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Oligopoly banking, risky investment, and monetary policy[☆]

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ABSTRACT

Oligopolistic competition in the banking sector and risk in the real economy are important characteristics of many economies. We build a model of monetary policy transmission that incorporates these characteristics which allows us to analyze the long-run consequences of variations in the degree of banking competition. We show theoretically that various equilibrium cases can occur, and that the effect of monetary policy varies greatly across equilibrium cases. We calibrate the model to the U.S. economy in 2016-2019 and find that monetary policy pass-through is incomplete under imperfect competition. Further, we show that a decrease in the policy rate during the calibration period would have increased expected welfare, but also bank default probability.

1. Introduction

In the recent financial crisis in the spring of 2023, the preferred reaction by policymakers to stabilize troubled banks was to let competitors take them over, as seen most prominently by the takeover of Credit Suisse by UBS in Switzerland, but also with some regional banks in the U.S. While this seems to be a reasonable approach to stabilize the sector in the short run, it will undoubtedly further reduce the degree of competition in an industry that is already characterized by imperfect competition and a high cost of entry.[1] Thus, the goal of this paper is to understand the long-run consequences of a variation in the degree of competition in the banking sector on macroeconomic outcomes, in particular on loans, welfare, the transmission of monetary policy, and also bank profitability and the probability of future bank default.

To do so, we build on papers such as Williamson (2012) and Altermatt (2022) that take the role of liquidity and the financial system in the transmission of monetary policy seriously. We extend them by introducing oligopolistic competition in the banking sector, and idiosyncratic and aggregate risk in the real economy. Doing so allows us to build a parsimonious model of the macroeconomy that highlights the pivotal roles banks play in the transmission of monetary policy to the real economy. In our model, there is a unique monetary equilibrium under standard regularity conditions. However, depending on parameter values, several equilibrium cases can occur, since banks may invest in multiple asset classes, fund themselves through deposits or equity, and face regulatory constraints which may or may not bind. We show that the effect of monetary policy and the effect of banking competition vary across the equilibrium cases. It is therefore pertinent to have a model where these cases may occur endogenously

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[1] See for example Drechsler et al. (2017) who document bank market power in the U.S.

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in order to analyze the effects of monetary policy. Furthermore, the equilibrium cases are easily identifiable empirically as they reflect scenarios such as the zero-lower bound, or regimes with either excess or scarce reserves.

We show that incorporating imperfect banking competition and risky investment into a model that takes liquidity and banks' portfolio choices seriously allows us to (1) match important empirical facts, and (2) improve our understanding of how monetary policy is transmitted to the real economy. Specifically, our model generates endogenous, positive spreads between the policy rate and the deposit rate, as well as between the loan rate and the policy rate. In line with empirical studies on banking competition such as Drechsler et al. (2017), our model predicts that both the quantities of loans and deposits and the pass-through from the policy rate to the deposit interest rate are decreasing with bank market power. In particular, we show that if bank market power is significant, changes in the policy rate have no effect at all on the deposit rate unless the policy rate is set relatively high (i.e., above 3% in our calibration). This finding implies that when the policy rate is low, the monetary authority loses control over inside money creation in the presence of bank market power. Finally, we find that while bank profits are increasing in the policy rate, its effect on bank default probability depends on the equilibrium cases. Nevertheless, according to our calibration to the U.S. economy in 2016–2019, bank default probability is decreasing in the policy rate.

Looking ahead, our findings suggest that the decrease in the degree of competition in the banking sector will lead to significant reductions in loans, deposits, and pass-through of monetary policy; however, it also reduces the probability of future bank failures, in part by increasing bank profitability and thereby increasing the safety cushion of banks against adverse shocks.

Model summary. Our model is based on the Lagos and Wright (2005) framework, and includes banks that perform liquidity transformation as in Altermatt (2022) or Keister and Sanches (2023). We introduce competition à la Cournot[2] in the banking sector and a risk-return trade-off for entrepreneurs as in Martinez-Miera and Repullo (2010). Each period is divided into a DM and a CM, and the economy is populated by three types of agents: households, banks, and entrepreneurs. In the DM, households meet with each other and there are gains from trade, but due to anonymity and limited commitment, households need liquid assets such as bank deposits and cash to complete trades. In the CM, entrepreneurs are the unique agents that have investment opportunities. If an entrepreneur invests, she faces a risk-return trade-off: she can choose a project with a high success rate but a low payoff, or one with low success rate but a high payoff. The quality of the project is heterogeneous across entrepreneurs. Specifically, an entrepreneur with a better project either receives a higher return given the success rate, or has a higher success rate given the project return. There is also aggregate risk, which affects the default probability of all entrepreneurs.

We assume banks are the unique agents that can enforce loans made to entrepreneurs. Banks can fund their lending either by raising equity or by issuing deposits. We assume that there is a fixed number of banks, and banks compete à la Cournot for loans and deposits. Besides lending to entrepreneurs, banks may purchase government bonds or hold reserves. Banks are also subject to reserve and capital requirements set by the government. While banks can fully protect themselves against the idiosyncratic risk choices of entrepreneurs through diversification, the aggregate shock introduces the possibility of bank default. If banks default, i.e. if the value of their assets is less than their outstanding deposits, their remaining assets are distributed to depositors by the government.

We find that there is a unique equilibrium, which can be divided in three cases depending on parameters. In Case I, banks are indifferent about raising additional equity, as the marginal cost of deposits equals the marginal cost of equity. Thus, the capital requirement is non-binding. Case I occurs when the demand for deposits by households is low relative to the demand for loans by entrepreneurs and the supply of government bonds. In Case II, the deposit rate is above the zero-lower bound, but the marginal cost of raising deposits lies strictly below the marginal cost of raising equity. Thus, banks prefer funding themselves with deposits over equity, so the capital requirement binds. Finally, Case III is characterized by the deposit rate being at the zero-lower bound. This case occurs when the demand for deposits by households is large relative to the demand for loans by entrepreneurs and the supply of government bonds.

Existing literature. The New Monetarist literature based on Lagos and Wright (2005) provides an excellent framework to study the transmission of monetary policy to the real economy. Within this literature, one of the first papers to take the role of banks into account is Berentsen et al. (2007). In this paper, perfectly competitive banks intermediate liquid assets from agents that do not want to consume during the DM to agents that want to consume. Dong et al. (2021) was among the first papers to study oligopolistic banking competition in this literature, but in their model, banks do not create liquid assets and extend loans to entrepreneurs as in our paper. In Williamson (2012), agents can use interest-bearing bonds to pay in some DM meetings, whereas they can only use fiat money in others. This creates a role for banks as they are able to insure agents against this uncertainty. Altermatt (2022) studies how monetary policy is transmitted to the real economy in a model where perfectly competitive banks perform liquidity creation by investing in illiquid assets such as loans to entrepreneurs and bonds and issue liquid assets. Gu et al. (2019) show, using a variety of different models, that banking is inherently unstable. Andolfatto et al. (2020) integrate banks a la Diamond (1997) in a New Monetarist framework and show that nominal deposit contracts combined with a central bank acting as a lender of last resort

[2] Bertrand competition may seem theoretically more appealing than Cournot to model competition in the banking industry since banks are not quantity-constrained in the same way other firms in the economy are. However, both of these approaches are imperfect since in reality, the way banks compete with each other is highly dynamic and thus described by a process far more complicated than either Cournot or Bertrand. We find that Cournot competition offers a tractable way to introduce important aspects of imperfect competition into the model, in particular bank profits and a positive spread between deposit and loan rates — neither of which Bertrand competition can generate.

allow for efficient liquidity insurance and a panic-free banking system.[3] Our paper contributes to the literature by extending models such as Williamson (2012) or Altermatt (2022) to allow for imperfect banking competition and bank default risk, and analyze what this implies for monetary policy. In this regard, our paper is close to Chiu et al. (2023), as they also introduce bank market power to such models. However, they focus primarily on bank market power in the context of central bank digital currency (CBDC) and abstract from risky investments, banking regulation, and bank assets such as government bonds.

In the broader literature on macroeconomics, Gertler and Karadi (2011) is an important paper on the transmission of monetary policy through the financial system. In this paper, an agency problem between banks and their depositors leads to endogenous constraints on the banks' leverage ratios.[4] Papers investigating similar questions as we do are Wang et al. (2022) and Corbae and Levine (2021), but the approaches taken in these papers vary significantly from ours. Wang et al. (2022) assume the risk in loans are exogenous and unaffected by agents' decisions, while in Corbae and Levine (2021) the demand for deposits is taken as given. We show that both endogenous risk choices and the deposit channel are crucial to our results. Another important difference is that we stress the importance of various equilibrium cases that may occur, with different implications for monetary policy and bank regulation, while such considerations are absent from both Wang et al. (2022) and Corbae and Levine (2021).

Imperfect banking competition has also been studied in microeconomic models of banking. Keeley (1990) is an early example of a theoretical framework on the relationship of banking competition and financial stability. The paper argues that a reduction in charter value and monopoly rents for banks leads to an increase in bank defaults. However, Allen and Gale (2004b) show that with incomplete markets, the efficiency gains from increased competition in the banking sector outweigh the losses resulting from a financial crisis.[5] In addition to reducing competition, another way to limit the agency problem is to enforce capital requirements on banks. While Marshall and Prescott (2001) show that capital requirements can indeed limit the agency problem, Hellmann et al. (2000) argue that such capital requirements can lead to Pareto-inefficient outcomes. Repullo (2004) shows that risk-based capital requirements can be used to effectively control risk-shifting incentives. Martinez-Miera and Repullo (2010) develop a Cournot banking competition model similar to ours. However, they abstract from banks' portfolio choices, monetary policy, as well as banking regulations. Kashyap et al. (2020) show that deposit funding and bank lending are not optimally chosen without regulation. While we focus on symmetric equilibria with equally-sized banks, Corbae and D'Erasmo (2021) study how regulatory policies affect bank failure rates as well as lending and interest rates in a model with an endogenous size distribution of banks. Our main contribution to this literature is that we micro-found money demand and inside money creation. By modeling liquidity demand and banks' portfolio choices explicitly, we discover equilibrium cases that are crucial for understanding the effects of monetary policy and banking regulations on loan supply, deposit supply, bank profits, and bank default probability.

Outline. The rest of the paper is as follows: Section 2 describes the model. Section 3 discusses the equilibrium. Section 4 explains the calibration strategy. Section 5 discusses the counterfactuals. Finally, Section 6 concludes the paper.

2. The model

2.1. Physical environment

Time is discrete and continues forever. Every period is divided into two subperiods: the DM (the decentralized market) and the CM (the centralized market). There is a measure two of infinitely-lived households, divided equally into buyers and sellers. In the DM, buyers consume a DM good that can only be produced by sellers. In the CM, sellers consume a CM good that can be produced by all agents. The CM good also serves as the numéraire. Both DM and CM goods are perishable and cannot be stored across periods. A buyer's utility is

$$u(q_t) - l_t \tag{2.1}$$

where $q_t \geq 0$ is the consumption of the DM good and l_t is the labor supplied in the CM. We assume $u'(.) > 0$, $u''(.) < 0$, and $-qu''(q)/u'(q) < 1$. A seller's utility is given by

$$x_t - h_t, \tag{2.2}$$

where x_t is the consumption of the CM good and h_t is the labor supplied in the DM. We assume that by using one unit of labor, sellers can produce one unit of the DM good, and all agents can produce one unit of the CM good. All households share the same discount factor β. We assume there is no discounting between subperiods.

Each period starts with the DM. At the beginning of the DM, a buyer is matched with a seller with probability one, and buyers make take-it-or-leave it offers to sellers.[6] Because buyers and sellers are anonymous in the DM, credit arrangements are not possible and a medium of exchange is necessary. Following Williamson (2012), we assume that a fraction $1-\eta$ of the meetings are *unmonitored*,

[3] Other related papers that study banks in similar framework include He et al. (2008), Gu et al. (2013), Imhof et al. (2018), Ait Lahcen and Gomis-Porqueras (2021), Head et al. (2022), and Choi and Rocheteau (2023). Andolfatto (2021) and Keister and Sanches (2023) study the roles of central bank digital currency in banking.

[4] See also Curdia and Woodford (2009), Gertler et al. (2012), Gertler and Kiyotaki (2015), Bianchi and Bigio (2022), and Brunnermeier and Koby (2018).

[5] See also Allen and Gale (2004a) and Boyd and De Nicolo (2005).

[6] Lagos and Wright (2005) explore the role of different bargaining powers, and Rocheteau and Wright (2005) explore the role of different trading protocols in similar models. We assume take-it-or-leave it offers in order not to complicate the model further, but our results are robust to other specifications.

which means buyers can only use cash as payment. The remaining meetings are *monitored*, which means buyers can pay with either cash or bank deposits. Differing from Williamson (2012), we assume that buyers learn the type of meeting they will be in during the next DM at the beginning of the previous CM, so there is no uncertainty regarding payments.

In addition to households, there is also a measure S of entrepreneurs who only live for one period. Each entrepreneur is born in the CM of period t and is endowed with a project that can produce CM goods in the CM of period $t+1$. Each project requires one unit of capital investment, which can be converted from one unit of the CM good. Before the end of the CM, each entrepreneur must choose a production technology. The production technology is represented by $R \in (0, R^i]$ where R^i is specific to entrepreneur i. The output given R^i, the entrepreneur's choice of R, and an aggregate shock s, is

$$y(R, R^i, s) = \begin{cases} 0 \text{ with probability } p(R, R^i, s), \\ R \text{ with probability } 1 - p(R, R^i, s). \end{cases} \tag{2.3}$$

Expression (2.3) says that if an entrepreneur chooses R, the production will yield R with probability $1 - p(R, R^i, s)$ and 0 otherwise. We assume that $p(R, R^i, s)$ is increasing in R and s and is decreasing in R^i, and that s is realized at the beginning of the next CM, when entrepreneurs' projects generate output.[7] Conditional on s, the realization of $y(R, R^i, s)$ is independent across all entrepreneurs. Note that R^i can be interpreted as an entrepreneur's innate ability. An entrepreneur with a larger R^i can choose a technology with higher potential yield and is more likely to succeed for any given R and s. We assume R^i is determined when an entrepreneur is born and follows a distribution on $[0, \bar{R}]$ characterized by a CDF $F(\cdot)$ and a PDF $f(\cdot)$. The aggregate shock s follows a distribution on $[\underline{s}, \bar{s}]$ characterized by a CDF $G(\cdot)$. Finally, after production, capital fully depreciates. Entrepreneurs only consume in the second CM of their lives, and they derive linear utility from consuming the CM good.

The economy also contains N banks that are born in the CM of period t and dissolve in the CM of period $t+1$. Unlike buyers, banks can commit to their liabilities, and therefore bank deposits are accepted as payment in the DM. Because entrepreneurs are not endowed with any CM good and they cannot work in the CM, they must borrow from other agents in the economy in order to invest. We assume banks can costlessly verify entrepreneurs' output in the CM while the cost for households is infinite. Hence, entrepreneurs can only borrow from banks. We assume banks can work in the CM using the same technology as buyers, and working generates linear disutility. This can be interpreted as giving banks the option to raise sweat equity. Alternatively, banks can fund their lending by issuing deposits. Banks receive linear utility from consuming during the second CM of their existence, and they discount at rate β between periods. Banks compete à la Cournot in both the loan market and the deposit market. We assume that the assets and liabilities of each bank can only be observed by the bank itself.[8] Deposits can be transferred to other agents in the DM, and in the next CM they can be redeemed for the CM good.

The economic activities in the CM are as follows. At the beginning of the CM, entrepreneurs born in the last period settle their debt with banks and consume. Then banks settle their debt with households and consume. We assume that both entrepreneurs and banks are subject to limited liability. That is, they can default on their liabilities if their net worth is negative. If an entrepreneur declares bankruptcy, a bank can seize the entrepreneur's output. If a bank defaults on its deposit liabilities, a government will seize the bank's assets and distribute them to its creditors.[9] Finally, after the old generation of entrepreneurs and banks consume, they are replaced with a new set of entrepreneurs and banks.

The government in the model economy controls the supply of fiat money and issues a one-period nominal bond. Let M_t^H and M_t^B denote the amount of money held by households and banks (i.e., reserves) in period t, respectively. We assume the total money supply grows at net rate μ, i.e., $M_{t+1}^H + M_{t+1}^B = (1+\mu)(M_t^H + M_t^B)$. We also assume that the government pays interest on reserves held by banks. Denote the nominal interest on reserves as i^R. Each unit of government bonds pays $1 + i_t^B$ units of money in the CM of period t. Let B_t denote the government bonds outstanding in period t. We assume government bonds cannot be used as payment in the DM.[10] The government also collects a lump-sum tax τ_t from households in the CM. The government's budget constraint is

$$\phi_t(M_t^H + M_t^B + B_t) + \tau_t = \phi_t\left(M_{t-1}^H + (1+i_t^R)M_{t-1}^B + (1+i_t^B)B_{t-1}\right) \tag{2.4}$$

where ϕ_t is the price of money in terms of CM good. We define $r_t^B = \frac{\phi_t(1+i_t^B)}{\phi_{t-1}}$ to be the gross real interest rate on government bonds, and we will also refer to it as the policy rate.

2.2. Household's problems in the DM and CM

In what follows, we restrict our attention to stationary equilibria, which implies $\frac{\phi_t}{\phi_{t+1}} = 1 + \mu$ and $r^B = \frac{1+i^B}{1+\mu}$. Now, let a denote the amount of real assets a buyer has at the beginning of the DM, which can be either cash or bank deposits. As is standard in

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[7] In Appendix A.1, we also consider the scenario where the aggregate shock s is realized in the DM. We show that as long as there exists a deposit insurance funded by actuarially fair premiums paid by banks, all results in this paper remain unchanged.

[8] This assumption ensures that households treat deposits issued by different banks as the same, which is necessary for Cournot competition in the deposit market. The opaqueness of the banking system is discussed extensively in Dang et al. (2017).

[9] Note that our timing assumptions are such that default occurs only after DM trade has occurred. Since households are risk-neutral in the CM, this in turn means that the banks' potential default has no welfare impact beyond the reduction in the expected deposit rate that results from the probability of default. As discussed in Footnote , our results are robust to changing the timing assumptions as long as one allows for actuarially fair deposit insurance.

[10] For example, government bonds may not be recognizable by sellers in the DM.

models following Lagos and Wright (2005), the households' value function in the CM is linear in their real wealth at the end of the DM. Hence, buyers solve

$$\max_{q}\{u(q)-q\} \quad \text{s.t. } q \leq a. \tag{2.5}$$

Let $q(a)$ denote the buyer's consumption in the DM. Then $q=\min\{a,q^*\}$, where $u'(q^*)=1$.

Next, we turn to the CM. Due to the quasilinearity of the household's utility function, the portfolio choice of buyers is independent of households' wealth. Let r^D denote the expected gross real deposit rate. Note that in a stationary equilibrium, the return on fiat money is equal to $1/(1+\mu)$. Let z, d, and b denote the amount of real fiat money balances, bank deposits, and bonds a monitored buyer chooses to carry. Monitored buyers in the CM choose their asset portfolio (z,d,b) to maximize their expected surplus.

$$\max_{z,d,b,q^d}\left\{-\frac{z}{\beta}-\frac{d}{\beta}-\frac{b}{\beta}+u(q^d)+\frac{z}{1+\mu}+r^Dd+r^Bb-q^d\right\} \text{ s.t. } q^d=\min\left\{\frac{z}{1+\mu}+r^Dd,q^*\right\}. \tag{2.6}$$

Monitored buyers will carry fiat money if and only if $1/(1+\mu)\geq r^D$. Monitored buyers will exhaust their money and deposits in the DM (i.e. $q^d=z/(1+\mu)+r^Dd$) unless $\max\{1/(1+\mu),r^D\}\geq 1/\beta$. Note that because government bonds cannot be used as payment in the DM, monitored buyers will hold bonds if and only if $r^B\geq 1/\beta$. Unmonitored buyers solve

$$\max_{\tilde{z},\tilde{d},\tilde{b},q^m}\left\{-\frac{\tilde{z}}{\beta}-\frac{\tilde{d}}{\beta}-\frac{\tilde{b}}{\beta}+u(q^m)+\frac{\tilde{z}}{1+\mu}+r^D\tilde{d}+r^B\tilde{b}-q^m\right\} \text{ s.t. } q^m=\min\left\{\frac{\tilde{z}}{1+\mu},q^*\right\}. \tag{2.7}$$

Unmonitored buyers will hold bonds if and only if $r^B\geq 1/\beta$. Since they cannot use deposits in the DM, they will also only hold deposits if $r^D\geq 1/\beta$. Similarly, sellers will hold money, bank deposits and government bonds if and only if $1/(1+\mu)\geq 1/\beta$, $r^D\geq 1/\beta$ and $r^B\geq 1/\beta$, respectively. We denote the total demand for government bonds from households as b^H.

2.3. Entrepreneur's problem in the CM

Let r^L denote the gross real lending rate. First, entrepreneurs choose the production technology by maximizing their expected value from producing, given the loan rate and R^i:

$$v(r^L,R^i)=\max_{R}\mathbb{E}\{[1-p(R,R^i,s)](R-r^L)\}, \tag{2.8}$$

where the expectation is taken over s. In words, (2.8) says that with probability $1-p(R,R^i,s)$, the entrepreneur succeeds, and her surplus after repaying the loan is $R-r^L$. It is clear that as long as $r^L>0$, entrepreneurs default if and only if their projects fail. Define

$$p(R,R^i)=\mathbb{E}[p(R,R^i,s)], \tag{2.9}$$

and let $R^*(r^L,R^i)$ denote the solution. Then

$$1-p(R^*(r^L,R^i),R^i)-p_R(R^*(r^L,R^i),R^i)[R^*(r^L,R^i)-r^L]\geq 0, \tag{2.10}$$

with equality when $R^*(r^L,R^i)\in(0,R^i)$. Note that entrepreneurs will never choose $R=0$. Now, suppose $p_{RR}(\cdot)\geq 0$. Then, there exists a unique solution as long as $R^i\geq r^L$, and $R^*(r^L,r^L)=r^L$. Furthermore, $R^*(r^L,R^i)$ is increasing in r^L and R^i. Finally, let

$$v(r^L,R^i)=[1-p(R^*(r^L,R^i),R^i)](R^*(r^L,R^i)-r^L). \tag{2.11}$$

It is clear that $v(r^L,R^i)$ is decreasing in r^L. Hence, entrepreneurs are willing to invest in projects (i.e., $v(r^L,R^i)\geq 0$) if and only if $R^i\geq r^L$. Note that the heterogeneity across entrepreneurs and their technology choice imply that both the extensive margin on loans and the riskiness of loans are endogenous in our model. This allows us to better understand how policy changes affect equilibrium outcomes, in particular bank default.

2.4. Bank's problem in the CM

Since only entrepreneurs with $R^i\geq r^L$ borrow from banks, the demand for loans is decreasing in r^L and is given by

$$L(r^L)=S[1-F(r^L)]. \tag{2.12}$$

Total demand for deposits $D(r^D)$ is increasing in r^D and given by

$$D(r^D)=\begin{cases}0, \text{ if } r^D<\frac{1}{1+\mu};\\ \eta(u')^{-1}\left(\frac{1}{\beta r^D}\right)/r^D, \text{ if } \frac{1}{1+\mu}\leq r^D<1/\beta;\\ \geq\beta\eta q^*, \text{ if } r^D\geq 1/\beta.\end{cases} \tag{2.13}$$

This expression follows from the solution to (2.6), and the interpretation is relatively straightforward: Whenever r^D is below the real return on cash (i.e., when the nominal deposit rate is negative), the demand for deposits is zero since all households prefer using cash to purchase q; when r^D is (weakly) higher than the real return on cash but strictly lower than $1/\beta$, the demand for deposits is increasing in r^D since a higher interest rate makes it cheaper for monitored buyers to purchase q; finally, if r^D is at least equal to $1/\beta$, deposits are costless to carry, so households carry enough to purchase q^* (and are indifferent to carrying more if $r^D = 1/\beta$, while they want to carry an infinite amount if $r^D > 1/\beta$).

Conditional on s, the expected probability of default on bank loans is

$$P(r^L, s) = \frac{1}{1 - F(r^L)} \int_{r^L}^{\bar{R}} p(R^*(r^L, R^i), R^i, s) f(R^i) \mathrm{d}R^i. \tag{2.14}$$

Note that with the aggregate shock, banks may default in equilibrium. Specifically, let l_j and d_j denote the quantities of loans and deposits originated from bank j. Let z_j and b_j denote the quantities of reserves and government bonds held by bank j. Let $\tilde{r}^D$ denote the deposit rate a bank pays if it does not default. Under limited liability, banks default if and only if

$$l_j r^L [1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B < d_j \tilde{r}^D. \tag{2.15}$$

Recall that r^D represents the expected deposit rate. Adjusted for the possibility of default, r^D is given by

$$d_j r^D = \mathbb{E}\left[\min\left\{ l_j r^L [1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B, d_j \tilde{r}^D \right\}\right], \tag{2.16}$$

where the expectation is taken over s.

Before moving on, we briefly discuss how we model bank default. By assumption, depositors are risk-neutral at the time bank default is realized, so the welfare effects of bank default in our model are negligible. This choice is deliberate, since we find it difficult to reliably measure the welfare cost of bank default. Hence, welfare in our model should be interpreted as an efficiency measure, and we leave it to policymakers to weigh improvements in welfare against reductions in the risk of bank default.

Based on the above analysis, banks' expected *payoff* in the CM is equal to

$$\mathbb{E}\left[\max\left\{ l_j r^L [1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j \tilde{r}^D, 0 \right\}\right]. \tag{2.17}$$

Note that if we add up (2.16) and (2.17), we get

$$\begin{aligned} & d_j r^D + \mathbb{E}\left[\max\left\{ l_j r^L [1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j \tilde{r}^D, 0 \right\}\right] \\ =& \mathbb{E}\left[l_j r^L [1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B \right] = l_j r^L [1 - P(r^L)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B, \end{aligned}$$

where $P(r^L) = \mathbb{E}[P(r^L, s)]$. Now, let e_j denote bank j's equity. Let $r^L(L)$ and $r^D(D)$ be the inverse demand functions for loans and deposits. Bank j solves

$$\max_{d_j \geq 0, e_j \geq 0, z_j \geq 0, b_j \geq 0} \Pi_j = -e_j + \beta\left[l_j r^L(L)[1 - P(r^L(L))] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D(D) \right], \tag{2.18}$$

$$\text{s.t. } l_j = d_j + e_j - z_j - b_j, \tag{2.19}$$

$$e_j \geq \gamma l_j, \tag{2.20}$$

$$z_j \geq \delta d_j, \tag{2.21}$$

with $L = l_j + \sum_{j' \neq j} l_{j'}$ and $D = d_j + \sum_{j' \neq j} d_{j'}$. Constraints (2.20) and (2.21) represent capital and reserve requirements on banks, respectively. Bank j takes the other banks' choices as given when choosing l_j and d_j. Note that all banks face the same expected deposit rate $r^D(D)$ because by assumption, the assets and liabilities of each bank can only be observed by the bank itself. This ensures that households treat deposits issued by different banks as the same.

We focus our attention on symmetric solutions where l_j and d_j are the same for all j. Now, define $H(L) = r^L(L)[1 - P(r^L(L))]$, the expected return per unit of loan. Let ζ and κ be the Lagrangian multipliers for constraints (2.20) and (2.21), respectively. The first order conditions w.r.t. z_j, b_j, d_j, and e_j are as follows:

$$l_j H'(L) + H(L) = \frac{1+i^R}{1+\mu} + \zeta\gamma + \kappa; \tag{2.22}$$

$$l_j H'(L) + H(L) = r^B + \zeta\gamma; \tag{2.23}$$

$$l_j H'(L) + H(L) = r^D(D) + d_j r^{D\prime}(D) + \zeta\gamma + \kappa\delta; \tag{2.24}$$

$$l_j H'(L) + H(L) = \frac{1}{\beta} - \zeta(1-\gamma). \tag{2.25}$$

We solve the equilibrium in the next section.

3. Equilibrium analysis

We restrict our attention to stationary equilibria. Recall the quantity of government bonds held by a household is b^H, and the total supply of government bonds is b. Below we define the equilibrium.

Definition 3.1. A stationary and symmetric equilibrium consists of the quantity of bank loans L, the quantity of deposits D, the quantity of equity E, the quantity of bank reserves z^B, the quantity of government bonds held by banks b^B, the quantity of government bonds held by households b^H, a loan rate r^L, a deposit rate r^D, and a government bond rate r^B such that

(1) L, D, E, z^B, and b^B satisfy (2.19) as well as the first order conditions (2.22) - (2.25) with $d_j = D/N$, $e_j = E/N$, $l_j = L/N$, $z_j = z^B/N$, and $b_j = b^B/N$ for all j;

(2) r^L is given by (2.12) and r^D is given by (2.13);

(3) r^B clears the bond market: $2b^H + b^B = b$.

Proposition 3.1. *Suppose $H'(L) < 0$ and $H''(L) < 0$ for all $L \in [0, S]$, and $r^{D\prime}(D) > 0$ and $r^{D\prime\prime}(D) \geq 0$ for all $D \geq 0$. Then, there exists a unique symmetric equilibrium.*

Proof. See Appendix C.1.

It is worth mentioning that the conditions $H'(L) < 0$ and $H''(L) < 0$ are standard for the existence and uniqueness of a symmetric solution to the oligopoly competition problem. See for example Tirole (1988) and Martinez-Miera and Repullo (2010). Since the equilibrium conditions are characterized by a number of inequalities that may or may not hold, different equilibrium cases may arise in this economy, as we discuss below in detail. Importantly, the empirical counterparts of these equilibrium cases can easily be identified. In our model, the economy ends up in these cases endogenously, depending on underlying parameters. While it is possible to give conditions for exogenous parameters under which a certain equilibrium case arises, we believe that this is not helpful since too many exogenous variables interact; Instead, we focus on variations in the real bond supply b throughout this section, keeping other variables constant, as this is closely related to the policy rate. We also point out for which degree of banking competition N an equilibrium case is more likely to occur, given other parameters. We group these equilibrium cases into Case I - III and discuss conditions under which they arise below. Case I captures the scenario where the supply of government bonds is plentiful, and the interest rates in the economy are high. In Case II, the supply of government bonds is less plentiful, and the government bond rate carries a liquidity premium. In Case III, government bonds are scarce, and the interest rates in the economy are low. In particular, the deposit rate is at its zero lower bound.

3.1. Case I: Government bonds are plentiful

Recall that banks can raise private equity at cost equal to $1/\beta$. This means that if the supply of government bonds b is sufficiently large, then r^B must be equal to $1/\beta$ so that banks are indifferent about holding government bonds at the margin. In such a case, the capital requirement (2.21) does not bind because the cost of raising equity is equal to the return from government bonds. Substitute $r^B = 1/\beta$ and $\zeta = 0$ into (2.23) to get

$$\frac{L}{N}H'(L) + H(L) = \frac{1}{\beta}. \tag{3.1}$$

The left-hand side of (3.1) is the marginal benefit of increasing loan quantity L, while the right-hand side is the marginal return on government bonds. Notice that because $H'(L) < 0$, the expected return per unit of loan, $H(L) = r^L(L)[1 - P(r^L(L))]$, is actually higher than the return on government bonds even after taking into account entrepreneurs' default risk $P(r^L(L))$. This is because when competition in the loan market is imperfect, banks are aware that increasing L will lead to a decrease in the loan rate (which is represented by the term $\frac{L}{N}H'(L)$), which will make all existing loans less profitable. Hence, despite that loans provide a higher expected return, banks only hold L units of loans while investing the rest of their assets in government bonds.

Next, on the deposit side, as long as the interest rate on reserves satisfies $\frac{1+i^R}{1+\mu} < \frac{1}{\beta}$, the reserve requirement (2.21) binds. Combine (2.22) and (2.24) to get

$$r^D(D) + \frac{D}{N}r^{D\prime}(D) = \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}. \tag{3.2}$$

The left-hand side of (3.2) is the marginal cost of deposit funding, while the right-hand side is the marginal return from bank investments. Note that an increase in $\frac{1+i^R}{1+\mu}$ does not translate into an increase of similar magnitude in the deposit rate r^D for two reasons. First, reserves only represent a fraction δ of bank assets funded by deposits. Second, even if $\delta = 1$, an increase in $\frac{1+i^R}{1+\mu}$ does not increase r^D one for one. This is because banks with market power in the deposit market are aware that increasing D will lead to an increase in the deposit rate (which is represented by the term $\frac{D}{N}r^{D\prime}(D)$), which will make all existing deposits more expensive for them. Consequently, banks choose to hold only D units of deposits even though the deposit rate they have to pay is strictly lower than the return from bank assets. For the same reason, banks are willing to raise excess equity (i.e., beyond the capital requirement) to fund some of their investment even if the cost of deposits is lower than the cost of equity.

Definition 3.2. The economy is in equilibrium Case I if conditions (3.1) and (3.2) are satisfied while $z^B = \delta D$, $E \geq \gamma L$, $b^B = E + D - L - z^B$, $r^D(D) \geq \frac{1}{1+\mu}$ and $b^B \leq b$.

From the formal definition, it can easily be seen that this equilibrium case is more likely to occur for higher b. Further, it is also more likely to occur for higher N. Empirically, this case can be identified through the capital requirement, as it is the only case in which it is slack.

Proposition 3.2. *In a Case I equilibrium, an increase in b has no effect on D, r^B, and L. An increase in i^R leads to an increase in D, but has no effect on L and r^B. An increase in N leads to increases in D and L.*

Proof. See Propositions A.1, - A.3 in Appendix A.

In equilibrium Case I, $1/\beta$ pins down the marginal return on loans, the marginal cost of deposits, and the real return on bonds, which is why changes in b have no effect on any of these variables. For i^R, the same is true for loans and the bond interest rate; deposits increase in i^R because a higher interest rate on reserves reduces the marginal cost of deposits stemming from the reserve requirement, so banks are willing to issue more. Finally, an increase in N implies that individual banks care less about the increase in the deposit rate and the decrease in the loan rate resulting from an increase in deposits and loans, so they issue more.

3.2. Case II: Liquidity premium on government bonds

Suppose that the supply of government bonds is less plentiful so that $r^B < 1/\beta$. This implies that government bond rates pay a liquidity premium. This is the case even though bonds are not liquid, but since banks can fund themselves through issuing liquid assets and invest in bonds, they attain a liquidity premium indirectly. Since the marginal return from bank assets is less than the cost of equity, the capital requirement (2.21) binds, but the reserve requirement may or may not bind. First, suppose the reserve requirement binds. We refer to this case as **Case IIA**. Combine (2.23) and (2.25) to get

$$\frac{L}{N} H'(L) + H(L) = (1-\gamma) r^B + \frac{\gamma}{\beta}. \tag{3.3}$$

And (2.22), (2.23), and (2.24) together imply that

$$r^D(D) + \frac{D}{N} r^{D\prime}(D) = (1-\delta) r^B + \frac{\delta(1+i^R)}{1+\mu}. \tag{3.4}$$

Further, because $E = \gamma L$ and $Z = \delta D$ when the reserve and the capital requirement both bind, we have

$$D = \big((1-\gamma)L + b\big)/(1-\delta) \tag{3.5}$$

Definition 3.3. The economy is in equilibrium Case IIA if r^B, L, and D solve Eqs. (3.3), (3.4), and (3.5) such that $r^D(D) \geq \frac{1}{1+\mu}$ and $r^B \geq \frac{1+i^R}{1+\mu}$.

This case occurs if the supply of bonds b is still relatively large, but not as plentiful as in Case I. Similarly, it occurs if N is not too low, but also not as high to get the economy into Case I. Empirically, Case IIA can be identified through binding reserve and capital requirements, while the interest rate on deposits is positive.

A crucial difference between this case and Case I is that r^B is no longer a constant but determined in equilibrium. Hence, a change in the supply of government bonds b affects r^B, L, and D, while an increase in i^R affects L through its effect on r^B. The following proposition summarizes the findings.

Proposition 3.3. *In a Case IIA equilibrium, an increase in b leads to increases in D and r^B and a decrease in L. An increase in i^R leads to increases in D and L and a decrease in r^B. An increase in N leads to increases in D and L.*

Proof. See Propositions A.1–A.3 in Appendix A.

For N, the intuition is the same as in Case I: When N increases, each individual bank has less market power and hence cares less about marginal effects of their actions on loan and deposit rates. An increase in b, which can be achieved through an open market sale of government bonds, raises the return on bonds and crowds out bank loans. To benefit from the higher return, banks also issue more deposits to fund more bond investments. In comparison, if the central bank raises i^R, bank loans will increase. In this case, because the reserve requirement binds, an increase in i^R lowers the cost of funding bank investments with deposits. Consequently, banks choose to issue deposits and invest in more loans and government bonds, which drives down their returns. Hence, we can conclude that when the reserve requirement binds, open market operations and interest on reserves have fundamentally different effects on bank loans: while open market sales are contractionary, raising i^R is expansionary.

There is, however, a limit to the expansionary effect of i^R on L. As i^R increases, the loan rate and r^B decrease. Once $r^B = \frac{1+i^R}{1+\mu}$, the reserve requirement will no longer bind. We refer to this case as **Case IIB**. Then, L and D are still determined by (3.3) and (3.4), respectively, but now with $r^B = \frac{1+i^R}{1+\mu}$. Thus, (3.3) and (3.4) simplify to

$$\frac{L}{N} H'(L) + H(L) = (1-\gamma)\frac{1+i^R}{1+\mu} + \frac{\gamma}{\beta} \tag{3.6}$$

$$r^D(D) + \frac{D}{N} r^{D\prime}(D) = \frac{1+i^R}{1+\mu}. \tag{3.7}$$

Definition 3.4. If r^B, L, and D solve Eqs. (3.3), (3.4), and (3.5) such that $r^B < \frac{1+i^R}{1+\mu}$ while $r^D(D) \geq \frac{1}{1+\mu}$, the economy is in equilibrium Case IIB. Then, $r^B = \frac{1+i^R}{1+\mu}$, and the equilibrium is given by L and D that solve Eqs. (3.6) and (3.7).

This case is more likely to occur for higher i^R; with respect to N, it is likely to occur if it is in a medium range while b is low relative to i^R (as otherwise the economy would be in case IIA). Empirically, Case IIB can be identified if we simultaneously observe binding capital requirements, slack reserve requirements, and positive interest rates on deposits.

Proposition 3.4. *In a Case IIB equilibrium, an increase in b has no effect in D, r^B or L. An increase in i^R leads to increases on D and r^B and a decrease in L. An increase in N leads to increases in D and L.*

Proof. See Propositions A.1–A.3 in Appendix A.

The reserve requirement not binding implies that banks hold excess reserves, and that government bonds and reserves are perfect substitutes. An increase in b therefore will only lead banks to substitute government bonds for reserves, but it has no effect on bank loans or deposits. An increase in i^R, on the other hand, has the opposite effects on bank loans depending on whether the reserve requirement binds or not. When the reserve requirement binds, reserves do not compete with bank loans as bank assets. An increase in i^R lowers the cost of raising deposits to fund bank investments and increases bank loans. When the reserve requirement does not bind, reserves compete with bank loans. An increase in i^R makes reserves more attractive compared to bank loans and government bonds. As a result, reserves crowd out bank loans instead. For an increase in N, the intuition is the same as in Cases I and IIA.

3.3. Case III: Nominal deposit rate at zero lower bound

Suppose that government bonds are scarce and $\frac{1+i^R}{1+\mu}$ is low so that the solution to (2.22)–(2.25) is characterized by $r^D(D) < \frac{1}{1+\mu}$. However, note that r^D cannot be smaller than $\frac{1}{1+\mu}$ in equilibrium, because buyers will hold cash instead. In such a case, r^D must be equal to $\frac{1}{1+\mu}$ instead, and the nominal deposit rate is zero. The marginal cost of issuing deposits is no longer represented by $r^D(D) + \frac{D}{N} r^{D\prime}(D)$ but by $\frac{1}{1+\mu}$. Hence, the first order condition (2.24) is replaced by

$$l_j H'(L) + H(L) \geq \frac{1}{1+\mu} + \zeta\gamma + \kappa\delta. \tag{3.8}$$

Suppose that the reserve requirement binds. We refer to this case as **Case IIIA**. Note that (2.23) implies that the inequality in (3.8) must be strict. Intuitively, although the marginal return from bank loans is strictly larger than the marginal cost of deposits, banks cannot increase D without discontinuously increasing the marginal cost of deposits from $\frac{1}{1+\mu}$ to $r^D(D) + \frac{D}{N} r^{D\prime}(D) > \frac{1}{1+\mu}$, which would then imply that the marginal cost of deposits is strictly *larger* than the marginal return from bank loans.

Definition 3.5. If r^B, L, and D solve Eqs. (3.3), (3.4), and (3.5) such that $r^D(D) < \frac{1}{1+\mu}$ while $r^B \geq \frac{1+i^R}{1+\mu}$, the economy is in equilibrium Case IIIA. In this case, D is given by $r^D(D) = \frac{1}{1+\mu}$, while L and r^B jointly solve Eqs. (3.3) and (3.5).

As already stated above, this equilibrium occurs if both b and i^R are low, but with b being high relative to i^R. It is also more likely to occur if the degree of banking competition N is low. Empirically, Case IIIA can be identified if we simultaneously observe binding capital and reserve requirements for banks, and a nominal deposit rate of zero.

Proposition 3.5. *In a Case IIIA equilibrium, an increase in b leads to an increase in r^B and a decrease in L, but it has no effect on D. An increase in i^R has no effect on D, L, or r^B. An increase in N has no effect on D and L.*

Proof. See Propositions A.1–A.3 in Appendix A.

When the nominal deposit rate is at the zero lower bound, open market operations or interest on reserves have no effect on D; i.e., there is no pass-through from policy to deposit rates. This is a direct result of the imperfect competition in the deposit market: despite the increases in bank assets' returns, banks are unwilling to increase the deposit rate and issue more deposits. Consequently, an increase in b only crowds out bank loans, but total bank investments remain unchanged, while an increase in i^R has no effect on equilibrium outcomes. In comparison, in a Case IIA equilibrium, an increase in i^R increases bank lending through lowering the cost of issuing deposits. A (small) increase in N has no effect on deposits and loans either: while an increase in N reduces the marginal cost of an increase in deposits from the point of view of an individual bank, the cost is still higher than the marginal return due to the discontinuity. Similarly, the increase in N increases the marginal return of loans for individual banks, but banks cannot issue more loans in this equilibrium without raising more deposits. Of course, if the increase in N is large enough, this does not hold anymore and the economy moves into case IIA instead.

Now, suppose that the reserve requirement does not bind. We refer to this case as **Case IIIB**. Combining (2.22), (2.23), and (2.25) to get

$$\frac{L}{N} H'(L) + H(L) = \frac{(1-\gamma)(1+i^R)}{1+\mu} + \frac{\gamma}{\beta}. \tag{3.9}$$

Table 1
Effects of N, monetary policies, and banking regulations on bank profits.

	Case I	Case IIA	Case IIB	Case IIIA	Case IIIB
Increasing N	↓	?	↓	?	↓
Increasing b	–	↑	–	↑	–
Increasing i^R	↑	?	↑	↑	↑

*Note: "↑": bank profit increases; "↓": decreases; "–": no change; "?": the effect is ambiguous.

Definition 3.6. If r^B, L, and D solve Eqs. (3.3), (3.4), and (3.5) such that $r^D(D) < \frac{1}{1+\mu}$ while also $r^B < \frac{1+i^R}{1+\mu}$, the economy is in equilibrium Case IIIB. In this case, D and r^B are given by $r^D(D) = \frac{1}{1+\mu}$ and $r^B < \frac{1+i^R}{1+\mu}$, respectively, while L is given by Eq. (3.9).

This case occurs for similar parameters as Case IIIA (i.e., low b, i^R, and N), but with the difference that now i^R needs to be high relative to b. Empirically, Case IIIB can be identified if we observe binding capital requirements, slack reserve requirements, and nominal deposit rates equal to zero.

Notice that unlike in Case IIIA, the equilibrium loan supply L now depends on i^R. This is because reserves compete with bank loans as a bank asset when the reserve requirement does not bind.

Proposition 3.6. *In a Case IIIB equilibrium, an increase in b has no effect on r^B, L, or D. An increase in i^R has no effect on D, but it decreases L and increases r^B. An increase in N has no effect on D but leads to an increase in L.*

Proof. See Propositions A.1–A.3 in Appendix A.

Similar to Case IIB, reserves and government bonds are perfect substitutes when the reserve requirement does not bind. Consequently, changing the supply of government bonds has no effect on equilibrium outcomes. An increase in i^R makes reserves more attractive and crowds out bank loans. Similar to Case IIIA, open market operations or interest on reserves have no effect on D. It is also worth noting that in both Cases IIIA and IIIB, the nominal deposit rate can reach the zero lower bound even when $i^R > 0$. The economy is not at zero lower bound in the sense that the policy rates (i^R and r^B) can be lowered further. Nevertheless, depending on whether the reserve requirement binds or not, either open market operations or interest on reserves is ineffective at stimulating bank lending. The intuition why an increase in N has no effect on D is the same as in Case IIIA. However, since banks are able to issue more loans in Case IIIB by holding more excess reserves, an increase in N leads to an increase in loans.

3.4. Bank profitability and bank default

In this section, we study how banking competition (i.e., the number of banks) and monetary policy (i.e., open market operations and interest on reserves) affect bank profitability and bank default. Profits per bank Π are given by

$$\Pi = -\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1-P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b^B}{N}r^B - \frac{D}{N}r^D(D)\right] \tag{3.10}$$

Proposition 3.7. *Table 1 shows how bank profits are affected by* (1) *increasing the number of banks,* (2) *open market sale of government bonds, and* (3) *increasing interest rate on reserves.*

Proof. See Propositions A.1–A.5 in Appendix A.

Unsurprisingly, higher banking competition tends to reduce profits per bank, although this is not necessarily true in Cases IIA and IIIA. The reason for this is that there may be externalities on the bond interest rate: As N increases, banks are willing to make less loans and in turn may reduce their demand for bonds, which then leads to an increase in the bond interest rate (which banks do not internalize). An increase in b leads to an increase in the policy rate r^B in equilibrium Cases IIA and IIIA, which in turn increases bank profits, as it essentially increases investment opportunities for banks. In equilibrium Cases IIB and IIIB, an increase in i^R also increases the policy rate and thus increases bank profits for similar reasons. In cases I, IIA, and IIIA, an increase in i^R has no effect on the policy rate r^B, but it reduces the marginal cost of reserves through the reserve requirement, which also tends to increase bank profits (except potentially in Case IIA, which is again due to externalities).

Next, we move on to bank default probability. Recall that the return on entrepreneurs' loans is subjected to an aggregate shock s, so banks may not have enough assets to meet their deposit liabilities. In such a case, banks default, and all of their assets are distributed evenly to creditors. Bank default probability is therefore given by $\Psi = 1 - G(\hat{s})$, where $\hat{s}$ solves

$$\frac{D}{N}r^D = \int_{\hat{s}}^{\bar{s}} \frac{L}{N}H(L,s)\mathrm{d}G(s) + G(\hat{s})\frac{L}{N}H(L,\hat{s}) + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b^B}{N}r^B. \tag{3.11}$$

On the left-hand side of (3.11), r^D is the risk-adjusted deposit rate, and $\frac{D}{N}r^D$ is the expected amount of liabilities that a bank will redeem, which takes into account the possibility of bank default. Note that $\hat{s}$ represents the realization of the aggregate shock such that the value of the bank's assets is equal to its liabilities. The right-hand side of (3.11) then represents the amount of bank assets

Table 2
Effects of N, monetary policies, and banking regulations on bank default.

	Case IIA	Case IIB	Case IIIA	Case IIIB
Increasing N	↑	↑	↓	↑
Increasing b	↓	–	↓	–
Increasing i^R	?	?	↓	↓

*Note: "↑": default probability increases; "↓": decreases; "–": no change; "?": the effect is ambiguous.

devoted to covering the bank's liabilities: when $s > \hat{s}$, the bank defaults, and the entirety of its assets is used to cover its liabilities; for all $s < \hat{s}$, the bank does not default, and the amount of assets used for deposit redemption is equal to $\frac{L}{N}H(L,\hat{s}) + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b^B}{N}r^B$. Recall that in Case I, banks are indifferent between raising more equity and purchasing more government bonds. As a result, bank default probability is not well-defined.

The effects of banking competition, monetary policies, and banking regulations on bank default probability depend crucially on the relationship between the expected return from bank loans and the total quantity of bank loans. Recall that $H(L) = r^L(L)[1 - P(r^L(L))]$ is the risk-adjusted loan rate, and $LH(L)$ is the total return from all bank loans, which affects bank default probability because in equilibrium, the profit from loans acts as a cushion against the aggregate shock. If $LH'(L) + H(L) < 0$, it means that increasing L lowers the total return from bank loans, which tends to increase bank default probability.

Assumption 3.1. Define $\underline{L}$ to be such that $\frac{\underline{L}}{N}H'(\underline{L}) + H(\underline{L}) = \frac{1}{\beta}$. Assume that $\underline{L}H'(\underline{L}) + H(\underline{L}) < 0$.

As discussed in Section 3, among all equilibrium cases, L is the lowest in Case I, where it is given by the first order condition $\frac{L}{N}H'(L) + H(L) = \frac{1}{\beta}$. Hence, if $LH'(L) + H(L) < 0$ is satisfied in Case I, then it is satisfied in all the other cases.

Proposition 3.8. *Suppose Assumption 3.1 holds. Table 2 shows how bank default probability is affected by* (1) *increasing the number of banks,* (2) *open market sale of government bonds, and* (3) *increasing interest rate on reserves.*

Proof. See Propositions A.1–A.5 in Appendix A.

When N increases, the return from bank loans decreases due to the increased competition. As a result, bank default probability in general increases. However, in Case IIIA, L is unaffected by N. Hence, the total return from loans, $LH(L)$, is unchanged. Bank default probability decreases because the government bond rate r^B increases in equilibrium, thus increasing the value of bank assets and providing a bigger cushion against the aggregate shock.

An open market sale that increases the supply of government bonds b has no effect in Cases IIB and IIIB, because in these cases, the reserve requirement does not bind, so an increase in b only leads to banks substituting reserves for government bonds. In Cases IIA and IIIA, a larger supply of government bonds increases banks' holdings of safe assets and decreases bank default probability.[11] Similarly, in Cases IIIA and IIIB, an increase in interest on reserves i^R lowers bank default probability. However, in Cases IIA and IIB, a higher i^R also leads banks to issue more deposits, increasing their liabilities. Hence, the effect is ambiguous.

Finally, if Assumption 3.1 does not hold, how bank default probability is affected by banking competition, monetary policies, and banking regulations is in general ambiguous. To resolve the ambiguity, in the following section, we calibrate the model to the U.S. economy.

4. Calibration

To quantify our results, we calibrate the model to the U.S. economy during 2016–2019. We choose this time period since the Federal Reserve started actively using the interest rate on (excess) reserves as a policy tool in 2016, and continued doing so until the onset of the COVID-19 pandemic in March 2020. All of our calibration targets are matched to moments from this time period, except for money demand, for which we need a longer time series and we thus calibrate separately, using data from 1959–2007.

We also assume that entrepreneur's ability R^i follows a uniform distribution on $[0, \bar{R}]$, while the aggregate shock s follows a truncated normal distribution. We set a period to one year and make the following assumptions on the buyers' utility function $u(q)$ and the success probabilities of entrepreneurs' projects $p(R, R^i, s)$:

$$u(q) = \frac{q^{1-\sigma} - 1}{1-\sigma}, \tag{4.1}$$

$$p(R, R^i, s) = s\left(\frac{R}{R^i}\right)^{\alpha}, \tag{4.2}$$

$$\mathbb{E}[p(R, R^i, s)] = p(R, R^i) = \left(\frac{R}{R^i}\right)^{\alpha}. \tag{4.3}$$

[11] If Assumption 3.1 does not hold, then the effects of b on bank default probability are in general ambiguous. See Appendix A.3 for more details.

Table 3
Externally calibrated parameters.

Parameter	Notation	Value	Calibration target
Inflation rate	μ	1.82%	Average inflation (CPI): 2016–19
Prop. of monitored DM meetings	η	0.85	Prop. of non-cash transactions: 2016
Nominal interest rate on reserves	i^R	1.40%	Average interest on reserves: 2016–19
Reserve requirement	δ	8.83%	Average reserve requirement: 2016–19
Capital requirement	γ	6%	Minimum Tier 1 capital ratio: 2016
Discount factor	β	0.96	Standard in the literature

There are 16 parameters to calibrate: inflation rate (μ), proportion of monitored meetings in the DM (η), nominal interest rate on reserves (i^R), reserve requirement (δ), capital requirement (γ), discount factor (β), buyer's preference (σ), the number of banks (N), upper bound of the distribution of entrepreneur's ability ($\bar{R}$), government bond supply (b), the measure of entrepreneurs (S), parameter α in $p(R, R^i)$, and four parameters that describe the distribution of the aggregate shock.

We pick the first five parameters directly to match the data, as summarized in Table 3. We obtain all data from Federal Reserve Economic Data (FRED) except for the proportion of non-cash transactions, which is from the 2016 Survey of Consumer Payment Choice.[12] We set the reserve requirement equal to the ratio between required reserves and total checkable deposits. Next, we follow Rocheteau et al. (2018) and calibrate σ by matching the semi-elasticity of money demand in the model to the data. Define $\imath = (1+\mu)/\beta$. The semi-elasticity of money demand in our model is given by the following expression:

$$\frac{\partial \log(q^m)}{\partial \imath} = -\frac{1}{\sigma \imath}, \tag{4.4}$$

where q^m is the output in unmonitored meetings (see Section 2.2). We calculate empirical money demand semi-elasticity using the new M1 series from Lucas and Nicolini (2015) and Moody's AAA corporate bond yield from FRED. We set the period for calibration to 1959–2007. We do not include data from 2008 and onward because the demand for currency increased during and after the financial crisis, likely for non-transactional reasons, such as store-of-value or flight-to-safety motives. Since our model does not include such reasons to hold currency, we find it best to exclude post-crisis data. Chiu et al. (2023) also argue that one should use pre-crisis data to estimate money demand elasticity. Our estimated semi-elasticity of money demand is −2.13, which corresponds to $\sigma = 0.44$.

The third step is to jointly calibrate N, $\bar{R}$, b, α, and S so that the expected return on loans $r^L(1 - P(r^L))$ (see Section 2.4), expected deposit rate r^D, reserve-to-loan ratio z^B/L, loan default probability $P(r^L)$, and loans-to-GDP ratio match their counterparts in the data, i.e., average return on loans, average deposit rate, average reserves-to-loan ratio, average loan default probability, and average commercial and industrial loans-to-GDP ratio. We do not target the supply of government bonds directly. The reason is that banks hold a lot more assets in reality than we allow for in the model. Thus, government bonds should be interpreted as encompassing a large array of safe assets that are traded on financial markets.

Table 4 summarizes the results. We calculate loan rates using loan interest income and loan quantities data from bank call reports data collected by Drechsler et al. (2017). The lowest 10% of the calculated loan rates are excluded as they are likely from safe loans. The rest of the data is from FRED. While the average interest checking account rate between 2016 and 2019 was 0.05% according to FRED, we choose to set the target for deposit rates to zero.[13] The reason for this choice is twofold: First, whether the deposit rate is exactly zero or slightly above it makes a significant difference in our model due to the existence of different equilibrium cases; Second, the FRED data may be skewed upward in the sense that most banks paid no interest on deposits during our calibration period, but while there were a few exceptions where positive interest was paid, none (or very few) exceptions with negative interest exist.

The final step is to calibrate the four parameters that describe the truncated normal distribution of the aggregate shock: $\hat{\mu}$, $\hat{\sigma}$, the lower bound $\hat{a}$, and the upper bound $\hat{b}$, where $\hat{\mu}$ and $\hat{\sigma}$ are the mean and variance of the corresponding normal distribution. We fix $\hat{a}$ to 0 and $\hat{b}$ to 50. Since we assume $\mathbb{E}[p(R, R^i, s)] = p(R, R^i)$, the mean of s must be 1. Hence, for a given $\hat{\sigma}$, $\hat{\mu}$ is given by

$$1 = \hat{\mu} + \frac{\phi\left(\frac{\hat{a}-\hat{\mu}}{\hat{\sigma}}\right) - \phi\left(\frac{\hat{b}-\hat{\mu}}{\hat{\sigma}}\right)}{\Phi\left(\frac{\hat{b}-\hat{\mu}}{\hat{\sigma}}\right) - \Phi\left(\frac{\hat{a}-\hat{\mu}}{\hat{\sigma}}\right)}, \tag{4.5}$$

where $\phi(.)$ and $\Phi(.)$ are the PDF and CDF of standard normal distribution, respectively. We use the ratio between bank failure cost to the Federal Deposit Insurance Corporation (FDIC) and total transactional deposits as a proxy for bank default probability in data. The average of this ratio between 2016 and 2019 is 0.02%. We then calibrate $\hat{\sigma}$ so that the default probability matches the data. We obtain $\hat{\sigma} = 3.08$ and $\hat{\mu} = 0.25$.

According to our calibration, the U.S. economy was in equilibrium Case IIIB during 2016–2019 — a scenario where banks hold excess reserves, and the nominal interest rate on deposits is zero. There were 24 banks in the U.S. economy according to our

[12] The value reported in the data is the number of transactions made with cash relative to the number of transactions without cash, which directly matches the parameter in our model.

[13] The deposit rate data we use is the interest checking account rate on non-jumbo deposits (less than $100,000).

Table 4
Internally calibrated parameters.

Parameter	Not.	Value	Calibration target
Number of banks	N	24	Avg. loan and deposit rate: 2016–19
Upper bound of ability dist.	$\bar{R}$	1.96	Avg. loan and deposit rate: 2016–19
Government bond supply	b	0.43	Avg. reserves-to-loan ratio: 2016–19
Parameter in $p(R, R^i)$	α	600	Avg. loan default rate: 2016–19
Measure of entrepreneurs	S	0.52	Avg. comm. & ind. loans-to-GDP ratio: 2016–19

Table 5
Model fit.

Target	Data	Model
Average nominal return on loans: 2016–19	5.47%	5.45%
Average nominal deposit rate: 2016–19	0.00%	0.00%
Average reserves-to-loan ratio: 2016–19	12.65%	12.65%
Average loan default rate: 2016–19	1.26%	1.20%
Average commercial and industry loans-to-GDP ratio: 2016–19	10.80%	10.80%
Average bank default probability: 2016–19	0.02%	0.02%

calibration. This result should not be taken literally — instead, our model implies that the banking sector behaved *as if* there were 24 banks with equal size and market power, while in reality banks differ in size.

The model fit is summarized in Table 5. Even though our model is highly nonlinear, all targets are matched very closely.

Before we end this section, we briefly discuss the choice of the bank default probability target. In the model, if default occurs, all banks default, whereas the target we use captures default of only some banks in the economy. However, we do not consider this as a serious issue for three reasons. First, there are no instances of system-wide bank default in recent history as it occurs in our model, meaning that there is no obvious calibration target in the data that fully matches what happens in our model. Second, extending the model to allow for heterogeneity across banks would come at the cost of losing tractability. Finally and most importantly, while it is true that default in our model affects all banks simultaneously, the economic effects are far less severe than they would be in reality, because our banks exist only for one period, and the economy goes back to normal in the period after bank default.

5. Counterfactual analysis

In this section, we use our calibration to conduct a counterfactual analysis. We first study how changes in the degree of banking competition would have affected economic outcomes in order to better understand how imperfect banking competition matters in this economy. We then move on to our main results, where we analyze how changes in the policy rate would have affected the U.S. economy during 2016–2019. As we have discussed in Section 3, the theoretical effects of changes in both the number of banks and the policy rate differ across equilibrium cases. Since equilibrium cases occur endogenously in our model, we can identify when a counterfactual policy moves the economy into another equilibrium case. The analysis in this section allows us to not only pin down the direction of the effects the policy changes have on other variables but also quantify them. In Appendix B, we discuss some more counterfactual experiments, and we show the effects of changes in all policy variables on equilibrium outcomes graphically.

In addition to the equilibrium outcomes discussed in Section 3, we also study the effects of policies on expected aggregate welfare W, which is simply an equal-weighted sum of all agents' expected utility:

$$\begin{aligned} W =& (1-\eta)[u(q^m)-q^m]+\eta[u(q^d)-q^d] \\ &+\beta S\mathbb{E}\left[\int_{r^L}^{\bar{R}} R^*(r^L,R^i)[1-p(R^*(r^L,R^i),R^i,s)]f(R^i)\mathrm{d}R^i\right]-L. \end{aligned} \tag{5.1}$$

First, $(1-\eta)[u(q^m)-q^m]+\eta[u(q^d)-q^d]$ is the total surplus from trade in the DM. Second, note that $R^*(r^L,R^i)$ is given by (2.10), and $\beta S\mathbb{E}[\cdot]$ is the discounted expected output by entrepreneurs, where the expectation is taken over s, the aggregate shock, and S is the measure of entrepreneurs. Recall that all agents derive linear utility (disutility) from consuming (producing) the CM good. Hence, the production of CM goods by agents other than entrepreneurs does not appear in the welfare definition. Similarly, bank profits do not enter the welfare definition since these profits are just a reallocation of goods to banks.

5.1. Banking competition

Table 6 reports the changes in macroeconomic outcomes relative to our baseline calibration with banking competition being twice and half as high as it effectively was, respectively. Both of these changes keep the economy in Case IIIB, so the deposit rate remains at zero and deposits remain unchanged in both counterfactuals. More banking competition is good for welfare but bad for banks: doubling the competition among banks increases welfare by 0.44%, but reduces profit per bank by 61.1%. Note that profit per bank would have only reduced by 50% (since N is doubled) had each bank simply reduced their assets and liabilities by 50%

Table 6
The effect of changes in N on equilibrium outcomes.

	$\frac{1}{2}N = 12$	$2N = 48$
W	−0.82%	+0.44%
L	−3.84%	+2.04%
D	unchanged %	unchanged
i^L	+3.56 pp	−1.89 pp
i^D	unchanged	unchanged
Π	+178.27%	−61.1%
Ψ	−0.0184 pp	+0.0566 pp
Case	IIIB	IIIB

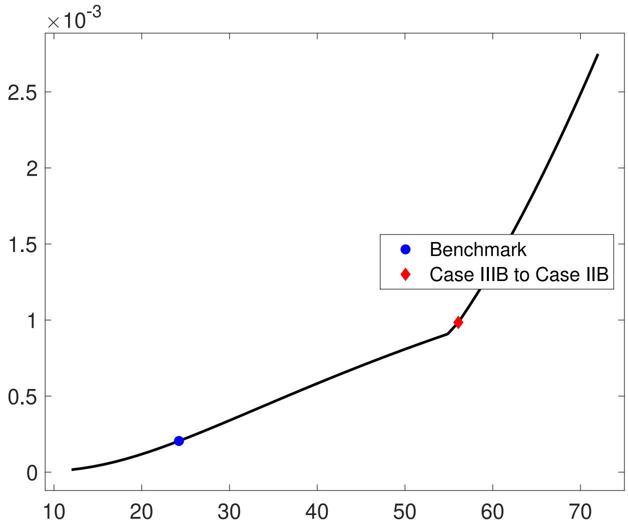


Fig. 1. Effects of N on Bank Default Probability.

while keeping the relative composition of their balance sheet unchanged. This shows that even with $N = 24$, bank market power is still significant and a further increase in banking competition has sizeable effects. The increase in welfare that follows an increase in N is driven by the reduction in the loan rate that occurs when banks compete more fiercely, which in turn increases investment.[14] A decrease in N has the opposite effects on welfare, bank profits, and loans, with all of the effects also being more pronounced since banks exert more market power the smaller N becomes.

In our theoretical analysis, we note that the effect of changes in N on the bank default probability Ψ is ambiguous in case IIIB. Our quantitative analysis shows that Ψ is increasing in N, so while stronger banking competition is good for welfare, it also increases the risk of bank failure, and it does so in a non-trivial way: Since the probability of bank failure is only about 0.02% according to our calibration, the increase by 0.057pp in Ψ that follows from doubling N implies that bank failure becomes more than 3 times more likely. Fig. 1 shows how bank default probability varies with N. Note that bank default probability is increasing even more strongly with N if N is large enough to move the equilibrium into case IIB.

5.2. Changes in the policy rate

Next, we study how varying the policy rate affects equilibrium outcomes. Table 7 shows how an increase or a decrease in the nominal interest rate on reserves i^R by 1pp affects the calibrated economy.[15] Recall that we calibrate the model to an interest rate on reserves of 1.4%, so the counterfactuals presented here correspond to economies with $i^R = 0.4\%$ and $i^R = 2.4\%$, respectively. The first main takeaway is the welfare effects of changes in interest on reserves. In general, increases in the deposit rate are good for welfare since they allow for more trades in the DM, while lower loan rates are good for welfare since they increase investment

[14] Away from the zero-lower bound on deposits, an increase in banking competition would also increase welfare through an increase in deposit rates which make holding liquidity less costly for buyers and increases their DM consumption. However, in our calibration the economy only moves away from this zero-lower bound if the number of banks is increased to $N = 57$.

[15] As discussed in Section 3, the policy rate r^B can be varied through changes in i^R when the reserve requirement is non-binding, which was the case in the U.S. during our calibration time period. While the U.S. Fed did in fact mainly use the interest rate on excess reserves to conduct monetary policy during this time period, an increase in the policy rate could also be implemented through (large enough) open-market operations. The results of this policy would have been very similar, as we discuss in Appendix B.3.

Table 7
The effect of changes in i^R on equilibrium outcomes.

	$i^R - 1$ pp	$i^R + 1$ pp
W	+0.21%	−0.21%
L	+0.99%	−0.99%
D	unchanged	unchanged
i^L	−0.92 pp	+0.92 pp
i^D	unchanged	unchanged
Π	−37.46%	+37.47%
Ψ	+0.229 pp	−0.0189 pp
Case	IIIB	IIIB

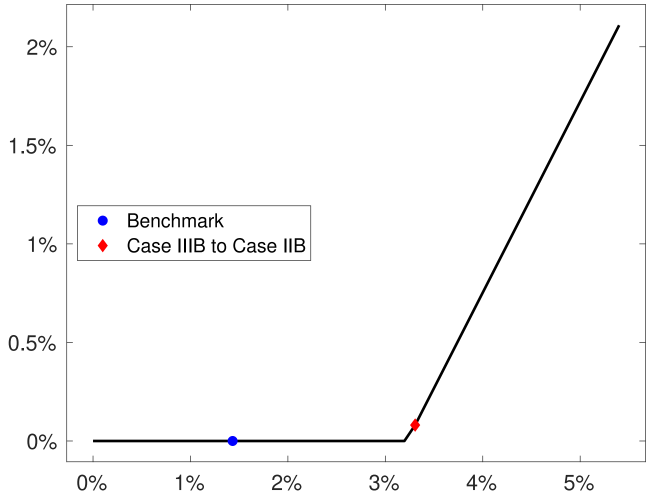


Fig. 2. Effects of i^R on the deposit interest rate.

and reduce risk-taking by entrepreneurs. Thus, the welfare effect of changes in the policy rate is in general ambiguous. However, in equilibrium Case IIIB, changes in the policy rate affect only the loan rate, since the deposit rate is equal to zero and does not react to marginal changes in the policy rate. Since our calibrated economy is in Case IIIB and also stays there in both counterfactuals, a decrease in the policy rate increases welfare through its effect on the loan rate, while an increase in the policy rate has the opposite effect.

Another important takeaway from Table 7 is that interest-rate pass-through is incomplete under imperfect competition. The loan rate changes by only 0.92pp following a 1pp change in the policy rate in either direction. For deposits, there is zero pass-through due to the economy starting at the zero-lower bound, and since even a 1pp increase in the policy rate is not enough to move the economy away from it. As Fig. 2 shows, the deposit rate does not react to changes in the policy rate until the policy rate reaches levels above 3%. Only at that level, the economy transitions into equilibrium Case IIB and there is positive pass-through from the policy rate to the deposit rate. Note that this implies the monetary authority has no control over (inside) money creation via the policy rate unless it is willing to increase it substantially.

The third main takeaway from Table 7 is the strong effect changes in the policy rate have on profits per bank. Fig. 3 shows in more detail how bank profits vary with the policy rate. While we have shown theoretically that an increase in the policy rate increases profits per bank in Cases IIB and IIIB, the figure shows that this effect is particularly strong in equilibrium Case IIIB. This is because while in general, the higher policy rate increases both the return on the banks' assets and the cost of their liabilities, the latter effect is nonexistent in Case IIIB as the deposit rate is at zero. Another way to think about this is that banks with market power would like to set a constant wedge between the loan rate and the deposit rate, but due to the zero lower bound, they cannot set the deposit rate arbitrarily low. This reduces their profit margin. In this sense, setting interest rates close to zero is a way for the central bank to reduce banks' market power.

The final result from Table 7 we want to highlight is the effect of policy rate changes on the bank default probability Ψ, which are generally ambiguous in the model. In our calibration, it turns out that increases in the policy rate reduce bank default probability, and as Fig. 4 shows, this effect is particularly strong for low policy rates. Hence, while a reduction in the policy rate would have increased welfare according to our calibration, this would have come at the cost of a significant increase in the probability of bank default.

We also assess how the degree of banking competition and the policy rate interact. The purpose is to compare how the effectiveness of monetary policy changes when N varies. Table 8 reports effects of an increase in i^R while Table 9 reports effects of a decrease in i^R. In both tables, the results in the $\frac{1}{2}N$ $(2N)$ column are the changes relative to an economy with $\frac{1}{2}N$ $(2N)$ but with the same baseline interest on reserves (i.e., 1.4%).

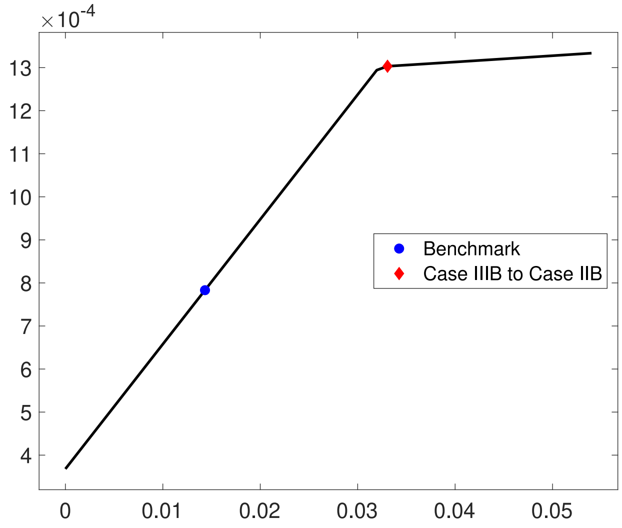


Fig. 3. Effects of i^R on bank profits.

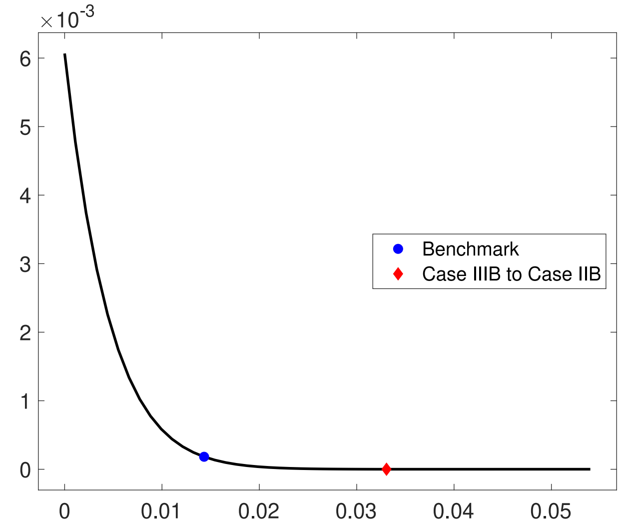


Fig. 4. Effects of i^R on Bank Default Probability.

From Tables 8 and 9, we can see how the pass-through to loan rates increases with N, which implies that the monetary authority gains stronger control over investment with higher banking competition. For the deposit rate, low banking competition ($N = 12$ or $N = 24$) makes it more likely that the economy is in equilibrium Case IIIB even for relatively high policy rates. This shows that low banking competition significantly reduces the central bank's control over inside money creation. The effects of policy rate changes on welfare and profits per bank interact in relatively straightforward ways with bank market power: Both welfare and profits per bank react slightly more strongly when banking competition is higher, and there is a stronger pass-through from monetary policy to other variables. Finally, the probability of bank default also tends to react more strongly to changes in the policy rate when banking competition is higher.

6. Conclusion

We build a dynamic general equilibrium model with oligopolistic banking competition and risky investment to study how these features affect the transmission of monetary policy to the real economy. We show that including these characteristics of developed economies is relevant to understand how policymakers can affect the real economy. Our model entails three equilibrium cases: One where banks are indifferent between raising deposits and sweat equity, one where the deposit rate is strictly positive, but the marginal cost of deposits is strictly below the marginal cost of equity, and one where the nominal deposit rate is equal to zero. The

Table 8
The effects of an increase in i^R by 1 pp with different N.

	$\frac{1}{2}N$	N	$2N$
W	−0.21%	−0.21%	−0.14%
L	−0.99%	−0.99%	−0.99%
D	unchanged	unchanged	+1.00%
i^L	+0.89 pp	+0.92 pp	+0.94 pp
i^D	unchanged	unchanged	+0.78 pp
Π	+26.36%	+37.47%	+11.22%
Ψ	−0.0015 pp	−0.0189 pp	−0.0296 pp
Case transition	IIIB unchanged	IIIB unchanged	IIIB to IIB

Table 9
The effects of a decrease in i^R by 1 pp with different N.

	$\frac{1}{2}N$	N	$2N$
W	+0.21%	+0.21%	+0.22%
L	+0.99%	+0.99%	+0.99%
D	unchanged	unchanged	unchanged
i^L	−0.89 pp	−0.92 pp	−0.94 pp
i^D	unchanged	unchanged	unchanged
Π	−26.35%	−37.46%	−48.69%
Ψ	+0.024 pp	+0.229 pp	+0.73 pp
Case transition	IIIB unchanged	IIIB unchanged	IIIB unchanged

latter two can be further distinguished depending on whether the reserve requirement does or does not bind. The theoretical effects of varying the number of banks in the economy and the policy rate differ among these cases, showing that it is important to have a model where these cases may occur endogenously. The calibration of our model to the U.S. economy during 2016–2019 shows that welfare could have been improved by either increasing competition in the banking sector or reducing the policy rate, but only at the cost of significantly increasing the probability of bank default. We also document that bank profits are increasing in the policy rate, with the effect being particularly strong close to the zero-lower bound, and that interest rate pass-through of monetary policy is incomplete under imperfect competition.

Appendix A. Additional theoretical results

A.1. Aggregate risk in the DM

In this appendix, we make two changes to the benchmark model. First, we assume that the aggregate shock, s, is realized at the beginning of the DM. The shock is observable to agents, and the realized value is common knowledge. Second, we assume that there exists a deposit insurance corporation that receives premium from the banks when they do not default and pays back households the value of their deposits when banks default.[16]

We assume the premium is actuarially fair, so the deposit insurance corporation earns an expected profit of zero. We believe this is a reasonable benchmark in the context of deposit insurance in the U.S based on a recent report published by the Federal Deposit Insurance Corporation (FDIC) (see FDIC, 2020). According to the report, the FDIC's deposit insurance premium is risk-based and is calculated using forward-looking statistic models and historical failure and loss rate data. The FDIC states their goal when designing the risk-based premium as "to be fair and easily understood, not to be unduly burdensome to weak institutions". The report shows that on the one hand, the FDIC avoids overcharging depository institutions so that the premium does not cause an already weak institution to fail. On the other hand, the FDIC stands to revise the premium rates when new data becomes available. For example, after the 2008–09 financial crisis, the FDIC increased significantly the overall premium in response to an increase in estimated losses from projected failures.

Now, we solve the equilibrium with deposit insurance and aggregate risk in the DM. Entrepreneur's problem in the CM remains unchanged (see Section 2). Recall that entrepreneurs are willing to invest in projects if and only if $R^i \geq r^L$. The demand for loans is given by

$$L(r^L) = S[1 - F(r^L)]. \tag{A.1}$$

[16] In practice, deposit insurance has a coverage limit. For example, in the US, the coverage limit is $250,000 (see https://www.fdic.gov/resources/deposit-insurance/faq/). However, most households keep way less money in their transactional accounts (e.g., checking accounts). According to Federal Reserve Board's 2022 Survey of Consumer Finances, in 2022 the average household checking account balance was $16,891, and the median was $2,800.

Conditional on s, the expected probability of default on bank loans is given by

$$P(r^L, s) = \frac{1}{1-F(r^L)} \int_{r^L}^{\bar{R}} p(R^*(r^L, R^i), R^i, s) f(R^i) \mathrm{d}R^i. \tag{A.2}$$

Because of the existence of the deposit insurance, deposits are risk-free to consumers. As a result, households' problem is also unchanged. Let r^D denote the risk-free gross real deposit rate. The total demand for deposits $D(r^D)$ is given by the following expression:

$$D(r^D) = \begin{cases} 0, \text{ if } r^D < \frac{1}{1+\mu}; \\ \eta(u')^{-1}\left(\frac{1}{\beta r^D}\right)/r^D, \text{ if } \frac{1}{1+\mu} \le r^D < 1/\beta; \\ \ge \beta\eta q^*, \text{ if } r^D \ge 1/\beta. \end{cases} \tag{A.3}$$

where μ is the inflation rate and β is the discount rate. Recall that deposit rate has a zero lower bound, so if r^D is lower than the return on cash, the demand will be zero. If r^D is larger than $1/\beta$, the demand will be infinite.

Next, let us consider the problem of the deposit insurance corporation. Let l_j and d_j denote the quantities of loans and deposits originated from bank j. Let z_j and b_j denote the quantities of reserves and government bonds held by bank j. Let $\mathcal{P}_j(s)$ denote the premium per unit of the deposits that bank j has to pay. It is worth noting that since the premium is risk-based and bank-specific, moral hazard problem does not appear in this environment. This is consistent with how the FDIC calculates bank's deposit insurance premium in practice.[17]

Under limited liability, banks j defaults if and only if

$$l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B < d_j[r^D + \mathcal{P}_j(s)], \tag{A.4}$$

where the left-hand side is the total value of bank assets, and the right-hand side is the total bank liabilities, which includes deposit insurance premium. The expected profit of the deposit insurance corporation is given by

$$\Pi^{DI} = \mathbb{E}\left[\min\left\{ l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D, d_j \mathcal{P}_j(s) \right\}\right] \tag{A.5}$$

where the expectation is taken over s. Note that $d_j\mathcal{P}_j(s)$ is the premium income if a bank does not default, and $l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D$ is the expected payment made to depositors if the bank defaults, after taking into account the value of the bank's assets.

Based on the above analysis, banks' expected profit in the CM is equal to

$$\mathbb{E}\left[\max\left\{ l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j[r^D + \mathcal{P}_j(s)], 0 \right\}\right]. \tag{A.6}$$

Note that if we add up (A.5) and (A.6), we obtain the following expression:

$$\begin{aligned} &\mathbb{E}\left[\min\left\{ l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D, d_j \mathcal{P}_j(s) \right\}\right] \\ &+ \mathbb{E}\left[\max\left\{ l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j[r^D + \mathcal{P}_j(s)], 0 \right\}\right] \\ &= l_j r^L[1 - P(r^L)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D, \end{aligned} \tag{A.7}$$

where $P(r^L) = \mathbb{E}[P(r^L, s)]$ is the expected default probability of bank loans. Since the premium is actuarially fair, $\Pi^{DI} = 0$. Hence, we have

$$\begin{aligned} &\mathbb{E}\left[\max\left\{ l_j r^L[1 - P(r^L, s)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j[r^D + \mathcal{P}_j(s)], 0 \right\}\right] \\ &= l_j r^L[1 - P(r^L)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D, \end{aligned} \tag{A.8}$$

Now, let e_j denote bank j's equity. Let $r^L(L)$ and $r^D(D)$ be the inverse demand functions for loans and deposits. Bank j solves

$$\max_{d_j\ge 0, e_j \ge 0, z_j \ge 0, b_j \ge 0} \Pi_j = -e_j + \beta\left[l_j r^L(L)[1 - P(r^L(L))] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B - d_j r^D(D) \right], \tag{A.9}$$

[17] For more details regarding how the FDIC calculates deposit insurance premium, see FDIC (2020) and https://www.fdic.gov/resources/deposit-insurance/deposit-insurance-fund/dif-assessments.html.

Table 10
Comparative statics for N across the different equilibrium regimes.

	I	IIA	IIB	IIIA	IIIB
L	↑	↑	↑	–	↑
D	↑	↑	↑	–	–
r^B	–	?	–	↑	–
Π	↓	?[a]	↓	?[b]	↓
Ψ		?[c]	?[c]	↓	?[c]

[a] Decreases if r^B decreases.
[b] Decreases if $\frac{2N+1}{N+1}\frac{L}{N}H'(L)+H(L)-\frac{\gamma}{\beta}>0$.
[c] Increases if $LH'(L)+H(L)<0$.

$$\text{s.t. } l_j = d_j + e_j - z_j - b_j, \tag{A.10}$$
$$e_j \geq \gamma l_j, \tag{A.11}$$
$$z_j \geq \delta d_j. \tag{A.12}$$

It is straightforward to see that bank's problem is unchanged compared to problem (2.18) in the benchmark case. Therefore, the solution to bank's problem is also unchanged. This means that all results in this paper remain valid when there is aggregate risk in the DM, so long as there exists a deposit insurance funded by actuarially fair premiums paid by banks.

It is worth noting that the above setup assumes that there is no cost to redistribute banks' assets when they default. In practice, default and bankruptcies can be very expensive, ranging from 14.7% to 30.5% of the market value of assets (Davydenko et al., 2012). Now, let $\hat{s}_j$ solves

$$l_j r^L[1-P(r^L,\hat{s}_j)] + \frac{z_j(1+i^R)}{1+\mu} + b_j r^B = d_j r^D. \tag{A.13}$$

Then, with probability $1-G(\hat{s}_j)$, a bank defaults. Let C denote the cost that the deposit insurance corporation incurs when a bank defaults and its assets need to be redistribute to depositors. The expected default cost is then $[1-G(\hat{s}_j)]C$. Recall that the default probability is in general increasing in banking competition (see Proposition 3.8). Then, a trade-off between efficiency and stability exists: policy makers have to weigh the benefits of more banking competition (i.e., increases in loan and deposit supplies) against the higher default cost $[1-G(\hat{s}_j)]C$.

A.2. Banking competition

Proposition A.1. *Consider an increase in N.*

(1) *L will increase in all cases except for Case IIIA.*
(2) *D will increase in all cases except for Case IIIA and IIIB.*
(3) *r^B remains unchanged in Case I, IIB, and IIIB. In Case IIA, r^B may increase or decrease. In Case IIIA, r^B will increase.*
(4) *Π will decrease in Case I, IIB, and IIIB. In Case IIA, Π will decrease if r^B decreases. In Case IIIA, Π will decrease if in equilibrium $\frac{2N+1}{N+1}\frac{L}{N}H'(L)+H(L)-\frac{\gamma}{\beta}>0$.*
(5) *Ψ will decrease in Case IIIA. The effect is ambiguous in Case IIA, IIB, and IIIB. However, if $LH'(L)+H(L)<0$, then Ψ will increase in Cases IIA, IIB, and IIIB. See Table 10.*

Proof. See Appendix C.2.

More banks implies less market power for each individual bank, as each bank has less impact on the loan rate and the deposit rate. This gives banks a higher incentive to issue more loans and bank deposits, so both loans and deposits increase with N in most equilibrium cases. However, in Case IIIA, D is determined by the amount of deposits households are willing to hold at the zero-lower bound, and since the number of bonds is fixed and the reserve and capital requirements are both binding, L is directly pinned down by D as well, so neither L nor D are affected by a small increase in N. In Case IIIB, the same is true for D, but since the reserve requirement is non-binding, banks can increase the amount of loans while keeping the amount of deposits unchanged by reducing reserve holdings, so L increases with N as banks care less about the marginal effects of extending more loans.

How r^B reacts to an increase in N depends on equilibrium cases. First, in Cases I, IIB, and IIIB, r^B is pinned down by exogenous variables ($\frac{1}{\beta}$ in Case I and $\frac{1+i^R}{1+\mu}$ in Cases IIB and IIIB), and is therefore unaffected by changes in N. In Case IIA, r^B is determined by the marginal return on loans, which may increase or decrease following an increase in N. This is because while L is larger (and hence the expected return on loans $H(L)$ is smaller), the impact of a marginal increase in L on the loan rate, $|\frac{L}{N}H'(L)|$, may be smaller. In Case IIIA, r^B is also determined by the marginal return on loans. However, since L does not change, r^B will be unambiguously larger. Intuitively, competition between banks lowers the impact of a marginal increase in L on the loan rate, $|\frac{L}{N}H'(L)|$, which makes issuing loans more attractive compared to investing in government bonds. As a result, the government bond rate must increase for bond investment to remain attractive.

Table 11
Comparative statics for b across the different equilibrium regimes.

	I	IIA	IIB	IIIA	IIIB
L	–	↓	–	↓	–
D	–	↑	–	–	–
r^B	–	↑	–	↑	–
Π	–	↑	–	↑	–
Ψ		?[a]	–	?[a]	–

[a] Decreases if $LH'(L) + H(L) < 0$.

In general, bank profits decrease when there is more competition. However, as we have shown, r^B may increase, which can increase bank profits. If r^B decreases in Case IIA, bank profits will decrease as well. In Case IIIA, the condition $\frac{2N+1}{N+1}\frac{L}{N}H'(L)+H(L)-\frac{\gamma}{\beta} > 0$ guarantees that the increase in r^B is sufficiently small so that bank profits will still decrease. Note that these conditions are sufficient but not necessary for bank profits to decrease.

Finally, as shown by (3.11), the default probability depends on the banking industry's total deposit liabilities (i.e., $Dr^D(D)$), total return from loans for any given s (i.e., $LH(L,s)$), total return from reserves (i.e., $\frac{z^B(1+i^R)}{1+\mu}$), and total return from government bonds (i.e., $b^B r^B$). The bank default probability decreases in Case IIIA because a higher return from government bonds (i.e., $b^B r^B$) provides a larger cushion against a negative shock. However, in Cases IIA and IIB, D and L will also change following an increase in N. An increase in D will increase default probability, because it increases bank's liabilities. An increase in L has ambiguous effects, because it may increase or decrease the total return from loans. If $LH'(L) + H(L) < 0$, however, the total return from loans will decrease following an increase in L, so bank default probability will increase.

A.3. Open-market operations

The quantity of bonds can be varied through open-market operations. For example, an increase in b can result from an open-market sale of government bonds by the central bank.

Proposition A.2. *Consider an increase in b.*

(1) *L will decrease in Cases IIA and IIIA and remain unchanged in the other cases.*
(2) *D will increase in Case IIA and remain unchanged in the other cases.*
(3) *r^B will increase in Cases IIA and IIIA and remain unchanged in the other cases.*
(4) *Π will increase in Cases IIA and IIIA and remain unchanged in the other cases.*
(5) *Ψ will remain unchanged in Cases IIB and IIIB. The effect is ambiguous in Cases IIA and IIIA. However, Ψ will decrease in Case IIIA if $LH'(L) + H(L) < 0$.*

Proof. See Appendix C.2.

Proposition A.2 and Table 11 summarize the effects of an increase in b in all equilibrium cases. As can easily be seen from the table, this has no real effects in equilibrium Cases I, IIB, and IIIB. The reason for this is as follows: In cases IIB and IIIB, banks hold excess reserves, which implies that bonds and reserves must be perfect substitutes. In turn, this implies that the real rates on bonds and reserves must be equal, and since the interest rate on reserves is given exogenously, it also pins down r^B. In Case I, some bonds are held by households, which are only willing to hold them at $r^B = \frac{1}{\beta}$, so this pins down the bond rate. Further, banks are indifferent about holding another unit of government bonds as the cost of raising equity equals the return on bonds.

In Cases IIA and IIIA, increasing b increases the policy rate r^b, so the government can use open-market operations to affect the economy in these cases. This also means that banks receive a higher return from government bonds. Further, the increase in r^B crowds out bank loans as investing in bonds becomes more attractive. In Case IIA, the increase in bond holdings by banks is not fully offset by reductions in loans, however, as banks also increase deposits to fund the additional bond investments. In Case IIIA, deposits are pinned down by the zero-lower bound and are unaffected by b.

Bank default probability may increase or decrease in Cases IIA and IIIA because of two opposing effects: On the one hand, a higher r^b and a larger supply of government bonds tend to decrease default probability through the increase in bank profits. On the other hand, a smaller L may increase or decrease the total return from loans, and a larger D will increase banks' liabilities. Consequently, the overall effect of increasing b on bank default is ambiguous in Case IIA. However, if $LH'(L) + H(L) < 0$, a smaller L will increase the return from loans. Since D does not change in Case IIIA, this means that the default probability will decrease.

A.4. Interest on reserves

Proposition A.3. *Consider an increase in i^R.*

(1) *L will increase in Cases IIA, decrease in Cases IIB and IIIB, and remain unchanged in the others.*
(2) *D will increase in Cases I, IIA, and IIB, and remain unchanged in the other cases.*
(3) *r^B will decrease in Cases IIA, increase in Cases IIB and IIIB, and remain unchanged in the others.*

Table 12
Comparative statics for i^R across the different equilibrium regimes.

	I	IIA	IIB	IIIA	IIIB
L	–	↑	↓	–	↓
D	↑	↑	↑	–	–
r^B	–	↓	↑	–	↑
Π	↑	?	↑	↑	↑
Ψ		?	?	↓	?[a]

[a] Decreases if $LH'(L) + H(L) < 0$.

(4) *Π will increase in all cases except Case IIA, in which the effect is ambiguous.*

(5) *Ψ will decrease in Case IIIA. In all the other cases, the effect is ambiguous. However, Ψ will also decrease in Case IIIB if $LH'(L) + H(L) < 0$.*

Proof. See Appendix C.2.

Proposition A.3 and Table 12 summarize the effects of an increase in i^R. The first thing to note is that the effect of increasing i^R in Cases IIB and IIIB is the same as increasing b in Cases IIA and IIIA, respectively. This is because while open-market operations are ineffective when banks hold excess reserves, the monetary authority can directly vary i^R to affect the policy rate in these cases. In particular, r^B increases with i^R in Cases IIB and IIIB since bonds and reserves are perfect substitutes at the margin. This in turn implies that banks earn a higher return from government bonds and reserves when i^R increases, so bank profits increase. Loans decrease with the increase in r^B since investing in bonds and reserves becomes relatively more attractive, and deposits increase in Case IIB as banks increase their balance sheets to benefit from the higher return on bonds and reserves. In Case IIIB, deposits are pinned down by the zero-lower bound, so they do not vary with i^R. Finally, the effect of an increase in i^R on bank default probability is ambiguous in Cases IIB and IIIB, since the increase in bank profits makes banks more resilient against default, but the increase in L may increase or decrease the total return from loans. Additionally, in Case IIB, the increase in D tends to increase bank default probability. In Case IIIB, if $LH'(L) + H(L) < 0$, a decrease in L implies that the total return from loans increases. Since D remains unchanged, bank default probability decreases.

In Cases I, IIA, and IIIA, i^R is not a good tool to vary the policy rate, but changes in the interest rate on reserves still have real effects since banks must hold some reserves due to the reserve requirement. In particular, an increase in i^R makes issuing deposits more attractive as the cost of satisfying the reserve requirement goes down. Thus, D increases with i^R in Cases I and IIA (but not in IIIA, where D is pinned down by the zero-lower bound). Consequently, this cost reduction also increases bank profits in Cases I, IIA, and IIIA. In Case IIA, banks also increase L because of the reduced cost of fulfilling the reserve requirement. In turn, r^B decreases. In Cases I and IIIA, the change in i^R has no effect on L because loans are directly pinned down by the discount factor β or the amount of deposits D, respectively. Finally, the effect on bank default probability is ambiguous in case IIA, while bank default probability unambiguously goes down with i^R in Case IIIA because of the increase in bank profits.

A.5. Comparative statics for the reserve requirement δ

The following proposition summarizes the effects of increasing the reserve requirement.

Proposition A.4. *Consider an increase in δ.*

(1) *L will decrease in Cases IIA and IIIA, and remain unchanged in the other cases.*

(2) *D will decrease in Case I, and remain unchanged in Cases IIB, IIIB, and IIIA. In Case IIA, the effect is ambiguous.*

(3) *r^B will increase in Cases IIA and IIIA, and remain unchanged in the other cases.*

(4) *Π will decrease in Case I, and remain unchanged in Cases IIB and IIIB. In Cases IIA and IIIA, the effect is ambiguous.*

(5) *Ψ remains unchanged in Cases IIB and IIIB. The effect is ambiguous in Cases IIA and IIIA. However, Ψ will decrease in Case IIIA if $LH'(L) + H(L) < 0$. See Table 13.*

Proof. See Appendix C.2.

In Cases IIB and IIIB, the reserve requirement does not bind. Hence, increasing the reserve requirement has no effect on the equilibrium. In Case IIA, the reserve requirement binds, and increasing the reserve requirement makes it more costly for banks to fund investment with deposits. Hence, L decreases and r^B increases. However, to comply with the new reserve requirement, banks also need to hold more reserves, which can be achieved through issuing more deposits. Consequently, in this case, the effect on D is ambiguous.

In Case I, a higher reserve requirement increases the cost of issuing deposits, which also happens in Cases IIA and IIIA. However, in the latter two cases, r^B will also increase, which will increase the income of banks. The mechanism is similar to when the government increases i^R: because banks take the government bond rate as given, they do not take the effect of their competition on the government bond rate into account. In this case, the lower demand for government bonds benefits banks. Consequently, the overall effect of increasing the reserve requirement on bank profits is ambiguous in Cases IIA and IIIA.

Table 13
Comparative statics for δ across the different equilibrium regimes.

	I	IIA	IIB	IIIA	IIIB
L	–	↓	–	↓	–
D	↓	?	–	–	–
r^B	–	↑	–	↑	–
Π	↓	?	–	?	–
Ψ		?	–	?[a]	–

[a] Decreases if $LH'(L) + H(L) < 0$.

Table 14
Comparative statics for γ across the different equilibrium regimes.

	I	IIA	IIB	IIIA	IIIB
L	–	?[a]	↓	↑	↓
D	–	↓	–	–	–
r^B	–	?[b]	–	↓	–
Π	–	↓	↓	↓	↓
Ψ		?	?[c]	?[d]	?[c]

[a] Increases if $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$ and $\frac{1-\sigma+\epsilon}{(1+\mu)(1-\sigma)} \geq \frac{1}{\beta}$.
[b] Decreases if $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$ and $\frac{1-\sigma+\epsilon}{(1+\mu)(1-\sigma)} \geq \frac{1}{\beta}$.
[c] Decreases if $LH'(L) + H(L) < 0$.
[d] Increases if $LH'(L) + H(L) < 0$.

Finally, in Cases IIA and IIIA, while increasing the reserve requirement makes banks hold more safe assets, it also forces banks to lower L, which may increase or decrease the total return from loans. As a result, increasing the reserve requirement has ambiguous effects on bank default probability. However, in Case IIIB, if $LH'(L)+H(L) < 0$, the decrease in L implies that the total return from loans increases. Since D remains unchanged, bank default probability will decrease.

A.6. Comparative statics for the capital requirement γ

Here, we study the effect of increasing the capital requirement.

Proposition A.5. *Consider an increase in γ.*

(1) *L will increase in Case IIIA, decrease in Cases IIB and IIIB, and remain unchanged in Case I. If $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$, then as long as $\frac{1-\sigma+\epsilon}{(1+\mu)(1-\sigma)} \geq \frac{1}{\beta}$ where $\epsilon = \frac{(1-\gamma)L}{(1-\delta)D}$, L will also increase in Case IIA.*

(2) *D will decrease in Case IIA, and remain unchanged in the other cases;*

(3) *r^B will decrease in Case IIIA and remain unchanged in Cases I, IIB, and IIIB. If $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$ and $\frac{1-\sigma+\epsilon}{(1+\mu)(1-\sigma)} \geq \frac{1}{\beta}$, then r^B will decrease in Case IIA.*

(4) *Π will remain unchanged in Case I and decrease in the other cases.*

(5) *The effect on Ψ is in general ambiguous. However, suppose that $LH'(L) + H(L) < 0$. Then Ψ will decrease in Cases IIB and IIIB but increase in Case IIIA. See Table 14.*

Proof. See Appendix C.2.

First, the capital requirement is nonbinding in Case I since banks are indifferent about raising more equity and investing it in bonds. In the other cases, an increase in γ implies that issuing loans becomes more expensive, since banks need to hold more equity. One may think that this will decrease the amount of loans banks make, but this is not true in every case. In Case IIA, banks reduce deposit supply following the increase in γ. Hence, although for any given deposit rate banks' funding costs are increasing in γ, the deposit rate is decreasing in γ. If banks' marginal funding cost decreases following the increase in γ, then bank loans will increase. In Case IIIA, D is pinned down by the corner solution and does not change. Because banks do not reduce deposits, they invest the new equity in bank loans since loans pay a higher marginal return than reserves. Finally, since r^B is equal to the marginal return on loans, in Case IIA, r^B increases if L decreases, and decreases if L increases. In Cases IIB and IIIB, the reserve requirement is slack, and therefore banks prefer to invest in more reserves if they are forced to raise more equity. Thus, L decreases in these cases, while D and r^B are unaffected.

Bank profits will decrease except in Case I, where the capital requirement does not bind, because compared to deposits, equity is more costly for banks to raise. In Cases IIA and IIIA, bank profits will further decrease due to the decrease in r^B.

Finally, the effect of γ on bank default probability is in general ambiguous. In Cases IIB and IIIB, a higher γ will increase bank's holdings of safe assets, but it will also lower L, which may increase or decrease the total return from loans. However, if

Table 15
The effects of an increase in b by 10% with different N.

	$\frac{1}{2}N$	N	$2N$
i^b	+1.76 pp	+2.79 pp	+1.84 pp
W	−0.36%	−0.52%	−0.26%
L	−1.75%	−2.77%	−1.83%
D	unchanged	+0.90%	+1.87%
i^L	+1.56 pp	+2.57 pp	+1.73 pp
i^D	unchanged	+0.70 pp	+1.44 pp
Π	+42.1%	+69.4%	+11.9%
Ψ	−0.0016 pp	−0.0199 pp	−0.0256 pp
Case transition	IIIB to IIIA	IIIB to IIA	IIIB to IIA

$LH'(L) + H(L) < 0$, since L decreases in Cases IIB and IIIB, the total return from loans will increase. Since D remains unchanged, bank default probability will decrease. In contrast, because L increases in Case IIIA, the total return from loans will decrease, and bank default probability will increase.

Appendix B. Additional quantitative results

B.1. Effectiveness of monetary policy with different N

In this section, we assess the effectiveness of monetary policy by varying government bond supply. We increase the bond supply by 10%, which corresponds to an open-market sale of assets by the central bank. We conduct this exercise with N, $\frac{1}{2}N$, and $2N$. Table 15 reports the results. We focus on open-market sales here because an open-market purchases has no effect in Case IIIB.

First, note that in all three scenarios, the economy transitions from a case where the reserve requirement does not bind to a case where it binds. This is necessary for open-market operations to have real effects, as we have shown in our theoretical analysis. Second, note that the effects of increasing b on the bond interest rate differ across the three economies, with the bond interest rate increasing the most in the baseline case. The effect on the loan rate is also strongest in the baseline case, while the effect on the deposit rate is increasing with N.

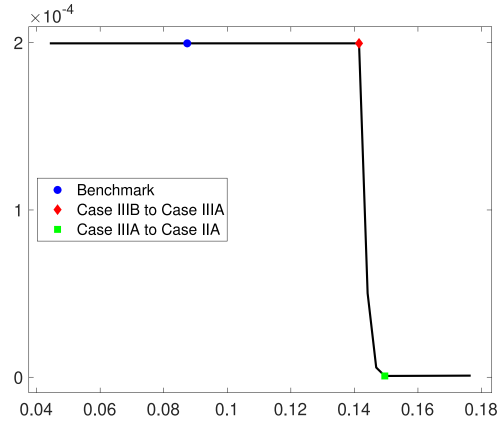
To understand these effects, first compare the baseline case with the $2N$ case since they involve the same equilibrium case transitions. When competition in the banking sector is high, banks also compete fiercely over available assets, so they are willing to hold the additional bonds even if they offer only a slightly higher return than the assets the banks already hold do. If banking competition is low, banks are more reluctant to change their balance sheet composition, and thus must be induced by much higher rates to hold the additional bonds. A similar story is going on with loans: With low banking competition and the high increase in the bond interest rate, banks reduce loans by a lot, which benefits them in two ways: First, they can get rid of the most risky loans as the loan rate increases and replace them with safe government bonds instead. Second, they earn more on their remaining loans as the loan rate goes up. With high banking competition, banks do not take the marginal effect on the loan rate into account in the same extent, and thus are much less willing to reduce loans made. Finally, if banking competition is high, banks are also more willing to raise additional funds to make use of the additional investment opportunities, which is why the deposit interest rate increases more strongly with $2N$.

The pattern in N is muddled when comparing the case with $\frac{1}{2}N$ to the baseline case since things are different at the zero-lower bound. In a sense, banks have too many funds in equilibrium Cases IIIA and IIIB, since they would in principle like to reduce the deposit rate further below zero but cannot do so without losing all their deposits. Hence, even if N is low, banks compete relatively fiercely for additional assets as they want to invest these excess funds, and thus i^B and i^L do not increase as strongly in this case.

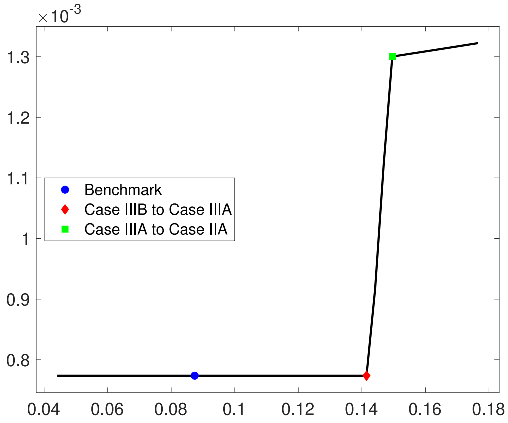
Note that the different results from the experiments on interest on reserves and open-market operations are *not* due to the policy we study, but to the way we design the experiments. As Appendix B.3 shows, if open-market operations are used to target an increase in the policy rate by 1pp, the results are very similar to those where the central bank increases i^R by 1pp. To put this differently, it does not matter which policy tool the central bank uses, but just whether an interest rate or an asset quantity is targeted. We think that both results are interesting and that the interest rate target is somewhat more natural with interest on reserves, while the asset quantity target can be more naturally studied with open-market operations (understanding that when central banks use open-market operations in practice, they typically also follow an interest rate target).

B.2. Changes in the reserve requirement

This section discusses how changing the reserve requirement δ affects the probability of bank failure Ψ and bank profits Π in our calibration. As we discussed in Appendix A.5, increasing the reserve requirement has ambiguous effects on these variables in equilibrium Cases IIA and IIIA. Fig. 5(a) shows how changing δ affects the probability of bank failure. Perhaps even more surprisingly, profits per bank is increasing in δ in our calibration, as can be seen in Fig. 5(b). This seems counter-intuitive, since banks could always hold more reserves if that allows them to increase profits. However, forcing all banks to hold more reserves is different from banks holding more reserves voluntarily, because it has effects on other assets, namely government bonds. When



(a) Effects of δ on Bank Default Probability



(b) Effects of δ on bank profits

Fig. 5. Effects of variations in δ.

Table 16
The effects of an increase in i^b by 1 pp with different N.

	$\frac{1}{2}N$	N	$2N$
b^B	+9.62% (+0.042)	+7.63% (+0.033)	+7.99% (+0.035)
W	−0.21%	−0.21%	−0.15%
L	−0.99%	−0.99%	−0.99%
D	unchanged	unchanged	+0.89%
i^L	+0.89 pp	+0.92 pp	+0.94 pp
i^D	unchanged	unchanged	+0.69 pp
Π	+23.91%	+33.99%	+11.06%
Ψ	−0.0015 pp	−0.0185 pp	−0.0292 pp
Case transition	IIIB to IIIA	IIIB to IIIA	IIIB to IIA

banks are forced to hold more reserves, their demand for bonds decreases. Since the amount of bonds available for banks to hold is independent of their demand, this leads to an increase in the bond interest rate. Thus, Π increases in δ because the positive externality from the higher bond rate dominates the cost of holding more reserves. It is important to note that this analysis assumes passive monetary policy. If the monetary authority simultaneously enacts open-market operations to keep the bond interest rate constant, the positive externality will vanish.

B.3. Open market operations

Instead of increasing the policy rate by increasing the interest rate on reserves i^R, the monetary authority could conduct open-market operations such that the policy rate increases by 1pp. Table 16 shows the effect of this policy with different N. By comparing with Table 8, we can determine to which extent it matters whether the monetary authority uses open-market operations or the interest rate on reserves to increase the policy rate by 1pp. For loans, loan rates, and the probability of default by entrepreneurs, it does not matter which rate is increased by 1pp, as the economy reacts in exactly the same way. For deposits, deposit rates, bank profits, and the probability of bank default, it matters slightly, because increasing the interest rate on reserves implies $i^R = i^b$ continues to hold as the bond interest rate increases endogenously and thus banks earn this (marginal) return on all their assets. Meanwhile, increasing the bond rate through open-market operations while keeping the interest rate on reserves constant implies $i^b > i^R$, which in turn makes the reserve requirement bind, and implies that banks earn a lower marginal return on their required reserves than on their remaining assets, which lowers their profits and their willingness to issue deposits. The most interesting result from Table 16 however is that the size of the open-market operation that is required to engineer an increase in i^b by 1pp varies with N, which mirrors the results presented in Table 15.

B.4. Additional counterfactuals

Figs. 6–10 show how changes in N, i^R, b, δ, and γ affect loan rates, deposit rates, nominal bond rates, loans, deposits, aggregate welfare, the probability of bank failure, and profits per bank.

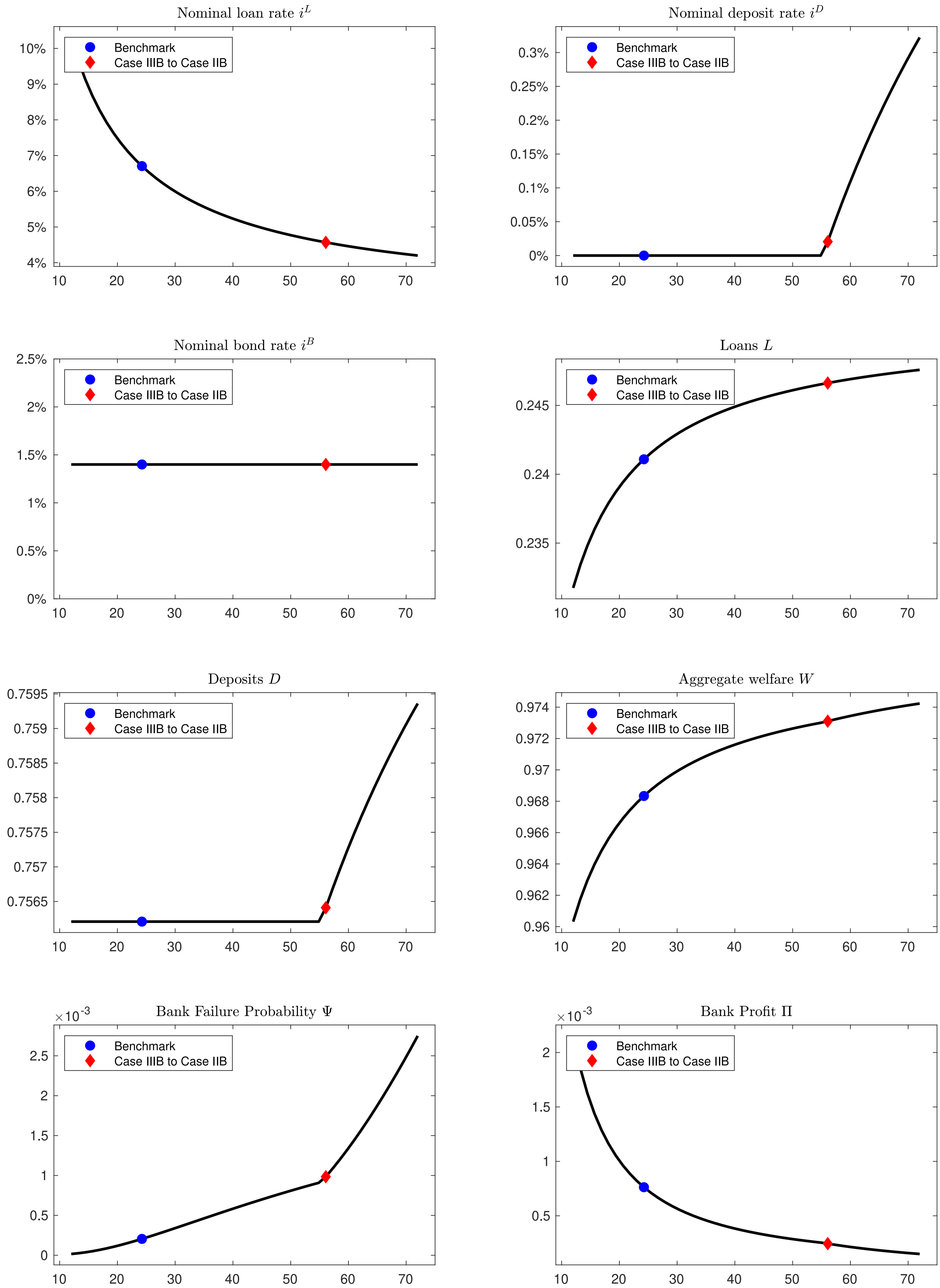


Fig. 6. Effects of N.

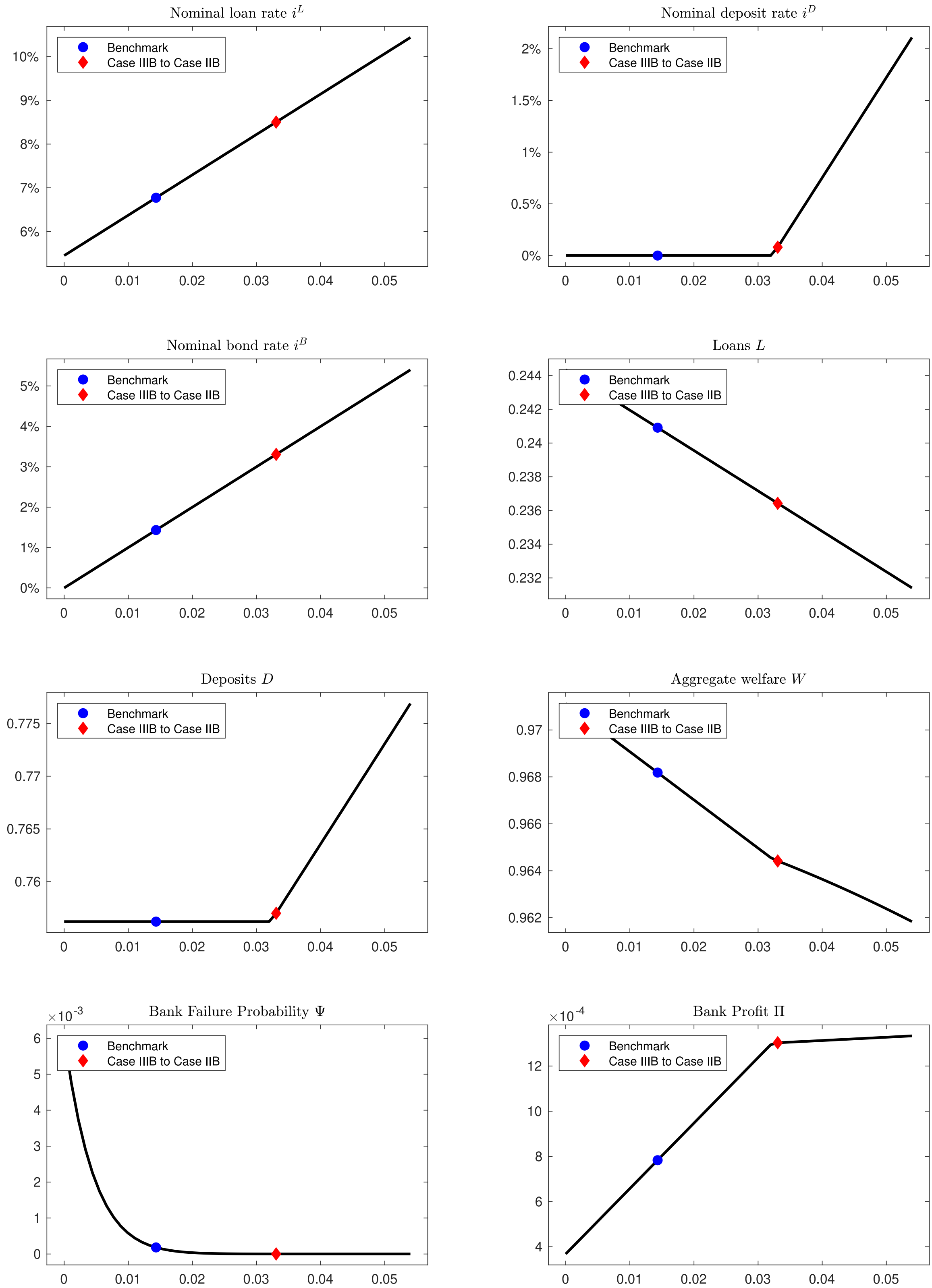


Fig. 7. Effects of i^R.

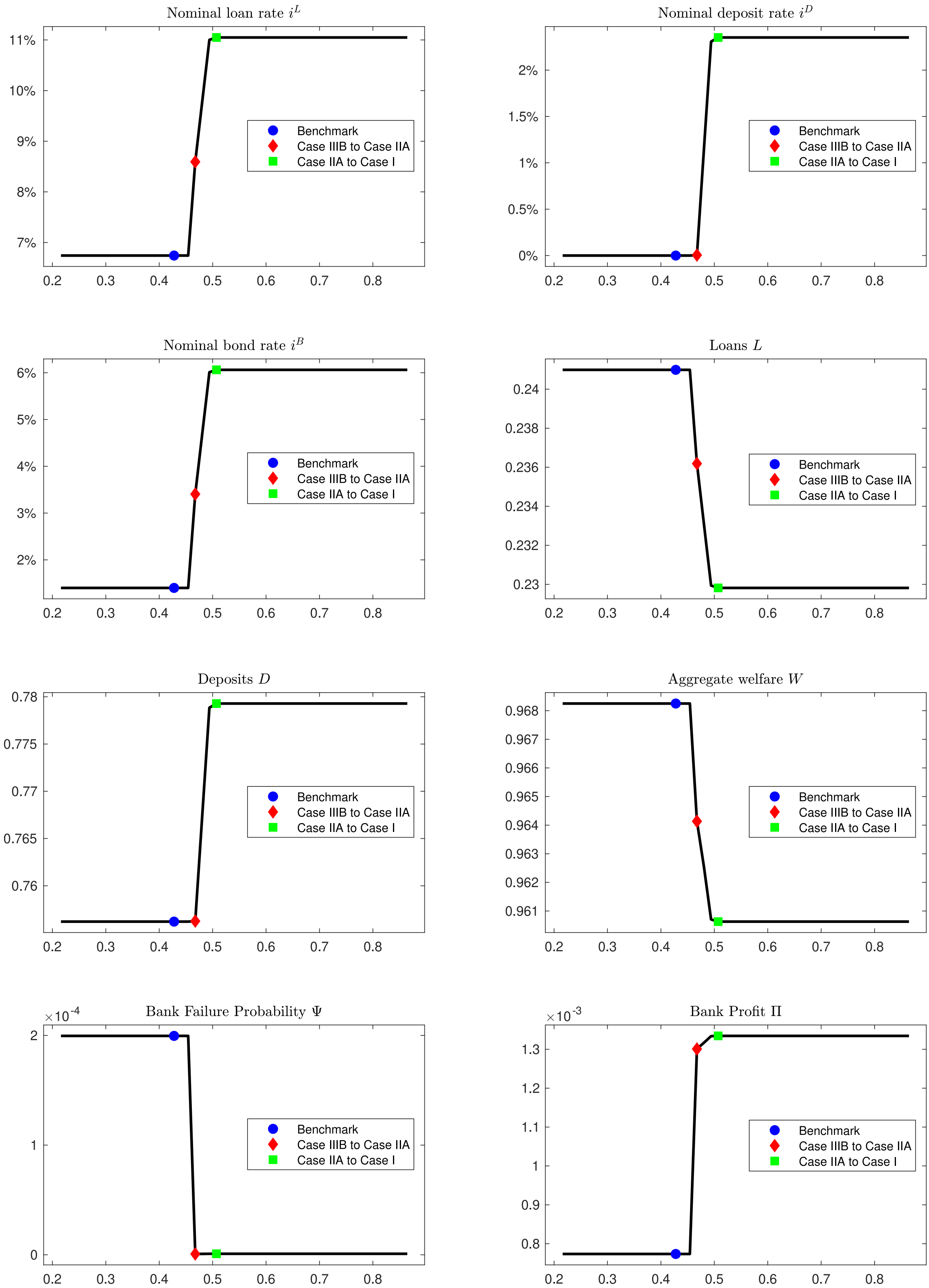


Fig. 8. Effects of b.

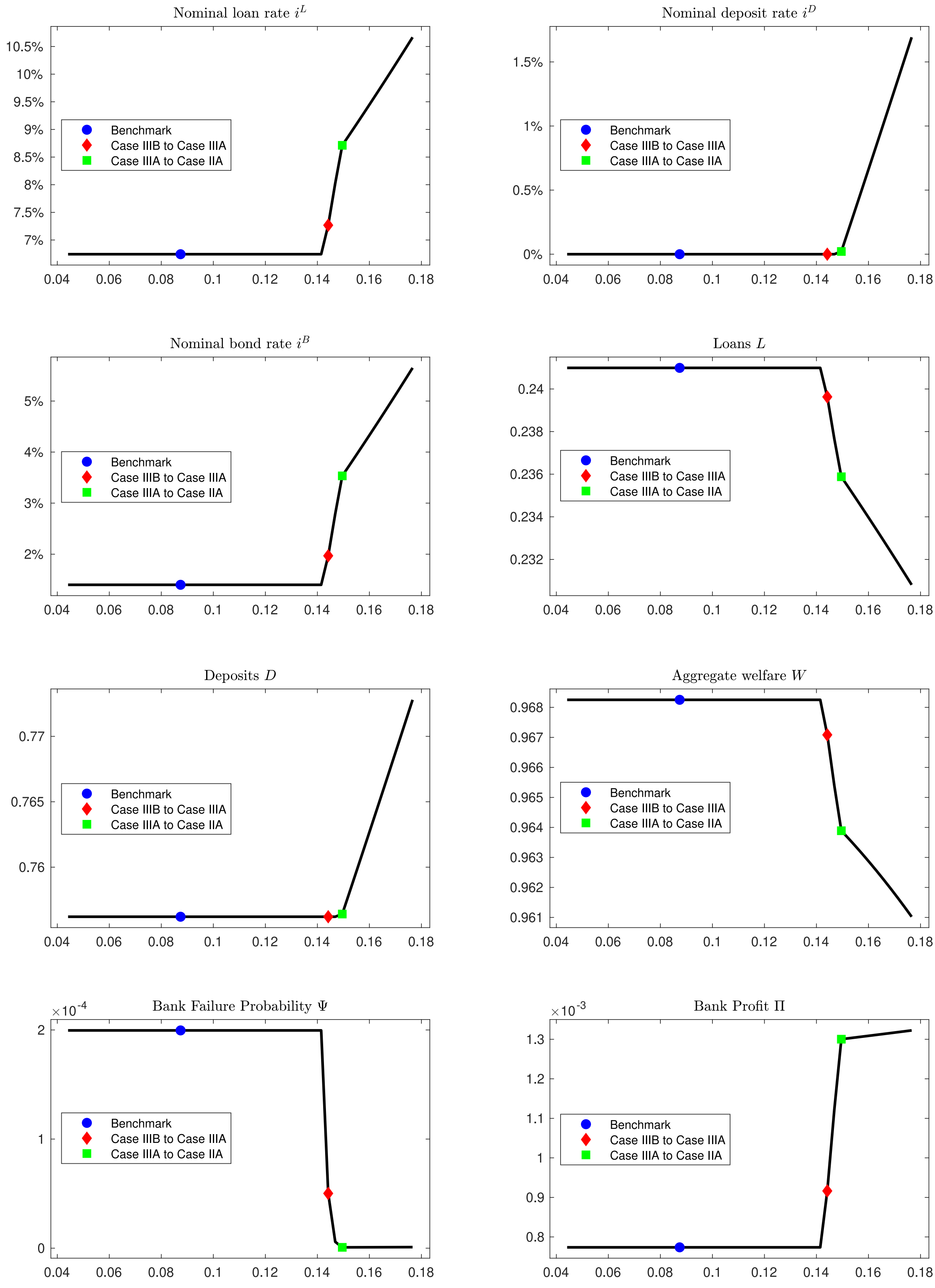


Fig. 9. Effects of δ.

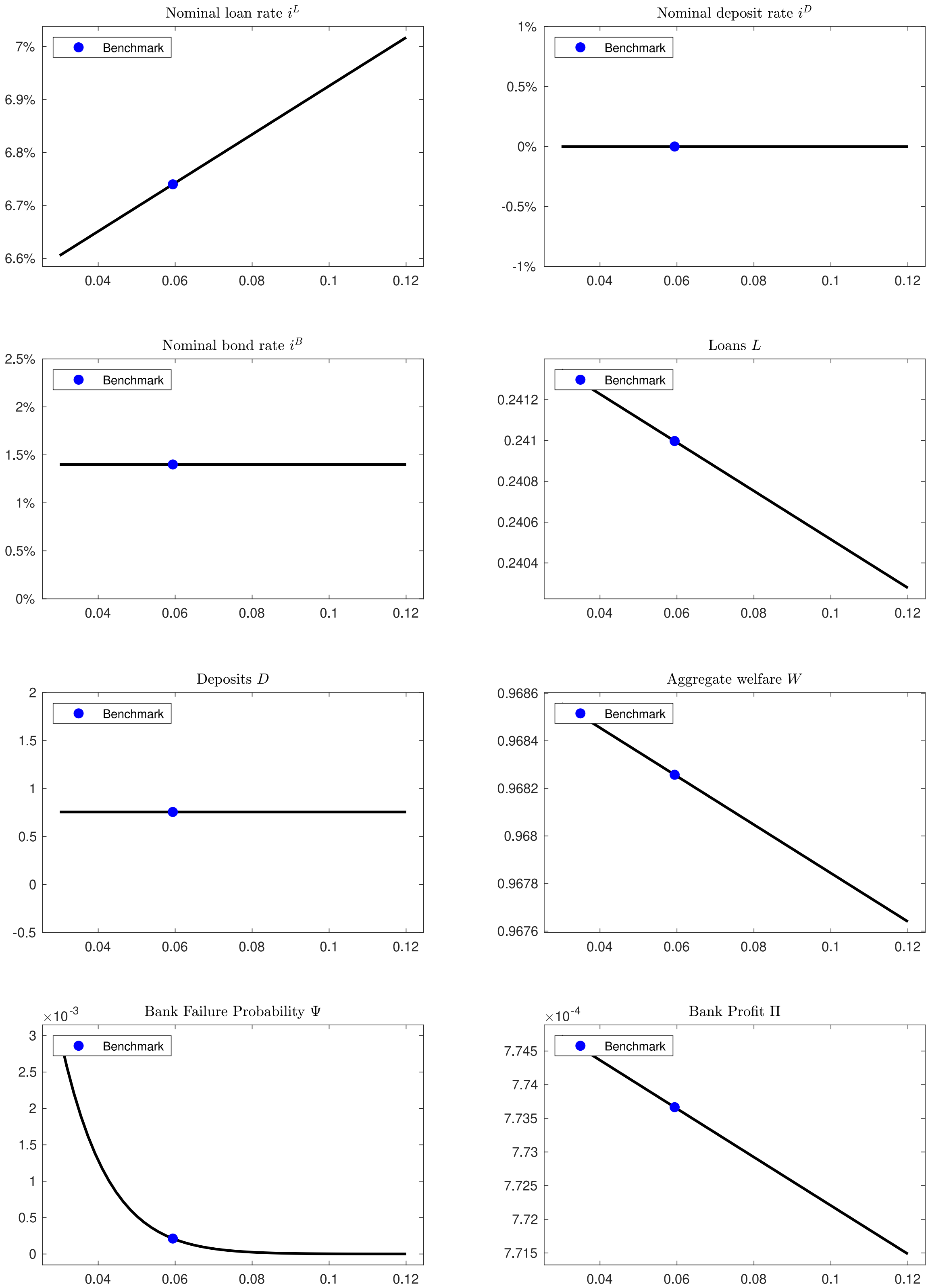


Fig. 10. Effects of γ.

Appendix C. Proofs

C.1. Proofs for Section 3

Proof of Proposition 3.1. It is clear that if $H'(L) < 0$ and $H''(L) < 0$ for all $L \in [0, S]$ and $r^{D\prime}(D) > 0$ and $r^{D\prime\prime}(D) \geq 0$ for all $D \geq 0$, then in each of the five cases (Case I, IIA, IIB, IIIA, and IIIB), there is a unique solution to bank's problem. We need only show that for any given parameter values, only one equilibrium case may exist.

First, suppose the parameter values are such that Case I exists and denote the equilibrium L and D as L^I and D^I. Now, consider Case IIA. If we plug in L^I and $D = ((1-\gamma)L^I + b)/(1-\delta)$ into the first order condition, we have

$$r^D(D) + \frac{D}{N} r^{D\prime}(D) > \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}, \tag{C.1}$$

which means L must be smaller to satisfy the first order condition. Then, we have $\frac{L}{N}H'(L) + H(L) > \frac{1}{\beta}$, which cannot happen in equilibrium because banks would have raised more equity and lend more to entrepreneurs. In other words, Case IIA cannot exist. Next, consider Case IIB. If it exists, because

$$\frac{(1-\gamma)(1+i^R)}{1+\mu} + \frac{\gamma}{\beta} < \frac{1}{\beta} \quad \text{and} \quad \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu} > \frac{1+i^R}{1+\mu}, \tag{C.2}$$

we have $L^{IIB} > L^I$ and $D^{IIB} < D^I$. Since $b^B \leq (1-\delta)D - (1-\gamma)L$, $L^{IIB} > L^I$ and $D^{IIB} < D^I$ mean that banks hold fewer government bonds, a contradiction. Finally, consider Cases IIIA and IIIB. In the first case, the existence of a solution would imply that L is smaller, which means $\frac{L}{N}H'(L) + H(L) > \frac{1}{\beta}$. In the second case, the existence of a solution would imply that b^B is smaller, a contradiction.

Second, suppose the parameter values are such that Case IIA exists. By the above arguments, Case I cannot exist. Note also that in Case IIA, we have

$$r^D(D) + \frac{D}{N} r^{D\prime}(D) = (1-\delta) r^B + \frac{\delta(1+i^R)}{1+\mu} > \frac{1+i^R}{1+\mu} \tag{C.3}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] > \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.4}$$

If Case IIB exists, then D will be smaller but L will be larger. This means b^B must be smaller, a contradiction. Similarly, Case IIIB cannot exist. Finally, if Case IIIA exists, then because Case IIA exists by assumption, banks could have increased their profits by increasing D and L, a contradiction.

Third, suppose the parameter values are such that Case IIB exists. By the above arguments, Case I and IIA cannot exist. If Case IIIB exists, because $r^D(D) = \frac{1}{1+\mu}$, it must be that

$$r^D(D) + \frac{D}{N} r^{D\prime}(D) < \frac{1+i^R}{1+\mu}. \tag{C.5}$$

Then banks could have increased their profits by increasing D and investing it into reserves, a contradiction. If Case IIIA exists, then banks could also have increased their profits by increasing D and investing it into reserves.

Finally, suppose the parameter values are such that Case IIIA exists. By the above arguments, Case I, IIA, and IIB cannot exist. Note that in Case IIIA,

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta} > \frac{1+i^R}{1+\mu} \tag{C.6}$$

If Case IIIB exists, then L would be larger so b^B would be smaller, a contradiction. This also means that if Case IIIB exists, none of the other cases exist. Hence, if there exists a unique equilibrium. □

C.2. Proofs for Appendix A

Before we prove Proposition A.1 to A.4, we prove the following lemma that will be useful later. Define $H(L, s) = r^L(L)[1 - P(r^L(L), s)]$.

Lemma C.1. *Assume there exists a random variable Z such that $|H_L(L, s)| \leq Z$ a.s. for all L and that $\mathbb{E}(Z) < \infty$. Then $H'(L) = \mathbb{E}[H_L(L, s)]$, where the expectation is taken over s.*

Proof. Note that

$$H'(L) = \lim_{t\to 0} \frac{H(L+t) - H(L)}{t} = \lim_{t\to 0} \mathbb{E}\left[\frac{H_L(L+t, s) - H_L(L, s)}{t}\right] = \lim_{t\to 0} \mathbb{E}[H_L(h(t), s)], \tag{C.7}$$

where $h(t) \in (L, L+t)$ exists because of the Mean Value Theorem. Since $|H_L(L,s)| \le Z$ a.s. for all L, then by the Dominated Convergence Theorem, we have

$$H'(L) = \lim_{t\to 0} \mathbb{E}[H_L(h(t), s)] = \mathbb{E}[H_L(L,s)], \tag{C.8}$$

which is the desired result. □

Proof of Proposition A.1. We consider the effects of increasing N to $N+1$ in all five cases.

Case I

In this case, the FOCs are

$$\frac{L}{N+1}H'(L) + H(L) = \frac{1}{\beta} \tag{C.9}$$

and

$$r^D(D) + \frac{D}{N+1}r^{D\prime}(D) = \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}. \tag{C.10}$$

Because $H'(L) < 0$ and $r^{D\prime}(D) > 0$, the left-hand side of (C.9) will be larger, while the left-hand side of (C.10) will be smaller than before. Hence, L and D must increase. Since $r^B = 1/\beta$, it is unaffected by the increase in N.

bank profits is given by

$$-\frac{E}{N+1} + \beta\left[\frac{L}{N+1}r^L(L)[1-P(r^L(L))] + \frac{z^B}{N+1}\frac{1+i^R}{1+\mu} + \frac{b^B}{N+1}\frac{1}{\beta} - \frac{D}{N+1}r^D(D)\right]. \tag{C.11}$$

If bank profits is equal to or larger than before, then one bank could have deviated and increase the loans and deposits it issues when there were N banks. This will give the deviating bank a strictly higher profit, a contradiction.

Case IIA

In this case, we have $D = ((1-\gamma)L + b)/(1-\delta)$ and

$$\frac{L}{N+1}H'(L) + H(L) + \frac{(1-\gamma)\delta(1+i^R)}{(1-\delta)(1+\mu)} = \frac{1-\gamma}{1-\delta}\left[r^D(D) + \frac{D}{N+1}r^{D\prime}(D)\right] + \frac{\gamma}{\beta}. \tag{C.12}$$

If the number of banks increases to $N+1$, the left-hand side will increase while the right-hand side will decrease. Hence, L must increase, which means D will increase as well. Note that r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N+1}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.13}$$

so (C.12) can be rewritten as

$$r^D(D) + \frac{D}{N+1}r^{D\prime}(D) = (1-\delta)r^B + \frac{\delta(1+i^R)}{1+\mu}. \tag{C.14}$$

Since the left-hand side may increase or decrease compared to the benchmark, r^B may decrease, increase, or remain unchanged. If r^B decreases or remains the same, then bank profits will be lower.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.15}$$

Note that in this case, r^B satisfies Eq. (C.13). Substitute this in and take total derivative to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\mathrm{d}D - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\mathrm{d}L\\ &= G(\hat{s})LH_s(L,\hat{s})\mathrm{d}\hat{s} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\mathrm{d}L.\end{aligned} \tag{C.16}$$

Note that $D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu} > 0$ and $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$. By Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Recall that an increase in N leads to increases in D and L. Hence, default probability will increase.

Case IIB

In this case, the FOCs are

$$r^D(D) + \frac{D}{N+1}r^{D\prime}(D) = \frac{1+i^R}{1+\mu}, \tag{C.17}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N+1}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.18}$$

Similar to Case I, L and D must increase. Since $r^B = \frac{1+i^R}{1+\mu}$, government bond rate is unaffected by the increase in N. Bank profits is given by

$$-\frac{E}{N+1} + \beta\left[\frac{L}{N+1} r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N+1}\frac{1+i^R}{1+\mu} + \frac{b}{N+1}\frac{1+i^R}{1+\mu} - \frac{D}{N+1} r^D(D)\right]. \tag{C.19}$$

Again, similar to Case I, if bank profits is equal to or larger than before, then one bank could have deviated and increase the loans and deposits it issues when there were N banks. This will give the deviating bank a strictly higher profit, a contradiction.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D-(1-\gamma)L)}{1+\mu}. \tag{C.20}$$

Take total derivative to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{1+i^R}{1+\mu}\right]\mathrm{d}D + \frac{(1+i^R)(1-\gamma)}{1+\mu}\mathrm{d}L \\ &= G(\hat{s})LH_s(L,\hat{s})\mathrm{d}\hat{s} + \left\{\int_{\hat{s}}^{\bar{s}} [LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\mathrm{d}L.\end{aligned} \tag{C.21}$$

Note that $D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{1+i^R}{1+\mu} > 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}} [LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Hence, default probability will increase.

Case IIIA

In this case, we have $(1-\gamma)L = (1-\delta)D' - b$, and

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.22}$$

Then increasing the number of banks to $N+1$ has no effect on L or D. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N+1}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.23}$$

it will increase following the increase in N.

Next, consider bank profits. Note that the bond rate when there are N banks is $\frac{L}{N}H'(L) + H(L) - \frac{\gamma}{\beta}$, and bond holding per bank is b/N. With $N+1$ banks, bond holding per bank becomes $b/(N+1)$. Now, suppose

$$\begin{aligned}&\frac{N+1}{N}\left\{\frac{L}{N}H'(L) + H(L) - \frac{\gamma}{\beta}\right\} - \left[\frac{L}{N+1}H'(L) + H(L) - \frac{\gamma}{\beta}\right] \\ =&\frac{1}{N}\left\{\frac{2N+1}{N+1}\frac{L}{N}H'(L) + H(L) - \frac{\gamma}{\beta}\right\} > 0.\end{aligned} \tag{C.24}$$

This means that

$$\frac{b}{N+1}\left[\frac{1}{1-\gamma}\left[\frac{L}{N+1}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}\right] < \frac{b}{N+1}\frac{N+1}{N}\left\{\frac{L}{N}H'(L) + H(L) - \frac{\gamma}{\beta}\right\} \tag{C.25}$$

In other words, for each bank, the income from government bonds, $b^B r^B$, will decrease when there are $N+1$ banks, despite that bond rate is higher. Hence, bank profits will decrease.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.26}$$

Since increasing N has no effect on L or D but r^B will increase, default probability will decrease.

Case IIIB

In this case, the FOCs are

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.27}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N+1}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.28}$$

Hence, L will increase while D remain unchanged. This means that banks will raise more equity and lower the investment in reserves. Since $r^B = \frac{1+i^R}{1+\mu}$, government bond rate is unaffected by the increase in N.

Similar to Case IIB, bank profits must decrease, because otherwise one bank could have deviated and increase the loans it issues. This will give the deviating bank a strictly higher profit, a contradiction.

Finally, consider bank default probability $1-G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D-(1-\gamma)L)}{1+\mu}. \tag{C.29}$$

Take total derivative to get

$$\begin{aligned}&\frac{(1+i^R)(1-\gamma)}{1+\mu}\mathrm{d}L\\ &= G(\hat{s})LH_s(L,\hat{s})\mathrm{d}\hat{s} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})]\right\}\mathrm{d}L.\end{aligned}$$

Note that by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})] < \mathbb{E}[LH'(L,s)+H(L,s)] = LH'(L)+H(L) < 0,$$

Hence, default probability will increase. □

Proof of Proposition A.2. We consider the effects of a marginal increase in b in all five cases.

Case I

In this case, because banks do not hold all government bonds, a marginal increase in b has no effect on L or D. Since $r^B = 1/\beta$, it is unaffected by the increase in b. Similarly, bank profits is not affected either.

Case IIA

In this case, we have $D = ((1-\gamma)L+b)/(1-\delta)$ and

$$\frac{L}{N}H'(L) + H(L) + \frac{(1-\gamma)\delta(1+i^R)}{(1-\delta)(1+\mu)} = \frac{1-\gamma}{1-\delta}\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] + \frac{\gamma}{\beta}. \tag{C.30}$$

Increasing b while holding L constant will increase D, which means the right-hand side will be larger than the left-hand side. This means that L must decrease for the equation to hold. Because L will be lower, the right-hand side will be larger than the benchmark, which means D will increase. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L)+H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.31}$$

it will increase after an increase in b.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1-P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b}{N}r^B - \frac{D}{N}r^D(D)\right]. \tag{C.32}$$

Since r^B is higher, bank profits is higher even if bank's assets and liabilities remain the same. Hence, bank profits must be higher.

Finally, consider bank default probability $1-G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.33}$$

Take the derivative w.r.t. b to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\frac{\partial D}{\partial b} - r^B - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial b}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial b} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})]\right\}\frac{\partial L}{\partial b}.\end{aligned} \tag{C.34}$$

Note that $\frac{\partial L}{\partial b} < 0$, $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$, and $\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\frac{\partial D}{\partial b} - r^B < 0$ from first order conditions and the fact that $\frac{\partial D}{\partial b} < 1$. Note that by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})] < \mathbb{E}[LH'(L,s)+H(L,s)] = LH'(L)+H(L) < 0,$$

Hence, default probability will increase. □

Case IIB

In this case, because the reserve requirement does not bind, a marginal increase in b will simply lead banks to substitute bonds for reserves. Hence, it will have no effect on L or D. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in b. Similarly, bank profits and default probability are not affected either.

Case IIIA

In this case, we have $(1-\gamma)L = (1-\delta)D' - b$, and

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.35}$$

Hence, L will decrease while D remain unchanged. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.36}$$

it will increase after an increase in b.

Similar to Case IIA, bank profits must increase, because r^B is higher so profit is higher even if bank's assets and liabilities remain the same. Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.37}$$

Take the derivative w.r.t. b to get

$$\begin{aligned} &-r^B - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial b} \\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial b} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial b}. \end{aligned} \tag{C.38}$$

Because $\frac{\partial L}{\partial b} < 0$ and by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

default probability will decrease.

Case IIIB

Similar to Case IIB, a marginal increase in b will lead banks to substitute bonds for reserves. Hence, it will have no effect on L or D. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in b. Similarly, bank profits and default probability are not affected either. □

Proof of Proposition A.3. We consider the effects of a marginal increase in i^R in all five cases.

Case I

In this case, the FOCs are

$$\frac{L}{N}H'(L) + H(L) = \frac{1}{\beta} \tag{C.39}$$

and

$$r^D(D) + \frac{D}{N}r^{D\prime}(D) = \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}. \tag{C.40}$$

Hence, L will remain unchanged while D will increase. Since $r^B = 1/\beta$, it is remains unchanged.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b^B}{N}\frac{1}{\beta} - \frac{D}{N}r^D(D)\right]. \tag{C.41}$$

Since i^R is higher, bank profits is higher even if bank's assets and liabilities remain the same. Hence, bank profits must be higher.

Case IIA

In this case, we have $D = ((1-\gamma)L + b)/(1-\delta)$ and

$$\frac{L}{N}H'(L) + H(L) + \frac{(1-\gamma)\delta(1+i^R)}{(1-\delta)(1+\mu)} = \frac{1-\gamma}{1-\delta}\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] + \frac{\gamma}{\beta}. \tag{C.42}$$

Hence, if L is held constant, the left-hand side will increase. This means that L must increase, which means D will increase as well. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.43}$$

it will decrease after an increase in i^R.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b}{N}r^B - \frac{D}{N}r^D(D)\right]. \tag{C.44}$$

Although i^R is higher, r^B is lower. Hence, the overall effect on bank profits is ambiguous.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.45}$$

Take the derivative w.r.t. i^R to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\left[\frac{1-\gamma}{1-\delta}\frac{\partial L}{\partial i^R}\right] - \frac{\delta D}{1+\mu} - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial i^R}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial i^R} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial i^R}.\end{aligned} \tag{C.46}$$

Note that $\frac{\partial L}{\partial i^R} > 0$, $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$, and $\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\left[\frac{1-\gamma}{1-\delta}\frac{\partial L}{\partial i^R}\right] > 0$. Also, $\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]$ is negative. Hence, the overall effect is ambiguous.

Case IIB

In this case, the FOCs are

$$r^D(D) + \frac{D}{N}r^{D\prime}(D) = \frac{1+i^R}{1+\mu}, \tag{C.47}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.48}$$

Hence, L will decrease while D will increase. Since $r^B = \frac{1+i^R}{1+\mu}$, it will increase following an increase in i^R.

bank profits is given by

$$-\frac{E}{N+1} + \beta\left[\frac{L}{N+1}r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N+1}\frac{1+i^R}{1+\mu} + \frac{b}{N+1}\frac{1+i^R}{1+\mu} - \frac{D}{N+1}r^D(D)\right]. \tag{C.49}$$

Similar to Case I, since i^R is higher, bank profits is higher even if bank's assets and liabilities remain the same. Hence, bank profits must be higher.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D-(1-\gamma)L)}{1+\mu}. \tag{C.50}$$

Take the derivative w.r.t. i^R to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{1+i^R}{1+\mu}\right]\frac{\partial D}{\partial i^R} - \frac{D-(1-\gamma)L}{1+\mu} + \frac{(1+i^R)(1-\gamma)}{1+\mu}\frac{\partial L}{\partial i^R}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial i^R} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial i^R}.\end{aligned} \tag{C.51}$$

Note that $\frac{\partial L}{\partial i^R} < 0$, $\frac{\partial D}{\partial i^R} > 0$, $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$, and $\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{1+i^R}{1+\mu}\right]\frac{\partial D}{\partial i^R} > 0$. Also, $\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]$ is negative. Hence, the overall effect is ambiguous.

Case IIIA

In this case, we have $(1-\gamma)L = (1-\delta)D' - b$, and

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.52}$$

Then a marginal increase in i^R has no effect on L or D. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.53}$$

it will also remain unchanged. However, since i^R is larger, profit will be higher. Finally, since D and L remain unchanged, a higher i^R lowers the default probability.

Case IIIB

In this case, we have

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.54}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.55}$$

Hence, L will decrease while D remains unchanged. Since $r^B = \frac{1+i^R}{1+\mu}$, it will increase.

Similar to Case IIB, since i^R is higher, bank profits is higher even if bank's assets and liabilities remain the same. Hence, bank profits must be higher.

Finally, consider bank default probability $1-G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D-(1-\gamma)L)}{1+\mu}. \tag{C.56}$$

Take the derivative w.r.t. i^R to get

$$\begin{aligned} &-\frac{D-(1-\gamma)L}{1+\mu} + \frac{(1+i^R)(1-\gamma)}{1+\mu}\frac{\partial L}{\partial i^R} \\ =&G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial i^R} + \left\{\int_{\hat{s}}^{\bar{s}} [LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})]\right\}\frac{\partial L}{\partial i^R}. \end{aligned} \tag{C.57}$$

Note that $\frac{\partial L}{\partial i^R} < 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}} [LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})] < \mathbb{E}[LH'(L,s)+H(L,s)] = LH'(L)+H(L) < 0,$$

Hence, default probability will decrease. □

Proof of Proposition A.4. We consider the effects of a marginal increase in δ in all five cases.

Case I

In this case, the FOCs are

$$\frac{L}{N}H'(L) + H(L) = \frac{1}{\beta} \tag{C.58}$$

and

$$r^D(D) + \frac{D}{N}r^{D\prime}(D) = \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}. \tag{C.59}$$

Hence, L will remain unchanged while D will decrease because $\frac{1+i^R}{1+\mu} < \frac{1}{\beta}$. Since $r^B = 1/\beta$, it is unaffected by the increase in δ.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1-P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b^B}{N}\frac{1}{\beta} - \frac{D}{N}r^D(D)\right]. \tag{C.60}$$

Since banks could have chosen a higher reserve ratio if it was profit maximizing, bank profits must decrease.

Case IIA

In this case, we have $D = ((1-\gamma)L + b)/(1-\delta)$ and

$$\frac{L}{N}H'(L) + H(L) + \frac{(1-\gamma)\delta(1+i^R)}{(1-\delta)(1+\mu)} = \frac{1-\gamma}{1-\delta}\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] + \frac{\gamma}{\beta}, \tag{C.61}$$

which can be rewritten as

$$(1-\gamma)\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] = (1-\delta)\left[\frac{L}{N}H'(L) + H(L) - \frac{\gamma}{\beta}\right] + \frac{(1-\gamma)\delta(1+i^R)}{1+\mu}. \tag{C.62}$$

Note that in Case IIA,

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta} > \frac{1+i^R}{1+\mu}. \tag{C.63}$$

Hence, increasing δ will make the right-hand side of (C.62) smaller and the left-hand side larger. This means that L must decrease. The effect on D, however, is ambiguous. Since L will decrease, r^B will increase.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1-P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b}{N}r^B - \frac{D}{N}r^D(D)\right]. \tag{C.64}$$

If r^B remains unchanged, then a higher δ will lower bank profits because banks could have chosen a higher reserve ratio if it was profit maximizing. However, r^B is also higher. Hence, the overall effect on bank profits is ambiguous.

Finally, consider bank default probability $1-G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.65}$$

Take the derivative w.r.t. δ to get

$$\begin{aligned} &\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\left[\frac{1-\gamma}{1-\delta}\frac{\partial L}{\partial \delta} + \frac{D}{1-\delta}\right] - \frac{(1+i^R)D}{1+\mu} - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial \delta} \\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \delta} + \left\{\int_{\hat{s}}^{\bar{s}} [LH_L(L,s)+H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s})+H(L,\hat{s})]\right\}\frac{\partial L}{\partial \delta}. \end{aligned} \tag{C.66}$$

Note that $\frac{\partial L}{\partial \delta} < 0$, $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$, and $D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu} > 0$. Also, $\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]$ is negative. Hence, the overall effect is ambiguous.

Case IIB

In this case, because reserve requirement does not bind, a marginal increase in δ has no effect on L or D. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in δ. Similarly, bank profits and default probability are not affected either.

Case IIIA

In this case, we have $(1-\gamma)L = (1-\delta)D' - b$, and

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.67}$$

Then increasing δ will decrease L but have no effect on D. Since L will decrease, r^B will increase. Similar to Case IIA, the higher r^B will increase bank profits while the higher δ will decrease bank profits. Hence, the overall effect is ambiguous.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.68}$$

Take the derivative w.r.t. δ to get

$$\begin{aligned}&-\frac{(1+i^R)D}{1+\mu} - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial \delta}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \delta} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial \delta}.\end{aligned} \tag{C.69}$$

Note that $\frac{\partial L}{\partial \delta} < 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Hence, default probability will decrease.

Case IIIB

Similar to Case IIB, because reserve requirement does not bind, a marginal increase in δ has no effect on L or D. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in δ. Similarly, bank profits and default probability are not affected either. □

Proof of Proposition A.5. We consider the effects of a marginal increase in γ in all five cases.

Case I

In this case, because capital requirement does not bind, a marginal increase in γ has not effects on L or D. Since $r^B = 1/\beta$, it is unaffected by the increase in γ. Similarly, bank profits is not affected either.

Case IIA

In this case, we have $D = ((1-\gamma)L + b)/(1-\delta)$ and

$$\frac{L}{N}H'(L) + H(L) + \frac{(1-\gamma)\delta(1+i^R)}{(1-\delta)(1+\mu)} = \frac{1-\gamma}{1-\delta}\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] + \frac{\gamma}{\beta}, \tag{C.70}$$

which can be rewritten as

$$(1-\gamma)\left[r^D(D) + \frac{D}{N}r^{D\prime}(D)\right] + \frac{\gamma(1-\delta)}{\beta} - \frac{\delta(1-\gamma)(1+i^R)}{1+\mu} = (1-\delta)\left[\frac{L}{N}H'(L) + H(L)\right]. \tag{C.71}$$

Then, we have

$$\frac{\partial L}{\partial \gamma} = \frac{r^D(D) + \frac{D}{N}r^{D\prime}(D) - \frac{1-\delta}{\beta} - \frac{\delta(1+i^R)}{1+\mu} + \frac{(1-\gamma)L}{1-\delta}\left[r^{D\prime}(D) + \frac{1}{N}r^{D\prime}(D) + \frac{D}{N}r^{D\prime\prime}(D)\right]}{(1-\gamma)^2\left[r^{D\prime}(D) + \frac{1}{N}r^{D\prime}(D) + \frac{D}{N}r^{D\prime\prime}(D)\right] - \frac{(1-\delta)^2}{1-\gamma}\left[\frac{1}{N}H'(L) + \frac{L}{N}H''(L) + H'(L)\right]} \tag{C.72}$$

and

$$\frac{\partial D}{\partial \gamma} = \frac{r^D(D) + \frac{D}{N}r^{D\prime}(D) - \frac{1-\delta}{\beta} - \frac{\delta(1+i^R)}{1+\mu} + \frac{(1-\delta)^2 D}{(1-\gamma)^2}\left[\frac{1}{N}H'(L) + \frac{L}{N}H''(L) + H'(L)\right]}{(1-\gamma)\left[r^{D\prime}(D) + \frac{1}{N}r^{D\prime}(D) + \frac{D}{N}r^{D\prime\prime}(D)\right] - (1-\delta)\left[\frac{1}{N}H'(L) + \frac{L}{N}H''(L) + H'(L)\right]} \tag{C.73}$$

Note that $r^D(D) + \frac{D}{N}r^{D\prime}(D) < \frac{1-\delta}{\beta} + \frac{\delta(1+i^R)}{1+\mu}$ in Case IIA. Hence, $\frac{\partial D}{\partial \gamma} < 0$. As for $\frac{\partial L}{\partial \gamma}$, assume $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$. Consider function $f(D)$.

$$\begin{aligned}f(D) &= r^D(D) + \frac{D}{N}r^{D\prime}(D) + \frac{(1-\gamma)L}{1-\delta}\left[r^{D\prime}(D) + \frac{1}{N}r^{D\prime}(D) + \frac{D}{N}r^{D\prime\prime}(D)\right]\\ &= \beta^{\frac{1}{\sigma-1}}\left(\frac{D}{\eta}\right)^{\frac{\sigma}{1-\sigma}}\frac{N-\sigma N+\sigma}{N-\sigma N}\frac{1+(\epsilon-1)\sigma}{1-\sigma},\end{aligned} \tag{C.74}$$

where $\epsilon = 1 - b/[(1-\delta)D]$. It is easy to see that $f'(D) > 0$. Note also that in Case IIA, $D > D^\dagger$ where $D^\dagger$ solves $r^D(D) = \frac{1}{1+\mu}$ Hence, $f(D) \geq f(D^\dagger)$, which is given by

$$f(D^\dagger) = \frac{1}{1+\mu}\frac{N - \sigma N + \sigma}{N - \sigma N}\frac{1 + (\epsilon - 1)\sigma}{1-\sigma}. \tag{C.75}$$

Then, $f(D) > 1/\beta$ as long as $\frac{1+(\epsilon-1)\sigma}{(1+\mu)(1-\sigma)} > \frac{1}{\beta}$. In such case, we have $\frac{\partial L}{\partial \gamma} > 0$. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.76}$$

r^B will decrease.

bank profits is given by

$$-\frac{E}{N} + \beta\left[\frac{L}{N}r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N}\frac{1+i^R}{1+\mu} + \frac{b}{N}r^B - \frac{D}{N}r^D(D)\right]. \tag{C.77}$$

If r^B remains unchanged, then a higher γ will lower bank profits because banks could have chosen a higher capital ratio if it was profit maximizing. If L increases, then r^B will be lower. Hence, bank profits must decrease.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.78}$$

Take the derivative w.r.t. γ to get

$$\begin{aligned}&\left[D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu}\right]\frac{\partial D}{\partial \gamma} - \frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial \gamma}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \gamma} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial \gamma}.\end{aligned} \tag{C.79}$$

Note that $\frac{\partial L}{\partial \gamma} > 0$ if $u(q) = \frac{q^{1-\sigma}-1}{1-\sigma}$ and $\frac{1+(\epsilon-1)\sigma}{(1+\mu)(1-\sigma)} > \frac{1}{\beta}$, $\frac{\partial D}{\partial \gamma} < 0$, $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$, and $D\hat{r}^{D\prime}(D) + \hat{r}^D(D) - \frac{(1+i^R)\delta}{1+\mu} > 0$. However, $\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]$ is negative. Hence, the overall effect is ambiguous.

Case IIB

In this case, the FOCs are

$$r^D(D) + \frac{D}{N}r^{D\prime}(D) = \frac{1+i^R}{1+\mu}, \tag{C.80}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.81}$$

Hence, L will decrease while D remains unchanged. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in γ.

bank profits is given by

$$-\frac{E}{N+1} + \beta\left[\frac{L}{N+1}r^L(L)[1 - P(r^L(L))] + \frac{z^B}{N+1}\frac{1+i^R}{1+\mu} + \frac{b}{N+1}\frac{1+i^R}{1+\mu} - \frac{D}{N+1}r^D(D)\right]. \tag{C.82}$$

Since banks could have raised more equity if it is profit maximizing, bank profits must decrease.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D - (1-\gamma)L)}{1+\mu}. \tag{C.83}$$

Take the derivative w.r.t. γ to get

$$\begin{aligned}&-\frac{(1+i^R)L}{1+\mu} + \frac{(1+i^R)(1-\gamma)}{1+\mu}\frac{\partial L}{\partial \gamma}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \gamma} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial \gamma}.\end{aligned} \tag{C.84}$$

Note that $\frac{\partial L}{\partial \delta} < 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Hence, default probability will decrease.

Case IIIA

In this case, we have $(1-\gamma)L = (1-\delta)D' - b$, and

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.85}$$

Then L will increase while D remains unchanged. Banks raise equity and invest them in loans. Since r^B is given by

$$r^B = \frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] - \frac{\gamma}{(1-\gamma)\beta}, \tag{C.86}$$

it is decreasing in both L and γ. Hence, r^B will decrease following an increase in γ. Hence, bank profits will also decrease.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)\delta D}{1+\mu} + br^B. \tag{C.87}$$

Take the derivative w.r.t. γ to get

$$\begin{aligned}&-\frac{b}{1-\gamma}\left[\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L)\right]\frac{\partial L}{\partial \gamma}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \gamma} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial \gamma}.\end{aligned} \tag{C.88}$$

Note that $\frac{\partial L}{\partial \delta} > 0$ and $\frac{H'(L)}{N} + \frac{L}{N}H''(L) + H'(L) < 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Hence, default probability will increase.

Case IIIB

In this case, the FOCs are

$$D' = \eta(u')^{-1}\left(\frac{1+\mu}{\beta}\right)(1+\mu). \tag{C.89}$$

and

$$\frac{1}{1-\gamma}\left[\frac{L}{N}H'(L) + H(L)\right] = \frac{1+i^R}{1+\mu} + \frac{\gamma}{(1-\gamma)\beta}. \tag{C.90}$$

Hence, L will decrease while D remains unchanged. Since $r^B = \frac{1+i^R}{1+\mu}$, it is unaffected by the increase in γ.

Similar to Case IIB, because banks could have raised more equity if it is profit maximizing, bank profits must decrease.

Finally, consider bank default probability $1 - G(\hat{s})$. Note that $\hat{s}$ is given by

$$Dr^D(D) = \int_{\hat{s}}^{\bar{s}} LH(L,s)\mathrm{d}G(s) + G(\hat{s})LH(L,\hat{s}) + \frac{(1+i^R)(D-(1-\gamma)L)}{1+\mu}. \tag{C.91}$$

Take the derivative w.r.t. γ to get

$$\begin{aligned}&-\frac{(1+i^R)L}{1+\mu} + \frac{(1+i^R)(1-\gamma)}{1+\mu}\frac{\partial L}{\partial \gamma}\\ &= G(\hat{s})LH_s(L,\hat{s})\frac{\partial \hat{s}}{\partial \gamma} + \left\{\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})]\right\}\frac{\partial L}{\partial \gamma}.\end{aligned} \tag{C.92}$$

Note that $\frac{\partial L}{\partial \gamma} < 0$. In addition, by Lemma C.1 we have

$$\int_{\hat{s}}^{\bar{s}}[LH_L(L,s) + H(L,s)]\mathrm{d}G(s) + G(\hat{s})[LH_L(L,\hat{s}) + H(L,\hat{s})] < \mathbb{E}[LH'(L,s) + H(L,s)] = LH'(L) + H(L) < 0,$$

Hence, default probability will decrease. □

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