

Received 28 August 2023, accepted 17 December 2023, date of publication 22 December 2023,  
date of current version 28 December 2023.

Digital Object Identifier 10.1109/ACCESS.2023.3346035

## RESEARCH ARTICLE

# On the Mates of Cross Z-Complementary Pairs for Training Sequence Design in Generalized Spatial Modulation

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The work of Zhen-Ming Huang, Chen Hu, and Chao-Yu Chen was supported by the National Science and Technology Council, Taiwan, under Grant NSTC 112-2221-E-006-122-MY3, Grant NSTC 112-2927-I-006-503, and Grant NSTC 112-2218-E-305-001. The work of Zilong Liu was supported in part by the U.K. Engineering and Physical Sciences Research Council under Grant EP/X035352/1 and Grant EP/Y000986/1, in part by the Royal Society under Grant IEC/R3/223079, and in part by the Research Council of Norway under Grant 311646/070.

**ABSTRACT** Generalized spatial modulation (GSM) as a novel multiple-antenna transmission technique is an extension of spatial modulation (SM). The GSM system provides flexibility in terms of spectral efficiency, energy efficiency, and the cost of radio-frequency (RF) chains. In the literature, cross Z-complementary pair (CZCP) has been proposed for optimal training sequence design in SM, but not for GSM. Due to its specific zero correlation zone (ZCZ) properties for autocorrelation sums and cross-correlation sums, optimal channel estimation performance in SM over frequency-selective channels can be achieved. In this paper, we propose an efficient construction for mutually orthogonal mate of a CZCP and provide a new construction of CZCPs. Based on the proposed CZCP and its mate, a novel training framework of GSM system is presented to achieve the minimum mean square error (MSE) channel estimation performance in frequency-selective channels. Numerical results demonstrate that the proposed CZCP-mate-based training for the GSM system can eliminate inter-symbol interference (ISI) and inter-antenna interference (IAI).

**INDEX TERMS** Cross Z-complementary pair (CZCP), zero correlation zone (ZCZ), generalized spatial modulation (GSM).

## I. INTRODUCTION

Sequences with good correlation properties have been widely utilized in communications, including multiple access, channel estimation, synchronization, etc. Among many others, one of the most celebrated sequence families is Golay complementary pairs (GCPs). Found by Marcel J. E. Golay in 1951 [1], [2], it was extended by Tseng and Liu to

The associate editor coordinating the review of this manuscript and approving it for publication was Pietro Savazzi<sup>1</sup>.

Golay complementary set (GCS) [3] where the aperiodic autocorrelations of the component sequences sum to zero at all non-zero shifts. A salient application of GCPs and GCSs is for the reduction of peak-to-average power ratio (PAPR) in orthogonal frequency-division multiplexing (OFDM) [4], [5], [6]. In [7], Fan et al. proposed the Z-complementary pair (ZCP) where the aperiodic autocorrelation sums take the zero values within a zero correlation zone (ZCZ). Compared to GCPs, ZCPs exist with more flexible lengths [8], [9].

On the other hand, multiple-input multiple-output (MIMO) technology has been playing an increasingly important role for high-rate and reliable wireless communication systems. Spatial modulation (SM) is one of the power-efficient multiple-antenna techniques with only one RF chain, whereby a single transmit antenna is activated at each time-slot [10], [11], [12]. In [13], a novel class of sequence pairs, called the cross Z-complementary pair (CZCP), has been proposed for the first time for optimal SM training sequences over frequency-selective channels. With their special autocorrelation and cross-correlation properties, CZCPs are capable of eliminating the inter-antenna interference (IAI) and inter-symbol interference (ISI) incurred by multipath propagation. This however may not be achieved by a generic GCP/ZCP.

Specifically, a pair of sequences is called a CZCP [13] if it satisfies the following two correlation properties: 1) the aperiodic autocorrelation sums have a front-end ZCZ and a tail-end ZCZ; 2) the aperiodic crosscorrelation sums have a tail-end ZCZ. A CZCP is referred to as perfect when its maximum ZCZ is attained. However, binary perfect CZCPs only exist for lengths  $2^{\alpha+1}10^\beta 26^\gamma$  where  $\alpha + \beta + \gamma \geq 0$  [13]. In [14] and [15], new CZCPs are presented to have flexible lengths of the forms  $10^\beta 26^\gamma$ ,  $2^{\alpha+1}10^\beta 26^\gamma + 2$ , and  $12 \times 2^\alpha 10^\beta 26^\gamma$  where  $\alpha$ ,  $\beta$ , and  $\gamma$  are nonnegative integers. In [16], more binary CZCPs with various lengths are constructed by applying Turyn’s method to some seed CZCPs and GCPs. In [17] and [18], new constructions of CZCPs based on ZCPs with different lengths are developed. Besides, there are also quadriphase CZCPs [18], [19] and polyphase CZCPs [20].

This paper aims to go one step further by studying the optimal training design in generalized spatial modulation (GSM) system. The basic idea of GSM is to simultaneously activate two or more antennas (depending on the number of RF chains) for data transmission [21], [22], [23]. Because of this, GSM is able to achieve higher spectral efficiency compared to SM. At the transmitter, similar to the case for SM, the input information bits are split into two portions: The first portion is used to activate transmitter antennas (TAs) through selected antenna patterns, while the second is used to select the constellation symbols for transmission. In a nutshell, GSM preserves the benefits of SM while offering greater flexibility between the spectral efficiency and the cost of RF chains.

Despite tremendous research attention on GSM, little is known on its training sequence design. Recently, Zhou et al. have proposed the symmetrical Z-complementary code set (SZCCS) for GSM training sequence design in [24]. However, their training framework requires additional overhead to mitigate the IAI by inserting a zero-padding sequence. As a result, their method leads to a reduced training efficiency. For more efficient training design of GSM, innovation is needed by properly dealing with the IAI of the activated antennas.

Therefore, we seek the aid of CZCP mates, each of which is not only mutually orthogonal to a CZCP, but also

exhibits certain cross-channel ZCZ property. We show that these novel properties can be fully exploited for efficient elimination of the IAI in GSM without reducing the training efficiency. We also provide a new construction of CZCPs based on shorter CZCP mates. Moreover, we present the training sequence design for GSM based on CZCP and its mate. Compared to the scheme in [24], our proposed design does not require additional zero-padding sequence. The proposed GSM training framework with CZCP and its mate can achieve the minimum mean square error (MSE) of the channel estimation over frequency-selective channels. Numerical results demonstrate that the proposed training design can eliminate ISI and IAI without incurring additional costs.

## II. PRELIMINARIES AND DEFINITIONS

Let  $\mathbf{p}_0 = (p_{0,0}, p_{0,1}, \dots, p_{0,L-1})$  and  $\mathbf{p}_1 = (p_{1,0}, p_{1,1}, \dots, p_{1,L-1})$  be two complex sequences of length  $L$ . We define the *aperiodic cross-correlation function* (ACCF) of  $\mathbf{p}_0$  and  $\mathbf{p}_1$  at displacement  $\mu$  as follows:

$$\hat{\rho}(\mathbf{p}_0, \mathbf{p}_1; \mu) = \begin{cases} \sum_{j=0}^{L-1-\mu} p_{0,j+\mu} p_{1,j}^*, & \text{for } 0 \leq \mu \leq L-1; \\ \sum_{j=0}^{L-1+\mu} p_{0,j} p_{1,j-\mu}^*, & \text{for } -L+1 \leq \mu < 0 \end{cases} \quad (1)$$

where  $(\cdot)^*$  denotes the complex conjugation. When  $\mathbf{p}_0 = \mathbf{p}_1$ ,  $\hat{\rho}(\mathbf{p}_0, \mathbf{p}_0; \mu) \triangleq \hat{\rho}(\mathbf{p}_0; \mu)$  is called the *aperiodic autocorrelation function* (AACF) of  $\mathbf{p}_0$ .

Moreover, the *periodic cross-correlation function* (PCCF) of  $\mathbf{p}_0$  and  $\mathbf{p}_1$  at displacement  $\mu$  is defined as follows:

$$\rho(\mathbf{p}_0, \mathbf{p}_1; \mu) = \begin{cases} \sum_{j=0}^{L-1} p_{0,(j+\mu)_{\text{Mod}L}} p_{1,j}^*, & \text{for } 0 \leq \mu \leq L-1; \\ \sum_{j=0}^{L-1} p_{0,j} p_{1,(j-\mu)_{\text{Mod}L}}^*, & \text{for } -L+1 \leq \mu < 0. \end{cases} \quad (2)$$

*Definition 1:* A pair of sequences  $\mathbf{p}_0$  and  $\mathbf{p}_1$  of length  $L$  is called the *cross Z-complementary pair*, denote by  $(L, Z)$ -CZCP, if it satisfies the following two conditions.

$$\hat{\rho}(\mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{p}_1; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_1 \cup \mathcal{D}_2; \quad (3-1)$$

$$\hat{\rho}(\mathbf{p}_0, \mathbf{p}_1; \mu) + \hat{\rho}(\mathbf{p}_1, \mathbf{p}_0; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_2 \quad (3-2)$$

where  $\mathcal{D}_1 \triangleq \{1, 2, \dots, Z\}$ ,  $\mathcal{D}_2 \triangleq \{L-Z, L-Z+1, \dots, L-1\}$ , and  $Z \leq L$ . Consequently, a CZCP displays specific correlation properties which consist of two zero autocorrelation zones (ZACZs) and one zero cross-correlation zone (ZCCZ).

*Lemma 1 ([13]):* For an  $(L, Z)$ -CZCP, we have  $Z \leq L/2$ . By *Lemma 1*, the CZCP is referred to as the perfect CZCP when the value of  $Z$  is  $L/2$ .

**Lemma 2 ([13]):** For an  $(L, Z)$ -CZCP  $(\mathbf{p}_0, \mathbf{p}_1)$ , we have  $p_{0,j} = p_{1,j}$  and  $p_{0,L-1-j} = -p_{1,L-1-j}$  for  $j = 0, 1, \dots, Z-1$ .

### III. GSM

Consider a single-carrier GSM (SC-GSM) system as depicted in Fig 1. For more information on SC-GSM and its applications in broadband large-scale antenna systems, refer to [21], [22], and [23]. Let  $N_t$ ,  $N_r$ , and  $N_A$  represent the quantities of TAs, receiver antennas (RAs), and RF chains, respectively. To establish connections between the RF chains and the TAs, an  $N_A \times N_t$  switch is required. During each time-slot  $k$ ,  $N_A$  of the  $N_t$  TAs are activated and the corresponding constellation symbols from QAM/PSK modulation  $\mathcal{M}$  transmitted on the activated TAs. The remaining  $N_t - N_A$  antennas remain silent.

Specifically,  $\lfloor \log_2 \binom{N_t}{N_A} \rfloor$  information bits, denoted by  $\mathbf{U}_k$  are used to select  $N_A$  antennas based on activation pattern mapping. In addition, we represent an  $N_t \times 1$  vector  $\mathbf{z} = [0 \dots 1 \dots 0 \dots 1]^T$  as an antenna activation pattern where 1's indicate the active antennas, and 0's indicate the silent antennas. Also,  $N_A \lfloor \log_2 |\mathcal{M}| \rfloor$  bits, represented as  $\mathbf{V}_k$ , are mapped onto a constellation using the alphabet  $\mathcal{M}$  through the  $N_A$  active antennas. Hence, the number of bits transmitted per symbol period is determined by  $\lfloor \log_2 \binom{N_t}{N_A} \rfloor + N_A \lfloor \log_2 |\mathcal{M}| \rfloor$ .

### IV. TRAINING FRAMEWORK FOR BROADBAND GSM

Consider a SC-GSM system with  $N_t$  TAs,  $N_r$  RAs, and  $N_A$  transmit RF chains. Each transmitted block includes a cyclic prefix (CP), the training sequence, and data payload as shown in Fig. 2. First, we consider that the training sequence  $\mathbf{x}_k = (x_{k,0}, x_{k,1}, \dots, x_{k,L'-1})$  is transmitted over the  $k$ -th TA and has equal energy  $E = \sum_{l=0}^{L'-1} |x_{k,l}|^2$ , for all  $k = 1, 2, \dots, N_t$ . In addition, we consider the complex additive white Gaussian noise (AWGN) with zero mean and variance  $\sigma^2$  and assume the quasi-static frequency-selective channel with the delay spread  $\lambda$ . For  $k = 1, 2, \dots, N_t$ , we assume that the channel impulse response from the  $k$ -th TA to the receiver is  $\mathbf{h}_k = [h_{k,0}, h_{k,1}, \dots, h_{k,\lambda}]^T$ . To formulate in matrix form, we let

$$\mathbf{X} = [\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_{N_t}]_{L' \times N_t(\lambda+1)} \quad (4)$$

where

$$\mathbf{X}_k = \begin{bmatrix} x_{k,0} & x_{k,L-1} & \cdots & x_{k,L-\lambda} \\ x_{k,1} & x_{k,0} & \cdots & x_{k,L-\lambda+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{k,L-1} & x_{k,L-2} & \cdots & x_{k,L-\lambda-1} \end{bmatrix}_{L' \times (\lambda+1)} \quad (5)$$

for  $k = 1, 2, \dots, N_t$ . Then, the complex received signal vector  $\mathbf{y} = [y_0, y_1, \dots, y_{L'-1}]^T$  at a RA can be expressed as

$$\mathbf{y} = \mathbf{X}\mathbf{h} + \mathbf{w} \quad (6)$$

where  $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \dots, \mathbf{h}_{N_t}^T]^T$  and  $\mathbf{w} = [w_0, w_1, \dots, w_{L'-1}]^T$  stands for the complex AWGN with zero mean and variance  $\sigma^2$ . By utilizing the least-square (LS) channel

estimator, the normalized MSE is derived as

$$\begin{aligned} \text{MSE} &= \frac{1}{N_t(\lambda+1)} \text{Tr} \left( E \left\{ (\mathbf{h} - \hat{\mathbf{h}}) (\mathbf{h} - \hat{\mathbf{h}})^H \right\} \right) \\ &= \frac{\sigma^2}{N_t\lambda + N_t} \text{Tr} \left( (\mathbf{X}^H \mathbf{X})^{-1} \right) \end{aligned} \quad (7)$$

in [13] and [25], and the minimum MSE of channel estimation can be achieved as

$$\text{minimum MSE} = \frac{\sigma^2}{E} \quad (8)$$

if and only if

$$\rho(\mathbf{x}_m, \mathbf{x}_n; \mu) = \begin{cases} 0, & \text{if } m \neq n, 0 \leq \mu \leq \lambda, \\ \text{or } m = n, 1 \leq \mu \leq \lambda; \\ E, & \text{if } m = n, \mu = 0. \end{cases} \quad (9)$$

### V. GSM TRAINING BASED ON CZCP MATE

In this section, we will propose the training framework employing the CZCPs for the GSM system. Also, we will describe the conditions of GSM training. Choosing two  $(L, Z)$ -CZCPs  $(\mathbf{p}_0, \mathbf{p}_1)$  and  $(\mathbf{q}_0, \mathbf{q}_1)$ , we generate a  $2B \times 2BL$  GSM training matrix with  $N_A = 2$  as follows:

$$\Psi = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \vdots \\ \mathbf{x}_{N_t-1} \\ \mathbf{x}_{N_t} \end{bmatrix} = \begin{bmatrix} \mathbf{p}_0 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{q}_0 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{q}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_0 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{p}_1 & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{q}_0 & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{q}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_1 \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{q}_0 & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{q}_1 \end{bmatrix} \quad (10)$$

where  $B = \lceil \frac{N_t}{2} \rceil$  and  $\mathbf{0}$  denotes a zero vector of length  $L$ . Each column of the training matrix  $\Psi$  contains  $N_A$  non-zero entries where  $N_A$  represents the number of activated TAs during each time-slot in the GSM system. Here, we consider even  $N_t$ . Note that if  $N_t$  is odd, we can select the first  $N_t$  rows of  $\Psi$  as the training sequences. For example, the GSM training matrix with  $N_t = 4$ ,  $N_A = 2$ , and  $B = 2$  is demonstrated in Fig. 3.

We will elaborate that the training matrix  $\Psi$  based on two  $(L, Z)$ -CZCPs can fulfill the condition in (9) if  $Z \geq \lambda$ . First of all, we have

$$\rho(\mathbf{x}_k, \mathbf{x}_k; \mu) = \begin{cases} \hat{\rho}(\mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{p}_1; \mu) = 0, \\ \text{for odd } k, 1 \leq |\mu| \leq Z; \\ \hat{\rho}(\mathbf{q}_0; \mu) + \hat{\rho}(\mathbf{q}_1; \mu) = 0, \\ \text{for even } k, 1 \leq |\mu| \leq Z \end{cases} \quad (11)$$

and

$$\rho(\mathbf{x}_{k+2}, \mathbf{x}_k; \mu) = \begin{cases} \hat{\rho}^*(\mathbf{p}_0; L - \mu) + \hat{\rho}^*(\mathbf{p}_1; L - \mu) = 0, \\ \text{for odd } k \leq N_t - 3, 1 \leq |\mu| \leq Z; \\ \hat{\rho}^*(\mathbf{q}_0; L - \mu) + \hat{\rho}^*(\mathbf{q}_1; L - \mu) = 0, \\ \text{for even } k \leq N_t - 2, 1 \leq |\mu| \leq Z \end{cases} \quad (12)$$

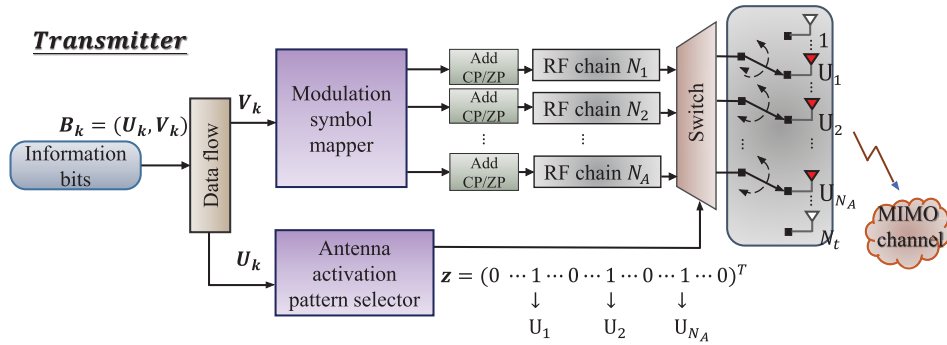


FIGURE 1. A generic transmitter structure of SC-GSM system.

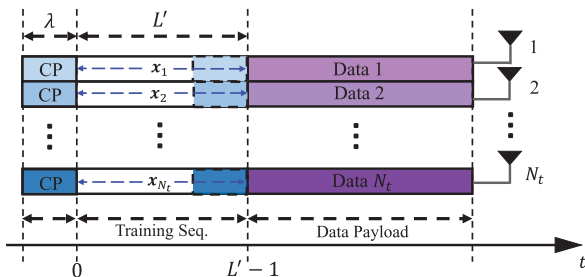


FIGURE 2. A training-based transmission structure with multiple antennas.

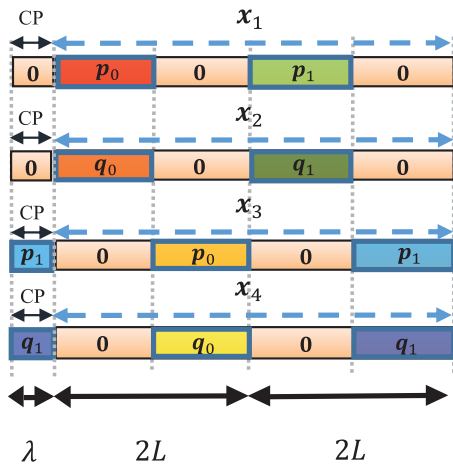


FIGURE 3. Training matrix with  $N_t = 4$  and  $N_A = 2$ .

since both  $(p_0, p_1)$  and  $(q_0, q_1)$  are  $(L, Z)$ -CZCPs. Also,

$$\rho(x_1, x_{N_t-1}; \mu) = \hat{\rho}^*(p_0, p_1; L - \mu) + \hat{\rho}^*(p_1, p_0; L - \mu) = 0 \quad (13)$$

and

$$\rho(x_2, x_{N_t}; \mu) = \hat{\rho}^*(q_0, q_1; L - \mu) + \hat{\rho}^*(q_1, q_0; L - \mu) = 0 \quad (14)$$

for  $|\mu| \leq Z$ . (11)-(14) indicate that the ISI of each TA, the IAI between the  $k$ -th and the  $(k + 2)$ -th TAs, the IAI between the 1st and the  $(N_t - 1)$ -th TAs, and the IAI between the 2nd and the  $N_t$  TAs can be eliminated, respectively, if the ZCZ width  $Z$  is larger than the delay spread. Besides, to eliminate the IAI between the  $k$ -th and the  $(k + 1)$ -th TAs and between the

$(k + 1)$ -th and the  $k$ -th TAs, we should have

$$\rho(x_k, x_{k+1}; \mu) = \hat{\rho}(p_0, q_0; \mu) + \hat{\rho}(p_1, q_1; \mu) = 0; \quad (15-1)$$

$$\rho(x_{k+1}, x_k; \mu) = \hat{\rho}(q_0, p_0; \mu) + \hat{\rho}(q_1, p_1; \mu) = 0; \quad (15-2)$$

for odd  $k \leq N_t - 1, 0 \leq |\mu| \leq Z$ . Also, we must have

$$\rho(x_{k+1}, x_k; \mu) = \hat{\rho}^*(q_0, p_0; L - \mu) + \hat{\rho}^*(q_1, p_1; L - \mu) = 0; \quad (16-1)$$

$$\rho(x_{k+2}, x_{k-1}; \mu) = \hat{\rho}^*(p_0, q_0; L - \mu) + \hat{\rho}^*(p_1, q_1; L - \mu) = 0, \quad (16-2)$$

for even  $k \leq N_t - 2, 1 \leq |\mu| \leq Z$ , to mitigate the IAI between the  $k$ -th and the  $(k + 1)$ -th TAs and between the  $(k - 1)$ -th and the  $(k + 2)$ -th TAs. Furthermore, we need to take into account the IAI between the 1st and the  $N_t$ -th TAs and that between the 2nd and the  $(N_t - 1)$ -th TAs, indicating

$$\rho(x_1, x_{N_t}; \mu) = \hat{\rho}^*(q_1, p_0; L - \mu) + \hat{\rho}^*(q_0, p_1; L - \mu) = 0 \quad (17)$$

and

$$\rho(x_2, x_{N_t-1}; \mu) = \hat{\rho}^*(p_0, q_1; L - \mu) + \hat{\rho}^*(p_1, q_0; L - \mu) = 0 \quad (18)$$

for  $|\mu| \leq Z$ . (15-1)-(18) mean that we need extra cross-correlation properties between the CZCPs  $(p_0, p_1)$  and  $(q_0, q_1)$ . Therefore, we define the CZCP mate to fit the requirements of training design for the GSM system.

**Definition 2:** For an  $(L, Z)$ -CZCP  $(p_0, p_1)$ , let us consider another  $(L, Z)$ -CZCP  $(q_0, q_1)$  satisfies the following conditions in (19-1) to (19-4). Then  $(p_0, p_1)$  and  $(q_0, q_1)$  are said to be CZCP mates<sup>1</sup> of each other.

$$\hat{\rho}(p_0, q_0; \mu) + \hat{\rho}(p_1, q_1; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_1 \cup \mathcal{D}_2 \cup \{0\}; \quad (19-1)$$

$$\hat{\rho}(q_0, p_0; \mu) + \hat{\rho}(q_1, p_1; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_1 \cup \mathcal{D}_2 \cup \{0\}; \quad (19-2)$$

$$\hat{\rho}(p_0, q_1; \mu) + \hat{\rho}(p_1, q_0; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_2; \quad (19-3)$$

$$\hat{\rho}(q_1, p_0; \mu) + \hat{\rho}(q_0, p_1; \mu) = 0, \quad \text{for } |\mu| \in \mathcal{D}_2. \quad (19-4)$$

<sup>1</sup>The well-known GCP mate [3] only satisfies (19-1) and (19-2) with  $Z = L$ , i.e.,  $\mathcal{D}_1 \cup \mathcal{D}_2 = \{0, 1, \dots, L\}$ . The CZCP mate can be regarded as an extension of the GCP mate.

*Theorem 1:* Suppose  $(\mathbf{p}_0, \mathbf{p}_1)$  is an  $(L, Z)$ -CZCP. One of its CZCP mate  $(\mathbf{q}_0, \mathbf{q}_1)$  can be obtained by  $\mathbf{q}_0 = \tilde{\mathbf{p}}_1^*$  and  $\mathbf{q}_1 = -\tilde{\mathbf{p}}_0^*$  where  $\tilde{\mathbf{p}}_0$  and  $\tilde{\mathbf{p}}_1$  denote the reverses of  $\mathbf{p}_0$  and  $\mathbf{p}_1$ , respectively.

*Proof:* From [13, Property 2], we have  $\hat{\rho}(\mathbf{p}_0, \tilde{\mathbf{p}}_1^*; \mu) + \hat{\rho}(\mathbf{p}_1, -\tilde{\mathbf{p}}_0^*; \mu) = 0$ , for  $|\mu| \in \mathcal{D}_1 \cup \mathcal{D}_2$  and  $\hat{\rho}(\mathbf{p}_1, \tilde{\mathbf{p}}_1^*; \mu) + \hat{\rho}(\mathbf{p}_0, -\tilde{\mathbf{p}}_0^*; \mu) = 0$ , for  $|\mu| \in \mathcal{D}_2$  inferring that (19-1) and (19-3) hold. Next, let us consider (19-2). For  $|\mu| \in \mathcal{D}_1 \cup \mathcal{D}_2$ , we have

$$\begin{aligned} & \hat{\rho}(\mathbf{q}_0, \mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{q}_1, \mathbf{p}_1; \mu) \\ &= \sum_{j=0}^{L-1-\mu} q_{0,j+\mu} p_{0,j}^* + \sum_{j=0}^{L-1-\mu} q_{1,j+\mu} p_{1,j}^* \\ &= \sum_{j=0}^{L-1-\mu} (p_{1,L-1-j-\mu} p_{0,j})^* - \sum_{j=0}^{L-1-\mu} (p_{0,L-1-j-\mu} p_{1,j})^* \\ &= \sum_{j=0}^{L-1-\mu} (p_{1,L-1-j-\mu} p_{0,j})^* - \sum_{j'=0}^{L-1-\mu} (p_{0,j'} p_{1,L-1-j'-\mu})^* \\ &= 0 \end{aligned}$$

inferring that (19-2) holds. Then, for  $|\mu| \in \mathcal{D}_2$ , we also have

$$\begin{aligned} & \hat{\rho}(\mathbf{q}_1, \mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{q}_0, \mathbf{p}_1; \mu) \\ &= \sum_{j=0}^{L-1-\mu} q_{1,j+\mu} p_{0,j}^* + \sum_{j=0}^{L-1-\mu} q_{0,j+\mu} p_{1,j}^* \\ &= \sum_{j=0}^{L-1-\mu} (-p_{0,L-1-j-\mu}^*) p_{0,j}^* + \sum_{j=0}^{L-1-\mu} p_{1,L-1-j-\mu}^* p_{1,j}^* \\ &= \sum_{j=0}^{L-1-\mu} p_{0,j}^* \left( \underbrace{p_{1,L-1-j-\mu}^* - p_{0,L-1-j-\mu}^*}_{=0} \right) = 0 \end{aligned}$$

inferring that (19-4) holds where the last equality comes from Lemma 2. Therefore, we obtain that  $(\mathbf{p}_0, \mathbf{p}_1)$  and  $(\mathbf{q}_0, \mathbf{q}_1)$  are CZCP mate of each other.  $\square$

Next, we propose a new construction of CZCPs.

*Theorem 2:* Let  $(\mathbf{z}_0, \mathbf{z}_1)$  be an  $(L, Z)$ -CZCP and  $(\mathbf{w}_0, \mathbf{w}_1)$  be an CZCP mate of  $(\mathbf{z}_0, \mathbf{z}_1)$ . We let

$$(\mathbf{p}_0, \mathbf{p}_1) = (\mathbf{z}_0 \star \mathbf{w}_0, \mathbf{z}_1 \star \mathbf{w}_1)$$

where  $\star$  represents the bit-interleaved operation. Then, the pair  $(\mathbf{p}_0, \mathbf{p}_1)$  is a  $(2L, 2Z)$ -CZCP.

*Proof:* Let us consider two cases to prove that the conditions in (3-1) and (3-2), respectively, are satisfied. Let  $\mathcal{D}_1^{\text{even}} = \{2, 4, \dots, 2Z\}$ ,  $\mathcal{D}_2^{\text{even}} = \{2L - 2Z, 2L - 2Z + 2, \dots, 2L - 2\}$ ,  $\mathcal{D}_1^{\text{odd}} = \{1, 3, \dots, 2Z - 1\}$ , and  $\mathcal{D}_2^{\text{odd}} = \{2L - 2Z + 1, 2L - 2Z + 3, \dots, 2L - 1\}$ .

*Case 1:* For  $|\mu| \in \mathcal{D}_1^{\text{even}} \cup \mathcal{D}_2^{\text{even}}$ , we have

$$\begin{aligned} & \hat{\rho}(\mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{p}_1; \mu) \\ &= \hat{\rho}\left(\mathbf{z}_0; \frac{\mu}{2}\right) + \hat{\rho}\left(\mathbf{w}_0; \frac{\mu}{2}\right) + \hat{\rho}\left(\mathbf{z}_1; \frac{\mu}{2}\right) + \hat{\rho}\left(\mathbf{w}_1; \frac{\mu}{2}\right) = 0 \end{aligned}$$

since the pairs  $(\mathbf{z}_0, \mathbf{z}_1)$  and  $(\mathbf{w}_0, \mathbf{w}_1)$  are  $(L, Z)$ -CZCPs and follow the condition in (3-1). For  $|\mu| \in \mathcal{D}_1^{\text{odd}} \cup \mathcal{D}_2^{\text{odd}}$ , we have

$$\begin{aligned} & \hat{\rho}(\mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{p}_1; \mu) \\ &= \hat{\rho}\left(\mathbf{w}_0, \mathbf{z}_0; \frac{\mu-1}{2}\right) + \hat{\rho}\left(\mathbf{z}_0, \mathbf{w}_0; \frac{\mu+1}{2}\right) \\ & \quad + \hat{\rho}\left(\mathbf{w}_1, \mathbf{z}_1; \frac{\mu-1}{2}\right) + \hat{\rho}\left(\mathbf{z}_1, \mathbf{w}_1; \frac{\mu+1}{2}\right) = 0 \end{aligned}$$

according to (19-1) and (19-2) since the pairs  $(\mathbf{z}_0, \mathbf{z}_1)$  and  $(\mathbf{w}_0, \mathbf{w}_1)$  are CZCP mates of each other. Therefore, we have  $\hat{\rho}(\mathbf{p}_0; \mu) + \hat{\rho}(\mathbf{p}_1; \mu) = 0$ , for  $|\mu| \in (\mathcal{D}_1^{\text{odd}} \cup \mathcal{D}_1^{\text{even}}) \cup (\mathcal{D}_2^{\text{odd}} \cup \mathcal{D}_2^{\text{even}})$ .

*Case 2:* For  $|\mu| \in \mathcal{D}_2^{\text{even}}$ , we have

$$\begin{aligned} & \hat{\rho}(\mathbf{p}_0, \mathbf{p}_1; \mu) + \hat{\rho}(\mathbf{p}_1, \mathbf{p}_0; \mu) \\ &= \hat{\rho}\left(\mathbf{z}_0, \mathbf{z}_1; \frac{\mu}{2}\right) + \hat{\rho}\left(\mathbf{w}_0, \mathbf{w}_1; \frac{\mu}{2}\right) + \hat{\rho}\left(\mathbf{z}_1, \mathbf{z}_0; \frac{\mu}{2}\right) \\ & \quad + \hat{\rho}\left(\mathbf{w}_1, \mathbf{w}_0; \frac{\mu}{2}\right) = 0 \end{aligned}$$

due to (3-2). For  $|\mu| \in \mathcal{D}_2^{\text{odd}}$ , we similarly have  $\hat{\rho}(\mathbf{p}_0, \mathbf{p}_1; \mu) + \hat{\rho}(\mathbf{p}_1, \mathbf{p}_0; \mu) = 0$  from (19-3) and (19-4). Therefore, we have

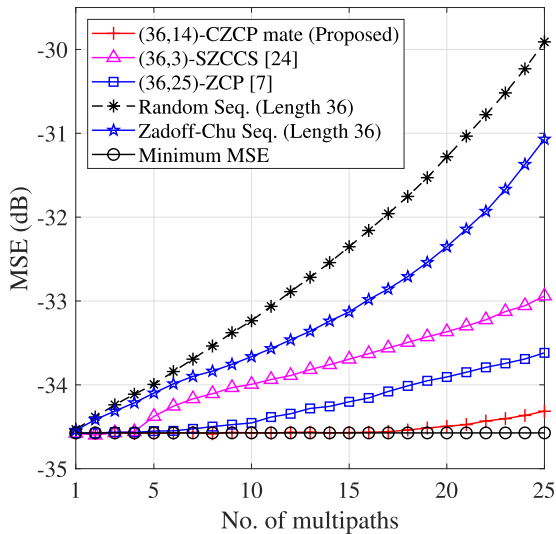
$$\hat{\rho}(\mathbf{p}_0, \mathbf{p}_1; \mu) + \hat{\rho}(\mathbf{p}_1, \mathbf{p}_0; \mu) = 0, \text{ for } |\mu| \in (\mathcal{D}_2^{\text{odd}} \cup \mathcal{D}_2^{\text{even}}).$$

From the above two cases, we obtain that  $(\mathbf{p}_0, \mathbf{p}_1)$  is a  $(2L, 2Z)$ -CZCP.  $\square$

*Example 1:* Given a  $(18, 7)$ -CZCP  $(\mathbf{z}_0, \mathbf{z}_1) = (+ + - + + + - - - - + + - + - +, + + - + + + + - - + + - - + - + -)$  with its CZCP mate  $(\mathbf{w}_0, \mathbf{w}_1) = (\tilde{\mathbf{z}}_1^*, -\tilde{\mathbf{z}}_0^*) = (- + - + - - + + - - + + + + - + +, - + - + - - + + + + - - - - + - -)$ , we can construct a  $(36, 14)$ -CZCP  $(\mathbf{p}_0, \mathbf{p}_1) = (\mathbf{z}_0 \star \mathbf{w}_0, \mathbf{z}_1 \star \mathbf{w}_1) = (+ - + + - - + + + - + + + - + - - - - + - + + + + - + - + + + - - - - + - - + + - - -)$  according to Theorem 2. Also,  $(\mathbf{p}_0, \mathbf{p}_1)$  and  $(\mathbf{q}_0, \mathbf{q}_1) = (\tilde{\mathbf{p}}_1^*, -\tilde{\mathbf{p}}_0^*)$  are CZCP mates of each other from Theorem 1.

## VI. SIMULATION RESULTS

The simulation setup for the GSM system includes  $N_t = 4$  TAs,  $N_r = 1$  RA, and  $N_A = 2$  RF chains. The  $(\lambda + 1)$ -paths channel is modeled as  $h[t] = \sum_{k=0}^{\lambda} h_k \delta[t - kT]$  where  $h_k$ 's are complex Gaussian random variables with zero mean and  $E(|h_k|^2) = 1/(\lambda + 1)$  for all  $k$ . We compare the performance of our proposed training matrix based on the CZCP mate with that of the SZCCS from [24], ZCPs, Zadoff-Chu sequences, and binary random sequences. Also, we define the training efficiency to show the effectiveness of the training framework. The training efficiency is modeled as  $T/T_{\text{total}}$  where  $T$  denotes the length of the interval on which the training sequences are transmitted and  $T_{\text{total}}$  denotes the length of the total training interval. If the training efficiency equals 1, it means the training sequences are transmitted on every time slot during the training interval. Compared with [24], our training efficiency is  $2BL/2BL = 1$  and that in [24] is  $2BL/(2BL + 2\lambda) < 1$  where  $\lambda$  is the delay spread.



**FIGURE 4.** Comparison of MSE for the GSM training using different sequences with 4 TAs.

Here, we have  $B = \lceil \frac{N_t}{2} \rceil = 2$  for  $N_t = 4$  and let  $L = 36$  as an example. Then, the training efficiency in [24] is only 0.74 when the delay spread  $\lambda$  is 25.

We utilize the binary (36, 14)-CZCP and its mate as given in Example 1 to generate the training matrix  $\Psi$  as shown in Fig. 3. For the training matrix based on ZCPs,  $(p_0, p_1)$  and  $(q_0, q_1)$  are both (36, 25)-ZCPs of length 36 and with ZCZ width 25 generated by the Kronecker product [7]. Then, the elements of  $p_0, p_1, q_0,$  and  $q_1$  are randomly generated from “-1” or “+1” for binary random sequences with length 36. For the training matrix based on Zadoff-Chu sequences, the sequences  $p_0, p_1, q_0,$  and  $q_1$  with low cross-correlations are obtained from Zadoff-Chu sequences of length 37 in which the last symbol is punctured. For the SZCCS, the training matrix  $\Psi'$  can be obtained by replacing  $p_0, p_1, q_0,$  and  $q_1$  by  $v_0^0, v_1^0, v_0^1,$  and  $v_1^1$ , respectively. Note that  $\{v_0^0, v_1^0\}$  and  $\{v_0^1, v_1^1\}$  are the two constituent sets of the (36, 3)-SZCCS with ZCZ width of 3 from [24, Th. 3]. In Fig. 4,  $E_b/N_0$  is fixed of 16 dB and the numbers of multipaths are given from 1 to 25. Our proposed CZCP-mate-based training matrix can achieve the minimum MSE when the number of multipaths is less than or equal to 15. In contrast, the MSE performance of the SZCCS-based training deteriorates as the number of multipaths increases, especially when it exceeds 4 since the ZCZ width of the SZCCS is only 3. For the ZCP-based training, the performance is worse since the IAI is not mitigated. This is because the ZCP does not consider the cross-correlation property although the ZCZ width of the ZCP is 25. Therefore, our proposed CZCP-mate-based training with specific cross-correlation properties for the GSM system effectively eliminates both ISI and IAI.

### VII. CONCLUSION

In this paper, we have studied CZCP mates for training sequence design in GSM. A construction of CZCP mates has been provided in Theorem 1. In addition, Theorem 2

provides a new construction of CZCPs based on CZCP mates. Moreover, we have presented a new training framework for the GSM system based on the proposed CZCP and its mate. It has been shown that the CZCP-mate-based training framework can achieve the minimum MSE over frequency-selective channels when the delay spread is not larger than the zero correlation zone (ZCZ) width. Our proposed CZCP-mate-based-training framework can activate two antennas at each time-slot; therefore, a possible future work is to extend the CZCP and its mate for the training in GSM system with more activated TAs.

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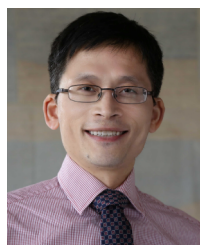


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