Research Repository

A Perceptual Computing Approach for Learning Interpretable Unsupervised Fuzzy Scoring Systems

Accepted for publication in IEEE Transactions on Artificial Intelligence.

A Perceptual Computing Approach for Learning Interpretable Unsupervised Fuzzy Scoring Systems

Prashant K. Gupta, Deepak Sharma, and Javier Andreu-Perez, *Senior Member, IEEE*

***Abstract*—Scoring the driver's behavior through the analysis of his/ her road trip data is an active area of research. However, such systems suffer from a lack of explainability, integration of expert bias in the calculated score, and ignoring the semantic uncertainty of various variables contributing to the score. To overcome these limitations, we have proposed a novel perceptual computing based unsupervised scoring system. The prowess of the proposed system has been exemplified in a case study of driver's scoring from telemetry data. Our proposed approach yields scores that showed a higher significant separability between drivers performing responsible or irresponsible (aggressive or drowsy) driving behaviours, than the formal method of computing these scores (a p value of 3.94×10^{-4} and 3.42×10^{-3}, respectively, in a Kolmogorov-Smirnov test). Further, the proposed method displayed higher robustness in the bootstrap test (where 30% of original data was omitted at random) by providing scores that were 90% similar to the original ones for all results within a confidence interval of 95%.**

***Index Terms*—Computing with Words, Fuzzy Logic, Perceptual Computing Systems, Unsupervised Scoring Systems.**

I. Introduction

DRIVING score estimation through the analysis of road trip data[1] is an active area of research. Various articles exist in the literature which has conducted studies for driving score estimation by incorporating various factors or conditions [1]–[7]. Independently, some works also focus on developing telematic devices [8] for calculating the score estimation on the basis of perceived driving behaviour. A case study was also conducted in [9]–[11], to estimate the score from the driving telemetry data, using a methodology similar to that commonly employed by the insurance or rental cars companies. The findings of this study have been presented in the form of a publicly available dataset. An outcome of these works has been to estimate the driver's score (behaviour) using the numeric values of the various imprecise variables (lane drifting, braking, etc.) and classify the behaviour linguistically (good, moderate, bad, etc.).

These driver's score estimation systems have demonstrated very good performances in their respective works; however, they have some shortcomings, too. They act like a 'black-box' and lack explainability. Often, such systems generally involve integrating several variables into a linear equation, which is purely based on the perception of the few individuals who tune or assign the score based on their subjective opinion. Such systems, therefore, hide the score calculation methodology from the end user. Also, they are seen as the subjacent family of supervised learning, where the experts provide apriori score labelling of the observations according to their subjective understanding. The scoring system then integrates the labelling bias of the experts in the learned model following a methodology akin to regression or multi-class problems. Another limitation of these systems is that they rely on the precise numeric values of various variables, which are semantically imprecise and vague. The perceptual understanding of the precise numerical values (or meanings) of these variables is rather soft (and uncertain) as it depends on the context (levels, places and so forth) and the person (a traffic officer, car mechanic, professional driver or insurance brokers [12]).

Thus, our position is that, in order to overcome the disadvantages of these scoring systems, we propose our novel *Perceptual Computing based Interpretable Unsupervised Fuzzy Scoring System*. The proposed system treats the scoring in an unsupervised way, computing with natural terms that express *perceptions* of the variables that make up the objective score. Perceptual Computing, the core methodology behind our scoring system, was proposed by Prof. Mendel in [13][2]. The means to achieve Perceptual Computing, is the framework called the *Perceptual Computer* or Per-C. The use of Per-C becomes reasonable whenever a computing system needs to process subjective linguistic information similar to the human cognitive process.

We have also demonstrated the utility of our proposed Per-C based scoring system using the telemetry data of [9]–[11]. We found that the driving scores obtained by our proposed system show a higher divergence between the ones obtained from responsible and irresponsible drivers (aggressive or drowsy), in a Kolmogorov-Smirnov test [16], with a higher significant value of $p = 3.94 \times 10^{-4}$ whereas the ones of a formal method of driver's scoring from telemetry [10] have $p = 3.42 \times 10^{-3}$. Further, a robustness analysis using a bootstrap test of 30% random removal of the original data showed that the resultant footprint of uncertainty (FOU) plots of[3] scores were 90%

Prashant K. Gupta is with Bennett University, Greater Noida, India. He is co-corresponding author. (e-mail: guptaprashant1986@gmail.com)

Deepak Sharma is with the Department of Computer Science, Christian-Albrechts-University Kiel, Kiel, Germany.

Javier Andreu-Perez is with the Centre for Computational Intelligence & School of Computer Science and Electronic Engineering, University of Essex, Colchester, United Kingdom; and Senior Research Fellow at the University of Jaen, Spain. He is co-corresponding author. (email: j.andreu-perez@essex.ac.uk)

[1]This data is generally curated through mobile apps and analysed using AI models.

[2]Perceptual Computing is one instantiation of Prof. Zadeh's novel Computing with Words (CWW) framework [14], [15]

[3]These FOU plots were generated in the Per-C.

similar to the original one using the full data, with a confidence interval of 95%. In sum, our proposed approach causes a better and more stable partitioning of the scores with respect to expected responsible (or otherwise) driving behaviours.

The rest of the paper is organized as follows: Section II discusses the related literature, Section III gives details of the novel Per-C based design of Unsupervised Scoring system and Section IV discusses the results obtained from its applicability to the telemetry data of [9]–[11]. Section V gives detailed discussions on the obtained results. Finally, section VI concludes this paper and throws light on its future scope. Details on important concepts are given in the Supplementary Materials (SMs).

II. Related Work

In this section, we present some of the literary works that motivated us to bring forth the research presented in this paper. We also discuss the basics of IT2 FSs and related concepts.

A. Literature review

With regards to our case study for drivers scoring in [7], the authors said that driver's behaviour assessment was a difficult task, especially in the insurance applications, due to numerous factors such as the trade-off between application cost and data accuracy, data uncertainty, noisy data, etc. They proposed a fuzzy treatment for driver behaviour assessment. The focus of this work was on modelling the data uncertainty, although explainability and unsupervised modelling were not prioritised. In [5], authors performed supervised regression to predict near-miss events. They used information such as vehicle usage, attitude toward speeding, and time and proportion of urban/nonurban driving from the telematics data, as well as additional information such as acceleration, braking, and cornering. They concluded general remarks such as night-time driving was associated with a lower risk of cornering events, urban driving increased the risk of braking events, and speeding was associated with acceleration events. Nonetheless, the non-fuzzy supervised approach did not elaborate on the importance or interrelations, using the 'everyday language' or explainable terms in each input variable, for example, how does low, intermediate, high, very high speeding, acceleration, both, or in combination with other terms, predicts few, some, several or many near-miss events. Some works have tried fuzzy approaches to develop score systems that consider inputs and outputs as 'natural language' imprecise terms. Sohn et al. [17] presented a fuzzy logistic regression method for credit scoring that processing inputs and outputs as T1 fuzzy numbers. This work pointed out the importance of considering the imprecision and vagueness of the input and output data. However, the inference approach (logistic regression) requires supervised training and although variable's data could be defined as fuzzy numbers, logistic regression is not CWW approach per se and depends on extra model parameters (coefficients) that are not fuzzy linguistic terms, hindering its overall interpretability and straightforward tuning. In another previous work, Hoffmann et al. [18] proposed a method to estimate a score, also, in this case, a financial scoring, based on a descriptive fuzzy-rule base (FRB) classifier. Rule-based models can be considered to be interpretable, provided that the fuzzy inputs and outputs can be adequately conceptualized into linguistic terms [19]. However, Hoffmann et. al learning mechanism is also supervised (viz. genetic fuzzy rule generation), and the initial linguistic terms are replaced by optimised membership functions to support classification accuracy of the FRB, at the expense of losing its initial perceived meaning hindering explainability.

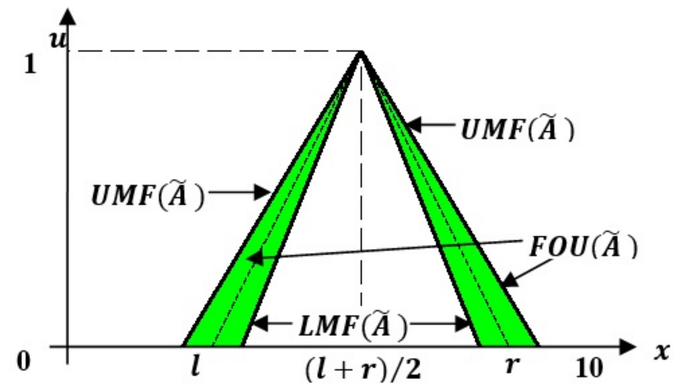


Figure 1: Membership functions of an IT2 FS [26]

Perceptual computing is a novel CWW approach. It has been used in various applications. The latest work, [20], uses perceptual computing for portfolio selection. Some latest works [21], [22] present a relation between granular computing and the CWW. CWW has also been used by Pratihar et. al for transportation [23]. The work [24] presents Python libraries for CWW methodologies.

B. A short primer on IT2 FS

The IT2 FSs were conceptualized by Prof. Zadeh in ([25]). They have a greater capability to model the semantics of linguistic information through the use of secondary membership degree. In the IT2 FSs, the secondary MF is assumed to be 1 everywhere.

An IT2 FS is pictorially shown in Fig. 1 and mathematically given in the form of Eq. (1) as:

$$\tilde{A} = \{(x, \mu(x), \mu_{\tilde{A}}(x, \mu(x)) = 1) \mid x \in U, 0 \leq \mu(x) \leq 1\} \tag{1}$$

here x is the data point, $\mu(x)$ isthe primary membership and $\mu_{\tilde{A}}(x, \mu(x))$ is the secondary membership. Also, in the Fig. 1, it can be seen that a T1 FS is shown inside the FOU of IT2 FS by a dashed line, whose ends rest on the x-axis at l and r. This T1 FS is called an embedded T1 FS. According to ([27]), the FOU of an IT2 FS can be considered as a union of all such embedded T1 FSs.

Additionally, the concept of the neighborhood is important. For any data point x lying inside an IT2 FS, its Neighborhood is defined as: $NH = \{\{\overleftarrow{NH}, \overrightarrow{NH}\} \mid \overleftarrow{NH} = \underset{cen \leq x}{\text{argmax}}\, d(cen - x), \overrightarrow{NH} = \underset{cen \geq x}{\text{argmin}}\, d(cen - x)\}$, cen : centroid value and $d(cen - x)$: distance between cen and x.

III. A Novel Per-C Based Unsupervised Scoring System with Type-2 Fuzzy Linguistic Terms

In classical Per-C, the semantics of problem variables are represented using IT2 FS models or FOU, which are constructed inside the encoder using the endpoint data intervals collected from a group of subjects. This data collection has limitations as a large amount of time is required for data collection and many users do not provide the data seriously [28]. Further, more often than not, each of problem variables, have an associated stream of numeric data values[4] [29]–[32], etc. This is also true for the original telemetry data [9]–[11].

Also, in the existing Per-C's CWW engine, linguistic weight is associated with a variable (and not individual linguistic terms associated with a variable). Further to it, a user chooses a linguistic term and elicits its respective associated linguistic weight at the time of aggregation. We feel that different linguistic terms of variable may have different connotations. For example, the amount of negative connotation attached to 'Very Low' may not be the same as the amount of positive connotation attached to 'Very High'. Thus, assigning the same weight to all the linguistic terms of a variable seems a little impractical.

Hence, for our proposed Per-C based unsupervised scoring system, we developed an encoder which disambiguates and conceptualises stream of numeric values, as the *fuzzy linguistic terms* using Fuzzy C-means (FCM) [33], [34]. They are later mapped into FOUs of the associated linguistic terms of a variable (Details are discussed in Section III-A). Also, in the CWW engine, the selection of the linguistic terms to be aggregated is data-driven. Each linguistic term is assigned a different linguistic weight instead of a variable. Finally, for human explainability, we generate linguistic recommendations from the decoder.

It is pertinent to mention that our proposed scoring system will define the boundaries of the linguistic terms in an unsupervised way. The scoring system only requires some prior information about the ordering of the linguistic terms of variables and whether these variables semantically support (or oppose) the score. This is all needed to convey the information from these variables into an overall score that will be interpretable (in essence). Our proposed scoring system also processes the linguistic information in same three steps of existing Per-C viz., encoder (Steps 1-3), CWW engine (Steps 4-6) and decoder (Steps 7-8), which are presented in the algorithm 1, and discussed next.

A. Encoder (Steps 1-3)

The Encoder of the novel Per-C based unsupervised scoring system consists of three steps as seen from Algo-1. Input to the encoder (please see Step 1) is V number of variables (which contribute to the score), each with an associated L number of linguistic terms and N associated numeric data values.

Then in the Step 2a, data cleaning is performed by removing the duplicates (if any) from the N number of data values to arrive at M surviving data values such that $M \leq N$. In Step 2b, the L centroids of these M data values are found (Linguistic terms, $LT_i, i = 1, 2, ..., L$) using the FCM, which also gives the degree of memberships of each M data value into the fuzzy boundaries around each L centroid. In Step 2c, for each data value say x, of the M data values, we define twin valued set called the neighborhood $NH = \{\overleftarrow{NH}, \overrightarrow{NH}\}$, where $\overrightarrow{NH}$ identifies the centroid which is closest to x and $\overleftarrow{NH}$ is the further centroid. Thus, NH enables the calculation of Upper Membership Function or UMF ($\bar{\mu}(x)$) and Lower Membership Function or LMF ($\underline{\mu}(x)$) values for each data point x. It is mentioned here that each of these M data values can belong to a maximum of two adjacent centroids with membership degrees $\bar{\mu}(x)$ and $\underline{\mu}(x)$, because these centroids are in one dimension. There is always more uncertainty about the boundary of the partition [35], and thus, we want membership degrees in a maximum of two adjacent centroids for any of the M data values.

In the Step 2d, the $\bar{\mu}(x)$ and $\underline{\mu}(x)$ are used for mapping each of the M data values into one of the interior or shoulder (left or right) FOUs. Consider a plot of the $\bar{\mu}$ and $\underline{\mu}$ for a data value, x (of M unique data values), belonging to a i^{th} linguistic term $LT_i, i = 1, 2, ..., L$, as shown in Fig. 2a. In the Fig., the blue colored curve is the $\bar{\mu}$ and orange colored is the $\underline{\mu}$. The term LT_i overlaps on the left side with LT_{i-1} and the right side with LT_{i+1}. From this Fig., 2a, we map the $\bar{\mu}(x)$ and $\underline{\mu}(x)$ into the UMF and LMF of the FOU parameters for the interior and shoulder IT2 FS word models, using (2)-(4). It is mentioned here that the resulting IT2 FS word model,as shown in Fig., 2b, is the interior FOU. However, the left shoulder of right shoulder FOU may also be obtained. We arrive at the FOU parameters as explained below.

a) UMF parameters: The UMF of the IT2 FS is defined by the parameters a, b, c and d (Please see Fig. 2b). To estimate their values through Fig. 2a, a is defined as the smallest x value for which $\bar{\mu} = 1$ in LT_{i-1} and $\underline{\mu} = 0$ in LT_i ($NH = \{LT_i, LT_{i-1}\}$); b is the smallest x that has $\bar{\mu} = 1$ in LT_i and $\underline{\mu} = 0$ in LT_{i-1} ($NH = \{LT_{i-1}, LT_i\}$); c is the largest x at which $\bar{\mu} = 1$ in LT_i and $\underline{\mu} = 0$ in LT_{i+1} ($NH = \{LT_{i+1}, LT_i\}$); and d is the largest x value for which $\bar{\mu} = 1$ in LT_{i+1} and $\underline{\mu} = 0$ in LT_i ($NH = \{LT_i, LT_{i+1}\}$). However, for left shoulder FOUs, $a = b = 0$ and for right shoulder FOUs, $c = d = 10$. It is mentioned here that wherever the maximum value of 1 and minimum of 0 of $\bar{\mu}$ or $\underline{\mu}$ are not possible, then the maximum possible values of $\bar{\mu}$ or $\underline{\mu}$ as obtained from FCM should be used.

b) LMF parameters: The LMF of the IT2 FS word model is defined by the parameters e, f, g and μ_f (Please see Fig. 2b). From Fig. 2a it follows that the value of parameter e is the average of all $x = e_q, q = 1, ..., j$, where $e_q, q = 1, 2, ..., j$ are the respective j^{th} data points satisfying the condition that $\underline{\mu}(x = e_q) = 0$ in LT_i and $\underline{\mu}(x = e_q + 1) \neq 0$ in LT_i, $e_q + 1$, being the immediately next data point to e_q, lying within the LT_i. Similarly, the value of g is the average of all $x = g_q, q = 1, 2, ..., j$, where $g_q, q = 1, ..., j$ are the respective j^{th} data points satisfying the condition that $\underline{\mu}(x = g_q) \neq 0$ in LT_i and $\underline{\mu}(x = g_q + 1) = 0$ in LT_i, $g_q + 1$, being the immediate successor of q_q, lying within the LT_i. The parameter f's value

[4] With the progression of Industry 4.0, sensors are being increasingly deployed in the environment which collects a stream of data values for the problem variables.

Algorithm 1 Novel Per-C based Unsupervised Scoring System

#Encoder

1: Input: V: Number of variables contributing to the score, L: Number of associated linguistic terms to each variable, N: Number of data values associated with each variable

2: For each Variable Repeat:

a: Remove the duplicates from the N number of data values to arrive at M unique data values. ▷ **Data Cleaning**

b: Subject these M data values to FCM to obtain L number of centroids $LT_i, i = 1, 2, ..., L$, as well as degree of memberships of each M data value into each of the LT_i centroids. ▷ **Data Processing**

c: $\forall x \in M$, define Neighborhood $NH = \{\{\overleftarrow{NH}, \overrightarrow{NH}\} \mid \overleftarrow{NH} = \underset{cen \leq x}{\text{argmax}}\, d(cen - x), \overrightarrow{NH} = \underset{cen \geq x}{\text{argmin}}\, d(cen - x)\}$, cen : centroid value and $d(cen - x)$: distance between cen and x. Calculate the UMF for x as: $\bar{\mu}(x) = max\{\mu_{NH} = \mu_{\overrightarrow{NH}}\}$ and LMF as: $\underline{\mu}(x) = min\{\mu_{NH} = \mu_{\overleftarrow{NH}}\}$.

d: For $\exists x \in M$, lying within i^{th} Linguistic term (LT_i), map it into interior or shoulder FOUs, where UMF of the FOU is defined by parameters: {a,b,c,d} and LMF by: {e,f,g,μ_f}, as ▷ **Mapping into FOU**

Left Shoulder FOU:

$$\begin{aligned}
&a = 0, b = 0, e = 0, g = \{\frac{\sum_{q=1}^{j} g_q}{q}, \mid \underline{\mu}(x = g_q) \neq 0 \cap \underline{\mu}(x = q_q + 1) = 0 \ \forall q_q, q = 1, 2, ..., j\}, f = 0, \mu_f = 1 \\
&c = max\{x \mid NH = \{LT_{i+1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, d = max\{x \mid NH = \{LT_i, LT_{i+1}\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}
\end{aligned} \tag{2}$$

Interior FOU:

$$\begin{aligned}
&a = min\{x \mid NH = \{LT_i, LT_{i-1}\}, \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, b = min\{x \mid NH = \{LT_{i-1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, \\
&c = max\{x \mid NH = \{LT_{i+1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, d = max\{x \mid NH = \{LT_i, LT_{i+1}\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\} \\
&e = \frac{\sum_{q=1}^{j} e_q}{q}, \mid \underline{\mu}(x = e_q) = 0 \cap \underline{\mu}(x = e_q + 1) \neq 0, g = \frac{\sum_{q=1}^{j} g_q}{q}, \mid \underline{\mu}(x = g_q) \neq 0 \cap \underline{\mu}(x = q_q + 1) = 0, \\
&f = \frac{\sum_{q=1}^{j} f_q}{q}, \mid max(\underline{\mu})\} \ \mu_f = \underline{\mu}(f_q) \cap q = 1, 2, ..., j
\end{aligned} \tag{3}$$

Right Shoulder FOU:

$$\begin{aligned}
&a = min\{x \mid NH = \{LT_i, LT_{i-1}\}, \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, b = min\{x \mid NH = \{LT_{i-1}, LT_i\} \cap \underline{\mu}(x) = 0, \bar{\mu}(x) = 1\}, \\
&c = 10, \ d = 10, e = \{\frac{\sum_{q=1}^{j} e_q}{q}, \mid \underline{\mu}(x = e_q) = 0 \cap \underline{\mu}(x = e_q + 1) \neq 0 \ \forall e_q, q = 1, 2, ..., j\}, \ f = 10, \ g = 10, \ \mu_f = 1
\end{aligned} \tag{4}$$

3: Determine the IT2 FS word models for linguistic terms of all the variables and store them in the codebook

#CWW Engine

4: Input $[n_1, n_2, ..., n_V]$: Data vector containing numeric values of each variable

5: For each $n_j, j = 1, 2, ..., V$ in Step 4, find out the linguistic term associated with the respective variable so that n_j has the highest degree of membership in it. Also, find out the associated linguistic weight of this term.

6: Extract the IT2 FS model $\tilde{X}_j$ of each linguistic term and its associated linguistic weight $\tilde{W}_j$ from the codebook (of Step 3) and aggregate them using LWA as:

$$\tilde{Y}_{LWA} = \frac{\sum_{j=1}^{V} \tilde{X}_j \tilde{W}_j}{\sum_{j=1}^{V} \tilde{W}_j} \tag{5}$$

#Decoder

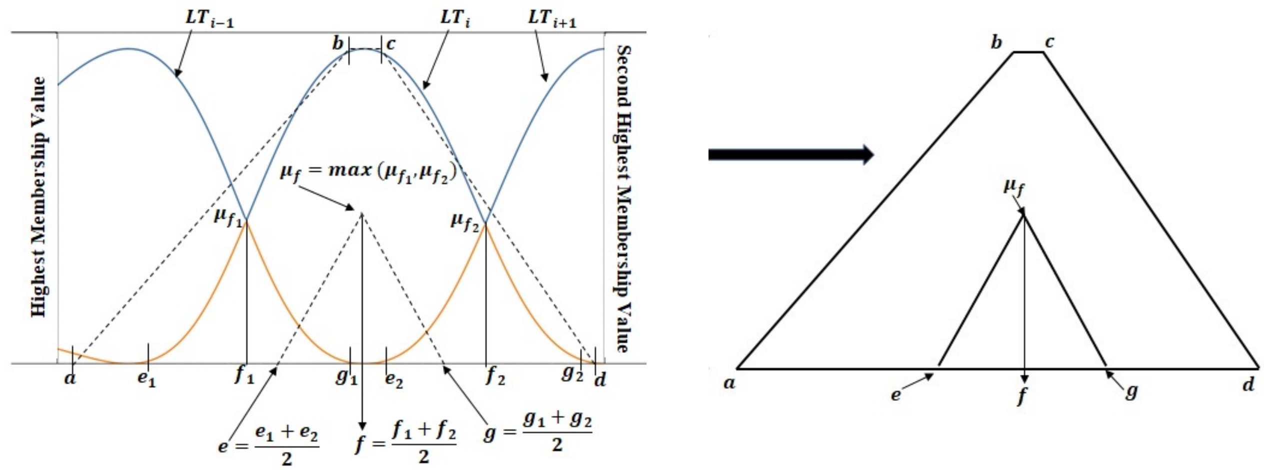
7: Numeric recommendation for the $\tilde{Y}_{LWA}$ is given based on c_l and c_r determined from the EKM algorithm, as:

$$c_{avg} = \frac{c_l + c_r}{2} \tag{6}$$

8: Linguistic recommendation for $\tilde{Y}_{LWA}$ is given using Jaccard's similarity measure as:

$$sm_j(\tilde{Y}_{LWA}, \tilde{L}_k) = \frac{\sum_{j=1}^{N} min(\bar{\mu}_{\tilde{Y}}(x_j), \bar{\mu}_{L\tilde{T}_k}(x_j)) + \sum_{j=1}^{N} min(\underline{\mu}_{\tilde{Y}}(x_j), \underline{\mu}_{L\tilde{T}_k}(x_j))}{\sum_{j=1}^{N} max(\bar{\mu}_{\tilde{Y}}(x_j), \bar{\mu}_{L\tilde{T}_k}(x_j)) + \sum_{j=1}^{N} max(\underline{\mu}_{\tilde{Y}}(x_j), \underline{\mu}_{L\tilde{T}_k}(x_j))} \tag{7}$$

$L\tilde{T}_k$ is a codebook linguistic term from Step 3 and x_j are equally spaced inside the support of $\tilde{Y}_{LWA} \cup L\tilde{T}_k$.



(a) $\bar{\mu}$ and $\underline{\mu}$ values of LT_i and its adjacent centroids LT_{i-1} and LT_{i+1}

(b) FOU parameters of the IT2 FS constructed from $\bar{\mu}$ and $\underline{\mu}$

Figure 2: Mapping the UMF ($\bar{\mu}$) and LMF ($\underline{\mu}$) from centroid LT_i into FOU parameters of the IT2 FS

is the average of all $x = f_q, q = 1, 2, ..., j$, where $f_q, q = 1, 2, ..., j$ are the respective j^{th} data points of highest $\underline{\mu}$, lying within LT_i and μ_{f_q} is the $\underline{\mu}$ values at $x = f_q$. However, for the left shoulder FOU, $e = f = 0$ and for the right shoulder FOU, $f = g = 10$. Also, $\mu_f = 1$ for both these shoulder FOUs. An important point to note in Fig. 2a is that it shows only two instances of each of e_q, q_q and f_q. The data can contain multiple of these data points.

Thus, the Step 2 is repeated for all the variables so that FOUs are obtained for all the linguistic terms corresponding to each of the variables, the FOUs as well as linguistic terms are stored in a codebook (Please see Step 3.

B. CWW Engine (Steps 4-6)

The CWW engine consists of three steps as seen from Algorithm-1. Step 4 inputs a data vector containing the numeric data values corresponding to each of the V variables into the Algo, where for each numeric value in the data vector, a linguistic term from codebook is found in which this numeric value has maximum membership degree, in Step 5 (Please refer Section SM-V). Then after finding the associated linguistic weight of each of these linguistic terms, the FOU data for each linguistic term and its associated weight is extracted from the codebook (constructed in Encoder) and aggregated to generate an IT2 FS $\tilde{Y}_{LWA}$ using (5), in Step 6 (Please refer [27] or Section SM-II.B).

C. Decoder (Steps 7-8)

The decoder of the proposed scoring system generates a numeric value, c_{avg} using (6) for the $\tilde{Y}_{LWA}$ using the switch points c_l and c_r of an IT2 FS (Please refer Section SM-II.C) in the Step 7. Finally, in the Step 8, a linguistic recommendation is generated for the $\tilde{Y}_{LWA}$ using the Jaccard's similarity measure using (7).

The complete block diagram of our proposed scoring system is shown in Fig. 3. From Fig., it can be seen that the input to our proposed scoring system is a stream of numeric data values, linguistic labels and linguistic weights. Inside the encoder, the FOUs are generated for the linguistic terms and their associated weights and stored in a codebook (please refer Section III-A. Then a numeric data vector causes the extraction of the linguistic terms' and their respective associated linguistic weights' FOUs to be extracted from the codebook based on the membership degree of each numeric value into the respective linguistic term from the codebook. These extracted FOUs of the linguistic terms and weights are subjected to LWA, which generates an aggregated nine point Y_{LWA} at the output of CWW engine (please refer Section III-B). This Y_{LWA} is fed as input to the decoder to generate a numeric score, c_{avg} using the EKM algorithm and a linguistic recommendation using the Jaccard's similarity (please refer Section III-C).

IV. Results: Real-world case study for driver's score estimation data

In this section we demonstrate the prowess of our Novel Per-C based Unsupervised Scoring System (from Section III) using the real-life telemetry data from [9]–[11]. In these works, the authors collected (a stream of) numeric data values for seven variables: acceleration (AC), braking (BR), car following (CF), lane drifting (LD), lane weaving (LW), over-speeding (OS) and turning (TU), from the road trips of six drivers. AC (BR) denotes a sudden increase (decrease) in the vehicle speed, CF is a measure of the safe distance between the personal vehicle and the one ahead, LD is measured as a deviation from the centre of the driving lane, LW is the number of lane changes, OS represents driving above the maximum allowed speed limit and TU means a sudden change in vehicle's direction.

The drivers conducted trips in the motorway and the secondary types of road. Motorway roads had 2 to 4 lanes in each direction and around 120 km/hr of maximum allowed speed. The secondary road had principally one lane in each direction with a maximum allowed speed of 90 km/hr. Each driver performed three trips on the motorway road (round-trip,

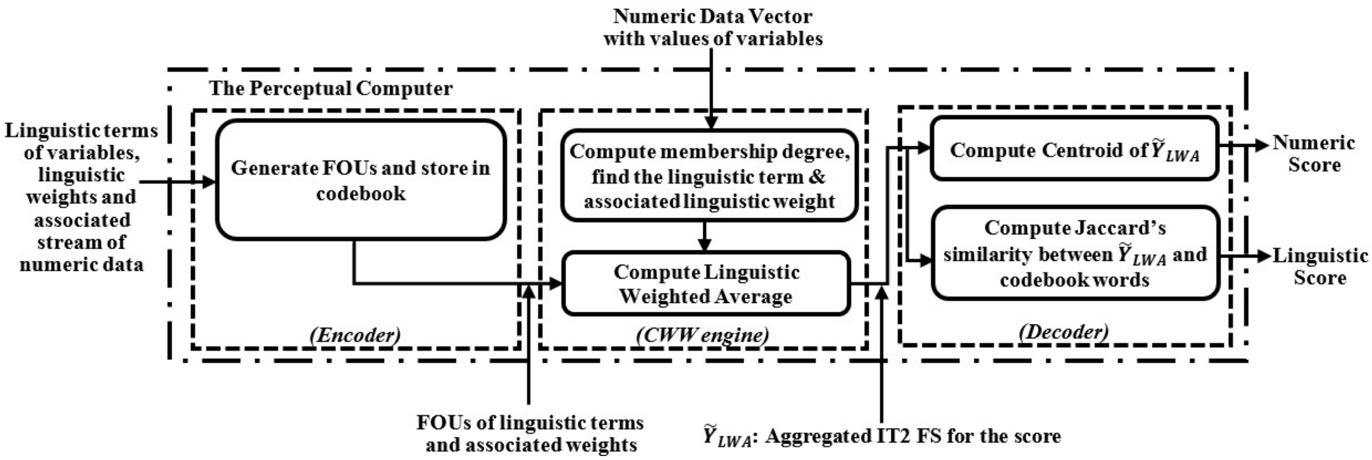


Figure 3: The proposed Per-C based unsupervised scoring system

around 25km each) and four trips on the secondary road (one-way, around 16km each).

We now present the working of our proposed novel Per-C based Unsupervised scoring system (from Section III) through this real-life telemetry data and use it to compute the overall trip score (Sections IV-A-IV-D). We also compare the results of our proposed system to those in the original telemetry data (Section V-A). We then give a robustness analysis of the scores obtained with our proposed scoring system (Section V-B).

A. Encoding telemetry data

Considering the same seven variables: AC, BR, CF, LD, LW, OS and TU [9]–[11], we associated five linguistic terms to each of these variables. Further, each linguistic term was allocated a linguistic weight (WT) (instead of variable), and WT was also assigned five linguistic terms:Very Low (WTV), Low (WTL), Medium (WTM), High (WTH) and Very High (WTE). The linguistic terms associated with the variables and the associated WT of each variable's linguistic term are listed in Table I.

For exemplifying the construction of (associated) linguistic terms' (of variables) FOU plots and those of the associated WT's, consider the variable AC in the motorway road. It had N = 16,631 associated numeric data values. Removal of duplicate values reduced them to M = 546 unique data values. As AC had five associated linguistic terms (refer Table I), we found L=5 centroids of these 546 numeric data values, as well as memberships degrees of each of these 546 values to every centroid's soft boundary, using FCM. Subsequently, we extracted the data points with the highest and second highest degrees of membership from these 546 values to estimate the FOU parameters.

Let's say the centroids are denoted as LT_1, LT_2, LT_3, LT_4 and LT_5. Consider LT_3, which will be used to define the linguistic variable ACM (please see Table I). We found that for LT_3, $x = 5.02$ is the smallest value at which the neighborhood $NH = LT_3, LT_2$ exists, with $\bar{(\mu)} = 0.89$ and $\underline{\mu} = 0.05$ (As stated in Section III-A, if $\bar{(\mu)} = 1$ and $\underline{\mu} = 0$ are not in the data set, then maximum available values of the two memberships should be used). The $x = 6.67$ is the minimum value at which $NH = LT_2, LT_3$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$, whereas $x = 6.78$ is the maximum value at which $NH = LT_4, LT_3$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. The $x = 8.06$ is the largest data value at which $NH = LT_3, LT_4$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. As LT_3, is the interior FOU, therefore, as seen from (3), $a = 5.02$, $b = 6.67$, $c = 6.78$ and $d = 8.06$. $x = 5.49$ and $x = 6.81$ satisfy the condition $\underline{\mu}(x = e_q) = 0$ in LT_3 and $\underline{\mu}(x = e_q + 1) \neq 0.0$ in LT_3. Therefore, $e_1 = 5.49$, $e_2 = 6.81$ and $e = \frac{e_1+e_2}{2} = 6.15$. Similarly, $x = 6.64$ and $x = 7.99$ give rise to $g = 7.32$. The $x = 6.07$ and $x = 7.4$ are the data values with the highest $\underline{\mu}$, lying within LT_3, and the membership degrees at both these points have a value of 0.45. Hence, $f = \frac{f_1+f_2}{2} = 6.74$ and $\mu_f = 0.45$. In this way, FOU parameters for the LT_3 are estimated. Similarly, FOU parameters of the interior FOUs of LT_2 and LT_4 are estimated.

For estimating the FOU parameters of left shoulder viz., LT_1, $x = 3.33$ is the maximum data value at which $NH = LT_2, LT_1$, $\bar{\mu} = 1$ and $\underline{\mu} = 0$. $x = 5.01$ is is the maximum data value at which $NH = LT_1, LT_2$, $\bar{\mu} = 0.89$ and $\underline{\mu} = 0.05$. Also, $x = 4.99$ is the only data point which satisfies the condition $\underline{\mu}(x = g_q) \neq 0$ in LT_3 and $\underline{\mu}(x = g_q + 1) = 0$ in LT_1. Therefore, using (2), $a = 0,\ b = 0,\ c = 3.33,\ d = 5.01,\ e = 0,\ f = 0,\ g = 4.99,\ \mu_f = 1$.

For estimating the FOU parameters of right shoulder viz., LT_5, $x = 8.07$ is the smallest data value at which neighborhood exists as $NH = LT_4, LT_5$, with $\bar{(\mu)} = 1$ and $\underline{\mu} = 0$. The $x = 9.34$ is the minimum data value at which $NH = LT_5, LT_4$, with $\bar{(\mu)} = 1$ and $\underline{\mu} = 0$. The $x = 8.16$ is the only data point which satisfy the condition $\underline{\mu}(x = e_q) = 0$ in LT_5 and $\underline{\mu}(x = e_q + 1) \neq 0$ in LT_5. Therefore, using (4), $a = 8.07,\ b = 9.34,\ c = 10,\ d = 10,\ e = 8.16,\ f = 10,\ g = 10,\ \mu_f = 1$. In this manner, the FOU parameters of all the linguistic terms of AC are determined and listed in Table II.

Similarly, the FOU plots for the associated linguistic terms of all other variables were computed. For generating the data stream for WT, all the data values associated with a variable were summed and processed, and FOU plots were generated. The obtained FOU plots along with the respective linguistic terms (of variables and WT) were stored in the form of a

Table I: Variables: Associated Linguistic Terms and Linguistic Weights.

Variables	Associated Linguistic Terms	Linguistic weight Motorway	Linguistic weight Secondary	Variables	Associated Linguistic Terms	Linguistic weight Motorway	Linguistic weight Secondary
Acceleration (AC)	Very Low (ACV)	Very Low (WTV)	Very High (WTE)	Braking (BR)	Very High (BRE)	Very Low (WTV)	Very High (WTE)
	Low (ACL)	Low (WTL)	High (WTH)		High (BRH)	Low (WTL)	High (WTH)
	Medium (ACM)	Medium (WTM)	Medium (WTM)		Moderate (BRM)	Medium (WTM)	Medium (WTM)
	High (ACH)	High (WTH)	Low (WTL)		Less (BRL)	High (WTH)	Low (WTL)
	Very High (ACE)	Very High (WTE)	Very Low (WTV)		Very Less (BRV)	Very High (WTE)	Very Low (WTV)
Car Following (CF)	Very small (CFV)	Very Low (WTV)	Very Low (WTV)	Lane Drifting (LD)	Very Large (LDE)	Very Low (WTV)	Very High (WTE)
	Small (CFS)	Low (WTL)	Low (WTL)		Large (LDL)	Low (WTL)	High (WTH)
	Average (CFA)	Medium (WTM)	Medium (WTM)		Average (LDA)	Medium (WTM)	Medium (WTM)
	Large (CFL)	High (WTH)	High (WTH)		Small (LDS)	High (WTH)	Low (WTL)
	Very Large (CFE)	Very High (WTE)	Very High (WTE)		Very small (LDV)	Very High (WTE)	Very Low (WTV)
Lane Weaving (LW)	Very High (LWE)	Very Low (WTV)	Very Low (WTV)	Over Speeding (OS)	Very High (OSE)	Very Low (WTV)	Very Low (WTV)
	High (LWH)	Low (WTL)	Low (WTL)		High (OSH)	Low (WTL)	Low (WTL)
	Medium (LWM)	Medium (WTM)	Medium (WTM)		Moderate (OSM)	Medium (WTM)	Medium (WTM)
	Low (LWL)	High (WTH)	High (WTH)		Less (OSL)	High (WTH)	High (WTH)
	Very Low (LWV)	Very High (WTE)	Very High (WTE)		Very Less (OSV)	Very High (WTE)	Very High (WTE)
Turning (TU)	Very Large (TUE)	Very Low (WTV)	Very Low (WTV)				
	Large (TUL)	Low (WTL)	Low (WTL)				
	Average (TUA)	Medium (WTM)	Medium (WTM)				
	Small (TUS)	High (WTH)	High (WTH)				
	Very small (TUV)	Very High (WTE)	Very High (WTE)				

codebook. The FOU data for the linguistic terms of all the variables and WT in the motorway is given in Table II. The corresponding FOU data for the secondary road, as well as FOU plots for the two types of road, are given in Table SM-I, Fig. SM-4 and Fig. SM-5.

B. Running CWW Engine

To exemplify the selection of linguistic terms for a variable in CWW engine, consider a data vector containing the variables' $\{AC, BR, CF, LD, LW, OS, TU\}$ values in the motorway as: {10, 9.18, 9.79, 7.8, 10, 9.32, 8.1}. For each of these values of the variables, we find out the respective highest membership degrees in the respective linguistic terms of the variables (Section SM-II.B). Thus, the linguistic terms corresponding to the values of the variable given in the vector are $\{ACE, BRV, CFE, LDS, LWV, OSV, TUV\}$ (For full forms of ACE, BRV, CFE, LDS, LWV, OSV and TUV, please see Table I). From Table I, the respective corresponding WT of the linguistic terms are found as: $\{WTE$, WTE, WTE, WTH, WTE, WTE, $WTE\}$, where WTE is Very High and WTH is a high.

Hence, the FOU data for the respective linguistic terms and associated WT from codebook (or Table II) is extracted and aggregated using (5), to generate nine points IT2 FS word model (described by its UMF and LMF) given as $\tilde{Y}_{LWA}$={7.01, 8.87, 9.78, 10, 7.24, 8.45, 9.77, 9.86, 0.46}, the first four points in the $\tilde{Y}_{LWA}$ describe the UMF and remaining the LMF.

C. Decoding Telemetry Data into Drivers Scores

In CWW engine, the values of linguistic terms of variables were linguistic weighted averaged to generate a nine point FOU for $\tilde{Y}_{LWA}$ (Please see section IV-B). This $\tilde{Y}_{LWA}$ corresponds to the output variable, the driver's score (DS). However, this FOU for $\tilde{Y}_{LWA}$ is decoded to a numeric score, $c_{avg} = 8.81$, as discussed in Section III-C.

Now, from the works [9]–[11], we extracted the numeric data values for DS and associated five linguistic terms to it: Terrible (DSV), Poor (DSL), Borderline (DSM), Fair (DSH) and Good (DSE). Then, we generated the FOUs for these linguistic terms using the encoder, section III-A. The FOU data for these linguistic terms are stored in the codebook and shown in Table II for a motorway. The corresponding FOU data for the secondary road, as well as FOU plots for the two types of road, are given in Table SM-I, Fig. SM-4 and Fig. SM-5.

To generate linguistic recommendations for $\tilde{Y}_{LWA}$, we find out the similarity between $\tilde{Y}_{LWA}$ and the FOUs of five linguistic terms associated with DS, using Jaccard's similarity measure from (7). Thus, the closest linguistic recommendation comes out to be Good (DSH).

D. Calculating the overall driver's score for one trip

From the telemetry data [9]–[11], we picked up the data vectors containing numeric data values of seven variables, accumulated per second and aggregated them through the encoder and CWW engine of our proposed scoring system (Sections IV-A-IV-B), to arrive at aggregated nine points Y_{LWA}'s. Then we weighted aggregated these Y_{LWA}'s based on their frequency in the complete trip of a driver, to generate a Y_{LWA} for the complete trip, denoted as $Y_{Overall-Trip_{LWA}}$. A linguistic recommendation for $Y_{Overall-Trip_{LWA}}$ was generated using the Jaccard's similarity (from (7) between $Y_{Overall-Trip_{LWA}}$ and FOUs of linguistic terms for DS (please see Table II). Thus, the scores were calculated for motorway and secondary roads (for the normal, aggressive and drowsy type of behaviours). All the results are given in Table III.

From the Table, it follows that the Normal driving behaviour, is scored as either Fair (DSH) or Good (DSE); however, the Aggressive or Drowsy driving behaviours, are rated as Borderline (DSM) or Poor (DSL), and very rarely as Fair (DSH). Thus, our proposed scoring system differentiates

Table II: FOU data for the Linguistic Terms of the Variables and the Driver's Score for Motorway

Variables, Linguistic weights and Driver's Score	Associated Linguistic terms	FOU data										
		UMF				LMF				Centroid		
		a	b	c	d	e	f	g	μ_f	c_l	c_r	c_{avg}
Acceleration (AC)	Very Low (ACV)	0.00	0.00	3.33	5.01	0.00	0.00	4.99	1.00	1.62	2.21	1.91
	Low (ACL)	0.00	5.34	5.48	6.74	4.46	5.21	5.82	0.45	2.84	5.68	4.26
	Medium (ACM)	5.02	6.67	6.78	8.06	6.15	6.74	7.32	0.45	6.16	7.13	6.65
	High (ACH)	6.75	8.02	8.13	10.00	7.49	8.07	8.66	0.46	7.68	8.74	8.21
	Very High (ACE)	8.07	9.34	10.00	10.00	8.16	10.00	10.00	1.00	9.27	9.40	9.34
Braking (BR)	Very High (BRE)	0.00	0.00	1.03	2.96	0.00	0.00	2.82	1.00	0.91	1.12	1.02
	High (BRH)	0.00	2.86	3.04	5.01	2.07	2.96	3.85	0.46	1.93	3.57	2.75
	Moderate (BRM)	2.97	4.93	5.10	7.06	4.12	5.01	5.92	0.44	4.40	5.63	5.02
	Less (BRL)	5.02	6.99	7.17	10.00	6.18	7.07	7.97	0.46	6.46	8.08	7.27
	Very Less (BRV)	7.07	9.00	10.00	10.00	7.21	10.00	10.00	1.00	8.89	9.10	9.00
Car Following (CF)	Very small (CFV)	0.00	0.00	0.60	2.82	0.00	0.00	2.77	1.00	0.90	1.01	0.96
	Small (CFS)	0.00	3.03	3.23	5.14	1.79	2.97	3.89	0.45	1.98	3.58	2.78
	Average (CFA)	2.83	5.05	5.22	7.13	4.26	5.14	6.02	0.45	4.40	5.73	5.07
	Large (CFL)	5.15	7.06	7.23	10.00	6.27	7.14	8.01	0.46	6.55	8.13	7.34
	Very Large (CFE)	7.14	9.02	10.00	10.00	7.27	10.00	10.00	1.00	8.92	9.12	9.02
Lane Drifting (LD)	Very Large (LDE)	0.00	0.00	3.53	4.92	0.00	0.00	4.82	1.00	1.56	2.23	1.89
	Large (LDL)	0.00	4.85	4.98	6.40	4.28	4.92	5.57	0.46	2.68	5.36	4.02
	Average (LDA)	4.93	6.34	6.47	7.88	5.76	6.41	7.06	0.44	5.96	6.85	6.41
	Small (LDS)	6.41	7.83	7.96	10.00	7.25	7.89	8.54	0.46	7.45	8.62	8.03
	Very small (LDV)	7.89	9.28	10.00	10.00	7.99	10.00	10.00	1.00	9.20	9.35	9.28
Lane Weaving (LW)	Very High (LWE)	0.00	0.00	1.30	2.50	0.00	0.00	2.38	1.00	0.77	1.02	0.90
	High (LWH)	0.00	2.38	2.50	4.00	1.87	2.58	3.19	0.47	1.61	2.95	2.28
	Medium (LWM)	2.63	3.91	4.00	6.36	3.38	4.17	4.86	0.36	3.64	4.92	4.28
	Low (LWL)	4.12	5.71	6.00	10.00	5.40	6.50	7.27	0.36	5.53	7.77	6.65
	Very Low (LWV)	6.67	8.89	10.00	10.00	6.67	10.00	10.00	1.00	8.75	8.92	8.84
Over-Speeding (OS)	Very High (OSE)	0.00	0.00	1.42	3.38	0.00	0.00	3.36	1.00	1.09	1.32	1.20
	High (OSH)	0.00	3.52	3.70	5.47	2.58	3.50	4.36	0.45	2.21	4.09	3.15
	Moderate (OSM)	3.39	5.39	5.55	7.33	4.66	5.48	6.29	0.45	4.82	6.03	5.42
	Less (OSL)	5.48	7.26	7.43	10.00	6.53	7.34	8.15	0.46	6.79	8.26	7.52
	Very Less (OSV)	7.34	9.09	10.00	10.00	7.46	10.00	10.00	1.00	9.00	9.18	9.09
Turning (TU)	Very Large (TUE)	0.00	0.00	1.00	2.89	0.00	0.00	2.74	1.00	0.89	1.09	0.99
	Large (TUL)	0.00	2.78	2.96	4.87	1.88	2.88	3.75	0.46	1.89	3.43	2.66
	Average (TUA)	2.90	4.8	4.97	6.86	4.01	4.88	5.76	0.44	4.28	5.48	4.88
	Small (TUS)	4.88	6.80	6.97	10.00	6.02	6.88	7.74	0.46	6.28	8.00	7.14
	Very small (TUV)	6.87	8.74	10.00	10.00	7.01	10.00	10.00	1.00	8.79	9.03	8.91
Linguistic Weight (WT)	Very Low (WTV)	0.00	0.00	1.00	2.94	0.00	0.00	2.80	1.00	0.91	1.11	1.01
	Low (WTL)	0.00	2.84	3.02	4.99	2.04	2.94	3.84	0.46	1.93	3.55	2.74
	Medium (WTM)	2.95	4.91	5.09	7.05	4.10	5.00	5.91	0.44	4.38	5.62	5.00
	High (WTH)	5.00	6.98	7.16	10.00	6.17	7.06	7.96	0.46	6.45	8.08	7.26
	Very High (WTE)	7.06	9.00	10.00	10.00	7.20	10.00	10.00	1.00	8.89	9.09	8.99
Driver's Score (DS)	Terrible (DSV)	0.00	0.00	5.28	6.32	0.00	0.00	6.28	1.00	2.03	3.05	2.54
	Poor (DSL)	0.00	6.30	6.40	7.41	5.78	6.35	6.82	0.45	3.06	6.64	4.85
	Borderline (DSM)	6.33	7.38	7.46	8.48	6.96	7.42	7.89	0.45	7.09	7.74	7.41
	Fair (DSH)	7.42	8.45	8.53	10.00	8.02	8.48	8.95	0.45	8.17	9.01	8.59
	Good (DSE)	8.49	9.49	10.00	10.00	8.55	10.00	10.00	1.00	9.43	9.53	9.48

between different types of driving behaviour and assigns a score in a proper manner.

V. Discussions

We now draw comparisons between the results of our proposed system to those in the original telemetry data [9]–[11]; provide a robustness analysis of the scores obtained with our proposed scoring system; provide a conceptual comparison between the existing Per-C and that of our proposed system; and discuss some of the interesting facts and findings from the work in this paper, which are applicable to any scoring systems in general. More discussions in the context of the driver's scoring systems are given in the Section SM-IV, due to paucity of space.

A. Comparison of scores obtained with proposed system and original telemetry data

In the works [9]–[11], the data vectors containing numeric values of the 7 variables were collected per second of the driver's trip. These data values were arithmetically averaged to calculate the overall driver's score for one trip. The scores were classified for the normal, aggressive and drowsy type of behaviour, where the driver in these trips was asked to simulate this type of behaviour. For reference, the respective scores for the driver, type of road and behaviour obtained with the telemetry data are listed in the last column of Table III. The table also lists the drivers' scores obtained with our proposed scoring system.

For comparison between the scores obtained with our proposed scoring system and original telemetry data, we ran the Kolmogorov-Smirnov statistics [16] on the respective streams

Table III: Comparison of drivers' trip scores in Motorway & Secondary: using proposed scoring system vs Telemetry Data

Driver*	Behavior**	Overall Score with proposed scoring system													Telemetry
		a	b	c	d	e	f	f	g	μ_f	c_l	c_r	c_{avg}	LS***	Score
		Type of Road: Motorway													
D1	N	6.52	8.65	9.61	9.93	6.93	8.3	9.6	9.73	0.48	8.31	8.86	8.58	Fair (DSH)	8.84
	A	6.41	8.50	9.33	9.89	6.92	8.15	9.31	9.54	0.46	8.11	8.76	8.44	Fair (DSH)	8.76
	D	5.45	8.14	9.14	9.88	6.34	7.63	9.14	9.41	0.42	7.58	8.52	8.00	Borderline (DSM)	7.52
D2	N	6.60	8.62	9.54	9.90	6.96	8.26	9.53	9.69	0.48	8.30	8.84	8.57	Fair (DSH)	9.08
	A	4.75	7.46	8.26	9.56	5.89	7.19	8.22	8.71	0.45	6.87	7.98	7.42	Borderline (DSM)	6.91
	D	5.96	8.28	9.19	9.89	6.62	7.96	9.19	9.44	0.47	7.86	8.62	8.24	Borderline (DSM)	7.87
D3	N	6.94	8.83	9.77	9.94	7.16	8.50	9.77	9.85	0.50	8.57	8.99	8.78	Fair (DSH)	9.62
	A	6.05	8.27	9.24	9.93	6.54	8.00	9.22	9.48	0.50	7.93	8.61	8.27	Fair (DSH)	7.54
	D	5.57	8.02	8.86	9.80	6.41	7.61	8.86	9.19	0.42	7.51	8.42	7.96	Borderline (DSM)	7.67
D4	N	7.14	8.97	9.91	10.00	7.29	9.36	9.91	9.94	0.78	8.84	9.10	8.97	Good (DSE)	9.84
	A	5.42	7.89	8.77	9.81	6.21	7.57	8.74	9.14	0.46	7.42	8.33	7.87	Borderline (DSM)	7.24
	D	6.09	8.44	9.36	9.90	6.77	7.99	9.36	9.55	0.43	7.98	8.73	8.35	Borderline (DSM)	8.40
D5	N	6.34	8.43	9.26	9.95	6.85	8.13	9.24	9.51	0.48	8.07	8.73	8.40	Fair (DSH)	8.44
	A	5.07	7.52	8.31	9.77	5.92	7.25	8.28	8.80	0.47	7.04	8.08	7.56	Borderline (DSM)	6.99
	D	5.13	8.05	9.18	9.94	6.10	7.59	9.15	9.45	0.45	7.49	8.46	7.97	Borderline (DSM)	6.61
D6	N	6.64	8.68	9.66	10.00	6.87	8.83	9.66	9.79	0.67	8.49	8.88	8.68	Fair (DSH)	8.66
	A	5.53	7.92	8.79	9.73	6.22	7.56	8.76	9.13	0.47	7.47	8.30	7.88	Borderline (DSM)	7.29
	D	5.38	7.77	8.65	9.72	6.15	7.28	8.72	9.06	0.38	7.29	8.26	7.78	Borderline (DSM)	9.96
		Type of Road: Secondary													
D1	N1	5.88	8.02	8.90	9.87	6.64	7.82	8.90	9.32	0.46	7.69	8.52	8.11	Fair (DSH)	8.40
	N2	6.36	8.35	9.27	9.85	6.91	8.13	9.25	9.55	0.48	8.08	8.72	8.40	Fair (DSH)	8.74
	A	5.38	7.77	8.82	9.65	6.13	7.44	8.81	9.18	0.45	7.40	8.25	7.82	Borderline (DSM)	7.28
	D	5.32	7.58	8.38	9.58	6.31	7.48	8.41	8.90	0.48	7.21	8.15	7.68	Borderline (DSM)	8.30
D2	N1	6.63	8.48	9.41	9.88	7.09	8.37	9.41	9.62	0.50	8.27	8.83	8.55	Fair (DSH)	9.18
	N2	6.27	8.29	9.33	9.81	6.87	8.41	9.17	9.57	0.62	8.10	8.69	8.39	Fair (DSH)	8.73
	A	4.89	7.25	8.41	9.37	5.64	7.01	8.44	8.91	0.47	7.00	7.85	7.42	Poor (DSL)	6.57
	D	5.07	7.77	8.78	9.55	6.15	7.45	8.76	9.11	0.45	7.25	8.21	7.73	Borderline (DSM)	7.85
D3	N1	7.19	8.90	9.88	10.00	7.43	8.93	9.88	9.94	0.6	8.77	9.14	8.95	Good (DSE)	9.69
	N2	6.13	8.13	9.16	9.70	6.66	7.47	9.24	9.42	0.30	7.76	8.60	8.18	Fair (DSH)	8.48
	A	5.15	7.51	8.66	9.64	5.89	7.34	8.64	9.13	0.50	7.26	8.10	7.68	Borderline (DSM)	7.20
	D	6	8.34	9.34	9.72	6.83	8.17	9.33	9.53	0.5	7.96	8.66	8.31	Fair (DSH)	9.08
D4	N1	7.04	8.76	9.73	10.00	7.36	9.06	9.73	9.85	0.69	8.67	9.06	8.87	Fair (DSH)	9.68
	N2	7.22	8.91	9.91	10.00	7.45	9.13	9.90	9.95	0.67	8.82	9.15	8.98	Good (DSE)	9.71
	A	5.40	7.72	8.71	9.67	6.46	7.68	8.60	9.12	0.50	7.38	8.31	7.85	Borderline (DSM)	8.08
	D	5.48	8.09	9.20	9.67	6.35	7.83	9.20	9.43	0.50	7.68	8.44	8.06	Borderline (DSM)	8.18
D5	N1	7.13	8.86	9.84	10	7.41	8.87	9.83	9.91	0.58	8.72	9.11	8.91	Good (DSE)	9.53
	N2	6.07	8.29	9.25	9.73	6.83	8.01	9.24	9.47	0.45	7.91	8.65	8.28	Fair (DSH)	8.87
	A	4.17	6.00	7.25	8.59	4.62	5.94	7.50	8.18	0.52	6.17	6.83	6.50	Poor (DSL)	5.72
	D	5.01	7.58	8.67	9.42	5.89	7.34	8.63	9.09	0.51	7.21	8.02	7.62	Borderline (DSM)	7.29
D6	N1	6.45	8.42	9.33	9.79	6.95	8.11	9.34	9.58	0.44	8.12	8.74	8.43	Fair (DSH)	9.21
	D	5.29	7.83	8.93	9.68	6.09	7.22	9.09	9.27	0.36	7.38	8.31	7.84	Borderline (DSM)	7.57

* Telemetry study [9]–[11] had 6 drivers, **N=Normal, A=Aggressive, D=Drowsy, N1=Normal 1, N2=Normal 2, ***Linguistic Value of Driver's Score

of scores (partitioned into normal and aggressive+drowsy behavior). We obtained respective p values of 3.94×10^{-4} and 3.42×10^{-3}. Thus, the lower p-value obtained with our proposed approach signifies that it is able to partition the normal driving scores from the aggressive+drowsy ones in a better manner compared to the telemetry data.

B. Robustness analysis using Bootstrap Sampling

We validated the drivers' scores generated by our proposed scoring system using the Bootstrap sampling. We heuristically filtered data vectors from Section IV-D for the driver's trip. Then, we randomly selected 70% of these data vectors (without replacement) for every driver and the trip, bifurcated by the type of road. We calculated the overall drivers' numeric as well as linguistic trip's score using our proposed scoring system. This was repeated 100 times for each driver, trip and type of road to obtain 100 nine point FOUs for the respective overall driver's score (Similar to the nine point FOUs for the overall score given in Table III).

We calculated the Jaccard's similarity between the respective driver's score for trip and type of road (taken from Table III) and these respective 100 nine point FOUs, to generate corresponding 100 Jaccard's similarity scores. From these 100 Jaccard's similarity scores, we found the similarity index value at 95% confidence. For e.g., for driver D1 in Normal driving behaviour on a motorway road, the similarity index at 95% confidence was found to be 93.84%. This similarity index is the robustness value of the score as it indicates how similar are the resultant FOUs from bootstrapping with respect to the original one. Similarly, we calculated the robustness values for all the drivers in different types of behaviours and both roads. These results are summarized in Table IV. We found that the proposed scoring system's scores are highly robust to significant removal of data (30%), with average robustness in its formulation of 90%, within a 95% confidence interval.

Table IV: Overall driver's score in a trip on Motorway and Secondary for robustness analysis

Driver	Behavior*	Robustness	Driver	Behavior*	Robustness	Driver	Behavior*	Robustness
Type of Road: Motorway								
	N	93.84		N	89.47		N	90.67
D1	A	93.03	D3	A	96.81	D5	A	83.26
	D	95.34		D	92.41		D	89.53
	N	92.56		N	97.87		N	95.19
D2	A	84.32	D4	A	84.26	D6	A	82.13
	D	91.57		D	92.14		D	88.53
Type of Road: Secondary								
	N1	86.78		N1	93.40		N1	89.68
D1	N2	79.03	D3	N2	86.54	D5	N2	87.04
	A	95.61		A	95.99		A	74.29
	D	89.33		D	84.59		D	93.20
	N1	89.62		N1	93.21		N1	80.60
D2	N2	85.37	D4	N2	97.20	D6	N2	-
	A	96.40		A	91.65		A	-
	D	93.50		D	88.97		D	80.57

*N=Normal, A=Aggressive, D=Drowsy, N1=Normal 1, N2=Normal 2

C. Existing Per-C vs Our proposed scoring system

As the data source of existing Per-C is a group of people vs the stream of numeric data value in our proposed scoring system, hence a fair comparison between the two is from a conceptual point of view. The differences and similarities between the two are listed in the Table V, from which it can be seen that the existing Per-C has some limitations, which are successfully overcome by the use of proposed scoring system.

D. FOUs with proposed scoring system vs other approaches

We have presented a novel way of obtaining FOUs from a stream of data values. There exist other methods for obtaining FOUs from data. Unfortunately, these other methods cannot be used straight for our proposed scoring system because we require the FOUs to follow a semantic ordering (bounds) based on prior information on the semantic partitioning (please see Section III). It is pertinent that the FOUs must fall within a set of pre-defined (bounded) linguistic terms. This is to ensure that each FOU represents the uncertainty (of the membership degree) associated with each data value belonging to the linguistic label and the maximum of one adjacent linguistic label. So, our method produces overlapping between the FOUs bounded to each linguistic term.

E. Interesting aspect on dealing with real constant data

In the encoder of our proposed scoring system, unique data values for finding the centroids were used because retention of the duplicate values introduced a bias towards long continuous sequences of uninformative observations. In fact, with the unique data values in the dataset, the percentile rank of the dataset ranged from 0 to 100. However, with the duplicate values in the dataset, the percentile rank did not reach 100, which inhibits the complete coverage of the information scale (0 to 10) in a natural way. Also, the effect of duplicate retention was quite pronounced on the shape of the obtained FOU plots. The duplicate values caused the centroid of the FOU to shift more towards them, thereby causing only a single linguistic term to cover almost 60% of the information scale and one or more of the other linguistic terms' FOUs to shrink in size and be limited in 40% of the scale. In our perception, this is not a fair scoring system.

In many passive real-world monitoring scenarios, a large amount of collected data is constant and lacks informativeness. Large sequences of constant data can cause the variables' frequency distributions to become skewed towards frequent duplicates. For example, irresponsible driving scores would be attenuated if the driver performed some risky driving but cruised during most of the trip. This is due to the linguistic variables mostly reflecting the variations around the most frequent values rather than capturing the impact of extreme behaviours. In the FOU generation, we used distinct values because they better generalise the information about variations, rendering evenly distributed IT2 word models that better reflect what can contextually happen during the driver's trip.

F. Semantic ordering

In score systems, not all variables affect the overall score with the same ordering and orientation, and we have accounted for that in our approach. There is a semantics ordering on the linguistic terms belonging to a variable. For example, we all know that semantically, $Very\ Less$ is smaller than $Moderate$, which in turn is smaller than $Very\ High$. With this notion in mind, we see that the orientation of the linguistic terms for variables AC and CF are opposite to those of other variables. The linguistic terms corresponding to the variable AC (as seen from Table I) are $Very\ Low(ACV)$, $Low(ACL)$, $Medium(ACM)$, $High(ACH)$ and $Very\ High(ACE)$. From the Fig. SM-4, we see that the FOU for ACV is located on a lower side of the scale, whereas the location order for FOUs of other linguistic terms follows the order $ACE > ACH > ACM > ACL$. Against this, consider the variable BR. The linguistic terms corresponding to BR are $Very\ High(BRE)$, $High(BRH)$, $Moderate(BRM)$, $Less(BRL)$ and $Very\ Less(BRV)$. However, from Fig. SM-4, we see that the FOUs of these terms follow the order

Table V: Comparison of the existing Per-C and proposed novel Per-C

Attributes	Existing Per-C (based on IA, EIA, HMA)	Proposed Novel Per-C
	Differences	
Generating word models?	Using data collected from a group of people or a single subject	From a stream of data values
Steps in the data part?	Bad data processing, outlier processing, tolerance limit processing, reasonable interval processing, etc.	Removing duplicate data values, finding the centroids using FCM and calculating the highest and second highest membership degree of each data point.
Mapping into FOU?	By mapping Embedded T1 FSs into the left shoulder, interior or right shoulder FOU	Estimating left shoulder, interior or right shoulder FOU parameters through highest and second highest membership function values of data points from FCM.
Linguistic terms for aggregation in CWW engine?	Generally elicited by the user	Found based on respective maximum degrees of memberships of the variables' numeric values from the input data vector
Associated linguistic weights?	Assigned to the variable	Can be assigned to the linguistic terms of the variable
	Similarities	
Aggregation operator in CWW Engine?	Interval weighted average, Fuzzy weighted average or Linguistic weighted average.	Interval weighted average, Fuzzy weighted average or Linguistic weighted average.
Decoding in the decoder?	By similarity measure, ranking, or subsethood	By similarity measure, ranking, or subsethood

$BRE < BRH < BRM < BRL < BRV$. This is so because the variable BR shows an opposite behaviour to that of variable AC.

VI. Conclusions and Future Scope

In this paper, we have proposed a novel Per-C based unsupervised scoring system. Our novel Per-C based system successfully models the word semantics through IT2 FS word models, which are automatically generated from a stream of numeric values, against other models which require collecting labelled data from people, which has its inherent limitations [27], [36].

We have also demonstrated the applicability of our proposed scoring system to the scenario of driver's score calculation using real-life data [9], [10]. We chose this scenario because in some countries, the driving quality (of a driver) is assessed using a telematics unit fitted inside the vehicle and that has a direct impact on their insurance premium. A high number of users seem dissatisfied with the respective score calculated by the 'black box' [37]. Consumer protection services have highlighted a growing high number of complaints on this system and the poor confidence of consumers in the predicted values [38]. On the contrary, with our proposed scoring system, drivers can naturally perceive, understand and peruse the relations between the various driving-related inputs linguistically, adding the desired explainability.

Further, more often than not, the scoring systems have been treated (in literature) in a supervised manner and have been developed based on subjective labelled samples coming from a single or several experts (section V-B). The pre-labelling of the data may induce bias in the final score. Our novel Per-C based system is unsupervised and yields objective scores that are purely data-driven (section V-A).

Nonetheless, the precise numeric data values pertaining to the variables (used in the system design) may have subjective interpretations. Their semantic uncertainty is ignored when the end user is provided with a single numeric data output. Thus, our novel Per-C based unsupervised scoring system overcomes this limitation by modelling the semantics of these numeric data linguistically, in the form of IT2 FS word models. Also, our proposed system generates linguistic recommendations, the linguistic information being subjective in nature as "words mean different things to different people" [27], [36].

When evaluating the resultant scores in a real-world scenario, it was found that these were able to exhibit higher differences between groups requiring divergent scores (responsible vs. irresponsible drivers) than a state-of-the-art method. Also, robustness analysis showed resiliency to loss of data. It is pertinent to mention that our proposed scoring system is quite general and can be applied to any scoring system or scenario where IT2 FS word models need to be generated from a stream of numeric data values, such as monitoring data.

In this work, we have mainly focused on the encoding and engine part of the Per-C system but in future works, we will pursue modifying the decoder part to allow other CWW applications by inter-relating linguistic variables in different ways. In the decoder section, future developments could encompass the use of higher-order fuzzy sets, such as those aimed at incorporating time-dependencies [39]. Another possible extension of the present work can be developing assessment metrics for the quality of generated explanations from a CWW system.

Acknowledgment

We sincerely thank Prof. Jerry M. Mendel for his invaluable discussions during the early stages of this research.

References

[1] S. Ghaemi, S. Khanmohammadi, and M. Tinati, "Driver's behavior modeling using fuzzy logic," *Mathematical Problems in Engineering*, vol. 2010, 2010.

[2] D. S. Hurwitz, H. Wang, M. A. Knodler Jr, D. Ni, and D. Moore, "Fuzzy sets to describe driver behavior in the dilemma zone of high-speed signalized intersections," *Transportation research part F: traffic psychology and behaviour*, vol. 15, no. 2, pp. 132–143, 2012.

[3] M. Eling and M. Kraft, "The impact of telematics on the insurability of risks," *The Journal of Risk Finance*, 2020.

[4] L. Pozueco, X. G. Pañeda, A. G. Tuero, G. Diaz, R. Garcia, D. Melendi, A. G. Pañeda, and J. A. Sánchez, "A methodology to evaluate driving efficiency for professional drivers based on a maturity model," *Transportation Research Part C: Emerging Technologies*, vol. 85, pp. 148–167, 2017.

[5] M. Guillen, J. P. Nielsen, A. M. Pérez-Marín, and V. Elpidorou, "Can automobile insurance telematics predict the risk of near-miss events?" *North American Actuarial Journal*, vol. 24, no. 1, pp. 141–152, 2020.

[6] A. Alaybeyoglu and B. C. Senel, "A mobile application using fuzzy sets to decrease road traffic accidents," *Arabian Journal for Science and Engineering*, vol. 43, no. 12, pp. 7853–7868, 2018.

[7] O. Derbel and R. Landry Jr, "Driver behavior assessment in case of critical driving situations," *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. 100, no. 2, pp. 491–498, 2017.

[8] "Accident forgiveness|safe driving bonus|deductible rewards," https://www.allstate.com/auto-insurance/safe-driver-savings.aspx, (Accessed on 08/24/2020).

[9] E. Romera, L. M. Bergasa, and R. Arroyo, "Need data for driver behaviour analysis? presenting the public uah-driveset," in *2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2016, pp. 387–392.

[10] E. Romera Carmena *et al.*, "Driver behavior evaluation by using smartphones," 2015.

[11] C. Arroyo, L. M. Bergasa, and E. Romera, "Adaptive fuzzy classifier to detect driving events from the inertial sensors of a smartphone," in *2016 IEEE 19th International Conference on Intelligent Transportation Systems (ITSC)*. IEEE, 2016, pp. 1896–1901.

[12] A. B. Ellison, S. P. Greaves, and M. C. Bliemer, "Driver behaviour profiles for road safety analysis," *Accident Analysis & Prevention*, vol. 76, pp. 118–132, 2015.

[13] J. M. Mendel, "The perceptual computer: An architecture for computing with words," in *10th IEEE International Conference on Fuzzy Systems.(Cat. No. 01CH37297)*, vol. 1. IEEE, 2001, pp. 35–38.

[14] L. Zadeh, "Fuzzy logic= computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 2, pp. 103–111, 1996.

[15] P. K. Gupta and J. Andreu-Perez, "A gentle introduction and survey on computing with words (cww) methodologies," *Neurocomputing*, vol. 500, pp. 921–937, 2022.

[16] "scipy.stats.ks_2samp scipy v1.5.4 reference guide," https://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.ks_2samp.html, (Accessed on 08/12/2020).

[17] S. Y. Sohn, D. H. Kim, and J. H. Yoon, "Technology credit scoring model with fuzzy logistic regression," *Applied Soft Computing*, vol. 43, pp. 150–158, 2016.

[18] F. Hoffmann, B. Baesens, C. Mues, T. Van Gestel, and J. Vanthienen, "Inferring descriptive and approximate fuzzy rules for credit scoring using evolutionary algorithms," *European journal of operational research*, vol. 177, no. 1, pp. 540–555, 2007.

[19] H. Hagras, "Toward human-understandable, explainable ai," *Computer*, vol. 51, no. 9, pp. 28–36, 2018.

[20] M. Karimi, H. Tahayori, K. Tirdad, and A. Sadeghian, "A perceptual computer for hierarchical portfolio selection based on interval type-2 fuzzy sets," *Granular Computing*, vol. 8, no. 1, pp. 23–43, 2023.

[21] Z. Chen, "Philosophical foundation for granular computing," *Granular, Fuzzy, and Soft Computing*, p. 177, 2023.

[22] S. Greco, B. Matarazzo, and R. Słowiński, "Granular computing and data mining for ordered data: The dominance-based rough set approach," *Granular, Fuzzy, and Soft Computing*, p. 117, 2023.

[23] J. Pratihar, A. Dey, A. Khan, P. Banerjee, and R. K. Pal, "Computing with words for solving the fuzzy transportation problem," *Soft Computing*, pp. 1–14, 2023.

[24] P. K. Gupta, "Python software libraries for computing with words (cww) methodologies," *Neurocomputing*, p. 126807, 2023.

[25] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning—i," *Information sciences*, vol. 8, no. 3, pp. 199–249, 1975.

[26] P. K. Gupta, *Computing with Words: For Power Management, Linguistic Optimization Decision Making*. LAP Lambert Academic Publishing, 2023.

[27] J. Mendel and D. Wu, *Perceptual computing: Aiding people in making subjective judgments*. John Wiley & Sons, 2010, vol. 13.

[28] J. M. Mendel and D. Wu, "Challenges for perceptual computer applications and how they were overcome," *IEEE computational intelligence magazine*, vol. 7, no. 3, pp. 36–47, 2012.

[29] X. Yao, J. Zhou, J. Zhang, and C. R. Boër, "From intelligent manufacturing to smart manufacturing for industry 4.0 driven by next generation artificial intelligence and further on," in *2017 5th international conference on enterprise systems (ES)*. IEEE, 2017, pp. 311–318.

[30] A. Rojko, "Industry 4.0 concept: background and overview," *International Journal of Interactive Mobile Technologies (iJIM)*, vol. 11, no. 5, pp. 77–90, 2017.

[31] O. Senvar and E. Akkartal, "An overview to industry 4.0," *International Journal of Information, Business and Management*, vol. 10, no. 4, pp. 50–57, 2018.

[32] S. S. Kamble, A. Gunasekaran, and R. Sharma, "Analysis of the driving and dependence power of barriers to adopt industry 4.0 in indian manufacturing industry," *Computers in Industry*, vol. 101, pp. 107–119, 2018.

[33] J. C. Bezdek, R. Ehrlich, and W. Full, "Fcm: The fuzzy c-means clustering algorithm," *Computers & Geosciences*, vol. 10, no. 2-3, pp. 191–203, 1984.

[34] N. R. Pal and J. C. Bezdek, "On cluster validity for the fuzzy c-means model," *IEEE Transactions on Fuzzy systems*, vol. 3, no. 3, pp. 370–379, 1995.

[35] J. M. Mendel, "Uncertain rule-based fuzzy systems," in *Introduction and new directions*. Springer, 2017, p. 684.

[36] J. M. Mendel, L. A. Zadeh, E. Trillas, R. Yager, J. Lawry, H. Hagras, and S. Guadarrama, "What computing with words means to me [discussion forum]," *IEEE computational intelligence magazine*, vol. 5, no. 1, pp. 20–26, 2010.

[37] "'black box' telematics insurance mistakes push up car insurance premiums," https://www.telegraph.co.uk/insurance/car/black-box-telematics-insurance-mistakes-push-car-insurance-premiums/, (Accessed on 02/19/2021).

[38] "Financial ombudsman service - annual review 2018/2019," https://www.financial-ombudsman.org.uk/news-events/annual-review-2018-2019, (Accessed on 08/24/2020).

[39] M. Kiani, J. Andreu-Perez, and H. Hagras, "A temporal type-2 fuzzy system for time-dependent explainable artificial intelligence," *IEEE Transactions on Artificial Intelligence*, 2022.

[40] L. A. Zadeh, "From computing with numbers to computing with words. from manipulation of measurements to manipulation of perceptions," *IEEE Transactions on circuits and systems I: fundamental theory and applications*, vol. 46, no. 1, pp. 105–119, 1999.

[41] G. Klir and B. Yuan, *Fuzzy sets and fuzzy logic*. Prentice hall New Jersey, 1995, vol. 4.

[42] L. Zadeh, "Fuzzy sets," *Information and Control*, vol. 8, pp. 338–353, 1965.

[43] P. K. Gupta, D. Sharma, and J. Andreu-Perez, "Enhanced linguistic computational models and their similarity with yager's computing with words," *Information Sciences*, vol. 574, pp. 259–278, 2021.

[44] J. M. Mendel, J. Lawry, and L. A. Zadeh, "Foreword to the special section on computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 437–440, 2010.

[45] Q. Liang and J. M. Mendel, "Interval type-2 fuzzy logic systems: theory and design," *IEEE Transactions on Fuzzy systems*, vol. 8, no. 5, pp. 535–550, 2000.

[46] J. M. Mendel and D. Wu, "Perceptual reasoning for perceptual computing," *IEEE Transactions on Fuzzy systems*, vol. 16, no. 6, pp. 1550–1564, 2008.

[47] N. N. Karnik and J. M. Mendel, "Centroid of a type-2 fuzzy set," *information Sciences*, vol. 132, no. 1-4, pp. 195–220, 2001.

[48] D. Wu and J. M. Mendel, "Enhanced karnik–mendel algorithms," *IEEE transactions on fuzzy systems*, vol. 17, no. 4, pp. 923–934, 2008.

[49] F. Liu and J. M. Mendel, "Encoding words into interval type-2 fuzzy sets using an interval approach," *IEEE transactions on fuzzy systems*, vol. 16, no. 6, pp. 1503–1521, 2008.

[50] D. Wu, J. M. Mendel, and S. Coupland, "Enhanced interval approach for encoding words into interval type-2 fuzzy sets and its convergence analysis," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 3, pp. 499–513, 2011.

[51] M. Hao and J. M. Mendel, "Encoding words into normal interval type-2 fuzzy sets: Hm approach," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 4, pp. 865–879, 2015.

[52] D. Wu and J. M. Mendel, "Computing with words for hierarchical decision making applied to evaluating a weapon system," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 3, pp. 441–460, 2010.

[53] ——, "Social judgment advisor: An application of the perceptual computer," in *International Conference on Fuzzy Systems*. IEEE, 2010, pp. 1–8.

[54] C. S. ANDERSEN, A. V. OLESEN, and L. BOLET, "Influence of road characteristics on density of accidents on secondary rural roads in denmark."

[55] P. Marchesini and W. A. M. Weijermars, *The relationship between road safety and congestion on motorways*. SWOV Institute for Road Safety Research Leidschendam, Netherlands, 2010.

[56] H. Berkes, "The Deadliest Roads Are Rural," *NPR*, Feb 2012. [Online]. Available: https://www.npr.org/2009/11/29/120716625/the-deadliest-roads-are-rural?t=1595840292214

[57] D. f. Transport, "Reported road accidents (RAS10)," *GOV*, Jul 2020. [Online]. Available: https://www.gov.uk/government/statistical-data-sets/ras10-reported-road-accidents#table-ras10001
[58] P. K. Gupta and J. Andreu-Perez, "Enhanced type-2 wang-mendel approach," *Journal of Experimental & Theoretical Artificial Intelligence*, pp. 1–26, 2022.

Prashant K Gupta is currently working as an Associate Professor at School of Computer Science Engineering & Technology, Bennett University, Greater Noida, India. He has previously worked as a Scientist at Leibniz Institute for Agriculture Engineering and Bioeconomy, Germany and as a Researcher at German Research Center for Artificial Intelligence, Germany. He received the B.Tech. and M.Tech. degrees in 2008 and 2012, respectively, both from Guru Gobind Singh Indraprastha University, India and the PhD degree in 2019 from South Asian University, India. He has authored several books on AI with International Publishers. He has contributed countless research publications in various journals, core A-ranked conferences and book chapters. Some of the top-tier venues where his publications appeared are IEEE Transactions on Fuzzy Systems, Neurocomputing (Elsevier), Information Sciences (Elsevier), Fuzzy Sets and Systems (Elsevier), Applied Soft Computing (Elsevier), Granular Computing (Springer), FUZZ- IEEE, IEEE-IJCNN, and IEEE SMC. He has been invited as a resource person to deliver various interesting talks on current AI technologies of which the latest one was on "Big Data Architecture & Big Data Tools". He has chaired special sessions at various core A-ranked conferences like FUZZ-IEEE, WCCI, etc. He is serving as a reviewer in various SCI and SCIE journals of international repute like Information Sciences (Elsevier), Expert Systems with Applications (Elsevier), Computers & Industrial Engineering (Elsevier), Knowledge-Based Systems (Elsevier), Applied soft computing (Elsevier), Electronic Commerce Research and Applications (Elsevier), Egyptian Informatics Journal (Elsevier), Complex & Intelligent Systems (Springer), etc. He is also a recipient of various reputed national fellowships from the government of India as well as from international bodies like IEEE Computational Intelligence Society. His research interests include fuzzy systems, computing with words, AI, E-Health, explainable AI, generative AI, hybrid learning and reasoning, hybrid learning and planning for collision-free navigation of autonomous cars, etc.

Deepak Sharma is currently working as a Scientific Researcher in the Department of Computer Science, Christian-Albrechts-Universität, Kiel, Germany. He received the B.E. and M.Tech. degrees from Bharati Vidyapeeth University, Pune, Maharashtra, India, in 2006 and 2012, respectively, and the PhD degree in Computer Science from the University of Delhi, New Delhi, in 2020. His research interests include data mining, natural language processing, fuzzy systems, text processing and image processing, etc. He has authored and co-authored research manuscripts in various reputed journals and conferences.

Javier Andreu-Perez (Senior Member, IEEE) received his PhD degree in Intelligent Systems, in 2012, from Lancaster University, United Kingdom, and MSc, in 2008, from the University of Granada, Spain. He is a Senior Lecturer (Associate Professor, tenured) at the School of Computer Science and Electronic Engineering (CSEE), Centre for Computational Intelligence and chair of the Smart Health Technologies Group at the University of Essex, United Kingdom. His expertise focuses on artificial intelligence, fuzzy methodologies computing-with-words approach for uncertainty modelling in highly noisy, non-stationary, high-dimensional data, bio-signals and other time-variant data analysis. Javier has published in journals edited by Elsevier, Springer-Nature, IEEE (TFS, TSMC, TBMI, JBHI, TDCS, etc.), and other venues in artificial intelligence and cognitive neuroscience. Javier's work in artificial intelligence, human informatics and biomedical engineering has attracted 4500+ citations. Javier is chair of the IEEE Computational Intelligence Society (CIS) task force on Extensions of Type-1 fuzzy sets (2018-2024). Javier acts as associate Editor-in-chief of the journal Neurocomputing (Elsevier), the EUSFLAT-sponsored International Journal of Computational Intelligence Systems, and other newer journals on emerging technologies. He's been invited as an expert reviewer for significant publishers such as Science, The Lancet, and BMC. He has served on the technical committee for IEEE WCCI on several occasions and as the workshop chair for FUZZ-IEEE. He has regularly led tutorials and a special session at this event. He was also the general chair for fNIRS-UK 2023. Javier's research has been founded by UK research councils, other funding schemes supported by the Royal Society, Welcome Trust, and big IT corporations such as Nvidia, Amazon, and Oracle. Javier has an extensive portfolio of completed successful knowledge transfer and collaboration with the industry. Dr. Andreu-Perez has received prestigious fellowships throughout his research career, including the Talentia Senior Fellowship from the Andalusia Scientific Council in 2020 and an international fellowship from the Japan Society for the Promotion of Science (JSPS) in 2022. His research interests focus on human informatics, symbiotic artificial intelligence, human-machine interaction, robotics, sensor engineering, bio/neuro-engineering, and life science research.