AirComp). To ensure the accuracy of data aggregation, this paper presents a mean-squared error (MSE) minimization problem via jointly optimizing the normalization factor, power splitting ratios of backscatter devices (BDs), and phase shifts of IRS. Inspired by the block coordinate descent technique, we propose an alternating optimization method to solve the formulated non-convex problem. Specifically, we derive the optimal normalization factor and power splitting ratios in closed-form expressions, and exploit the variable substitution technique and semi-definite relaxation method to address the phase shift optimization subproblem. Simulation results demonstrate the significant MSE reduction achieved by the proposed method, in comparison with other benchmark methods.**

***Index Terms*—Intelligent reflecting surface, over-the-air computation, backscatter communications, mean-squared error.**

I. Introduction

In the foreseeable future, a massive number of Internet of Things (IoT) devices will be deployed to collect environment data and to enable various smart applications, such as industrial automation, intelligent transport systems, smart agriculture, and so forth. Nevertheless, the restricted spectrum resource imposes great pressure on wireless data aggregation from ubiquitous IoT devices. In recent years, over-the-air computation (AirComp) technique is proposed to support fast and efficient data aggregation via utilizing the superposition property of wireless channels [1]–[4]. So far, AirComp technique has been extensively investigated in existing works [5]–[7]. In [5], Liu *et al.* investigated the computation-optimal strategy for a typical AirComp system, and they revealed the ergodic performance and scaling laws for large-scale AirComp systems. In [6], the authors studied the optimal design for the broadband AirComp systems, where the sensor device can transmit their data to the fusion node over multiple selected frequency channels. The authors in [7] developed the optimal AirComp policy considering spatial-and-temporal correlated sensing signals.

Besides the data aggregation, the convenient energy supply is another bottleneck for massive low-power IoT devices. Ambient backscatter communication is recognized as a candidate solution for realizing sustainable IoT networks [8]–[10]. On the one hand, backscatter devices can reduce their power consumption by modulating and reflecting wireless signals to carry their own information to intended receivers, instead of generating radio-frequency signals independently. In addition, backscatter devices are able to split one part of received signals for harvesting energy to prolong their lifetime. However, the double pathloss of backscatter communication restricts its applicability in IoT networks.

Recently, intelligent reflecting surface (IRS) has been envisioned as an innovative technique that can improve the performance of wireless communications, via adjusting its amplitudes and phase shifts to reconstruct the wireless transmission environment intelligently [11]–[13]. In [14], the authors proposed an optimization framework for IRS-assisted backscatter communication systems, where the IRS was properly equipped to enhance the communication performance between the tag and the reader. They further presented a joint transmit and reflect beamforming optimization strategy to minimize the system power consumption. In [15], Shi *et al.* investigated the mean-squared error (MSE) minimization problem for an IRS-empowered wireless powered Aircomp systems. To reduce the cost of channel estimation, Zhai *et al.* in [16] proposed a two-timescale optimization method to solve the average MSE minimization problem for IRS-assisted Aircomp systems. In particular, the transmit power of IoT devices and receive beamforming of access point were optimized in the short timeslot based on the real-time channel state information (CSI), and the reflect beamforming of IRS was optimized in the long frame based on the channel statistics. As described above, the existing works utilized the IRS to improve the performance of backscatter communications or AirComp. Nevertheless, to be best of our knowledge, the efficient integration of IRS, backscatter communications and AirComp have not been investigated in existing works. It can be expected that the optimal design for such an integration system can achieve fast data aggregation and sustainable energy supply in IoT.

Therefore, this article proposes a novel design framework for IRS-assisted backscatter sensor networks with AirComp.

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To balance the data aggregation and energy supply, we formulate a MSE minimization problem under the power harvesting requirements of BDs, through jointly optimizing the normalization factor, power splitting ratios of BDs, and phase shifts of IRS. Inspired by the block coordinate descent technique, we develop an alternating optimization algorithm to solve the joint optimization problem, which is strictly non-convex due to the complex objective function and highly coupled optimization variables. In particular, the optimal normalization factor and power splitting ratios are derived in closed-form expressions for reducing the algorithm's computational complexity. Besides, the variable substitution technique and the semi-definite relaxation method are exploited to solve the phase shift optimization subproblem. Numerical results demonstrate that the proposed method can reduce the MSE significantly compared with existing benchmark methods.

The rest of this article is summarized as follows. Section II introduces the system model and the formulated MSE minimization problem. In Section III, we develop a low-complexity alternating optimization method with proved convergence. Simulation results in Section IV will reveal the performance of our proposed scheme. This article is summarized in Section V.

Notations: This article adopts the lowercase letter, boldface lowercase letter, and boldface uppercase letter to indicate the scalar, vector and matrix, respectively. Re(x) stands for the real component of complex number x. $\mathbf{x}^H$ and diag$(\mathbf{x})$ indicates the hermitian transpose and diagonalization of vector $\mathbf{x}$, respectively. Rank$(\mathbf{X})$ and Tr$(\mathbf{X})$ represent the rank and trace of matrix $\mathbf{X}$, respectively.

II. System Model and Problem Formulation

A. System Model

As shown in Fig. 1, this paper considers an IRS-assisted backscatter sensor networks including a power beacon (PB), an access point (AP), an IRS with M reflection units, and K backscatter devices (BDs). The index sets of BDs and IRS's reflection units are denoted by $\mathcal{K} = \{1, 2, \cdots, K\}$ and $\mathcal{M} = \{1, 2, \cdots, M\}$, respectively. Each BD is able to receive and backscatter the signal from the power beacon, for carrying its own information to the AP. To alleviate the effect of double pathloss of backscatter communications, an IRS is equipped to assist the backscatter communications between BDs and AP. Moreover, the channel coefficients between the power beacon and the k-th BD, between the k-th BD and the IRS, between the IRS and the AP, and between the k-th BD and the AP are indicated by $g_{D,k} \in \mathbb{C}^{1\times1}$, $\mathbf{h}_{T,k}^H \in \mathbb{C}^{1\times M}$, $\mathbf{h}_R \in \mathbb{C}^{M\times1}$, and $h_{D,k} \in \mathbb{C}^{1\times1}$, respectively. According to [17], it is supposed that the perfect CSI can be acquired by utilizing the existing channel estimation methods for IRS-assisted communication systems.

In this scenario, each BD is equipped with a sensor to collect some environmental parameters (e.g., humidity, temperature, and so forth), and the AP aims to obtain the target function of aggregated data from all BDs, instead of recovering the individual data of each BD. Specifically, the AirComp technique is adopted to improve the data aggregation

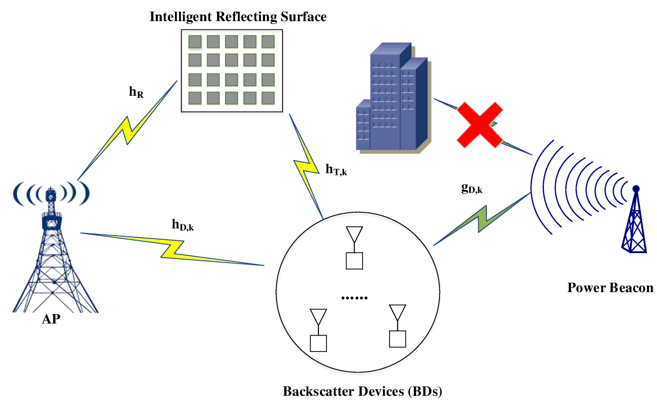

Fig. 1: IRS-assisted backscatter sensor networks with over-the-air computation.

efficiency. Defining x_k as the data measured by the k-th BD, thus the target function at the AP is given by

$$f = \phi\left(\sum_{k=1}^{K} \psi_k(x_k)\right), \tag{1}$$

where ψ and ϕ represent the pre-processing function at the k-th BD and post-processing function at the AP, respectively. We denote $s_k = \psi_k(x_k)$ as the symbol backscattered by the k-th BD, which satisfies $\mathbb{E}(s_k) = 0$, $\mathbb{E}(s_k s_k^H) = 1$ and $\mathbb{E}(s_k s_m^H) = 0$ for $k \neq m$. Besides, this paper considers a case that the AP desires to obtain the target function $f = \sum_{k=1}^{K} s_k$ [15]. It should be noted that other target functions can also be computed by using proper pre-processing and post-processing functions.

In the considered time block, the signal received at the k-th BD can be expressed as

$$y_{T,k} = g_{D,k}\sqrt{P_t}e, \forall k \in \mathcal{K}, \tag{2}$$

where P_t is the transmit power at the power beacon, and e denotes the unmodulated downlink energy signal with unit power. Then, the BDs split a part of the received signal for harvesting energy, and modulate and backscatter the rest part for carrying their own information to the AP [18]. Defining $\nu_k \in [0, 1]$ as the power splitting ratio and $\eta_k \in (0, 1]$ as the linear power conversion factor, so the power harvested by the k-th BD is given by

$$E_k = \eta_k(1 - \nu_k)|g_{D,k}|^2 P_t, \forall k \in \mathcal{K}. \tag{3}$$

Furthermore, the received signal at the AP from all BDs will be

$$y = \sum_{k=1}^{K} g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t}s_k e + n, \tag{4}$$

where $n \sim \mathcal{CN}(0, \delta^2)$ denotes the additive white Gaussian noise (AWGN), and $\Gamma_B = \text{diag}\{e^{j\alpha_1}, e^{j\alpha_2}, \cdots, e^{j\alpha_M}\}$ stands for the phase shift matrix of IRS, where $\alpha_m \in (0, 2\pi]$ represents the phase shifts of m-th reflection unit on IRS. After receiving the signal y, the AP utilizes the normalization factor θ to recover the target function f. Therefore, the

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estimated function at the AP is expressed as

$$\hat{f} = \frac{y}{\sqrt{\theta}} = \frac{\sum_{k=1}^{K} g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t} s_k e + n}{\sqrt{\theta}}. \quad (5)$$

B. Problem Formulation

Despite the benefits brought by the AirComp technique, it also introduces severe computation distortion due to the noise signal and the unequal channel coefficients. In general, the computation error is quantified by the mean-squared error (MSE) between the target function f and the estimated function $\hat{f}$. Therefore, this paper concentrates on minimizing the MSE via jointly optimizing the normalization factor, power splitting ratios of BDs, and phase shift matrix of IRS. It is worth to noting that the MSE has been extensively adopted as a performance indicator to evaluate the signal distortion in AirComp-enabled networks [15], [16], and it can be calculated as

$$\begin{aligned} \mathrm{MSE}(f, \hat{f}) &= \mathbb{E}\left[|f - \hat{f}|^2\right] = \mathbb{E}\left[\left|\frac{y}{\sqrt{\theta}} - \sum_{k=1}^{K} s_k\right|^2\right] \\ &= \sum_{k=1}^{K}\left|\frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t}}{\sqrt{\theta}} - 1\right|^2 + \frac{\delta^2}{\theta}. \end{aligned} \quad (6)$$

Therefore, the MSE minimization problem can be formulated as

$$\begin{aligned} \underset{\{\nu_k, \theta, \Gamma_B\}}{\text{minimize}} \quad & \sum_{k=1}^{K}\left|\frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t}}{\sqrt{\theta}} - 1\right|^2 + \frac{\delta^2}{\theta} \\ \text{s.t.} \quad & \text{C1: } \eta_k(1-\nu_k)|g_{D,k}|^2 P_t \geq E_{\min,k}, \forall k \in \mathcal{K}, \\ & \text{C2: } 0 \leq \alpha_m \leq 2\pi, \forall m \in \mathcal{M}, \\ & \text{C3: } 0 \leq \nu_k \leq 1, \forall k \in \mathcal{K}, \\ & \text{C4: } \theta > 0, \end{aligned} \quad (7)$$

where C1 indicates that the power harvested by BDs should be larger than the minimum power requirement $E_{\min,k}$, C2 represents the phase shift constraints of IRS, C3 restricts the power splitting ratios of BDs, and C4 is the normalization factor constraints. It is observed that (7) is a strictly non-convex problem due to the highly coupled optimization variables, such as Γ_B and ν_k. Therefore, there is no standard method to tackle it. In the following section, an alternating optimization method is developed to obtain the optimal solution of (7).

Remark 1 (Scalability of Backscatter AirComp): In practice, the proposed IRS-aided AirComp can also be exploited in large-scale backscatter sensor networks. while the computation accuracy will be degraded due to the challenge in aligning the signals from massive BDs by a common reflection beamformer. Nevertheless, the issue may be alleviated in the future ultra-dense networks, since a larger number of multiple-antenna AP and IRS will be deployed, and it will lead to higher possibility to align the signals from BDs.

Remark 2 (Extension to Broadband Systems): In the broadband backscatter network with multiple frequency channels, the estimated function at the AP is expressed as the sum of multiple single-channel estimated functions, which will further lead to the similar-structure MSE expression and optimization problem. Therefore, the proposed method in this paper can be exploited in the scenario with multiple frequency channels. Moreover, it is quite challenging to obtain the CSI of all channels for IRS-aided broadband backscatter networks, especially when IRS is equipped with a large number of reflection units. Hence, the performance gain introduced by multiple frequency channels may be counteracted by the huge costs of channel estimation. To tackle this issue, our future work will investigate the statistical CSI-based method for IRS-aided broadband backscatter sensor networks with AirComp.

III. Proposed Solution

Based on the block coordinate descent (BCD) technique, we propose an alternating optimization method to solve the non-convex MSE minimization problem (7), where the normalization factor θ, power splitting ratios $\{\nu_k, \forall k \in \mathcal{K}\}$ of BDs, and phase beamforming matrix Γ_B of IRS are optimized alternately until convergence. In particular, we derive the optimal normalization factor and power splitting ratios of BDs in closed-form expressions, and the variable substitution technique and semi-definite relaxation method are adopted to obtain the optimal phase beamforming matrix of IRS.

A. Normalization Factor Optimization

Given $\{\nu_k^*, \Gamma_B^*\}$, we reformulate the problem (7) by optimizing the normalization factor θ as

$$\underset{\theta > 0}{\text{minimize}} \quad \sum_{k=1}^{K}\left|\frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B^* \mathbf{h}_R + h_{D,k})\sqrt{\nu_k^*}\sqrt{P_t}}{\sqrt{\theta}} - 1\right|^2 + \frac{\delta^2}{\theta}. \quad (8)$$

Defining $\beta = \frac{1}{\sqrt{\theta}}$ and $b_k = g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B^* \mathbf{h}_R + h_{D,k})\sqrt{\nu_k^* P_t}$, (8) will be converted to

$$\underset{\beta > 0}{\text{minimize}} \quad \sum_{k=1}^{K} |b_k \beta - 1|^2 + \delta^2 \beta^2. \quad (9)$$

In the following *Theorem 1*, we will derive the optimal normalization factor in closed-form expression.

Theorem 1: The optimal normalization factor θ^* is given by

$$\theta^* = \frac{1}{(\beta^*)^2} = \left(\frac{\delta^2 + \sum_{k=1}^{K} |b_k|^2}{\sum_{k=1}^{K} \mathrm{Re}(b_k)}\right)^2. \quad (10)$$

Proof: According to the convex optimization theory, (9) can be proved as a typical convex problem, and its optimal solution can be found in the stationary point. The first derivation of objective function in (9) to β can be derived as

$$\frac{\partial(\sum_{k=1}^{K} |b_k \beta - 1|^2 + \delta^2 \beta^2)}{\partial \beta} = \sum_{k=1}^{K}(2\beta|b_k|^2 - 2\mathrm{Re}(b_k)) + 2\delta^2\beta. \quad (11)$$

By setting (11) to zero, we can obtain the optimal normalization factor as given in (10). ■

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B. Power Splitting Optimization

Given $\{\theta^*, \Gamma_B^*\}$, (7) will be reduced to the following problem

$$\underset{\{\nu_k\}}{\text{minimize}} \quad \sum_{k=1}^{K} \left| \frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B^* \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t}}{\sqrt{\theta^*}} - 1 \right|^2 \tag{12a}$$

$$\text{s.t.} \quad \eta_k(1-\nu_k)|g_{D,k}|^2 P_t \geq E_{\min,k}, \forall k \in \mathcal{K}, \tag{12b}$$

$$0 \leq \nu_k \leq 1, \forall k \in \mathcal{K}. \tag{12c}$$

Problem (12) can be decoupled into K subproblems to optimize the power splitting ratio $\{\nu_k\}$ for each backscatter device. The k-th subproblem is given by

$$\underset{\nu_k}{\text{minimize}} \quad \left| \frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B^* \mathbf{h}_R + h_{D,k})\sqrt{\nu_k}\sqrt{P_t}}{\sqrt{\theta^*}} - 1 \right|^2 \tag{13a}$$

$$\text{s.t.} \quad \eta_k(1-\nu_k)|g_{D,k}|^2 P_t \geq E_{\min,k}, \tag{13b}$$

$$0 \leq \nu_k \leq 1. \tag{13c}$$

let $m_k = \frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B^* \mathbf{h}_R + h_{D,k})\sqrt{P_t}}{\sqrt{\theta^*}}$ and $u_k = \sqrt{\nu_k}$, (13) can be rewritten as the following convex problem

$$\underset{u_k}{\text{minimize}} \quad |m_k|^2 u_k^2 - 2\text{Re}(m_k) u_k \tag{14a}$$

$$\text{s.t.} \quad 0 \leq u_k \leq \sqrt{1 - \frac{E_{\min,k}}{\eta_k |g_{D,k}|^2 P_t}}. \tag{14b}$$

Next, we will derive the optimal power splitting ratio in the following *Theorem 2*.

Theorem 2: The optimal power splitting ratio $\{\nu_k^*\}$ of problem (14) is expressed as

$$\nu_k^* = (u_k^*)^2 = \min\left\{ \left(\frac{\text{Re}(m_k)}{|m_k|^2}\right)^2, 1 - \frac{E_{\min,k}}{\eta_k |g_{D,k}|^2 P_t} \right\}, \forall k \in \mathcal{K}. \tag{15}$$

Proof: Since the optimal solution of convex problem (14) can be found in the stationary point or the boundary point, so we can derive the optimal power splitting ratio as described in (15). ■

C. Phase Beamforming Optimization

For given $\{\nu_k^*, \theta^*\}$, (7) is simplified to the phase beamforming optimization subproblem

$$\underset{\Gamma_B}{\text{minimize}} \quad \sum_{k=1}^{K} \left| \frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k^*}\sqrt{P_t}}{\sqrt{\theta^*}} - 1 \right|^2 \tag{16a}$$

$$\text{s.t.} \quad 0 \leq \alpha_m \leq 2\pi, \forall m \in \mathcal{M}. \tag{16b}$$

We set $c_k = \frac{g_{D,k}\sqrt{\nu_k^*}\sqrt{P_t}}{\sqrt{\theta^*}}$, $\mathbf{v} = [e^{j\alpha_1}, \cdots, e^{j\alpha_M}]^T$, and $\mathbf{e}_k = \mathbf{h}_{T,k}^H \text{diag}(\mathbf{h}_R)$, which make $\frac{g_{D,k}(\mathbf{h}_{T,k}^H \Gamma_B \mathbf{h}_R + h_{D,k})\sqrt{\nu_k^*}\sqrt{P_t}}{\sqrt{\theta^*}} = c_k(\mathbf{e}_k \mathbf{v} + h_{D,k})$. Since the objective function of (16) is still complex, so we further define the following expressions

$$\begin{aligned} \mathbf{d}_k &= c_k \mathbf{e}_k, \\ \mathbf{f}_k &= \mathbf{d}_k^H c_k h_{D,k} - \mathbf{d}_k^H. \end{aligned} \tag{17}$$

Based on above expressions, problem (16) is rewritten as

$$\underset{\mathbf{v}}{\text{minimize}} \quad \sum_{k=1}^{K} (\mathbf{v}^H \mathbf{d}_k^H \mathbf{d}_k \mathbf{v} + \mathbf{v}^H \mathbf{f}_k + \mathbf{f}_k^H \mathbf{v}) \tag{18a}$$

$$\text{s.t.} \quad [\mathbf{v}\mathbf{v}^H]_{mm} = 1, \forall m \in \mathcal{M}. \tag{18b}$$

Due to the quadratically equality constraint, (18) is still nonconvex. We further define $\hat{\mathbf{v}} = [\mathbf{v}^T, 1]^T$ and $\hat{\mathbf{V}} = \hat{\mathbf{v}}\hat{\mathbf{v}}^H$. (18) is then transformed to

$$\underset{\hat{\mathbf{V}}}{\text{minimize}} \quad \text{Tr}\left(\hat{\mathbf{V}}\mathbf{G}\right) \tag{19a}$$

$$\text{s.t.} \quad [\hat{\mathbf{V}}]_{mm} = 1, \forall m \in \{\mathcal{M}, M+1\}, \tag{19b}$$

$$\hat{\mathbf{V}} \succeq 0, \tag{19c}$$

$$\text{Rank}(\hat{\mathbf{V}}) = 1, \tag{19d}$$

where

$$\mathbf{G} = \sum_{k=1}^{K} \begin{bmatrix} \mathbf{d}_k^H \mathbf{d}_k & \mathbf{f}_k \\ \mathbf{f}_k^H & 0 \end{bmatrix}. \tag{20}$$

Based on the semi-definite relaxation technique, we drop the rank-one constraint (19d), and the relaxed version of (19) can be solved by CVX. It is worth noting that the obtained $\hat{\mathbf{V}}^*$ may be not rank-one. Therefore, we make $\hat{\mathbf{v}}^* \approx \sqrt{\lambda_{\max}^*}\mathbf{u}_{\max}^*$, where $\lambda_{\max}^*$ and $\mathbf{u}_{\max}^*$ denote the maximum eigenvalue and its corresponding eigenvector of $\hat{\mathbf{V}}^*$.

In summary, the proposed alternating optimization method is illustrated in Algorithm 1. Next, we will prove the convergence of Algorithm 1 in the following *Theorem 3*, and analyze its computational complexity.

Algorithm 1: Alternating optimization method for solving (7)

Initialize: Setting $(\nu_k^{(0)}, \Gamma_B^{(0)})$, and the iteration factor $n = 1$.
Repeat:
  Calculating the normalization factor $\theta^{(n)}$ by utilizing *Theorem 1*;
  Obtaining the optimal power splitting ratio $\{\nu_k^{(n)}\}$ according to *Theorem 2*;
  Acquiring the optimal phase shift matrix $\hat{\mathbf{V}}^{(n)}$ by solving the relaxation version of (19);
  Recovering $\hat{\mathbf{v}}^{(n)} \approx \sqrt{\lambda_{\max}^{(n)}}\mathbf{u}_{\max}^{(n)}$;
  Update the iteration factor $n = n + 1$;
Until convergence.
Obtaining optimal solution $(\theta^*, \nu_k^*, \Gamma_B^*)$.

Theorem 3: The proposed alternating optimization-based Algorithm 1 can converge to the optimal solution within several iterations.

Proof: Defining $\text{MSE}(\theta, \nu_k, \Gamma_B)$ as the objective function

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of (7), we have

$$\begin{aligned}\mathrm{MSE}(\theta^{(n)},\nu_k^{(n)},\Gamma_B^{(n)}) &\overset{\text{(c1)}}{\geq} \mathrm{MSE}(\theta^{(n+1)},\nu_k^{(n)},\Gamma_B^{(n)}) \\ &\overset{\text{(c2)}}{\geq} \mathrm{MSE}(\theta^{(n+1)},\nu_k^{(n+1)},\Gamma_B^{(n)}) \\ &\overset{\text{(c3)}}{\geq} \mathrm{MSE}(\theta^{(n+1)},\nu_k^{(n+1)},\Gamma_B^{(n+1)}),\end{aligned} \tag{21}$$

where (c1), (c2) and (c3) hold because the optimal MSE are obtained from (8), (12) and (16), respectively. Therefore, $\mathrm{MSE}(\theta^{(n+1)},\nu_k^{(n+1)},\Gamma_B^{(n+1)})$ is not larger than $\mathrm{MSE}(\theta^{(n)},\nu_k^{(n)},\Gamma_B^{(n)})$. Since the value of MSE is limited, we can derive that Algorithm 1 will coverage to an optimal solution after several iterations according to the Cauchy theory. ■

The computational complexity of Algorithm 1 is divided into two parts, including the iteration number and per-iteration complexity. In each iteration, the method needs to solve three subproblems, i.e., normalization factor optimization subproblem, power splitting optimization subproblem, and phase beamforming optimization subproblem. Since we derive the optimal normalization factor and power splitting ratios in closed-form expressions, thus the corresponding computational complexity is expressed as $\mathcal{O}(K+1)$. The interior point method-based solver is adopted to solve the phase beamforming optimization subproblem (19) with $(M+1)^2$ optimization variables and $M+2$ constraints, so the corresponding computational complexity can be expressed as $\mathcal{O}(((M+1)^2+M+2)(M+1)^4\sqrt{M+2}\log(1/\epsilon))$, where ϵ stands for the tolerance factor. Defining I_n as the number of iterations of Algorithm 1, the total computational complexity of Algorithm 1 is expressed as $\mathcal{O}(I_n(K+1+((M+1)^2+M+2)(M+1)^4\sqrt{M+2}\log(1/\epsilon)))$.

IV. Performance Evaluation

This section provides the simulation results to reveal the MSE achieved by the proposed IRS-assisted backscatter sensor networks with AirComp, in comparison with the following two benchmark methods:

- *Without IRS:* In this scheme, no IRS is equipped to assist the backscatter communications between BDs and AP.
- *Random phase shift:* In this scheme, the phase shifts of reflection units on IRS are selected from $[0, 2\pi]$ randomly, and the normalization factor and power splitting ratios of BDs are optimized by exploiting the proposed method in this article.

Furthermore, we consider two scenarios with $K = 2$ BDs and $K = 4$ BDs, respectively. The AP, power beacon, IRS, and BDs are located at (0,0,10), (6,0,8), (3,1,5), and [(2,-1,0),(4,0,0),(4,1,0),(3,2,0)], respectively. The channels are modeled as

$$\mathbf{C_c} = \sqrt{P_l d^{-\xi}}\left(\sqrt{\frac{1}{M_r+1}}\mathbf{C}^{\mathrm{NLoS}} + \sqrt{\frac{M_r}{M_r+1}}\mathbf{C}^{\mathrm{LoS}}\right), \tag{22}$$

where $P_l = -30$ dB represents the path-loss at the unit distance, d indicates the distance between communication nodes, and ξ denotes the path-loss factor (i.e., $\xi_1 = \xi_2 = \xi_3 = 2.5$ and $\xi_4 = 4$ stand for the path-loss factor for the PB-BD, BD-IRS, IRS-AP, BD-AP links, respectively), $M_r = 2$ is the Rician factor, and $\mathbf{C}^{\mathrm{NLoS}}$ and $\mathbf{C}^{\mathrm{LoS}}$ represent the Rayleigh fading and Line-of-Sight (LoS) components, respectively. Moreover, we set the other simulation parameters as follows: $\eta_k = 0.8$, $P_t = 5$ W, and $\delta^2 = 10^{-9}$ W.

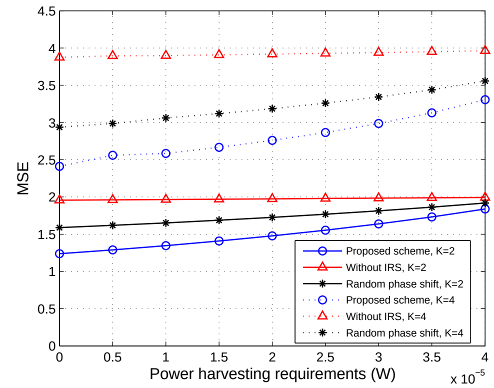

Fig. 2: Power harvesting requirements versus MSE with $M = 60$.

Fig. 2 shows the MSE versus the power harvesting requirement $E_{\min,k}$. It is seen from this figure that the MSE increases with the power harvesting requirements of BDs. As $E_{\min,k}$ increases, each BD must split a higher ratio of received signal for harvesting energy, and it will lead to a higher MSE of data aggregation. This observation also reveals the tradeoff between the data aggregation and energy supply for considered systems. Besides, it is also observed that the proposed method can achieve a smaller MSE in comparison with the *Without IRS* scheme and the *Random phase shift* scheme. This is due to the fact that the optimal design of IRS's phase shift has great potential to reduce the MSE by enhancing the communication quality between the AP and BDs. Furthermore, the considered system exhibits a higher MSE when a larger number of BDs are deployed, since it is hard to align all the channels from BDs to the AP by utilizing a single reflection beamformer at the IRS.

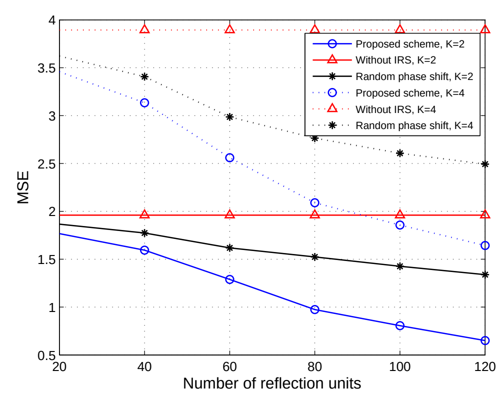

Fig. 3: Number of reflection units versus MSE with $E_{\min,k} = 5$uW.

Fig. 3 shows the MSE versus the number of reflection units integrated on the IRS. We observe that the MSE achieved by the proposed scheme decreases rapidly with the size of reflection units at the IRS. Because the additional reflection units contribute to extra degrees of freedom for improving the accuracy of data aggregation. Meanwhile, it is also found that the proposed scheme can achieve higher performance gain than benchmark methods when the IRS is integrated with a larger number of reflection elements. Fig. 4 depicts the convergence of Algorithm 1. As observed, Algorithm 1 can converge to the optimal MSE within a few iterations under different K and M. It verifies that the proposed alternating optimization method has good convergence properties and low computational complexity.

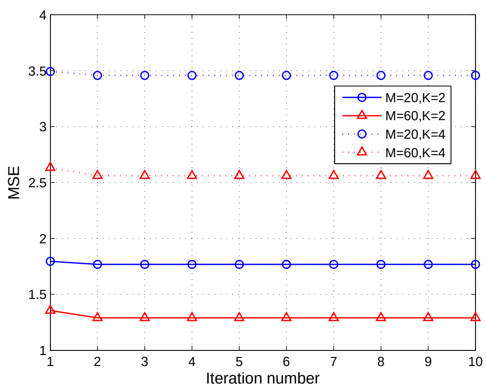

Fig. 4: Convergence of Algorithm 1 with $E_{\min,k} = 5$uW.

V. Conclusion

In this article, we investigated the MSE minimization problem for IRS-assisted backscatter sensor networks with over-the-air computation, via jointly optimizing the normalization factor, power splitting ratios of BDs, and phase shifts of IRS. In order to tackle the coupled optimization variables, an alternating optimization method was presented to solve the non-convex MSE minimization problem. In particular, the optimal normalization factor and power splitting ratios were derived in closed-form expressions, and the semi-definite relaxation method and variable substitution technique were adopted to solve the phase shift optimization subproblem. Finally, numerical results demonstrated the significant MSE reduction achieved by our proposed IRS-assisted strategy, as compared with existing benchmark methods.

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