A Perceptual Computing Approach for Learning Interpretable Unsupervised Fuzzy Scoring Systems

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***Abstract*—Scoring the driver's behavior through the analysis of his/ her road trip data is an active area of research. However, such systems suffer from a lack of explainability, integration of expert bias in the calculated score, and ignoring the semantic uncertainty of various variables contributing to the score. To overcome these limitations, we have proposed a novel perceptual computing based unsupervised scoring system. The prowess of the proposed system has been exemplified in a case study of driver's scoring from telemetry data. Our proposed approach yields scores that showed a higher significant separability between drivers performing responsible or irresponsible (aggressive or drowsy) driving behaviours, than the formal method of computing these scores (a p value of 3.94×10^{-4} and 3.42×10^{-3}, respectively, in a Kolmogorov-Smirnov test). Further, the proposed method displayed higher robustness in the bootstrap test (where 30% of original data was omitted at random) by providing scores that were 90% similar to the original ones for all results within a confidence interval of 95%.**

***Index Terms*—Computing with Words, Fuzzy Logic, Perceptual Computing Systems, Unsupervised Scoring Systems.**

I. Introduction

DRIVING score estimation through the analysis of road trip data[1] is an active area of research. Various articles exist in the literature which has conducted studies for driving score estimation by incorporating various factors or conditions [1]–[7]. Independently, some works also focus on developing telematic devices [8] for calculating the score estimation on the basis of perceived driving behaviour. A case study was also conducted in [9]–[11], to estimate the score from the driving telemetry data, using a methodology similar to that commonly employed by the insurance or rental cars companies. The findings of this study have been presented in the form of a publicly available dataset. An outcome of these works has been to estimate the driver's score (behaviour) using the numeric values of the various imprecise variables (lane drifting, braking, etc.) and classify the behaviour linguistically (good, moderate, bad, etc.).

These driver's score estimation systems have demonstrated very good performances in their respective works; however, they have some shortcomings, too. They act like a 'black-box' and lack explainability. Often, such systems generally involve integrating several variables into a linear equation, which is purely based on the perception of the few individuals who tune or assign the score based on their subjective opinion. Such systems, therefore, hide the score calculation methodology from the end user. Also, they are seen as the subjacent family of supervised learning, where the experts provide apriori score labelling of the observations according to their subjective understanding. The scoring system then integrates the labelling bias of the experts in the learned model following a methodology akin to regression or multi-class problems. Another limitation of these systems is that they rely on the precise numeric values of various variables, which are semantically imprecise and vague. The perceptual understanding of the precise numerical values (or meanings) of these variables is rather soft (and uncertain) as it depends on the context (levels, places and so forth) and the person (a traffic officer, car mechanic, professional driver or insurance brokers [12]).

Thus, our position is that, in order to overcome the disadvantages of these scoring systems, we propose our novel *Perceptual Computing based Interpretable Unsupervised Fuzzy Scoring System*. The proposed system treats the scoring in an unsupervised way, computing with natural terms that express *perceptions* of the variables that make up the objective score. Perceptual Computing, the core methodology behind our scoring system, was proposed by Prof. Mendel in [13][2]. The means to achieve Perceptual Computing, is the framework called the *Perceptual Computer* or Per-C. The use of Per-C becomes reasonable whenever a computing system needs to process subjective linguistic information similar to the human cognitive process.

We have also demonstrated the utility of our proposed Per-C based scoring system using the telemetry data of [9]–[11]. We found that the driving scores obtained by our proposed system show a higher divergence between the ones obtained from responsible and irresponsible drivers (aggressive or drowsy), in a Kolmogorov-Smirnov test [16], with a higher significant value of $p = 3.94 \times 10^{-4}$ whereas the ones of a formal method of driver's scoring from telemetry [10] have $p = 3.42 \times 10^{-3}$. Further, a robustness analysis using a bootstrap test of 30% random removal of the original data showed that the resultant footprint of uncertainty (FOU) plots of[3] scores were 90% similar to the original one using the full data, with a confidence interval of 95%. In sum, our proposed approach causes a

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[1]This data is generally curated through mobile apps and analysed using AI models.

[2]Perceptual Computing is one instantiation of Prof. Zadeh's novel Computing with Words (CWW) framework [14], [15]

[3]These FOU plots were generated in the Per-C.

better and more stable partitioning of the scores with respect to expected responsible (or otherwise) driving behaviours.

The rest of the paper is organized as follows: Section II discusses the related literature, Section III gives details of the novel Per-C based design of Unsupervised Scoring system and Section IV discusses the results obtained from its applicability to the telemetry data of [9]–[11]. Section V gives detailed discussions on the obtained results. Finally, section VI concludes this paper and throws light on its future scope. Details on important concepts are given in the Supplementary Materials (SMs).

II. Related Work

In this section, we present some of the literary works that motivated us to bring forth the research presented in this paper. We also discuss the basics of IT2 FSs and related concepts.

A. Literature review

With regards to our case study for drivers scoring in [7], the authors said that driver's behaviour assessment was a difficult task, especially in the insurance applications, due to numerous factors such as the trade-off between application cost and data accuracy, data uncertainty, noisy data, etc. They proposed a fuzzy treatment for driver behaviour assessment. The focus of this work was on modelling the data uncertainty, although explainability and unsupervised modelling were not prioritised. In [5], authors performed supervised regression to predict near-miss events. They used information such as vehicle usage, attitude toward speeding, and time and proportion of urban/nonurban driving from the telematics data, as well as additional information such as acceleration, braking, and cornering. They concluded general remarks such as night-time driving was associated with a lower risk of cornering events, urban driving increased the risk of braking events, and speeding was associated with acceleration events. Nonetheless, the non-fuzzy supervised approach did not elaborate on the importance or interrelations, using the 'everyday language' or explainable terms in each input variable, for example, how does low, intermediate, high, very high speeding, acceleration, both, or in combination with other terms, predicts few, some, several or many near-miss events. Some works have tried fuzzy approaches to develop score systems that consider inputs and outputs as 'natural language' imprecise terms. Sohn et al. [17] presented a fuzzy logistic regression method for credit scoring that processing inputs and outputs as T1 fuzzy numbers. This work pointed out the importance of considering the imprecision and vagueness of the input and output data. However, the inference approach (logistic regression) requires supervised training and although variable's data could be defined as fuzzy numbers, logistic regression is not CWW approach per se and depends on extra model parameters (coefficients) that are not fuzzy linguistic terms, hindering its overall interpretability and straightforward tuning. In another previous work, Hoffmann et al. [18] proposed a method to estimate a score, also, in this case, a financial scoring, based on a descriptive fuzzy-rule base (FRB) classifier. Rule-based models can be considered to be interpretable, provided that the fuzzy inputs and outputs can be adequately conceptualized into linguistic terms [19]. However, Hoffmann et. al learning mechanism is also supervised (viz. genetic fuzzy rule generation), and the initial linguistic terms are replaced by optimised membership functions to support classification accuracy of the FRB, at the expense of losing its initial perceived meaning hindering explainability.

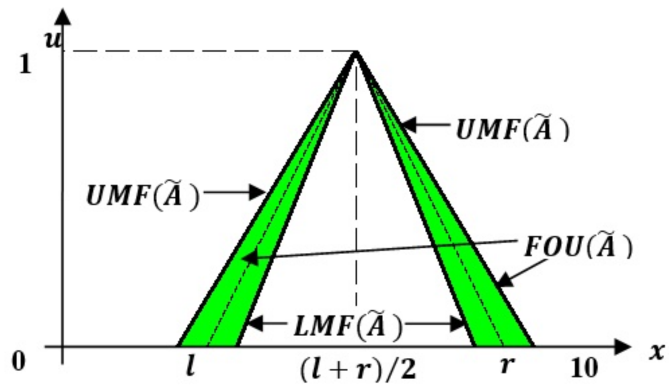


Figure 1: Membership functions of an IT2 FS [26]

Perceptual computing is a novel CWW approach. It has been used in various applications. The latest work, [20], uses perceptual computing for portfolio selection. Some latest works [21], [22] present a relation between granular computing and the CWW. CWW has also been used by Pratihar et. al for transportation [23]. The work [24] presents Python libraries for CWW methodologies.

B. A short primer on IT2 FS

The IT2 FSs were conceptualized by Prof. Zadeh in ([25]). They have a greater capability to model the semantics of linguistic information through the use of secondary membership degree. In the IT2 FSs, the secondary MF is assumed to be 1 everywhere.

An IT2 FS is pictorially shown in Fig. 1 and mathematically given in the form of Eq. (1) as:

$$\tilde{A} = \{(x, \mu(x), \mu_{\tilde{A}}(x, \mu(x)) = 1) \mid x \in U, 0 \leq \mu(x) \leq 1\} \tag{1}$$

here x is the data point, $\mu(x)$ isthe primary membership and $\mu_{\tilde{A}}(x, \mu(x))$ is the secondary membership. Also, in the Fig. 1, it can be seen that a T1 FS is shown inside the FOU of IT2 FS by a dashed line, whose ends rest on the x-axis at l and r. This T1 FS is called an embedded T1 FS. According to ([27]), the FOU of an IT2 FS can be considered as a union of all such embedded T1 FSs.

Additionally, the concept of the neighborhood is important. For any data point x lying inside an IT2 FS, its Neighborhood is defined as: $NH = \{\{\overleftarrow{NH}, \overrightarrow{NH}\} \mid \overleftarrow{NH} = \underset{cen \leq x}{\operatorname{argmax}}\, d(cen - x), \overrightarrow{NH} = \underset{cen \geq x}{\operatorname{argmin}}\, d(cen - x)\}$, cen : centroid value and $d(cen - x)$: distance between cen and x.

III. A Novel Per-C Based Unsupervised Scoring System with Type-2 Fuzzy Linguistic Terms

In classical Per-C, the semantics of problem variables are represented using IT2 FS models or FOU, which are constructed inside the encoder using the endpoint data intervals collected from a group of subjects. This data collection has limitations as a large amount of time is required for data collection and many users do not provide the data seriously [28]. Further, more often than not, each of problem variables, have an associated stream of numeric data values[4] [29]–[32], etc. This is also true for the original telemetry data [9]–[11].

Also, in the existing Per-C's CWW engine, linguistic weight is associated with a variable (and not individual linguistic terms associated with a variable). Further to it, a user chooses a linguistic term and elicits its respective associated linguistic weight at the time of aggregation. We feel that different linguistic terms of variable may have different connotations. For example, the amount of negative connotation attached to 'Very Low' may not be the same as the amount of positive connotation attached to 'Very High'. Thus, assigning the same weight to all the linguistic terms of a variable seems a little impractical.

Hence, for our proposed Per-C based unsupervised scoring system, we developed an encoder which disambiguates and conceptualises stream of numeric values, as the *fuzzy linguistic terms* using Fuzzy C-means (FCM) [33], [34]. They are later mapped into FOUs of the associated linguistic terms of a variable (Details are discussed in Section III-A). Also, in the CWW engine, the selection of the linguistic terms to be aggregated is data-driven. Each linguistic term is assigned a different linguistic weight instead of a variable. Finally, for human explainability, we generate linguistic recommendations from the decoder.

It is pertinent to mention that our proposed scoring system will define the boundaries of the linguistic terms in an unsupervised way. The scoring system only requires some prior information about the ordering of the linguistic terms of variables and whether these variables semantically support (or oppose) the score. This is all needed to convey the information from these variables into an overall score that will be interpretable (in essence). Our proposed scoring system also processes the linguistic information in same three steps of existing Per-C viz., encoder (Steps 1-3), CWW engine (Steps 4-6) and decoder (Steps 7-8), which are presented in the algorithm 1, and discussed next.

A. Encoder (Steps 1-3)

The Encoder of the novel Per-C based unsupervised scoring system consists of three steps as seen from Algo-1. Input to the encoder (please see Step 1) is V number of variables (which contribute to the score), each with an associated L number of linguistic terms and N associated numeric data values.

Then in the Step 2a, data cleaning is performed by removing the duplicates (if any) from the N number of data values to arrive at M surviving data values such that $M \leq N$. In Step 2b, the L centroids of these M data values are found (Linguistic terms, $LT_i, i = 1, 2, ..., L$) using the FCM, which also gives the degree of memberships of each M data value into the fuzzy boundaries around each L centroid. In Step 2c, for each data value say x, of the M data values, we define twin valued set called the neighborhood $NH = \{\overleftarrow{NH}, \overrightarrow{NH}\}$, where $\overrightarrow{NH}$ identifies the centroid which is closest to x and $\overleftarrow{NH}$ is the further centroid. Thus, NH enables the calculation of Upper Membership Function or UMF ($\bar{\mu}(x)$) and Lower Membership Function or LMF ($\underline{\mu}(x)$) values for each data point x. It is mentioned here that each of these M data values can belong to a maximum of two adjacent centroids with membership degrees $\bar{\mu}(x)$ and $\underline{\mu}(x)$, because these centroids are in one dimension. There is always more uncertainty about the boundary of the partition [35], and thus, we want membership degrees in a maximum of two adjacent centroids for any of the M data values.

In the Step 2d, the $\bar{\mu}(x)$ and $\underline{\mu}(x)$ are used for mapping each of the M data values into one of the interior or shoulder (left or right) FOUs. Consider a plot of the $\bar{\mu}$ and $\underline{\mu}$ for a data value, x (of M unique data values), belonging to a i^{th} linguistic term $LT_i, i = 1, 2, ..., L$, as shown in Fig. 2a. In the Fig., the blue colored curve is the $\bar{\mu}$ and orange colored is the $\underline{\mu}$. The term LT_i overlaps on the left side with LT_{i-1} and the right side with LT_{i+1}. From this Fig., 2a, we map the $\bar{\mu}(x)$ and $\underline{\mu}(x)$ into the UMF and LMF of the FOU parameters for the interior and shoulder IT2 FS word models, using (2)-(4). It is mentioned here that the resulting IT2 FS word model,as shown in Fig., 2b, is the interior FOU. However, the left shoulder of right shoulder FOU may also be obtained. We arrive at the FOU parameters as explained below.

a) UMF parameters: The UMF of the IT2 FS is defined by the parameters a, b, c and d (Please see Fig. 2b). To estimate their values through Fig. 2a, a is defined as the smallest x value for which $\bar{\mu} = 1$ in LT_{i-1} and $\underline{\mu} = 0$ in LT_i ($NH = \{LT_i, LT_{i-1}\}$); b is the smallest x that has $\bar{\mu} = 1$ in LT_i and $\underline{\mu} = 0$ in LT_{i-1} ($NH = \{LT_{i-1}, LT_i\}$); c is the largest x at which $\bar{\mu} = 1$ in LT_i and $\underline{\mu} = 0$ in LT_{i+1} ($NH = \{LT_{i+1}, LT_i\}$); and d is the largest x value for which $\bar{\mu} = 1$ in LT_{i+1} and $\underline{\mu} = 0$ in LT_i ($NH = \{LT_i, LT_{i+1}\}$). However, for left shoulder FOUs, $a = b = 0$ and for right shoulder FOUs, $c = d = 10$. It is mentioned here that wherever the maximum value of 1 and minimum of 0 of $\bar{\mu}$ or $\underline{\mu}$ are not possible, then the maximum possible values of $\bar{\mu}$ or $\underline{\mu}$ as obtained from FCM should be used.

b) LMF parameters: The LMF of the IT2 FS word model is defined by the parameters e, f, g and μ_f (Please see Fig. 2b). From Fig. 2a it follows that the value of parameter e is the average of all $x = e_q, q = 1, ..., j$, where $e_q, q = 1, 2, ..., j$ are the respective j^{th} data points satisfying the condition that $\underline{\mu}(x = e_q) = 0$ in LT_i and $\underline{\mu}(x = e_q + 1) \neq 0$ in LT_i, $e_q + 1$, being the immediately next data point to e_q, lying within the LT_i. Similarly, the value of g is the average of all $x = g_q, q = 1, 2, ..., j$, where $g_q, q = 1, ..., j$ are the respective j^{th} data points satisfying the condition that $\underline{\mu}(x = g_q) \neq 0$ in LT_i and $\underline{\mu}(x = g_q{+}1) = 0$ in LT_i, $g_q{+}1$, being the immediate successor of q_q, lying within the LT_i. The parameter f's value

[4] With the progression of Industry 4.0, sensors are being increasingly deployed in the environment which collects a stream of data values for the problem variables.

Algorithm 1 Novel Per-C based Unsupervised Scoring System

#Encoder

1: Input: V: Number of variables contributing to the score, L: Number of associated linguistic terms to each variable, N: Number of data values associated with each variable

2: For each Variable Repeat:

a: Remove the duplicates from the N number of data values to arrive at M unique data values. ▷ **Data Cleaning**

b: Subject these M data values to FCM to obtain L number of centroids $LT_i, i = 1, 2, ..., L$, as well as degree of memberships of each M data value into each of the LT_i centroids. ▷ **Data Processing**

c: $\forall x \in M$, define Neighborhood $NH = \{\{\overleftarrow{NH}, \overrightarrow{NH}\} \mid \overleftarrow{NH} = \underset{cen \le x}{\operatorname{argmax}}\, d(cen - x), \overrightarrow{NH} = \underset{cen \ge x}{\operatorname{argmin}}\, d(cen - x)\}$, cen : centroid value and $d(cen - x)$: distance between cen and x. Calculate the UMF for x as: $\bar{\mu}(x) = max\{\mu_{NH} = \mu_{\overrightarrow{NH}}\}$ and LMF as: $\underline{\mu}(x) = min\{\mu_{NH} = \mu_{\overleftarrow{NH}}\}$.

d: For $\exists x \in M$, lying within i^{th} Linguistic term (LT_i), map it into interior or shoulder FOUs, where UMF of the FOU is defined by parameters: {a,b,c,d} and LMF by: {e,f,g,μ_f}, as ▷ **Mapping into FOU**

Left Shoulder FOU:

$$\begin{aligned}
&a = 0, b = 0, e = 0, g = \{\frac{\sum_{q=1}^{j} g_q}{q}, \mid \underline{\mu}(x = g_q) \neq 0 \cap \underline{\mu}(x = q_q + 1) = 0 \ \forall q_q, q = 1, 2, ..., j\}, f = 0, \mu_f = 1 \\
&c = max\{x \mid NH = \{LT_{i+1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, d = max\{x \mid NH = \{LT_i, LT_{i+1}\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}
\end{aligned} \tag{2}$$

Interior FOU:

$$\begin{aligned}
&a = min\{x \mid NH = \{LT_i, LT_{i-1}\}, \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, b = min\{x \mid NH = \{LT_{i-1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, \\
&c = max\{x \mid NH = \{LT_{i+1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, d = max\{x \mid NH = \{LT_i, LT_{i+1}\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\} \\
&e = \frac{\sum_{q=1}^{j} e_q}{q}, \mid \underline{\mu}(x = e_q) = 0 \cap \underline{\mu}(x = e_q + 1) \neq 0, g = \frac{\sum_{q=1}^{j} g_q}{q}, \mid \underline{\mu}(x = g_q) \neq 0 \cap \underline{\mu}(x = q_q + 1) = 0, \\
&f = \frac{\sum_{q=1}^{j} f_q}{q}, \mid max(\underline{\mu})\} \ \mu_f = \underline{\mu}(f_q) \cap q = 1, 2, ..., j
\end{aligned} \tag{3}$$

Right Shoulder FOU:

$$\begin{aligned}
&a = min\{x \mid NH = \{LT_i, LT_{i-1}\}, \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, b = min\{x \mid NH = \{LT_{i-1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, \\
&c = 10, \ d = 10, e = \{\frac{\sum_{q=1}^{j} e_q}{q}, \mid \underline{\mu}(x = e_q) = 0 \cap \underline{\mu}(x = e_q + 1) \neq 0 \ \forall e_q, q = 1, 2, ..., j\}, \ f = 10, \ g = 10, \ \mu_f = 1
\end{aligned} \tag{4}$$

3: Determine the IT2 FS word models for linguistic terms of all the variables and store them in the codebook

#CWW Engine

4: Input $[n_1, n_2, ..., n_V]$: Data vector containing numeric values of each variable

5: For each $n_j, j = 1, 2, ..., V$ in Step 4, find out the linguistic term associated with the respective variable so that n_j has the highest degree of membership in it. Also, find out the associated linguistic weight of this term.

6: Extract the IT2 FS model $\tilde{X}_j$ of each linguistic term and its associated linguistic weight $\tilde{W}_j$ from the codebook (of Step 3) and aggregate them using LWA as:

$$\tilde{Y}_{LWA} = \frac{\sum_{j=1}^{V} \tilde{X}_j \tilde{W}_j}{\sum_{j=1}^{V} \tilde{W}_j} \tag{5}$$

#Decoder

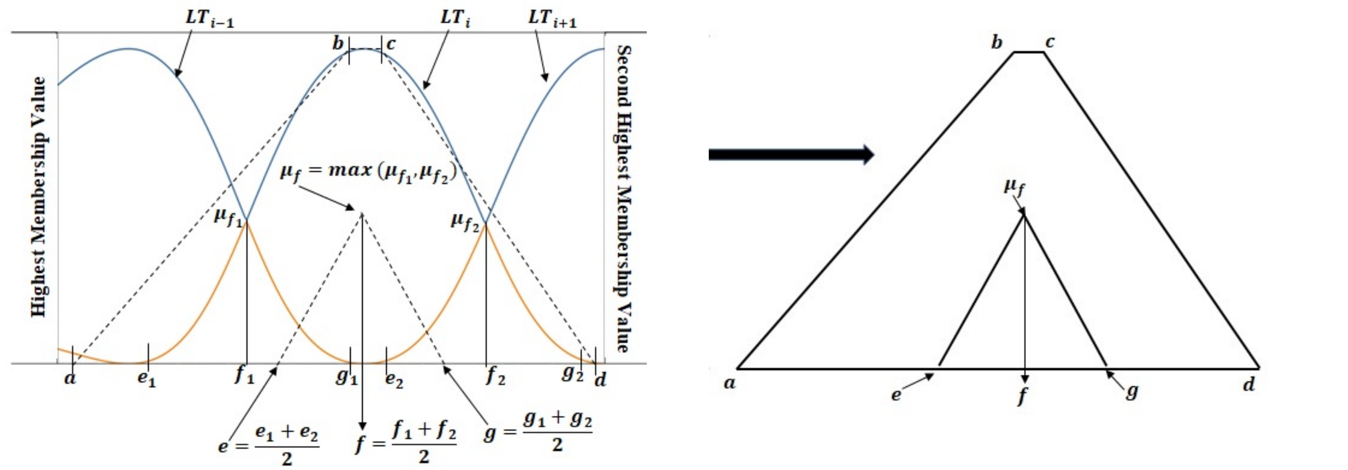
7: Numeric recommendation for the $\tilde{Y}_{LWA}$ is given based on c_l and c_r determined from the EKM algorithm, as:

$$c_{avg} = \frac{c_l + c_r}{2} \tag{6}$$

8: Linguistic recommendation for $\tilde{Y}_{LWA}$ is given using Jaccard's similarity measure as:

$$sm_j(\tilde{Y}_{LWA}, \tilde{L}_k) = \frac{\sum_{j=1}^{N} min(\bar{\mu}_{\tilde{Y}}(x_j), \bar{\mu}_{L\tilde{T}_k}(x_j)) + \sum_{j=1}^{N} min(\underline{\mu}_{\tilde{Y}}(x_j), \underline{\mu}_{L\tilde{T}_k}(x_j))}{\sum_{j=1}^{N} max(\bar{\mu}_{\tilde{Y}}(x_j), \bar{\mu}_{L\tilde{T}_k}(x_j)) + \sum_{j=1}^{N} max(\underline{\mu}_{\tilde{Y}}(x_j), \underline{\mu}_{L\tilde{T}_k}(x_j))} \tag{7}$$

$\tilde{LT}_k$ is a codebook linguistic term from Step 3 and x_j are equally spaced inside the support of $\tilde{Y}_{LWA} \cup \tilde{LT}_k$.



(a) $\bar{\mu}$ and $\underline{\mu}$ values of LT_i and its adjacent centroids LT_{i-1} and LT_{i+1}

(b) FOU parameters of the IT2 FS constructed from $\bar{\mu}$ and $\underline{\mu}$

Figure 2: Mapping the UMF ($\bar{\mu}$) and LMF ($\underline{\mu}$) from centroid LT_i into FOU parameters of the IT2 FS

is the average of all $x = f_q, q = 1, 2, ..., j$, where $f_q, q = 1, 2, ..., j$ are the respective j^{th} data points of highest $\underline{\mu}$, lying within LT_i and μ_{f_q} is the $\underline{\mu}$ values at $x = f_q$. However, for the left shoulder FOU, $e = f = 0$ and for the right shoulder FOU, $f = g = 10$. Also, $\mu_f = 1$ for both these shoulder FOUs. An important point to note in Fig. 2a is that it shows only two instances of each of e_q, q_q and f_q. The data can contain multiple of these data points.

Thus, the Step 2 is repeated for all the variables so that FOUs are obtained for all the linguistic terms corresponding to each of the variables, the FOUs as well as linguistic terms are stored in a codebook (Please see Step 3.

B. CWW Engine (Steps 4-6)

The CWW engine consists of three steps as seen from Algorithm-1. Step 4 inputs a data vector containing the numeric data values corresponding to each of the V variables into the Algo, where for each numeric value in the data vector, a linguistic term from codebook is found in which this numeric value has maximum membership degree, in Step 5 (Please refer Section SM-V). Then after finding the associated linguistic weight of each of these linguistic terms, the FOU data for each linguistic term and its associated weight is extracted from the codebook (constructed in Encoder) and aggregated to generate an IT2 FS $\tilde{Y}_{LWA}$ using (5), in Step 6 (Please refer [27] or Section SM-II.B).

C. Decoder (Steps 7-8)

The decoder of the proposed scoring system generates a numeric value, c_{avg} using (6) for the $\tilde{Y}_{LWA}$ using the switch points c_l and c_r of an IT2 FS (Please refer Section SM-II.C) in the Step 7. Finally, in the Step 8, a linguistic recommendation is generated for the $\tilde{Y}_{LWA}$ using the Jaccard's similarity measure using (7).

The complete block diagram of our proposed scoring system is shown in Fig. 3. From Fig., it can be seen that the input to our proposed scoring system is a stream of numeric data values, linguistic labels and linguistic weights. Inside the encoder, the FOUs are generated for the linguistic terms and their associated weights and stored in a codebook (please refer Section III-A. Then a numeric data vector causes the extraction of the linguistic terms' and their respective associated linguistic weights' FOUs to be extracted from the codebook based on the membership degree of each numeric value into the respective linguistic term from the codebook. These extracted FOUs of the linguistic terms and weights are subjected to LWA, which generates an aggregated nine point Y_{LWA} at the output of CWW engine (please refer Section III-B). This Y_{LWA} is fed as input to the decoder to generate a numeric score, c_{avg} using the EKM algorithm and a linguistic recommendation using the Jaccard's similarity (please refer Section III-C).

IV. Results: Real-world case study for driver's score estimation data

In this section we demonstrate the prowess of our Novel Per-C based Unsupervised Scoring System (from Section III) using the real-life telemetry data from [9]–[11]. In these works, the authors collected (a stream of) numeric data values for seven variables: acceleration (AC), braking (BR), car following (CF), lane drifting (LD), lane weaving (LW), over-speeding (OS) and turning (TU), from the road trips of six drivers. AC (BR) denotes a sudden increase (decrease) in the vehicle speed, CF is a measure of the safe distance between the personal vehicle and the one ahead, LD is measured as a deviation from the centre of the driving lane, LW is the number of lane changes, OS represents driving above the maximum allowed speed limit and TU means a sudden change in vehicle's direction.

The drivers conducted trips in the motorway and the secondary types of road. Motorway roads had 2 to 4 lanes in each direction and around 120 km/hr of maximum allowed speed. The secondary road had principally one lane in each direction with a maximum allowed speed of 90 km/hr. Each driver performed three trips on the motorway road (round-trip,

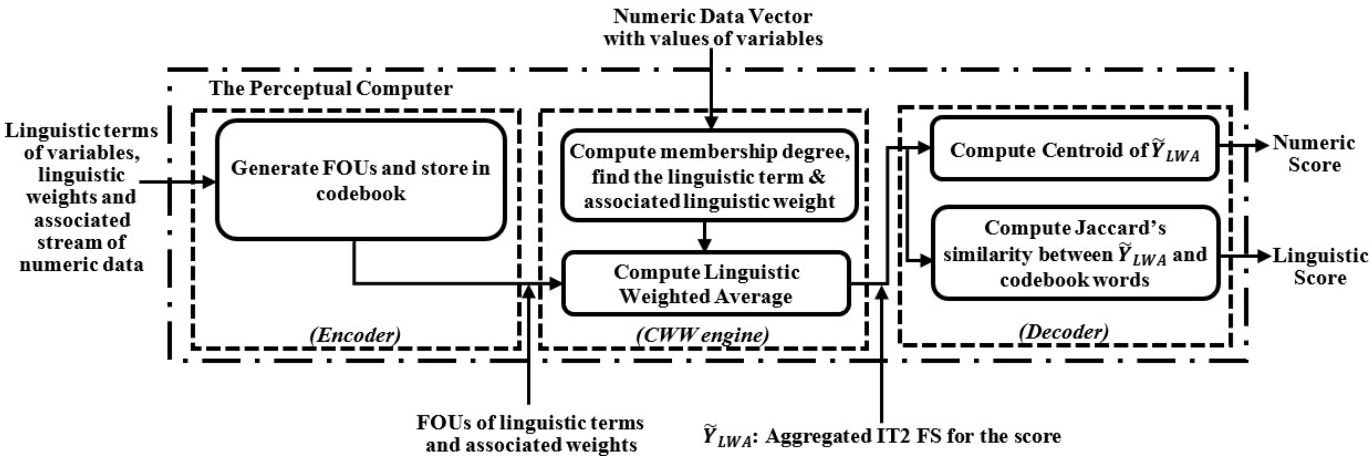


Figure 3: The proposed Per-C based unsupervised scoring system

around 25km each) and four trips on the secondary road (one-way, around 16km each).

We now present the working of our proposed novel Per-C based Unsupervised scoring system (from Section III) through this real-life telemetry data and use it to compute the overall trip score (Sections IV-A-IV-D). We also compare the results of our proposed system to those in the original telemetry data (Section V-A). We then give a robustness analysis of the scores obtained with our proposed scoring system (Section V-B).

A. Encoding telemetry data

Considering the same seven variables: AC, BR, CF, LD, LW, OS and TU [9]–[11], we associated five linguistic terms to each of these variables. Further, each linguistic term was allocated a linguistic weight (WT) (instead of variable), and WT was also assigned five linguistic terms:Very Low (WTV), Low (WTL), Medium (WTM), High (WTH) and Very High (WTE). The linguistic terms associated with the variables and the associated WT of each variable's linguistic term are listed in Table I.

For exemplifying the construction of (associated) linguistic terms' (of variables) FOU plots and those of the associated WT's, consider the variable AC in the motorway road. It had N = 16,631 associated numeric data values. Removal of duplicate values reduced them to M = 546 unique data values. As AC had five associated linguistic terms (refer Table I), we found L=5 centroids of these 546 numeric data values, as well as memberships degrees of each of these 546 values to every centroid's soft boundary, using FCM. Subsequently, we extracted the data points with the highest and second highest degrees of membership from these 546 values to estimate the FOU parameters.

Let's say the centroids are denoted as LT_1, LT_2, LT_3, LT_4 and LT_5. Consider LT_3, which will be used to define the linguistic variable ACM (please see Table I). We found that for LT_3, $x = 5.02$ is the smallest value at which the neighborhood $NH = LT_3, LT_2$ exists, with $\bar{(}\mu) = 0.89$ and $\underline{\mu} = 0.05$ (As stated in Section III-A, if $\bar{(}\mu) = 1$ and $\underline{\mu} = 0$ are not in the data set, then maximum available values of the two memberships should be used). The $x = 6.67$ is the minimum value at which $NH = LT_2, LT_3$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$, whereas $x = 6.78$ is the maximum value at which $NH = LT_4, LT_3$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. The $x = 8.06$ is the largest data value at which $NH = LT_3, LT_4$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. As LT_3, is the interior FOU, therefore, as seen from (3), $a = 5.02$, $b = 6.67$, $c = 6.78$ and $d = 8.06$. $x = 5.49$ and $x = 6.81$ satisfy the condition $\underline{\mu}(x = e_q) = 0$ in LT_3 and $\underline{\mu}(x = e_q + 1) \neq 0.0$ in LT_3. Therefore, $e_1 = 5.49$, $e_2 = 6.81$ and $e = \frac{e_1+e_2}{2} = 6.15$. Similarly, $x = 6.64$ and $x = 7.99$ give rise to $g = 7.32$. The $x = 6.07$ and $x = 7.4$ are the data values with the highest $\underline{\mu}$, lying within LT_3, and the membership degrees at both these points have a value of 0.45. Hence, $f = \frac{f_1+f_2}{2} = 6.74$ and $\mu_f = 0.45$. In this way, FOU parameters for the LT_3 are estimated. Similarly, FOU parameters of the interior FOUs of LT_2 and LT_4 are estimated.

For estimating the FOU parameters of left shoulder viz., LT_1, $x = 3.33$ is the maximum data value at which $NH = LT_2, LT_1$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. $x = 5.01$ is is the maximum data value at which $NH = LT_1, LT_2$, $\bar{\mu} = 0.89$ and $\underline{\mu} = 0.05$. Also, $x = 4.99$ is the only data point which satisfies the condition $\underline{\mu}(x = g_q) \neq\ 0$ in LT_3 and $\underline{\mu}(x = g_q + 1) = 0$ in LT_1. Therefore, using (2), $a = 0,\ b = 0,\ c = 3.33,\ d = 5.01,\ e = 0,\ f = 0,\ g = 4.99,\ \mu_f = 1.$

For estimating the FOU parameters of right shoulder viz., LT_5, $x = 8.07$ is the smallest data value at which neighborhood exists as $NH = LT_4, LT_5$, with $\bar{(}\mu) = 1$ and $\underline{\mu} = 0$. The $x = 9.34$ is the minimum data value at which $NH = LT_5, LT_4$, with $\bar{(}\mu) = 1$ and $\underline{\mu} = 0$. The $x = 8.16$ is the only data point which satisfy the condition $\underline{\mu}(x = e_q) = 0$ in LT_5 and $\underline{\mu}(x = e_q + 1) \neq\ 0$ in LT_5. Therefore, using (4), $a = 8.07,\ b = 9.34,\ c = 10,\ d = 10,\ e = 8.16,\ f = 10,\ g = 10,\ \mu_f = 1$. In this manner, the FOU parameters of all the linguistic terms of AC are determined and listed in Table II.

Similarly, the FOU plots for the associated linguistic terms of all other variables were computed. For generating the data stream for WT, all the data values associated with a variable were summed and processed, and FOU plots were generated. The obtained FOU plots along with the respective linguistic terms (of variables and WT) were stored in the form of a

Table I: Variables: Associated Linguistic Terms and Linguistic Weights.

Variables	Associated Linguistic Terms	Linguistic weight Motorway	Linguistic weight Secondary	Variables	Associated Linguistic Terms	Linguistic weight Motorway	Linguistic weight Secondary
	Very Low (ACV)	Very Low (WTV)	Very High (WTE)		Very High (BRE)	Very Low (WTV)	Very High (WTE)
	Low (ACL)	Low (WTL)	High (WTH)		High (BRH)	Low (WTL)	High (WTH)
Acceleration	Medium (ACM)	Medium (WTM)	Medium (WTM)	Braking	Moderate (BRM)	Medium (WTM)	Medium (WTM)
(AC)	High (ACH)	High (WTH)	Low (WTL)	(BR)	Less (BRL)	High (WTH)	Low (WTL)
	Very High (ACE)	Very High (WTE)	Very Low (WTV)		Very Less (BRV)	Very High (WTE)	Very Low (WTV)
	Very small (CFV)	Very Low (WTV)	Very Low (WTV)		Very Large (LDE)	Very Low (WTV)	Very High (WTE)
Car	Small (CFS)	Low (WTL)	Low (WTL)	Lane	Large (LDL)	Low (WTL)	High (WTH)
Following	Average (CFA)	Medium (WTM)	Medium (WTM)	Drifting	Average (LDA)	Medium (WTM)	Medium (WTM)
(CF)	Large (CFL)	High (WTH)	High (WTH)	(LD)	Small (LDS)	High (WTH)	Low (WTL)
	Very Large (CFE)	Very High (WTE)	Very High (WTE)		Very small (LDV)	Very High (WTE)	Very Low (WTV)
	Very High (LWE)	Very Low (WTV)	Very Low (WTV)		Very High (OSE)	Very Low (WTV)	Very Low (WTV)
Lane	High (LWH)	Low (WTL)	Low (WTL)	Over	High (OSH)	Low (WTL)	Low (WTL)
Weaving	Medium (LWM)	Medium (WTM)	Medium (WTM)	Speeding	Moderate (OSM)	Medium (WTM)	Medium (WTM)
(LW)	Low (LWL)	High (WTH)	High (WTH)	(OS)	Less (OSL)	High (WTH)	High (WTH)
	Very Low (LWV)	Very High (WTE)	Very High (WTE)		Very Less (OSV)	Very High (WTE)	Very High (WTE)
	Very Large (TUE)	Very Low (WTV)	Very Low (WTV)				
	Large (TUL)	Low (WTL)	Low (WTL)				
Turning	Average (TUA)	Medium (WTM)	Medium (WTM)				
(TU)	Small (TUS)	High (WTH)	High (WTH)				
	Very small (TUV)	Very High (WTE)	Very High (WTE)				

codebook. The FOU data for the linguistic terms of all the variables and WT in the motorway is given in Table II. The corresponding FOU data for the secondary road, as well as FOU plots for the two types of road, are given in Table SM-I, Fig. SM-4 and Fig. SM-5.

B. Running CWW Engine

To exemplify the selection of linguistic terms for a variable in CWW engine, consider a data vector containing the variables' $\{AC$, BR, CF, LD, LW, OS, $TU\}$ values in the motorway as: {10, 9.18, 9.79, 7.8, 10, 9.32, 8.1}. For each of these values of the variables, we find out the respective highest membership degrees in the respective linguistic terms of the variables (Section SM-II.B). Thus, the linguistic terms corresponding to the values of the variable given in the vector are $\{ACE$, BRV, CFE, LDS, LWV, OSV, $TUV\}$ (For full forms of ACE, BRV, CFE, LDS, LWV, OSV and TUV, please see Table I). From Table I, the respective corresponding WT of the linguistic terms are found as: $\{WTE$, WTE, WTE, WTH, WTE, WTE, $WTE\}$, where WTE is Very High and WTH is a high.

Hence, the FOU data for the respective linguistic terms and associated WT from codebook (or Table II) is extracted and aggregated using (5), to generate nine points IT2 FS word model (described by its UMF and LMF) given as $\tilde{Y}_{LWA}$={7.01, 8.87, 9.78, 10, 7.24, 8.45, 9.77, 9.86, 0.46}, the first four points in the $\tilde{Y}_{LWA}$ describe the UMF and remaining the LMF.

C. Decoding Telemetry Data into Drivers Scores

In CWW engine, the values of linguistic terms of variables were linguistic weighted averaged to generate a nine point FOU for $\tilde{Y}_{LWA}$ (Please see section IV-B). This $\tilde{Y}_{LWA}$ corresponds to the output variable, the driver's score (DS). However, this FOU for $\tilde{Y}_{LWA}$ is decoded to a numeric score, $c_{avg} = 8.81$, as discussed in Section III-C.

Now, from the works [9]–[11], we extracted the numeric data values for DS and associated five linguistic terms to it: Terrible (DSV), Poor (DSL), Borderline (DSM), Fair (DSH) and Good (DSE). Then, we generated the FOUs for these linguistic terms using the encoder, section III-A. The FOU data for these linguistic terms are stored in the codebook and shown in Table II for a motorway. The corresponding FOU data for the secondary road, as well as FOU plots for the two types of road, are given in Table SM-I, Fig. SM-4 and Fig. SM-5.

To generate linguistic recommendations for $\tilde{Y}_{LWA}$, we find out the similarity between $\tilde{Y}_{LWA}$ and the FOUs of five linguistic terms associated with DS, using Jaccard's similarity measure from (7). Thus, the closest linguistic recommendation comes out to be Good (DSH).

D. Calculating the overall driver's score for one trip

From the telemetry data [9]–[11], we picked up the data vectors containing numeric data values of seven variables, accumulated per second and aggregated them through the encoder and CWW engine of our proposed scoring system (Sections IV-A-IV-B), to arrive at aggregated nine points Y_{LWA}'s. Then we weighted aggregated these Y_{LWA}'s based on their frequency in the complete trip of a driver, to generate a Y_{LWA} for the complete trip, denoted as $Y_{Overall-Trip_{LWA}}$. A linguistic recommendation for $Y_{Overall-Trip_{LWA}}$ was generated using the Jaccard's similarity (from (7) between $Y_{Overall-Trip_{LWA}}$ and FOUs of linguistic terms for DS (please see Table II). Thus, the scores were calculated for motorway and secondary roads (for the normal, aggressive and drowsy type of behaviours). All the results are given in Table III.

From the Table, it follows that the Normal driving behaviour, is scored as either Fair (DSH) or Good (DSE); however, the Aggressive or Drowsy driving behaviours, are rated as Borderline (DSM) or Poor (DSL), and very rarely as Fair (DSH). Thus, our proposed scoring system differentiates

Table II: FOU data for the Linguistic Terms of the Variables and the Driver's Score for Motorway

Variables, Linguistic weights and Driver's Score	Associated Linguistic terms	FOU data										
		UMF				LMF				Centroid		
		a	b	c	d	e	f	g	μ_f	c_l	c_r	c_{avg}
Acceleration (AC)	Very Low (ACV)	0.00	0.00	3.33	5.01	0.00	0.00	4.99	1.00	1.62	2.21	1.91
	Low (ACL)	0.00	5.34	5.48	6.74	4.46	5.21	5.82	0.45	2.84	5.68	4.26
	Medium (ACM)	5.02	6.67	6.78	8.06	6.15	6.74	7.32	0.45	6.16	7.13	6.65
	High (ACH)	6.75	8.02	8.13	10.00	7.49	8.07	8.66	0.46	7.68	8.74	8.21
	Very High (ACE)	8.07	9.34	10.00	10.00	8.16	10.00	10.00	1.00	9.27	9.40	9.34
Braking (BR)	Very High (BRE)	0.00	0.00	1.03	2.96	0.00	0.00	2.82	1.00	0.91	1.12	1.02
	High (BRH)	0.00	2.86	3.04	5.01	2.07	2.96	3.85	0.46	1.93	3.57	2.75
	Moderate (BRM)	2.97	4.93	5.10	7.06	4.12	5.01	5.92	0.44	4.40	5.63	5.02
	Less (BRL)	5.02	6.99	7.17	10.00	6.18	7.07	7.97	0.46	6.46	8.08	7.27
	Very Less (BRV)	7.07	9.00	10.00	10.00	7.21	10.00	10.00	1.00	8.89	9.10	9.00
Car Following (CF)	Very small (CFV)	0.00	0.00	0.60	2.82	0.00	0.00	2.77	1.00	0.90	1.01	0.96
	Small (CFS)	0.00	3.03	3.23	5.14	1.79	2.97	3.89	0.45	1.98	3.58	2.78
	Average (CFA)	2.83	5.05	5.22	7.13	4.26	5.14	6.02	0.45	4.40	5.73	5.07
	Large (CFL)	5.15	7.06	7.23	10.00	6.27	7.14	8.01	0.46	6.55	8.13	7.34
	Very Large (CFE)	7.14	9.02	10.00	10.00	7.27	10.00	10.00	1.00	8.92	9.12	9.02
Lane Drifting (LD)	Very Large (LDE)	0.00	0.00	3.53	4.92	0.00	0.00	4.82	1.00	1.56	2.23	1.89
	Large (LDL)	0.00	4.85	4.98	6.40	4.28	4.92	5.57	0.46	2.68	5.36	4.02
	Average (LDA)	4.93	6.34	6.47	7.88	5.76	6.41	7.06	0.44	5.96	6.85	6.41
	Small (LDS)	6.41	7.83	7.96	10.00	7.25	7.89	8.54	0.46	7.45	8.62	8.03
	Very small (LDV)	7.89	9.28	10.00	10.00	7.99	10.00	10.00	1.00	9.20	9.35	9.28
Lane Weaving (LW)	Very High (LWE)	0.00	0.00	1.30	2.50	0.00	0.00	2.38	1.00	0.77	1.02	0.90
	High (LWH)	0.00	2.38	2.50	4.00	1.87	2.58	3.19	0.47	1.61	2.95	2.28
	Medium (LWM)	2.63	3.91	4.00	6.36	3.38	4.17	4.86	0.36	3.64	4.92	4.28
	Low (LWL)	4.12	5.71	6.00	10.00	5.40	6.50	7.27	0.36	5.53	7.77	6.65
	Very Low (LWV)	6.67	8.89	10.00	10.00	6.67	10.00	10.00	1.00	8.75	8.92	8.84
Over-Speeding (OS)	Very High (OSE)	0.00	0.00	1.42	3.38	0.00	0.00	3.36	1.00	1.09	1.32	1.20
	High (OSH)	0.00	3.52	3.70	5.47	2.58	3.50	4.36	0.45	2.21	4.09	3.15
	Moderate (OSM)	3.39	5.39	5.55	7.33	4.66	5.48	6.29	0.45	4.82	6.03	5.42
	Less (OSL)	5.48	7.26	7.43	10.00	6.53	7.34	8.15	0.46	6.79	8.26	7.52
	Very Less (OSV)	7.34	9.09	10.00	10.00	7.46	10.00	10.00	1.00	9.00	9.18	9.09
Turning (TU)	Very Large (TUE)	0.00	0.00	1.00	2.89	0.00	0.00	2.74	1.00	0.89	1.09	0.99
	Large (TUL)	0.00	2.78	2.96	4.87	1.88	2.88	3.75	0.46	1.89	3.43	2.66
	Average (TUA)	2.90	4.8	4.97	6.86	4.01	4.88	5.76	0.44	4.28	5.48	4.88
	Small (TUS)	4.88	6.80	6.97	10.00	6.02	6.88	7.74	0.46	6.28	8.00	7.14
	Very small (TUV)	6.87	8.74	10.00	10.00	7.01	10.00	10.00	1.00	8.79	9.03	8.91
Linguistic Weight (WT)	Very Low (WTV)	0.00	0.00	1.00	2.94	0.00	0.00	2.80	1.00	0.91	1.11	1.01
	Low (WTL)	0.00	2.84	3.02	4.99	2.04	2.94	3.84	0.46	1.93	3.55	2.74
	Medium (WTM)	2.95	4.91	5.09	7.05	4.10	5.00	5.91	0.44	4.38	5.62	5.00
	High (WTH)	5.00	6.98	7.16	10.00	6.17	7.06	7.96	0.46	6.45	8.08	7.26
	Very High (WTE)	7.06	9.00	10.00	10.00	7.20	10.00	10.00	1.00	8.89	9.09	8.99
Driver's Score (DS)	Terrible (DSV)	0.00	0.00	5.28	6.32	0.00	0.00	6.28	1.00	2.03	3.05	2.54
	Poor (DSL)	0.00	6.30	6.40	7.41	5.78	6.35	6.82	0.45	3.06	6.64	4.85
	Borderline (DSM)	6.33	7.38	7.46	8.48	6.96	7.42	7.89	0.45	7.09	7.74	7.41
	Fair (DSH)	7.42	8.45	8.53	10.00	8.02	8.48	8.95	0.45	8.17	9.01	8.59
	Good (DSE)	8.49	9.49	10.00	10.00	8.55	10.00	10.00	1.00	9.43	9.53	9.48

between different types of driving behaviour and assigns a score in a proper manner.

V. Discussions

We now draw comparisons between the results of our proposed system to those in the original telemetry data [9]–[11]; provide a robustness analysis of the scores obtained with our proposed scoring system; provide a conceptual comparison between the existing Per-C and that of our proposed system; and discuss some of the interesting facts and findings from the work in this paper, which are applicable to any scoring systems in general. More discussions in the context of the driver's scoring systems are given in the Section SM-IV, due to paucity of space.

A. Comparison of scores obtained with proposed system and original telemetry data

In the works [9]–[11], the data vectors containing numeric values of the 7 variables were collected per second of the driver's trip. These data values were arithmetically averaged to calculate the overall driver's score for one trip. The scores were classified for the normal, aggressive and drowsy type of behaviour, where the driver in these trips was asked to simulate this type of behaviour. For reference, the respective scores for the driver, type of road and behaviour obtained with the telemetry data are listed in the last column of Table III. The table also lists the drivers' scores obtained with our proposed scoring system.

For comparison between the scores obtained with our proposed scoring system and original telemetry data, we ran the Kolmogorov-Smirnov statistics [16] on the respective streams

Table III: Comparison of drivers' trip scores in Motorway & Secondary: using proposed scoring system vs Telemetry Data

Driver*	Behavior**	Overall Score with proposed scoring system													Telemetry
		a	b	c	d	e	f	f	g	μ_f	c_l	c_r	c_{avg}	LS***	Score
		Type of Road: Motorway													
D1	N	6.52	8.65	9.61	9.93	6.93	8.3	9.6	9.73	0.48	8.31	8.86	8.58	Fair (DSH)	8.84
	A	6.41	8.50	9.33	9.89	6.92	8.15	9.31	9.54	0.46	8.11	8.76	8.44	Fair (DSH)	8.76
	D	5.45	8.14	9.14	9.88	6.34	7.63	9.14	9.41	0.42	7.58	8.52	8.00	Borderline (DSM)	7.52
D2	N	6.60	8.62	9.54	9.90	6.96	8.26	9.53	9.69	0.48	8.30	8.84	8.57	Fair (DSH)	9.08
	A	4.75	7.46	8.26	9.56	5.89	7.19	8.22	8.71	0.45	6.87	7.98	7.42	Borderline (DSM)	6.91
	D	5.96	8.28	9.19	9.89	6.62	7.96	9.19	9.44	0.47	7.86	8.62	8.24	Borderline (DSM)	7.87
D3	N	6.94	8.83	9.77	9.94	7.16	8.50	9.77	9.85	0.50	8.57	8.99	8.78	Fair (DSH)	9.62
	A	6.05	8.27	9.24	9.93	6.54	8.00	9.22	9.48	0.50	7.93	8.61	8.27	Fair (DSH)	7.54
	D	5.57	8.02	8.86	9.80	6.41	7.61	8.86	9.19	0.42	7.51	8.42	7.96	Borderline (DSM)	7.67
D4	N	7.14	8.97	9.91	10.00	7.29	9.36	9.91	9.94	0.78	8.84	9.10	8.97	Good (DSE)	9.84
	A	5.42	7.89	8.77	9.81	6.21	7.57	8.74	9.14	0.46	7.42	8.33	7.87	Borderline (DSM)	7.24
	D	6.09	8.44	9.36	9.90	6.77	7.99	9.36	9.55	0.43	7.98	8.73	8.35	Borderline (DSM)	8.40
D5	N	6.34	8.43	9.26	9.95	6.85	8.13	9.24	9.51	0.48	8.07	8.73	8.40	Fair (DSH)	8.44
	A	5.07	7.52	8.31	9.77	5.92	7.25	8.28	8.80	0.47	7.04	8.08	7.56	Borderline (DSM)	6.99
	D	5.13	8.05	9.18	9.94	6.10	7.59	9.15	9.45	0.45	7.49	8.46	7.97	Borderline (DSM)	6.61
D6	N	6.64	8.68	9.66	10.00	6.87	8.83	9.66	9.79	0.67	8.49	8.88	8.68	Fair (DSH)	8.66
	A	5.53	7.92	8.79	9.73	6.22	7.56	8.76	9.13	0.47	7.47	8.30	7.88	Borderline (DSM)	7.29
	D	5.38	7.77	8.65	9.72	6.15	7.28	8.72	9.06	0.38	7.29	8.26	7.78	Borderline (DSM)	9.96
		Type of Road: Secondary													
D1	N1	5.88	8.02	8.90	9.87	6.64	7.82	8.90	9.32	0.46	7.69	8.52	8.11	Fair (DSH)	8.40
	N2	6.36	8.35	9.27	9.85	6.91	8.13	9.25	9.55	0.48	8.08	8.72	8.40	Fair (DSH)	8.74
	A	5.38	7.77	8.82	9.65	6.13	7.44	8.81	9.18	0.45	7.40	8.25	7.82	Borderline (DSM)	7.28
	D	5.32	7.58	8.38	9.58	6.31	7.48	8.41	8.90	0.48	7.21	8.15	7.68	Borderline (DSM)	8.30
D2	N1	6.63	8.48	9.41	9.88	7.09	8.37	9.41	9.62	0.50	8.27	8.83	8.55	Fair (DSH)	9.18
	N2	6.27	8.29	9.33	9.81	6.87	8.41	9.17	9.57	0.62	8.10	8.69	8.39	Fair (DSH)	8.73
	A	4.89	7.25	8.41	9.37	5.64	7.01	8.44	8.91	0.47	7.00	7.85	7.42	Poor (DSL)	6.57
	D	5.07	7.77	8.78	9.55	6.15	7.45	8.76	9.11	0.45	7.25	8.21	7.73	Borderline (DSM)	7.85
D3	N1	7.19	8.90	9.88	10.00	7.43	8.93	9.88	9.94	0.6	8.77	9.14	8.95	Good (DSE)	9.69
	N2	6.13	8.13	9.16	9.70	6.66	7.47	9.24	9.42	0.30	7.76	8.60	8.18	Fair (DSH)	8.48
	A	5.15	7.51	8.66	9.64	5.89	7.34	8.64	9.13	0.50	7.26	8.10	7.68	Borderline (DSM)	7.20
	D	6	8.34	9.34	9.72	6.83	8.17	9.33	9.53	0.5	7.96	8.66	8.31	Fair (DSH)	9.08
D4	N1	7.04	8.76	9.73	10.00	7.36	9.06	9.73	9.85	0.69	8.67	9.06	8.87	Fair (DSH)	9.68
	N2	7.22	8.91	9.91	10.00	7.45	9.13	9.90	9.95	0.67	8.82	9.15	8.98	Good (DSE)	9.71
	A	5.40	7.72	8.71	9.67	6.46	7.68	8.60	9.12	0.50	7.38	8.31	7.85	Borderline (DSM)	8.08
	D	5.48	8.09	9.20	9.67	6.35	7.83	9.20	9.43	0.50	7.68	8.44	8.06	Borderline (DSM)	8.18
D5	N1	7.13	8.86	9.84	10	7.41	8.87	9.83	9.91	0.58	8.72	9.11	8.91	Good (DSE)	9.53
	N2	6.07	8.29	9.25	9.73	6.83	8.01	9.24	9.47	0.45	7.91	8.65	8.28	Fair (DSH)	8.87
	A	4.17	6.00	7.25	8.59	4.62	5.94	7.50	8.18	0.52	6.17	6.83	6.50	Poor (DSL)	5.72
	D	5.01	7.58	8.67	9.42	5.89	7.34	8.63	9.09	0.51	7.21	8.02	7.62	Borderline (DSM)	7.29
D6	N1	6.45	8.42	9.33	9.79	6.95	8.11	9.34	9.58	0.44	8.12	8.74	8.43	Fair (DSH)	9.21
	D	5.29	7.83	8.93	9.68	6.09	7.22	9.09	9.27	0.36	7.38	8.31	7.84	Borderline (DSM)	7.57

* Telemetry study [9]–[11] had 6 drivers, **N=Normal, A=Aggressive, D=Drowsy, N1=Normal 1, N2=Normal 2, ***Linguistic Value of Driver's Score

of scores (partitioned into normal and aggressive+drowsy behavior). We obtained respective p values of 3.94×10^{-4} and 3.42×10^{-3}. Thus, the lower p-value obtained with our proposed approach signifies that it is able to partition the normal driving scores from the aggressive+drowsy ones in a better manner compared to the telemetry data.

B. Robustness analysis using Bootstrap Sampling

We validated the drivers' scores generated by our proposed scoring system using the Bootstrap sampling. We heuristically filtered data vectors from Section IV-D for the driver's trip. Then, we randomly selected 70% of these data vectors (without replacement) for every driver and the trip, bifurcated by the type of road. We calculated the overall drivers' numeric as well as linguistic trip's score using our proposed scoring system. This was repeated 100 times for each driver, trip and type of road to obtain 100 nine point FOUs for the respective overall driver's score (Similar to the nine point FOUs for the overall score given in Table III).

We calculated the Jaccard's similarity between the respective driver's score for trip and type of road (taken from Table III) and these respective 100 nine point FOUs, to generate corresponding 100 Jaccard's similarity scores. From these 100 Jaccard's similarity scores, we found the similarity index value at 95% confidence. For e.g., for driver D1 in Normal driving behaviour on a motorway road, the similarity index at 95% confidence was found to be 93.84%. This similarity index is the robustness value of the score as it indicates how similar are the resultant FOUs from bootstrapping with respect to the original one. Similarly, we calculated the robustness values for all the drivers in different types of behaviours and both roads. These results are summarized in Table IV. We found that the proposed scoring system's scores are highly robust to significant removal of data (30%), with average robustness in its formulation of 90%, within a 95% confidence interval.

Table IV: Overall driver's score in a trip on Motorway and Secondary for robustness analysis

Driver	Behavior*	Robustness	Driver	Behavior*	Robustness	Driver	Behavior*	Robustness
Type of Road: Motorway								
	N	93.84		N	89.47		N	90.67
D1	A	93.03	D3	A	96.81	D5	A	83.26
	D	95.34		D	92.41		D	89.53
	N	92.56		N	97.87		N	95.19
D2	A	84.32	D4	A	84.26	D6	A	82.13
	D	91.57		D	92.14		D	88.53
Type of Road: Secondary								
	N1	86.78		N1	93.40		N1	89.68
D1	N2	79.03	D3	N2	86.54	D5	N2	87.04
	A	95.61		A	95.99		A	74.29
	D	89.33		D	84.59		D	93.20
	N1	89.62		N1	93.21		N1	80.60
D2	N2	85.37	D4	N2	97.20	D6	N2	-
	A	96.40		A	91.65		A	-
	D	93.50		D	88.97		D	80.57

*N=Normal, A=Aggressive, D=Drowsy, N1=Normal 1, N2=Normal 2

C. Existing Per-C vs Our proposed scoring system

As the data source of existing Per-C is a group of people vs the stream of numeric data value in our proposed scoring system, hence a fair comparison between the two is from a conceptual point of view. The differences and similarities between the two are listed in the Table V, from which it can be seen that the existing Per-C has some limitations, which are successfully overcome by the use of proposed scoring system.

D. FOUs with proposed scoring system vs other approaches

We have presented a novel way of obtaining FOUs from a stream of data values. There exist other methods for obtaining FOUs from data. Unfortunately, these other methods cannot be used straight for our proposed scoring system because we require the FOUs to follow a semantic ordering (bounds) based on prior information on the semantic partitioning (please see Section III). It is pertinent that the FOUs must fall within a set of pre-defined (bounded) linguistic terms. This is to ensure that each FOU represents the uncertainty (of the membership degree) associated with each data value belonging to the linguistic label and the maximum of one adjacent linguistic label. So, our method produces overlapping between the FOUs bounded to each linguistic term.

E. Interesting aspect on dealing with real constant data

In the encoder of our proposed scoring system, unique data values for finding the centroids were used because retention of the duplicate values introduced a bias towards long continuous sequences of uninformative observations. In fact, with the unique data values in the dataset, the percentile rank of the dataset ranged from 0 to 100. However, with the duplicate values in the dataset, the percentile rank did not reach 100, which inhibits the complete coverage of the information scale (0 to 10) in a natural way. Also, the effect of duplicate retention was quite pronounced on the shape of the obtained FOU plots. The duplicate values caused the centroid of the FOU to shift more towards them, thereby causing only a single linguistic term to cover almost 60% of the information scale and one or more of the other linguistic terms' FOUs to shrink in size and be limited in 40% of the scale. In our perception, this is not a fair scoring system.

In many passive real-world monitoring scenarios, a large amount of collected data is constant and lacks informativeness. Large sequences of constant data can cause the variables' frequency distributions to become skewed towards frequent duplicates. For example, irresponsible driving scores would be attenuated if the driver performed some risky driving but cruised during most of the trip. This is due to the linguistic variables mostly reflecting the variations around the most frequent values rather than capturing the impact of extreme behaviours. In the FOU generation, we used distinct values because they better generalise the information about variations, rendering evenly distributed IT2 word models that better reflect what can contextually happen during the driver's trip.

F. Semantic ordering

In score systems, not all variables affect the overall score with the same ordering and orientation, and we have accounted for that in our approach. There is a semantics ordering on the linguistic terms belonging to a variable. For example, we all know that semantically, $Very\ Less$ is smaller than $Moderate$, which in turn is smaller than $Very\ High$. With this notion in mind, we see that the orientation of the linguistic terms for variables AC and CF are opposite to those of other variables. The linguistic terms corresponding to the variable AC (as seen from Table I) are $Very\ Low(ACV)$, $Low(ACL)$, $Medium(ACM)$, $High(ACH)$ and $Very\ High(ACE)$. From the Fig. SM-4, we see that the FOU for ACV is located on a lower side of the scale, whereas the location order for FOUs of other linguistic terms follows the order $ACE > ACH > ACM > ACL$. Against this, consider the variable BR. The linguistic terms corresponding to BR are $Very\ High(BRE)$, $High(BRH)$, $Moderate(BRM)$, $Less(BRL)$ and $Very\ Less(BRV)$. However, from Fig. SM-4, we see that the FOUs of these terms follow the order

Table V: Comparison of the existing Per-C and proposed novel Per-C

Attributes	Existing Per-C (based on IA, EIA, HMA)	Proposed Novel Per-C
	Differences	
Generating word models?	Using data collected from a group of people or a single subject	From a stream of data values
Steps in the data part?	Bad data processing, outlier processing, tolerance limit processing, reasonable interval processing, etc.	Removing duplicate data values, finding the centroids using FCM and calculating the highest and second highest membership degree of each data point.
Mapping into FOU?	By mapping Embedded T1 FSs into the left shoulder, interior or right shoulder FOU	Estimating left shoulder, interior or right shoulder FOU parameters through highest and second highest membership function values of data points from FCM.
Linguistic terms for aggregation in CWW engine?	Generally elicited by the user	Found based on respective maximum degrees of memberships of the variables' numeric values from the input data vector
Associated linguistic weights?	Assigned to the variable	Can be assigned to the linguistic terms of the variable
	Similarities	
Aggregation operator in CWW Engine?	Interval weighted average, Fuzzy weighted average or Linguistic weighted average.	Interval weighted average, Fuzzy weighted average or Linguistic weighted average.
Decoding in the decoder?	By similarity measure, ranking, or subsethood	By similarity measure, ranking, or subsethood

$BRE < BRH < BRM < BRL < BRV$. This is so because the variable BR shows an opposite behaviour to that of variable AC.

VI. Conclusions and Future Scope

In this paper, we have proposed a novel Per-C based unsupervised scoring system. Our novel Per-C based system successfully models the word semantics through IT2 FS word models, which are automatically generated from a stream of numeric values, against other models which require collecting labelled data from people, which has its inherent limitations [27], [36].

We have also demonstrated the applicability of our proposed scoring system to the scenario of driver's score calculation using real-life data [9], [10]. We chose this scenario because in some countries, the driving quality (of a driver) is assessed using a telematics unit fitted inside the vehicle and that has a direct impact on their insurance premium. A high number of users seem dissatisfied with the respective score calculated by the 'black box' [37]. Consumer protection services have highlighted a growing high number of complaints on this system and the poor confidence of consumers in the predicted values [38]. On the contrary, with our proposed scoring system, drivers can naturally perceive, understand and peruse the relations between the various driving-related inputs linguistically, adding the desired explainability.

Further, more often than not, the scoring systems have been treated (in literature) in a supervised manner and have been developed based on subjective labelled samples coming from a single or several experts (section V-B). The pre-labelling of the data may induce bias in the final score. Our novel Per-C based system is unsupervised and yields objective scores that are purely data-driven (section V-A).

Nonetheless, the precise numeric data values pertaining to the variables (used in the system design) may have subjective interpretations. Their semantic uncertainty is ignored when the end user is provided with a single numeric data output. Thus, our novel Per-C based unsupervised scoring system overcomes this limitation by modelling the semantics of these numeric data linguistically, in the form of IT2 FS word models. Also, our proposed system generates linguistic recommendations, the linguistic information being subjective in nature as "words mean different things to different people" [27], [36].

When evaluating the resultant scores in a real-world scenario, it was found that these were able to exhibit higher differences between groups requiring divergent scores (responsible vs. irresponsible drivers) than a state-of-the-art method. Also, robustness analysis showed resiliency to loss of data. It is pertinent to mention that our proposed scoring system is quite general and can be applied to any scoring system or scenario where IT2 FS word models need to be generated from a stream of numeric data values, such as monitoring data.

In this work, we have mainly focused on the encoding and engine part of the Per-C system but in future works, we will pursue modifying the decoder part to allow other CWW applications by inter-relating linguistic variables in different ways. In the decoder section, future developments could encompass the use of higher-order fuzzy sets, such as those aimed at incorporating time-dependencies [39]. Another possible extension of the present work can be developing assessment metrics for the quality of generated explanations from a CWW system.

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SUPPLEMENTARY MATERIALS

SM-I. Background on CWW and Perceptual Computing

The concept of CWW was put forward in the research community by Prof. Zadeh through his seminal works [14], [40]. In [14], he equated fuzzy logic to the CWW and provided a methodology through which it is possible to process the linguistic information by a computer in a manner similar to that done by human beings. An important aspect of CWW is modeling the semantics of linguistic information. Various approaches have been proposed in the literature that model semantics of linguistic information using different mechanisms such as type-1 (T1) Fuzzy Sets (FSs) [41], [42] and ordinal term sets. In [43], authors have proposed various CWW methodologies.

There have been other notable works on CWW also. This article [44], focuses on partitioning the domain of CWW into basic and advanced. This discussion article [36], presents different viewpoints about the CWW, from different researchers, as there seems to be no generalized agreement throughout the research community on what exactly the CWW is. Mendel also introduced a CWW framework for making subjective judgments using subjective concepts or “perceptions” called the *Per-C* and is composed of 3 main components: 1) encoder; 2) CWW engine; 3) decoder (see Fig. SM-1).

SM-II. Perceptual Computer: The machinery behind the Perceptual Computing

Perceptual Computing models the word semantics using the IT2 FSs [45]. A novel architecture for achieving perceptual computing is the Per-C. The design of a generic Per-C was proposed by Prof. Mendel in 2001 [13], as consisting of three components viz., encoder, CWW engine [27], [46] and decoder [47], [48] (please see Fig. SM-1). Let’s discuss each of the components viz., encoder, CWW engine and decoder, in detail.

A. Encoder

In the encoder, a vocabulary of application dependent linguistic terms (or words) is decided, and data intervals are collected about them from a group of subjects on a scale of 0 to 10. These data intervals are then processed using the Interval Approach (IA) [49], Enhanced Interval Approach (EIA) [50] or Hao-Mendel Approach (HMA) [51] to generate IT2 FS word models and store them in the form of a codebook, inside the encoder.

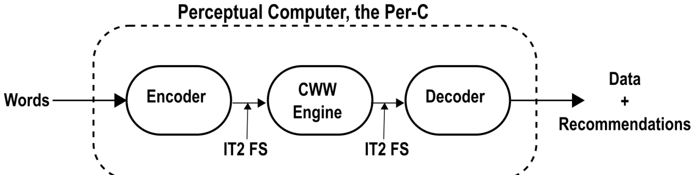

Figure SM-1: Perceptual Computer (Per-C) [27]

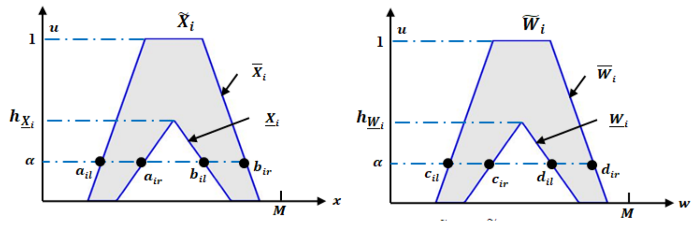

Figure SM-2: α-cut decomposition of the $\tilde{X}_i$ and the $\tilde{W}_i$ [27]

B. CWW Engine

The task of the CWW engine is to aggregate the FOU plots (obtained from the codebook) of the linguistic terms corresponding to the user inputs. The result of this aggregation is a nine point IT2 FS word model, which generally differs from the ones stored in the codebook. The commonly used aggregation operators in the CWW engine are interval weighted average (IWA), fuzzy weighted average (FWA), linguistic weighted average (LWA), or if-then rules, etc. Details of the same can be found in [27], [52], [53]. Of these, we have used the LWA in this paper. Using the LWA approach, the aggregated FOU plot of the linguistic terms corresponding to the user’s inputs is given as $\tilde{Y}_{LWA}$, in (5).

The computation of $\tilde{Y}_{LWA}$, can be performed using the α-cut decomposition of the $\tilde{X}_i$ and $\tilde{W}_i$. Consider an α-cut decomposition of the $\tilde{X}_i$ and $\tilde{W}_i$, shown in Fig. SM-2. In the Fig. SM-2, it can be seen that $\tilde{X}_i \in [a_i, b_i]$ and $\tilde{W}_i \in [c_i, d_i]$. Also the quantities a_{il}, a_{ir} are the ends of α-cut on the UMF of $\tilde{X}_i$, whereas b_{il}, b_{ir} are corresponding ends on the LMF. Similarly, c_{il}, c_{ir}, d_{il}, and d_{ir} can also be defined for $\tilde{W}_i$. The quantities a_{il}, a_{ir}, b_{il}, b_{ir}, c_{il}, c_{ir}, d_{il}, and d_{ir}, are used in determining the α-cut parameters $y_{Lr}(\alpha)$, $y_{Rl}(\alpha)$, $y_{Ll}(\alpha)$ and $y_{Rr}(\alpha)$, which are subsequently used to determining the interval ends $\underline{Y}_{LWA}$ and $\bar{Y}_{LWA}$ of the $\tilde{Y}_{LWA}$. All these calculations are given in (SM-1)-(SM-7). Further, the α-cut decomposition of the $\tilde{Y}_{LWA}$ and semantics of parameters used in (SM-1)-(SM-7) are shown in Fig. SM-3.

$$\tilde{Y}_{LWA} = \left[\underline{Y}_{LWA}, \bar{Y}_{LWA}\right] \tag{SM-1}$$

$$\underline{Y}_{LWA} = \left[y_{Lr}(\alpha), y_{Rl}(\alpha)\right], \alpha \in [0, h_{min}] \tag{SM-2}$$

$$\bar{Y}_{LWA} = \left[y_{Ll}(\alpha), y_{Rr}(\alpha)\right], \alpha \in [0, 1] \tag{SM-3}$$

$$y_{Lr}(\alpha) = \frac{\sum_{i=1}^{L_r} a_{ir}(\alpha)d_{il}(\alpha) + \sum_{i=L_r+1}^{n} a_{ir}(\alpha)c_{ir}(\alpha)}{\sum_{i=1}^{L_r} d_{il}(\alpha) + \sum_{i=L_r+1}^{n} c_{ir}(\alpha)}, \quad \alpha \in [0, h_{min}] \tag{SM-4}$$

$$y_{Rl}(\alpha) = \frac{\sum_{i=1}^{R_l} b_{il}(\alpha)c_{ir}(\alpha) + \sum_{i=R_l+1}^{n} b_{il}(\alpha)d_{il}(\alpha)}{\sum_{i=1}^{R_l} c_{ir}(\alpha) + \sum_{i=R_l+1}^{n} d_{il}(\alpha)}, \quad \alpha \in [0, h_{min}] \tag{SM-5}$$

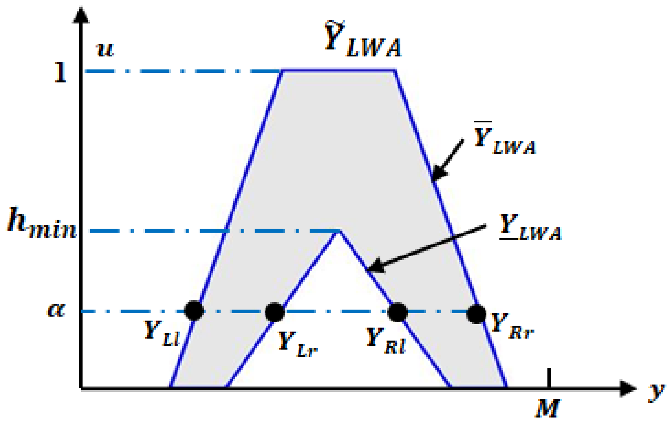

Figure SM-3: α-cut decomposition of the $\tilde{Y}_{LWA}$ [27]

$$y_{Ll}(\alpha) = \frac{\sum_{i=1}^{L_l} a_{il}(\alpha)d_{ir}(\alpha) + \sum_{i=L_l+1}^{n} a_{il}(\alpha)c_{il}(\alpha)}{\sum_{i=1}^{L_l} d_{ir}(\alpha) + \sum_{i=L_l+1}^{n} c_{il}(\alpha)}, \quad \alpha \in [0,1] \tag{SM-6}$$

$$y_{Rr}(\alpha) = \frac{\sum_{i=1}^{R_r} b_{ir}(\alpha)c_{il}(\alpha) + \sum_{i=R_r+1}^{n} b_{ir}(\alpha)d_{ir}(\alpha)}{\sum_{i=1}^{R_r} c_{il}(\alpha) + \sum_{i=R_r+1}^{n} d_{ir}(\alpha)}, \quad \alpha \in [0,1] \tag{SM-7}$$

Here L_r, R_l, L_l, and R_r are the switch points determined using the Enhanced Karnik Mendel (EKM) algorithm [48]. It is mentioned here that h_{min} used in (SM-2), (SM-4) and (SM-5) is the height of LMF of $\tilde{Y}_{LWA}$ as shown in Fig. SM-3.

An important point to note here is that if m α-cuts are performed on the $\tilde{Y}_{LWA}$, then calculations in (SM-4)-(SM-7) are performed for all these m α-cuts. Thus, to determine the values $\underline{Y}_{LWA}$ all the left coordinates viz., $(y_{Lr}(\alpha_i), \alpha_i), i = 1, \ldots, m$ and right coordinates $(y_{Rl}(\alpha_i), \alpha_i), i = 1, \ldots, m$ are joined. Similarly, to determine the value $\bar{Y}_{LWA}$, all the left coordinates viz., $(y_{Ll}(\alpha_i), \alpha_i), i = 1, \ldots, m$ and right coordinates $(y_{Rr}(\alpha_i), \alpha_i), i = 1, \ldots, m$ are joined.

C. Decoder

The task of the decoder is to generate recommendations in the form of rankings, words, or classes. The rankings are generally calculated on the basis of averaged center values, c_{avg}, given by (6). In this equation, the $[c_l, c_r]$ is the center of the IT2 FS word model, calculated based on the switch points determined from the EKM algorithm. The values c_l, c_r and c_{avg} are given in (SM-8)-(6) as:

$$c_l = \frac{\sum_{i=1}^{L} x_i \bar{\mu}_{\tilde{A}}(x_i) + \sum_{i=L+1}^{N} x_i \underline{\mu}_{\tilde{A}}(x_i)}{\sum_{i=1}^{L} \bar{\mu}_{\tilde{A}}(x_i) + \sum_{i=L+1}^{N} \underline{\mu}_{\tilde{A}}(x_i)} \tag{SM-8}$$

$$c_r = \frac{\sum_{i=1}^{R} x_i \underline{\mu}_{\tilde{A}}(x_i) + \sum_{i=R+1}^{N} x_i \bar{\mu}_{\tilde{A}}(x_i)}{\sum_{i=1}^{R} \underline{\mu}_{\tilde{A}}(x_i) + \sum_{i=R+1}^{N} \bar{\mu}_{\tilde{A}}(x_i)} \tag{SM-9}$$

In (SM-8)-(SM-9), L and R are the switch points determined iteratively using the EKM algorithm. Also, the $\bar{\mu}_{\tilde{A}}(x_i)$ and the $\underline{\mu}_{\tilde{A}}(x_i)$ are the UMF and LMF, respectively at x_i.

The word recommendations are generated using the Jaccard's similarity measure, given in (7).

SM-III. FOU plots for the linguistic terms of motorway and secondary as well as FOU data for the Linguistic Terms of the Variables and the Driver's Score for Secondary

In this Section, we have presented the FOU data for the linguistic terms of variables, linguistic weights and driver's score for the secondary type of road in Table SM-I. Further, we have presented the FOU plots of the respective linguistic terms of variables, linguistic weights and driver's score in the motorway type of road as well as secondary in the Fig. SM-4 and Fig. SM-5, respectively.

SM-IV. Extended Discussions

In this section, we present some facts and findings with respect to the driver's scoring system, obtained from the work presented in this paper.

- In the works [9]–[11], the authors asked the drivers to perform trips on the motorway and secondary, simulating three types of driving behaviours viz., "Normal", "Aggressive", and "Drowsy". We identified the data vectors corresponding to the respective type of driving behaviour (13781 data vector for motorway and 12038 for secondary in Section IV.C) and compared them against the linguistic scores generated by our Per-C based novel methodology. The respective number of samples corresponding to each type of behaviour and road with linguistic scores are given in Table SM-II. From Table SM-II, it can be seen that for normal driving in motorways, 88% of the samples correspond to the Good (DSE) and Fair (DSH), taken together. Against this, in secondary roads, 82% alone is taken by Fair (DSH). For both motorway and secondary types of road, Fair (DSH) and Borderline (DSM) together contribute to 88% and 85%, respectively. For drowsy behaviour in motorways, 86% of samples correspond to the Fair (DSH). Against this, 83% samples are occupied by Fair (DSH) and Borderline (DSM), together for drowsy behaviour in secondary roads. Further, the variable that contributed most to the observed behaviour in the respective road, as found by our novel Per-C based Unsupervised scoring system and the original dataset from [9]–[11], is shown in the last four columns of the Table SM-II. From Table SM-II, it can be clearly seen that the scoring system based on the original dataset from [9]–[11] clearly ignores the contribution by individual variables in the overall score. This leads to all the variables giving equal maximum contribution in the attainment of the respective linguistic scores. Thus, the equal weighting scheme of [9]–[11] is not a feasible approach. On the contrary, our novel Per-C based Unsupervised scoring system finds one variable contributes most to the linguistic score.

Table SM-I: FOU data for the Linguistic Terms of the Variables and the Driver's Score for Secondary

Variables, Linguistic weights and Driver's Score	Associated Linguistic terms	FOU data: UMF a	b	c	d	LMF e	f	f	g	μ_f	Centroid c_l	c_r	c_{avg}
	Very Low (ACV)	0.00	0.00	4.49	5.89	0.00	0.00	0.00	5.89	1.00	1.91	2.73	2.32
	Low (ACL)	0.00	6.35	6.45	7.44	5.48	6.15	6.15	6.63	0.45	3.09	6.55	4.82
Acceleration (AC)	Medium (ACM)	5.95	7.39	7.47	8.48	6.99	7.45	7.45	7.90	0.45	6.92	7.75	7.34
	High (ACH)	7.45	8.45	8.53	10.00	7.98	8.50	8.50	8.95	0.45	8.18	9.00	8.59
	Very High (ACE)	8.49	9.48	10.00	10.00	8.45	10.00	10.00	10.00	1.00	9.43	9.5	9.46
	Very High (BRE)	0.00	0.00	0.76	2.96	0.00	0.00	0.00	2.96	1.00	0.96	1.08	1.02
	High (BRH)	0.00	3.38	3.57	5.39	2.20	3.21	3.21	4.08	0.45	2.09	3.89	2.99
Braking (BR)	Moderated (BRM)	2.97	5.24	5.40	7.24	4.52	5.35	5.35	6.20	0.45	4.57	5.91	5.24
	Less (BRL)	5.40	7.23	7.40	10.00	6.45	7.28	7.28	8.13	0.46	6.73	8.22	7.48
	Very Less (BRV)	7.25	9.08	10.00	10.00	7.44	10.00	10.00	10.00	1.00	8.96	9.17	9.07
	Very small (CFV)	0.00	0.00	0.97	3.11	0.00	0.00	0.00	3.03	1.00	0.98	1.16	1.07
	Small (CFS)	0.00	3.19	3.39	5.41	2.22	3.20	3.20	4.15	0.45	2.09	3.87	2.98
Car Following (CF)	Average (CFA)	3.13	5.30	5.48	7.31	4.48	5.40	5.40	6.29	0.44	4.68	5.95	5.32
	Large (CFL)	5.42	7.47	7.63	10.00	6.58	7.43	7.43	8.23	0.47	6.83	8.30	7.57
	Very Large (CFE)	7.34	9.16	10.00	10.00	7.63	10.00	10.00	10.00	1.00	9.00	9.23	9.12
	Very Large (LDE)	0.00	0.00	2.23	3.92	0.00	0.00	0.00	3.82	1.00	1.24	1.64	1.44
	Large (LDL)	0.00	3.86	4.01	5.71	3.15	3.93	3.93	4.71	0.46	2.36	4.46	3.41
Lane Drifting (LD)	Average (LDA)	3.93	5.64	5.79	7.48	4.94	5.72	5.72	6.49	0.44	5.17	6.25	5.71
	Small (LDS)	5.72	7.42	7.57	10.00	6.72	7.49	7.49	8.26	0.46	6.96	8.35	7.66
	Very small (LDV)	7.49	9.16	10.00	10.00	7.61	10.00	10.00	10.00	1.00	9.05	9.23	9.14
	Very High (LWE)	0.00	0.00	1.00	2.00	0.00	0.00	0.00	2.00	1.00	0.65	0.81	0.73
	High (LWH)	0.00	1.00	2.00	3.33	1.67	2.67	2.67	3.08	0.16	1.00	2.71	1.85
Lane Weaving (LW)	Medium (LWM)	3.33	4.00	5.00	6.67	4.17	4.67	4.67	5.41	0.19	4.03	5.65	4.84
	Low (LWL)	5.00	6.66	6.67	10.00	6.67	8.00	8.00	8.33	0.19	6.29	8.58	7.44
	Very Low (LWV)	8.33	9.16	10.00	10.00	8.33	10.00	10.00	10.00	1.00	9.32	9.46	9.39
	Very High (OSE)	0.00	0.00	0.88	3.10	0.00	0.00	0.00	3.09	1.00	1.00	1.14	1.07
	High (OSH)	0.00	3.56	3.74	5.49	2.32	3.37	3.37	4.23	0.45	2.18	4.02	3.10
Over-Speeding (OS)	Moderate (OSM)	3.12	5.40	5.56	7.33	4.68	5.49	5.49	6.30	0.45	4.70	6.04	5.37
	Less (OSL)	5.50	7.27	7.43	10.00	6.54	7.34	7.34	8.15	0.46	6.80	8.26	7.53
	Very Less (OSV)	7.34	9.09	10.00	10.00	7.47	10.00	10.00	10.00	1.00	8.99	9.18	9.09
	Very Large (TUE)	0.00	0.00	0.94	2.77	0.00	0.00	0.00	2.58	1.00	0.84	1.05	0.94
	Large (TUL)	0.00	2.63	2.78	4.70	1.90	2.74	2.74	3.59	0.46	1.8	3.32	2.56
Turning (TU)	Average (TUA)	2.78	4.63	4.80	6.70	3.84	4.71	4.71	5.59	0.44	4.14	5.31	4.73
	Small (TUS)	4.71	6.62	6.79	10.00	5.84	6.71	6.71	7.57	0.46	6.11	7.91	7.01
	Very small (TUV)	6.71	8.58	10.00	10.00	6.83	10.00	10.00	10.00	1.00	8.71	8.97	8.84
	Very Low (WTV)	0.00	0.00	1.00	2.94	0.00	0.00	0.00	2.80	1.00	0.91	1.11	1.01
	Low (WTL)	0.00	2.84	3.02	4.99	2.04	2.94	2.94	3.84	0.46	1.93	3.55	2.74
Linguistic Weight (WT)	Medium (WTM)	2.95	4.91	5.09	7.05	4.10	5.00	5.00	5.91	0.44	4.38	5.62	5.00
	High (WTH)	5.00	6.98	7.16	10.00	6.17	7.06	7.06	7.96	0.46	6.45	8.08	7.26
	Very High (WTE)	7.06	9.00	10.00	10.00	7.20	10.00	10.00	10.00	1.00	8.89	9.09	8.99
	Terrible (DSV)	0.00	0.00	5.21	6.26	0.00	0.00	0.00	6.23	1.00	2.02	3.01	2.51
	Poor (DSL)	0.00	6.25	6.34	7.38	5.80	6.28	6.28	6.77	0.44	2.98	6.62	4.80
Driver's Score (DS)	Borderline (DSM)	6.27	7.34	7.42	8.45	6.91	7.38	7.38	7.86	0.44	7.04	7.70	7.37
	Fair (DSH)	7.39	8.42	8.51	10.00	7.99	8.46	8.46	8.94	0.46	8.15	8.99	8.57
	Good (DSE)	8.46	9.48	10.00	10.00	8.53	10.00	10.00	10.00	1.00	9.42	9.52	9.47

Table SM-II: Number of Samples for respective Driver's Linguistic Score simulating different behaviors

Driving Behavior	Linguistic Score	Number of Samples for linguistic score in: Motorway	Secondary	Variable Contributing maximum to the score in: Motorway, Per-C based model	Motorway, Original dataset	Secondary, Per-C based model	Secondary, Original dataset
	DSE	837	963	LD	LW	AC	BR, LW
Normal	DSH	5888	5804	AC	AC	AC	AC
	DSM	939	270	LD	LW	CF	LW
	DSL	-	8	-	-	LW	BR, CF, LD, LW, OS, TU
	DSE	15	5	AC	AC, LW	AC	AC, BR, CF, LD, LW, TU
Aggressive	DSH	922	1293	LW	LW	LW	LW
	DSM	1100	828	LW	LW	LW	LW
	DSL	270	374	LW	LW	LW	LW
	DSH	3258	854	OS	CF	AC	CF
Drowsy	DSM	552	1230	CF	CF	AC	CF
	DSL	-	409	-	-	CF	CF

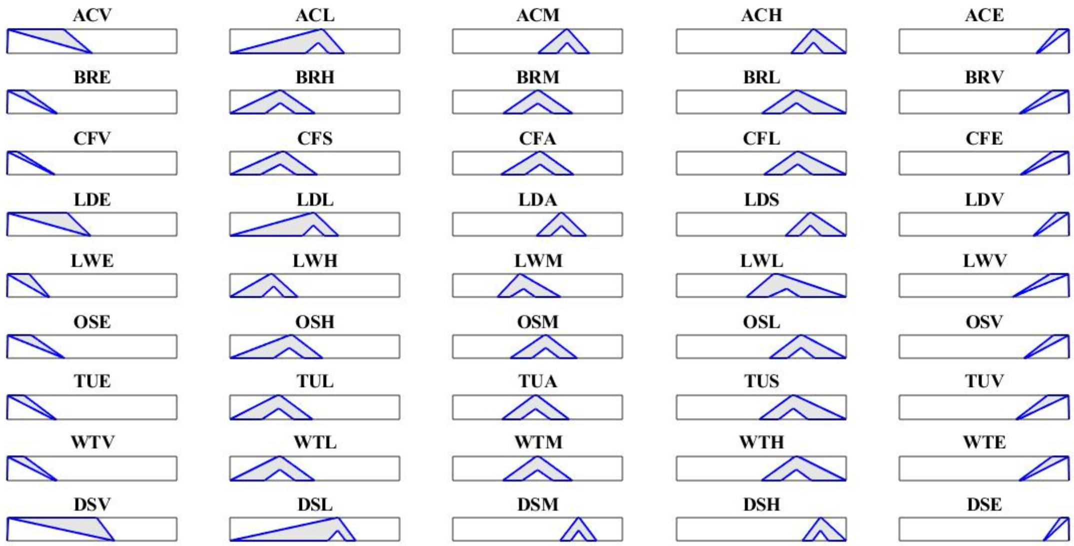

Figure SM-4: FOU plots for the words in the codebook: Motorway

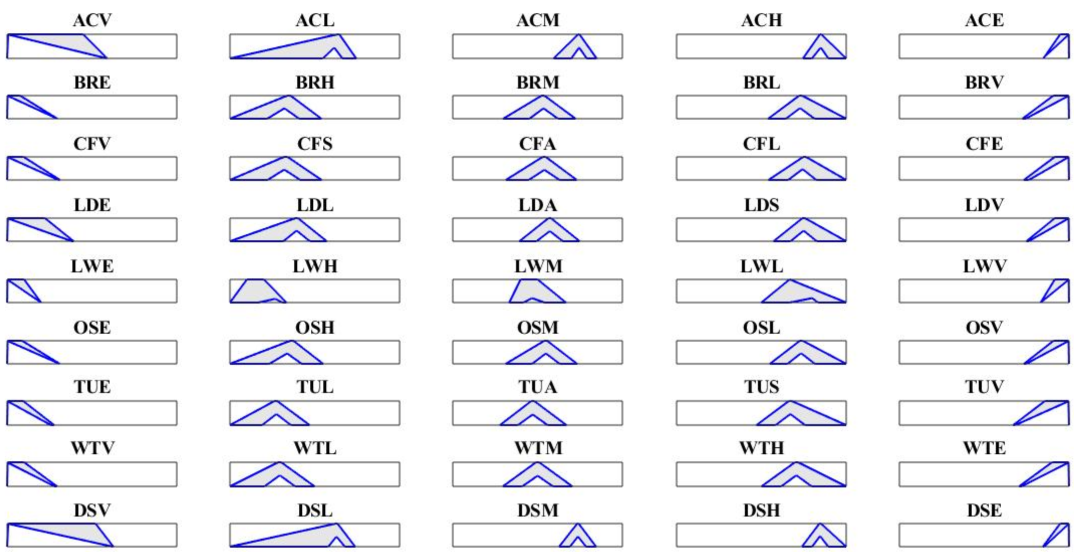

Figure SM-5: FOU plots for the words in the codebook: Secondary

- There is higher support for the low sets in secondary roads (i.e. rural), that drivers tend to drive worse on those roads (most accidents and penalties happen on such roads) [54]. Domain coverage for lower in the motorways is higher (not always, but in some cases) as driving on these roads is more homogeneous. If one sees the FOUs for the Motorway (Fig. SM-4) and Secondary (Fig. SM-5), the low set is shifted to the right of the domain, while for small roads (medium) is closer to the left side. This means that an acceleration that is low in a motorway could be medium or high in a small road, which explains a source of uncertainty.
- Further, it can be appreciated visually from Figs. SM-4 and SM-5 that braking is actually more frequent in a secondary type of road than in a motorway, as this is somehow obvious. One has to break more in secondaries to go through curves, intersections and so on. Therefore, a ‘moderate’ braking in a secondary should cover for higher values (on scale 0 to 10).
- The uncertainty of the parameters to measure the score of a driver on the motorway is higher; however, on the secondary roads, there is less uncertainty that someone

who drives well could be a good driver. People who are cautious and good drivers on the riskier secondary roads are potentially good drivers regardless of their car model, road characteristics, weather, or other co-factors generating uncertainty [54].

- Because motorways are more constantly straight and have better signalization, pavement markings, lightings, a good driver needs to stick to the rules of the road; bad drivers here are purposely reckless or aggressive drivers, and factors such as the car model, road congestion etc. [55], comes in to play up to what respect the impact for parameters such as braking or acceleration.
- The parameters that are high in a secondary road have a more precise high weighting because if you drive well on secondary roads, then there is a higher chance you are a good driver.
- The uncertainty of the parameters to measure the score of a driver on the motorway is higher (other co-factors come into play). However, there is less uncertainty on the secondary roads than someone who drives well and is a good driver. People who are cautious in (‘riskier’) secondary roads could be good drivers regardless of other co-factors.
- Indeed, the high uncertainty on the high weight parameter should pull back the high scores for that parameter. The weight of the parameters is uncertain in motorway because there are several lines, speed and acceleration also will depend on the car model etc, viz. it doesn’t mean a driver is irresponsible if a driver is just faster. A motorway is a better kind of road. For someone who drives well on secondary roads in a parameter with a high weight, their scores should be positively high and that’s something that our model implements. If the FOU for the weight high in secondary would be bigger, it would pull down their score instead.
- According to the results, in secondary roads, the distinction between drivers’ quality driving expertise is much more pronounced than in motorways. It is worth mentioning that previous studies have shown that secondary roads are the ones where more accidents and penalties occur [56], [57]. Therefore, someone driving well on a secondary road could be potentially a good driver. They should be, therefore, rewarded more if they drive properly there.

SM-V. Calculating the membership degree of a data point in an IT2 FS [13], [58]

The IT2 FSs (used in this paper) are described by two bounding functions UMF and LMF. Consider an IT2 FS, $(\tilde{B})$, whose UMF is trapezoidal shaped and the LMF are triangular. Considering a data point x_1 lying inside $(\tilde{B})$, as shown in Fig. SM-6. The membership value $\mu(x_1)$ of this point x_1 in the IT2 FS $(\tilde{B})$, is given as an interval, given as:

$$\mu(x_1) = \left[\underline{\mu}(x_1), \bar{\mu}(x_1)\right] \tag{SM-10}$$

Here, $\bar{\mu}(x_1)$ is the membership value of x_1 in the UMF, shown as y-intercept in Fig. SM-7. Also, $\underline{\mu}(x_1)$ is the mem-

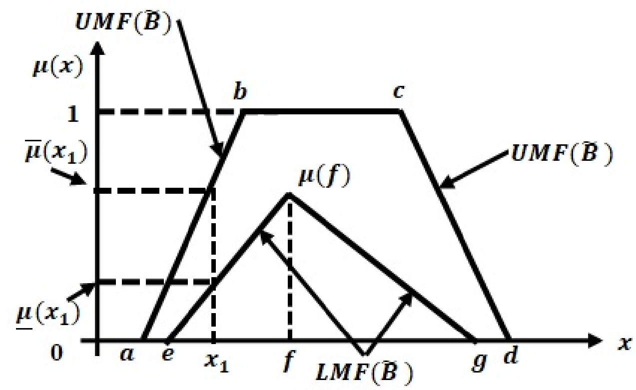

Figure SM-6: An IT2 FS $(\tilde{B})$ containing the data point x_1

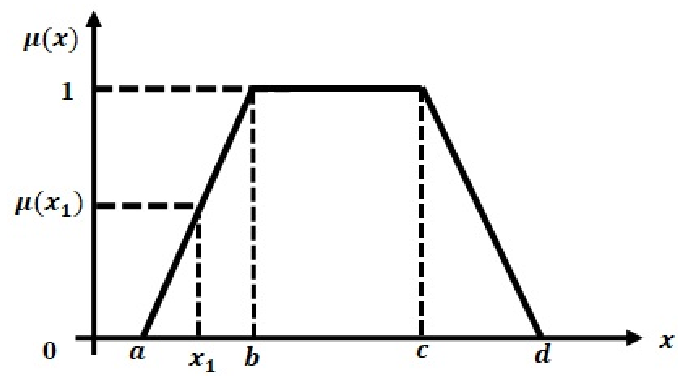

Figure SM-7: Intercept of point x_1 on Trapeziodal UMF

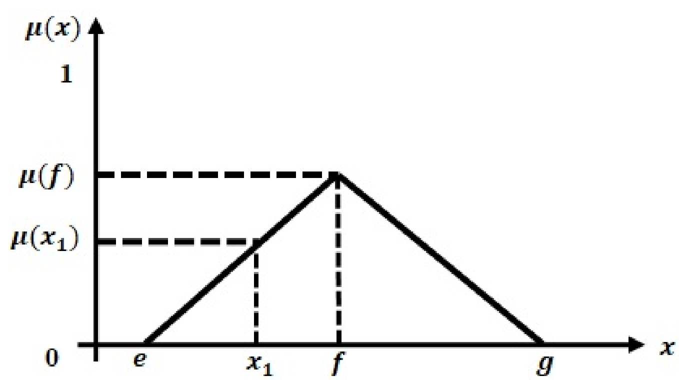

Figure SM-8: Intercept of point x_1 on Triangular LMF

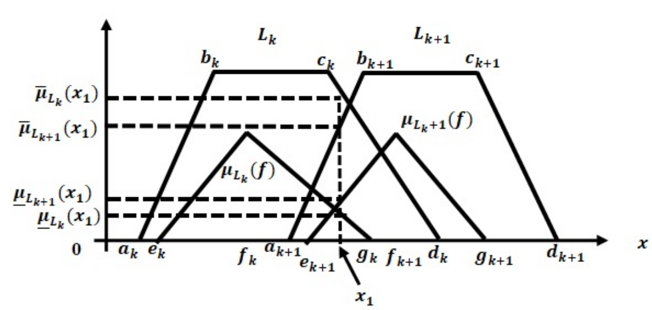

Figure SM-9: Data point x_1 lying in terms LT_k and LT_{k+1}

bership value of x_1 in the LMF, shown as y-intercept in Fig. SM-8. These values $\bar{\mu}(x_1)$ and $\underline{\mu}(x_1)$ are given by (SM-11) and (SM-12), respectively.

$$UMF : \bar{\mu}(x_1) = \begin{cases} 0, \; x_1 < a \\ \frac{x_1 - a}{b - a}, \; a \leq x_1 < b \\ 1, \; x_1 < b \leq x_1 < c \\ \frac{d - x_1}{d - c}, \; c \leq x_1 < d \\ 0, \; x_1 \geq d \end{cases} \tag{SM-11}$$

$$LMF : \underline{\mu}(x_1) = \begin{cases} 0, \; x_1 < e \\ \frac{\mu(f)(x_1 - e)}{f - e}, \; e \leq x_1 < f \\ \frac{\mu(f)(g - x_1)}{g - f}, \; f \leq x_1 < g \\ 0, \; x_1 \geq g \end{cases} \tag{SM-12}$$

Here, a,b,c,d are the points defining the trapezoidal UMF (please see Fig. SM-7) and e, f, g, $\mu(f)$ are the ones defining triangular LMF (please see Fig. SM-6).

Thus, a case may arise when a point x_1, may belong simultaneously to two adjacent linguistic terms LT_k and LT_{k+1}, each described by two IT2 MFs. The situation is shown in Fig. SM-9. Here, the MF of the data point x_1 is found in both LT_k and LT_{k+1}, using (SM-10). Let's say these values are:

$$\mu_{LT_k}(x_1) = \left[\underline{\mu}_{LT_k}(x_1), \bar{\mu}_{LT_k}(x_1) \right] \tag{SM-13}$$

$$\mu_{LT_{k+1}}(x_1) = \left[\underline{\mu}_{LT_{k+1}}(x_1), \bar{\mu}_{LT_{k+1}}(x_1) \right] \tag{SM-14}$$

Then we calculate the centre of gravity (COG) for the membership degree of x_1 in both LT_k and LT_{k+1} as an average of the UMF and LMF values as:

$$\mu_{LT_k}^{COG}(x_1) = \frac{1}{2} \left[\underline{\mu}_{LT_k}(x_1) + \bar{\mu}_{LT_k}(x_1) \right] \tag{SM-15}$$

$$\mu_{LT_{k+1}}^{COG}(x_1) = \frac{1}{2} \left[\underline{\mu}_{LT_{k+1}}(x_1) + \bar{\mu}_{LT_{k+1}}(x_1) \right] \tag{SM-16}$$

Next we compare the values $\mu_{LT_k}^{COG}(x_1)$ and $\mu_{LT_{k+1}}^{COG}(x_1)$. If $\mu_{LT_k}^{COG}(x_1) > \mu_{LT_{k+1}}^{COG}(x_1)$, then x_1 belongs to LT_k, else it belongs to LT_{k+1}.

Acknowledgment

We want to sincerely thank Dr. Deepa Sharma and Prof. Jerry M. Mendel for engaging in initial discussions at the inception of this work.