ISSN 1755-5361

University of Essex

Department of Economics

Discussion Paper Series

No. 629 April 2007

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Marginal contribution, reciprocity and equity in segregated groups: Bounded rationality and self-organization in social networks

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This paper is forthcoming in the Journal of Economic Dynamics and Control
June 2007

Abstract

We study the formation of social networks that are based on local interaction and simple rule following. Agents evaluate the profitability of link formation on the basis of the Myerson-Shapley principle that payoffs come from the marginal contribution they make to coalitions. The NP-hard problem associated with the Myerson-Shapley value is replaced by a boundedly rational 'spatially' myopic process. Agents consider payoffs from direct links with their neighbours (level 1) which can include indirect payoffs from neighbours' neighbours (level 2) and up to M-levels that are far from global. Agents dynamically break away from the neighbour to whom they make the least marginal contribution. Computational experiments show that when this self-interested process of link formation operates at level 2 neighbourhoods, agents self-organize into stable and efficient network structures that manifest reciprocity, equity and *segregation* reminiscent of hunter gather groups. A large literature alleges that this is incompatible with self-interested behaviour and market oriented marginality principle in the allocation of value. We conclude that it is not this valuation principle that needs to be altered to obtain segregated social networks as opposed to global components, but whether it operates at level 1 or level 2 of social neighbourhoods. Remarkably, all M>2 neighbourhood calculations for payoffs leave the efficient network structures identical to the case when M=2.

Keywords: Social networks, self organisation, stability, efficiency, Myerson-Shapley Value.

JEL: C70, D01, D63, D85

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Acknowledgements: Sheri Markose is grateful for discussions with Peyton Young, Sanjeev Goyal and Mathew Jackson.

1. Introduction

The importance of social networks in economics has been emphasised in the recent economics literature. The idea that people interact through specific links rather than solely through anonymous market mechanisms is one which has been inherited from the literature in other fields, particularly sociology. In standard economic models consideration of inter-relational network structures for economic outcomes has been regarded as irrelevant since a central coordinating device achieved the necessary coordination. [1] The intrinsic difficulties with this approach have been avoided by resorting to an extreme version of methodological individualism. This "representative agent approach" holds that aggregate economic activity can be described as the behaviour of some "average individual" and thus denies the importance of any direct interaction between individuals. This vision has been widely criticised, (see e.g. Kirman (1992)) and more recently a considerable literature on economic networks and the importance of local interaction has developed, (for an excellent survey see Jackson (2005)). We will review below the strands of literature on social networks that have a bearing on the objectives of the paper and the computational experiments that we design for this.

Although the nature of links is often left at an abstract level, in reality they may involve genealogy, kinship, friendship, trade (barter, supply chains, credit and inter bank relationships), traffic, disease (genetic or infectious), criminality, political or interest based alliances, communication, information and knowledge transfer (scientific collaboration, co-authorship) and so on. Each of these interactions, some of which may be voluntary and some involuntary, can lead to different types of network structure. Some socio-economic network structures are characterised by intimate social groupings or clusters that manifest principles of reciprocity, equity and punishment by ostracism. Such structures have dominated long periods of human

[1] In the Arrow-Debreu general equilibrium framework, though heterogeneous consumers and producers are possible, no coordination through local interaction is necessary because of the existence of equilibrium prices via a centralized Walrasian price setting process. A recent critique of this problem in Markose (2005) and Axtell (2005) argues that the original rationale that the existence of the Walrasian equilibrium prices can achieve Pareto efficiency solely from self-interested behaviour and without central command is an example of the invisible hand argument or self-organization is not entirely valid. The Axtell (2005) view is that as Walrasian price determination is NP-hard, it is unlikely to be an invisible hand or self-organizing process. There is a strong assumption that self-organizing processes that resort to simple or adaptive local calculations have arisen precisely to avoid NP-hard calculations or those that are non-computable.

evolution associated with the hunter gather stage as well as in non-market interrelationships (see, Kranton (1996), Bowles and Gintis (2000, a,b)). These clusters are, paradoxically, still found even on large freely accessible markets such as commodity exchanges, Aboulafia (1997), or on large wholesale markets, Weisbuch *et. al.* (2000). These structures are very different from the large social networks which involve looser social connections such as those found in information and communication networks. It has also been argued that much of the coordination of aggregate economic activity depends on "cooperation with strangers", Seabright (2004). Between these two views falls the "small worlds" model of Watts and Strogatz (1998), Watts (1999) and Albert and Barabási (2002).[2] In that perspective dense local clusters are linked through some long connections. The latter decrease the distance between any two individuals whilst retaining the essentially local nature of groups. However, small world networks are characterised by so-called "scale free" or power law distribution in terms of connections (in-degrees) and payoffs to some individual nodes, (see Barabási and Albert (1999)), which make them structurally different from the more balanced or egalitarian in-degree distribution found in close society networks.[3] Methodologically, it has been the study of scale free networks, especially in the context of the www, that has stimulated the use of computer simulation models and the analysis of self-organization in network structures based on low rationality models of link formation.[4] While there is a close conceptual link with what economists originally called the 'invisible hand' process, self-organization today refers to a much wider category of phenomena than those associated with the Walrasian model. These involve situations, where coherent patterns of network structures or other globally identifiable regularity can be seen to emerge without

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[2] This is named after the work of the sociologist Stanley Milgram (1967) on the six degrees of separation or that everybody is linked to every body else in a communication type network by no more that six indirect links

[3] With the concept of the 'small world' network now firmly attached to the description of large networks with highly skewed in-degrees, we will use the term 'close society', with no derogatory implication, to refer to the more intimate segregated social networks with balanced in-degrees.

[4] Barabasi and Albert (1999) have shown that a constantly increasing population and preferential attachment are sufficient conditions for generating stationary scale free networks with power law skewed in-degree distribution. The mechanical nature of process involved in producing the final network structures in these studies have also extended to sophisticated schemes such as the assignment of weights to network formation (Yook *et. al.* 2001) and assortative mixing in networks when high degree nodes attach to high degree nodes (see, Newman (2002) and Newman and Park (2000)). Economists, in contrast, have attempted to give more behavioural foundations for the social network formation process.

central direction and solely as a result of rule following by agents who interact locally without full system wide knowledge of its structure and components (see, Markose, 2005, for a recent critique). We subscribe to the necessity for analysing self-organization in networks and the use of computer simulations from this strand of the literature on large social networks. We will, however, seek more intentionality and a less mechanistic process behind the link formation process.

The underlying idea for all the analysis on social networks is that the individuals involved stand to gain from being connected. Yet the major question as to how these networks evolve has to be answered. There has been extensive theoretical and empirical work done by sociologists, anthropologists, neuro-physiologists, psychologists, physicists, mathematicians and economists to uncover the socio-economic behavioural, neuro-cognitive and mathematical rules that govern and sustain social network structures. However, to date, there is no model of social networks that gives a mathematical, dynamical or behavioural process that can account for the observed topological and social value related characteristics of *both* small group network dynamics *and* more anonymous large communications network.

We will draw on an early literature that combined theories of neuro-cognition with social anthropology which gave rise to the so called social brain hypothesis (see, Dunbar,1998, 1993). There it is postulated that the coevolutionary growth of the human neo cortex and its capacity to process social information places constraints on the number of close associates individuals can have and this implies that there is a clear relation between the numbers of links individuals have at different social distances, Zhou *et. al.*(2004). An important aspect of the social brain hypothesis, which we take into account, is the significance of what is called the level 1 support clique to an individual. Hence, when an individual considers his position vis-à-vis his level 1, or immediate neighbours, he has to also consider their support clique, that is, individuals who are at distance 2 from himself. This is what will be referred to as the individual's level 2 neighbourhood. It will be seen that what is critical about the identity and stability of intimate social groups is whether self-interested relational calculations made by individuals operate at level 1 or level 2 and above. That is, if all agents consider their neighbourhoods to be of level 2 or more, then any agent relating to his direct neighbour will have to be concerned also about his indirect impact on his neighbours' neighbours up to a *minimum* of 3 or more levels from himself. Remarkably, as we will see no more than level 2 neighbourhood calculations of 'payoffs' to each agent from direct neighbours are required to achieve the social values and network structures associated with intimate social groupings. In contrast, an analysis which restricts dealings to level 1, viz. direct neighbours' reactions will generate network structures that can incorporate large global components but with looser connections. This inverse relationship between intimacy and group size is, therefore, an issue of the cognitive costs behind social relational links. This is in keeping with the idea that individuals reflecting on the reactions of other individuals to whom they are only distantly linked is a move towards the complete game theorising which as Binmore (1990) and others have shown poses logical as well as calculability problems (see Axtell (2005)).

We will now discuss which class of payoff function is appropriate to explain the dynamic behind social network formation that can produce both large components and segregated clusters. In this and in the design of the model behind the computational experiments on the dynamic behind social network formation, we face a conundrum that is prominent in the extant economics literature on social networks. A large well established literature appears to argue that the principles of equity and reciprocity observed in close societies which manifest 'other regarding' preferences and cooperation are fundamentally different and often in conflict ('strong' dichotomy hypothesis) with those based on self-interest. In the latter payoffs are based on individual marginal calculations which are found in large global networks involving impersonal market oriented interactions. The basic argument here made influential by Olson (1965) and also from discussions on the one shot n-person Prisoner's Dilemma models starting with Hardin (1971, 1982)[5] is that, in the absence of coercion, self-regarding individuals, except in small intimate groups, cannot be made to perform cooperative and other regarding activities even if they perceive these to be beneficial to themselves in the longer term. This contrasts with the development of the social networks literature (see, Durlauf and Young (2001)) which challenges the primacy in economics of models based on individual rationality which ignore the impact of social networks on choices made by individuals.[6] Likewise, a large experimental economics literature, which we do not deal with here, attempts to see the

[5] See, Ostrom (2000) for a survey.
[6] Durlauf and Young (2001) consolidate and extend the view that methodological individualism cannot explain socio-economic problems such as the perpetuation of low aspirations and poverty while 'group think' or mimetic conformity influenced by local interactions in the social networks is a better

extent to which self and other regarding values 'inform' behaviour and choices of people from different cultures under controlled conditions.[7]

A part of the literature that is relevant to us seeks to explain 'deeper' roots for the dynamic behind how equity and reciprocity in the context of group formation can become norms among selfish individuals. A number of theoretical models use evolutionary game theory which either postulate natural selection of carriers of the cooperative 'gene', viz. a gene based model (see, Bergstrom (1995) or a culture based model on the evolution of cooperation as in repeated Prisoner's Dilemma, Axelrod (1984). The latter is seminal in that it gave the first demonstration of how self-regarding behaviour can 'morph' into other regarding behaviour. Thus, it is not surprising that in the few papers, for example, Haag and Lagunoff (2005) and Bowles and Gintis (2000,a), that use local interaction network models to understand the dynamic behind close society values of reciprocity, equity (within the group) and exclusivity or segregation, have done so within the context of the pay off/incentive function implied by the repeated Prisoner's Dilemma. In contrast to this, in the axiomatic framework of the Shapley value in cooperative game theory, Shapley (1953), a certain concept of equity is shown to be consistent with the allocation of value or payoffs to a player in terms of his marginal contribution in the context of group formation. There is also a somewhat looser literature which has recently revived insights from as far back as Adam Smith (see, Ashraf *et. al.* (2005)) that there is no inconsistency between the self-regarding wealth creating activities in markets and precepts of civil society which are manifestly socially orientated. Ashraf *et. al.* (2005) emphasize the Adam Smith view that a spontaneous or invisible hand process arising from cultural reinforcement results in other regarding values of civility that include fairness and trust which become an innate or deontic virtue that individuals in such cultures will follow without external compulsion. These two ideas are pivotal to our inquiry. Thus, as noted by Winter (2002) what is most appealing in the payoff function of the Shapley value is that it is a " synonym for the principle of marginal

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explanation for individuals who turn their back on better opportunities for themselves. Akerlof (1987) gives a good introduction to these issues.

[7] These experiments began with the classic one designed by Roth *et. al.* (1991), followed by other influential papers by Rabin (1993) and Fehr and Gächter (2000). Within the context of the set piece of games such as the ultimatum game, it was found that participants from market societies as opposed to more tribal ones showed as much or greater willingness for equitable allocations (see, Henrich *et. al.* 2004). This can be viewed as evidence against the 'strong' dichotomy hypothesis.

contribution – a time-honoured principle in economic theory" of markets .[8] It is interesting to see which social network systems will self-organize if all players attempt to maximize their Shapley style payoffs by choosing appropriate neighbours within a local interaction framework**.** In particular, can the formation of network structures with characteristics of close society values be engendered by self-interested behaviour and payoff/value functions based on the marginality principle ?

The rest of the paper is organized as follows. Section 2.1 first sets up the rationale for a spatio-socially myopic implementation of the payoff function based on the principle of marginal contribution in the network oriented framework of Myerson (1977). Section 2.2 gives the mathematical prerequisites on the use of graph theory in social network modelling and also introduces the notion of neighbourhood components which defines the nature of local interactions. Section 3.1 gives the analytical framework for a boundedly rational implementation for the Myerson-Shapley value for networks in a dynamic setting. The definition of a stable and efficient network is developed along the lines of neighbourhood stability. The results from the simulations are given in Section 3.2. The concluding section summarizes the results and discusses future work.

2 Spatially Myopic Implementation of Myerson-Shapley Style Payoffs in Social Networks: Network Theory and Neighbourhoods

2.1 Payoff Functions and Social Network Dynamics

Any dynamical process considered so far in the cooperative game theory framework of network formation potentially has had to contend with the computational problem that the Shapley value depends on an NP- hard problem of working out, *ex ante*, players' contribution to *all* possible 2^{N-1} coalitions. [9] Nevertheless, it is not unfair to say that network game theory started with the Myerson (1977) extension of the Shapley value, referred to as the Myerson-Shapley value, wherein each player's

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[8] Young (1985) raised the important issue on how far a deviation from the Shapley value is feasible if the principle that agent's payoffs are based on their marginal contributions is not abandoned. The answer appears to be not a lot.

[9] A problem is computationally intractable if it only has exponential time algorithms that have computation times that vary exponentially with the problem size (given by an integer N), for example, r^N, for some r >1. A polynomial (P) time algorithm is said to be tractable if its computation time varies proportional to the problem size raised to some integer power, d, as in N^d. Exponential functions grow strictly faster than polynomial ones. The significance of P-class problems is that they coincide with those that can be realistically solved by computers or economic agents. Deng and Papadimitriou (1994) were among the first to discuss the NP-hard problem when payoff functions are in general defined by 2^N coalitional values. In special cases, as we will also see, such as when agents consider payoffs only from direct level 1 neighbours, the Shapley value can be easily computed.

marginal contribution is not just a function of the members of the coalition but also the communication network structure involving these members. Myerson (1977) assumes a framework of undirected graphs where a link formation requires the mutual agreement of both the players on the premise that a link can be formed only if there is equitable division of the payoffs.[10] In the models that followed from Aumann and Myerson (1988) where non-cooperative dynamic processes have been modelled to see if the Myerson–Shapley value will be obtained, bilateral bargaining and behavioural norms based on equity are introduced *apriori* rather than shown to be the consequence of self-regarding behaviour guiding the choice of optimal coalition.

In general, if we consider the theoretical literature on network formation, it typically assumes that there is a value for any individual of a given network.[11] When contemplating the creation or severing of a link, the individual involved can calculate the change in his pay-off which follows. To do this, he has to know the mapping from *all* network structures to individual pay-offs. This is already a strong assumption. It is that adopted by Jackson and Watts (2002) for example. In this framework one tries to find equilibrium situations in which no individual has any incentive to change the links in which he is involved. One might want to go further and ask how one arrives at such an equilibrium. The procedure generally adopted, (see Bala and Goyal (2000)) is to consider a protocol which determines in which order each player appears and makes a proposition or forms links. This is important as a simplifying device but is somewhat artificial. Furthermore, it leaves open the horizon of the individuals. When making propositions, how many reactions in the future can they anticipate?

We shall take a different line on both of these points. First, as we have argued, given the significance of market oriented individualistic determination of value, we

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[10] Myerson (1977) argues for the condition of equity, symmetry or fairness to hold in an *apriori* fashion: ….“ Unequal allocations … would seem unfair and therefore unlikely to most observers. If players have an extra-utilitarian ethic against being exploited or taken advantage of in the cooperation process, then equal gains split must be the most likely outcome for this game. Certainly, we would expect an impartial arbitrator to suggest an equal split based on considerations of symmetry or equity”.

[11] Apart from the payoff frameworks of Prisoner’s Dilemma and the Myerson-Shapley value that have been used in social network models, the payoff functions that have become well known are those that have given rise to two main classes of network models developed by Jackson and Wolinsky (1996). In both models, agents benefit from their direct connections but in the first, often called friendship model, indirect connections generate positive externalities, while in the second, they generate negative externalities. The latter, often referred to as the Jackson -Wolinksy coauthorship model, is now recognized to be flawed. This is because in the real world coauthorship networks, in various subject fields such as bio-medicine, physics, and mathematics, show little evidence of what is implied in the Jackson-Wolinsky co-authorship model that the majority of authors are organized in small segregated components of researchers. Rather, as summarized in Newman (2004) and also Goyal (2007) ,

will use the Myerson-Shapley payoff structure which is based on an agent's marginal contributions to coalitions he belongs to. Thus, when an agent contemplates joining a coalition or forming a link to a neighbour he anticipates, as payment, what his arrival will add to the value of that coalition/neighbourhood. We propose using this approach to evaluate which neighbours individuals will keep over time as they dynamically break away from the neighbour to whom they make the least contribution. In connection with the second point, we will introduce a form of myopia which is spatio-social rather than temporal. We will not allow individuals to calculate the impact of their decision to form a link on agents far from them in the network. Agents evaluate their marginal contribution to their level 1 neighbours and their respective neighbourhoods up to a limited number of further levels, with a remarkable result that agents effectively need not look at payoffs from their neighbours with more than level 2 neighbourhood components. Further levels cannot improve on the efficient and stable network structures that emerge. Thus, by adopting spatio-social myopia, we can evaluate the impact of self-interested behaviour on the type of network which forms without facing the full brunt of computational intractability often assumed away in the axiomatic framework.

In summary, there is a long standing economics literature which when considered with the topological differences between the "small world" and "close society" networks has led to strongly held views. Firstly, it is argued that large scale communication networks in globalized market societies often underpin impersonal or anonymous relationships and these operate on profit maximization marginal efficiency principles. Given this, and the observation that close society networks display equity and reciprocity in the division of value it is argued that other regarding principles in such networks are inconsistent with the principle of self interested calculations of maximizing payoffs based on marginal contribution. The conundrum the literature, therefore, poses is as follows. Field and experimental data and in particular in the axiomatic literature on social coalition formation involving the Shapley value suggests that there is no inconsistency between equity and symmetry or reciprocity and self-regarding value functions based on the marginality principle. However, the axiomatic approach to social group formation and the dynamic models that have followed are for the most part analytical and have not taken into account the

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scientific collaboration manifests what are called giant components of maximally connected peers which account for 80% of all authors and with in degree distribution showing power law.

NP-hardness of the decision problem involved for the individuals nor paid any attention to the trade off between size of social components and the level of social closeness. A way round this is to develop models of self-organization based on local interaction with somewhat more intentionality than the low level rationality decision rules such as those used in the extensive literature on the Strogatz and Watts genre of large scale communication networks.

The objective of this paper is to construct computational experiments to combine a computational feasible boundedly rational approach to group formation which involves local interaction. This leads to the self-organisation of network structures through mutually self-interested behaviour that is based on payoffs determined by the Myerson-Shapley principle of marginal contribution. No *apriori* assumptions of equity, reciprocity or symmetry are made.

2.2. The graph theoretic framework for social network formation

In graph theory representations of socio-economic relations, nodes stand for agents or players and edges are connective links. There is a fixed and finite set of players, N = {1,2,3,…..,n}, with n > 3. Myerson (1977) first made the distinction between all manner of groupings ie. subsets of N, $\{S \subseteq N, S \neq \emptyset\}$, called coalitions on N and the network structures, that he called cooperation structures. The network structures will be denoted as g and optimal network structures maximize payoffs from coalitions. At each time t, t= 1,2,….., as will be explained presently, the network will be altered by agents making and breaking links, and hence its dynamics will be denoted by g_t.

Let i and j be two members of the set N and note that when a link between them can exist in either or both directions, it will be denoted simply as (i,j). When a direct link originates with i and ends with j, viz. an out degree for i, this will be denoted by $(\overrightarrow{i, j})$. If vice versa, we have $(\overleftarrow{i, j})$. The latter yields an in degree for i from j. If the links exist in both directions we will denote it as $(\overleftrightarrow{i, j})$. Note, an agent’s out degrees is denoted by k_i. We will use directed graphs, as we aim to model agents as having complete discretion over the initiation of any link that they may choose to form. Further, a connection to another agent may generate value even without the active consent of that agent. The notion of active reciprocity requires a link to be explicitly initiated by an agent if there is one to him from another agent. In a system of linkages modelled by undirected graphs, the relationships between N agents when viewed in NxN matrix form will produce a symmetric matrix as a link between two agents will produce the same outcome whichever of the two partners initiated it. In contrast, directed graphs are useful to study relative asymmetries and imbalances in link formation.

It is assumed that each agent i can not send more than one out degree to another agent j, $i \neq j$. We define a complete network

$$g^N = \{ (\overleftrightarrow{i, j}) \mid i \in N, j \in N, i \neq j \} \qquad (1)$$

as the set of all subsets of N of size 2, where all players are connected to all others. Let $g \subseteq g^N$ be an arbitrary collection of links or a network on N. We define

$$G = \{g \mid g \subseteq g^N \} \qquad (2)$$

as the finite set of all possible networks between the n agents. The *empty network* denoted as $g^{\emptyset}$ is such that it contains no links. $g^{\emptyset}$ (i,j), $g^{\emptyset}$(S) for any S S, $\subseteq$ N, imply respectively that there are no direct links between the pair (i,j) or and zero links in the set S. Singleton sets are also empty networks.

Example 1: Consider a 3 player game with N = {1,2,3} and $g = \{ (\overleftrightarrow{1,2}), (\overleftrightarrow{2,3}) \}$. This means that there are reciprocal or two way links between (1, 2) and (2 ,3) and no direct links between 1 and 3, ie $g^{\emptyset}$ (1,3) . Player 2 has two out degrees and two in degrees while players 1 and 3 only have one of each.

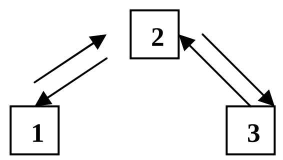

Figure 1 Example of a directed graph $g = \{ (\overleftrightarrow{1,2}), (\overleftrightarrow{2,3}) \}$.

Throughout the paper we will adopt the symbol \ to denote the removal of a member from the set of players or of a link in any or both directions (a,b) from a graph. Thus:

$$S \backslash a = \{ i \mid i \in S, i \neq a \} \qquad (3.a)$$

$$g \backslash (a,b) = \{(i,j) \mid (i,j) \in g, (i,j) \neq (a,b) \} \qquad (3.b)$$

To study properties of networks on some special subsets $S \subseteq N$ we will define the set of links in g that is obtained by eliminating links involving players outside S, ie. the complement set of S, $S^{\neg} = (a \in N \mid a \notin S \}$. Thus,

$$g(S) = \{ (i, j) \mid (i, j) \in g, \; i \in S \text{ and } j \in S \} \quad . \qquad (4)$$

In what follows, g(S\i), i ∈ S, will be important when each of the i players have to assess their topological significance which will be defined as their marginal value to the group or coalition on the network g.

Indirect Connections and Components

A *strong path (weak path)* in a network g ∈G between players i and j is a sequence of links between distinct players with i= i_1 followed by i_2 , ……, i_M = j such that for the successive pairs we have $(\overrightarrow{i_m, i_{m+1}})$ $((\overleftarrow{i_m, i_{m+1}}))$ and $(i_m, i_{m+1}) \in g$ for each m ∈ {1, …….., M}. A variant which combines properties of the strong and weak path , referred to as a *reciprocal path* is one where in at least one of the successive pairs *m* and *m+1*, defined above, we have *both* $(\overrightarrow{i_m, i_{m+1}})$ *and* $(\overleftarrow{i_m, i_{m+1}})$. In the case of reciprocal paths, we have loops and sequences of M links need not have distinct members. Thus, let $(i \rightarrow_g j)$, $(i \rightarrow_{g^-} j)$, and $(i \leftrightarrow_g j)$ respectively be the set of paths that strongly, weakly or reciprocally connect i and j on the graph g. The length of the path is the number of links m in it, whilst counting only 1 link between any successive pair, (i_m, i_{m+1}), for the strong and the weak paths that belong to g. The set of strong (weak) *shortest paths* between i and j on g will have minimum path lengths. This is called the *geodesic distance* between two agents and is denoted as $d^*(\overrightarrow{i, j})$ and $d^*(\overrightarrow{i, j}^{\,-})$, respectively in the strong and weak paths between i and j. In the case of reciprocal paths, agents at distance M from from i will be denoted $d(\overleftrightarrow{i, j})$=M where $d^*(\overrightarrow{i, j})$ and $d^*(\overrightarrow{i, j}^{\,-})$ operate to track the shortest paths in the two directions. When there is no path between (i,j), then conventionally, their geodesic distance is taken to be infinite: d(i,j) = ∞.

A graph g ⊆ G is said to be *connected* if there exits at least a weak path between any two nodes of g. For any g, we define η(g) ={i| $\exists j$ *and* $(i, j) \in g$ }, the set of agents who have at least one link in the network g and the cardinality of this set denoted by |η(g)| gives the number of players involved in g and all the links in η(g) is given by $k = \sum_{i \in \eta(g)} k_i$.

A weak *component* of a network g on the set η(g), is a non-empty sub-network g′ ⊂ g which we will denote as c(g′ (S))):

$$c(g'(S)) = \{ i \leftrightarrow_{g'} j \text{ for all } i \in \eta(g') \text{ and } j \in \eta(g'),\ i \neq j \text{ and if } i \in \eta(g') \text{ and } (i, j) \in g, \textit{ then } (i, j) \in g' \} \quad (5)$$

The set of all sub-components of any g is denoted as C(g) where $g = \cup_{g' \in C(g)} g'$.

It is useful to define a strong component implemented by g on subsets or coalitions $S \subset N$, given by g(S) above, which we will denote as

$$c^*(g(S)) = \{(i \rightarrow_g j) \quad \text{for } \textit{all } i \in S \text{ and } j \in S,\ i \neq j \}. \quad (6)$$

2.3 Neighbours and M-level neighbourhoods

We will be particularly concerned with components that are defined by *neighbourhoods*. Placing an agent i at level 0, i's first level or direct neighbours taken from this point are defined by

$$\Xi_i^1 = \{j \in N \mid (\overrightarrow{i, j}) \in g \}, \quad (7)$$

and the cardinality of this set denoted by $k_i = | \Xi_i^1 |$ equals agent i's out degrees. Note, the 2nd and 3rd or the Mth. level neighbours of agent i will be denoted by Ξ_i^2, Ξ_i^3, Ξ_i^m respectively. Thus,

$$\Xi_i^m = \{ j \in N \mid \forall\ i \rightarrow_g j\ ,\ d^*(\overrightarrow{i, j}) = M\} . \quad (8.a)$$

Example 2: For purposes of illustration, **Figure 2** gives an arbitrary weakly connected graph defined by the set η(g) = {1,2,3,4,5,6, 7, 8, 9}. Using the network given in **Figure 2,** agent 1's level 2 neighbours along strong paths are given by (8) is :

$$\Xi_1^2 = \{5,6\} .$$

In contrast, we will refer to an agent i's M-level neighbourhood , denoted by Ξ_i^{M+}, as one which includes all direct neighbours of i and *their* neighbours upto and including those at strong/weak geodesic length of M . Note, as agent i can be the neighbour of any member j in the M length sequence, we have reciprocal paths and

$$\Xi_i^{M+} = \{ j \in N \mid \forall\ i \leftrightarrow_g j\ ,\ d(\overleftrightarrow{i, j})_{\leq M} \} . \quad (8.b)$$

Thus, this set can include agent i.

Example 3: Agent 1's level 2 neighbourhood from the network given in **Figure 2** as defined in (8.b) yields:

$$\Xi_i^{2+} = \{2,3,1,5,6\}.$$

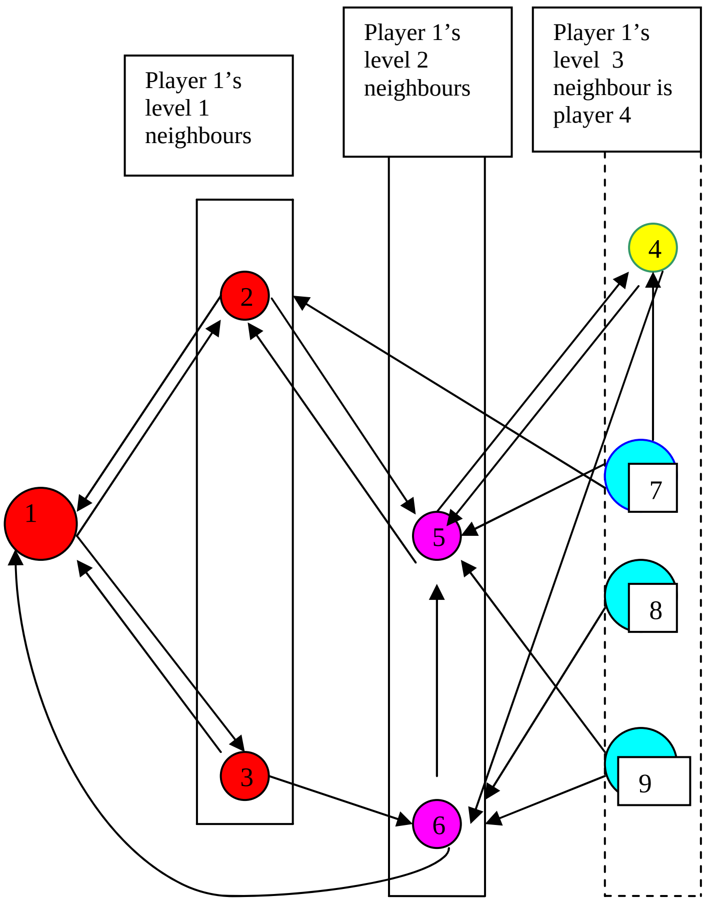

Figure 2: Example of a weakly connected network defined by the set $\eta(g) = \{1,2,3,4,5,6, 7, 8, 9\}$

In the network given in **Figure 2**, we will identify a so called level-2 neighbourhood, Ξ^2, as one where *all* connected individuals in the network is a level-2 neighbour of at least one member of $\eta(g)$.

$$\Xi^2 = \{ i \in \eta(g) \mid i \in \Xi_j^2 \text{ for at least one } j \in \eta(g), i \neq j\} \qquad (9)$$

Thus, for the $\eta(g) = \{1,2,3,4,5,6, 7, 8, 9\}$in **Figure 2**, $\Xi^2 = \{1,2,3,4,5,6\}$.

Note, agents {7,8,9}, do not belong to Ξ^2 as they are not neighbours of any member of the given $\eta(g)$. In the sort of boundedly rational framework that we will consider, spatio-social myopia restricts all agents to payoffs from *their* M-level neighbourhood networks denoted as g (Ξ_i^{M+}). At initial time t=0, using **Figure 2** and from Example 3 agent 1's $\Xi_1^{2+} = \{2,3,1,5,6\}$. Note when counting the links in Ξ_1^{2+} which are at strong geodesic distance of upto and including 2 from agent 1, viz. $d(\overrightarrow{1,j}) \leq 2$, the out degrees of agents {1,2,3} are counted and not those of agents {5,6}. Thus ,

$$|g_{t=0}(\Xi_1^{2+})| = 6 . \qquad (10)$$

The generalized formulae for the cardinality of links in g (Ξ_i^{M+}) viz. of a M-level neighbourhood network for any i, is given presently in section 3.2.

Each agent is assumed to obtain payoffs from each of their direct/level 1 neighbours based on the latters' respective M-level neighbourhoods. Thus, the set of M-level neighbourhood subcomponents of agent i, denoted as C(g(Ξ_i^{M+})), can first be decomposed to include the agent i's direct neighbours Ξ_i^1 and then given in terms of the M-level neighbourhood of agent i's level 1 neighbours denoted as $j_{q,1} \in \Xi_i^1$ with the k_i neighbours at level 1 listed as {q, q_{+1}, … k_i} in ascending order of the numerical indexes given for the k_i agents, Thus,

$$C(g(\Xi_i^{M+})) = \Xi_i^1 \bigcup \{ \bigcup_q^{k_i} \Xi^{M+}_{\overrightarrow{i,j_{q,1}}} \} . \qquad (11)$$

The notation $\Xi^{M+}_{\overrightarrow{i,j_{q,1}}}$ denotes the M-level neighbourhood of agent i's neighbour , $j_{q,1}$. For each of i's level 1 neighbours $j_{q,1}$, $\Xi^{M+}_{\overrightarrow{i,j_{q,1}}}$ can be recursively defined one level at a time. Thus,

$$\Xi^{M+}_{\overrightarrow{i,j_{q,1}}} = \bigcup_{m=1}^{M} \bigcup_{j_{q,m} \in \{\Xi_{j_{q,m-1}}^{m}\}} \Xi_{j_{q,m}}^{m+1} , \qquad (12)$$

with $\Xi_{j_{q,0}}^1 \equiv \Xi_i^1$. Here, $j_{q,m} \in \{\Xi_{j_{q,m-1}}^{m}\}$ defines membership of the set of neighbourhoods of agent $j_{q,1}$ at geodesic distance m from agent i whose level 1 neighbour $j_{q,1}$ is.

Example 3: Consider the example in which spatio-social myopia restricts agents to M=2 neighbourhoods. Then for agent 1 in **Figure 2,** his level 1 neighbours are given by Ξ_1^1 = {2,3}. Equation (11) which yields the set for M=2 neighbourhood subcomponents for agent 1 for its level 1 neighbours, agents 2 and 3, is as follows:

$$C(g(\Xi_1^{2+})) = \Xi_1^1 \bigcup \{ \bigcup_q^{k=2} \Xi^{2+}_{\overrightarrow{1,j_{q,1}}} \} = \{2,3\} \bigcup \Xi^{2+}_{\overrightarrow{1,2}} \bigcup \Xi^{2+}_{\overrightarrow{1,3}} . \qquad (13)$$

Using the recursion in (12) and setting $j_{q,1}$ to be agent 2 in **Figure 2**, in (13) we have,

$$\Xi^{2+}{}_{\overrightarrow{1,2}} = \bigcup_{m=1}^{2} \bigcup_{j_{q,m} \in \{\Xi_{j_{q,m-1}}{}^{m}\}} \Xi_{j_{q,m}}{}^{m+1} = \{\Xi_2{}^2 = \{1,5\},\ \Xi_1{}^3 = \{2,3\},\ \Xi_5{}^3 = \{2,4\}\}\ .^{12} \qquad (14)$$

That is, we have agent 2's direct neighbours who are agents 1 and 5 and then their respective neighbours who are now at relational level 3 from agent 1.

Likewise, on setting $j_{q+1\ ,1}$ to be agent 3 in **Figure 2**, in (13) we have:

$$\Xi^{2+}{}_{\overrightarrow{1,3}} = \{\Xi_3{}^2 = \{1,6\},\ \Xi_1{}^3 = \{2,3\},\ \Xi_6{}^3 = \{1,5\}\}\ . \qquad (15)$$

Then on combining equations (14) and (15) by substitution into (13), the full set of agent 1's level 2 neighbourhood subcomponents in terms of its direct neighbours agents 2 and 3 can be obtained. As we will see later, each i gets payoffs from its marginal contribution to each of its k_i neighbours and their respective neighbourhoods up to level M, recursively defined as above. In the case of M=2, for agent 1 his marginal contribution to neighbours 2 and 3 are calculated from (14) and (15) respectively. When M=1, in the above example, agent 1's marginal contributions to each of its two neighbours agents 2 and 3 are respectively calculated on the basis if the following sets :

$\Xi^{1+}{}_{\overrightarrow{1,2}} = \Xi_2{}^2 = \{1,5\}$ and $\Xi^{1+}{}_{\overrightarrow{1,3}} = \Xi_3{}^2 = \{1,6\}$.

3. Modelling Dynamics in Social Networks

3.1 Payoffs based on marginal contribution

The principle of an agent's payoffs being a function of the vector of his marginal contributions to the components (viz. maximally connected coalitions) he is part of is the fundamental aspect of the Shapley-Myerson value function. In general, the total value function of a network is represented by

$$v: \{g\ |g \subset g^N\} \rightarrow \mathbf{R}, \text{ with } v(g^{\emptyset}) = 0. \qquad (16)$$

The set of all such functions is denoted by V.

The vector valued payoff allocation rule, Φ (v, g) = (ϕ_1(v,g), ϕ_1(v,g), ……. ϕ_n(v,g)) in $\mathbf{R}^n$ determines the payoffs of each of the n players.

The Myerson-Shapley Value (MSV), ϕ_i^{MSV} (v(g)) , allocation rule is given in terms of a global *ex ante* calculation of the expected payoff to player i from the network g. This will be based on the marginal contribution of player *i* to all subsets, that contain i

[12] In the recursion, when M=2 in $j_{q,m} \in \{\Xi_{j_{q,m-1}}{}^{m}\}$, we have $j_{q,2} \in \{\Xi_2{}^2 = \{1,5\}\}$, that is 2's direct neighbours are agents 1 and 5. This implies that at M= 2+1=3 strong/weak geodesic distance

, $S \subseteq N$ and $i \in S$, and in terms of its component, viz. the graph that contains all direct and indirect links to i for all $j \in S$, $j \neq i$:

$$\phi_i^{MSV} (v(c(g(S)) = \sum_{S \subseteq N:\, i \in S} p_{i,S} [v(c(g(S)) - v(c(g(S \setminus i))] \ . \qquad (17)$$

Here, the probability weights for each of the g-components of size |S| where all permutations of members are equally likely is defined by:

$$p_{i,s} = \frac{(|S|-1)!\,(|N|-|S|)!}{|N|!} \ , \quad \text{if } i \in S. \qquad (18)$$

The marginal contribution of a player to the component that includes him for a given set S is given by the term in square brackets (17)

$$MC^i (v(c(g(S)) = [v(c(g(S)) – v(c(g(S \setminus i))]. \qquad (19)$$

The marginal contribution of agent i is given by the total value of the component less the value of the component without agent i.

3.2 Boundedly Rational and Computationally Feasible Implementation

We now replace the NP-hard problem associated with the calculation of the Myerson-Shapley value of networks by a computationally feasible procedure based on a simple boundedly rational experiential learning process that agents in the system can reasonably implement. In a dynamic framework of network formation, time t is taken to be discrete and the network at time t will be denoted as g_t, t = {0,1,……..}. Rather than starting with a network with no links, we will in all cases assume that agents have a fixed number of out degrees and initially these are randomly connected to others. In a boundedly rational framework of local interaction, we consider M- level neighbourhood networks for player i at time t , $g_t(\Xi_i^{M+})$, as the component of interest for the value function. The payoff to an agent is a function of his marginal contribution to the M-level neighbourhoods of each of his direct neighbours. The rule for retaining or breaking away from his neighbours is the simple one of self-interest, viz, of eliminating the neighbour who yields him the least payoff or equivalently to whom he makes the least marginal contribution. The 'weakest' neighbour is replaced randomly by another who yields a better payoff. What are the characteristics of the weak neighbours ? What are the properties of the efficient networks, ie, those which maximize the total payoffs to an agent from all the neighbourhood components which constitute his network $g_t(\Xi_i^{M+})$?

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from agent 1who is at level 0, we have agent 2 's neighbours' neighbours, viz.

For each i and each t, we will use a simple value function that is link based and is given by the number of links, determined by all the out degrees in an agent's M- level neighbourhood network :

$$v(g_t(\Xi_i^{M+})) = |g_t(\Xi_i^{M+})| = K_{it} . \qquad (20)$$

Benchmark case equal out degrees for all agents: k≥2

The cardinality of links in players i's M-level neighbourhood with identical out degrees, k, for all i is given by

$$|g_t(\Xi_i^{M+})| = \sum_{m=1}^{M} k^m = (k + k^2 + k^3 + \ldots k^M) = K_{it}. \qquad (21)$$

Note, the M relational levels is not along strong paths but can involve reciprocal paths. Hence, in the former case a level 3 relationship for an agent i will involve 14 other nodes while with reciprocal links where i becomes his neighbour's own direct neighbour, a 3-level neighbourhood calculation can contain fewer distinct nodes than 14. However, to show a non-trivial difference between the efficient network structures that arise from payoff functions based on M=1 and M≥2 neighbourhood calculations, we need to have the number of agents to be N≥6 with outdegrees k≥ 2. In general where k_i is heterogeneous, the recursive relationship discussed earlier in (12) can be adopted. To obtain the cardinality of all out degrees we sum over q=1,…k_i, ie. all neighbours of i yields

$$K_{it} = |g_t(\Xi_i^{M+})| = \sum_{q=1}^{k_i} \sum_{m=0}^{M} \sum_{j_{q,m} \in \{\Xi_{j_{q,m-1}}^{m}\}} |g_t(\Xi_{j_{q,m}}^{m+1})| , \qquad (22)$$

with $|g_t(\Xi_{j_{q,0}}^{1})| = |g_t(\Xi_i^1)|$ *and* $|\Xi_{j_{q,-1}}^{0}| = 0$.

At each time t, every agent i evaluates its marginal contribution, denoted as $MC_{\overline{i,j_{q,1}}}(g_t(\Xi_i^{M+}))$ to each of its level 1 neighbours $j_{q,1} \in \Xi_i^1$ and their respective M level neighbourhoods. This yields

$$MC_{\overline{i,j_{q,1}}}(g_t(\Xi_i^{M+})) = |g_t(\Xi_i^{M+})| - \sum_{m=1}^{M} \sum_{j_{q,m} \in \{\Xi_{j_{q,m-1}}^{m}\}} |g_t(\Xi_{j_{q,m}}^{m+1} \setminus i)| . \qquad (23)$$

The terms involved in the summation of $|g_t(\Xi_{j_{q,m}}^{m+1} \setminus i)|$ yields the cardinality of i's of the out degrees of neighbour $j_{q,1}$ and his M-level neighbours' neighbours' without i in each of the m-levels. Note, the number of links in $\Xi_i \setminus i$ is zero or the

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$\Xi_1^3 = \{2,3\}$, $\Xi_5^3 = \{2,4\}$

number of links in i's direct neighbourhood without i is zero. Thus, $| g_t(\Xi_{j_{q,m}}^{m+1} \setminus i) | = 0$. The total payoff , TP_i, to each i at each t is given by the sum of i's marginal contribution to each of i's direct neighbours and their respective M-level neighbourhoods:

$$TP_i\ (g_t) = \sum_{q=1}^{k_i} MC_{\overline{i, j_{q,1}}}\ (g_t(\Xi_i^{M+}))\ . \qquad (24)$$

Example 4: With M=1, i's marginal contribution is given by:

$$MC_{\overline{i, j_{q,1}}}\ (g_t(\Xi_i^{1+})) = |g_t(\Xi_i^1)| - | g_t(\Xi_{j_{q,1}}^{2} \setminus i) | . \qquad (25)$$

In (25) we set i to be agent 1 in **Figure 2**, and his level 1 neighbour $j_{q,1}$ to be agent 2. Agent 1's marginal contribution to agent 2's level-1 neighbourhood given that agents are confined to level 1 neighbourhoods is evaluated as follows. The value of the network, $|g_t(\Xi_1^1)| = 2$, viz. agent 1's out degrees $k_1 = 2$. As agent 2's neighbours are agents 1 and 5, $| g_t(\Xi_2^{2} \setminus i) | = 1$, that is if agent 1 is removed from agent 2's network, the number of links remaining is only 1. This yields a marginal contribution to agent 2's neighbourhood to be, (2-1) =1. Likewise, agent 1's marginal contribution to the direct neighbour 3 is also 1 making the total payoff to 1, $TP_1 = 2$.

3.3 Dynamic Improvements In Network Payoffs and Efficient Networks

The dynamics behind the evolution of the network structure is driven by self-interested behaviour in that at each t, each agent i evaluates the vector of marginal contributions $\{ MC_{\overline{i, j_{q,1}}}(g_t(\Xi_i^{M+}))$, ….., $MC_{\overline{i, j_{k_i,1}}}(g_t(\Xi_i^{M+})) \}$ and will break away from the neighbour to whom he makes the least contribution if another agent b, $b \notin \Xi_i$, and b's M-level neighbourhood enables i to make a greater marginal contribution. [13] So what sort of neighbours does it pay for i to drop according to the rule in (23) ?

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[13] This is a simultaneous game and the strategies of each agent will be executed at the same time at each time step. Each agent *i* executes the following operations:

(i)computes $MC_{\overline{i, j_{q,1}}}(g_t(\Xi_i^{M+}))$, $\forall\ j_q,\ j_q \in \Xi_i$ using equation (23)

(ii)finds $MC_i^{\min} = \min_{j_q \in \Xi_i} \{MC_{\overline{i, j_{q,1}}}(g_t(\Xi_i^{M+}))\}$

(iii) randomly chooses another node $b \notin \Xi_i$ and agent *i* will detach his link to the neighbour with $MC_i^{\min}$ and replace him with *b* if and only if: $\alpha MC_{\overrightarrow{i,b}} > MC_i^{\min}$. Here $\alpha > 1$ is the perturbation

For any agent i, his marginal contribution to m- level neighbourhoods of his direct neighbour $j_{q,1}$ given in (23) has component wise evaluations given by :

$$| g_t (\Xi_{j_{q,m}}^{m+1} \setminus i) | = \begin{cases} | \Xi_{j_{q,m}}^{m+1} | & if \quad i \notin \Xi_{j_{,q,m}}^{m+1} \qquad (26.a) \\ | \Xi_{j_{q,m}}^{m+1} | -1 & if \quad i \in \Xi_{j_{q,m}}^{m+1} \qquad (26.b) \end{cases}$$

This implies that marginal contributions and hence payoffs are enhanced for each i if the neighbour, $j_{q,1}$, and the latter's m-level neighbours reciprocate to i, ie., i is also their neighbour's neighbours at each component level as in (26.b). Intuitively, it is clear that one's marginal contribution to those who are not linked back to one will be limited.

Stable and efficient neighbourhood networks structures including each i denoted as $g^*(\Xi_i^* \cup i)$ are such that total payoff from it,

$$TP_i(g^*(\Xi_i^* \cup i)) \geq TP_i(g), \forall g \in G \quad . \qquad (27)$$

Three remarkable results followed from the self-organization process in terms of the network dynamics, g_t, while converging to efficient network structures. The low level rationality in the search process does not enable the agent to 'knowingly' seek those who reciprocate. Indeed, all this is achieved by a self-regarding process by all of the agents following steps set out in footnote (13). The main results can be summarized as follows:

(i) The only critical difference in the efficient network structures occur at M=1 and M=2.

(ii) For all values of M≥2, for equal out degrees k for all i, the efficient networks yielded the same complete, symmetric, reciprocal and *segregated* network structure with k+1 members. Thus, starting from initial random in-degree distribution and network structure of a fully connect graph as shown in **Figure 3a**, the final self-organized efficient network structures are given in **Figure 3b**. The only consequence of M > 2, and for k >2 is to scale payoffs according to **Table 1b**. All members of every segregated efficient network with nodes equal to k+1 achieve the same total payoff. [14] For example the total payoff for each agent in **Figure 3b** can be read off **Table 1b** with k=3 and M=2 to give 6x3 =18.

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coefficient which is used to speed up the discovery of better neighbours. Neighbourhood stability is determined when agents find no advantageous link changes as per (iii) and total payoffs $TP_i(g_t)$ in (24) stabilize.

[14] In the experiments done with a distribution of agents with different k_i , the self regarding process that leads to efficient networks resulted in agents with same k_i to cluster together in segregated groups , with M≥ 2.

(iii) For M=1, k=2 an efficient network which displays reciprocity arises within a global component as in **Figure 4.** The latter appears in both populations of agents with homogenous out-degrees and in the heterogeneous case. No break away segregated groups ever appear. Link based level 1 spatio-social total payoffs are equalized for all identically endowed agents. In order to achieve this, the efficient stable network has to be a giant component where all 100% of agents are connected. Note, the initial graph for this experiment is omitted as it is a similar graph with random in-degree distribution to that given in **Figure 3a**.

Table 1a gives the numerical values for the calculation of marginal contribution based payoffs from the formula in (23) for the benchmark case of equal out degrees k, and all agent i's neighbours and m-level neighbour's reciprocate in an efficient network equilibrium.

Table 1a Payoff Formula In An Efficient Network Equilibrium (Based on Marginal Contribution (MC_i) and Total Payoff (TP_i); Case: Equal out degrees k >2)

	M= 1 Out Degree k=2	**M=2 Out Degree 2 k=2**	**M=3 Out Degree k=2**	**Level M Out Degree k=2**	**Level M Out Degree k**
MC Eq. (23)	2-1=1	$(2+2^2)-(1+1)=4$	$(2+2^2+2^3)-(1+1+1)=11$	$(2+2^2+\ldots+2^m)-(1+1^2+\ldots+1^m)$	$(k+k^2+\ldots\ldots+k^m)-[(k-1)^2+(k-1)^2+\ldots\ldots.+(k-1)^m]$
TP Eq. (25)	*1+1 = 2*	*4+4=8*	*11+11=22*	*2*MC*	*k*MC*

Table 2b Marginal Contribution based Payoff values for each agent based on Table 1a
(For Total Payoff for each agent multiply entries in the Table by k)

	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9	M=10
k=1	1	2	3	4	5	6	7	8	9	10
k=2	1	4	11	26	57	120	247	502	1013	2036
k=3	1	6	25	90	301	966	3025	9330	28501	86526
k=4	1	8	45	220	1001	4368	18565	77540	320001	1309528
k=5	1	10	71	440	2541	14070	75811	400900	2091881	10808930
k=6	1	12	103	774	5425	36456	238267	1527258	9651829	60352380
k=7	1	14	141	1246	10277	81270	624877	4710062	34985973	256995046
k=8	1	16	185	1880	17841	162336	1435945	12448360	106312481	897579056
k=9	1	18	235	2700	28981	298278	2984095	29253600	282456361	1356198184
k=10	1	20	291	3730	44681	513240	5730271	62683550	675263061	675263061

Note, **Table 2b** also serves to show how quickly the size of calculations required to determine payoffs based on marginal contributions grow as the level M of neighbourhood increases beyond 2 and as the number of out degrees, k, grows.

Figure 3a :Initial graph with random in-degree distribution (M=2 ;Nodes 32; k=3)

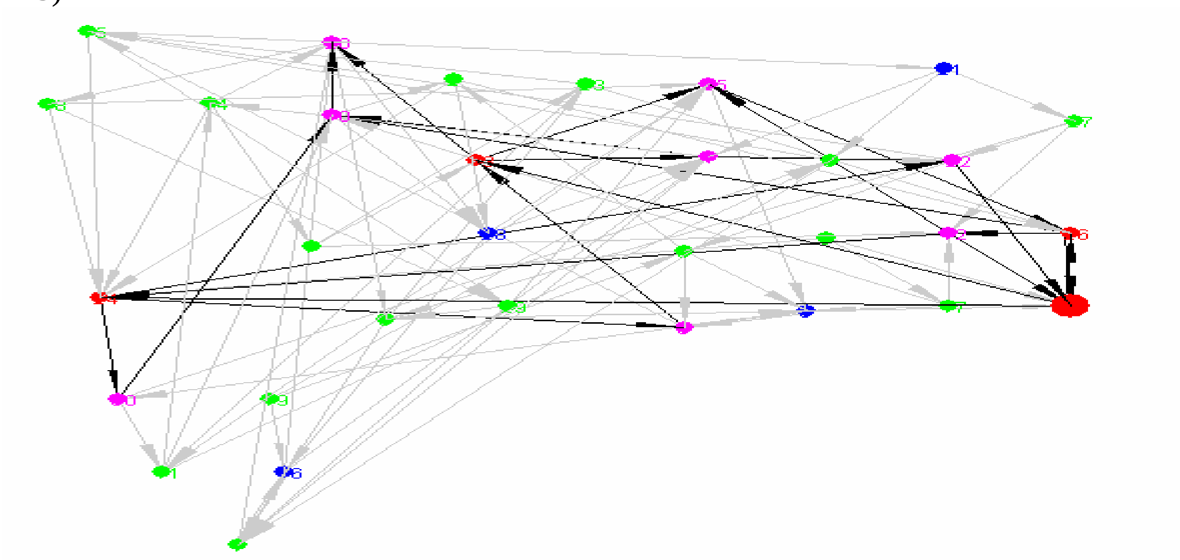


Figure 3b Final Graph: Segregated Efficient Networks with reciprocity, symmetry and equity (M=2; Nodes=32; k=3)

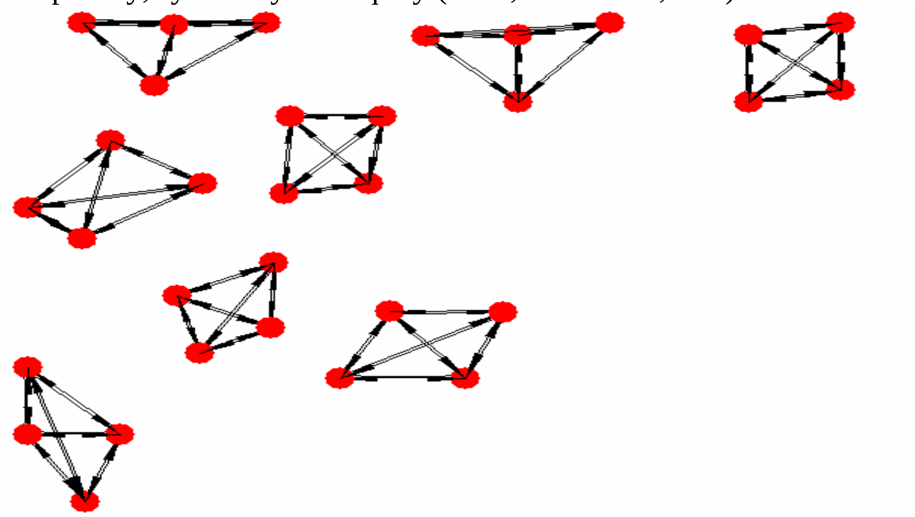
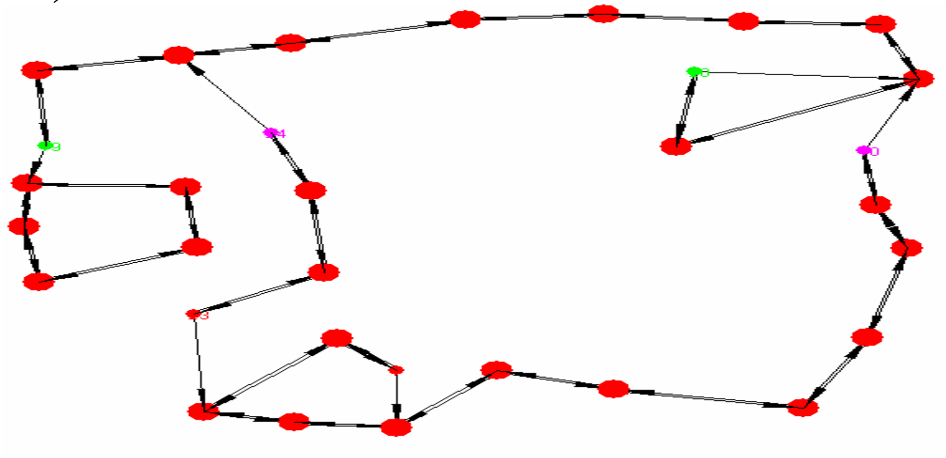


Figure 4 :Final graph : Global Wheel (M=1 ; Nodes = 32; k=2)



4. Concluding Remarks

There have been a number of explanations for the prevalence of close knit bands based on reciprocity and equity amongst one's kind that have been the bed rock of social order from the hunter gatherer stage and continues into modern times. There has been a long standing conundrum about the apparent inconsistency between reciprocity and equity in close knit groups and self-regarding behaviour and marginality based payoff functions in large market oriented networks. The principle that payoffs are determined on the basis of one's marginal contribution to the group or neighbourhood one belongs to, has been considered to be the most attractive feature of the Shapley allocation rule. However, the potentially NP-hard nature of the evaluation of such calculations for all possible coalitions one can belong to has led to the exercise reported here on a boundedly rational calculation of how agents may dynamically choose neighbourhoods to maximize their payoffs with efficient network structures that self-organize from local interaction over time.

Placing credence on the 'social brain' hypothesis that individuals have support cliques and that there is an inverse relationship between the size of stable social network structures and social distance, we have proposed a model of spatio-social myopia. In order to incorporate the support clique, individuals consider neighbourhoods of level 2 or more. Thus, on evaluating one's marginal contribution to a neighbour one needs to consider *his* neighbour's support clique, thus, involving evaluations at a minimum of (geodesic) distance of 3 for each individual. Constraining individuals to operate relationally at level 1 neighbourhoods, on the other hand, corresponds to methodological individualism. The self regarding criterion that an agent seeks to replace the neighbour from whom he receives the smallest payoff (and who is found to least reciprocate) when simultaneously applied by all agents in level 2 or more neighbourhoods produced an outcome that was not anticipated by us - viz. stable social network structures that can be identified as the benchmark social unit of a close knit, segregated cluster with perfect symmetry, reciprocity and equity. These close knit social structures maximize individual, group and global payoffs. In contrast, the exact same self-regarding application of the Shapley payoff principle based on marginal contribution at level 1 neighbourhoods produces a global wheel or a giant component.[15] Thus, what is remarkable is that while marginality principle in the payoff function is relentless in equalizing payoffs among identically endowed players the critical differences between the efficient and stable networks structures occur at level 1 and level 2 neighbourhoods. That all calculations beyond level 2 did not alter the above result on segregated cliques with symmetry, reciprocity and equity among identically endowed agents highlights the self-organising properties of the self-regarding dynamic to maximize the given payoff function based on myopic local interaction and without running into computational intractability. In the model, endowments were the number of out degrees an agent possessed. Experiments done at level 2 or higher with agents with heterogeneous numbers of out degrees, resulted in the case of 'birds of a feather flock together' with a hierarchy of cliques of different sizes where k+1 agents with k out degrees got together. The self-regarding implementation of the marginality based payoff function does not produce skewed in degree distributions.

In conclusion, our computational experiments show that both close knit segregated network structures and global network components naturally self-organize. These simulation results are important in that they overturn the widely held view that principles other than self-interested behaviour based on maximizing marginality based payoffs are needed to obtain close knit segregated social networks that manifest reciprocity and equity.

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[15] It is intuitively easy to understand why level 2 payoff calculations will force agents with k outdegrees each to break away from large networks and form reciprocal and complete network formations of k+1 members. At level 2 , an agent i can maximize his payoffs from his direct neighbour only if all of the neighbour's neighbours also has an in degree to agent i. However, as all i have similar pressures, and each i with k outdegrees can only link to k others , one to all and all to one becomes the norm and groups of k+1 members will break off from larger networks. In contrast, payoffs that come from level 1 relationships implies that agents is satisfied if his direct neighbour reciprocates and is not concerned by whether or not a neighbour's neighbour is linked to one.

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