Explainable Strategic Optimisation of Grand Scale Problems

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Publications and Awards Arising from this Work

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Conference Papers

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Abstract

Explainable Strategic Optimisation of grand scale problems aims to identify solutions that provide long term planning advantages to problems that cannot undergo traditional optimisation techniques due to their level of complexity. Usually, optimisation tasks focus on improving a limited number of objectives in the pursuit of obviously immediate target. However, this methodology, when applied to grand scale problems is found to be insufficient; a major reason for this is the inherent complexities typical of problems such as utility optimisation and massive logistical operations. One approach to these problems is Generational Expansion Planning that typically addresses long-term planning of country/county-wide utility problems.

This thesis draws influence from the Generational Expansion Planning field; a significant field in relation to this work as it typically focuses on large scale optimisation problems. Problems such as the improvement and maintenance of national utility operations. However, this thesis takes a novel approach that places empathises on an abstract strategic planning method that concerns itself with the extraction of design insights that can guide an experts understanding of an unrelentingly complex problem.

The proposed system was developed with data from British Telecom (BT) and was developed within their organisation in which its deployment is being planned. The techniques behind the proposed systems presented in this thesis are shown to improve the popular many-objective Non-Dominated Sorting Genetic Algorithm II in a series of experiments in which the improved Type-2 dominance method outperformed the traditional dominance method by 59%.

Several component parts are brought together within this thesis so that the unique optimisation of varied regions that exist inside the United Kingdom’s Access Network can be explored. The

proposed system places great import on the interpretability of the system and the solutions that it produces. As such, an Explainable Artificial Intelligent (XAI) system has been implemented in the hope that with greater interpretability, AI systems will be able to provide solutions with greater context, nuance, and confidence, particularly when the decision of an AI model has a direct impact on a person or business. This thesis will explore the related material and will explore the proposed framework; which brings together a multitude of technologies, such as, novel fuzzy many-objective optimisation, fuzzy explainable artificial intelligence, and strategic analysis. These technologies have been approached and combined in order to develop a novel system capable of dealing with complex grand scale problems, which traditionally are tackled as piecemeal optimisation problems.

The proposed systems were shown to improve the optimisation of focused scenarios; in these experiments the proposed system was able to provide solutions for the optimisation of telecommunication networks that outperformed the current methodology for the planning/upgrading of the access network. The proposed systems were tested on rural, mixed, and urban regions of a simulated United Kingdom; it was observed that when the proposed systems were used the network solutions produced were 51.99% cheaper for rural regions, in which a combination of technologies were used as opposed to only FTTP. It was also observed that solutions produced by the proposed system in mixed regions were 54.16% cheaper while still providing the customer broadband requirements.

These results identify how an expansive system such as the novel system proposed in this thesis is able to provide sound business solutions to complex real-world problems that consists of an ever growing number of variables, constraints, and objectives. Additionally, the proposed systems are capable of producing greater understanding of design principles/choices in network solutions, which in turn provides BT and users with a greater level of trust in the solutions and the system. This is a major obstacle that must be overcome when the problem domain that is being considered is incredible vast, uncertain, and extremely vital to the success of a company.

The results of this thesis identify how the proposed systems can be developed and implemented to provide an insight into the planning and execution of an access network not required for decades to come. This is a significant change from the current reactive approach to a proactive approach that provides insight into the ever changing variables and needs of the network. The proposed systems are able to instil the confidence that allow a more thoughtful approach to be taken that is beneficial to both company and customer.

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List of Acronyms

ACO: Ant Colony Optimisation

ADSL: Asymmetric Digital Subscriber Line

AI: Artificial Intelligence

BT: British Telecom

FDR: Fuzzy Dominance Rules

FLC: Fuzzy Logic Controller

FOU: Footprint of Uncertainty

FTTC: Fibre-to-the-cabinet

FTTP: Fibre-to-the-premise

GA: Genetic Algorithm

HD: High-Definition

HVI: Hypervolume Indicator

MOO: Many Objective Optimisation

PCP: Personal Connection Points

SA: Simulated Annealing

SI: Swarm Intelligence

TSP: Travelling Salesman Problem

UK: United Kingdom

Chapter 1. Introduction

1.1 An introduction to strategic access network planning

Ensuring that all have access to sufficient and reliable bandwidth for computer/telecommunication processes has become an increasingly important task, as a greater and greater importance is placed upon this technology's use within everyday life. Between 2015 and 2020, the bandwidth consumption of Europe has risen from 49Tbit/s to 152Tbit/s [1]. This comes as natural growth in response to increased bandwidth from new technologies/services (HD streaming, VR systems, etc.), but has also been exacerbated by the Covid-19 lockdowns.

The access network, maintained by British Telecom (BT), is responsible for guaranteeing that all UK consumers can use basic services, run businesses, and benefit from the telecommunication network. The access network makes use of many delivery technologies, such as copper cables, fibre-optic cables, and wireless/cellular networks in order to supply bandwidth. However, as bandwidth consumption continues to grow, the pre-existing technology (which makes up the bulk of the access network architecture) starts to become obsolete, as it can no longer supply the required bandwidth. In response, a great reactive campaign must be undertaken to replace this technology and ensure that services are unaffected. This process is doomed to be repeated while bandwidth consumption continues to grow, and delivery technologies are limited in what they can supply.

The current approach to the problem is too broad and more akin to maintenance than to a solution. The work of this thesis attempts to identify a proactive method of improving, predicting, and understanding the access network through the modelling of the various geographical areas, demographics, and other network inputs like business competition and technology longevity.

Optimisation strategies are widely used to find solutions to network design problems. Plethora of techniques have evolved alongside telecommunication technology; Markov Decision Tree-based optimisation [2], Self-Organising Networks (SON) [3], and Fuzzy Reinforcement Learning [4] are examples of techniques used in the optimisation of the dynamic cellular Long Term Evolution (LTE) network. However, many of these strategies seek solutions for isolated problems, which alone do not take into consideration the numerous objectives, constraints, and variations that make up the entire access network and the uncertainty that surrounds real-world business problems.

Many optimisation strategies also look to identify solutions to current network problems and do not consider future potentialities, such as much more demanding bandwidth services/technologies that would drastically affect a network's topology. Many-objective strategic planning optimisations can be implemented in an attempt to solve these problems. Adjustments can be made to allow any objective, constraint, development, and potential uncertainties to play a role in the network solutions that are identified. This can be taken further with the inclusion of XAI and interpretable solutions which allow solutions to provide greater context and understanding to problems.

1.2 Research questions and contributions to science

It is the aim of this thesis to seek a solution and implement a system for the optimisation of the BT access network. It is therefore important to recognise the type of solutions that would be beneficial to telecommunications providers in their pursuit to achieve a series of long-term business objectives. There are serval objectives that must pursued - these include; improving customer satisfaction by providing a reliable connection that fulfils their bandwidth consumption demands, reducing spending costs accumulated through the installation and maintenance of network equipment, and finally predicting future network requirements and providing insight to experts as to why particular network configurations are successful in achieving their objectives. This final objective is primarily interested with providing end users with greater interpretability of the solutions produced by the novel framework discussed in this thesis. These objectives can be more comprehensively represented as:

I. Identifying and simulating differences between demographics in terms of their bandwidth requirements. Thus improving customer satisfaction and reducing costs through targeted developments.
II. Understanding and representing the business interests of BT, such as competition and institutional responsibilities.
III. Delivering a Pareto front of solutions, each representing a network that has excelled at a particular objective.
IV. Gleaming design insights from network solutions that improve understanding and ensure confidence in the system.

The coordination of many systems is required in order to achieve the desired objectives. The systems that are required are:

- An elitist many-objective optimisation approach to the network optimisation problem.
- A novel simulation approach for the creation of varying population demographics.
- A novel fuzzy logic based approach to the improvement of the many-objective optimisation algorithms.
- A novel approach to the extraction of explainable data from the solutions of the Pareto front, with emphasis placed on the interpretability of the solutions produced.

- The development of a novel unified system that ties together all the required features, as determined from the objectives, both business and academic.

BT is currently progressing with the nationwide rollout of full-fibre broadband, which was pushed as a key governmental pledge during the last UK general election. However, this initial ambitious task has struggled to receive the large sums of funding required and has now been modified, with the government revising its commitment to using full fibre [5]. The objectives aforementioned identify why an indiscriminate role out of full fibre broadband is not only pointlessly expensive, but also does little to elevate the issues faced by the access network. That is why the work of this thesis aims to provide a targeted method to plan, upgrade, understand, and safeguard the future reliance of the access network.

1.3 Thesis Layout

This thesis will be structured as follows; Chapter 2 gives an overview of the problems and the current approaches to these problems. Several features will be explored such as the topology of the BT access network, the breakdown of the proposed problems and the novel system to solve them. This chapter will identify in detail the varying aspects that BT consider to play a vital role in the optimisation and prediction of future access networks, such as; population density, bandwidth demands, geographical regions, competition, technology redundancy, and how these can be combined to form a strategic long-term plan.

Chapter 3 will provide an overview of optimisation algorithms. Descriptions will be provided that identify common optimisation algorithms, such as Genetic Algorithms (GAs), Simulated Annealing, and Ant Colony Optimisation (ACO), and how they are most optimally deployed. From more common single objective optimisation algorithms, this chapter will then give a review of many-objective optimisation, with specific attention given to the Non-Dominated Sorting Genetic Algorithm (NSGA-II).

Chapter 4 will give an overview of fuzzy logic systems, describing the unique process and uses of fuzzy logic; focusing specifically on its advantages when dealing with real-world business problems and data. Type-1, interval, and general Type-2 fuzzy logic will be explored and their implementations detailed.

Chapter 5 addresses the growing concern towards black box systems and the inability for both designer and user to understand why particular solutions/conclusions have been made. This chapter will explore the growing field of explainable artificial intelligence (XAI), reviewing the current implementation methods and showing how it can play a vital role in creating a strategic plan that provides understanding and meaning for the solutions created by black box systems.

Chapter 6 presents the proposed modified Type-2 fuzzy NSGA-II algorithm; the implementation of this system is explained, and a comparison is made between other optimisation algorithms and their results based on several common open source optimisation problems. This chapter also provides insight into how the modified Type-2 fuzzy NSGA-II system provides benefits over the proposed plan of installing full-fibre indiscriminately across the UK. This chapter identifies how several of the proposed objectives can be improved with this system. This chapter also discusses why a more nuanced approach is beneficial to the problems proposed.

Chapter 7 provides an overview of the proposed explainable fuzzy system. This chapter explores how it is implemented and how fuzzy rule extraction can be used in order to provide insight for the expert user of a system. This chapter also explores the proposed explainable Type-2 for topographical optimisation problems and the interpretable GUI that accompanies it. The chapter identifies how a combination of fuzzy modified systems can extract information from Pareto front solutions in order to provide intuitive understanding of complex many-objective optimisation problems. This chapter also explores how this system can be extracted using modified fuzzy optimisation techniques. The proposed system is compared to XAI systems currently implemented and explores theory surrounding knowledge extraction.

Finally, chapter 8 presents the conclusion of this thesis, the real world impact and the potential future work that can be undertaken.

Chapter 2. An Overview of Strategic Planning and Access Network Optimisation

2.1 An Overview of Current Grand Scale Problem Optimisation

The optimisation of numerous grand scale problems has been explored in literature for decades. However, a large sum of this research is focused on the optimisation of specific, highly detailed subsections of an overall problem. Some methods, such as that in [6], share similarities with the work of this thesis; the research in [6] is focused on the optimisation of Swedish electricity and heating systems but can be applied to any energy system that can be modelled in a linear method. However, despite considering a number of constraints this method differs, as it is not considering long term planning and it is attempting to optimise a single objective; the minimisation of capital costs.

The work detailed in [7] addresses many of the questions considered by this thesis; the work proposes a multi-criteria optimisation model for the planning of Uganda's national power systems. Significantly, this work is interested in the effects of long term planning, considering periods of fifteen years or longer, which is one of the major focuses of this thesis. Too often, grand scale problems are tackled in a fragmentary manner. Importantly, the work in [7] considers many of the potentialities that could affect the design of the power networks and places significance on constraints such as costs, demand, emissions, and distribution. It is also important to note that the proposed method in [7] allowed the experts to gleam insight about power generation methods from the designs in the Pareto front. However, these insights were extracted by users from the end data.

Another approach to be considered is the work of [8], which focuses again on generational expansion planning, this time investigating the optimisation of power plants over a 14 and 24-year period. It is important to note that the questions proposed in this thesis are not restricted solely to utility problems; the same characteristics present in utility-based problems can be found in any sufficiently large logistical problem such as those found in [9], [10], [11], and [12].

Another considerable field of research, interested in the optimisation of medical and disease treatment problems, need be discussed. There exists many approaches to the optimisation of treatment targeting varying forms of cancers such as those in [13], [14] and [15]. All of the research in this field has great importance to vast numbers of the population; the problems are also fraught with complexities that must be overcome. Expanding the score of this project into the medical field opens up the possibility of explainable tools, such as those in [16], that are able to remove the barriers related to using AI in a typically meticulous field, to expand medical knowledge.

Typically, the optimisation of any problem focuses on the adjustment of variables and parameters in the pursuit of a solution that outperforms existing implementations. However, as the problems being optimised become more and more complex, not only does the accuracy of an optimised solution decline, also the interpretability of solutions and the design choices behind them declines. This aspect of optimisation, notably within grand scale projects, is sorely missing. A system capable of providing understanding and reassurance concerning the planning, maintaining, and evolution of enormous capital related problems would be a great boon for any entity seeking to advance their current position.

As mentioned, traditionally optimisations problems care little for the interpretability of a solution, however it is vitally important to this thesis and the optimisation field that a greater import is placed on this feature. Gaining a greater understanding of the numerous variables, constraints, and objectives that may exist in a given problem will most certainly improve the results of given solutions which is why interpretability is so important for optimisation problems.

2.2 Uncertainty in Real-World Problems

Uncertainty can be found in any real-world problem and even within most fields of research. This uncertainty can take many forms when a real-world problem is being considered; for example, the data may be incomplete or utterly unknown, it may also suffer from inaccuracy of measurements/recordings. Left unattended the uncertainty found in data will cause inaccuracies in the solutions produced by optimising systems. Therefore, in order to deal with the uncertainty the data must be represented by probability distributions or through statistical information [17]. The uncertainty of this data can also be handled using fuzzy set theory that will be explored in detail in section 4.

Another example of uncertainty in real-world problems is found in model uncertainty, especially when grand scale projects are considered. When the plethora of inputs for any such project are examined, it is evident that any mathematical modelling used is mere approximation of the true system inputs. These model uncertainties can be divided between "*lack of knowledge"* uncertainties, which as the name suggests consists of some process or feature that is not understood or can be measured. The second type of model uncertainty is "*disregard of knowledge"* uncertainty, which encompasses processes understood but deemed too difficult or inconsequential to model [18]. Both of these forms of uncertainty are present in the real-world problems this thesis is interested in exploring. There are circumstances where exact values cannot be determined, and so the prediction of future broadband delivery/consumption technologies, is an impossibility to anything more than an approximation. Some of these uncertainties can be dealt with through probabilistic or deterministic frameworks as explored in [19] and [20].

It is important to recognise these varying forms of uncertainty in models and data. Where possible, the uncertainty should be remedied; where it is not possible, designs should recognise the flaws and ensure they are modelled around them. There are others forms of uncertainty that are centred on the human aspects of design; the proposed system is deeply interested in the abstract analysis of telecommunication networks which can be rapidly adjusted and modelled around a set of potentialities conceived by the system user. The system should produce a set of solutions along a Pareto front; this Pareto front, however, does not contain a solution of the perfect network. Instead, the solutions of this front span the optimality of a user's interests. This front returns an ambiguous set of solutions that each present some aspect of interest to the experts whom are seeking to gain a greater understanding of the hugely uncertain environment in which the problem domain resides.

2.3 Infrastructure of the Access Network and its Objectives

Strategic planning within the telecommunication domain has become necessary to ensure that networks supplying broadband are capable of being designed and updated proactively in order to meet the requirements of societies that are increasingly reliant on it. However, these networks are mired in many, often contradictory, objectives. Is it possible to improve customer satisfaction while reducing costs and ensuring redundancy? In order to answer these questions we must first examine the network responsible for delivering broadband to customers.

The access network supplies over 27.7 million homes and businesses with broadband across the UK [21]. The access network starts at the exchange; perhaps the most important part of the core network, which for the most part is outside the scope of the proposed problem. There are 5.5k exchanges distributed across the UK that service every major town and surrounding area. Each exchange provides an interface for the telecommunication infrastructure that delivers broadband to customers; it is this infrastructure and the technology upon it that is most significant to this problem. There are several methods through which broadband is delivered. However, the most common are now fibre-to-the-cabinet (FTTC) and fibre-to-the-premise (FTTP) which have mostly replaced the original copper network that can no longer supply the required bandwidth. FTTC and FTTP make use of over 150k cabinets and nearly 5m distribution points [21].

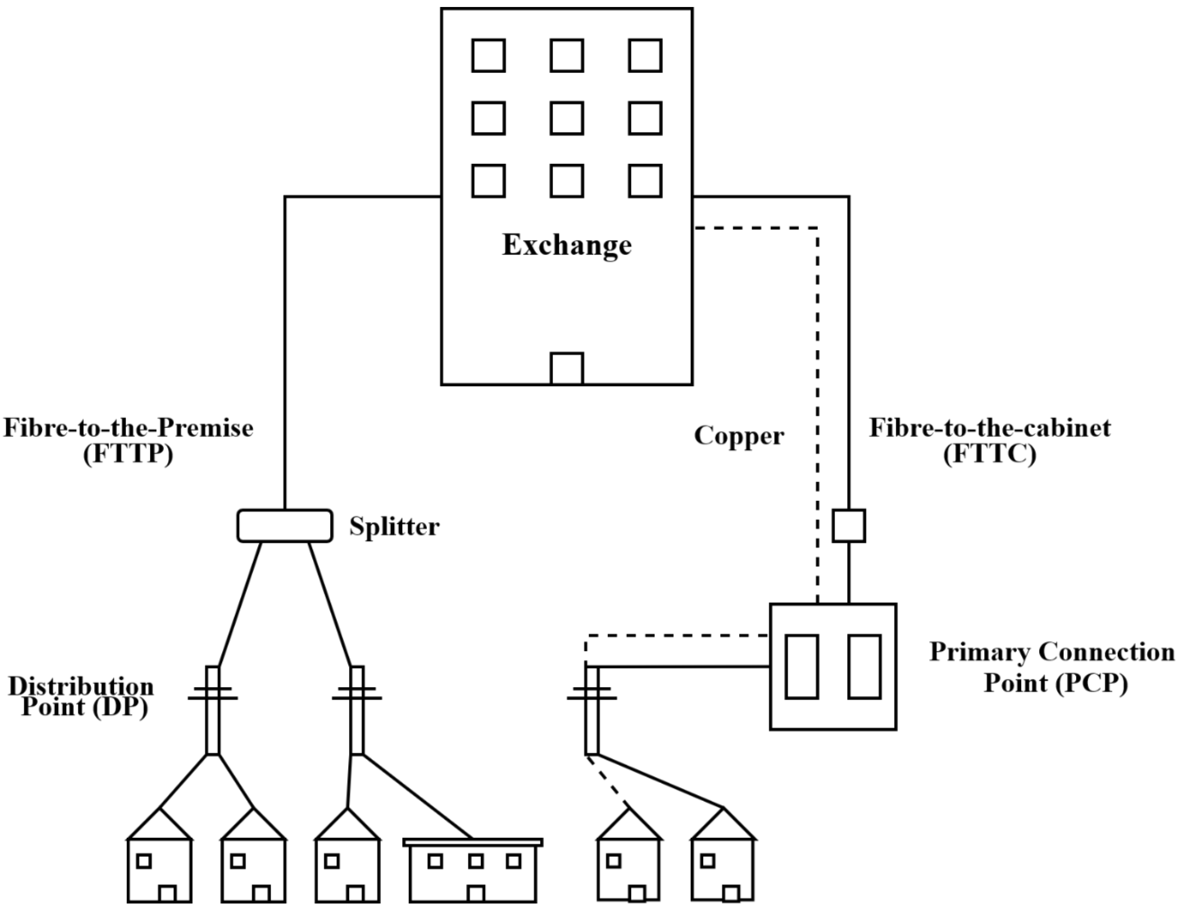


Fig. 2.1 Openreach Local Access Network

Much of the access network is immutable when considered in terms of an optimisation problem; the exchanges, for example, cannot be moved or changed. Therefore, in consideration of the proposed objectives, the delivery technologies are optimised. The proposed system allows broadband delivery technologies to be quickly implemented and included in the designed networks; this ensures the system can simulate the current infrastructure of the access network as well as any potential disruptive technologies developed henceforth. With the broadband technologies modelled, the optimisation between distribution points and customer premises can then be considered. The system is most interested in reducing the costs of the networks while ensuring that as many premises as possible are connected to broadband, and that the undelivered bandwidth is as small as possible.

2.3.1 Delivery Technology

There are a host of considerations to be made when it comes to determining the correct broadband delivery method. Typically, the most important aspect is broadband speed, which can be varying depending on the technology used, the distance from the origin point (exchange building), and the geographical topography. However, it is not always the case that a premise/customer requires the fastest broadband speeds; analysis of demographics identifies variations in their requirements. Some customers will require a much larger share of bandwidth; for example, a young family of four with multiple devices capable of streaming Ultra HD video simultaneously will have a requirement of 30 Mbps minimum (and likely much higher), whereas a retired couple that only use broadband to read the news will use approximately 1 Mbps. These large variations in customer requirements is a significant reason the proposed system is so vital in the upgrading and understanding of the access network.

The choice of technology is also influenced by a geographical region. In sparsely populated rural regions, particularly those that have low bandwidth requirements, it seemingly does not make practical or financial sense to ensure every premise has a direct fibre connection to the exchange. In circumstances like this, wireless technologies may play a more vital role.

2.3.1.1 Copper

Copper is used to deliver two forms of broadband; these are Asymmetric Digital Subscriber Line (ADSL) and ADSL2+. Between the two, ADSL2+ is the most prominent, having replaced the existing network in order to obtain improved broadband speeds of up to 24 Mbps. There has been a concentrated effort to phase out the existing copper network with fibre technologies as copper is now seen to be redundant. However, arguments can be made that in some circumstances it is a better financial and business decision to continue to use this method of delivery to certain premises.

As with most delivery technologies, the copper network originates from the exchange buildings and makes it way to distribution points before reaching the customer. Physical limitations of copper cables are the main reason for its undoing as a delivery technology; broadband speeds are severely affected by the size, both length and thickness, of the cable being used. The further a customer is from the exchange, the lower their bandwidth speeds will be. This can be overcome to some extent by increasing the thickness of the cable, however, this improvement is itself limited by cost and design.

2.3.1.2 Fibre-to-the-Cabinet

Fibre-to-the-cabinet (FTTC), also called Superfast Broadband, is the most prominent broadband delivery method and the first method discussed here that implements fibre optic broadband. This technology utilises fibre optic cables that emanate from the exchange and travel to personal connection points (PCP); from here, copper cables connect to premises. FTTC is a massive improvement over purely copper in terms of maximum broadband speeds, potentially reaching 80 Mbps.

FTTC removes one of the major drawbacks of the copper network, which is the varying distances between premises and the exchange. The fibre optic cables that start at the exchange do not suffer from decreasing performance in relation to their length, therefore eliminating the inherent weakness of the copper network. However, broadband speeds can still be negatively affected by the distance between the PCP and the premise.

FTTC can be modified through the changing of frequency connections; this method is termed GFast and it can potentially deliver broadband speeds up to 330 Mbps. This technology serves approximately 2.8 million premises [21]. However, its improvement over Superfast Broadband is limited as it can only be implemented over short distances and therefore can only be provided to premises located close to the exchange.

2.3.1.3 Fibre-to-the-Premise

Fibre-to-the-Premise, or Ultrafast Full Fibre, is the technology that BT Openreach are currently committed to installing nationwide. This technology makes use of fibre optic cables from exchange to premise and can potentially provide customers with broadband speeds up to 1 Gbps.

FTTP overcomes the drawbacks of copper cables; this is because the broadband speeds provided by a full fibre system are not dependent on the length of the cable. Another benefit of the technology is that it is implemented as a passive optical network in which multiple premises can be supplied by unpowered splitters; this removes another potential weakness present in the copper and FTTC technologies as the intermediate powered cabinets (PCPs) are no longer required.

The drawback of this technology is that its implementation is relatively new and therefore is not currently available nationwide in the UK. Its implementation also requires an entire new network to be laid atop the existing copper network, which may already be sufficient in many cases. An important consideration is that the improved service is likely to cost customers more money and may therefore reduce customer satisfaction if it is not required.

2.3.1.4 Wireless

BT currently offer 4G broadband connection to customers unable to receive sufficiently fast broadband speeds in their location. This technology can supply customers with a maximum speed of 30 Mb. Wireless technology offers a unique solution for ensuring that all premises, regardless of how isolated they are, are capable of receiving a broadband connection.

Broadband speeds offered by the next generation of mobile networks (5G) identify it as a key technology when considering the planning of the access network. It may play a more significant role within the topology of the network, particularly if the bandwidth demands of rural regions outpace the existing network technology in the area.

It is also important to recognise other wireless network technology such as satellites. This technology, while in use, may not be suitable as a broadband delivery method for the UK access network; however, being able model this technology allows all design options to be considered.

2.3.2 Constraints

A constraint is defined as "something that limits or controls what you can do" [22]. If this is applied to the real-world problem being considered, we are able to identify a number of limitations that are placed upon any solution that might be constructed by an optimiser. For example, solutions may be required to remain under a set budget or to take into consideration the environmental impact the installation and construction of network equipment may cause.

It is important to recognise that a problem becomes more complex as the number of inputs and objectives increase, inevitably so too do the number of constraints. Some constraints may be modelled as part of the network technology/equipment; others may be enforced by the genetic algorithm during the operational process. There also exists methods to handle constraints such as Deb's Constrained-Domination Principle (DCDP) and the Infeasible Elitists – Stochastic Ranking Selection (IE-SRS) that implements a ranking system to ensure the preservation of non-dominated solutions to the next generation [23].

When a problem becomes complex enough it becomes prudent to implement several methods to deal with constraint breaking solutions in a population. For example, as mentioned, some constraints may be modelled out, others will be dealt with during the ranking process of a genetic algorithm; however, it is not always possible to ensure a solution does not break any constraint. However, the removal of every constraint breaking solution may hamper the search for an optimal solution; it is therefore possible to maintain solutions that break constraints and rank them depending on their degree of infeasibility.

2.3.3 Objectives

There are numerous objectives to be considered and the modular characteristic of the proposed system ensures that any number of them can be optimised at any given time. It is important that greater complexity can be introduced into the system as our understanding of the problem develops. In this way, the system is able to incorporate the fluid nature of a real-world business problem.

Three of the main objectives are:

1. **The minimisation of costs**

This first objective is perhaps the most obvious; every business is interested in reducing their expenses in order to make greater profits. The minimisation of costs can take several forms; for example, the clearest reduction in cost would be the cheapest combination of technology that does not break any of the aforementioned constraints. Costs could also be reduced by the selection of technology that has the greatest longevity and reliability and so reduces long-term upkeep costs.

2. **The maximisation of connected premises**

This objective ensures that as many premises as possible are connected to the access network; this may be constrained between a range depending on enforced requirements or competition in a local area. Allowing solutions to be connected to varying numbers of premises allows the explorative nature of the proposed system to identify unique design insights that might otherwise be missed.

3. **The minimisation of undelivered bandwidth**

This final objective is interested in producing solutions that do not undersupply the required broadband requirements of individual premises. Each premise modelled will possess a bandwidth requirement that is calculated from demographic averages; this objective differentiates solutions that connect with many premises but routinely fail to deliver or meet the broadband speed requirements due to the combination of technology chosen.

2.4 Discussion

Throughout this chapter, we have explored the infrastructure that constitutes the relevant aspects of the UK access network. The broadband delivery technologies have been detailed, with their strengths and weaknesses being discussed. The objectives and constraints of the proposed system have also been discussed and an explanation given for the approach and reasoning of them. The modularity surrounding the modelling of broadband delivery technologies was discussed in order to highlight the robust nature of the proposed system.

This chapter also discussed the current methods that are commonly implemented when the optimisation of grand scale projects is approached. Examples of Generation Expansion Planning were outlined for a number of utility problems, and the scope of the proposed system was expanded to include any sufficiently complex optimisation methods such as large scale logistic and construction problems.

A discussion was also made in this chapter, concerning the uncertainty and ambiguity that is present in all real-world problems of which this problem is no exception. This uncertainty is significant as it plays a prominent role in the explainable system presented; it affects both the data/modelling and the outputs produced.

In the following chapter an overview of optimisation algorithms will be given in which a progression from single-objective optimisation to many-objective genetic algorithm techniques will be presented. A number of popular algorithms, A* Search and Simulated Annealing, are inspected. The NSGA-II algorithm in particular will be presented, the variations that set it apart will be discussed and its role within this project are highlighted.

Chapter 3. An Overview of Optimisation Algorithms

The process of optimising a problem in order to determine the best possible solutions has a considerable history with many techniques being employed. This chapter will examine the differences between varying optimisation techniques and the division between single-objective and many-objective optimisation.

Single-objective optimisation techniques come in many forms. For example, those that use gradient-based methods of which there are many variants, such as batch, stochastic, and mini-batch [24]. There are also those that implement heuristic-based search techniques, which aims to find solutions that are deemed capable enough rather than globally optimal. The reason for this may be due to any number of problem constraints, such as time complexity, computational complexity or costs. A-star (A*) and Greedy Best-First Search are examples of two heuristic search algorithms that are commonly used in the optimisation of telecommunication networks [25] [26].

As is often true with innovation, the field of optimisation has drawn considerable influence from processes existing in nature. Evolutionary algorithms encompass many approaches to optimisation - some of the most popular methods include; Genetic Algorithms [27], Genetic Programming [28], Simulated Annealing [29], and Big Bang Big Crunch [30]. Ant Colony Optimisation [31] is a form of Swarm Intelligence (SI) [32] that also draws influence from nature and has become a general-purpose optimiser - but it is often described tackling the Traveling Salesman Problem (TSP).

3.1 Single-Objective Optimisation

3.1.1 A* Search

The A* Search algorithm was first developed in the 1960's by a group of Stanford researchers [33]. As mentioned previously, this method implements a system of heuristics to guide its search and improve its performance. A* is commonly used for pathfinding and graph traversal [34] [35]. This search algorithm is often compared to and seen as an extension to Dijkstra's algorithm. A* is able to outperform Dijkstra's due to its goal guided heuristic search.

However, the very heuristics that ensure A* outperforms Dijkstra's must be carefully determined to be *admissible* and *consistent*. If a heuristic function is chosen and found not to be admissible, the search may overestimate the cost required to reach the goal state and never find the optimal path. On the other hand, the heuristic must also be consistent. If it is not, the time/space complexity of the algorithm will rise as a result of the search exploring non-optimal paths to the goal state [36]. To ensure that the heuristic is admissible, it should be implemented as follows:

$$h(n) \leq dist(n, t) \tag{3.1}$$

This guarantees that the heuristic is never greater than the cost of moving from the current node to the goal. Therefore, the search never overestimates the required distance and always finds the optimal path. In order to establish consistency:

$$f(n) = g(n) + h(n) \tag{3.2}$$

Where $g(n)$ is the accumulated cost to reach the current node and $h(n)$ is the estimated cost from the node to the goal.

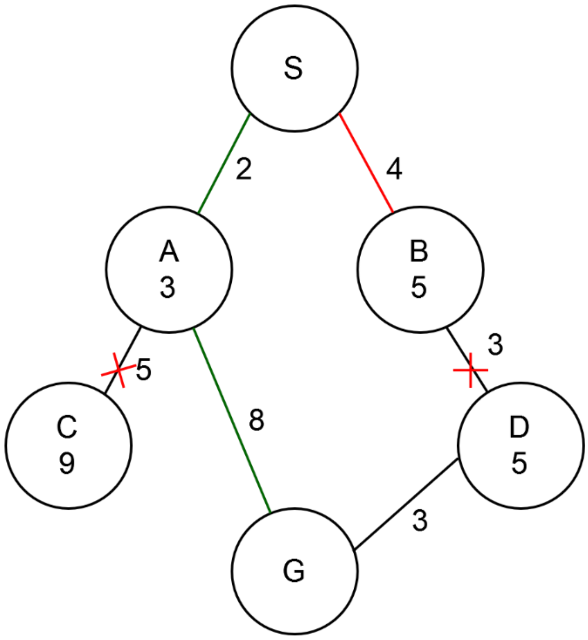


Fig. 3.1 A Search Example (S) Start Node, (G) Goal Node*

3.1.2 Simulated Annealing

Simulated annealing has a long history. First being introduced in 1983 [37], it is inspired by a technique that exists within metallurgy in which the malleability of materials is increased through the application of heat. As a heated material begins to cool, it becomes increasingly difficult to make changes to - this same idea was adapted in order to solve combinatorial problems.

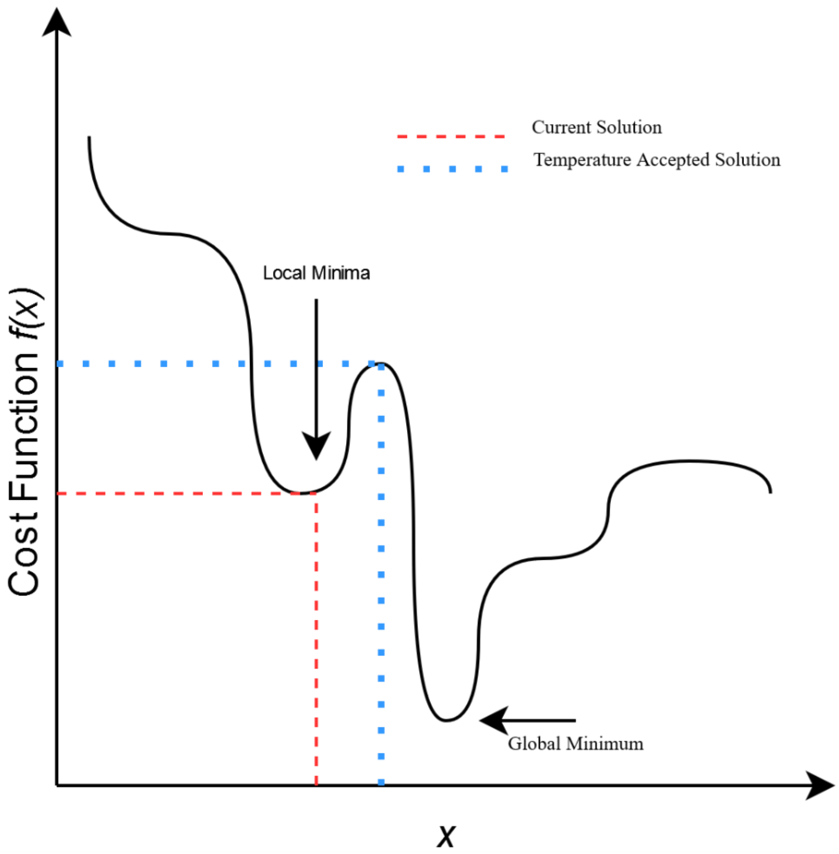


Fig. 3.2 Simulated Annealing Overcoming Local Minima Problem [38]

An appealing benefit to the simulated annealing process is its ability to overcome the local minima issue shown in Fig 3.2 above. Simulated annealing incorporates temperature as a mechanic for the exploration of solutions, and as long as temperature is nonzero, it is possible to overcome local optimum [37]. As this expression of temperature decreases, the system finds a new equilibrium at each step and becomes increasingly unlikely to accept the changes made to the solution.

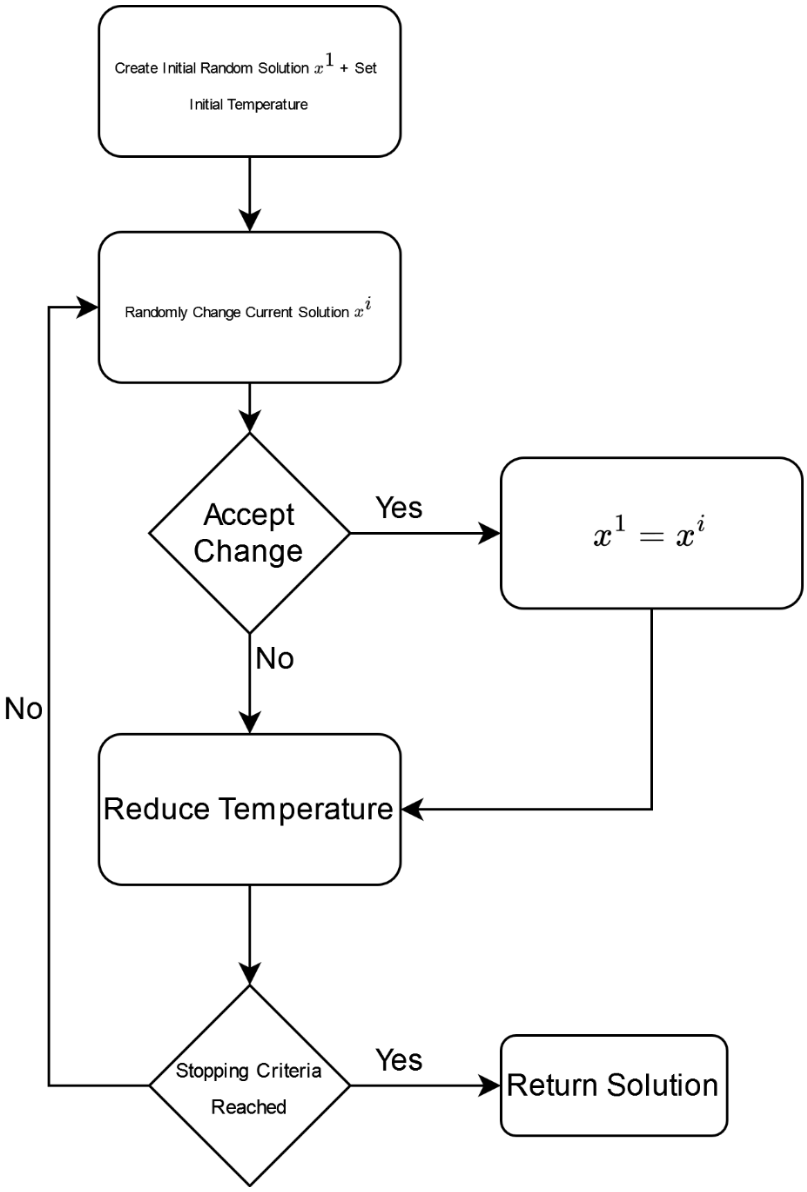


Fig. 3.3 Flowchart of Simulated Annealing

Fig 3.3 above describes the simulated annealing process. An initial candidate solution is chosen at random along with the initial temperature. The initial temperature of the system should be carefully considered; if it is too high, the algorithm will regress to random search around the solution space. However, if the value is too low it will cause the algorithm to become too greedy in its consideration of solutions, and it will more likely succumb to the local minima problem. The choice for this initial temperature therefore should be some value that allows exploration and exploitation of the solution space in the correct amount. In [29] it is suggested that the initial temperature should allow 80% of explorative moves to be accepted; whatever acceptance value is chosen. One method for calculating the initial temperature is by measuring the variance in cost for a small sample of neighbourhood moves [39] [40].

With the initial variables set, the algorithm now moves into its main loop - the first stage of which produces a new solution through random change of the current solution. The new solution is chosen over the previous one if it is classified as better, in regards to the cost function and the solutions fitness. However, it is also possible that the new solution meets the second evaluation criteria, and is selected. This criterion is:

$$e^{\left(\frac{-\delta}{t}\right)} \tag{3.3}$$

In which δ is the change in the cost function between the two solutions and t is the system temperature. If either condition is true, the new solution is chosen and the previous one discarded. If neither are true, the new solution is discarded instead.

The final step is to reduce the temperature; this is controlled by a cooling schedule. The cooling schedule is considered very significant to the performance of the algorithm and is therefore widely investigated. The work in [37] makes use of a linear cooling schedule in which the temperature is calculated through a constant reduction, whereas [29] makes use of a cooling factor constant in order to calculate the new temperature. There is a great deal more research surrounding the topic; [41] [42] again offer their own solutions to the cooling problem and great consideration is given to how the cooling schedule will affect the behaviour of the system. As the temperature of the system decreases, it becomes less explorative of the solution space and only fine adjustments will be made to the solution before it becomes set.

This process is repeated until an optimal solution is presented. Many implementations of simulated annealing will continue the main loop for a fixed number of times, corresponding to the interval drop of the temperature.

3.2 Swarm Intelligence

Swarm intelligence (SI) is another series of commonly implemented artificial intelligent models that often draws inspiration from nature. Swarm intelligence attempts to produce intelligent, complex behaviours from simplistic unionised agents. These behaviours are common to insect and other animal life; they have inspired algorithms such as the Ant Colony Optimisation [43], Artificial Bee Colony [44], and Particle Swarm Optimisation [45].

Each of these popular approaches models a particular swarm as a combination of individual units/agents. These agents may take many forms and can be considered “animate, mechanical, computational, or mathematical; they can be insects, birds, or human beings; they can be array elements, robots, or standalone workstations” [46].

Swarm intelligence has many variations and implementations; it is commonly used to solve planning and logistical problems, its successful performance applied to Travelling Salesman type problems. The work in [47], [48] and [49] present relevant examples of swarm intelligence being implemented to solve complex logistical network, constrained optimisation, and large-scale city design problems.

3.2.1 Ant Colony Optimisation (ACO)

Ant Colony Optimisation draws inspiration from the foraging behaviour of some ant species. Ants of said species make use of pheromones in order to communicate favourable pathways to other ants of the colony. This mechanism is enabled through the concentration of copied behaviour, each time an ant finds a favourable pathway and highlights it with pheromones the process is repeated by each ant/agent that follows.

Ant Colony Optimisation is considered a metaheuristic optimisation method with similarities to the simulated annealing algorithm discussed previously in [43]. There are many variations of the ACO algorithm that are successful; one of its resounding successes is that is easily manipulated to suit any optimisation problem. The algorithm that was initially published is referred to as the Ant System. This system makes use of a population of artificial ants/agent; these ants are each responsible for generating a solution to the optimisation problem at hand. If this is considered in relation to the Travelling Salesman Problem, each ant must determine a solution that is a pathway through a graph.

Alongside the generation of a solution, each ant is responsible for determining the pheromone rate, which is dependent on the successfulness of a solution. With each vertices of a graph explored, the ants must update their pheromone rates, which in turn guides the path of successive ants. The update of the pheromone τ_{ij}, identifies how the pheromone rate of the edge between the two vertices i and j is updated, it is as follows:

$$\tau_{ij} \leftarrow (1-\rho) \cdot \tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k}, \tag{3.4}$$

Included in equation 3.4 in the evaporation rate ρ; this pheromone evaporation rate is responsible for reducing the likelihood of an ant choosing a poor solution by reducing the pheromone rate as iterations pass. Also represented in this equation is m, which refers to the amount of ants, and lastly, $\Delta\tau_{ij}^{k}$ which refers to the rate of pheromone ant k places on the edge ij.

The amount of pheromone placed is defined as:

$$\Delta\tau_{ij}^{k} = \begin{cases} Q/L_k & if\ ant\ k\ used\ edge\ ij, \\ 0 & Otherwise \end{cases} \tag{3.5}$$

In which, Q is a defined constant and L_k is the length of the solution created by ant k.

With the ability to place pheromones, the final part of the Ant System concerns the ants' selection process in relation to the chosen pathway. Each ant will determine its destination vertex as follows; if an ant is in vertex i, the probability that the ant will move towards vertex j is determined by the following equation:

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta}}{\sum_{c_{il} \in N(s^p)} \tau_{il}^{\alpha} \cdot \eta_{il}^{\beta}} & if\ c_{ij} \in \boldsymbol{N}(S^p), \\ 0 & otherwise, \end{cases} \tag{3.6}$$

This equation says that $N(S^p)$, contains pathways with vertices not yet visited by the ant. Moreover, the final piece of information is determined by the α, β parameters that distribute the importance of the pheromone values in comparison to the heuristic information [43].

As mentioned previously, there exists many variations of the Ant Colony Optimisation algorithm. Perhaps the most popular being the Min-Max ACO algorithm that takes a greater exploitative attitude; this algorithm does this by placing pheromones on only the fittest/most successful solutions in a population.

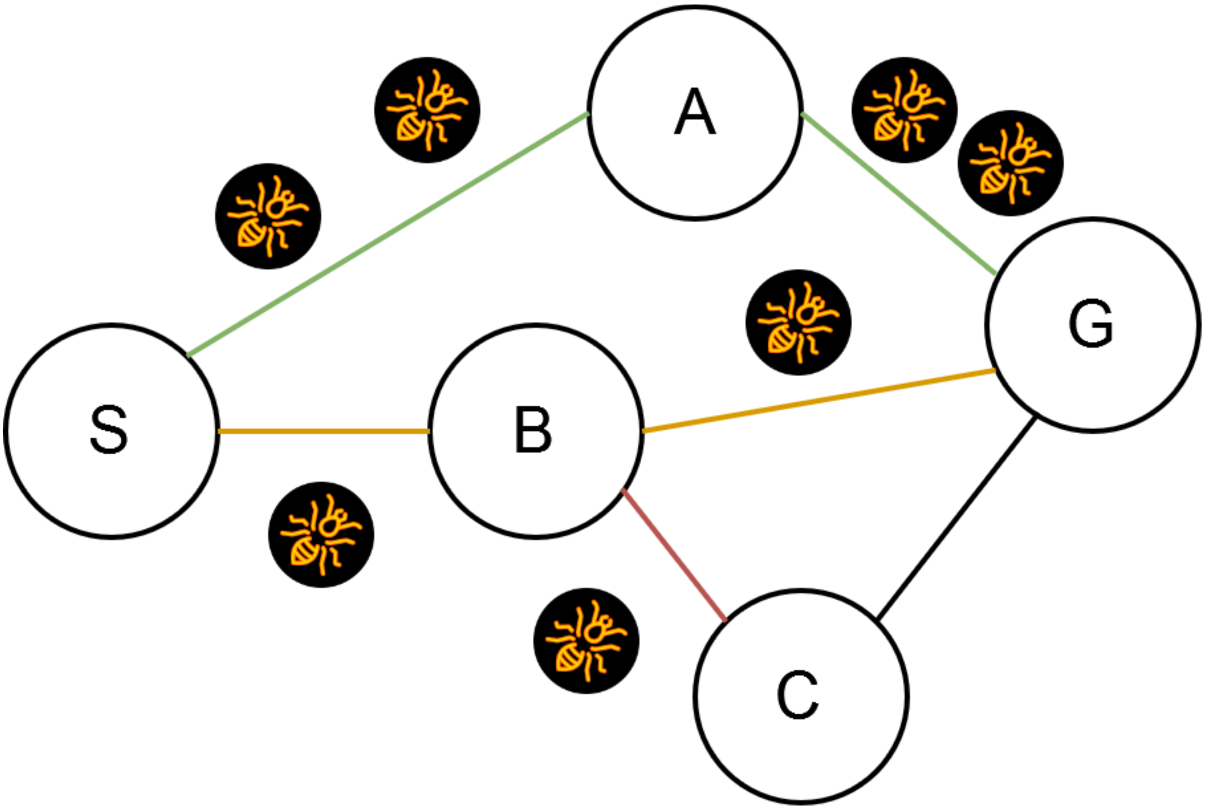


Fig. 3.4 ACO representation with a (S)tarting node and a (G)oal node, in which the best paths are visited by more ants and receive greater pheromone rates.

3.3 Genetic Algorithms

Genetic algorithms are a prominent solution to optimisation problems. GAs "mimic mechanisms of biological evolution in order to develop powerful algorithms" [50]. Genetic algorithms employ the Darwinian evolutionary theory that is survival of the fittest [51] and the biological concept of *fitness,* so that members of the population can iteratively improve as generations pass.

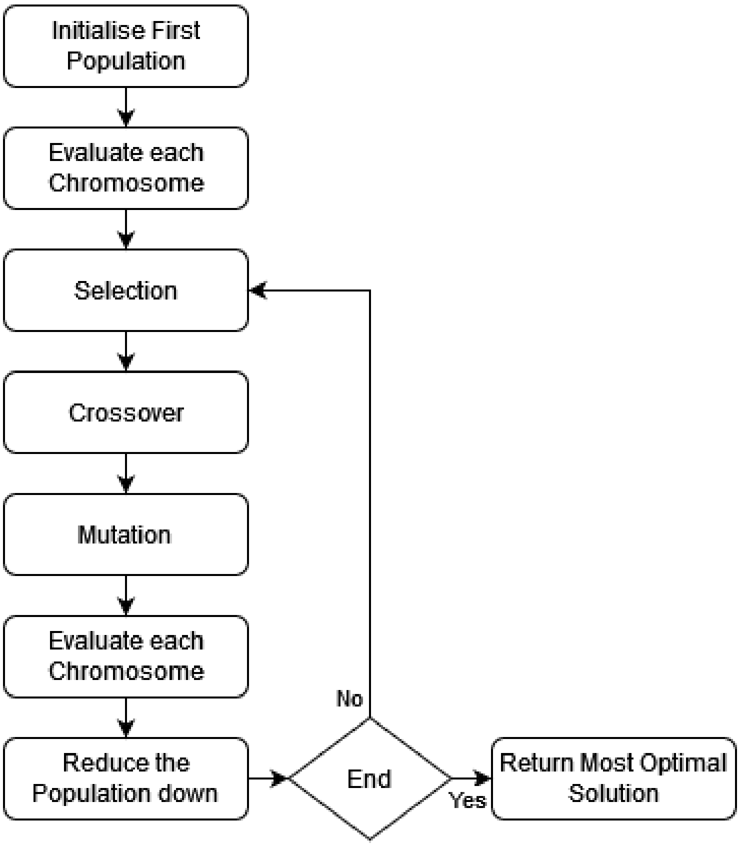


Fig. 3.5 Flowchart of a Genetic Algorithm

Genetic algorithms manipulate a population of chromosomes, which each represent a potential solution to a problem. The chromosomes are evaluated by their fitness, often calculated by a cost/objective function, and a series of genetic operators inspired by nature to ensure that the best scoring/fittest solutions are given the greater chance to propagate their genes to subsequent generations.

Genetic algorithms have found a use across many fields of research and development. For instance, they can be found solving scheduling problems [52], optimising engineering problems [53], and data mining tasks [54]. Genetic algorithms see such widespread use because they can be manipulated in order to deal with the often-complex constraints found in both academic research and real-world problems.

3.3.1 Population

The population of a genetic algorithm is comprised of individual member chromosomes, generated randomly during the first step of the GA process. The amount of chromosomes in the initial population is often problem specific. However, the amount and distribution of the initial population can play a major role in the algorithms ability to converge on an optimal solution [55], and its initialisation is often ignored. Research into the initial population identifies different generation methods other than the traditional pseudo random method. For example, the work in [56] explores the use of different point generators to generate the initial population and found that alternate methods will have a varying effect on different aspects of the GA. It is also possible to design the initial population from expert knowledge to suit a particular problem.

3.3.1.1 Chromosomes

The chromosomes of a genetic algorithm represent the collection of genetic material that are its genes. Each chromosome can be considered an individual solution existing within the domain of a particular problem. In many cases, chromosomes are equally fixed length structures that allow for easy combination of genetic material. However, it is also possible to design chromosomes of variable length.

It is upon the chromosomes that the algorithm conducts the processes of natural selection. The chromosomes play both the vital roles that are parent and child; if they are found to be successful, they will have a chance to combine with other successful chromosomes to produce children that are a combination of the parent's genes, thus ensuring their successful genes survive.

3.3.1.2 Genes

Genes or alleles are perhaps the most significant aspect of the GAs population breakdown. Genes are encodings that determine aspects of the solution; genes can be encoded in many ways to suit the problem at hand. For example, it is possible to represent genes as strings, binary values, real values and problem specific values as shown in Fig 3.6 below.

A	V	D	T	U	L	P	T

0	1	1	0	0	0	1	1

2	4.8	5	37	3	9	4.5	4

Fig. 3.6 Chromosome Representation and Gene Encoding

The genes of a chromosome are responsible for the solutions evaluated fitness value, and it is upon these genes that genetic operators like mutation act. The variable nature of genes allows variation and exploration of the domain space to occur.

3.3.1.3 Generations

In order for optimal solutions to be found, steady progress must be introduced into the population of solutions. In order to simulate this, genetic algorithms loop through the processes outlined in Fig 3.5. Each complete loop represents a single generation; the amount of generations is often problem specific and is often the final criteria which ends the optimisation process if some early stopping criteria has not ended the process already.

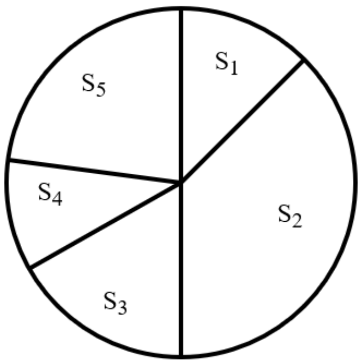
Each generation affords the genetic algorithm the opportunity to remove the least successful solutions from the population and ensure that the solutions that remain have the greatest probability of finding an optimal solution.

3.3.2 Genetic Operators

3.3.2.1 Selection

Before chromosomes can be selected to become parents to the next generation, they must be evaluated and given a fitness score. This fitness score represents how successful a chromosome is in relation to the given task. If, for example, the encoded genes that make up a chromosome are particularly good at minimising the objective function, the chromosome will achieve a dominating fitness score.

When the entire population has been evaluated (a process that is problem specific), it is possible to rank them in order of the successfulness (fitness). Considerable research has investigated methods of selection; some of the most used methods are Tournament Selection [57], Roulette Wheel Selection [58], and Linear Rank Selection [59].



	Fitness	**Proportional Fitness**
S_1	2	0.1
S_2	7	0.35
S_3	4	0.2
S_4	2	0.1
S_5	5	0.25

Fig. 3.7 Roulette Wheel Selection

The Fig 3.7 above is a representation of the Roulette Wheel Selection method. This method works by assigning each member of the population a selection probability that is proportional to its fitness. The selection probability of an individual is calculated as follows:

$$P(i) = \frac{f(i)}{\sum_{j=1}^{n} f(j)} \tag{3.7}$$

Where n is the size of the population and $f(i)$ is the fitness of an individual. However, there is another common approach to this method. The second approach maps the individual's proportional fitness to a number line; this time, when a random number is generated, the segment that it falls upon denotes the individual that has been selected. This second approach is analogous to the weighted roulette wheel shown in Fig 3.7 above. One issue common to this method is the risk that the fittest solution will dominate the parent population and cause the optimisation to converge before it has a chance to find the optimal solution.

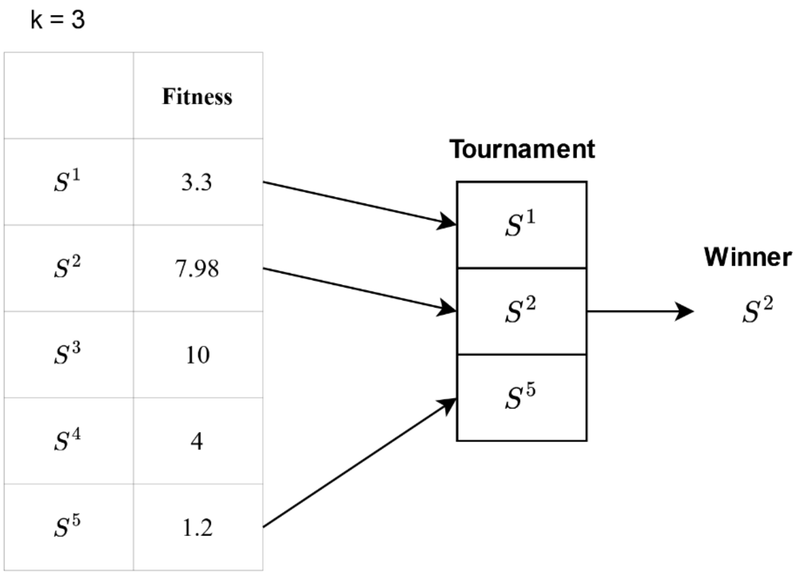


Fig. 3.8 Tournament Selection

The second selection method depicted above in Fig 3.8 is that of Tournament Selection. This method of selection can be implemented in various ways. However, the most common is to select a set of k individuals randomly from the population. The selected individuals are then ranked in relation to one another and the one with the greatest fitness *wins* the tournament. The winner is selected and this process can then be repeated until the parent population size has been reached.

3.3.2.2 Crossover

Crossover is extremely important to the successfulness of a genetic algorithm; the crossover process can be both exploitative and explorative of the solution domain. During crossover, the selected chromosomes from a parent population are combined to create a child chromosome. The selection method will determine the types of chromosomes that are combined and if a genetic algorithm is more exploitative, explorative, or balanced in its attempt to find an optimal solution.

There are a great many crossover methods, such as single point, multi-point, and arithmetic [60]; the best crossover method is often problem specific and can also be influenced by the type of gene encoding.

The crossover methods listed above are implemented as such:

- Single Point Crossover

Single point crossover takes two parent chromosomes and decides on a splitting point (either randomly or by design), and creates two new child chromosomes that are opposite mixtures of the parents genes. In some populations/problems, chromosomes may contain a varying number of genes. For most crossover methods, this is not a problem and can be solved as shown in Fig 3.9 below.

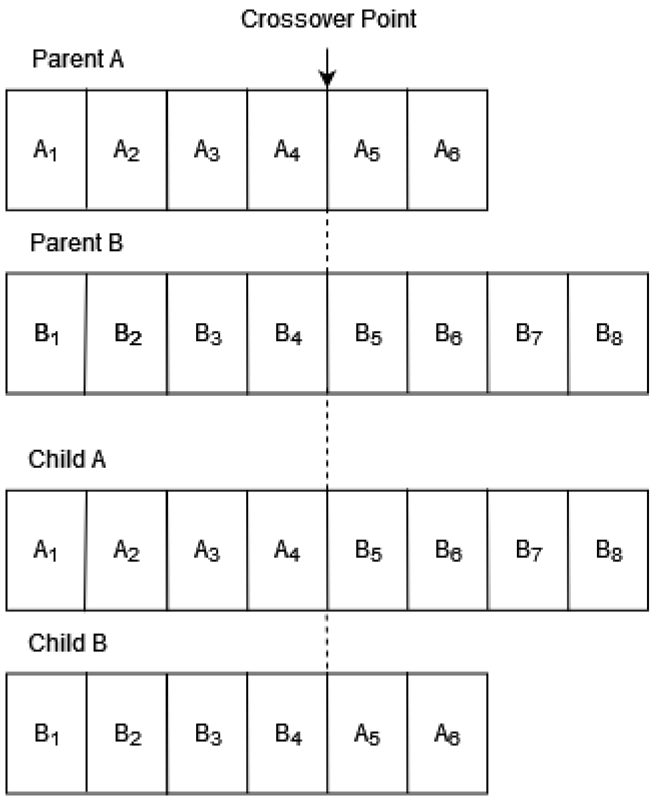


Fig. 3.9 Variable Length Single Point Crossover

- Multi-Point Crossover

Multi-point crossover works in much the same way as single point; *k* points are chosen, indicating the multiple points in which the genes of parent chromosomes must be swapped in order to generate the children chromosomes. Multi-point crossover can be beneficial if a problem requires particular features to be present in chromosomes; in this circumstance, the exploitative nature of this crossover method comes into effect.

However, multi-point crossover suffers some drawbacks; the amount of splitting points will greatly affect the exploitative/explorative nature of the method. As more splitting points are added, the method becomes more explorative. Research suggests, however, that at some point too many splitting points becomes disruptive and a balance must be found [61].

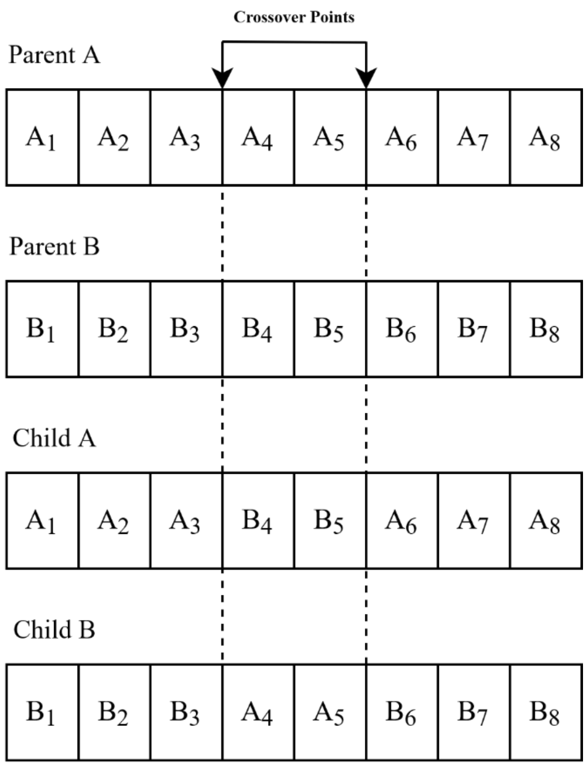


Fig. 3.10 Multi-Point Crossover

- Arithmetic Crossover

Arithmetic crossover is applied to real value coded genes. In this method, only one chromosome child is created from two parents. Each gene of the child chromosome is constructed using the following [62]:

$$x_i = \alpha a_i + (1 - \alpha) b_i, \; \alpha \in [0,1] \tag{3.8}$$

Where a_i and b_i are the ith gene of the parent chromosomes.

Significant research investigates the best crossover practises and implementations. Another intriguing aspect of crossover is deciding the probability in which it occurs and how this probability affects the relationship with the mutation operator [63].

$$\alpha = 0.35$$

Parent A

3	4	1	2	1	6	4	4

Parent B

2	7	3	7	8	2	9	1

Child

2.35	5.95	2.3	5.25	5.55	3.4	7.25	2.05

Fig. 3.11 Arithmetic Crossover

3.3.2.3 Mutation

While crossover tends to favour the creation of new chromosomes from the genes that already exist within a population, mutation allows completely new genes to enter the genetic pool. Methods for mutation vary according to problem and encoding, as does the mutation probability/rate. However, the mutation probability is usually set to a relatively small value in order to introduce new solutions into the population without causing too much disruption to the search.

The most common method of mutation for binary encoded genes is bit flip mutation, which involves the simple flipping of binary gene bits. For real coded genes, uniform mutation is most common; a random gene of a chromosome is selected and set to a new value between defined values.

Along with the mutation probability, it is also important to consider the type of problem being optimised and identify solutions that do not satisfy the constraints of a problem domain. One solution to the crossover/mutation probability ratios is dynamic adaptation strategies, which are introduced in order to maintain the diversity of a population and improve the flexibility of a GA [64].

3.4 Many-Objective Optimisation

There has been a long history of development and research focused on multi-criteria/objective problems. For example, stemming from the early work of Zadeh [65], which introduced the weighted sum method. This method, which uses a set of variable weights in order to generate multiple solution points, is still used extensively; [66] explores how an individual solution point can be determined from a particular set of weights. This is to gain a greater understanding of the method and the significance of the chosen weight values.

Another solution to Many-objective Mathematical Programming (MOMP) is explored in [67]. This method can generate a subset of non-dominated vectors of multiple objectives through linear integer programming. While this method can be implemented to generate a whole set of non-dominated vectors, the results suggest that due to computational requirements it would be inefficient to do so. Instead, this solution is better implemented in order to obtain a well-dispersed subset of non-dominated vectors, making it suitable for decision-making processes.

Many-objective optimisation has not been confined to a single problem domain and is being implemented in logistics-based problems such as [68], which uses a many-objective mathematical model that has been explored previously. [69] Is another logistics-based implementation; it uses a modified many-objective artificial bee colony algorithm in order to reduce economic costs, enlarge customer coverage, and weaken environmental influences.

Many-objective optimisation is also popular in the engineering field with its principles being applied to robotics [70] [71], gas/water distribution [72] [73], and a host of other engineering problems [74]. There are many variations of many-objective optimisers; some of the most popular being NSGA-II [75], many-objective differential evolution (MODE) [76], and self-organising many-objective evolutionary algorithms [77].

These implementations identify the necessity of many-objective optimisation; any significantly challenging real world problem will have numerous objectives and constraints that simply cannot be solved using single objective optimisers. Since many of the objectives will be conflicting, such as, the improvement of one objective comes at the detriment of another, it does not make sense to return a single solution. It is for this reason that many-objective optimisation makes use of Pareto dominance.

3.4.1 Dominance and Pareto Optimality

As previously alluded to, many-objective optimisation must use other methods in order to determine the rank of solutions in a population. Solutions are ranked by their dominance over other members of the population. To determine dominance between two solutions, the objective functions of the solutions must be compared, and it is found that (in the case of all objectives being that of minimisation).

Given two solutions $X_i, X_j \in \mathcal{F}$, we say that X_i Pareto-dominates X_j ($X_i \prec {}_p X_j$) if and only if [78] :

$$\forall m \in \{1,2,\ldots,M\}: f_m(X_i) \leq f_m(X_j) \wedge \qquad (3.9)$$
$$\exists m \in \{1,2,\ldots,M\}: f_m(X_i) < f_m(X_j)$$

If, therefore, none of solutions X_i's objectives, are worse than those of X_j's, and at least one of solutions X_i's objectives is better, it is said to dominate solution X_j.

Minimisation

Solutions	Function 1: Minimise	Function 2: Minimise
A	1	2
B	5	1
C	3	3
D	3	4
E	0	5

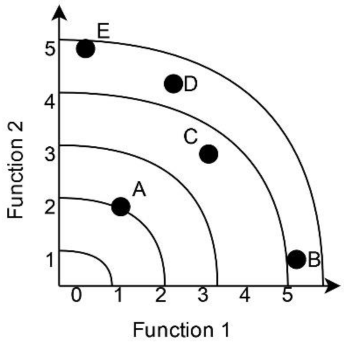


Fig. 3.12 Representation of Solutions in a Many-objective (2) Space [79]

Fig 3.12 above is a representation of the objectives belonging to solutions of the same population; each objective in this example is that of minimisation. This representation identifies important aspects of the dominance rules described above; when solution A is compared against solution C in Fig 3.12, it is clear that both of solution A's objectives are better and therefore it can be said that solution A is dominant over solution C. However, Fig 3.12 also identifies how a solution can outperform another on one objective while being outperformed on the other, as is the case between solutions A and E. In this example, not all of the dominance rules are fulfilled and so neither solution can be said to be dominant over the other.

When approaching a many-objective optimisation problem, objectives are often considered to be of equal importance. For this reason, any population of solutions will eventually be sorted by dominance rules into *sets/fronts*. Each solution is placed in a front based on how often it is dominated, with each subsequent front being dominated by more and more solutions. Under these circumstances, front one illustrated in Fig 3.13, will only contain non-dominated solutions. This first front is termed the Pareto-Optimal and it is understood that any objective belonging to a solution cannot be improved without causing detriment to another.

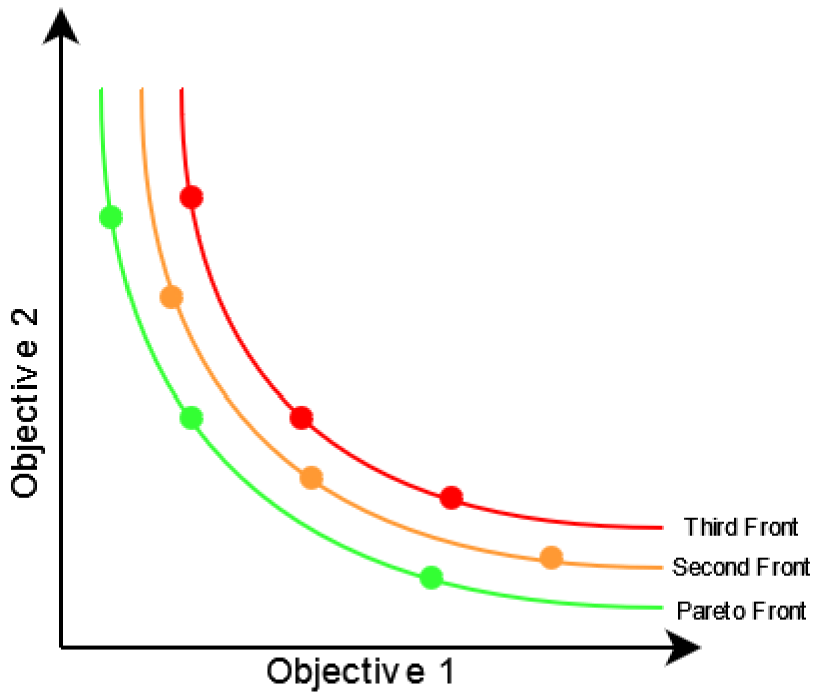


Fig. 3.13 Multiple Fronts in Many-objective Problem [80]

3.4.2 NSGA-II

The Non-Dominated Sorting Genetic Algorithm II [75] is a popular approach to solving many-objective optimisation problems. NSGA-II has been successfully deployed across a range of sections and problems; the work in [81] identifies a way of coordinating the multiple objectives and constraints of a real-world business problem.

NSGA-II makes use of the same genetic operators that single objective GA's use, with selection, crossover, and mutation implemented in comparable fashion. However, NSGA-II diverges from the traditional evaluation of solutions, instead using the dominance rules (as described in section 3.3.1). This dominance method is the focus of much research and it can be further improved with the implementation of fuzzy dominance, which is explored in the work of [82]. In [82], they identified improvements over typical dominance methods and it was observed in [83] that the NSGA-II algorithm could be improved using modified dominance also.

3.4.2.1 Crowding Distance

During the typical GA processes, the NSGA-II algorithm produces a child population, thereby doubling the size of the population. In the reduction of the population, back to its initial size, the algorithm must determine which of the solutions are kept. In most instances, the solutions will be chosen based on their evaluated ranks. However, NSGA-II also differs in its use of a crowding distance metric; as the capacity of a population is being approached, the crowding distance metric is used to determine the choice between comparable solutions that share the same rank.

There are two steps to the crowding distance metric, which are:

1. **Density Estimation**

The density estimation requires the calculation of the average distance between one solutions objectives and the objectives of the solutions that are its nearest neighbours. With the initial distances set to zero for all solutions, the first step is to order the solutions. For each objective from best to worst. Those solutions that now reside at either end of the sorted list for each objective should now be considered the bounds and have their distances set to ∞. For all other solutions, the distance between themselves and their nearest neighbours can now be calculated as follows using the solutions objective values:

$$dis^i = dis^i + \frac{f_{n+1}^k - f_{n-1}^k}{f_{max}^k - f_{min}^k} \tag{3.10}$$

2. Crowded-Comparison Operator

Now that the solutions have been assigned distances, it is possible to determine the solutions that should be selected as successful. The comparison should be made as follows:

1. If solution *A* has a better rank than that of solution *B*
2. If both solutions share the same rank and solution *A's* crowding distance is better than solutions *B*.

A major benefit of using crowding distance as an evaluation method is that there is an increased likelihood of solutions in less crowded areas of the solutions space being chosen to remain in the population. This therefore leads to greater explorative capabilities and a greater uniformity in the Pareto-Optimal front.

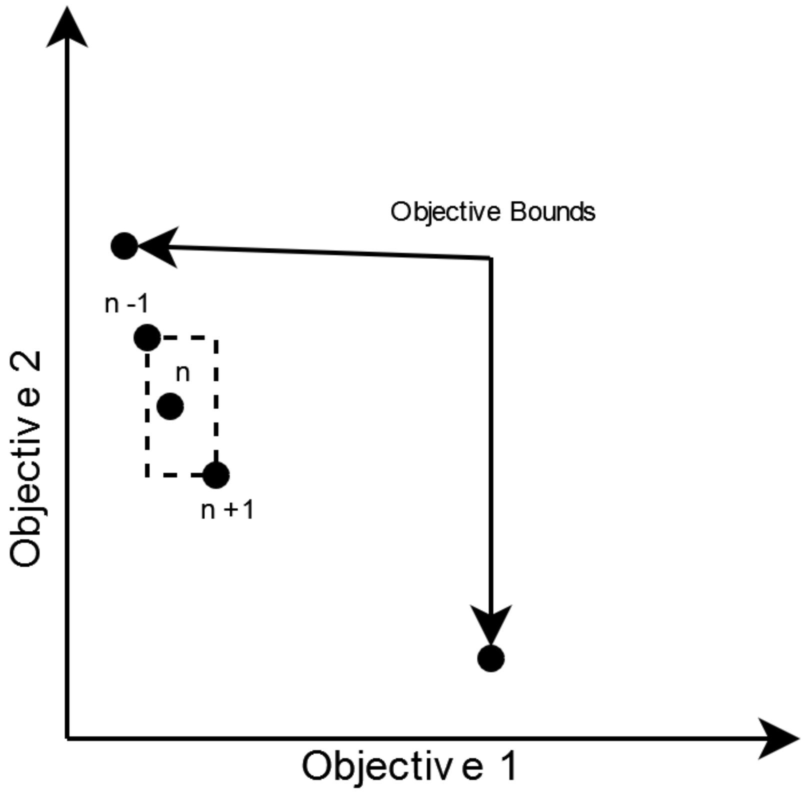


Fig. 3.14 Crowding Distance [79]

3.4.3 Hypervolume

Many-objective Genetic Algorithms (MOGA) are interested in producing a Pareto-Optimal set containing many solutions, with multiple objectives and constraints. It can often be difficult to determine the optimality of that final set. Therefore, the hypervolume metric [84] is a comparison method that can be used to determine the proficiency of many-objective optimisation algorithms.

The Hyper Volume metric provides a measure of the solution space that is dominated by the Pareto-Optimal front; a large hypervolume value reveals a Pareto Front that is distributed with greater uniformity across the entire solution space. Therefore, conclusions can be drawn showing that MOGA's, which produce Pareto-Fronts with large hypervolumes, have had greater success exploring the solution space and returning a more optimal set of solutions.

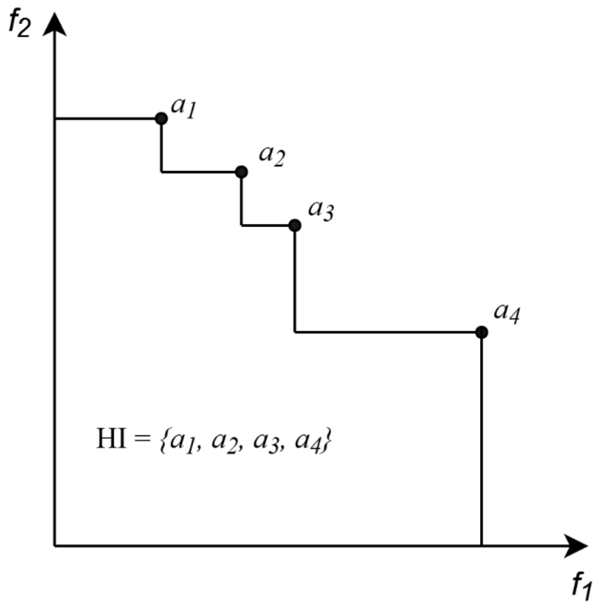


Fig. 3.15 2D Representation of Hypervolume Indicator [85]

The hypervolume indicator uses the position of the solutions in the Pareto front as well as a reference point, which is often set as the anti-optimal point in the objective space. The hypervolume metric is widely implemented for its ability to identify both diversity and performance of Pareto sets. However, there are some drawbacks to the hypervolume metric; its computational complexity can become unfeasible if the number of objectives or solutions in the Pareto set are too high.

3.5 Discussion

In this chapter, an overview of optimisation algorithms was presented. Several single objective optimisation algorithms were explored from A* Search and simulated annealing, an overview was given of swarm intelligences with empathises on Ant Colony Optimisation (ACO), before genetic algorithms were described in detail.

From single objective optimisation the scope was expanded to include many-objective optimisation in which the strengths of this form of optimisation were detailed and considered in regards to complex real-world and academic problems consisting of many, often contradictory objectives. A brief explanation of hypervolume was explored in order to understand how the quality of many-objective optimisation results could be analysed.

Finally, the NSGA-II algorithm was outlined and its variations from a typical genetic algorithm were highlighted.

In the next chapter, fuzzy logic systems will be discussed; an explanation will cover both Type-1 and Type-2 interval fuzzy logic systems and their design methods.

Chapter 4. An Overview of Fuzzy Logic Systems

The field of Fuzzy Logic dates back to the 1960s, when Lotfi Zadeh published a paper [86] that proposed a method of assigning a membership value within the unit interval to objects; therefore introducing ambiguity into class membership in pursuit of modelling the uncertainty of real world conditions.

To provide an example of this logic in action, take for example the speed of a car. The speed of the car could be defined by several classes, such as slow, average, and fast. However, problems arise if strict values are assigned for each; for example, how is each boundary value decided? If a value is situated around a boundary value by a minuscule amount, does it make sense to claim it is a member of one set over another? It is in scenarios like this that Fuzzy Logic excels - values can instead exist to multiple classes to varying membership degrees; a speed can instead be defined as any combination of overlapping classes, such as average and fast. In doing this, Fuzzy Logic provides greater accuracy and context to an ambiguous and uncertain system that has no single definition.

Fuzzy Logic Systems are prefaced by a number; for example, Type-"1" and Type-"2". Both of these systems follow the same Fuzzy practises, however, each subsequent Type-*n* system allows greater representation of uncertainty through more complex set modelling [87]. Due to complexity in both design and computation, it is Type-1 and Type-2 Interval Fuzzy Logic Systems that are most frequently implemented.

Fuzzy Logic is implemented across a spectrum of fields, such as robotic control [88] [89], image processing [90] [91], evolutionary optimisation [92] [93], and a host of other industrial and academic applications.

4.1 Type-1 Fuzzy Logic Systems

Type-1 Fuzzy Logic systems contain four significant parts; these are the fuzzifier, rule base, inference engine, and the defuzzifier. These parts, illustrated in Fig 4.1 below, are examined in detail throughout the remainder of this chapter. The system as a whole is responsible for transforming raw crisp values into modified values that take into consideration the ambiguity of the problem being solved.

One criticism of Type-1 Fuzzy Logic, and by extension all fuzzy logic, is that whenever hard borders are defined for Fuzzy sets it is counter to the entire purpose of the theory, which is to remove rigid definitions of data and sets [87]. However, it is evident from the countless implementations of Fuzzy Logic and the benefits that it provides, especially to control theory, that this is a trivial criticism.

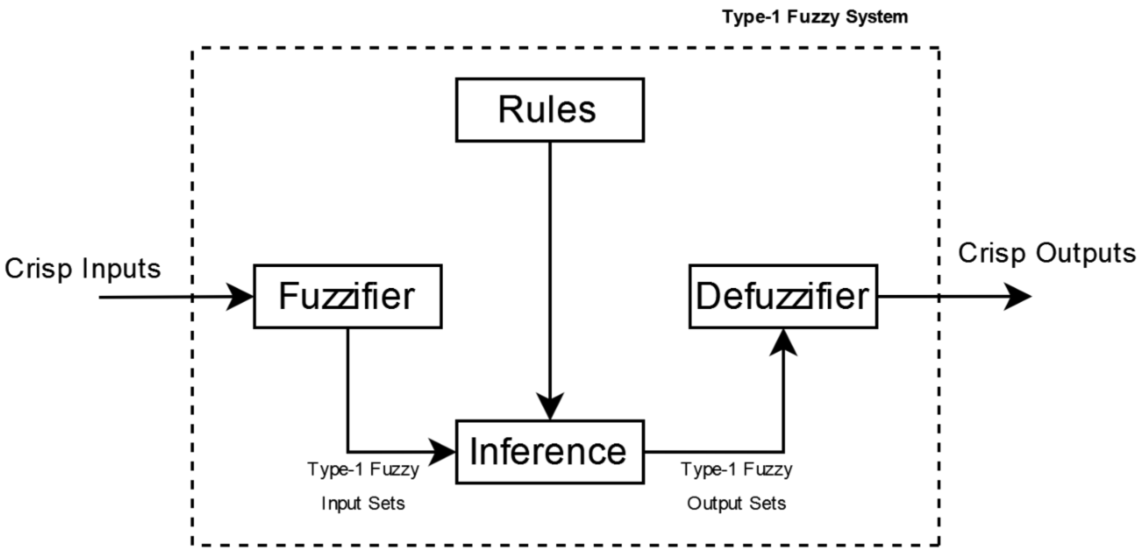


Fig. 4.1 Type-1 Fuzzy Logic System [88]

4.1.1 Linguistic Variables

One feature of Fuzzy Logic is its use of linguistic variables. These linguistic variables allow numerical values to be defined in a way which is neither "*exact nor very inexact*" [94]. Through this method, Fuzzy Logic is able to represent specific numerical data with a greater consideration of the uncertainty that is found in the universe of discourse.

One benefit to this system is that numerical values can be contextualised; and due to the nature of membership within fuzzy logic, can be represented by more than one term belonging to a linguistic variable, if overlap exists between them. Fig 4.2 below identifies how a crisp numerical value (68 mph) falls between the two linguistic terms, *quick* and *fast*, of the linguistic variable *speed* and therefore belongs to both sets to some degree.

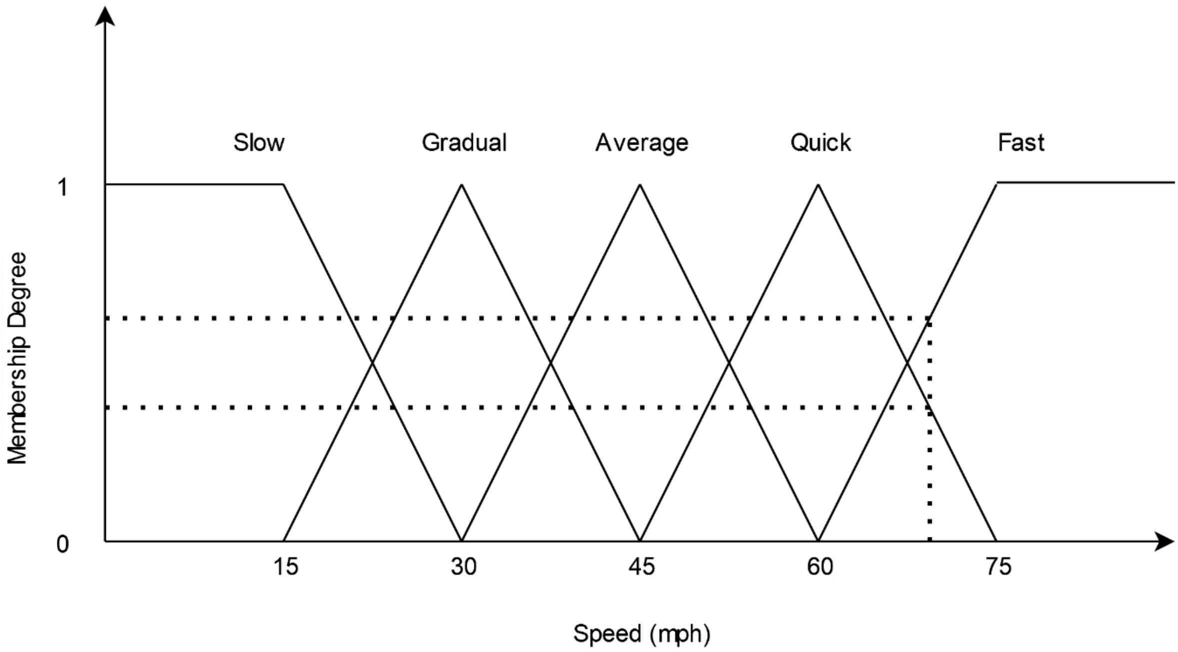


Fig. 4.2 Linguistic Variable Speed and the corresponding Membership Functions

In [94], Zadeh represents each linguistic variable with a quintuple $(L, T(L), U, G, M)$. Within this L defines the name of the linguistic variable; $T(L)$ is the collection of linguistic terms belonging to the linguistic variable; U is the universe of discourse in question; G is a syntactic rule, used to generate the linguistic terms; and M is semantic rule that assigns each linguistic term its meaning.

With this notation, it is possible to define the linguistic variable illustrated in Fix 4.2; the linguistic variable (L) = *Speed (Mph)*. The linguistic terms belong to it $T(L) =$

$Slow, Gradual, Average, Quick, Fast.$ The Universe of Discourse is $U = [0\ mph, 100mph]$. *G* is ensuring that the linguistic variables and terms make linguistic sense, *Slow Speed*, *Average Speed*, etc. Finally, *M* is defined by the design of the fuzzy sets for each linguistic term.

4.1.2 Membership Functions

Linguistic variables are accompanied by a membership function, which delineates the values belonging to the individual linguistic terms in a system. Membership functions use the mathematic notation $\mu_F(x)$ and they are commonly represented by triangular, trapezoid, singleton, and Gaussian shaped sets as shown in Fig 4.3 below. The best shape for the membership functions is often problem specific, with the implementation often being selected by the designer due to personal preferences. Whichever shapes are used; membership functions offer both simplicity and intuitive design for problems containing some level of ambiguity.

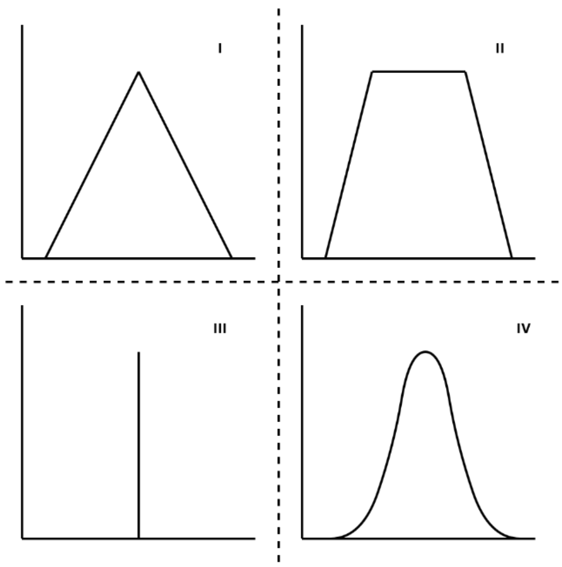


Fig. 4.3 Traditional Membership Function Shapes

Abundant literature is dedicated to the optimisation of the membership functions; the research is primarily interested in the performance improvement of fuzzy logic systems through the fine-tuning of membership function positions. The work in [95] and [96] are examples of both genetic algorithms and particle swarm techniques being used to tune membership functions.

The membership function defines not only the range of values that are coupled to linguistic terms, but also the degree to which those values are considered members of their own set. The calculation of this membership degree is dependent on the shape of the membership function; for those illustrated in Fig 4.3, it is as follows [87]:

$$\mu_F(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b < x \leq c \\ 0, & Otherwise \end{cases} \tag{4.1}$$

Equation 4.1 defines the triangular membership function.

$$\mu_F(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ 1, & b < x \leq c \\ \frac{d-x}{d-c}, & c < x \leq d \\ 0, & Otherwise \end{cases} \tag{4.2}$$

Equation 4.2 defines the trapezoidal membership function.

$$\mu_F(x) = \begin{cases} 1, & x = a \\ 0, & Otherwise \end{cases} \tag{4.3}$$

Equation 4.3 defines the singleton membership function.

$$\mu_F(x) = e^{\left(-0.5\left(\frac{x-a}{\theta}\right)^2\right)} \tag{4.4}$$

Equation 4.4 defines the Gaussian membership function.

Whichever membership functions are chosen, the best performance is observed when there is overlap between them. Theoretically, each defined membership function will improve the performance/accuracy of the fuzzy system at the cost of both computational and design complexities; traditionally, therefore, linguistic variables are defined by 3-5 membership functions. This is the typical amount chosen as it lends itself to the interpretable nature of fuzzy logic systems; an inflation of membership functions leads to a loss of interpretability of the design and an explosion of ineffectual rules. Illustrated in Fig 4.2, a crisp input of 68 mph belongs to both the *quick* and *fast* linguistic terms; it has membership values of $\mu_{quick}(68) = 0.4\dot{6}$ and $\mu_{fast}(68) = 0.5\dot{3}$.

4.1.3 Fuzzy Rules

The desired outputs of a fuzzy logic system are controlled by a series of IF-THEN rules that together form a rule base. Each rule can be defined by its constituent parts; there is the antecedent section which contains the IF component of the rule and the consequent that defines the THEN component [87]. A typical rule will have the following structure [87]:

$$R^1 : IF\ x_1\ is\ F_1^l\ and \dots and\ x_p\ is\ F_p^l, THEN\ y\ is\ G^l \quad l = 1, \dots, M \tag{4.5}$$

Each fuzzy rule can take many forms; however, it is most common to encounter rules consisting of multiple antecedents and single consequents. Rules also make use of varying connectives, the example above making use of the "*and*" connective. However, the "*or*" connective is also commonplace. It is also possible to observe "*mixed*" rules that use a combination of connectives.

Rule Number	*Sensor One*	*Sensor Two*	*Steering*
1	Close	Close	Left
2	Close	Medium	Left
3	Close	Far	Left
4	Medium	Close	Right
5	Medium	Medium	Straight
6	Medium	Far	Right

7	Far	Close	Right
8	Far	Medium	Right
9	Far	Far	Right

Table 4.4.1 Example Right Wall Following Rule base

Table 4.1 gives an example of a set of fuzzy rules developed to ensure a robot is able to follow along a wall located on its right. The rules for this system enable a coordination of behaviours that improve the performance of the system. As discussed, it is possible that a crisp input belongs to more than one membership function; therefore, multiple rule conditions can be met simultaneously. For example, it would be possible for rules 4 and 5 to fire together if it is assumed there is overlap between the *close* and *medium* membership functions. Importantly, because these rules have differing consequents, the output of the fuzzy system will become a blend of the two.

The research surrounding the rules of a fuzzy system is multi-faceted; one particular area of interest is the extraction of rules found in the work of [97] [98]. This research is focused on techniques that can derive the fuzzy rules for a system from sample data.

4.1.4 Inference

Within inference (commonly referred to as the fuzzy inference engine), fuzzy input sets are mapped to fuzzy output sets [87]. In order to interpret the rules of a fuzzy system, a fuzzy implication method must be chosen. This work makes use of the Mamdani methods for implication, which are the most commonly implemented variations.

In order to calculate the system output, it is required to calculate the weight of each fired rule; these weights are more often referred to as firing strengths and the implication method is dependent on the rule connective that is in use. For example, if *"and"* is the sole connective in use, the t-norms of minimum and product can be implemented [87]:

$$\mu_{A\to B}(x,y) \equiv \min[\mu_A(x), \mu_B(y)] \tag{4.6}$$

$$\mu_{A\to B}(x,y) \equiv \mu_A(x)\mu_B(y) \tag{4.7}$$

However, if the *"or"* connective is in use the t-conorm maximum is used [87]:

$$\mu_{A\to B}(x,y) \equiv \max[\mu_A(x), \mu_B(y)] \tag{4.8}$$

4.1.5 Defuzzification

The defuzzification process is the final step in the Type-1 fuzzy logic system, responsible for producing a crisp output from the fuzzy sets represented in the inference engine. Numerous methods exist for the realisation of defuzzification. For each of those discussed in the ensuing section, output sets, akin to the membership functions discussed previously, must be constructed. Illustrated in Fig 4.4 below are the output sets previously defined in Table 4.1.

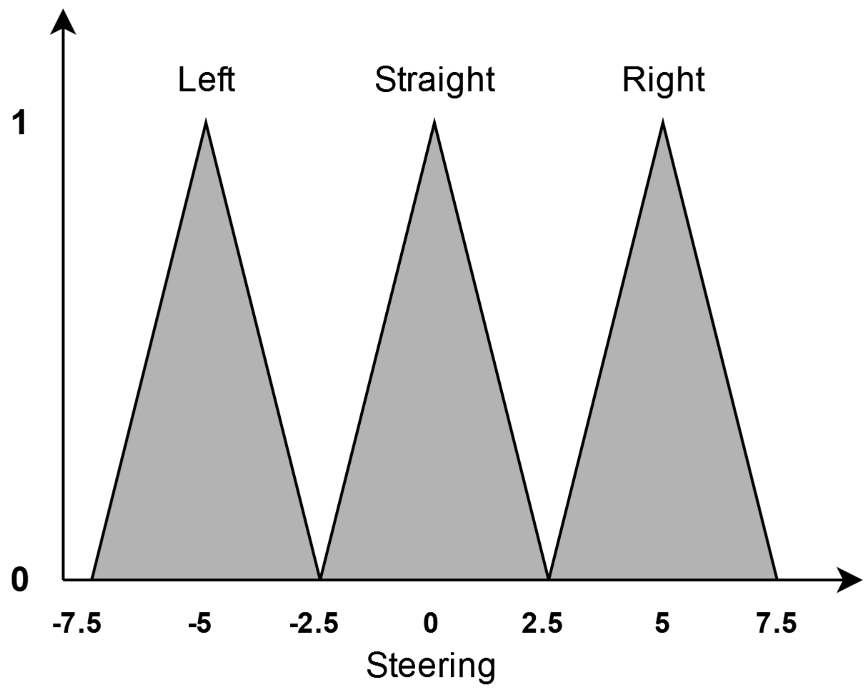


Fig. 4.4 Output Membership Functions for Robot Wall Following Example

4.1.5.1 Centroid Defuzzifier

The centroid defuzzification method performs a union on the relevant fuzzy output sets. The centroid of this union is used alongside the firing strengths calculated in the previous inference stage. Centroid defuzzification was a particularly early method; its implementation however, has become less prevalent due to computational complexities that stem from the union calculation of sets.

Equation 4.9 below describes the centroid defuzzification method [87]:

$$y_c(x') = \frac{\sum_{i=1}^{N} y_i \mu_B(y_i|x')}{\sum_{i=1}^{N} \mu_B(y_i|x')} \tag{4.9}$$

This method requires the discretisation at N points of the membership function for output set B.

4.1.5.2 Height Defuzzifier

In the height defuzzification method, the output sets of the fired rules are replaced by a series of singleton points that have the maximum membership for each rules consequent. These singleton points are then used to calculate the centroid of a new set that is a combination of them all. This defuzzification method is describe in equation 4.10 below [87].

$$y_h(x') = \frac{\sum_{l=1}^{M} \bar{y}^l \mu_{B^l}(\bar{y}^l|x')}{\sum_{l=1}^{M} \mu_{B^l}(\bar{y}^l|x')} \tag{4.10}$$

4.1.5.3 Centre-of-sets (COS) Defuzzifier

The centre-of-sets defuzzification method is the most commonly implemented. This method replaces each rule consequent set with a singleton located at the centroid. The amplitude of this singleton is equal to the firing strength of the rule. As with the height defuzzification method, the centre-of-sets method then constructs a new centroid from the combination of singletons. The expression of this method is given in equation 4.11 below [87].

$$y_{cos}(x') = \frac{\sum_{l=1}^{M} COG(G^l) f^l(x')}{\sum_{l=1}^{M} f^l(x')} = \frac{\sum_{l=1}^{M} c^l f^l(x')}{\sum_{l=1}^{M} f^l(x')} \tag{4.11}$$

4.2 Type-2 Fuzzy Logic Systems

As discussed in the previous section, Type-1 fuzzy logic has limitations on the extent with which it is able to deal with uncertainty in data and the ambiguous characteristics of real-world problems. Generally, both types of fuzzy logic share core principles; however, Type-2 fuzzy logic systems modify these principles in order to model greater uncertainty that leads to improved accuracy and results. To do this, Type-2 fuzzy logic adapts the fuzzy sets in such a way that membership values are no longer crisp values between $[0, 1]$.

There are two comprehensive implementations of Type-2 fuzzy logic; these are general and interval Type-2 fuzzy logic systems. These implementations differ primarily in the type of fuzzy sets used. Of the two methods, interval Type-2 fuzzy logic is the most typically implemented and it will be the focus for the following sections. Interval Type-2 fuzzy logic is more commonly implemented for a number of reasons; firstly, it is able to deal with greater levels of uncertainty and yields greater results than type-1 fuzzy logic [92]. Secondly, due to the computational complexity of using general Type-2 fuzzy sets, most people only use interval type-2 fuzzy sets as the computations associated with them are very manageable; making them practical [99].

4.2.1 Interval Type-2 Fuzzy Logic Systems

The interval Type-2 fuzzy logic system illustrated in Fig 4.5 below identifies the additional type reduction step that is not present in Type-1 fuzzy logic. The type reduction process, as the name suggests, is responsible for reducing the Type-2 fuzzy output sets to Type-1 fuzzy sets, from which the output process from Type-1 fuzzy logic can be followed.

The type reduction step and the Type-2 fuzzy sets (for input and output) are the two major differences between the systems. The other steps observe some differences; however, the main process in which a crisp value is fuzzified and a crisp value is returned remains. Each of the steps deemed to differ significantly from Type-1 will be explored.

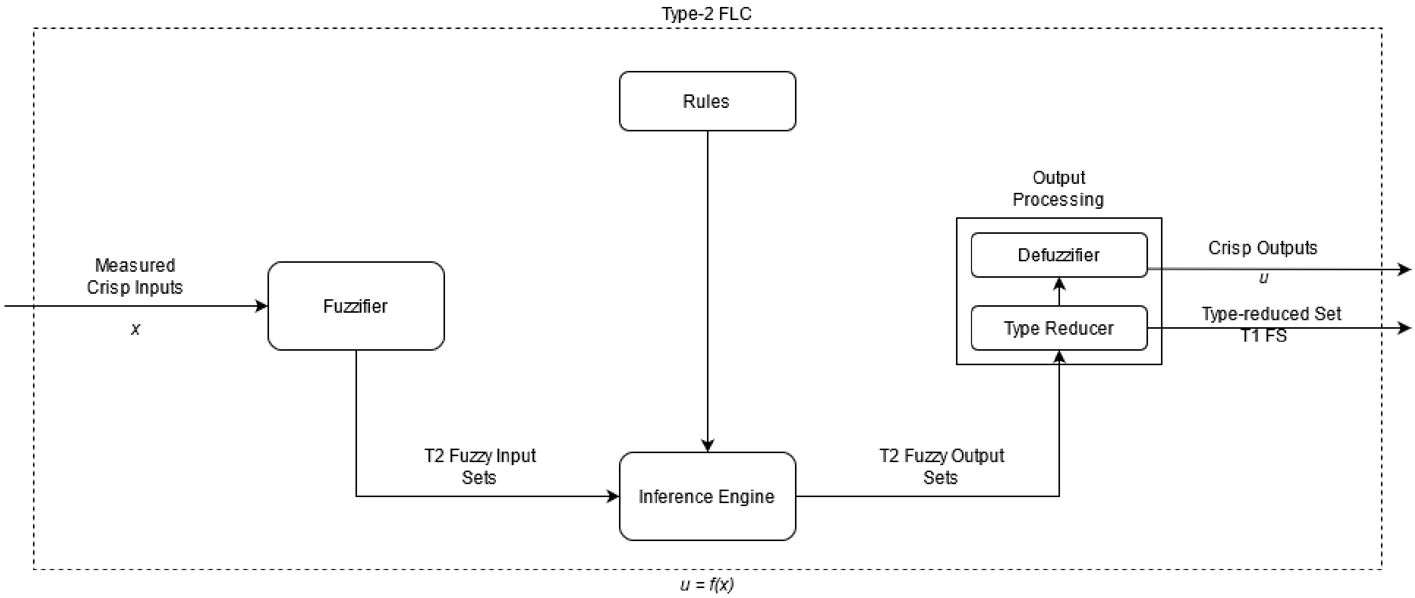


Fig. 4.5 Type-2 Fuzzy Logic System [87]

4.2.2 Interval Type-2 Membership Functions

The most significant change occurs with the fuzzy sets; the Type-2 fuzzy sets can be viewed as a combination of Type-1 fuzzy sets. These Type-2 fuzzy sets, illustrated in Fig 4.6 below, possess a footprint of uncertainty (FOU) that facilitates the visualisation of these Type-2 sets in two-dimensions [100] [101].

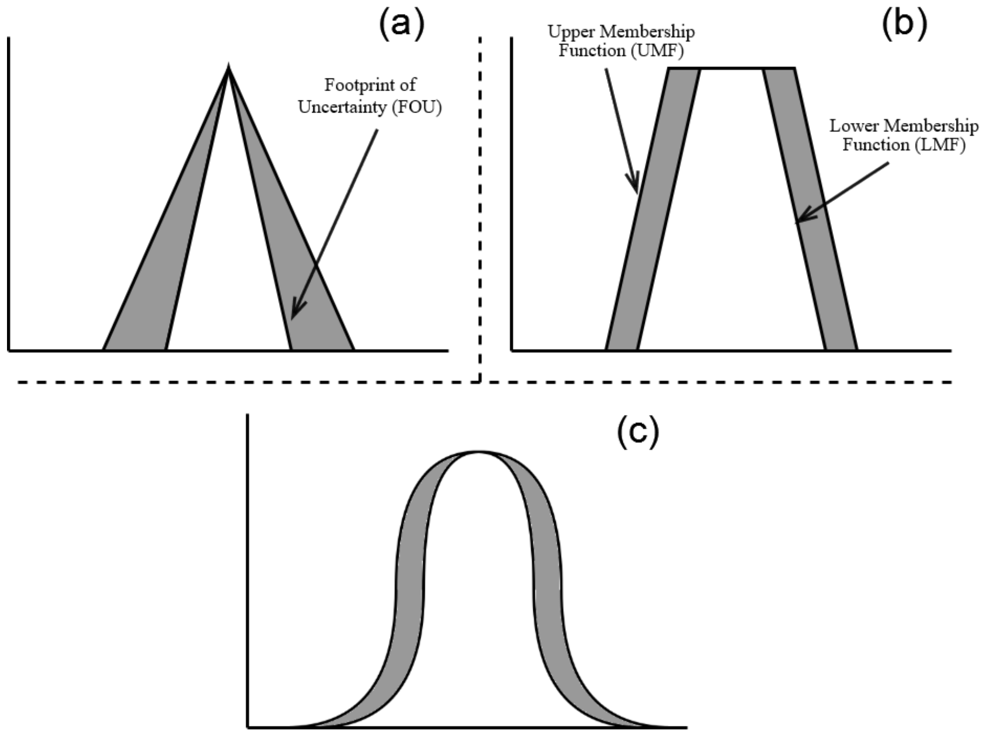


Fig. 4.6 Type-2 Membership Functions (a) Triangular (b) Trapezoidal (c) Gaussian

The FOU of these Type-2 sets ensures that a membership value will no longer be a single real value that is an element of $[0, 1]$; instead, membership values are now an interval of real values within this range. For each region, this interval is referred to as the upper and lower membership values; a Type-2 fuzzy set, denoted $\tilde{A}$, is characterised by a Type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$. This is described in equations 4.12/4.13 and illustrated by Fig 4.7 below.

$$\tilde{A} = \{(x,u), \mu_{\tilde{A}}(x,u)\} \mid \forall x \in X, \forall u \in J_x \subseteq [0,1] \qquad (4.12)$$

In which $0 \leq \mu_{\tilde{A}}(x,u) \leq 1$. $\tilde{A}$ can also be expressed as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \quad J_x \subseteq [0,1] \qquad (4.13)$$

Where $\iint$ denotes union over all admissible *x* and *u* [88].

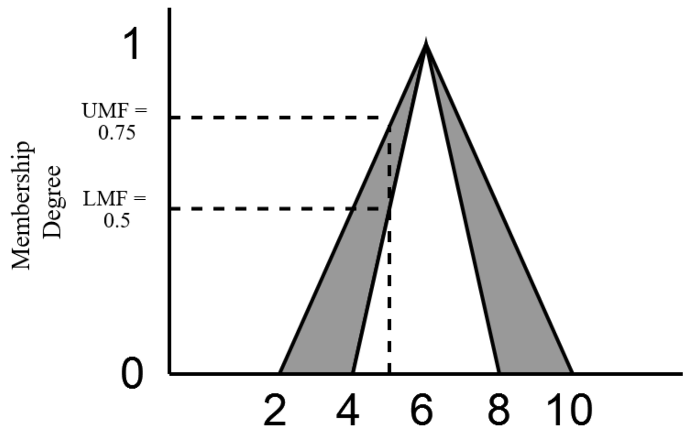


Fig. 4.7 Interval Type-2 Triangular Membership Function

Not only does the FOU allow for greater accuracy and improved modelling of uncertainty, another benefit is that each Type-2 fuzzy set is able to cover a greater measure of the problem domain, therefore fewer membership functions are required overall. This is a benefit because as discussed previously, a reduction of membership functions reduces both computational and design complexities.

4.2.3 Type Reduction and Defuzzification

As with Type-1, the end goal of Type-2 fuzzy logic is to return a crisp number as an output. However, in order to reach this point, type reduction must be undertaken in whichType-2 fuzzy output sets must first be reduced to Type-1 before defuzzification can take place. The process of type reduction and the improvement of the computational times necessary for it has been, and continues to be a significant area of research within the fuzzy logic field. The initial process of type reduction, explored in the remainder of this chapter, was introduced by Karnik and Mendel (KM) [102]. The Wu-Mendel (WM) Uncertainty Bound Method [103] follows a similar process to KM method, the output of an interval Type-2 set is computed by equation 4.18, however:

$$y_l = \frac{\underline{y}_l + \overline{y}_l}{2}, \quad y_r = \frac{\underline{y}_r + \overline{y}_r}{2} \tag{4.14}$$

In which:

$$\overline{y}_l = \min\left\{\underline{y}^{(0)}, \underline{y}^{(N)}\right\}, \quad \underline{y_r} = \max\left\{\overline{y}^{(0)}, \overline{y}^{(N)}\right\} \tag{4.15}$$

Unlike the KM method explored in the remainder of this section the WM method does not require the sorting of $\{\underline{y}^n\}$ and $\{\overline{y}^n\}$ in order for the final output to be calculated. However, the minimum and maximum of $\{\underline{y}^n\}$ and $\{\overline{y}^n\}$ is still required [104]. Experiments identify the improvements of the WM algorithms over the traditional KM algorithms, however, it should be noted that for rule bases that are not overly expansive the KM method is sufficient in the reduction of sets and calculation of final output [88].

A number of type reduction methods exist, each responsible for representing the mapping between Interval Type-2 and Type-1 Fuzzy sets. The most common methods include centre of sets, centroid, and height. The centre of sets method will be described here, as it is the method implemented throughout this work.

The type reduced set is represented by an interval set, which has been illustrated in equation 4.14 below.

$$Y_{TR} = [y_l, y_r] \tag{4.14}$$

Equation 4.15, in which Y_{COS} represents an interval set, described the method for the centre of sets type reducer.

$$Y_{COS} = [y_l, y_r] = \int y^1 \in [y_l, y_r] \ldots \int y^m \in [y_l^m, y_r^m] \int f^1 \in \left[\underline{f}^1, \overline{f}^1\right] \ldots \int f^1 \in \left[\underline{f}^M, \overline{f}^M\right] \; 1/\frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \tag{4.15}$$

The centroids of the Type-2 interval consequence set, $\tilde{G}^i$, must be calculated ahead of $Y_{COS(x)}$ using the following:

$$C_{\tilde{G}^i} = \int \theta_1 \in J_{y1} \dots \int \theta_1 \in J_{yN} \, 1/\frac{\sum_{i=1}^{N} y_i \theta_i}{\sum_{i=1}^{N} \theta_i} = [y_l^i, y_r^i] \qquad (4.16)$$

With these steps completed, it is now possible to defuzzify and obtain the final output from the interval Type-2 system. As mentioned, the type reduced set consists of two numbers and so the final crisp value is the average of this set.

$$Y(x) = \frac{y_l + y_r}{2} \qquad (4.17)$$

4.3 Discussion

In this chapter, an overview of Fuzzy Logic Systems was presented. The process is outlined; initiated with a crisp input, into the fuzzification, onto inference of rules and the defuzzification back to a crisp output.

The differences between the types of fuzzy logic are presented along with the common criticisms. The strengths of fuzzy logic are also presented and its ability to deal with the uncertainty and ambiguity common in real world problems is detailed.

The variations between the different fuzzy logic system, such as type reduction and the design of fuzzy sets in explored in detail to identify the strengths of Type-2 fuzzy logic while acknowledging the potential drawbacks.

The expression of Interval Type-2 Fuzzy Logic has been explored in greater detail, with the variation of type reduction that has been implemented within this project presented. The variations of type reduction methods are listed and the commonly implemented Wu-Mendel Uncertainty Bound Method is discussed as the extension to this projects implemented Karnik-Mendel method.

The following chapter explores the progression to explainable artificial intelligence and how understanding the AI models typically implemented can lead to a greater understanding of problem domains. The chapter also explores the most common methods currently in place for the implementation of XAI.

Chapter 5. An Overview of Explainable Artificial Intelligence (XAI)

Many-objective optimisation algorithms and 'black-box' AI models traditionally provide extremely limited information regarding the *how* or *why* concerning solution optimality. The interest surrounding the thought processes of intelligent machines harkens back to Alan Turing and his infamous paper "the imitation game" [105]. Turing reflected on whether machines were capable of thinking in human-relatable terms. The questions posed by Turing, and the concept of the imitation game, are typically applied to artificial intelligences that are specialised in natural language processing or the fabled artificial general intelligence. For many artificial intelligent models today, the interest lies not in whether they can think; instead, the focus has shifted, and now we are curious as to why a model produces a particular solution.

The two common classifications for AI are *white box* and *black box* models as discussed in the following subsections

5.1 White Box Models

White box AI techniques include linear regression, which can be used fit models to data sets and identify the relationship between variables. Decision trees are another white box model that can be used for classification and regression tasks.

The commonality between these techniques is their relative simplicity; this simplicity is both a pro and a con that is often present in white box methods. Because of their simplicity, white box models tend to be easily interpretable. Their functions often produce solutions that are clear and understandable, with key features being easily visualised. However, this same simplicity is responsible for the inherent weakness/lack of accuracy present in white box models. Increasing the number of inputs to linear/logistic regression and Decision Trees models can convert them to opaque models which cannot be easily understood, analysed and augmented by the relevant stake holders.

5.2 Black Box Models

Black box AI techniques include techniques like deep neural networks, support vector machines, Xgboost, etc. Such techniques are flexible in their implementations and are highly successful in producing accurate models. Commonly, black box models contain high levels of complexity. The workings of these models are often unknowable and their ability to deal with problem complexity typically means they are not easily interpretable to users.

There exists numerous AI techniques, each capable of achieving successes in specific problem domains. The consideration regarding the choice of a white box or black box model is problem specific; the problem domain must be considered and a trade-off between accuracy and interpretability is typically made. One of the key challenges with XAI is finding the desired coordination between interpretability and accuracy; it is often found that the more understandable a system is made to be, the less accurate the results of that system will be [106].

The role of artificial intelligence continues to grow across all fields; it is therefore important to design novel AI methods that are explainable across all fields, but in particular, when it has an impact on people's lives. The interpretability of AI not only provides confidence in solutions, it can also provide an insight to a problem domain that has not yet been explored. Our ability to understand AI models is critical to the continued growth of the field and its wider implementation and acceptance.

The adoption of AI models by organisations has more than doubled since 2017, from 20 to 50 percent [107]. As newer approaches and AI models have been developed, organisations have revaluated what can be achieved through AI and the way in which they incorporate it into their systems. However, there have already been found to exist incorrect or unethical implementations of AI such as those discussed in [108] and [109]; in each of these examples, a deeply significant aspect of a person's life is determined utterly at the whims of misunderstood, uninterpretable, or poorly communicated AI models. It is in circumstances like this that XAI can be most evidently implemented; the ability to understand and explain the findings of an AI model is invaluable. This interpretability can be expanded to organisations using AI models in practical real-world domains.

There are several aspects of research that explainable AI is particularly interested in, which include [106] [110]:

1) Transparency

As previously mentioned, if AI is playing a significant role in a person's life, they deserve to know how that model reaches its conclusions; this should be readily and easily available to them in formats that can be understood. The implementation of this transparency can be built into the AI models themselves, or be the result of external model decomposition if it is capable of being expressive enough to pass as "human-understandable" [110]. AI systems that provide aid to the decision making of significant decisions in a person's life will benefit greatly from increased transparency, for example, providing context to whether or not a loan should be granted due to the specific considerations of the request.

2) Causality

What underlying features can be learned from a generated model? The extraction of explainable material from problem domains should lead to a greater understanding of the fundamental nature of the interest area. For example, understanding which features are most important for a health practitioner to investigate in an analysis of data that concerns the risk factors for cervical cancer [111].

3) Bias

Explainable AI should be able to provide insights to AI models that exhibit forms of bias or prejudice. AI models may contain this bias through weaknesses in the collection of data or from failings of the algorithm itself. In these circumstances, XAI should provide engineers the insight required to address the source of the bias. For example, an XAI system can be implemented in order to reduce the bias and discrimination commonly found in hiring practises which perpetuate an unfair and unequal procedure [112].

4) Fairness

XAI techniques should ensure that the decisions and solutions arrived at by AI models are both impartial and do not favour any one relationship in a problem space. It is very important that where AI systems impact significant fields, such as the medical field, the model design and system data accounts for all demographics and is not influenced by societal bias which may provide incorrect output [113].

5) Safety and Trust

XAI should provide confidence in regards to the reliability of implemented AI models. It could be argued that solutions that are sub-optimal with a greater explanation are more valuable to an end user than optimal solutions that are not explainable. For example, [114] explores a problem concerning a planetary rover and describes how a safe AI system (SAFE-AI) is capable of providing an expert engineer with solutions that improve the mission outcomes.

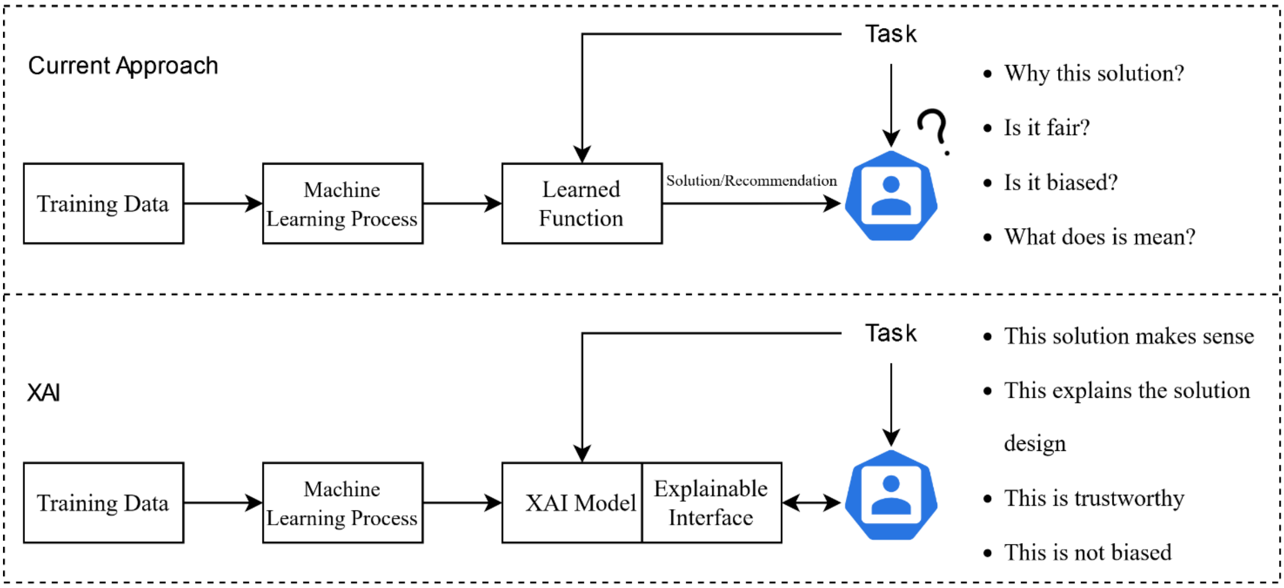


Fig. 5.1 XAI Concept [115]

5.3 Current XAI Methods

The field of explainable artificial intelligence (XAI) attempts to increase transparency on typical 'black-box' style AI. There are increasingly more methods in which this is being pursued. For example, the work of [116] produced accurately predicted models referred to as Bayesian Rule Lists (BRL); these produced interpretable antecedent/consequent rules (if/then) which aided the understanding gained by domain experts that used the system. Bayesian Rule Lists share many similarities with generated fuzzy rule bases; the implementation of BRL in [116] provides an insight into the strengths of the method, for example, the computation complexity of BRLs is truncated by its use of pre-mined rules that reduces the permutations of rules that can be produced. While this is a strength of BRLs it is a weakness in the consideration of our problem, this is because our problem requires an investigation of all possible and uncertainty features that does not scale with the implementation of this method.

A different system that is proposed in [117] describes an attention-based model that makes use of a long short-term memory network (recurrent neural network) to caption images. The work identifies through visualisation how the system determines a caption for an image, in which sections of an image play a vital role in the outcome and where mistakes are made. The LSTM makes use of an attention mechanism that focuses on the salient parts of an image in order to produce a useful image caption. Due to the scale of our problem this method would not be able to encompass the detail of design features and would overlook potential insights in its pursuit to generate a concise and understandable caption of a network solution. This loss of information means that this method would not be suitable for our problem.

Another interesting system is described in [118]; this system proposes a Super Parse Linear Integer Model (SLIM) which consists of a data-driven scoring system for the medical field. The paper suggests that SLIM can be extended to produce specialised rule-based systems, introduced as M-of-N rule tables which provide interpretability to the solutions derived from SLIM. This method makes use of a loss function that penalises misclassification, an interpretability penalty that provides qualities that are desirable, an interpretability set that is responsible for ensuring the qualities that are absolutely required, and a trade-parameter that determines the balance between accuracy and qualities. If the training data consists of binary rules this system is able to produce M-of-N rule tables, however, due to the rigidity of these rules bases they're unable to deal with the complex uncertainty of our problem.

There are several classification for XAI models that are commonly implemented, these include deep explanation, interpretable, and model induction methods. Each of these classifications contain many XAI models, often with similar accuracy or default transparency, and each with a particular method that they implement in order to explain problem features. These classifications are explored below:

I. Deep Explanation/Classification

These are some of the varying implementations of XAI; however, a greater range of models can be grouped under more defined terms. For example, 'Deep Explanation', systems defined under this model attempt to extract explainable features from deep learning models [119].

[120], [121] and [122] can each be termed as deep explanation models. These three examples all investigate the potential to extract useful information from visualisation tasks. [120] Proposes a novel reinforcement learning based loss method in order to influence the interpretability of their 'justification explanation systems' which aim to provide more detail to non-experts of a system.

II. Interpretable Models

Another model is coined, 'Interpretable Models'. This model attempts to understand the structure and interpretability of designed systems. Examples of this implementation are described in [116], [123], [124], and [125].

[123] Presents a unified framework for interpreting predictions. The method named SHAP implements additive feature attribution and assigns an importance value to each feature that represents the effect on the model prediction of including that feature.

In [124] a Knowledge Distribution Framework is explained, which coordinates a teacher model and one or more student models. The 'teacher' model is responsible for generating large labelled data sets, which the student models can then use to produce explainable results through logistic regression and gradient boosting trees.

III. Model Induction

The final model that will be explored is 'Model Induction'. Model induction attempts to produce an interpretable model from any AI system that is defined as a black box. Examples of two of these systems are found in [126] and [127]. The main difficulty, and therefore the main aim of model induction, is to glimpse an understanding of a system, which is essentially closed to the user.

[127] Approaches model induction by extracting a standardised rule-based knowledge representation from a systems input-output behaviour. The system creates a 'Rule Matrix', which enables users to visualise the rules of the black-box model without necessarily needing expert knowledge of the system.

The approach to XAI in [127] and many of the implementations outlined above focus significantly on methods that will provide greater understanding to non AI experts. This enormously important aspect must be respected, to ensure the continued research and uptake of AI models across as many fields as possible.

Many current approaches to XAI are interested in the visualisation of output. This means that XAI systems seek methods of translating typically crisp output to visually interpretable output, such as graphs, explainable descriptions, and explainable if-then rule bases. The work of [128] investigates a novel method to extract an explainable rule base, which can be further designed and presented to end users in order to deal with the complexity of the problem at hand.

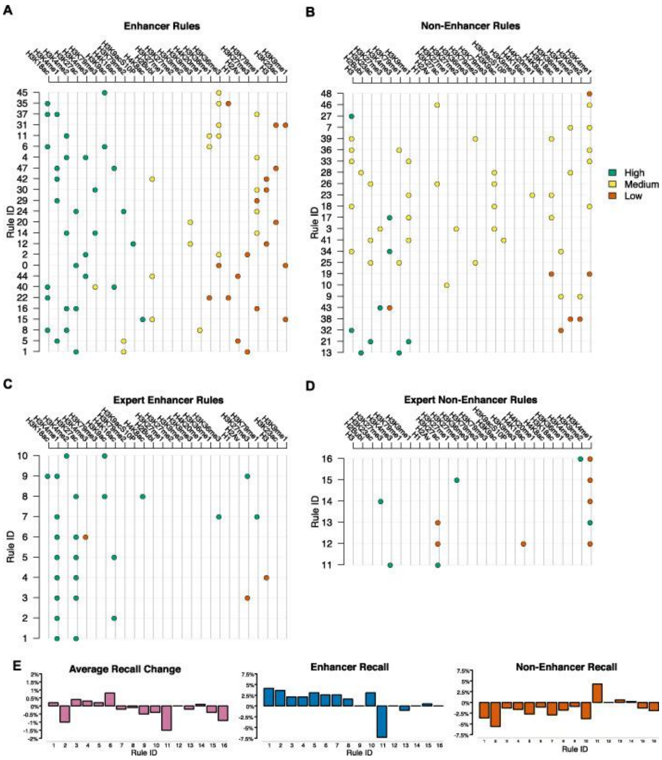


Fig. 5.2 Representation of extracted rules for enhancer and non-enhancer classification [128]

5.4 Explainable Fuzzy Logic

Fuzzy Logic systems are another method that could be implemented to produce an explainable model. Fuzzy Logic systems could potentially supply systems in which information is presented to end users in human-understandable ways, based on the way in which they attempt to mimic human thinking [106]. Fuzzy logic systems contain the innate ability to represent complex real world scenarios in explainable ways, which makes them ideal as XAI methods.

The linguistic IF-THEN rules discussed in section 4.1.3 enable fuzzy logic systems the ability to represent data in interpretable ways such as in Fig 5.2. A process of fuzzy rule extraction can be implemented in order to generate a rule base that can provide insight into the outputs of a system.

5.4.1 Fuzzy Rule Extraction

There are multiple approaches to fuzzy rule extraction. In this section, we will explore the improved Wang-Mendel method [129] [130]. Fuzzy rule extraction is a one-pass technique that uses a systems data to generate fuzzy rules. This method can be used to extract multiple-input-multiple-output rules from interval Type-2 fuzzy sets, which describe the relationships present in the data. For example:

$$IF\ x_1\ is\ \tilde{A}_1^l\ \dots and\ x_n\ is\ \tilde{A}_n^l\ THEN\ y_1\ is\ \tilde{B}_1^l\ and\ y_k\ is\ B_k^l \tag{5.1}$$

In order to describe the input and output sets of this rule, the following is detailed. $l = 1, 2, \dots, M$, where M is the number of rules and l the index of those rules. There are V_i interval Type-2 fuzzy sets $\tilde{A}_s^q$, $q = 1, \dots, V_i$ defined for each input x_s where $s = 1, \dots, n$. For the output it can be described by having V_o interval Type-2 fuzzy sets $\tilde{B}_c^h$, where $h = 1, \dots, V_o$ defined for each output y_c in which $c = 1, \dots, k$.

Each V_i and V_o input and output fuzzy sets can be extracted from data as in [130] or be designed in the traditional manner as discussed in section 4. The following method describes the simplified notation for extracting single output fuzzy rules from data.

1. **Step One**

The first step requires that each input-output pair present in the data $(x^{(t)}, y^{(t)})$ follow the process described in section 4.2.2 in which the upper and lower membership values for each fuzzy set and variable are calculated. Both $\overline{\mu}\tilde{A}_s^q(x_s^{(t)})$ and $\underline{\mu}\tilde{A}_s^q(x_s^{(t)})$ represent the upper and lower membership values for each fuzzy set $q = 1, \dots, V_i$ and for each of the input variables $s = 1, \dots, n$.

Therefore, for each fuzzy set it is required to find $q* \in \{1, \dots, V_i\}$ in which:

$$\mu^{cg}_{\tilde{A}^{q*}_s}(x^{(t)}_s) \geq \mu^{cg}_{\tilde{A}^{q}_s}(x^{(t)}_s) \tag{5.2}$$

Where $\mu^{cg}_{\tilde{A}^{q}_s}(x^{(t)}_s)$ is the centre of gravity of the interval membership $\tilde{A}^{q}_s$ at $x^{(t)}_s$. This is calculated by the following equation:

$$\mu^{cg}_{\tilde{A}^{q}_s}\left(x^{(t)}_s\right) = \frac{1}{2}\,[\overline{\mu}\bar{A}^{q}_s(x^{(t)}_s) + \underline{\mu}\tilde{A}^{q}_s(x^{(t)}_s)] \tag{5.3}$$

For each rule derived in this manner, it can be said that the rule was generated by $(x^{(t)}, y^{(t)})$ in such a way that:

$$IF\ x_1\ is\ \tilde{A}^{q*(t)}_1 \dots and\ x_n\ is\ \tilde{A}^{q*(t)}_n\ THEN\ y\ is\ centered\ at\ y^{(t)} \tag{5.4}$$

In this manner, a rule can be generated for each input variable x_s characterised by an interval Type-2 fuzzy set. Therefore, it is possible to generate a maximum of V^n_i possible rules. However, it is possible that not every rule will be generated by this step; this is because a generated rule is dependent on a data point falling within the dominant region of a fuzzy set. To expand upon this, a single rule is generated for each input-output data pair, where it is the fuzzy set that obtains the maximum membership value for the particular data point that is selected as the antecedent IF section of the rule.

However, this is not the deciding factor that determines the final rule set. For each rule that is generated using the previous step, its weight is calculated as follows:

$$wi^{(t)} = \prod\nolimits^{n}_{s=1} \mu^{cg}_{\tilde{A}^{q}_s}(x_s(t)) \tag{5.5}$$

Which describes the weight of a rule $wi^{(t)}$, as the measure of firing strengths of each point $x^{(t)}$ that belongs to the fuzzy region covered by the rule.

This step is repeated for all of the data points *t* from *one to N* in order to obtain *N* data generated rules that are formatted as shown in Equation 5.1. Due to the intersecting nature of the data used to generate the rules, many generated in this step share the antecedent IF section and can be considered conflicting.

2. Step Two

This step is concerned with reducing and simplifying the number of conflicting rules. Rules found to share the same antecedent IF section are combined into a single rule. If it is assumed that there exists M groups, it can be defined that group l has N_l rules that are presented in the following format:

$$IF\ x_1\ is\ \tilde{A}_1^l \dots and\ x_n\ is\ \tilde{A}_n^l\ THEN\ y\ is\ centered\ at\ y^{(t_u^l)} \tag{5.6}$$

Where $u = 1, \dots, N_l$ and t_u^l is the index of the data points for group l. It is from this point that the weighted average, consisting of all of the rules that are present in the conflicting group is calculated with the following equation:

$$av^{(l)} = \frac{\sum_{u=1}^{N_l} y^{(t_u^l)} wi^{(t_u^l)}}{\sum_{u=1}^{N_l} wi^{(t_u^l)}} \tag{5.7}$$

With this, it is now possible to combine and simplify the conflicting rules into a single rule that takes the following form:

$$IF\ x_1\ is\ \tilde{A}_1^l\ and\ x_n\ is\ \tilde{A}_n^l\ THEN\ y\ is\ \tilde{B}^l \tag{5.8}$$

This describes the fuzzy output set $\tilde{B}^l$ as the chosen set from all of the interval fuzzy output sets and this is determined by:

$$\mu_{\tilde{B}^{h*}}^{cg}(av^{(l)}) \geq \mu_{\tilde{B}^h}^{cg}(av^{(l)}) \tag{5.9}$$

The rules that have been generated now represent a variation of the fuzzy system that could be generated by a user's data. These rules can play a role in the fuzzy systems ability to find solutions. However, there is a more significant role for these rules in relation to explainable

AI; the generated rule base now presents the user with an easily interpretable representation of the relationship between a datasets inputs and outputs.

The second step is therefore very significant to a user's ability to understand what is being presented. Without the second step, it would not be uncommon to present an overwhelming number of rules that are mostly the same and only lead to reducing the interpretability of the generate rule base. It is therefore important to reduce the generated rules down to a concise set that presents a clear interpretation of a system.

However, there still exists a problem with this method. If the data used to generate the rule base has inconsistencies or missing regional values, it is possible that certain mappings between the input and output pairs are not present and so do not generate a potential rule.

5.4.2 Similarity Metric

To ensure that each input triggers a rule from an incomplete rule base, even when there is no mapping to a direct fuzzy output set, a similarity check can be made upon the fuzzy rules. With this method, an input found not to combine with any output is compared against the rules already generated. The relationship between the input and the existing rules is measured as a distance function.

If an input variable x_1 is found to have no output, it is given a compatibility grade with the existing rules that is a measure of its similarity [131]. Of the existing rules, the one found to be most similar is chosen to classify the input. In order to find the most similar rule, the fuzzy set that achieves the maximum membership value for the data point is selected:

$$A_s^{q*}\left(x_s(t)\right), where\ q = 1, \dots, V_i; s = 1, \dots, n \tag{5.10}$$

Therefore, a rule can then be obtained for the input $x^{(t)}$ in which the antecedent part of the rule is written as follows:

$$IF\ x_1^t\ is\ \tilde{A}_1^{q*}\ and\ x_n^t\ is\ \tilde{A}_n^{k*} \tag{5.11}$$

Following this, the similarity between the antecedent of the generated rule and the pre-existing rules is calculated as a distance function that is represented as $\mathfrak{D}(\tilde{A}_i^{q*}, \tilde{A}_i^{(q)})$. With this, it is possible to define a distance that identifies the difference between the linguistic labels of the generated and pre-existing rules. To expand upon this, an example from Fig 4.2 would identify the distance between variables as such $\mathfrak{D}(Slow, Gradual) = 1$ or $\mathfrak{D}(Slow, Quick) = 3$.

With the distance between the sets calculated, the final similarity between the generated rule and the pre-existing rules can be calculated as follows:

$$S\left(x^{(t)}, R_q\right) = \frac{\sum_{i=1}^{n}(1 - \frac{\left(A_i^{(t)}, A_i^{(q)}\right)}{V-1})}{n} \tag{5.12}$$

Where $S\left(x^{(t)}, R_q\right) \epsilon\ [0,1]$ and *n* is the number of antecedents of the rule.

When the generated rule is compared against each of the existing rules in the rule base, the similarity distance, which is found to be the highest, corresponds to the higher degree of similarity between rules. Subsequently, for each rule R_q a similarity metric should be calculated against the generated rule, such that:

$$S\left(x^{(t)}, R_{q*}\right) \geq\ S\left(x^{(t)}, R_q\right), for\ all\ q = 1, \dots, M \tag{5.13}$$

This finally evaluates with the generated rule being assigned the consequent section of rule R_{q*}.

Through these methods, it is possible to generate a rule base that enables end users the ability to interpret the relationships and mappings that exist within the universe of discourse for a particular problem. It is also possible with these methods to ensure that the rule base generated does not exhibit bias and represents fairness in regards to the relationships present in the problem space.

5.5 Discussion

In this chapter, an overview of explainable artificial intelligence was presented. The history surrounding our collective interest of the inner workings of AI was discussed and an argument was presented that considered how our interest in the interpretability of AI had changed.

A discussion was also held that considered what the most important features of XAI currently are, before an insight was given to the current methods that are being implemented in an attempt to gain a better understanding of black box AI models.

This chapter then explained in detail the strengths of implementing a fuzzy based system as a form of XAI and gave an overview of how fuzzy rules can be generated from data in order to produce a level of interpretability about a solution or problem.

In the following chapter, the proposed many-objective optimisation algorithm and its ability to improve the accuracy of generated access networks will be explored.

Chapter 6. The Proposed Type-2 Modified Approach for the Improvement of Access Network Objectives Optimisation

The proposed method uses the Non-Dominated Sorting Genetic Algorithm II as the many-objective optimisation technique; a representation of this proposed system is presented in Fig. 6.1. The proposed method combines the many-objective optimisation technique alongside both fuzzy optimisation and interval type-2 fuzzy dominance. Together these techniques forge a novel method capable of optimising solutions to complex grand scale real-world many objective problems; the two forms of fuzzy logic, integrated into the many-objective optimisation algorithm allow this approach to tackle problems typically too complex to solve with traditional optimisation.

In this system, an initial population of a variable size is created. This population is made up of chromosomes representing an individual solution. The chromosomes in turn are made up of genes. For our real-world problem, these gene objects represent a tree structure of network equipment; each comprising gene is responsible for the geographical position of a piece of network equipment, such as a copper only distribution point, or a fibre optic splitter.

Each chromosome from the initial population is then evaluated based on the problems objective functions and ranked based on the dominance system in place. A child population is then created via selection, crossover, and mutation, each explored in detail in Chapter 3. Tournament Selection and variable length crossover are the implemented methods in regards to the proposed system.

Within the context of the real-world business problem, the chromosomes do not have a fixed size, as such; variable length crossover has been implemented in order to produce suitable chromosomes for the child population. This process of variable length crossover is illustrated in Fig. 3.9. The reason that each chromosome in the population may contain a varying number of genes is to allow solutions to consist of different combinations of network equipment. Each gene, that is representative of a piece of delivery network technology, can exist in this manner due to the network constructor that evaluates the validity of a network. This network constructor operates on each chromosome of the population and ensures that when new genetic material is introduced into them via crossover or mutation, the new chromosome does not violate design constraints such as double connections to premises, or non-connections between technology and network exchange.

The child population is evaluated and combined with the initial parent population. Then the entire population is ranked via the dominance rules before being reduced back down to the initial population size. At this point, if the stopping criteria is reached, the final Pareto-set is supplied to the user; otherwise, the process produces a new child population in order to explore the solution space.

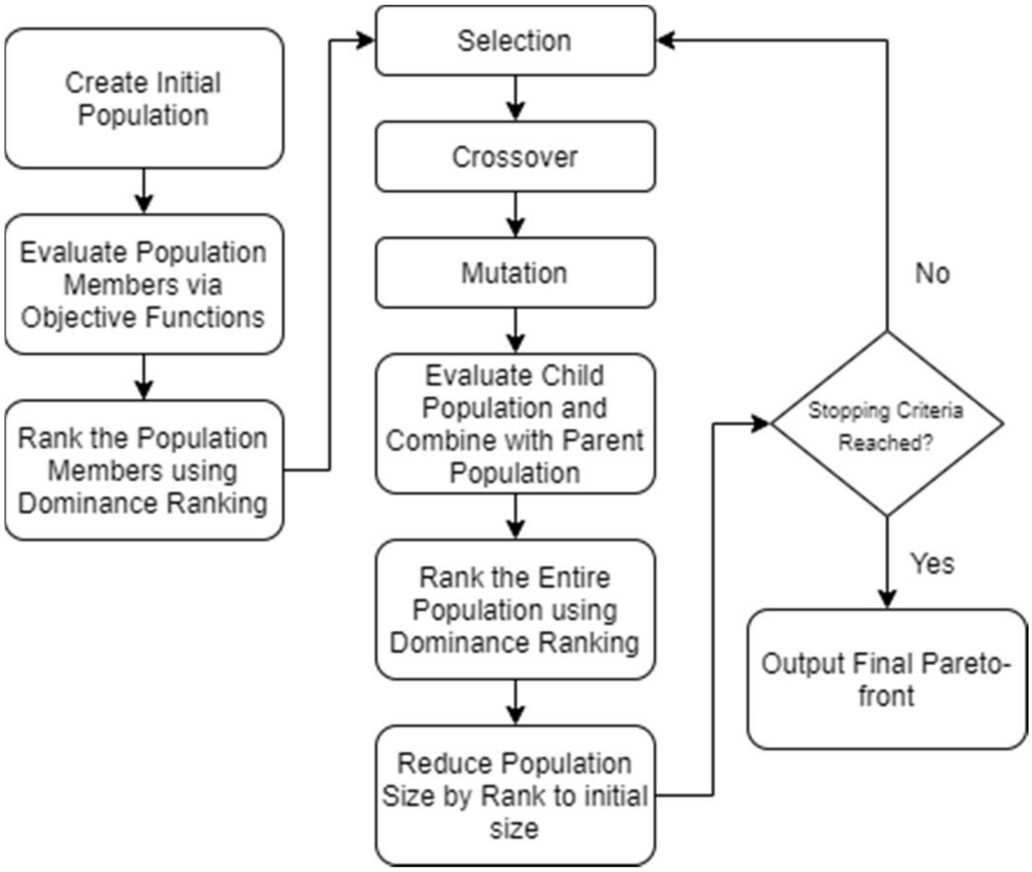


Fig. 6.1Many-Objective Optimisation

6.1 The proposed Type-2 Fuzzy Logic Dominance System

As the number of objectives in a problem increases, traditional dominance rules are found to be inadequate at converging on a truly dominating Pareto-front. Crisp dominance evaluation leads to an oversaturation of the Pareto-front. For this reason, we have implemented a Type-2 Fuzzy Logic System in order to identify the truly dominant solutions.

The proposed system considers the dominance of solutions as described in [82]. In [82], they determine that for one solution to be dominant over another, it must have no objective value that is worse, and at least one objective value that is better than the corresponding objective values in the solution to which it is being compared. However, it is here that the proposed system introduces the Fuzzy Logic System. The traditional method implements crisp objective values to determine a solution rank; each chromosome in the population is assigned n objective values that depend on the fitness of the solution. The proposed system, using a Fuzzy Logic

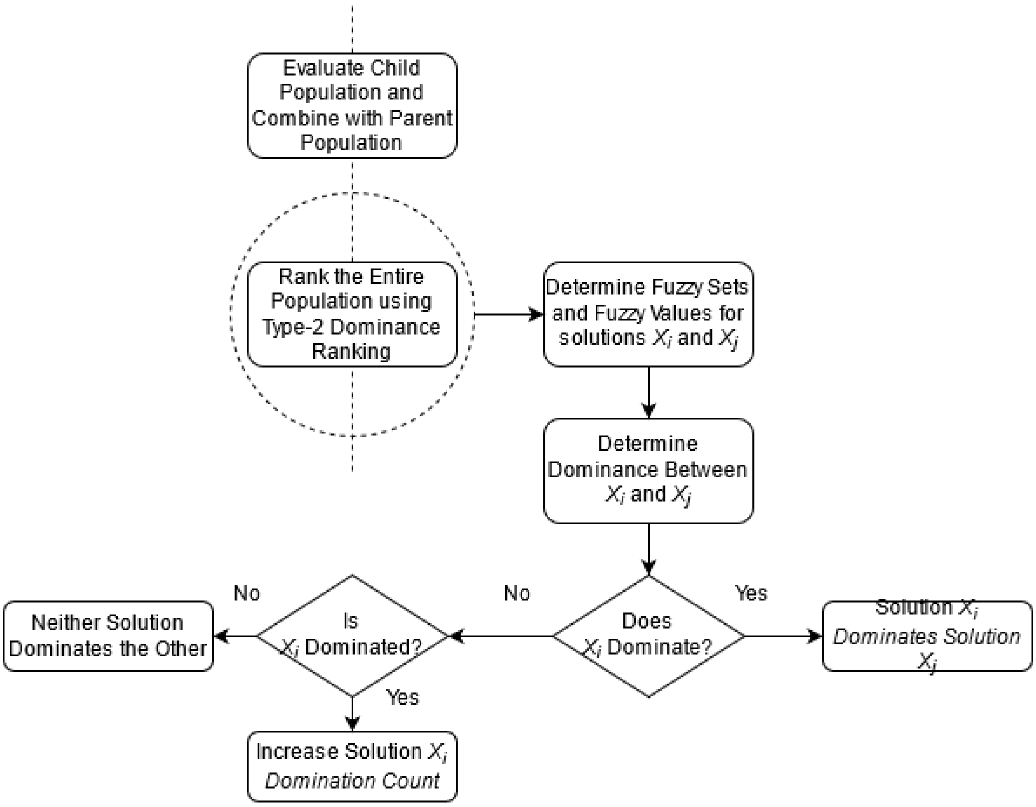


Fig. 6.2 Proposed NSGA-II system with modified dominance ranking

System, converts the crisp objective values to fuzzy; this is done through the interval fuzzy methods that are outlined starting at Chapter 4.2.1. One reason that an interval Type-2 fuzzy logic system is implemented is that the FOU allows for greater accuracy and improved modelling of uncertainty. In order to benefit from the fuzzy system, the fuzzified values must go through the defuzzification process that involves type reduction. This is one concern of the system, as many processes must be undertaken as the increased complexity of Type-2 fuzzy logic could cause potential computational problems. However, as the approach to this problem is considered grand scale in size and timeframe, the additional computational complexities are offset by the increase in solution accuracy.

The entire process of the proposed system is identified in Fig. 6.2. In this figure, we can see that during the normal execution of the many-objective optimisation algorithm, each member of the population is compared against one another and used to produce varying interval Type-2 FLS. A final crisp value is returned for each objective value, for each member of the population; when determining the dominance of one solution over another, these crisp values are considered in total, alongside the assertion about dominance stated earlier in this Section. If the crisp outputs determine that a solution has no objective values that are worse than the corresponding crisp value of another solution, and it has at least one objective value that is better, it can be considered dominant over the other solution. If this assertion is not evaluated as true, then the proposed system can identify dominance in the opposite direction, if the comparable solution follows the same assertion. If this too is not the case, then it can be said that neither solution dominates the other. This is valuable to the system as the NSGA-II algorithm must keep track of the dominance and dominated counts of a solution; these two values are required in order to determine which front each solution will be placed.

There are many features within the FLS. For example, Fig. 6.3 illustrates how the fuzzy input sets are generated for each solutions objective values. Each set is bounded between the minimum and maximum values that exist in the universe of discourse; it is important to ensure that the fuzzy sets are normalised between these bounds to ensure an overlapping comparison can be made between fuzzified objective values. In the proposed system there are two input sets, each generated using the corresponding objective values from each solution. One input set is used to compare the objective values of one solution against another; the other input set is used for the reverse of this comparison. The linguistic terms, ‘Smaller’, ‘Equal’, and ‘Larger’ have been chosen in order to identify which of the objective values dominate, regardless of the type of objective function which is being considered (i.e. minimisation/maximisation objectives). For our proposed system, the fuzzy sets can be intuitively determined from consideration of the varying solutions objectives values. However, Type-2 fuzzy sets can be constructed through various methods such as c-means clustering as described in [132].

The proposed system also consists of an output fuzzy set, which is depicted, in Fig. 6.4. This output set is responsible for determining if the objective values of solution X_i are smaller, equal to, or larger than the objective values of solution X_{i+1}. These linguistic terms have been chosen as they gain meaning depending on which type of objective function is being evaluated. The fuzzy output set in Fig. 6.4 shows the range of values for which a crisp value

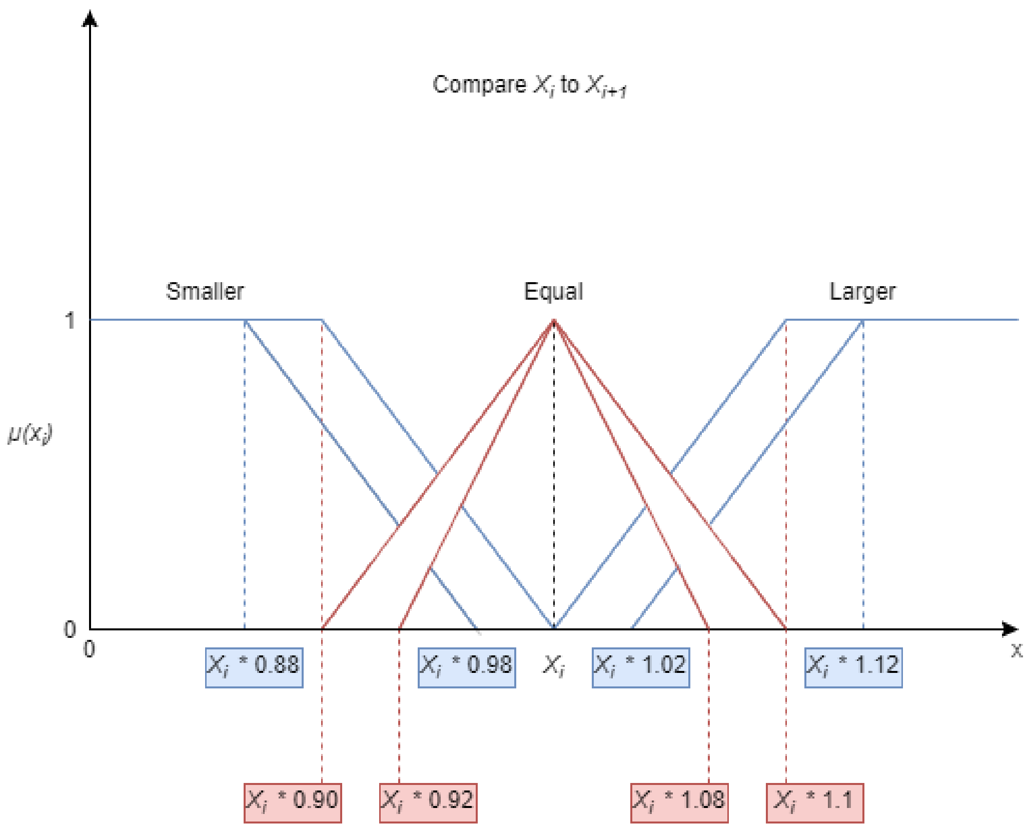


Fig. 6.3 Interval Type-2 Fuzzy Input Set for Variable Objective Values

may be returned by the FLS. For example, for a minimisation objective, a crisp value of 20 or more suggests that the objective value of solution X_i dominates the objective value of X_{i+1}.

However, for a maximisation problem the inverse is true, a crisp value of 20 or more suggests that the objective value of solution X_i is dominated by that of solution X_{i+1}. The final output is calculated through the Centre of Sets (COS) method after type reduction has taken place; COS is used to defuzzify the information into a crisp value, which can be interpreted by the proposed optimiser.

The final aspect of the proposed FLS that should be highlighted is its rule base, which is given in Table 6.1. Each of the fuzzy inputs and outputs are represented by three fuzzy sets, as shown in Fig. 6.4; the rules displayed in Table 6.1 show two comparison columns which are the representation of these fuzzy sets. The column 'Compare X_i to X_{i+1}' is a representation of

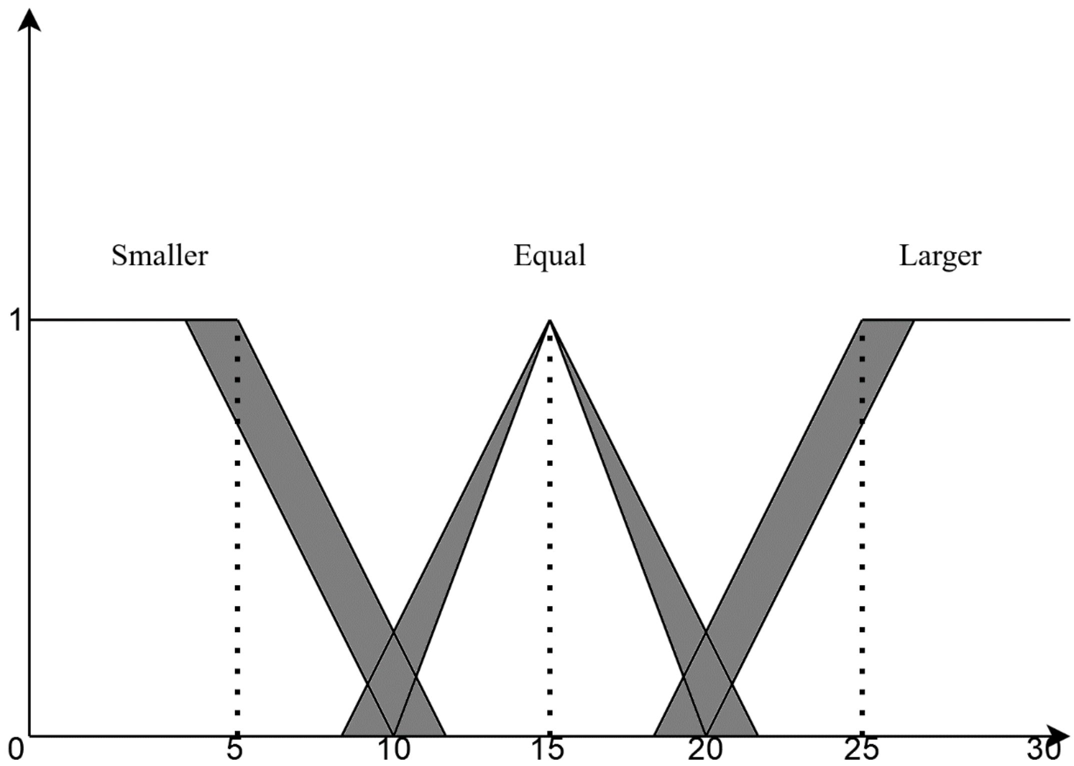


Fig. 6.4 Interval Type-2 Fuzzy Output Sets

solution X_i being compared to solutions X_{i+1}. The overlap between these sets means that solutions with a minute difference are seen as equal rather than dominate, this is an improvement over traditional and type-1 dominance. The second column 'Compare X_{i+1} to X_i' shows the reverse comparison between the solutions. It is this two-way validation of the solutions dominance values that strengthens the final dominance decision.

The rules for the FLS have been designed to ensure that a greater distinction can be made between two comparable solutions in the population, in which one may dominate the other and would not be identified as such using traditional crisp dominance rules. This is the considerable advantage to using Type-2 fuzzy logic, because it allows for greater complexity.

Rule Number	*Compare* X_i *to* X_{i+1}	*Compare* X_{i+1} *to* X_i	*Output (*X_{i+1} *is…)*
1	Smaller	Smaller	Equal
2	Smaller	Equal	Smaller
3	Smaller	Larger	Smaller
4	Equal	Smaller	Larger
5	Equal	Equal	Equal
6	Equal	Larger	Smaller
7	Larger	Smaller	Larger
8	Larger	Equal	Larger
9	Larger	Larger	Equal

Table 6.1 Rule base for proposed Type-2 interval system

6.1.2 Experiments and Results

The experiments for our proposed system test the NSGA-II algorithm on a series of many-objective problems in order to compare the variations in the final Pareto-fronts. The NSGA-II algorithm will be considered in three variations, NSGA-II Normal Dominance, NSGA-II Type-1 Fuzzy Dominance Rules (NSGAIIFDR), and the proposed NSGA-II Type-2 Fuzzy Dominance Rules (NSGAIIT2FDR).

We can compare the various Pareto-fronts by evaluating their convergence and diversity compared to a true Pareto front, as described in [133] and [134]. Typically, fronts, which return smaller values for the two metrics, indicates a better set of solutions in the non-dominated front. We calculate the convergence metric by taking a sample of uniformly spread solutions from a true Pareto front. We then calculate the minimal Euclidean distances between the chosen sample solutions and the solutions on our generated non-dominated fronts. It is the average of these distances, which identify the convergence metric.

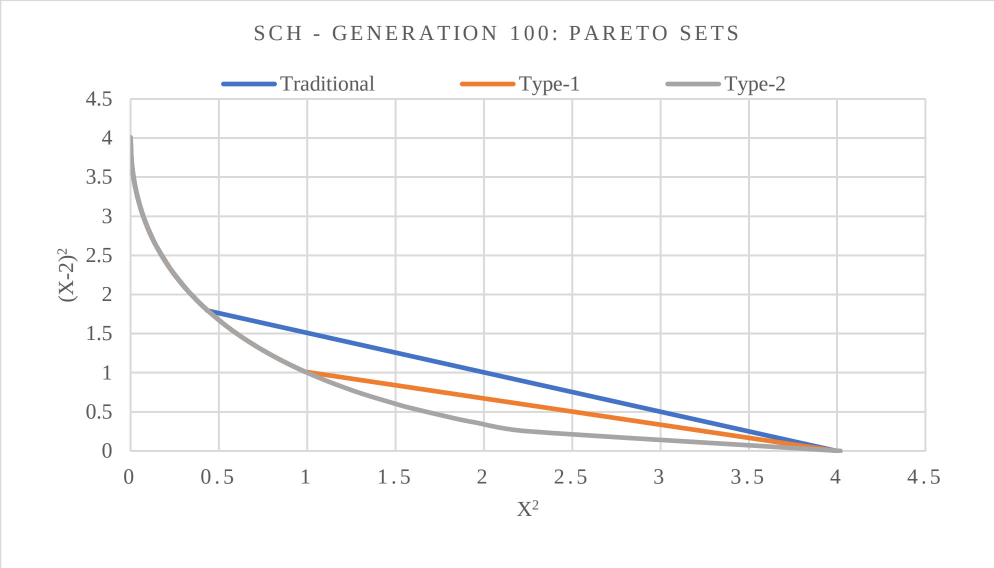


Fig. 6.5 The Pareto Sets Generated from the SCH Test Problem for the Three Different Variations of the NSGA-II Algorithm

The diversity metric is obtained by calculating the Euclidean distance between the boundary solutions of our non-dominated fronts and the extreme solutions of the true Pareto front d_f andd_l. The Euclidean distances between neighbouring solutions is also calculated d_i along with the average of those distances $\bar{d}$ [133].

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} |d_i - \bar{d}|}{d_f + d_l + (N-1)\bar{d}} \tag{6.1}$$

The Pareto sets generated from the SCH test problem for the three different variations of the NSGA-II algorithm.

In order to test the validity of the proposed system, the first experiment tested the three variations of the NSGA-II algorithm on a well-known many-objective test problem, which is described in [133]; the Schaffers' Study (SCH), gives a comparison between the various forms of the NSGA-II algorithm on a problem with only two objective functions. The objective functions are both minimisation problems, which are labelled in Fig. 6.5. Due to both objective functions being minimisation, we would expect to see the most optimal solutions located in the bottom left hand corner of the plot. For this test problem, a population of 100 chromosomes propagated for 100 generations before the Pareto sets were retrieved. We can identify from Fig. 6.5 that the proposed system, which is implementing interval Type-2 fuzzy dominance rules, produces a non-dominated front, which contains a greater number of optimal and diverse solutions across the Pareto front than the systems with traditional and Type-1 fuzzy dominance rules. This can be identified from the results in Table II. These results show that the proposed interval Type-2 fuzzy dominance rules were able to produce a smaller value for both the convergence and diversity metrics. The diversity value of the NSGA-II algorithm with Type-1 fuzzy dominance rules was 8% better than that of crisp dominance rules. The proposed system was 46% better than that of the Type-1 fuzzy dominance rules and 59% better than crisp dominance rules; these results can be interpreted from the shapes of the produced Pareto fronts in Fig. 6.5. The convergence figure between the three systems varies only slightly, however, as the convergence metric is a measure of solution spread in relation to a known pareto-optimal front, this metric becomes less valuable when applied to real-world business problems. Therefore, while the proposed system still provides the best convergence value, it is more significant that the proposed system also achieves a much greater diversity value; which identifies greater information concerning the obtained solutions.

	NSGA-II	**NSGA-II FDR**	**NSGA-II IT2FDR**
CONVERGENCE (Υ)	0.03083	0.03082	0.03071
DIVERSITY (Δ)	0.04672	0.04297	0.02924

Table 6.2 Convergence and Diversity Metrics on SCH Test Problem

Following the standard many-objective test, we then applied the same systems to a real-world business problem. The problem at hand consists of three objective functions, which are the minimisation of spending costs, the maximisation of connected premises, and the minimisation of unfulfilled bandwidth, all of which have been described in greater detail in Chapter 2.

The objective functions relate to a set of goals that we want a network structure to achieve. The complexity and strategic approach taken towards the problem means that variation of the objectives and constraints allows for vastly different network structures to be found. This is significant because we are interested in finding design insights for this problem in which parameters can easily fluctuate.

The NSGA-II algorithm has been modified to produce a population of network chromosomes. These network chromosomes are comprised of varying network equipment responsible for the delivery of bandwidth. For our problem, a defined region is populated by premises each with a variable bandwidth demand. The network chromosomes are responsible for creating a viable network structure that can connect to the available premises while attempting to balance the objective functions of cost, coverage, and unfulfilled bandwidth demand. The proposed system generated a population of 50 network chromosomes, each a network of its own, and had them propagate for 100 generations in order to produce their Pareto fronts.

The structure of the network chromosomes further increases the complexity of the problem. For this reason, the solution fronts tend to converge around small clusters of solutions, which reduces the diversity of solutions in the final non-dominated front. However, we have found that the proposed interval Type-2 fuzzy dominance system can produce both greater diversity and more optimal solutions into its final front. Fig. 6.6 is one example of the proposed system producing more optimal final fronts than the traditional and Type-1 FLS dominance rules. The front produced by our proposed system is closer to a true Pareto-front. The shape of the Type-2 front in Fig. 6.6 shows that the produced network solutions were more able to minimise the cost and unfulfilled bandwidth demand, while maximising the network coverage. The proposed system was able to produce solutions that more readily identified the most optimal network structure and equipment placement than the compared systems. In this way, a solution could reduce its costs by constructing a network with less equipment that is more efficiently placed. This results in greater coverage and reduced unfulfilled bandwidth by targeting premises with higher broadband demands.

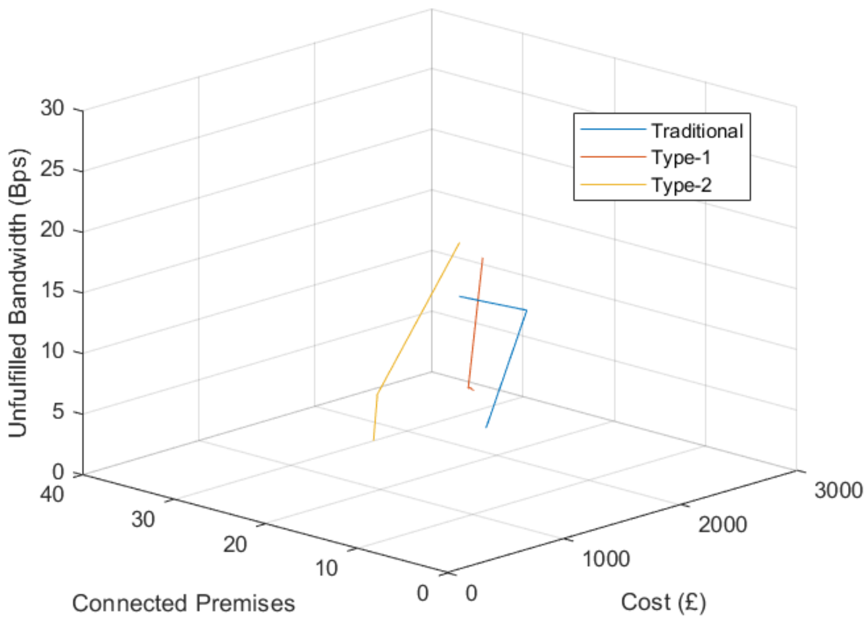


Fig. 6.6 Pareto Fronts Generated from the Real-World Access Network Problem for each NSGA-II Variation

6.2 Construction of Viable Cost Effective Networks

6.2.1 Modelling of varying UK regions

To expand upon these experiments, additional regions of the United Kingdom were modelled; these regions were modelled from a collection of open source UK data alongside BT data. Combinations of population densities, broadband requirements, and demographics were used to design rural, mixed, and urban population centres; these modelled regions were created following procedural generation methods and can be represented as fully connected graphs. It was significant for the project that the various regions of the United Kingdom could be modelled independently and, if required, stitched together to form increasingly greater, yet accurate, models of the UK topology. These regions could be designed to represent a range of typical real conditions; the population density of these regions could be adjusted so that regions of varying sparseness can be tested. Of the regions tested, three designations were used that included:

Urban – Varying levels of densely populated cities/large towns with a greater average broadband requirement.

Rural – Varying levels of sparsely populated villages/hamlets with a lower average broadband requirement.

Mixed – A designation created for the system; consists of a mixture of a large town with densely populated centres and also the expanded rural outskirts found between towns.

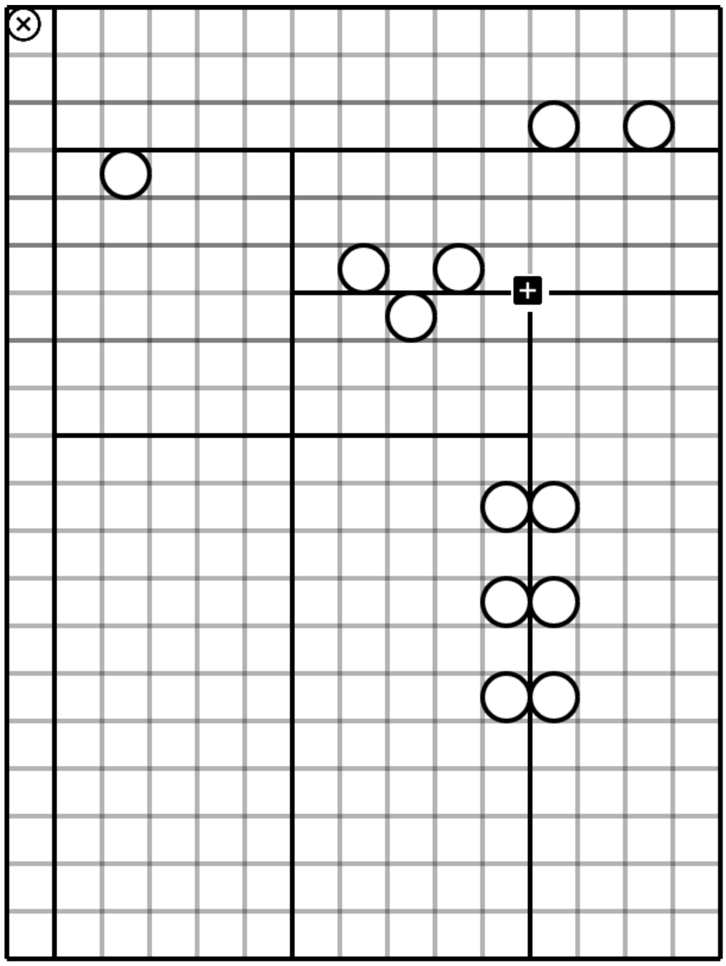


Fig. 6.7 Example of sparsely populated rural region, consisting of few premises (O), a Telephone Exchange (x), and an example distribution point (+)

Fig 6.7 gives a representation of the style of region that might be constructed by the procedural generation system. The region in Fig 6.7 would be considered a very sparse area, consisting of few premises and roads guaranteed to lead to the exchange building. In order to reduce modelling/optimisation complexities and simulate real-world practises, the optimisation and network constructor makes use of the start and end of streets as location that can contain network distribution equipment. This guarantees that network equipment will ultimately be able to connect back to the telephone exchange via the constructed roads and also models the typical position for telephone poles/distribution points.

Each of the individual regions generated with this method were standardised to $1Km^2$ so that average population densities can be modelled depending on the rural/urban classification process [135].

6.2.2 Additional Experiments and Results

The additional experiments placed greater importance on identifying network architectures that were able to satisfy customers requirements while reducing costs by not over delivering on bandwidth needs. This was a significant experiment to undertake as it identifies how the proposed system can be implemented in order to explore the possibilities of access network design that may be in contrast to the current approach. This argument was alluded to in Chapter 1; the current upgrade plan is influenced to some degree by political motivations. However, the proposed system suggests that network designs can meet the operation requirements and reduce costs that would be spent on unnecessary upgrades that would far surpass a customer's broadband requirements, and potentially reduce customer satisfaction through increased service costs.

A series of experiments were undertaken to see whether the proposed system could produce solutions that identified the argument just given; the system was tested on the three defined regions previously discussed. In order to test our hypothesis, the network constructor and many-objective optimiser are modifiable and so can be adjusted for each experiment to allow specific network technologies to be used in the delivery of broadband. This meant that the optimisation of networks could be done solely with FTTP technology, mixtures of FTTP and FTTC, and wireless technology alongside FTTP/FTTC; and so this was carried out.

The first experiment tested the optimisation of rural networks. Rural regions in the UK can be identified within six categories, ranging from hamlets and isolated dwellings to towns and fringe (for the purposes of these experiments the mixed regions represented the towns and fringes/in-sparse settings) [136]. For this reason, the population density could be modified to represent the variety in rural UK regions. Network solutions were created through the proposed system that contained the different combinations of network technologies. A population of 100 chromosomes were allowed to propagate for 1000 generations - this was repeated 10 times. With each test, the solution in the final Pareto front that obtained both the lowest costs and undelivered bandwidth was selected. It was identified that using the proposed system could save on average 51.99% on spending costs when both FTTP and FTTC technologies are used in the construction of a rural network over only FTTP, with both being able to meet the broadband demand.

RURAL REGION	
COST (£)	
FTTC; FTTP	**FTTP**
80126.46	152546.78
68959.34	143683.59
78692.69	105270.18
80947.91	160181.03
88893.82	115287.70
79524.04 (£)	**(£) 135393.86**
51.99%	

Table 6.3 Rural Optimisation results, Comparison between FTTC/FTTP combinations over FTTP.

This experiment was extended to consider the mixed regions previously discussed. These regions typically have greater variation between the interior and outskirts; as such they were modelled accordingly. The regions constructed represent towns with increasingly dense centres that become more sparsely populated on the outskirts; the procedural generation of these models allowed for this variation to be exhibited. As the population density of regions increases, as it does between the rural and mixed regions, there is also an increase in the amount of flats that are present. The premises that are present in more densely populated regions need be considered and so the models also take this into account. It was found that the proposed system was capable of producing solutions that saved on average 54.16% on spending costs for mixed region networks when the combination of FTTC and FTTP technologies were used in comparison to only FTTP, and again both combinations were able to supply the broadband demand.

MIXED REGION	
COST (£)	
FTTC; FTTP	FTTP
137410.35	312679.97
137011.95	259597.56
138930.81	260503.35
132809.56	332331.09
131071.15	312387.30
135446.76 (£)	**(£) 295499.85**
54.16%	

Table 6.4 Mixed Optimisation results, Comparison between FTTC/FTTP combinations over FTTP.

The final experiment investigated urban population centres; alongside the vastly increased population was a sizeable increase in broadband requirements. The modelling of these regions consisted of highly populated homogenous areas; the ratio between low-density residential housing and high-density residential flats shifted in favour of the latter. It was in this experiment that we found the combination of FTTP and FTTC technology to be cheaper by an average of 34.48%, yet worse performing in its ability to deliver the broadband demand by an average of 69.33%. However, it was also noted that, although FTTP was able to provide premises with an above average broadband demand, the vast majority of premises could have their requirements met by FTTC technology. This is significant because as previously mentioned, simply providing the highest broadband rates may not identify the optimal solution or provide the best service to customers that may now be required to pay additional costs for a service they do not require. During the optimisation of urban regions a check was made each time a premise was connected to broadband delivery equipment; each premises connected via FTTP was investigated and the broadband requirements of each premise was compared against FTTCs ability to provide the required broadband demand. It was found that approximately 14.23% of all premises connected via FTTP could have been adequately supplied with FTTC. When scaled nationwide this presents a concerning waste of resource for premises in all regions of the UK; it is particularly wasteful in very sparsely populated rural regions that present additional challenges in the building of new network equipment.

URBAN REGION	
COST (£)	
FTTC; FTTP	**FTTP**
142293.35	269778
155889.18	203958.4
148160.11	222399.2
141073.91	206291.4
150801.05	224243.9
147643.52 (£)	**(£) 225334.2**
34.48%	

Table 6.5 Urban Optimisation results, Comparison between FTTC/FTTP combinations over FTTP.

From these experiments, we are able to identify a number of significant conclusions. Firstly, it is apparent that it is possible to use the proposed system to produce solutions that reduce the cost of potential network upgrades without relying on unnecessary upgrades that have the potential to harm customer satisfaction. However, we can also identify that as a region develops in complexity and population it is more difficult to supply a suitable bandwidth with older technologies. However, this is from the consideration of current technology and broadband demand. The experiment identifies how the proposed system can benefit the long-term strategic development of the access network. The simplicity with which technology can be modelled, and the ease with which varying UK regions are created further highlights how the proposed system can be implemented in the extraction of ideas and planning of this problem.

6.3 Discussion

This work presents a solution to the inherent difficulties faced in modelling the vast complexities of real-world many-objective optimisation problems. The optimisation of problems in which layers of complexity can continually be added leaves traditional dominance rules inadequate in their ability to produce a truly dominating Pareto-front. This chapter described a system, which attempts to address the current limitation of dominance rules, in which interval Type-2 fuzzy logic is integrated with dominance rules in order to model the greater complexity of a given problem.

We validated the proposed system on a many-objective optimisation test (SCH), alongside crisp and Type-1 fuzzy dominance rules. We found the proposed system outperformed the other two systems in both diversity and convergence of solutions towards the true Pareto front.

We continued to test the proposed system on a real-world many-objective problem, which required a population of network solutions to be constructed under the constraints of three objective functions; namely, cost minimisation, customer coverage maximisation, and unfulfilled bandwidth minimisation. We found that the complexities of the problem limited the solution fronts which were generated; even so, the proposed system was more capable at producing a more optimal non-dominated front than traditional dominance rules, which have been found to struggle when the number of objectives and complexity of a problem increases.

To extend upon the experiments that were undertaken a system was created that allowed for the creation of modifiable and modular regions inspired by the actuality of regions in the United Kingdom. We found that experiments that focused on the individual regions of the UK highlighted flaws in a reactive approach to network upgrading that could be elevated when greater importance is placed on the differences that are found across the country. This means that more attention can be given to different demographics and how they vary region to region, as well as what the best approach is to reduce spending costs while ensuring customer satisfaction.

From these additional experiments, we also found that a mixture of technologies is the best approach to the problem in rural and mixed regions. However, we also found that regions of increasing complexity and population density are not satisfied with older technology.

In future work, it would be beneficial to identify a method of batch optimisation that can take many of the modular regions generated by the procedural generation and stitch them together once optimisation is complete. This would allow for a more detailed picture of the UK as a whole and would potentially highlight relationships and design insights not yet observed, for example, in larger networks in which more than one exchange is present and there is an overlap between technologies and regions.

The numerous experiments that were undertaken identify that the proposed method, that made use of Interval Type-2 Dominance rules, was able to produce results that were 59% better than those produced by traditional dominance rules and 46% better than results produced by Type-1 Dominance rules. Additional experiments investigated the performance of the proposed method on more complex networks that had distinct regional attributes. These experiments found that the proposed method was able to produce less expensive network solutions while maintaining the customer's broadband demand. It was found that in rural regions the proposed method was able to produce solutions that were approximately 51.99% less expensive than network solutions produced using the current methodology that requires only FTTP technology to be used. It was also found that the proposed method was able to produce solutions that were approximately 54.16% cheaper than current solutions in mixed regions. It should also be noted that in urban regions where the proposed method produced solutions that were 34.88% cheaper, the underperformance of broadband delivery can be offset, as the current methodology over delivers broadband, to customers that do not required the additional rates. This can be identified from our experiments that identified approximately 14.23% of all premises connected via FTTP during the urban optimisation were being oversupplied and could instead be supplied adequately by FTTC. This presents a waste of resources that could also alienate customers forced to pay more for a service they do not require.

The following chapter explores how the proposed method can be integrated with explainable artificial intelligence and how an explainable system can be generated from the solution data of our previous network optimisations. The chapter explores the optimisation of the explainable systems membership functions, the extraction and refinement of rule bases, and the combination of these systems into a unified explainable system that can be deployed in order to generate solutions of a higher quality to those produced by the current methodology. The chapter also identifies how the user interface can play a vital role in the combination of each proposed system and provide users with a greater understanding of the solutions, both current and future, that are created.

Chapter 7. A Proposed Explainable Type-2 System for Topographical Optimisation Problems

7.1 XAI System

The introduction of interpretability into AI systems can offer new insights about particular tasks/problems; it can identify potential issues/bugs with a system, and for real-world problems it can reduce risk and give experts confidence in system accuracy. As discussed in Chapter 5, there are many approaches to XAI that can be undertaken. The proposed system makes use of the interpretable nature of Fuzzy Logic Systems, which provides understanding and insight into black box and real-world problems [137].

There are several aspects to the XAI system that need be discussed. Firstly, the development of the fuzzy system that is responsible for interpreting network solutions is undertaken by an iterative process, in which a genetic algorithm extracts the initial interval Type-2 membership functions and rules that are then optimised in sequence. During this process, the individual points of the membership functions are optimised by a standard GA and the final extracted rule base is simplified to maintain its interpretability.

The proposed XAI system is implemented in coordination with the proposed many-objective optimisation algorithm that was outlined in the previous chapter. A novel framework has been produced; in which a new approach to the optimisation and extraction of an explainable artificial intelligent system, in coordination with a many-objective optimiser, has been found. This novel approach introduces a new method for the optimisation of the fuzzy XAI system itself, along with a completely novel approach to the extraction of real-world design insights for grand scale optimisation problems.

The Pareto front of solutions generated by that system are deconstructed with this XAI system in order to identify interpretable features. Features of an interpretable user interface are also presented, this is in order to increase the user's ability to gain understanding from the set of solutions in the Pareto front.

The entirety of the proposed framework is displayed in Fig 7.1. This figure contains the proposed NSGA-II optimiser described previously in Chapter 6; it is the initial phase of the proposed framework and can be seen in the first and smaller dotted box of Fig. 7. It was the objective of this section of the novel framework to provide a new approach to the optimisation of many objective grand scale problems, and obtain improved results while improving the diversity of solutions produced. The second portion of the framework is contained within the second, larger dotted box. The objective of this portion was to provide a novel approach to the optimisation of an XAI system and extraction of interpretable information from real-world problems.

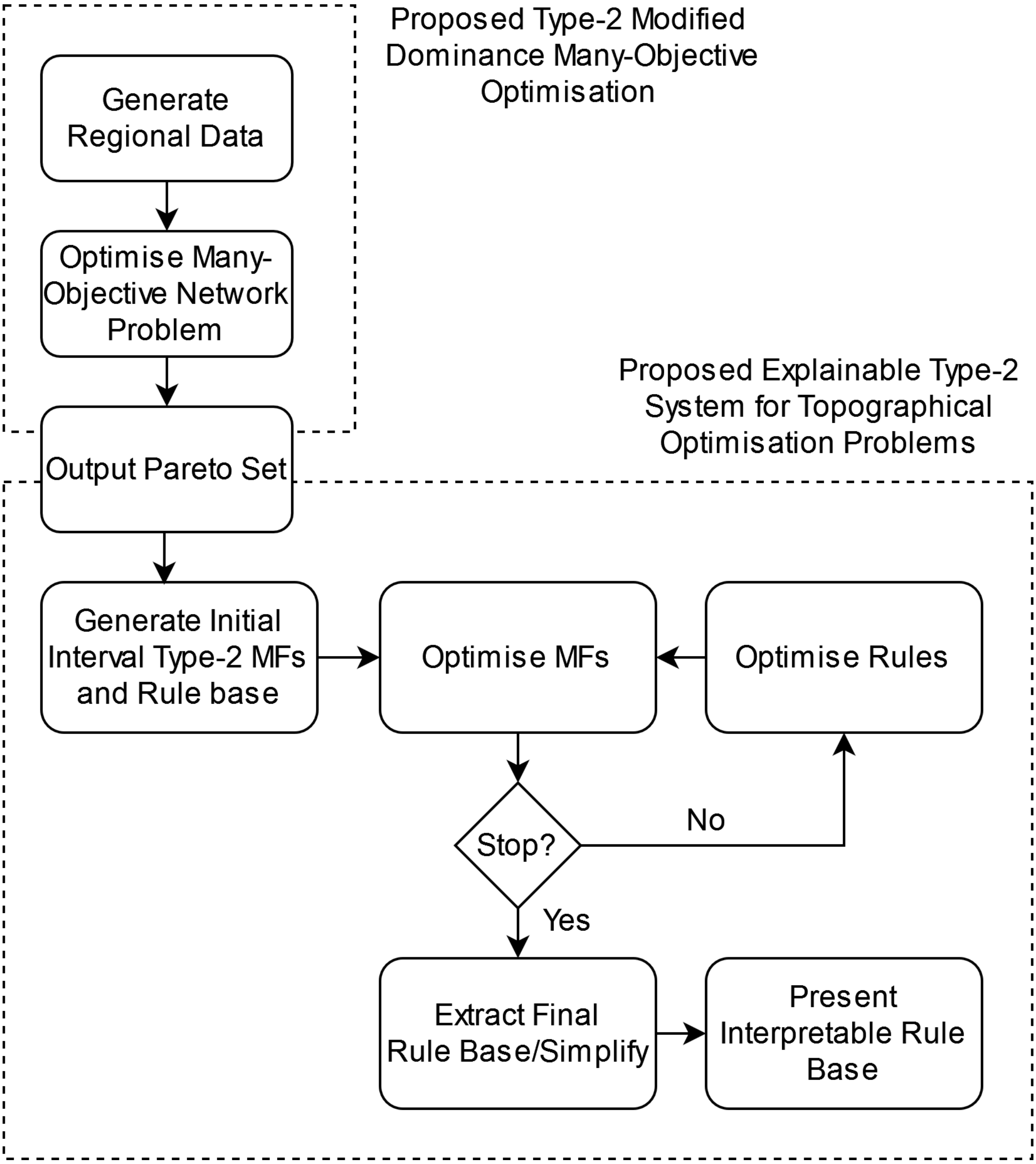


Fig. 7.1 Flowchart of Proposed Systems

7.1.1 Genetic Algorithm Fuzzy Membership Extractions

In order to generate the fuzzy systems interval Type-2 membership functions, an initial configuration of the individual points must be selected. The easiest way to accomplish this is to uniformly space the membership function points across a 0-1 normalised range. For the proposed system, a genetic algorithm is used to optimise the membership functions. This GA follows the principles outlined in Chapter 3; a population of chromosomes are used to represent the membership functions. The chromosomes in turn are represented by a number of genes that are equal to the amount of parameters present in the membership function. For example, when considering interval Type-2 membership functions there are several methods for determining the amount of parameters that are required (this example represents three membership functions for two inputs and one output):

- **Completely unrestrained –**

This method offers the most freedom for the exploration of the universe of discourse. In this method, a parameter has complete freedom; this allows the membership functions to take any configuration as long as they do not break the inherent constraints placed upon fuzzy membership functions.

For this example, the inputs and output use the same membership function configurations (left shoulder trapezoid, trapezoid, and right shoulder trapezoid) and therefore 48 parameters are needed to represent all fuzzy sets. A representation of these parameters is given below in Fig. 7.2.

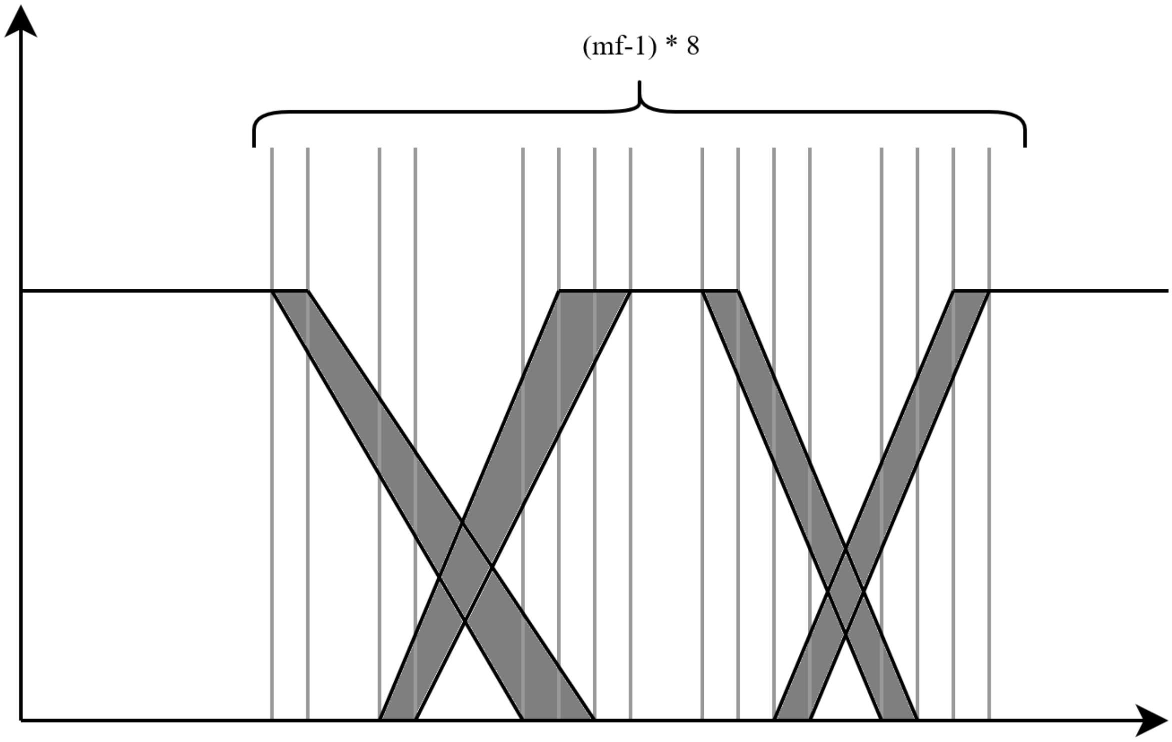


Fig. 7.2 Representation of 16 unrestricted parameters

In this example, it is not required to have parameters that represent the start and end points. For this reason, the amount of parameters that are always required to represent the current configuration of membership functions is equal to the following:

$$Parameters = (mf - 1) * 8 \tag{7.1}$$

Where *mf* is equal to the amount of membership functions for the fuzzy set.

Although this method theoretically allows for the most optimal configuration of parameters its flexibility is also its greatest drawback. The large number of parameters increase computational complexity and also require constant checks to make sure that no invalid membership functions are produced.

- **Fixed –**

In order to reduce both the complexity of the optimisation and amount of checks that must be undertaken, several of the parameters can be fixed in place with the previous membership function parameters. For our given configuration, only 18 parameters are required to represent every fuzzy set - an example of this can be seen in Fig 7.3 below:

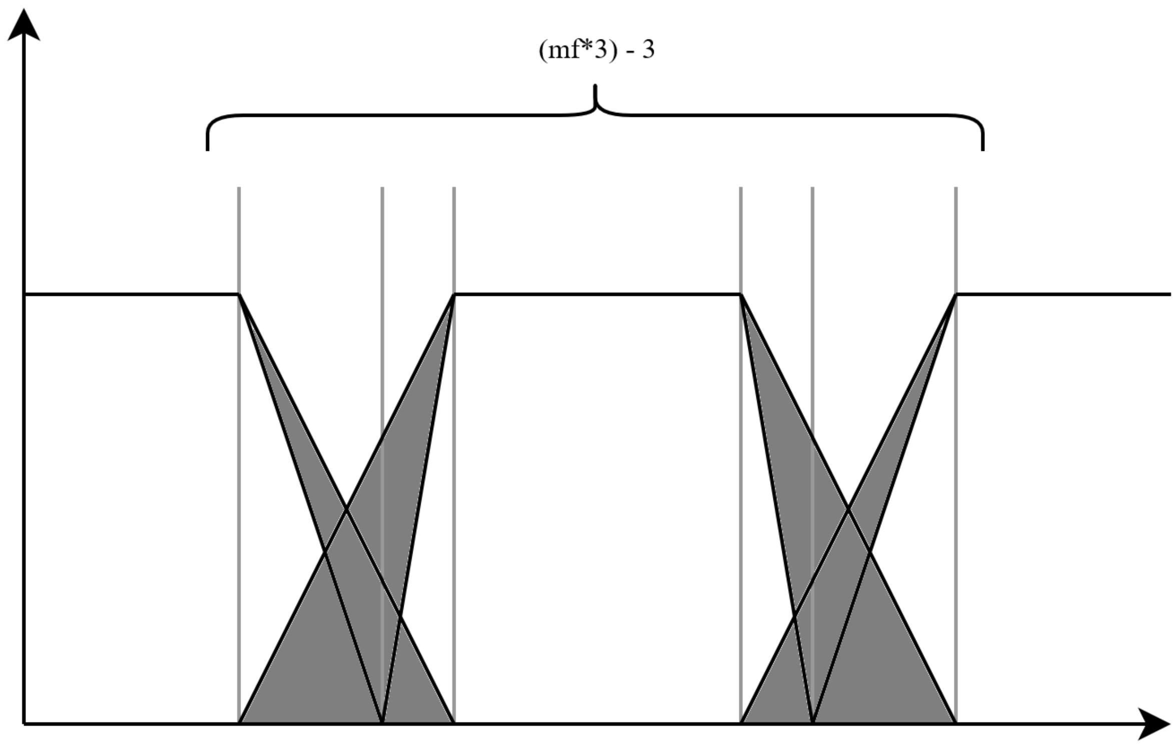


Fig. 7.3 Representation of six fixed parameters

$$Parameters = (mf * 3) - 3 \tag{7.2}$$

This method removes much of the complexity found in the previous method. This method can also easily remove the need for any checks if bounds are placed upon key parameters to ensure that only valid membership functions are produced. The drawback of this method is of course the reduced freedom given to the parameters.

This method has similarities to the two-tuple tuning method. However, the two-tuple method does not manipulate individual parameters; instead, membership functions maintain their shape and are translated across the universe of discourse [138].

In order to maintain interpretability, traditional Fuzzy Logic Systems will consist of between 3-5 membership functions per fuzzy input/output set. Keeping the number of membership functions between these values has a twofold affect; firstly, it reduces the computational complexity. Secondly, and more significantly to interpretability, is that it maintains linguistic consistency and readability. A Fuzzy System consisting of a large number of linguistic variables reduces its interpretable nature; another benefit of keeping the number of linguistic variables between three and five is the intuitive design that is possible. For example, Fig. 7.4 below is a representation of the initial inputs and output membership functions prior to their optimisation.

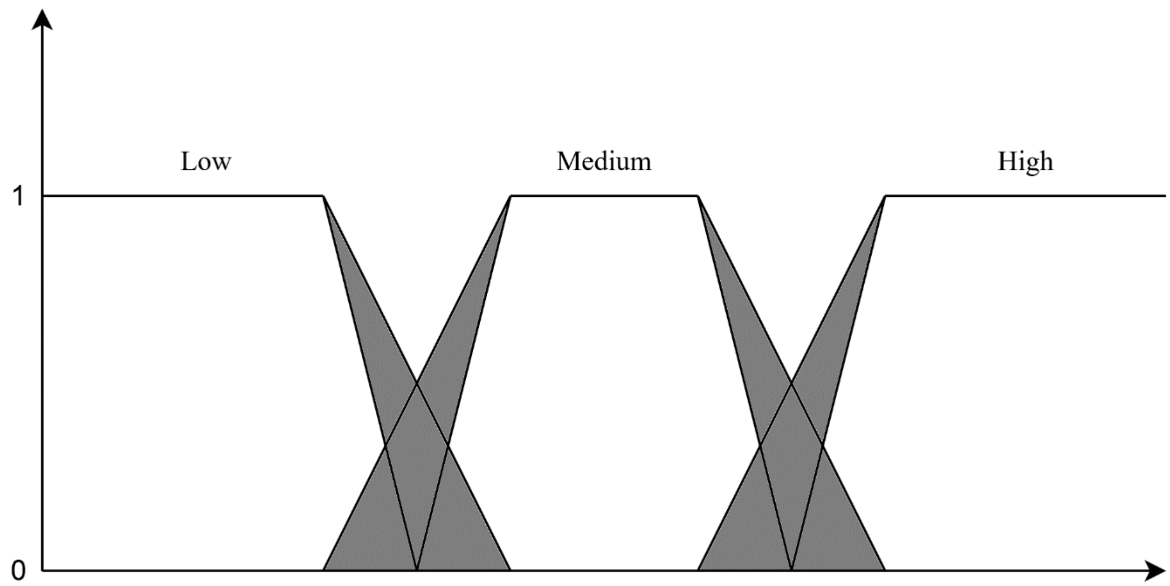


Fig. 7.4 Representation of input/output membership functions prior to optimisation

The linguistic terms "Low", "Medium", and "High" are both easily designed and intuitively understood. Another benefit of the two tuple or fixed system described above is their ability to maintain membership function consistency. Maintaining this shape consistency is a benefit when considering interpretability. The unrestrained method, while enjoying greater freedom, has the potential to produce membership functions that when viewed do not make intuitive sense, which is something that is guaranteed when the other methods are implemented.

7.1.2 Unrestrained vs Fixed Experiments

Due to the complexities of the problem being addressed, it is important to consider the computational complexities of each feature of the proposed systems. The advantages and drawbacks of each system has been discussed; invalid membership functions created from the freedom offered by the unconstrained system outweigh the potential benefits of the system.

In Fig 7.5, illustrated below, a series of Type-1 membership functions, consisting of varying parameter amounts were optimised 25 times each, so that an average count of the invalid membership functions could be made. In this initial experiment, the unconstrained system explored the construction of membership functions with three parameter variations; these were, 24, 36, and 48 parameters respectively.

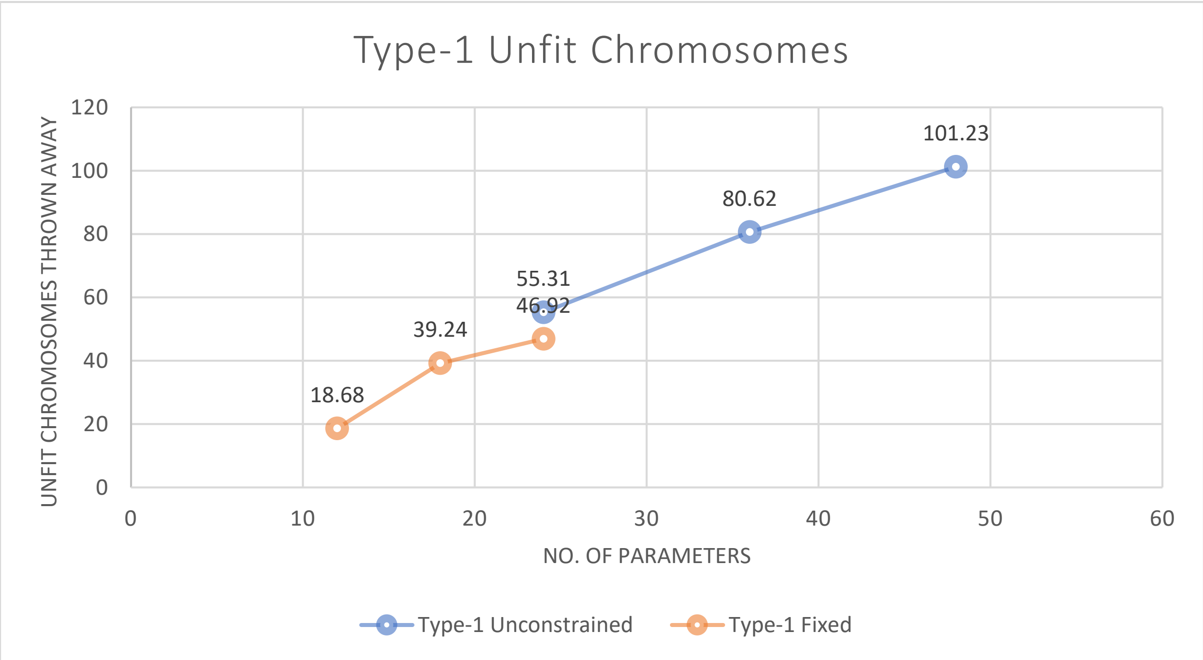


Fig. 7.5 Unfit Tyep-1 Membership Function Chromosomes Discarded During Optimisation for Unconstrained and Fixed Systems

The membership functions took a similar configuration to those shown in the previous section. For example, the chromosome containing 24 parameters consisted of one interior trapezoid and a left/right shoulder trapezoid. For an unconstrained Type-1 membership function this could be represented as follows:

$$Parameters = (mf - 1) * 4 \tag{7.3}$$

A representation of this configuration can be observed in Fig 7.6 below. As each chromosome represents both input membership functions and the output membership function (all of the same configuration) the parameter count is repeated three times, hence why 24 parameters are required in total.

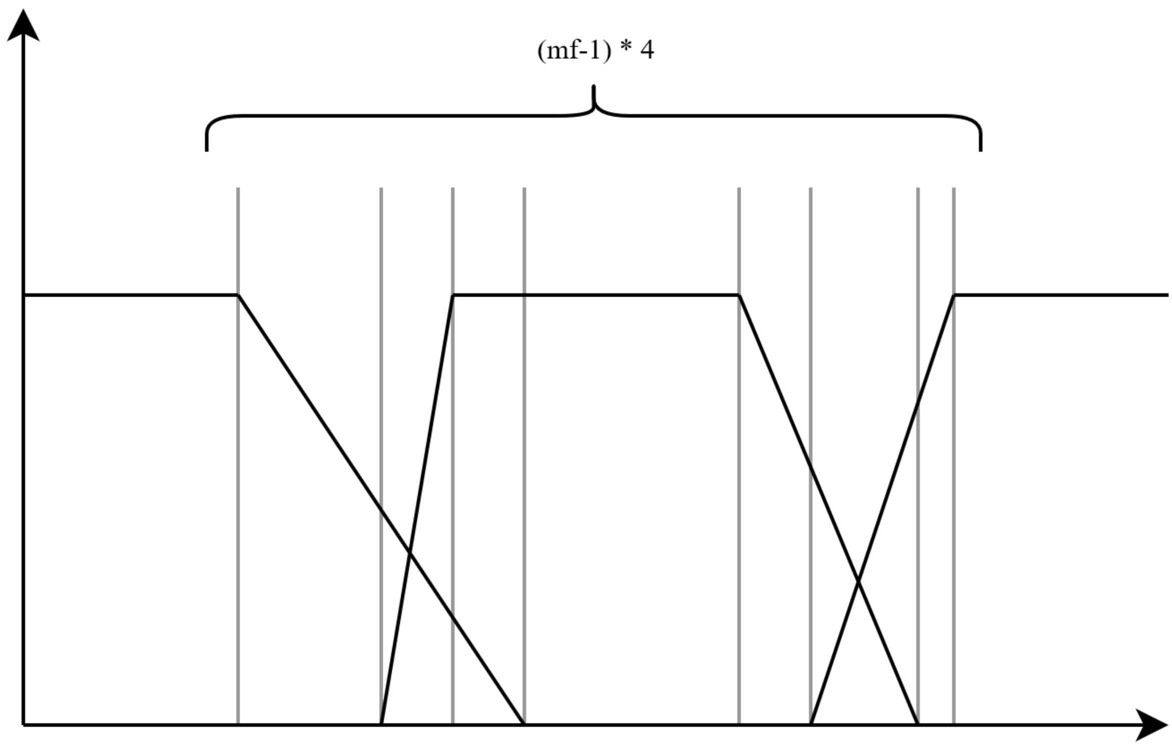


Fig. 7.6 Representation of 8 parameters for unconstrained Type-1 membership functions

The configuration of 36 parameters consisted of an additional interior trapezoid so that each fuzzy set contained four membership function for a total of 12. The configuration of 48 parameters contained another interior trapezoid and so consisted of 5 membership functions per fuzzy set, for a total of 15.

While the fixed system shared the number of membership functions per experiment it was able to represent them with far fewer parameters. Fixed Type-1 membership functions could be represented as described in equation 7.4 and Fig. 7.7 below:

$$Parameters = (mf - 1) * 2 \tag{7.4}$$

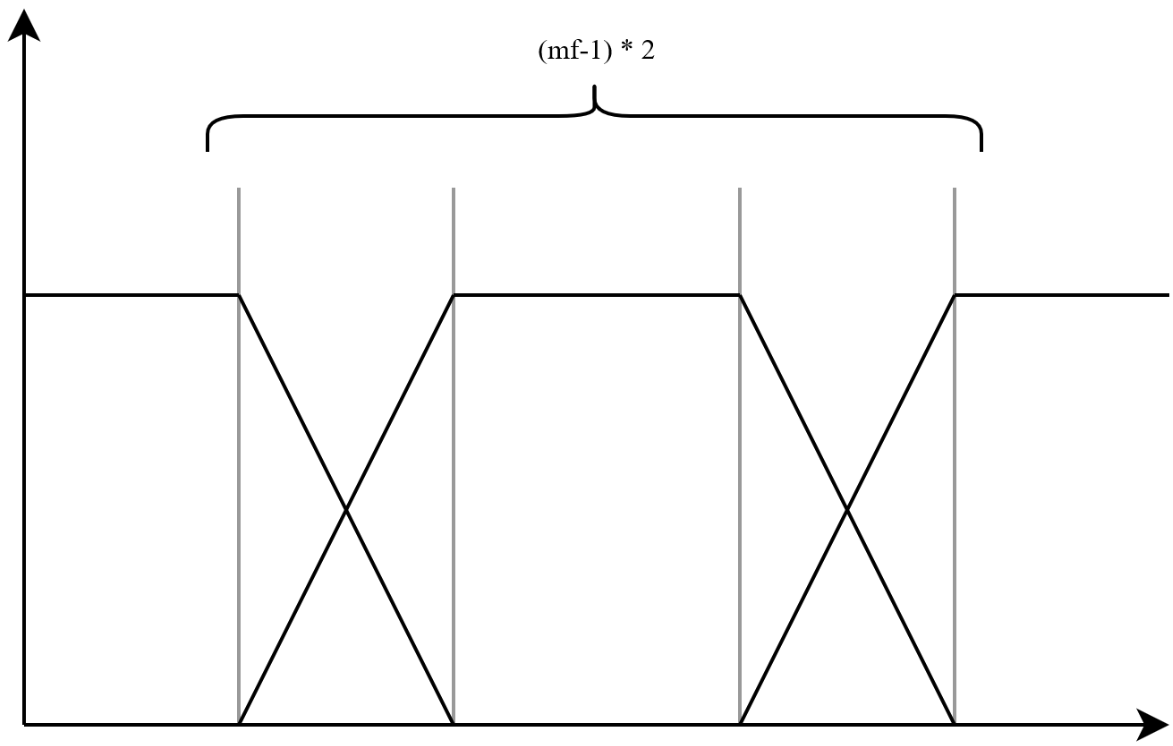


Fig. 7.7 Representation of 4 parameters for Fixed Type-1 membership functions

In this initial experiment it was therefore possible to represent the same fuzzy sets (consisting of 3, 4, and 5 membership functions respectively) using a total of 12, 18, and 24 parameters with the fixed system.

In each test a population of 50 chromosomes was used, and the average count for unfit chromosomes due to invalid membership functions was recorded. In each iteration the genetic algorithm responsible for optimising the membership functions would check for constraints being broken. An example of an unfit membership function is illustrated in Fig. 7.7 below.

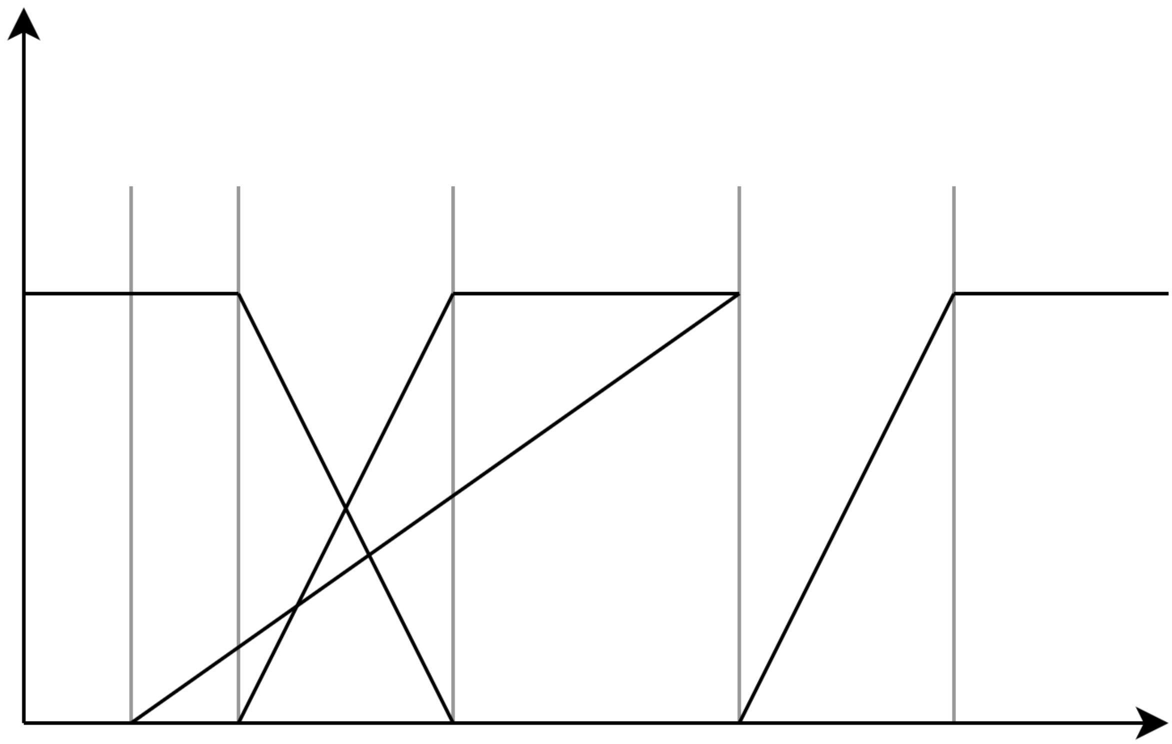


Fig. 7.8 An invalid membership function where a constraint has been broken

This is one example of how an invalid membership function may be organised when unrestricted freedom is given to every parameter. The results displayed in Fig. 7.5 identify how despite the freedom provided by the unconstrained system in regards to the universe of discourse in which parameters can be optimised, a greater number of parameters are required for the representation of membership functions and an even greater number must be thrown away during the optimisation. Both of these negatives increase the computational complexity of the proposed system; therefore, when the fixed system is used, despite a small loss in freedom, the computational complexity is reduced due to fewer parameters and less constraints being broken. The fixed system is also more likely to maintain human interpretable membership functions by the end of the optimisation process.

There were a number of reasons as to why it was significant to reduce the computational complexity of the system. Firstly, due to the huge scale of the problem being addressed, any reduction to the computational complexity of the proposed system would translate to massively reduced time and requirements. This is particularly significant due to the real-world nature of the problem at hand; the novel framework produced, had to be responsible for providing improved solutions that were actionable and achievable. Any reduction in computational complexity would reduce running times and the huge costs that coincide with computation complex systems.

This experiment was expanded to also include interval Type-2 membership functions. In these experiments three variations were also tested; these variations were configured as outlined in the previous section and contained 48, 72, and 96 parameters respectively when configured using the unconstrained system. These fuzzy sets took the same form as the initial experiment and so always contained an interval Type-2 left/right shoulder trapezoid and 1, 2, or 3 interior interval Type-2 trapezoidal membership functions.

For the fixed system these representations were configured using 18, 27 and 36 parameters. The same experiment was undertaken with the new representation of the fuzzy sets and Fig. 7.9 below identifies how the computational complexity is massively impacted when the unconstrained system is implemented over the fixed system due to the massive increase in parameters.

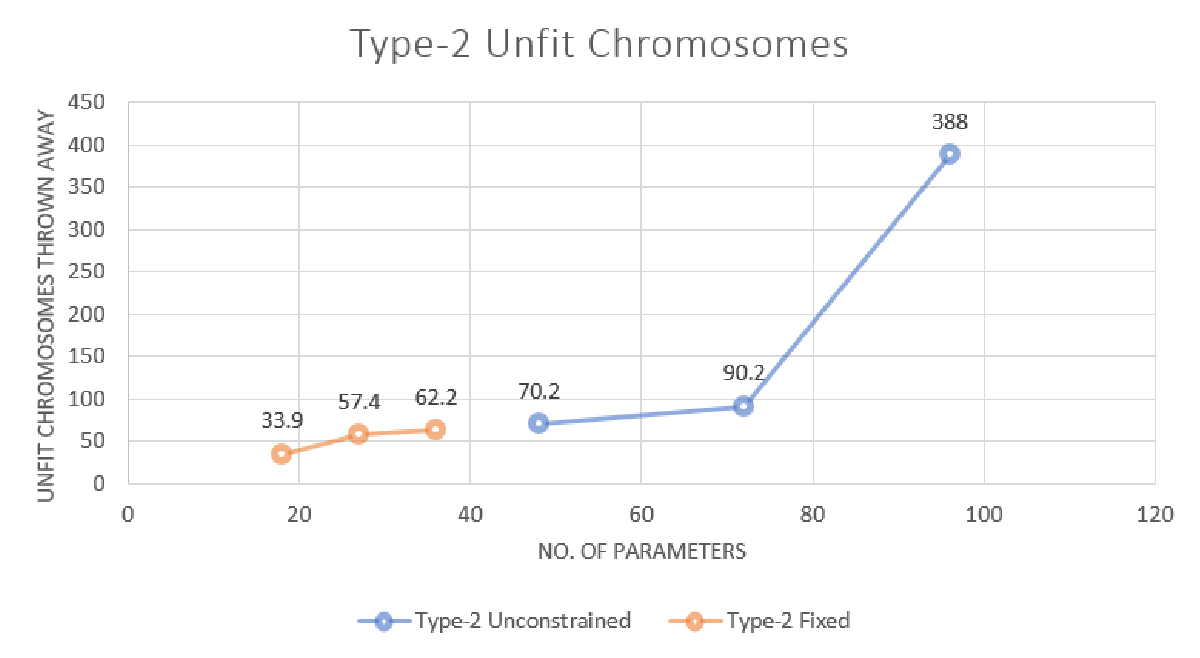


Fig. 7.9 Unfit Tyep-2 Membership Function Chromosomes Discarded During Optimisation for Unconstrained and Fixed Systems

Fig. 7.9 shows us that as the number of parameters increases with respect to membership function representation a greater number of chromosomes must be thrown away by the genetic algorithm due to the breaking of constraints. To reduce the computational complexity of the proposed system and maintain accurate results the fixed system is implemented to ensure that some membership function constraints are not broken.

7.1.3 Fuzzy Rule Extraction

The solutions that are most successful and secure their position in the Pareto front of a particular optimisation scenario can be extracted and used by the proposed system to generate a visual representation of the network and an interpretable rule base. The rule base is generated by following the fuzzy rule extraction process that is outlined in detail in Chapter 5; however, the rules generated via this system take the form of many antecedent, many consequents which is represented by Equation 5.1.

In order for the rule base to be generated, each of the solutions in a Pareto front are deconstructed at the end of the optimisation process; at this point an additional file is created that describes the optimised network as input and output data for the Fuzzy Logic System. The proposed system takes the final objective values and population density of each solution as the four inputs in the system (Population density, costs, connected premises, undelivered bandwidth). For the fuzzy outputs, the system takes the relative percentage of network equipment that is FTTC and FTTP. During the extraction of data each of the values are normalised between $0 - 1$.

Table 7.1 below is a partition of the generated rules that were produced from the optimisation of a rural designated region. The optimisation searched for a series of solutions to the optimsation of a rural network in which both FTTP and FTTC are used. One of the initial concerns with the generated rule base is the repeated generation of identical rules; this problem can be overcome easily through the merging of such rules. However, there still exists the problem of similarity. Many of the extracted rules can be considered similar or conflicting, i.e. the antcednets of the rules are the same while the consequents are different. In order to further reduce the generated rule base and deal with conflicting rules, confidence and support values are introduced for each rule.

Rule base
… *Rule 5: if Pop Density is Medium and Unfulfilled Bandwidth is Medium and Cost is High and Premises Connected is Medium then FTTC % is High and FTTP % is Low* *Rule 6: if Pop Density is Medium and Unfulfilled Bandwidth is Medium and Cost is Low and Premises Connected is Medium then FTTC % is High and FTTP % is Low* *Rule 7: if Pop Density is Medium and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low* *Rule 8: if Pop Density is Medium and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low* *Rule 9: if Pop Density is High and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low*

Table 7.1 Partition of the generated rule from the optimisation of rural designated access network

The confidence value determines the confidence with which a particular output fuzzy membership function is the valid output for the set of identical antecedents. This confidence value is calculated as follows [139]:

$$c\left(\tilde{A}_q \Longrightarrow C_q\right) = \frac{\sum x_s \epsilon \; class \; c_q \, fs^{jt}(x_s)}{\sum_{j=1}^{m} fs^{jt}(x_s)} \tag{7.5}$$

Where C_q is the given output class for the collection of antecedents $\widetilde{A}_q$. Equation 7.5 identifies how the summation of the scaled firing strengths fs^{jt}, is divided by the summation of the firing strengths for all of the conflicting rules.

The second value determined is the support for a rule. The support represents the coverage of the data in relation to the following $\tilde{A}_q \Longrightarrow C_q$. The value for the confidence of a rule is calculated as follows:

$$s\left(\tilde{A}_q \Longrightarrow C_q\right) = \frac{\sum x_s \epsilon \; class \; c_q \, fs^{jt}(x_s)}{m} \tag{7.6}$$

Where *m*, is the total number of data points in our training data. The final step needed to reduce the rule base is the calculation of rule dominance. This is calculated using the previously calculated confidence and support and is done as follows:

$$d\left(\tilde{A}_q \Longrightarrow C_q\right) = c\left(\tilde{A}_q \Longrightarrow C_q\right) * s\left(\tilde{A}_q \Longrightarrow C_q\right) \tag{7.7}$$

From Equation 7.7 it is now possible to determine which of the rules should be kept, the one that has the highest average dominance value is selected and the others removed.

The reduction of the rule base can often be very significant; due to distribution of training data, it is common to produce hundreds of rules, many of which will be conflicting. It is therefore possible to reduce a very large rule base down to a handful of rules. This reduction in rules is an important aspect of the explainable system; this is because the interpretable nature of the Fuzzy Logic System is lost when a user is presented with an incomprehensible number of rules.

Rule base

Rule 1: if Pop Density is Low and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low

Rule 2: if Pop Density is Low and Unfulfilled Bandwidth is Low and Cost is Medium and Premises Connected is High then FTTC % is High and FTTP % is Low

Table 7.2 Reduced Extracted Rule Base for a Single Rural Network Solution

Table 7.2 above is an example of how a rule base consisting of 50+ rules can be reduced to solidify the interpretability of the rule base. The example rule base is completely subjective to the parameters that define the optimisation problem, which concerning the proposed systems can be easily manipulated to allow a user to investigate various designs.

The rule base from Table 7.2 has been extracted from a rural optimisation of the access network that made use of both the FTTP and FTTC technologies. The rules then present a clear understanding surrounding the network that has been designed, creating easily interpretable links between the fundamental features of the network and the ratio of technologies that have been selected to create it.

Alongside the extracted fuzzy rules is a user interface; this gives the user a visual representation of the network composition. This composition is a combination of the topology created by the procedural generation system that includes the roadways and premises unique to the generated region, and the network equipment that was placed during the optimisation.

The user interface application allows the user to review all of the network solutions that make up the Pareto-front of a particular optimisation. The application provides the user with the ability to explore the boundaries of a region, zoom in/out to inspect network configurations, and gain a visual understanding of the solutions that have been generated.

Fig 7.10 presents the network from which the rules were generated and displayed in Tables 7.1/2. This visualisation of the network solution provides a detailed account of how the premise/road configuration, technology choices, constraints, and regional parameters affect solutions produced via optimisation. Figure 7.10 and the rule base allow the user to infer from the solution how low-density residential housing (blue circles) are the sole premise type in this rural town. The way in which they are being supplied primarily by FTTC technology (FTTP/FTTC distribution point grey/yellow square) which is suitable when pursuing reduced spending costs for a low/medium density area and a lower than average bandwidth requirement.

Fig. 7.11 is another representation of user interface presenting a solution. This example of a network is of a mixed region and features an increasing quantity of higher-density residential premises (red circles). The rules generated from this network identify how this mixed region with its variations of premises and high population density, are best supplied in this circumstance by FTTC in the pursuit of lower costs, where broadband demands can still be met. These rules are in line with the results that were presented in Chapter 6.

Rule base

Rule 1: if Pop Density is Medium and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low

Rule 2: if Pop Density is Medium and Unfulfilled Bandwidth is Low and Cost is Medium and Premises Connected is High then FTTC % is High and FTTP % is Medium

Rule 3: if Pop Density is High and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is High then FTTC % is High and FTTP % is Low

Table 7.3 Reduced rule base for mixed region network

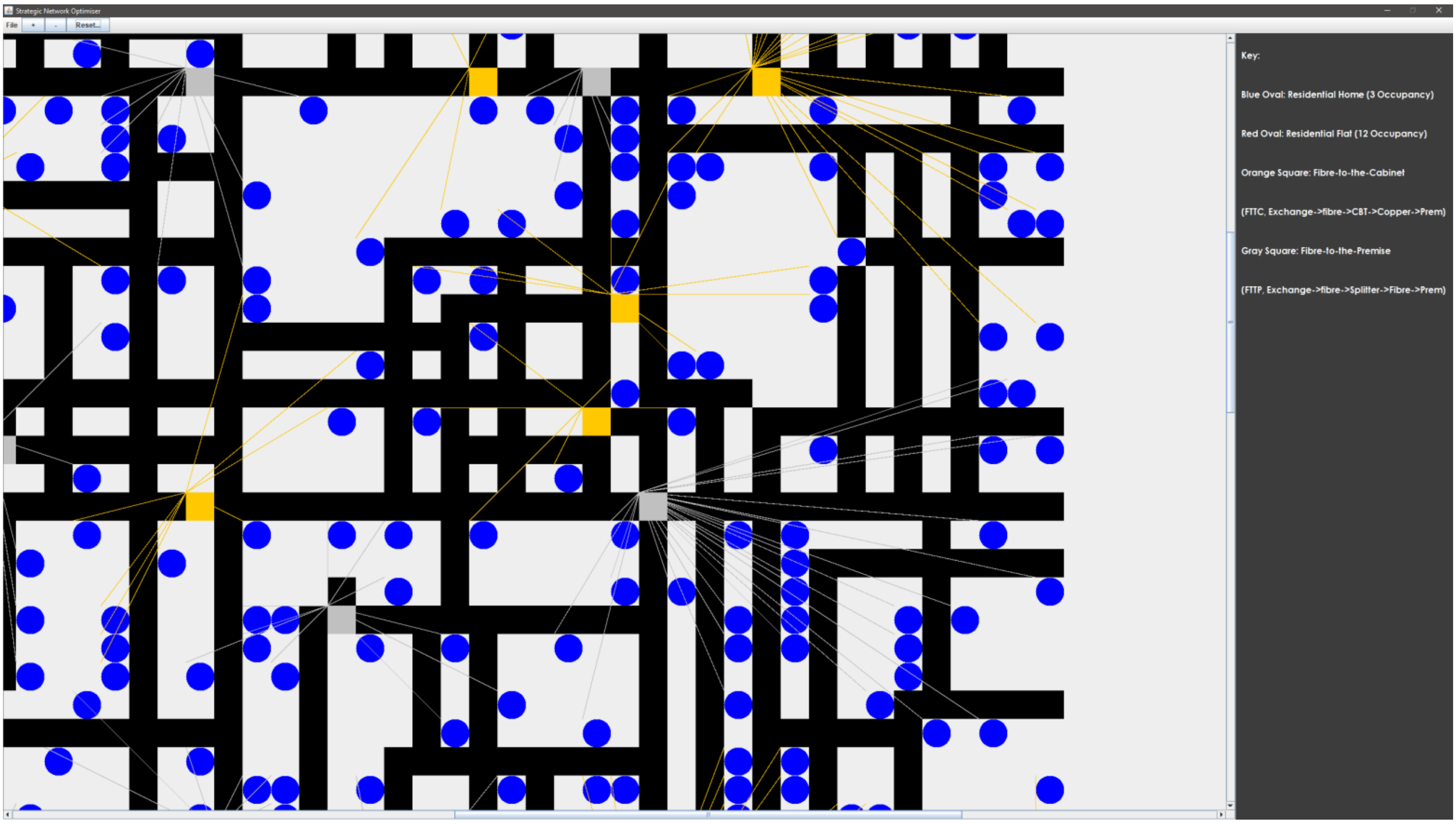


Fig. 7.10 User interface provided alongside extracted fuzzy rules

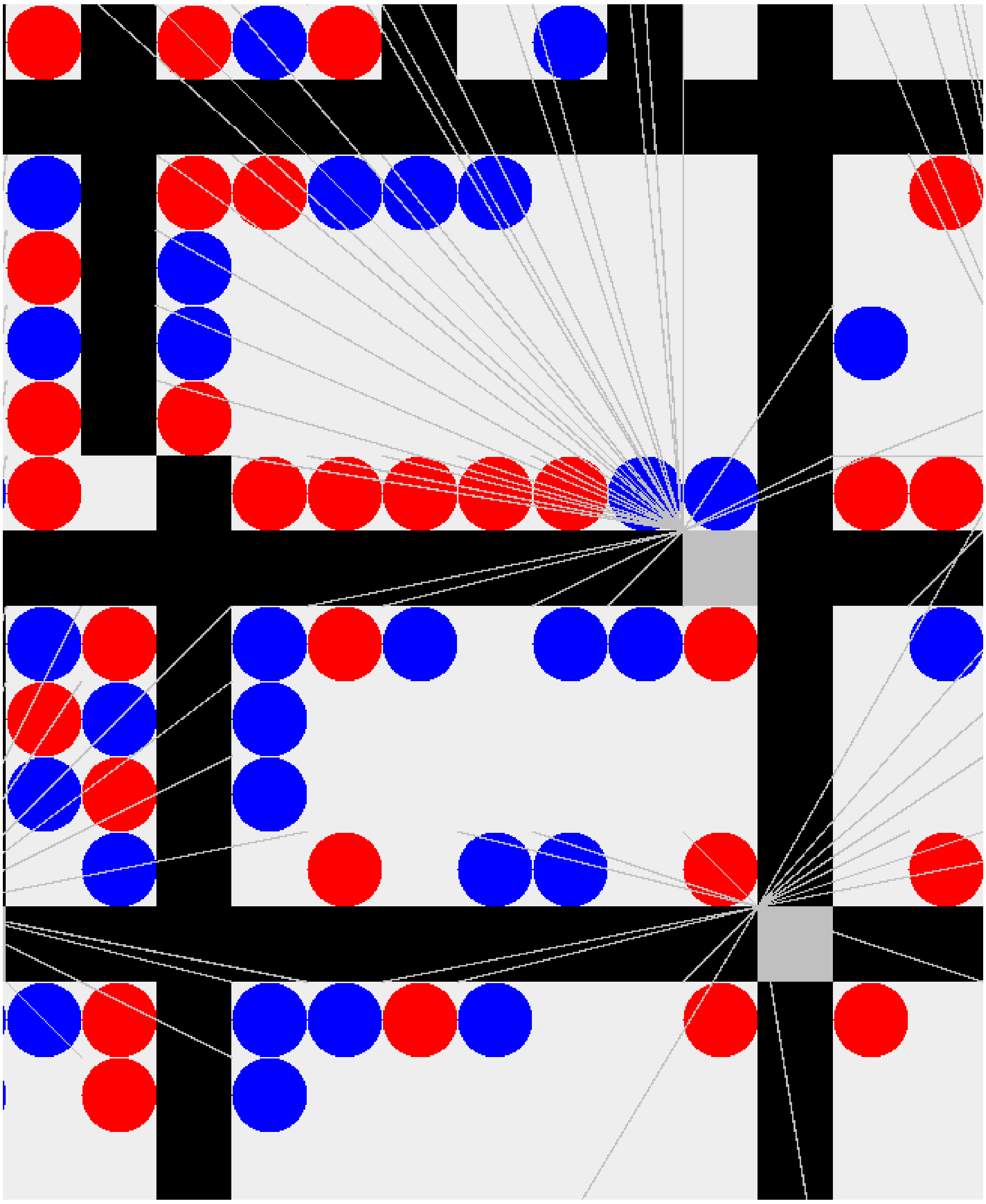


Fig. 7.11 Snapshot of densly populated mixed region interior

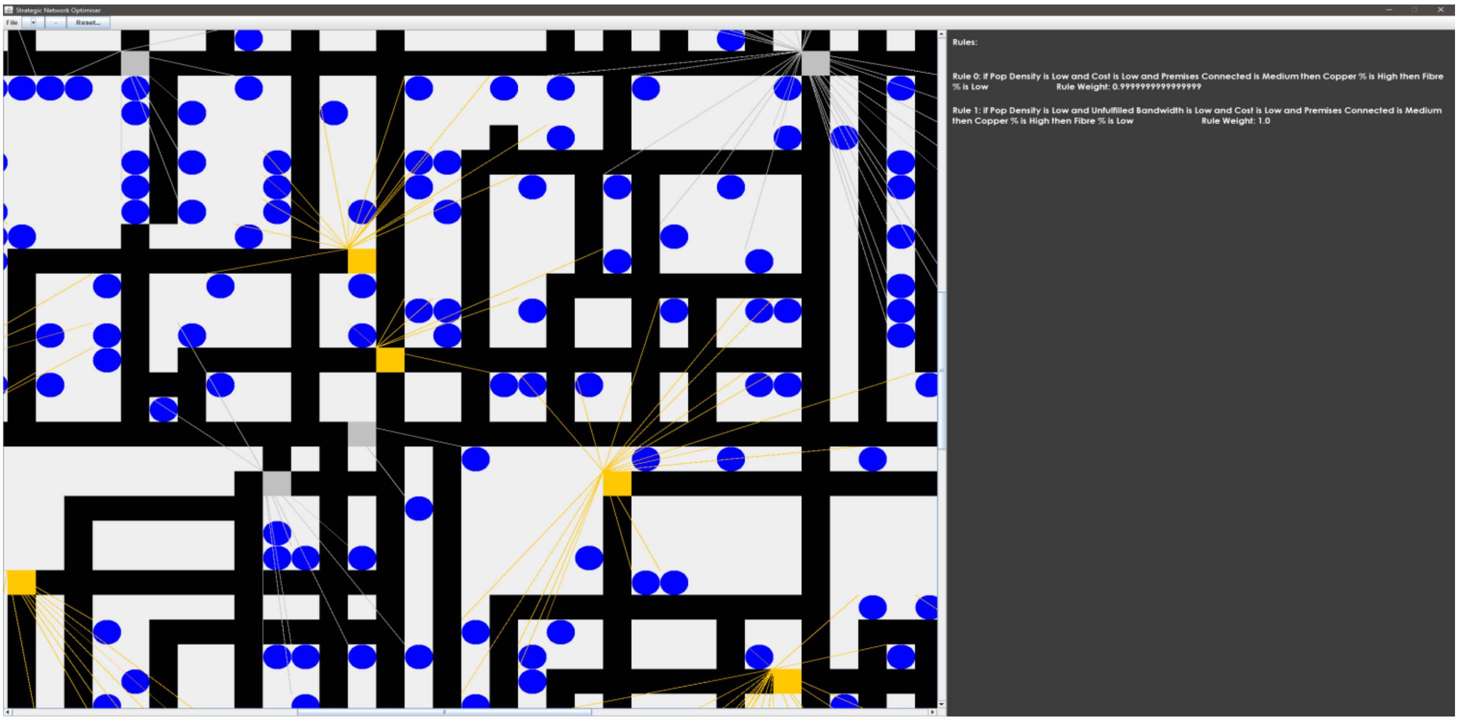


Fig. 7.12 Rural Network Solution Presented Alongside Extracted Rules (Right)

Fig. 7.12 is another example of a rural region optimisation; this particular solution is one of many taken from a final Pareto front. This network solution maintains the observations that have already been outlined in our results. This rural network consists of a sparse population whose broadband demand is supplied predominantly by FTTC technology, with FTTP playing a minor role.

In Fig.7.12 the user is provided a visual representation of the topology and the network solution alongside the extracted rule base that is shown on the right hand panel. This rule base explains that:

Rule base

Rule 1: if Pop Density is Low and Cost is Low and Premises Connected is Medium then FTTC % is High and FTTP % is Low

Rule 2: if Pop Density is Low and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is Medium then FTTC % is High and FTTP % is Low

Table 7.4 Extracted rule base for rural network solution of Fig. 7.12

Providing the rule base within the user interface grants users a more instinctive understanding of the particular solution that is being inspected.

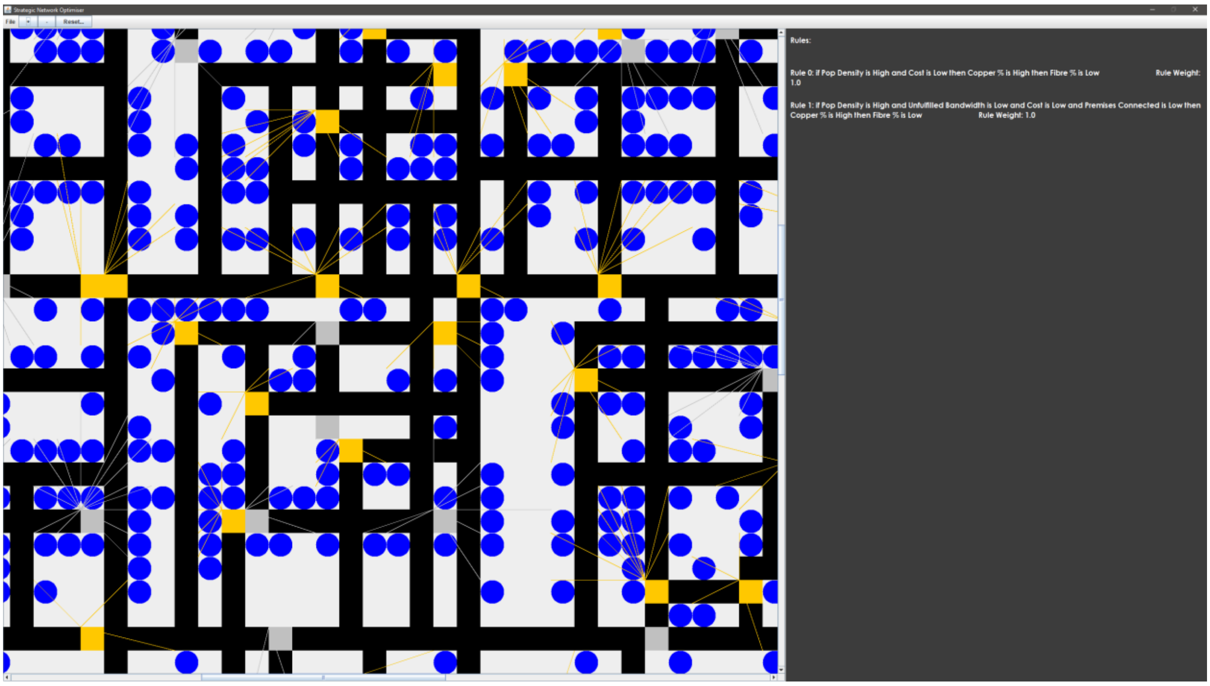


Fig. 7.13 Mixed Network Solution Presented Alongside Extracted Rules (Right)

Fig. 7.13 provides another example of an optimised network solution. In this figure a mixed region is presented, as outlined in Chapter 6, mixed regions were implemented in order to model regions of the United Kingdom that are progressing from rural to urban. An example of this, as shown in Fig. 7.13, is that of a middling/large town composed mostly of low density residences. As we have shown from our results it is in regions like this that the proposed system is able to produce the most valuable solutions; solutions generated in this manner were on average 54.16% cheaper than those produced while implementing the current methodology. The rules for the network presented in Fig. 7.13 are as follows:

Rule base
Rule 1: if Pop Density is Low and Cost is Low and Premises Connected is Medium then FTTC % is High and FTTP % is Low
Rule 2: if Pop Density is Low and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is Medium then FTTC % is High and FTTP % is Low

Table 7.5 Extracted rule base for mixed network solution of Fig. 7.13

The strength of the user interface is even more evident when solutions between regions are compared. The rule bases and visualisation of the network solutions offer clarity and provide trust in the results produced by the proposed system in our experiments.

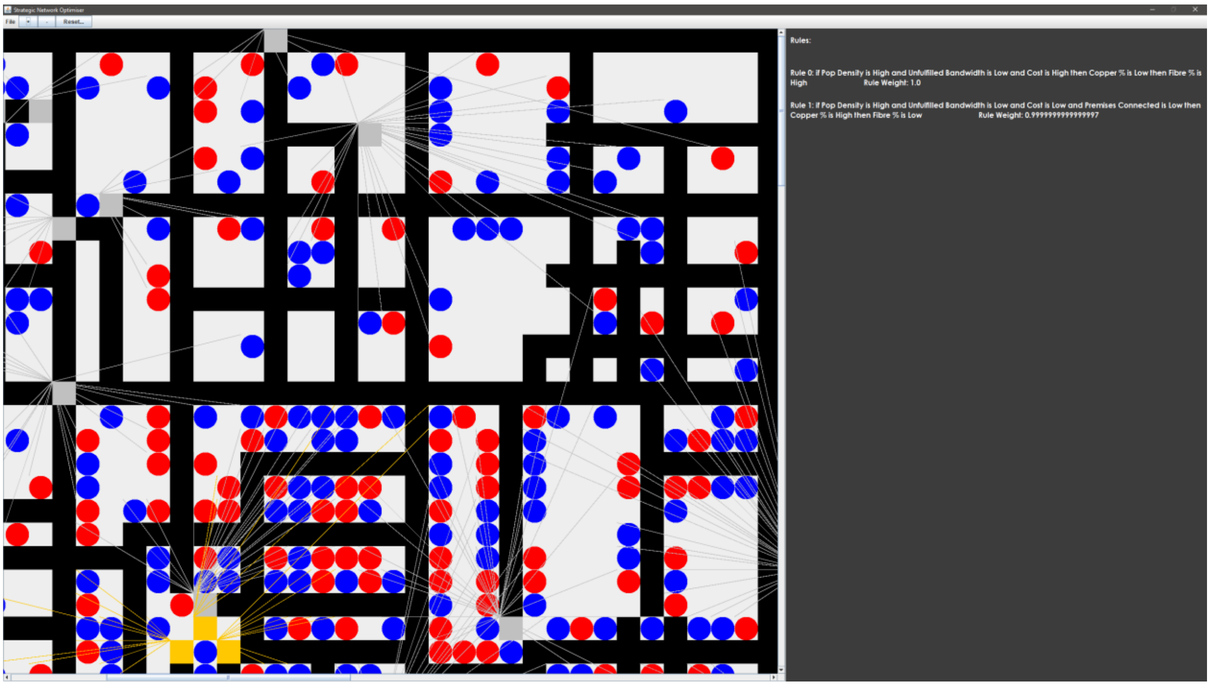


Fig. 7.14 Urban Network Solution Presented Alongside Extracted Rules (Right)

An example of a more densely populated region is presented in Fig. 7.14. This region is a representation of an urban area, there is a greater population density and an increase in higher density residences such as flats. From our results, and mirrored in the visual representation and extracted rules, it is understood that FTTP is able to provide a more dominant role in these types of regions, that can vary from large towns to bustling cities. However, as discussed previously, our experiments identify that network solutions that only implement FTTP are more capable at providing customers broadband demands but are also more expensive and often oversupply broadband to customers that do not require the additional supply. In scenarios like this customer will be forced to pay extra for a service they do not require, leading to reduced customer satisfaction. The rules generated for this network are shown on the right hand side and are as follows:

Rule base

Rule 1: if Pop Density is High and Unfulfilled Bandwidth is Low and Cost is High and Premises Connected is Medium then FTTC % is Low and FTTP % is High *Rule 2: if Pop Density is High and Unfulfilled Bandwidth is Low and Cost is Low and Premises Connected is Low then FTTC % is High and FTTP % is Low*

Table 7.6 Extracted rule base for urban network solution of Fig. 7.14

7.2 Discussion and Future Work

The work presented in this chapter identifies the approaches that are necessary when it is vitally important to have a better understanding of the solutions presented to us from artificial intelligent models. This is particular true when we are considering real-world problems and their ever-growing complexities.

This chapter identifies the strengths of Fuzzy Logic Systems. It highlights how the unique aspects of the theory can be readily manipulated in order to achieve the goals of a problem, which typically would be considered outside the scope of traditional fuzzy logic implementations. Fuzzy logic is able to incorporate the large amounts of uncertainty that will always be a prevalent factor in real-world business problems. Fuzzy Logics ability to handle both uncertainty and grand scale problems, consisting of many variables and objectives, demonstrate how it is the most applicable solution in our approach to XAI. Other methods of XAI such as the implementation of logistic regression discussed in [124] are able to provide great clarity and understanding to users, particularly non experts, which is vitally important. However, methods that implement logistic regression or Shapely values are often not scalable to projects of this size. This is because the additional and ever growing complexities cause these method to lose their transparency and therefore are unable to provide clarity to users.

In order to test the validity of the proposed system experiments were run on simulated regional locations. With optimised solutions populating Pareto-fronts, the proposed system deconstructed the networks so that a series of data points could be used to extract and optimise the fuzzy rule base and membership functions. Firstly, the optimisation or tuning of the membership functions was addressed. The genetic algorithm that was used was presented in detail in Chapter 3, however, several methods were explored for the chromosomal encoding and their strengths and weakness were discussed. Following this, a detailed account concerning the reduction of conflicting rules was given once they had been extracted via the method outlined in Chapter 5.

This chapter concluded with a presentation of the reduced interpretable fuzzy IF-THEN rule bases alongside the visual representation of the optimised as shown by the user interface application. The rule bases and user interface were shown to provide a link between how the uninterpretable raw data of an optimised solution can be transformed into human-relatable network solutions.

Going forward, it would be ideal to consider continuous learning integration. This is one area of research that is particularly interesting for the improvement of grand scale optimisation systems; a system that incorporates continuous learning would be able to learn and reason as new data/problems/technologies are presented to it, and then communicate naturally with users [140].

Another extension to this work that would be beneficial would be the advancement of visual elements involved in XAI. For example, the presentation of Fuzzy Rules as shown in Fig. 5.2 [128] would enable a greater level of interpretability and understanding to be conveyed to the user.

The final chapter of this thesis will give an overview of this thesis, provide an analysis of both academic and industrial questions that have been pursued and completed, and give a direction to the future development of the proposed systems.

Chapter 8. Conclusions and Future Work

In this thesis, a novel many-objective Type-2 explainable fuzzy logic system for the strategic optimisation of grand scale real world problems has been presented. The proposed system has been presented in two forms, both modified with fuzzy logic; the first represents a system capable of improving the optimality of results in any grand scale problem. The second outlines a novel method for approaching these grand scale problems from a strategic viewpoint, in which we are more interested in the abstract and planning insights that can be learned.

8.1 Summary

In Chapter 1, an introduction to the problem was given along with the aims of the project and goals of this thesis. A general overview of the thesis was also presented.

Chapter 2 described in detail the problem at hand that concerned the optimisation and planning of the UK access network. This chapter gave a detailed account of the approaches that are currently implemented concerning the optimisation and planning of grand scale problems, typically those of national utility problems such as gas, water, and electrical grid optimisation/generational planning. The importance of uncertainty in both real world and academic problems was discussed and emphasis was placed on how the varying forms of uncertainty would affect this project. This chapter concluded with a description of the technology that is currently in use and how it affects the objectives and constraints of the project.

Chapter 3 presents an overview of optimisation in general and of optimisation algorithms. This chapter presented a series of optimisation algorithms in succession. Firstly, some common search optimisation techniques were reviewed that included A* search and simulated annealing. The popular swarm intelligence, ant colony optimisation, was also presented.

Afterwards, a detailed account of genetic algorithms and genetic operators was given that led into a discussion of many-objective optimisation algorithms. The chapter concluded with a description of the Non-Dominated Sorting Genetic Algorithm II and the hypervolume metric that can be used to evaluate the performance of many-objective optimisers.

Chapter 4 was concerned with detailing fuzzy logic - its strengths, weaknesses, and implementations. An overview was given of both Type-1 and interval Type-2 fuzzy logic. The history and implementations of the theory was outlined, before a detailed account of the design process and its inclusion in this thesis concluded the chapter.

In Chapter 5, artificial intelligence is considered alongside the perspective shift that has occurred since its foundation. This chapter considers how the development of powerful optimising models has shifted the interest from whether AI is capable of thought, to why particular solutions are arrived at by AI models. The main aspects of interest for XAI are described and an overview concerning the current implementation methods is given. This chapter is concluded with an in-depth look into how fuzzy logic can be used as an XAI system and how it is possible to obtain interpretable solutions to complex problems through fuzzy rule extraction.

Chapter 6 introduced the proposed genetic Type-2 fuzzy logic many-objective system for the planning and optimisation of the UK access network. Firstly, the system is outlined with a brief description given for each of the aspects that have been combined for the overall development. This chapter proceeds to describe the modifications to the dominance system that is present in the Non-Dominated Sorting Genetic Algorithm II; also presented is the initial experiments that tested the proposed system on a typical many-objective optimisation problem alongside the optimisation and comparison of access network solutions.

Another investigation is presented in which the proposed system is shown to produce network solutions that outperform the current national approach to upgrading the access network. This experiment is interested in the strategic approach that requires the objectives highlighted in Chapter 2 to be met.

The dominance system that is presented in these experiments considers the traditional NSGA-II method, a Type-1 and an interval Type-2 fuzzy logic dominance method. In these experiments, it was found that each iteration of fuzzy logic provided improved results; Type-1 outperformed the traditional method and was itself outperformed by Type-2 fuzzy logic. The proposed system improved the results of the experiments by 46% over Type-1 fuzzy logic and 59% over the traditional method.

In Chapter 7, the proposed system is further modified in order to incorporate an explainable AI system. The optimisation of varying modularly designed regions of the United Kingdom allowed design insights to be extracted in the form of interpretable fuzzy IF-THEN rules. The XAI method was incorporated into the proposed system in order to identify how the manipulation of varying network constraints such as population density, bandwidth requirement, technology dependency etc. can be conveyed to a user with fuzzy rules.

This chapter gives an explanation to the genetic algorithm that is responsible for optimisation of membership function configuration. The GA is responsible for identifying the membership functions as a set of points that define the rules that can be extracted from the solutions present in the final Pareto-front. The chromosomes present in the GA population consist of varying numbers of real coded genes that each represent a position of a membership function. They are ranked based on their fitness in regards to their ability to produce optimally segmented fuzzy input sets.

This chapter concludes with an investigation into the presentation of optimised access network solutions, and how the explainable interface and the explainable AI system can be incorporated together in order to produce interpretable solutions.

8.2 Conclusion

The intention of this thesis was to present a novel system capable of encapsulating the grand scale of telecommunications network optimisation. The aim was to provide a new approach to the organisation, optimisation, and visualisation of social and economic, countrywide scale problems. It was important to consider the development in relation to British Telecom's objectives regarding the project; these objectives can be defined as follows:

1) Provide a meaningful and modular approach to the generation of statistically relevant regional data.
2) Provide a system capable of dealing with the complexities of a grand scale problem that takes into account the ever-changing nature of the field and technology.
3) Identify how a system could be employed to find improvements to the current methodology that is set out for network optimisation.
4) Consider how to present optimised networks to users with emphasis on the interpretability of a solution and what insights can be gleaned from it.

These objectives were fulfilled through a series of research, system development, and application creation. These aspects were alluded to in Chapter 1; and have been explored in detail throughout this thesis. These developments can be explored as follows:

- A novel simulation approach for the creation of varying population demographics.

This objective was significant as it determined the overarching philosophy of the proposed systems and the novel approach required to tackle the strategic planning of grand scale problems. This aspect of the proposed problem was discussed in section 6.2.1 and was achieved by using government available statistics concerning the population density for regions of the UK, alongside BTs data concerning the broadband requirements of UK demographics. The combination of freely available data and business data meant that a system could be created that procedurally generated subsections of the United Kingdom. This gave freedom and flexibility for continuous and targeted optimisation objectives that would otherwise be constrained and time consuming to produce.

- Construction of an elitist many-objective optimisation approach to the network optimisation problem that included a novel fuzzy logic based approach to the improvement of said algorithm.

This was achieved through the modification of the NSGA-II algorithm (as seen in Chapter 6). Experiments found that modifications of the traditional dominance method found in NSGA-II improved both the convergence and diversity of solutions. An improvement could be identified for each iteration of fuzzy logic that was implemented with the proposed interval Type-2 system having a 46% improvement over Type-1 and a 59% improvement over the traditional crisp dominance method. The reasoning for the improvement over the traditional method was discussed and the true strengths of Fuzzy Logic was addressed. Further experiments focusing on specific regions of the UK were carried out and they identified that the proposed system was able to improve the optimisation of targeted, constraint heavy, optimisation scenarios. In these experiments, it was found that the proposed system was able to produce a solution that outperformed the current methodology concerning the upgrading of the access network. It was noted that when rural regions were targeted networks could be produced that were on average 51.99% cheaper than the current approach and 54.16%

cheaper for mixed regions, while still being capable of supplying customer broadband requirements. In urban regions we further identified that solutions could be produced that were cheaper on average 34.48% of the time but struggled to compete on the delivery of broadband. However, we also found that approximately 14.23% of premises in urban regions were unnecessarily connected to FTTP when they could have been supplied adequately by FTTC.

- A novel approach to the extraction of explainable data from the solutions of the Pareto front.

This was achieved through the development of an XAI system explored in Chapters 5/7. The proposed system outlined in these chapters identifies how fuzzy logic, with its natural interpretability can be implemented in order to gain a better understanding of the system design and solutions produced. The XAI system is responsible for bringing clarity to an obtuse system that, alone, offers little to no understanding or interpretability. A series of experiments were run on the optimised solutions for rural, mixed, and urban population centres. In these experiments, the proposed system used the data collected from the solutions in order to produce initial membership functions and rules for the XAI system. Both of these are optimised until a system exists that is capable of extracting a refined set of rules from the solutions data. It is then shown that these rules provide the user with information and understanding concerning the raw solutions produced by the optimisation task.

- The development of a novel unified system that ties together all the required features, as determined from the objectives, both business and academic.

This was achieved through the development of a user interface that combined each of the other proposed systems. This system allows the user to carry out their unique optimisation specifications and receive a Pareto-set of solutions, which are instantly available in visual representation and accompanied by their extracted fuzzy rule base. The user interface also offers another interpretability aspect to the project and allows users to combine their understanding gained from extracted fuzzy rules with the exploration gained from the user interface.

With the successful implementation of these individual objectives, the focus of this thesis has been achieved. A new approach to the organisation, optimisation, and visualisation of social and economic, countrywide scale problems was developed.

8.3 Real-World Impact

The performance and interpretability concerning the optimisation of the access network has been improved in consideration of BTs goals for this thesis. No approach existed for the strategic optimisation of the access network, only a reactive method concerning the maintenance or continual upgrading of the network existed. The only method that was available meant that new build areas and failing network regions would receive an upgrade that did not consider the greater range of issues, objectives, and constraints. Now a solution exists that will enable the long term planning of the access network, which can also be modified to target any grand scale problem. The proposed systems as an entirety is being considered as part of a larger scale patent by BT, as it is believed to be an idea worth protecting.

The proposed system is not yet integrated into the BT systems; however, it is being considered as an additional system to accompany a series of other tools being developed by BT research that exist within the universe of discourse.

With a potential £5bn [141] being injected into the upgrading of the access network and concession surrounding the cost of providing FTTP broadband to all regions of the United Kingdom, the proposed solution promises to elevate these very issues, saving spending costs and providing confidence in the network solutions produced. For these reasons, and those discussed in Chapter 5, it is clear that AI technology is having a profound impact on real-world and academic problems. The proposed systems integrate technologies such as genetic algorithms, many-objective optimisation, and XAI in order to improve business objectives and decision-making.

8.4 Future Work

In regards to future work, there are several areas of research that would be able to provide improvements to the proposed systems. As mentioned previously a form of continuous learning would drastically affect the way in which grade optimisation challenges are tackled. The integration of this form of technology would allow an even greater number of objectives, constraints, and complexity to be modelled. This would allow for drastically new insights not yet considered and it would massively increase our understanding of both the system and the solutions it produces.

If continuous learning could be combined with an improvement to the visualisation of interpretable fuzzy IF-THEN, rules this would provide an even greater level of understanding for users. Another benefit to this would be a much greater sense of confidence as users would have a much greater understanding of how typically opaque AI systems work.

Another potential improvement to the proposed systems is the development of optimisation methods for interval Type-2 fuzzy membership functions. The development of a system in which greater freedom is provided to the parameters being optimised, that does not drastically increase the computational or design complexities, would be considered as a noticeable improvement on the current system.

A considerable improvement could be the implementation of general Type-2 fuzzy logic, this form of fuzzy logic could act as another improvement to the solutions produced by the proposed framework. General Type-2 fuzzy logic could be deployed across multiple sections of the framework and could therefore improve both the optimisation of results as well as the extraction of interpretable information. General Type-2 fuzzy logic would likely be a very noticeable improvement as it is more capable than interval type-2 fuzzy logic at dealing with uncertainty and complexity, both of which are common to this problem.

Another extension to the work could see improvements made to the many-objective optimisation algorithm being used to produce solutions. The NSGA-II algorithm could be further extended to the newer NSGA-III algorithm which has been shown to outperform NSGA-II. The Heated Stack [142] algorithm is another potential optimisation algorithm that could be used to improve results as it has also been tested against both NSGA algorithms in relation to multi-objective optimisation problems and has been found to be an improvement over them within this problem domain.

Lastly, the population of candidate solutions could be initially manipulated in order to guide the future generation of solutions towards an end goal pre-conceived by a user, or potentially identified by an earlier iteration of the proposed system.

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