

# BER Analysis of SCMA-OFDM Systems in the Presence of Carrier Frequency Offset

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**Abstract**—Sparse code multiple access (SCMA) building upon orthogonal frequency division multiplexing (OFDM) is a promising wireless technology for supporting massive connectivity in future machine-type communication networks. However, the sensitivity of OFDM to carrier frequency offset (CFO) poses a major challenge because it leads to orthogonality loss and incurs intercarrier interference (ICI). In this paper, we investigate the bit error rate (BER) performance of SCMA-OFDM systems in the presence of CFO over both Gaussian and multipath Rayleigh fading channels. We first model the ICI in SCMA-OFDM as Gaussian variables conditioned on a single channel realization for fading channels. The BER is then evaluated by averaging over all codeword pairs considering the fading statistics. Through simulations, we validate the accuracy of our BER analysis and reveal that there is a significant BER degradation for SCMA-OFDM systems when the normalized CFO exceeds 0.02.

**Index Terms**—Sparse code multiple access (SCMA), orthogonal frequency division multiplexing (OFDM), carrier frequency offset (CFO), bit error rate (BER)

## I. INTRODUCTION

Non-orthogonal multiple access (NOMA) has been extensively studied in recent years to meet the stringent requirements such as massive connectivity and higher spectral efficiency in future machine-type communication networks [1]. In a NOMA system, multiple users communicate simultaneously to achieve an overloading factor larger than one [2], [3]. As an important variant of code division multiple access, sparse code multiple access (SCMA) has emerged as a promising NOMA technique [4]. In SCMA, the incoming message bits of each SCMA user are directly mapped to a multi-dimensional sparse codeword drawn from a carefully designed codebook [5]. To deal with asynchronous wireless transmissions, one can build SCMA upon orthogonal frequency division multiplexing (OFDM) [6]–[8], where the resultant system is called SCMA-OFDM. This is because the cyclic prefix (CP) in an OFDM system can help accommodate the inter-user asynchrony and intersymbol interference in uplink channels. In SCMA-OFDM, multi-dimensional sparse codewords are sent over a number of orthogonal subcarriers (SCs), also known as resource elements (REs).

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Due to its multicarrier feature, nonetheless, an OFDM system may suffer from significant inter-carrier interference (ICI) due to the carrier frequency offset (CFO) caused by inaccurate oscillator clocks. As a matter of fact, CFO may be large and widely present for an array of low-cost and low-end communication sensor devices. In addition, similar to the CFO effect, the deleterious Doppler shifts in moving environments may also lead to ICI. Extensive research activities have been conducted in recent years to understand the detrimental effects of CFO in OFDM systems [9]–[12]. For example, in [9], the ICI was approximated to be a Gaussian distributed noise. In [10], the authors extended the approximation method of [9] to frequency-selective channels in order to derive an improved bit error rate (BER) analysis framework. It should be noted that these works generally assume that the subcarriers in an OFDM symbol are independent. However, this may not hold in SCMA-OFDM because CFO simultaneously affects those nonzero codeword elements located in a frequency sub-band. Aiming to address this problem, we carry out a systematic study to analyze the impact of CFO on SCMA-OFDM.

Despite a rich body of literature on SCMA, there is a paucity of studies on its accurate error rate performance analysis. The major difficulty lies in the multidimensional sparse codebook nature of SCMA that involves multi-user and multi-carrier transmissions. The error performances of uplink and downlink SCMA over Rayleigh fading channels with a maximum likelihood decoder were analyzed in [13], and a similar study was presented in [14] for uplink SCMA with multiple receive antennas.

In this letter, we present a BER analysis for the CFO impaired SCMA-OFDM systems over Gaussian and multipath Rayleigh fading channels. First, the ICI in SCMA-OFDM is approximated by two uncorrelated Gaussian variables and the signal-to-interference-plus-noise ratio (SINR) conditioned on a single channel realization is evaluated. Then, the conditional pair-wise error probability (PEP) is obtained by using the conditional SINR. Finally, the BER is calculated by averaging over the fading statistics and all the transmitted codewords. Our analysis shows that the PEP of SCMA systems affected by CFO can be expressed as the product of a few integrals, where each integral can be expressed by a series of generalized hypergeometric functions. Interestingly, our numerical evaluations indicate that the BER performance of SCMA-OFDM system degrades significantly when the normalized CFO exceeds 0.02, thus providing a useful guideline for practical system design.

**Notations:**  $x$ ,  $\mathbf{x}$  and  $\mathbf{X}$  denote scalar, vector and matrix, respectively.  $\text{diag}(\mathbf{x})$  represents a diagonal matrix with the

main diagonal vector  $\mathbf{x}$ .  $\mathbf{X}^T$  and  $\mathbf{X}^H$  denote the transpose and the Hermitian transpose, respectively.  $E\{\cdot\}$  denotes the expectation operator.  $\mathbb{B}$  and  $\mathbb{C}$  denote the set of binary numbers and complex numbers, respectively.

## II. SYSTEM MODEL

### A. SCMA Communication Model

We consider a downlink SCMA system, where  $J$  users communicate over  $K$  resource elements, where  $J > K$ . The overloading factor defined as  $J/K$  is thus larger than one. In SCMA, each user is assigned with an unique codebook, denoted by  $\mathcal{X}_j = \{\mathbf{x}_{j,1}, \mathbf{x}_{j,2}, \dots, \mathbf{x}_{j,M}\} \in \mathbb{C}^{K \times M}$ ,  $j \in \{1, 2, \dots, J\}$ , consisting of  $M$  codewords with a dimension of  $K$ . During transmissions, each user maps  $\log_2(M)$  binary bits to a length- $K$  codeword  $\mathbf{x}_j$  drawn from the  $\mathcal{X}_j$ . The mapping process can be expressed as [15]

$$f_j : \mathbb{B}^{\log_2 M \times 1} \rightarrow \mathcal{X}_j \in \mathbb{C}^{K \times M}, \text{ i.e., } \mathbf{x}_j = f_j(\mathbf{b}_j), \quad (1)$$

where  $\mathbf{b}_j = [b_{j,1}, b_{j,2}, \dots, b_{j,\log_2 M}]^T \in \mathbb{B}^{\log_2 M \times 1}$  represents the input binary message of the  $j$ th user. The  $K$ -dimensional complex codewords in the SCMA codebook are sparse vectors with  $V$  non-zero elements and  $V < K$ . The factor graph can be used to represent the connections between user nodes (UNs) and resource nodes (RNs) in SCMA. The sparse structure of the  $J$  SCMA codebooks can also be represented by the indicator matrix  $\mathbf{F} \in \mathbb{B}^{K \times J}$ . An element of  $\mathbf{F}$  is defined as  $f_{k,j}$  which takes the value of 1 if and only if  $j$ th UN ( $u_j$ ) is connected to  $k$ th RN ( $r_k$ ) and 0 otherwise. Fig. 1 illustrates an SCMA indicator matrix and corresponding factor graph with  $J = 6$ ,  $K = 4$  and  $V = 2$ .

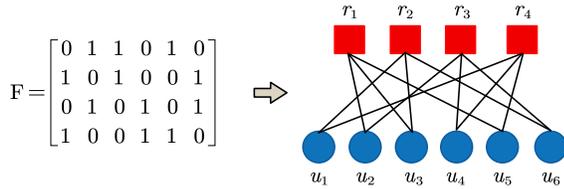


Fig. 1. Factor graph representation for an SCMA system with  $J = 6$ ,  $K = 4$ ,  $V = 2$ .

### B. Downlink SCMA-OFDM Impaired by CFO

In SCMA-OFDM systems, the SCMA multiplexing symbols are transmitted over multiple OFDM SCs. The block diagram of a downlink SCMA system is shown in Fig. 2, where  $N$  represents the number of SCs in an OFDM system. We assume that  $N = QK$ , where  $Q$  is the total number of SCMA blocks, each consisting of  $K$  REs. In each SCMA block, users' data are first superimposed over  $K$  REs, and then allocated to OFDM SCs. The superimposed codewords at the  $q$ th SCMA block is denoted as  $\mathbf{w}_q = \sum_{j=1}^J \mathbf{x}_{j,q} \in \mathbb{C}^{K \times 1}$ . We introduce two allocation schemes, namely the localized and interleaved allocations.

In the case of localized transmission, the superimposed codewords are mapped to a number of consecutive OFDM SCs. Therefore, the resultant signal is obtained as  $\mathbf{s} = [\mathbf{w}_1^T, \mathbf{w}_2^T, \dots, \mathbf{w}_Q^T]^T$ .

In the case of interleaved transmission, the  $N$  SCs are divided into  $K$  groups, with each group consists of  $Q$  SCs. Then, the  $k$ -th entry of  $\mathbf{x}_q$  is transmitted at the  $q$ -th position of group  $k$  over the OFDM symbol. As a result, the mapped signal is obtained as  $\mathbf{s} = [\mathbf{r}_1^T, \mathbf{r}_2^T, \dots, \mathbf{r}_K^T]^T$ ,

where  $\mathbf{r}_k = [w_{k,1}, w_{k,2}, \dots, w_{k,Q}]$ , and  $w_{k,q}$  denotes the  $k$ th entry of  $\mathbf{w}_q$ .

Afterwards, the transmitted signal in the time domain is obtained by performing the inverse fast Fourier transform (IFFT) and adding a CP. Let  $\mathbf{h} = [h_0, h_1, \dots, h_{P-1}]^T$  be the multipath fading channels of  $P$  paths. We assume that the CP length is not less than  $P$ . At the receiver side, in the presence of CFO<sup>1</sup>, the received signal after down-sampling and removing the CP can be expressed in matrix form as

$$\mathbf{y} = \mathbf{D}\mathbf{H}\mathbf{F}^H \mathbf{s} + \mathbf{n}, \quad (2)$$

where  $\mathbf{D}$  is an  $N \times N$  diagonal matrix defined by  $[\mathbf{D}]_{n,n} = e^{j2\pi\varepsilon(n-1)/N}$ ,  $\varepsilon$  is the CFO normalized to the subcarrier spacing,  $\mathbf{H}$  is the circular channel matrix given by  $[\mathbf{H}]_{n,m} = h_{(n-m) \bmod N}$ ,  $h_i = 0$ ,  $P \leq i \leq N-1$ , and  $\mathbf{n}$  is the noise vector with the Gaussian noise with variance of  $\sigma_0^2$ . After performing FFT at the receiver, (2) can be expressed as

$$\mathbf{z} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{s} + \mathbf{v}, \quad (3)$$

where  $\mathbf{\Phi} = \mathbf{F}\mathbf{D}\mathbf{F}^H$  denotes the circular matrix that yields the ICI,  $\mathbf{v} = \mathbf{F}\mathbf{n}$  represents the noise term, and  $\mathbf{\Lambda} = \mathbf{F}\mathbf{H}\mathbf{F}^H = \text{diag}(\boldsymbol{\lambda})$  is the channel diagonal matrix with elements expressed by

$$\lambda = \sqrt{N}\mathbf{F}\mathbf{h}, \quad (4)$$

where  $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_N]^T$  denotes the channel vector in frequency domain. With these definitions, it is straightforward to verify that

$$[\mathbf{\Phi}]_{n,m} = \frac{\sin(\pi((m-n) \bmod N + \varepsilon))}{N \sin(\frac{\pi}{N}((m-n) \bmod N + \varepsilon))} e^{j\pi \frac{N-1}{N}((m-n) \bmod N + \varepsilon)}. \quad (5)$$

Obviously, the matrix  $\mathbf{\Phi}$  includes a phase-shift term  $e^{j\pi \frac{N-1}{N}\varepsilon}$  that applies to all SCs. In this paper, we assume perfect channel state information at the receiver.

## III. PERFORMANCE ANALYSIS OF SCMA-OFDM IN THE PRESENCE OF CFO

In this section, we consider the interleaved transmission scheme due to its larger frequency diversity. Let  $\xi_N = \{1, 2, \dots, N\}$  be the set of OFDM SCs. After the SC de-allocation, the received  $q$ th SCMA block is denoted by  $\mathbf{z}_q = \{z_{q,k}\} \in \mathbb{C}^{K \times 1}$ . In the following, we use the subscripts  $k$  and  $n$  for SCMA REs and OFDM SCs, respectively. For interleaved transmission, we have  $n = Q(k-1) + q$ . In order to evaluate the error probability and without loss of generality, we focus on the first SCMA block in the received OFDM symbol and drop the block index  $q$  in  $\mathbf{w}_q$  for the sake of simplicity. Therefore,  $z_k$ ,  $1 \leq k \leq K$  can be expressed as

$$z_k = \phi_{n,n} \lambda_n w_k + \underbrace{\sum_{m \in \xi_N \setminus n} \phi_{n,m} \lambda_m s_m}_{\text{ICI}_k} + v_n, \text{ for } n = \text{Ind}_k, \quad (6)$$

<sup>1</sup>Similar to [9], we assume the CFO is the same for each path.

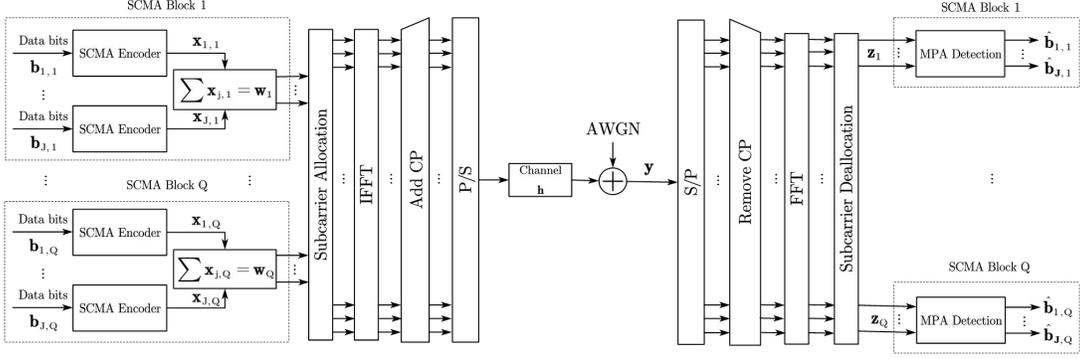


Fig. 2. Downlink SCMA system based on OFDM.

where  $\text{Ind}_k = Q(k-1) + 1$ ,  $\xi_{N \setminus n} = \xi_N \setminus \{n\}$  denotes the REs after removing the  $n$ th RE in  $\xi_{N \setminus n}$ ,  $w_k \in \mathbf{w}$  is the  $k$ th transmitted codeword over the  $n$ th OFDM SC, and  $\phi_{n,m} = [\Phi]_{n,m}$  represents the ICI coefficient from subcarriers  $\xi_{N \setminus n}$  to the subcarrier  $n$ . In the rest of the paper, unless otherwise stated,  $n = \text{Ind}_k$ . Assume that the energy of the transmitted codeword is normalized to unit. For Gaussian channels,  $\text{ICI}_k$  can be approximated as a Gaussian distributed random variable with zero mean and variance given by [16]

$$\begin{aligned} E\{|\text{ICI}_k|^2\} &= \frac{J}{K} \left( 1 - E\left\{ \left| \sum_{m \in \xi_{N \setminus n}} \phi_{n,m} \right|^2 \right\} \right) \\ &= \frac{J}{K} \left( 1 - \frac{\sin^2 \pi \varepsilon}{N^2 \sin^2 \frac{\pi \varepsilon}{N}} \right). \end{aligned} \quad (7)$$

In general, (7) is tightly held as  $N$  increases. In the following, we present the PEP in the SCMA-OFDM system and the ICI approximation for fading channels. Due to CFO and additive white Gaussian noise, the transmitted signal  $\mathbf{w}$  is assumed to be erroneously decoded to another codewords  $\hat{\mathbf{w}}$ . Denote the PEP between  $\mathbf{w}$  and  $\hat{\mathbf{w}}$  as  $\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} | \boldsymbol{\lambda}\}$ , which is conditioned on the channel fading vector  $\boldsymbol{\lambda}$ , then the average BER for SCMA-OFDM systems with a maximum likelihood (ML) detector can be approximated as [4]

$$\begin{aligned} \text{BER} &\leq \frac{1}{M^J J \log_2(M)} \\ &\sum_{\mathbf{w}} \sum_{\hat{\mathbf{w}} \neq \mathbf{w}} n_e(\mathbf{w}, \hat{\mathbf{w}}) \int_{\boldsymbol{\lambda}} \Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} | \boldsymbol{\lambda}\} f_{\Lambda}(\boldsymbol{\lambda}) d\boldsymbol{\lambda}, \end{aligned} \quad (8)$$

where  $n_e(\mathbf{w}, \hat{\mathbf{w}})$  denotes number of the erroneous bits when  $\hat{\mathbf{w}}$  is decoded. It is noted that the evaluation of (8) needs to deal with the  $N$ -dimensional probability density function (pdf)  $f_{\Lambda}(\boldsymbol{\lambda})$ . As the frequency domain channel is obtained by combining  $P$  random variables, the variables in  $\boldsymbol{\lambda}$  are correlated with one another. Consequently, the pdf  $f_{\Lambda}(\boldsymbol{\lambda})$  cannot be factorized as the product of  $N$  independent one-dimensional pdfs. To address this problem, we re-write the multidimensional integration as  $f_{\Lambda}(\boldsymbol{\lambda}) = f_{\Lambda_n | \lambda_{N \setminus n}}(\lambda_{N \setminus n} | \lambda_n) f_{\lambda_n}(\lambda_n)$ , where  $\lambda_{N \setminus n}$  denotes the channel vector  $\boldsymbol{\lambda}$  after removing the  $n$ th entry and  $f_{\Lambda_n | \lambda_{N \setminus n}}(\lambda_{N \setminus n} | \lambda_n)$  is the conditional pdf of

$\lambda_{N \setminus n}$  given in  $\lambda_n$ . Hence, the PEP can be re-written as

$$\begin{aligned} \Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}}\} &= \int_{\boldsymbol{\lambda}} \Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} | \boldsymbol{\lambda}\} f_{\Lambda}(\boldsymbol{\lambda}) d\boldsymbol{\lambda} \\ &= \prod_{k=1}^K \int_{\lambda_{N \setminus n}} \int_{\lambda_n} \Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} | \lambda_n, n = \text{Ind}_k\} \\ &\quad \times f_{\Lambda_n | \lambda_{N \setminus n}}(\lambda_{N \setminus n} | \lambda_n) f_{\lambda_n}(\lambda_n) d\lambda_{N \setminus n} d\lambda_n. \end{aligned} \quad (9)$$

When the channel experiences Rayleigh fading, the conditional pdf  $f_{\Lambda_n | \lambda_{N \setminus n}}(\lambda_{N \setminus n} | \lambda_n)$  can be approximated as a Gaussian distribution for large  $N$  with mean  $\eta_{\lambda_{N \setminus n} | \lambda_n}$  and covariance  $\mathbf{C}_{\lambda_{N \setminus n} | \lambda_n}$  expressed as [11]

$$\begin{aligned} \eta_{\lambda_{N \setminus n} | \lambda_n} &= \lambda_n c_{\lambda_n \lambda_n}^{-1} \mathbf{c}_{\lambda_{N \setminus n} \lambda_n}, \\ \mathbf{C}_{\lambda_{N \setminus n} | \lambda_n} &= \mathbf{C}_{\lambda_{N \setminus n} \lambda_{N \setminus n}} - c_{\lambda_n \lambda_n}^{-1} \mathbf{c}_{\lambda_{N \setminus n} \lambda_n} \mathbf{c}_{\lambda_{N \setminus n} \lambda_n}^H, \end{aligned} \quad (10)$$

where  $c_{\lambda_m \lambda_n} = E\{\lambda_m \lambda_n^*\}$ ,  $\mathbf{c}_{\lambda_{N \setminus n} \lambda_n} = [c_{\lambda_1 \lambda_n}, \dots, c_{\lambda_{n-1} \lambda_n}, c_{\lambda_{n+1} \lambda_n}, \dots, c_{\lambda_N \lambda_n}]^T$ , and  $\mathbf{C}_{\lambda_{N \setminus n} \lambda_{N \setminus n}}$  is the  $(N-1)$ -dimensional square matrix obtained by removing the  $n$ th row and column of the frequency-domain channel covariance matrix  $\mathbf{C}_{\lambda \lambda} = E\{\boldsymbol{\lambda} \boldsymbol{\lambda}^H\}$ . Here, we define the conditional random variable  $t_k = z_k | \lambda_n$ , and from (6), we have

$$t_k = \phi_{n,n} \lambda_n w_k + \underbrace{\sum_{m \in \xi_{N \setminus n}} \phi_{n,m} \xi_m s_m}_{\text{Conditional ICI}_k} + v_n, \quad (11)$$

where the conditional random variable  $\xi_m = \lambda_m | \lambda_n$  is Gaussian with its mean and variance given by  $\eta_m = \lambda_n c_{\lambda_n \lambda_n}^{-1} c_{\lambda_m \lambda_n}$  and  $\sigma_{\xi_m}^2 = [\mathbf{C}_{\lambda_{N \setminus n} | \lambda_n}]_{m-1, m-1}$ ,  $m = 1, \dots, N, m \neq n$ , respectively. Consequently, by defining the zero-mean random variable  $\zeta_m = \xi_m - \eta_m$ , we obtain

$$t_k = \phi_{n,n} \lambda_n w_k + \alpha_n \lambda_n + \beta_n + v_n, \quad (12)$$

where by means of (10),  $\alpha_n$  and  $\beta_n$  are expressed as follows

$$\begin{aligned} \alpha_n &= c_{\lambda_n \lambda_n}^{-1} \sum_{m \in \xi_{N \setminus n}} \phi_{n,m} c_{\lambda_m \lambda_n} s_m, \\ \beta_n &= \sum_{m \in \xi_{N \setminus n}} \phi_{n,m} \zeta_m s_m. \end{aligned} \quad (13)$$

It is obvious that  $\alpha_n$  and  $\beta_n$  do not depend on the specific value of  $\lambda_n$  that characterizes the channel realization, but on the statistical characterization of the channel in the frequency

domain. As can be seen from (12)-(13), the ICI is divided into two parts,  $\lambda_n \alpha_n$  is proportional to the channel amplitude  $\lambda_n$  of the useful signal, while the second part  $\beta_n$  is independent of  $\lambda_n$ . The power of  $\lambda_n \alpha_n$  and  $\beta_n$  can be expressed as

$$\begin{aligned}\sigma_{\alpha_n}^2 &= |c_{\lambda_n \lambda_n}^{-1}|^2 \sum_{m \in \xi_{N \setminus n}} |\phi_{n,m} c_{\lambda_m \lambda_n}|^2, \\ \sigma_{\beta_n}^2 &= \sum_{m \in \xi_{N \setminus n}} |\phi_{n,m}|^2 [C_{\lambda_{N \setminus n} \lambda_n}]_{m-1, m-1}.\end{aligned}\quad (14)$$

We now turn to the evaluation of  $\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}}\}$  in (9). The conditioned PEP is given by

$$\begin{aligned}\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} \mid \lambda_n, n = \text{Ind}_k\} \\ &= \Pr\left\{\sum_{k=1}^K |z_k - \phi_{n,n} \lambda_n \hat{w}_k|^2 \leq \sum_{k=1}^K |z_k - \phi_{n,n} \lambda_n w_k|^2\right\} \\ &= \Pr\left\{\sum_{k=1}^K \left(\phi_{n,n}^2 \lambda_n^2 \Delta_k^2 + 2\phi_{n,n} \lambda_n \Delta_k (\text{ICI}_k + v_n) \leq 0\right)\right\},\end{aligned}\quad (15)$$

where  $\Delta_k = w_k - \hat{w}_k$ . To proceed, let us define

$$\chi = \sum_{k=1}^K \phi_{n,n} \lambda_n \Delta_k (\text{ICI}_k + v_n).\quad (16)$$

From (10)-(14), it is shown that the ICI at  $k$ th SCMA RE can be modeled as Gaussian random variable with variance  $|\lambda_n|^2 \sigma_{\alpha_n}^2 + \sigma_{\beta_n}^2$  based on central limit theorem. Hence,  $\chi$  is also a Gaussian random variable with its variance given by

$$\sigma_{\chi}^2 = \sum_{k=1}^K \phi_{n,n}^2 \lambda_n^2 \Delta_k^2 (|\lambda_n|^2 \sigma_{\alpha_n}^2 + \sigma_{\beta_n}^2 + \sigma_v^2).\quad (17)$$

Let  $A = \frac{1}{2} \sum_{k=1}^K \phi_{n,n}^2 \lambda_n^2 \Delta_k^2$ , then (15) can be reformulated as

$$\begin{aligned}\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} \mid \lambda_n, n = \text{Ind}_k\} &= \Pr(\chi \geq A) = Q(A/\sigma_{\chi}) \\ &= Q\left(\frac{\sum_{k=1}^K \phi_{n,n}^2 \lambda_n^2 \Delta_k^2}{2\sqrt{\sum_{k=1}^K \phi_{n,n}^2 \lambda_n^2 \Delta_k^2 (|\lambda_n|^2 \sigma_{\alpha_n}^2 + \sigma_{\beta_n}^2 + \sigma_v^2)}}\right) \\ &\stackrel{i}{\approx} Q\left(\sqrt{\frac{1}{4} \sum_{k=1}^K \gamma(\lambda_n) \Delta_k^2}\right),\end{aligned}\quad (18)$$

where  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt$  is the  $Q$ -function, and

$$\gamma(\lambda_n) = \frac{|\lambda_n|^2 |\phi_{n,n}|^2}{|\lambda_n|^2 \sigma_{\alpha_n}^2 + \sigma_{\beta_n}^2 + \sigma_0^2},\quad (19)$$

is the conditional SINR. Step  $i$  is obtained based on the fact that the term  $|\lambda_n|^2 \sigma_{\alpha_n}^2 + \sigma_{\beta_n}^2 + \sigma_0^2$  is approximately equal for each RE. It is worth noting that, when the  $N$  SCs fade simultaneously within an OFDM symbol duration, we have  $\mathbf{c}_{\lambda_{N \setminus n} \lambda_n} = \mathbf{0}_{N-1 \times 1}$  and  $\sigma_{\alpha_n}^2 = 0$ . However, the condition of independent fading is not realistic for all the SCs as it is not compatible with the usual hypothesis that the CP length is shorter than the OFDM symbol. By applying the

approximation  $Q(x) \simeq \frac{1}{12} \exp(-x^2/2) + \frac{1}{4} \exp(-2x^2/3)$  [1], (18) can be approximated as

$$\begin{aligned}\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}} \mid \lambda_n, n = \text{Ind}_k\} \\ \simeq \frac{1}{12} \exp\left(-\frac{\sum_{k=1}^K \gamma(\lambda_n) \Delta_k^2}{8}\right) \\ + \frac{1}{4} \exp\left(-\frac{\sum_{k=1}^K \gamma(\lambda_n) \Delta_k^2}{6}\right).\end{aligned}\quad (20)$$

Then, the unconditional PEP can be obtained by averaging over the channel statistics:

$$\begin{aligned}\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}}\} &\simeq \int_0^{\infty} \left(\frac{1}{12} \exp\left(-\frac{\sum_{k=1}^K \gamma(\lambda_n) \Delta_k^2}{8}\right) \right. \\ &\left. + \frac{1}{4} \exp\left(-\frac{\sum_{k=1}^K \gamma(\lambda_n) \Delta_k^2}{6}\right)\right) \prod_{k=1}^K f_{\lambda_n}(\lambda_n) d\lambda_n.\end{aligned}\quad (21)$$

For Rayleigh fading channels, we have  $f_{\lambda_n}(\lambda_n) = \frac{2|\lambda_n|}{c_{\lambda_n \lambda_n}} \exp\left(-\frac{|\lambda_n|^2}{c_{\lambda_n \lambda_n}}\right)$ . In this case, (21) can be estimated by Monte Carlo methods or be expressed as the series expansion [17]

$$\begin{aligned}\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}}\} &\leq \frac{1}{12} \prod_{k=1}^K \left(1 - \frac{1}{8v} e^{\frac{4-\mu}{8v}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{1}{8v}\right)^m \right. \\ &\quad \left. \left(\frac{\mu}{v}\right)^{\frac{m}{2}} \mu^{\frac{m+2}{2}} W_{-\frac{2-m}{2}, \frac{m+1}{2}}\left(\frac{1}{v}\right)\right) \\ &+ \frac{1}{4} \prod_{k=1}^K \left(1 - \frac{1}{6v} e^{\frac{3-\mu}{6v}} \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{1}{6v}\right)^m \right. \\ &\quad \left. \left(\frac{\mu}{v}\right)^{\frac{m}{2}} \mu^{\frac{m+2}{2}} W_{-\frac{2-m}{2}, \frac{m+1}{2}}\left(\frac{1}{v}\right)\right),\end{aligned}\quad (22)$$

with  $\mu$  and  $v$  given by

$$\mu = \frac{c_{\lambda_n \lambda_n} \phi_{n,n}^2 \Delta_k^2}{\sigma_{\beta_n}^2 + \sigma_0^2}, v = \frac{c_{\lambda_n \lambda_n} \sigma_{\alpha_n}^2}{\sigma_{\beta_n}^2 + \sigma_0^2},\quad (23)$$

respectively, and  $W(a, b, z) = e^{-z/2} z^{b+1/2} U(b-z+1/2, 1+2b, z)$ ,  $U(c, d, x) = \frac{1}{\Gamma(c)} \int_0^1 \frac{t^{c-1}}{(1-t)^d} e^{-xt} dt$ , and  $\Gamma(\cdot)$  is the Gamma function.

Following a similar derivation, we can obtain the PEP for Gaussian channels, which has the same form as (20), but with different expression of  $\gamma(\lambda_n)$  given by

$$\bar{\gamma}(\lambda_k) = \frac{1}{\frac{J}{K} \left(1 - \frac{\sin^2 \frac{\pi \epsilon}{N}}{N^2 \sin^2 \frac{\pi \epsilon}{N}}\right) + \sigma_0^2}.\quad (24)$$

Finally, by substituting  $\Pr\{\mathbf{w} \rightarrow \hat{\mathbf{w}}\}$  into (8), we obtain the BER of the OFDM-SCMA with CFO under both Gaussian and multipath Rayleigh fading channels.

#### IV. NUMERICAL RESULTS

In this section, numerical results are presented to evaluate the BER performance for the CFO affected SCMA-OFDM systems. Specifically, we consider a frequency-selective fading channel model consisting of  $P = 8$  paths, each with an independent Rayleigh distribution with delay and power profiles given by [1, 2, 4, 6, 9, 11, 15, 20] and [0.36, 0.24, 0.15, 0.10, 0.06, 0.04, 0.025, 0.017], respectively. Such model is suitable for simulating the propagation

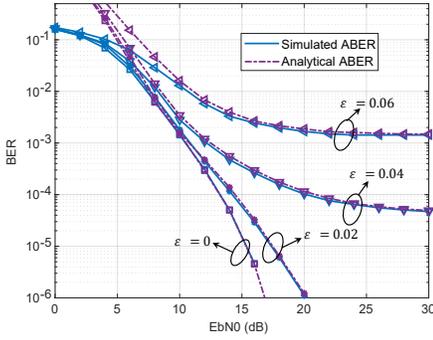


Fig. 3. BER performance of SCMA-OFDM over Gaussian channels.

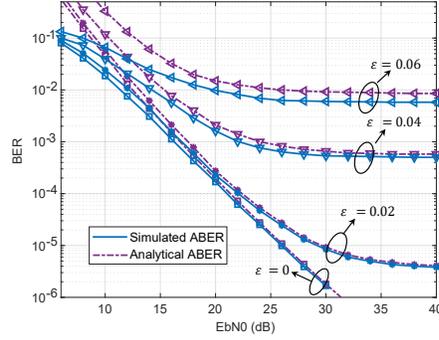


Fig. 4. BER performance of SCMA-OFDM over multipath Rayleigh fading channels.

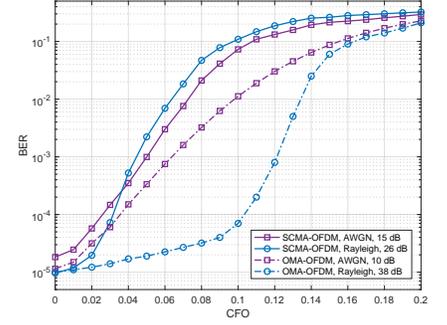


Fig. 5. Simulated BER performance of SCMA-OFDM and OMA-OFDM with different CFO values.

path responses in a scattering environment. The SCMA indicator matrix that presented in Fig. 1 with  $J = 6$ ,  $K = 4$  and  $V = 2$  is considered. We employ the codebook proposed in [18] with  $M = 4$  and a message passing algorithm detector for the ML performance [3]. The number of OFDM subcarriers and the length of CP are  $N = 1024$  and  $N_{cp} = 32$ , respectively.

Fig. 3 and Fig. 4 depict the BER performance of SCMA-OFDM in Gaussian and multipath Rayleigh fading channels, respectively. Notably, all the simulated BERs match well with the theoretical analysis at medium-to-high SNR values for various normalized CFOs for  $\epsilon \leq 0.04$ . However, at low SNR values or large  $\epsilon$ , a discrepancy between the approximated and simulated BER is observed. This is due to the fact that the union bound in (8) is not tight in low SNRs. In addition, it is also observed that the BER performances of SCMA-OFDM systems deteriorate significantly for CFO larger than 0.02 in both Gaussian and multipath Rayleigh fading channels.

Fig. 5 shows the simulated BER performance of SCMA-OFDM and the OFDM with orthogonal multiple access (OMA-OFDM) with different CFO values in Gaussian and multipath Rayleigh fading channels. Specifically, the QPSK constellation with a maximum likelihood decoder for a single user OMA-OFDM system is considered. Obviously, SCMA-OFDM is more sensitive to CFO than that of the OMA-OFDM systems, especially in Rayleigh fading channels. It is also observed that the SCMA-OFDM systems in Rayleigh fading channels are slightly more sensitive to CFO than in Gaussian channels. The BER performance of SCMA-OFDM system deteriorates shapely for CFOs larger than 0.02, whereas the OMA-OFDM deteriorates significantly for CFOs larger than 0.1 over Rayleigh fading channels.

## V. CONCLUSION

In this paper, we have investigated the impact of CFO impairments on the performance of SCMA-OFDM systems. Although SCMA-OFDM offers high spectral efficiency and enables massive connectivity, it is sensitive to the effects of CFO. We have observed that the BER performance degrades significantly when the normalized CFO exceeds 0.02. Therefore, the impact of CFO should be taken into account when designing SCMA-OFDM systems. As a future work, other existing modulation schemes that are robust to CFO effects,

such as orthogonal time frequency space, can be explored for the SCMA systems.

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