Uniform-Distributed Constellation Codebook Design for High-Capacity Visible Light Communications

Lisu Yu, Jiajia Qian, Yiming Gui, Zilong Liu, Pei Xiao, Yuhao Wang, and Zhenghai Wang

***Abstract*—This letter studies high-capacity visible light communication (VLC) based on code-domain non-orthogonal multiple access (CD-NOMA) with the goal of enabling the future machine-type communication networks. To fulfill the non-negative signal constraint and mitigate/suppress the nonlinear effects and shot noise, a novel uniform-distributed constellation codebook is developed for lower peak power and larger minimum Euclidean distance (MED). The simulation results demonstrate that our proposed codebooks give rise to significantly improved error rate performance compared to existing codebooks. In addition, codebooks with larger overloading factors are presented to achieve high-capacity communication.**

***Index Terms*—Non-orthogonal multiple access (NOMA), visible light communication (VLC), uniform-distributed constellation, shot noise, minimum Euclidean distance.**

I. Introduction

In recent years, visible light communication (VLC) has attracted increasing research attention owing to its great advantages of zero electromagnetic interference, high-level safety, unlicensed spectrum, and easy installation [1], [2]. With tens of billions of communication devices, sensors, and robots connecting into the beyond-5G machine-type communication networks, high-rate and high-capacity VLC systems are widely recognized as an efficient means of meeting the stringent & diverse quality-of-service data requirements. However, VLC systems suffer from certain intrinsic problems such as nonlinear effects and shot noise [3]. In order to further improve the performance of VLC systems, non-orthogonal multiple access (NOMA) is studied for supporting massive connectivity in the next generation mobile systems.

Specifically, a major nonlinear problem is the high peak-to-average power ratio (PAPR) problem caused by orthogonal frequency division multiplexing (OFDM) technology, making the power amplifier at the transmitter working in the nonlinear region. Because the light-emitting diode (LED) radiates photons randomly, the noise power depends on the input signal in the VLC system. In a practical VLC system, one needs to deal with both the shot noise and thermal noise. In an indoor environment, typical lighting and communication scenarios provide a very high Signal-to-noise ratio (SNR), resulting in a significant amount of shot noise caused by large received power.

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As far as NOMA technology is concerned, there are power domain NOMA (PD-NOMA) and code domain NOMA (CD-NOMA) [4]. The main principle of NOMA is to use different codebooks (CBs) or power resources to provide data services for multiple users using the same resource block. Among many others, sparse code multiple access (SCMA) is widely considered as an attractive CD-NOMA scheme owing to its coding and constellation shaping gains. In the current literature on SCMA-assisted VLC, the main research focus is on the design of CBs. Most constellation design schemes only consider the impact of minimum Euclidean distance (MED) using SCMA [5] or optical OFDM [6], without however considering the signal correlation noise (SDN). [7] designed a new codebook with large MED using star quadrature amplitude modulation (star-QAM) signal constellation. Based on the phase distribution of the proposed SCMA constellation, the bit error rate (BER) of the SCMA system on the downlink Rayleigh fading channel was obtained in a closed form. For SCMA-VLC system affected by SDN, [8] and [9] proposed modulation based on rotated MED (R-MED) and distance-range-oriented (DRO) respectively considering different shot noises in the VLC systems By considering the impact of shot noise on the system, [10] proposed a new low-complexity CB design with 150% overloading factor. However, the codebook design for a higher overloading factor (e.g., 300%) is still missing. Following the existing design approaches, the main issue is that the BER performance could deteriorate sharply with the increase of user number. This is because the amplitude of the superimposed codeword may exceed the linear range of LED, causing amplified shot noise.

In this paper, we target at reducing the amplitude of superimposed constellations by carefully designing a sparse codebook so as to improve BER performance. The key idea is to reduce the peak power of transmitting signals by overlapping codewords of the mother constellation (MC) and constructing a uniform constellation. These methods enable the transmitting signals to fall in the linear interval as much as possible and reduce the variance of shot noise. In addition, we show that the MED of the superimposed constellation can be enhanced to improve BER performance.

II. System Model

A downlink indoor VLC system using the CD-NOMA scheme is considered in this letter. The K resource elements (REs) are deployed for the concurrent communication of $J\,(J > K)$ users, thus providing a $\lambda = J/K$ overloading factor. As seen from Fig. 1, the incoming $\log_2 M$ bits of each

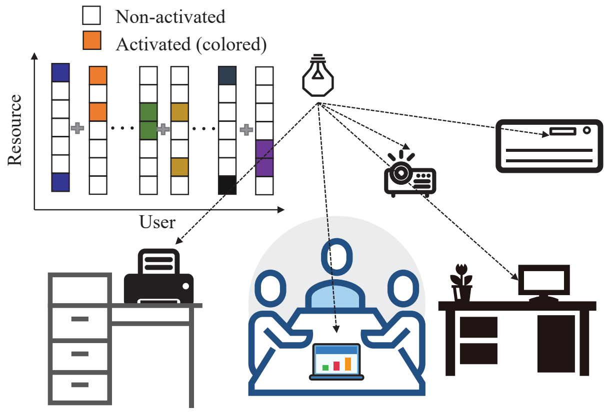

Fig. 1. High-capacity VLC system model based on sparse codebooks.

user are mapped to a K-dimensional sparse codeword at the transmitter, where M denotes the number of sparse codewords per user. Owing to the constraint of VLC, all the codewords are real and not negative. Codewords from different users are overlaid to drive the LED light for transmitting information.

The photodetector (PD) receiver monitors the intensity of optical signals and converts them into electrical ones. Two types of noise should be properly dealt with: shot noise and thermal noise. Shot noise is caused by the fluctuating current and is therefore linked to the signal itself. Moreover, the thermal motion of the electron carriers induces thermal noise and this is independent of the transmit signal. Ultimately, the decoder retrieves the transmit bits from the corrupted signal by using the message passing algorithm (MPA) [11].

Assuming synchronous transmission [10], the received signal after the VLC channel can be expressed as

$$\boldsymbol{y} = \mathrm{diag}\left(\boldsymbol{h}\right) f\left(\boldsymbol{w}\right) + \left(\mathrm{diag}\left(\boldsymbol{h}\right) f\left(\boldsymbol{w}\right)\right)^{\frac{1}{2}} \boldsymbol{n}_{sh} + \boldsymbol{n}_{th}, \tag{1}$$

where $\boldsymbol{h}$ denotes the channel fading vector and $\boldsymbol{w} = \sum_{j=1}^{J} \boldsymbol{x}_j$ denotes the superimposed signal of all transmitting users. The shot noise is $\boldsymbol{n}_{sh} \sim \mathcal{N}_{\mathbb{R}}\left[\mathbf{0}, \zeta^2\sigma^2 \mathrm{I}\right]$, and thermal noise is $\boldsymbol{n}_{th} \sim \mathcal{N}_{\mathbb{R}}\left[\mathbf{0}, \sigma^2 \mathrm{I}\right]$. ζ^2 denotes the ratio of shot noise to thermal noise power. $f\left(\cdot\right)$ indicates a non-linear conversion function from voltage to current. In this letter, the Rapps function is adopted, which can be shown as

$$f\left(v\right) = \frac{v}{\left(1 + \left(v/i_{\max}\right)^{2t}\right)^{1/2t}}, \tag{2}$$

where the smooth factor t controls the degree of a smooth transition from linear to saturated regions and $i_{\max}$ denotes the maximum current allowed through the LED.

A high-capacity VLC system can be achieved by careful design of sparse codebooks with a larger overloading factor. In this letter, the throughput of a single RE is adopted as a performance metric which can be written as [12]

$$\mathrm{Throughput} = \left(1 - \mathrm{BER}\right) \cdot \log_2 M \cdot J/K, \tag{3}$$

where the throughput is linearly related to the overloading factor $\lambda = J/K$ when different codebooks have identical BER and M.

III. Uniform-distributed Constellation Codebook Design

It is noted that MC is essential for the design of sparse codebooks. First, we generate a one-dimension based constellation $\boldsymbol{\mathcal{Z}}_T = \left[z_1, \cdots, z_T\right]$ with $T = \sqrt[N]{M}$ distinct point. Furthermore, we obtain an MC by N-dimension Cartesian product over $\boldsymbol{\mathcal{Z}}_T$

$$\boldsymbol{\mathcal{C}}_{N\times M} = \overbrace{\boldsymbol{\mathcal{Z}}_T \times \cdots \times \boldsymbol{\mathcal{Z}}_T \times \cdots \times \boldsymbol{\mathcal{Z}}_T}^{N}. \tag{4}$$

In this letter, we adopt the following $\boldsymbol{\mathcal{C}}_{N\times M}$ with $N = 2$, i.e.,

$$\boldsymbol{\mathcal{C}}_{2\times M} = \boldsymbol{\mathcal{Z}}_T \times \boldsymbol{\mathcal{Z}}_T, \tag{5}$$

where $\boldsymbol{\mathcal{Z}}_T = \left[0, 1, \cdots, T-1\right]$ is proposed in this paper. In addition, the N-dimension Cartesian product ensures overlapped codewords in one RE are distinguished in another RE.

According to the Latin matrix, the following signature matrix $\boldsymbol{\mathcal{S}}_{4\times 6}$, $\boldsymbol{\mathcal{S}}_{5\times 10}$ and $\boldsymbol{\mathcal{S}}_{6\times 15}$ are adopted to obtain sparse codebooks with different overloading factors. Due to limited space, $\boldsymbol{\mathcal{S}}_{7\times 21}$ is not presented in this letter.

$$\boldsymbol{\mathcal{S}}_{4\times 6} = \begin{bmatrix} 0 & s_0 & s_1 & 0 & s_2 & 0 \\ s_1 & 0 & s_2 & 0 & 0 & s_0 \\ 0 & s_1 & 0 & s_0 & 0 & s_2 \\ s_0 & 0 & 0 & s_2 & s_1 & 0 \end{bmatrix}, \tag{6}$$

$$\boldsymbol{\mathcal{S}}_{5\times 10} = \begin{bmatrix} s_0 & s_1 & s_2 & s_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ s_3 & 0 & 0 & 0 & s_0 & s_1 & s_2 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 & s_3 & 0 & 0 & s_0 & s_1 & 0 \\ 0 & 0 & s_1 & 0 & 0 & s_2 & 0 & s_3 & 0 & s_0 \\ 0 & 0 & 0 & s_0 & 0 & 0 & s_1 & 0 & s_2 & s_3 \end{bmatrix}, \tag{7}$$

$$\boldsymbol{\mathcal{S}}_{6\times 15} = \begin{bmatrix} s_0 & s_1 & s_2 & s_3 & s_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ s_1 & 0 & 0 & 0 & 0 & s_2 & s_3 & s_4 & s_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 & 0 & s_3 & 0 & 0 & 0 & s_4 & s_0 & s_1 & 0 & 0 & 0 \\ 0 & 0 & s_3 & 0 & 0 & 0 & s_4 & 0 & 0 & s_0 & 0 & 0 & s_1 & s_2 & 0 \\ 0 & 0 & 0 & s_4 & 0 & 0 & 0 & s_0 & 0 & 0 & s_1 & 0 & s_2 & 0 & s_3 \\ 0 & 0 & 0 & 0 & s_0 & 0 & 0 & 0 & s_1 & 0 & 0 & s_2 & 0 & s_3 & s_4 \end{bmatrix}, \tag{8}$$

where $s_i = T^i, \forall i = 0, \cdots, d_f - 1$. d_f denotes the number of users that superimpose over a resource node.

Finally, the codebook $\boldsymbol{\mathcal{X}}_j$ of user j can be obtained by

$$\mathcal{X}_j = ezc\left(\mathrm{diag}\left(\mathcal{S}^j_{K\times 1}\right)\right)\mathcal{C}_{2\times M}, \tag{9}$$

where $ezc\left(\mathrm{diag}\left(\boldsymbol{\mathcal{S}}^{\mathrm{j}}_{\mathrm{K}\times 1}\right)\right)$ denotes the resultant matrix after removing the all-zero columns.

For an indoor downlink VLC system using CD-NOMA, the simultaneous transmission of codewords by J users is carried out over K orthogonal resources. Therefore, there are overlaid M^J codewords constituting a superposed constellation. Let us define the element-wise distance $\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}\left(k\right) = \left|\sum_{j\in\xi_k}\left(x_{k,j} - \hat{x}_{k,j}\right)\right|^2$ and Euclidean distance $\delta_{\boldsymbol{w}\to\hat{\boldsymbol{w}}} = \sum_{k=1}^{K}\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}\left(k\right)$. The MED is a key indicator of the BER performance for sparse codebooks, and the computational complexity is thus largely dominated by the calculation for $M^J(M^J-1)/2$ mutual distances between the M^J superimposed codewords. The calculation of MED can be shown as

$$\Xi_{\text{min}} = \min\left\{\delta_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}|\forall \boldsymbol{w} \neq \hat{\boldsymbol{w}}\right\}. \tag{10}$$

Let $\boldsymbol{\Psi}^k_{\text{sum}}$ denote the superimposed constellation at the k-th RE after removing the overlapped constellation points, which can be written as

$$\boldsymbol{\Psi}^k_{\text{sum}} = \sum_{i=0}^{d_f-1} s_i \boldsymbol{\mathcal{Z}}_T,\ \forall k = 1,\cdots,K. \tag{11}$$

In the following, we prove that

$$\Xi_{\text{min}} = \min \boldsymbol{\Psi}^k_{\text{sum}} \tag{12}$$

For all $\boldsymbol{w}$ and $\hat{\boldsymbol{w}}$, we have $\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}(k) \geq \min \boldsymbol{\Psi}^k_{\text{sum}}$ with $w^k \neq \hat{w}^k$ or $\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}(k) = 0$ with $w^k = \hat{w}^k$. Since $\boldsymbol{w}$ and $\hat{\boldsymbol{w}}$ are two distinct points, we have $\Xi_{\text{min}} \geq \min \boldsymbol{\Psi}^k_{\text{sum}}$. In addition, the codebook is obtained by a Cartesian product and we can achieve $\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}(k) > 0$ and $\tau_{\boldsymbol{w}\to\hat{\boldsymbol{w}}}\left(\tilde{k}\right) = 0, \forall \tilde{k} \neq k$ for all possible $\boldsymbol{w}$ and $\hat{\boldsymbol{w}}$. $\Xi_{\text{min}} = \min \boldsymbol{\Psi}^k_{\text{sum}}$ are obtained.

Therefore, we focus on improving the performance of the sparse codebook by maximizing the $\boldsymbol{\Psi}^k_{\text{sum}}$ of a single RE rather than Ξ_{min}. The computation complexity significantly reduces from $M^J(M^J-1)/2$ to T^{d_f} and this enables us to obtain various codebooks with higher overloading factors.

To facilitate the understanding of the uniformly distribution of constellation $\boldsymbol{\Psi}^k_{\text{sum}}$, we rewrite (11) as

$$\boldsymbol{\Psi}^k_{\text{sum}} = \left\{\sum_{i=0}^{d_f-1}\sum_{t=0}^{T-1} \gamma^q_{i,t} t T^i | q = 1,\cdots,T^{d_f}\right\}, \tag{13}$$

where $\gamma^q_{i,t} = 1$ or 0. $\sum_{t=0}^{T-1}\gamma^q_{i,t} = 1$, $i = 1,\cdots,d_f$. According to (13), the superimposed codewords of $\boldsymbol{\Psi}^k_{\text{sum}}$ correspond to the T-base notation with d_f-bits. The step of T-base notation is one, meaning uniform distribution of $\boldsymbol{\Psi}^k_{\text{sum}}$. In Fig. 2, the Euclidean distances of all adjacent points in the superimposed constellations $\boldsymbol{\Psi}^k_{\text{sum}}$ are equal, which can significantly improve energy efficiency. In other words, this method can significantly improve the MED under the same energy constraint.

IV. Simulation Results

In this section, we compare our proposed codebooks with the existing low-complexity codebooks [10] with the overloading factor of $\lambda = 150\%$. The average power of sparse codewords is 1. Moreover, we propose sparse codebooks for VLC with a larger overloading factor, including $\lambda = 200\%$, $\lambda = 250\%$, and $\lambda = 300\%$. $M = 4$ is adopted for all numerical simulations.

Table I compares both PAPR and MED of different codebooks, two key performance indicators for VLC. One can see that both lower peak power and higher MED are distinctive features of our proposed codebook. Compared to the codebooks in [10] with $\lambda = 150\%$, our obtained codebooks achieve 3/5 of its peak power and 5.7 times of its MED. With an increase of λ, the peak power grows and the MED reduces.

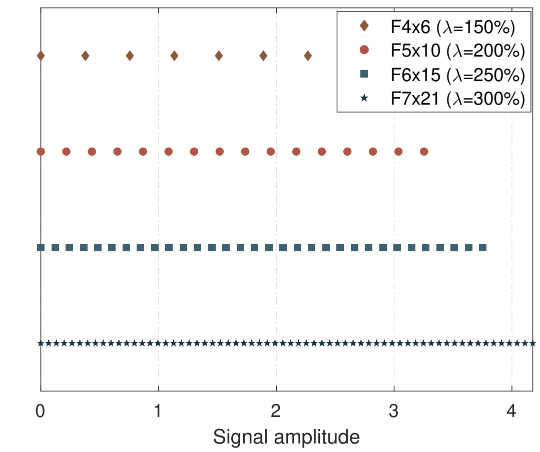

Fig. 2. Superimposed constellation on single RE for codebooks with different overloading factors.

TABLE I
THE PAPR COMPARISON OF DIFFERENT CODEBOOKS

	Peak	Average	PAPR	MED
LowCom. [10]($\lambda = 150\%$)	4.3489	1.3275	3.2759	0.0661
Prop.($\lambda = 150\%$)	2.6458	1.3229	2.0000	0.3780
Prop.($\lambda = 200\%$)	3.2540	1.6270	2.0000	0.2169
Prop.($\lambda = 250\%$)	3.7538	1.8769	2.0000	0.1221
Prop.($\lambda = 300\%$)	4.1769	2.0884	2.0000	0.0663

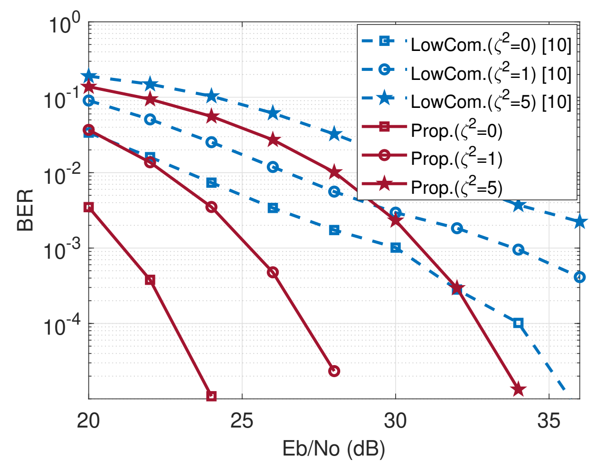

Fig. 3. BER performance comparison of different codebooks for $K = 4$, $J = 6$ and $\lambda = 150\%$.

The BER performance of codebooks with $K = 4$ and $J = 6$ is depicted in Fig. 3. From the results, one can see that as ζ^2 increases, the BER performance gets deteriorated drastically. When $\zeta^2 = 0$, the proposed codebooks achieve about 11.3 dB over the codebooks in [10] at BER = 1e-4. Moreover, the substantial performance gains can be achieved by the proposed codebooks in the presence of shot noise with $\zeta^2 = 1$ and $\zeta^2 = 5$.

Fig. 4 compares the BER performance of different codebooks in the presence of non-linear deterioration where smooth factor t is used to express the impact of different nonlinear deterioration. Due to the presence of higher peak power

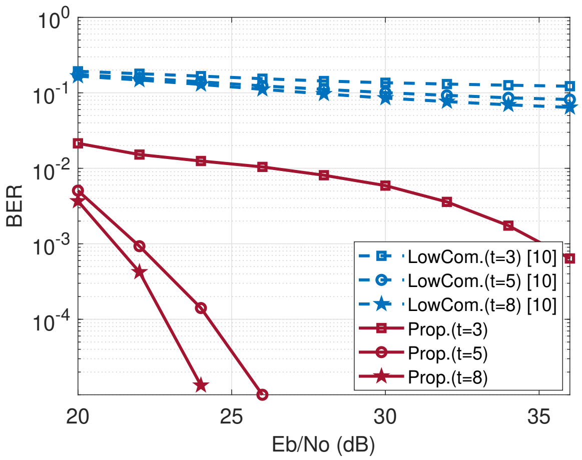

Fig. 4. BER performance with non-linear deterioration for $K = 4$, $J = 6$ and $\lambda = 150\%$.

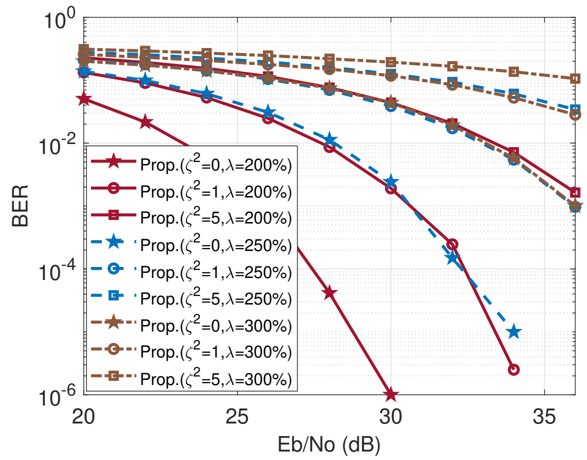

Fig. 5. BER performance of proposed codebook with higher overloading factors (up to 300%).

and PAPR in low-complexity codebook [10], there is a significant degradation of BER performance compared to our proposed codebooks and as the smooth factor t changes, BER performance is relatively stable. In this letter, our obtained codebooks have a smaller peak power and PAPR, leading to minimum degradation of BER performance compared to the low-complexity codebook and BER performance varies greatly with t. As the nonlinear coefficient t decreases, the BER performance gets improved significantly. When t increases, although performance degradation may occur, it is still acceptable and superior to the low-complexity codebooks in [10].

The BER performance with higher overloading factors is shown in Fig. 5. It is noted that the Eb/N0 of our proposed codebooks with $\lambda = 200\%$ is 28.8 dB at BER = 1e-5 and 24 dB for $\lambda = 150\%$. The minimum Eb/N0 for codebooks with $\lambda = 250\%$ to reach BER = 1e-4 is 32.3 dB. Compared with the low-complexity codebooks with $\lambda = 150\%$, 1.7 dB gain is obtained. With the increase of overloading factors, the gain loss is inevitable within a certain range of BER. For instance, the minimum Eb/N0 for codebooks with $\lambda = 300\%$ to reach BER = 1e-4 is 37.7 dB. Compared with the low-complexity codebooks with $\lambda = 150\%$, 3.7 dB gain loss is obtained.

Fig. 6 depicts the throughput of a single RE as a function of Eb/N0 obtained by (3). The overloaded VLC system can meet the need for more users under the same spectrum resource constraint. One can see that the throughput is a monotonically increasing function with respect to the overloading factor λ. As Eb/N0 increases, the system capacity gradually increases and there is an Eb/N0 threshold. When Eb/N0 exceeds the threshold, the system capacity tends to stabilize. When $\lambda = 150\%$, the proposed codebooks have the same threshold (23dB) as the low-complexity codebooks in [10], and when Eb/N0 is below the threshold, the system capacity based on the proposed codebook gets improved significantly. As the λ increases, the Eb/N0 threshold of the system will also increases, and so does the system capacity. Thus, a high-capacity VLC system can be achieved by designing a uniform-distributed constellation codebook with improved BER performance.

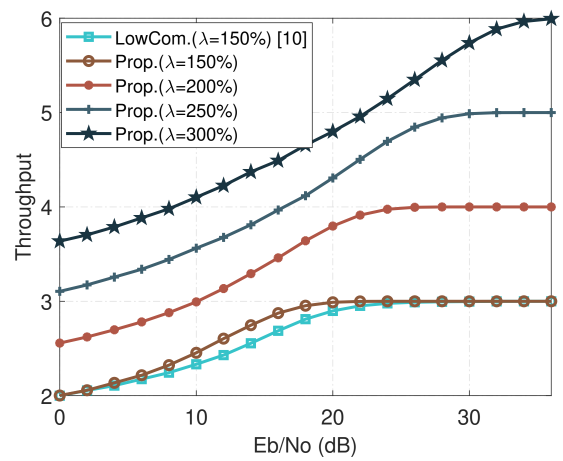

Fig. 6. Throughput of different overloading factor λ using (3).

V. Conclusions

In this work, we have proposed a novel class of sparse codebooks to improve the BER performance of VLC systems that suffer from nonlinear degradation and shot noise. It is shown that the proposed design scheme significantly reduces the peak power and improves MED by achieving a uniform distribution of superimposed constellations on RE. The simulation results demonstrate that the proposed codebooks deliver substantial BER gains over existing codebooks in the VLC channel. In addition, sparse codebooks with larger overloading factors are proposed to further improve transmission capacity at the desired BER performance.

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