Ambiguity and ambiguity attitude

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Abstract

Ambiguity refers to scenarios where probabilities are not known nor objectively specified as opposed to scenarios of risk where probability are objectively available. Scenarios with ambiguity are widespread in economics. Based on early thought experiments, subsequent literature has found that many individuals choices are different under ambiguity from those implied by the standard subjective expected utility model. This has led to significant extensions and advances in models of decision-making. In turn these models have been useful to accommodate a rich pattern of choices and to help understand a wide range of economic phenomena.

Introduction

The following example of a decision problem is known as the Ellsberg 3-color urn example and provides a stark illustration of the phenomenon of "ambiguity aversion" under the Ellsberg paradox (Ellsberg (1961)). An urn contains 90 balls of 3 color – red, blue, and yellow. 30 of the balls are known to be red, while the composition of the remaining 60 balls between blue and yellow is unknown. Individuals are offered the following choices, involving the colour of a ball to be drawn from the urn. The first choice is between a bet on the ball drawn being red versus a bet on the ball drawn being yellow. The second choice is between two different bets namely, a bet on the ball drawn being red or blue versus a bet on the ball drawn being blue or yellow. Experimental evidence found that many individuals strictly preferred a bet on red in the first choice and a bet on blue or yellow in the second choice (Camerer and Weber (1992), Machina and Siniscalchi (2014), Trautmann and van de Kuilen (2015)).

These preferences exhibit "ambiguity aversion" and contradict the Savage (1954) subjective expected utility (SEU) hypothesis under which individual beliefs are represented by a single probability measure over states, which in the example here correspond to the colour of the ball drawn, and individual tastes are represented by a utility over outcomes. In fact, these preferences contradict the weaker hypothesis that individual beliefs are represented by a single probability measure. A strict preference for a bet on red over a bet on green in the first choice implies a strictly higher likelihood of red than green. On the other hand, a strict preference for a bet on green or blue over a bet on red or blue in the second choice implies that the likelihood of the event red or blue is strictly lower than that of the event green or blue. However, if beliefs are represented by a probability measure then the strictly higher likelihood of the event green or blue over the event red or blue implies that the likelihood of red is strictly lower than that of green, contradicting the implication of the preference in the first choice.

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The incomplete composition of the urn is a scenario of 'ambiguity', where probabilities are not known nor objectively specified. A conceptual distinction has been drawn between scenarios that can be described by known or calculable probabilities, termed scenarios involving 'risk' and scenarios where probabilities are neither known nor calculable , termed scenarios of 'uncertainty' or 'ambiguity' following Ellsberg (1961) since at least the work of Knight (1921) and Keynes (1921). The preferences specified in the ambiguous urn scenario above suggest that individuals may exhibit "ambiguity aversion", i.e. prefer the ultimate outcome of their choice to depend on events whose probabilities they know or can calculate rather than on events with unknown probabilities. On the other hand, if individuals had preferences opposite to those in each of the choice scenarios above, then they would be exhibiting "ambiguity loving".

1 Theoretical extensions to SEU

To accommodate ambiguity-sensitive preferences like these, new axiomatic representations have been developed. Some of the leading axiomatic representations of ambiguity-sensitive preferences are the following. More detailed discussions of these and many other representations can be found in Gilboa and Marinacci (2012), Machina and Siniscalchi (2014), Wakker (2010).

Non-additive probability Schmeidler (1989) axiomatized the Choquet expected utility (CEU) representation as an alternative to the SEU representation, independently of the Ellsberg (1961) paradox. The key difference under the CEU representation is that beliefs are represented by non-additive measures or capacities, which can accommodate the preferences expressed under the paradox. With a capacity, the "probability" of the event red or blue (respectively, green or blue) does not need to be sum of the individual "probability" of red and of blue (respectively, of green and of blue). This in turn means that contradiction under SEU with a additive probability measure does not arise for CEU. The non-additive probability representation also allows for the ambiguity-sensitive preferences to be represented via the prospect theory framework which allows for a separation of ambiguity attitudes across gains and losses similar to the case of risk (Wakker (2010)).

Multiple-priors Gilboa and Schmeidler (1989) axiomatized a multiple-priors maxmin expected utility (MEU) representation. The key difference of this representation to the SEU representation is that beliefs are represented by a set of (additive) probability measures instead of just a probability measure and the utility from a choice is the minimal expected utility over the set of measures. This accommodates the paradoxical choices above if the set of measures includes one probability measure where the probability of red is strictly higher than that of green and one probability measure where the probability of red is strictly less than that of green.

Smooth model Klibanoff, Marinacci, and Mukerji (2005) axiomatized a representation where the individual's beliefs are represented via a 'first order' set of probability measures and a 'second order' probability measure with support over the first order set. The utility from a choice is expressed as follows: each expected utility under the first order set is weighting using a possibly non-linear 'second order' utility function and then these weighted expected utilities are aggregated using the second order probability measure similarly to SEU. This representation aims to separate ambiguity attitude, captured by the second order utility, from ambiguity perception, captured by the second order probability and its support (the set of first order probabilities).

2 Implications for economic decision-making

The study of ambiguity and ambiguity attitude and the implications of ambiguity-sensitivity in economics continues and is too extensive fully to cover here. Subsequent work has found richer patterns of ambiguity attitude than universal ambiguity aversion (Trautmann and van de Kuilen (2015)): ambiguity aversion is commonly found for choices involving moderate or high likelihood gains or low likelihood losses while ambiguity seeking or ambiguity neutrality is commonly found for choices involving low likelihood gains or moderate or high likelihood losses. Machina (2009, 2014) proposed choice problems related to Ellsberg (1961) which posed challenges for many of the widely-used ambiguity-sensitive preference representations (Machina and Siniscalchi (2014)).

Moving away from the SEU representation also raises questions regarding the dynamic or intertemporal choice problems. Two key issues that emerge involve how beliefs are updated and whether dynamic consistency and consequentalism can both be preserved when considering dynamic choices for ambiguity-sensitive preferences (Gilboa and Marinacci (2012), Machina and Siniscalchi (2014)).

Allowing for ambiguity-sensitive preference representations has proved to be very useful in understanding a wide range of phenomena. This includes a wide range of empirical phenomena and issues in macroeconomics and in finance (Epstein and Schneider (2010), Guidolin and Rinaldi (2013) Ilut and Schneider (2023)). A number of applications have also considered strategic interactions between individuals with ambiguity-sensitive preferences (Mukerji and Tallon (2001), Carroll (2019), Hanany, Klibanoff, and Mukerji (2020)).

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