Channel Hardening in RIS-Assisted Short-Packet Communication Systems

Aritra Basu, Mohsen Naseri, Leila Musavian, and Sonia Aïssa

***Abstract*—Finite blocklength information theory plays a pivotal role in the design and analysis of short-packet communications systems, especially for applications requiring ultra reliability and low latency. Recently, leveraging reconfigurable intelligent surfaces (RISs) for such type of communications has proven to be very promising. However, investigating the channel hardening in such systems remains an open issue. In this context, this paper examines the channel hardening in RIS-assisted communication with finite blocklength. After investigating the properties of the achievable rate term, the paper analyzes the scaling law of the signal-to-noise power ratio with respect to the RIS size, and introduces a metric for quantifying the channel hardening. Results demonstrate the presence of hardening, and illustrate the effects of the main system parameters on the hardening property.**

I. Introduction

The advent of beyond 5G systems has led the way for seamless and reliable data transmission, surpassing the capabilities of previous-generation technologies. These systems aim to support cutting-edge applications, including applications that demand ultra-reliable low-latency communications (URLLC), where latency should be reduced to an unprecedented 1ms while simultaneously providing reliability in the 99.9% range [1]. This stringent requirement, which is crucial in mission-critical applications, e.g., industrial automation and autonomous vehicles [2], necessitates the use of short-packet communication (SPC) [3]. In SPC, the data packets are finite, in contrast to the long blocklength in traditional communication systems. However, relying solely on conventional technologies, such as multiple-input multiple-output (MIMO), may not adequately meet the stringent quality-of-service (QoS) of SPC applications, especially when the line(s) of sight between the transmitter(s) and receiver(s) are obstructed.

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To meet the QoS demands, recent research efforts explored the use of reconfigurable intelligent surfaces (RISs) [4], [5]. While the integration of RISs in SPC has promising advantages from a performance viewpoint, the overhead in estimating and acquiring channel state information (CSI) is a major issue. To alleviate the instantaneous CSI requirements, the potential hardening of the channels, i.e., deterministic behavior, can be leveraged. In fact, the hardening allows the use of statistical CSI, which reduces overhead and allows an efficient management of the resources in RIS-aided SPC systems [6].

By harnessing a multitude of passive reflectors controlled by integrated circuits, RISs can intelligently manipulate their incident waves, which can help in transforming the propagation environment from highly probabilistic to partially deterministic [7]. In the state of the art, recent studies underscored the potential of RISs in realizing channel hardening [8]. Investigations demonstrated the presence of channel hardening in diverse scenarios, ranging from single-input single-output (SISO) configurations to environments with multiple RISs and Rician fading channels. In particular, the work in [9] and [10] explored the impact of the RIS size and the fading severity on channel hardening, and the findings in [10] revealed that the channels of a MIMO communication system aided with passive RIS tend to harden, with the degree of hardening influenced by the physical dimension of the RIS.

In the realm of SPC, significant efforts were directed toward evaluating the reliability performance, in terms of block error rate (BLER), while fewer studies delved into analyzing the achievable rate due to the inherent complexity. For instance, [11] presented asymptotic and approximate expressions for the average BLER in MIMO SPC systems employing non-orthogonal multiple access. In [12], the authors derived approximate expressions for the BLER in relay-aided MIMO SPC systems, considering various transmit/receive diversity techniques. The performance of MIMO SPC over Rayleigh fading channels was also investigated in [13], where a tight lower bound on the average achievable rate was derived. The work in [14], on the other hand, explored the trade-off between the transmission rate,

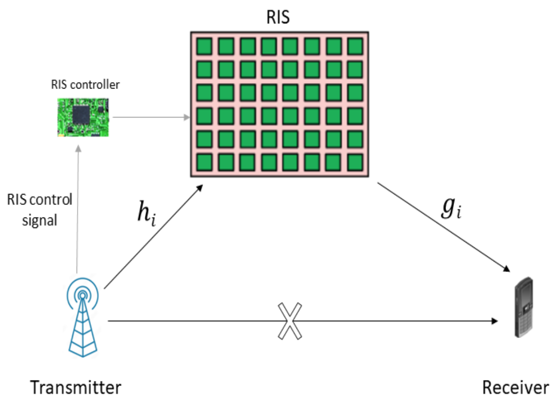

Fig. 1: The RIS-aided SPC model.

the decoding error probability, and the transmission latency in finite blocklength regime. In [15], the error probability and throughput of a point-to-point MIMO communication system with isotropically distributed codewords was evaluated, assuming no CSI at the transmitter. Therein, the authors considered a simplified version of the SPC rate dispersion term.

Despite the extensive research in the areas of RIS-assisted communications and SPC, there remains a gap in the literature regarding the analysis of the achievable rates and channel hardening properties in RIS-assisted systems employing short packets. While [8] investigated the channel hardening in SISO RIS-aided communication with infinite blocklength, no prior work has tackled RIS-assisted communications with finite blocklength from the viewpoint of channel hardening. This paper fills this gap by investigating the channel hardening property of RIS-aided SPC and the conditions under which the hardening holds. Assuming a SISO antenna model in this communication system, the paper proves that the channel hardens for a given error probability, packet blocklength, and channel dispersion, and analyzes the impact of the main system parameters on the hardening property.

The following content of the paper is organized as follows. First, section II describes the SPC model, and formulates the problem under consideration. In section III, we investigate the channel hardening properties of the communication system. Numerical results and discussions are provided in section IV, followed by concluding remarks in section V.

II. System and Channel Models

The RIS-aided SPC system under study is illustrated in Fig. 1. The system is assumed to operate in an isotropic scattering environment, where the direct path between the transmitter and the receiver is blocked and, hence, the information transfer takes place through the RIS. The latter is a uniform planar array consisting of N elements aligned in the yz-plane, with element spacing d_y and d_z along the y and z axes, respectively.

Accordingly, the received signal at the end-user receiver can be expressed as

$$y = \sqrt{P_{\rm t}} \sum_{i=1}^{N} h_i \exp\{\jmath\phi_i\} g_i x + w\,, \tag{1}$$

where $h_i = \alpha_{h_i} \exp\{-\jmath\theta_{h_i}\}$ denotes the channel coefficient between the transmitter and RIS element i, the parameter $g_i = \alpha_{g_i} \exp\{-\jmath\theta_{g_i}\}$ is the channel coefficient between the i-th RIS element and the receiver, ϕ_i represents the phase shift introduced for the reflection from the i-th element of the metasurface, x denotes the transmitted signal, and w represents the additive white Gaussian noise (AWGN) encountered by the system, which follows a complex Gaussian distribution with zero mean and variance σ^2, i.e., $w \sim \mathcal{CN}(0, \sigma^2)$. The envelope or magnitude of the channel coefficients, α_{h_i} and α_{g_i}, are assumed to be independent but not identically distributed (i.n.i.d.) according to a Nakagami model with the same shape parameter m and different spread parameters, namely, Ω_{h_i} and Ω_{g_i}, respectively. The phases θ_{h_i} and θ_{g_i} are uniformly distributed.

Considering the availability of real-time CSI at the RIS [9], the phase delay of the i-th RIS element can be altered at any given point of time according to $\phi_i = \theta_{h_i} + \theta_{g_i}$. Thus, the received signal will be

$$y = \sqrt{P_{\rm t}} \sum_{i=1}^{N} \alpha_{h_i} \alpha_{g_i} x + w. \tag{2}$$

Therefore, the end-to-end signal-to-noise power ratio (SNR) can be expressed as

$$\gamma = \rho \left(\sum_{i=1}^{N} \alpha_{h_i} \alpha_{g_i} \right)^2, \tag{3}$$

where $\rho = \frac{P_{\rm t}}{\sigma^2}$ is the transmit SNR.

To infer the statistical properties of the end-to-end SNR shown in (3), we follow two identities, namely, $\mathrm{E}[\alpha_{h_i}\alpha_{g_i}] = \mathrm{E}[\alpha_{h_i}]\mathrm{E}[\alpha_{g_i}]$ and $\mathrm{Var}[\alpha_{h_i}\alpha_{g_i}] = \mathrm{Var}[\alpha_{h_i}]\mathrm{Var}[\alpha_{g_i}] + \mathrm{Var}[\alpha_{h_i}]\mathrm{E}^2[\alpha_{g_i}] + \mathrm{E}^2[\alpha_{h_i}]\mathrm{Var}[\alpha_{g_i}]$, which are based on the statistical independence of the random variables α_{h_i} and α_{g_i}, with $\mathrm{E}[\cdot]$ representing the expectation operator and $\mathrm{Var}[\cdot]$ indicating the variance operator.

Now, assuming large RIS size, i.e., $N \gg 1$, and utilizing the central limit theorem (CLT), $\sqrt{\gamma}$ can be modeled as a Gaussian with mean and variance given by [16]

$$\mu_{\sqrt{\gamma}} = N\frac{\sqrt{\Omega_{h_i}\Omega_{g_i}}}{m}\frac{\Gamma^2(m+\frac{1}{2})}{\Gamma^2(m)}\sqrt{\rho}, \tag{4}$$

$$\sigma^2_{\sqrt{\gamma}} = \rho N \Omega_{h_i}\Omega_{g_i}\left[1-\left(\frac{\Gamma^2(m+\frac{1}{2})}{m\Gamma^2(m)}\right)^2\right], \tag{5}$$

where $\Gamma(\cdot)$ is the Gamma function. Therefore, the end-to-end SNR, i.e., γ, follows a non-central chi-squared distribution.

III. Channel Hardening

A. Achievable Rate Analysis

For a finite blocklength communication between a single-antenna transmitter and a single-antenna receiver, the achievable rate can be expressed as

$$R(n,\epsilon) \simeq C(\gamma) - \sqrt{\frac{V(\gamma)}{n}}Q^{-1}(\epsilon), \tag{6}$$

where n is the blocklength, ϵ denotes the BLER, $C(\cdot)$ is the capacity term, $V(\cdot)$ represents the dispersion term, and $Q^{-1}(\cdot)$ denotes the inverse Q-function [3].[1] The terms $C(\gamma)$ and $V(\gamma)$ are given by

$$C(\gamma) = \log(1+\gamma), \tag{7}$$

$$V(\gamma) = 1 - \frac{1}{(1+\gamma)^2}. \tag{8}$$

To investigate the presence of channel hardening in the RIS-aided SPC under study, one can rely on the analysis of the probability density function (PDF) of the achievable rate defined in (6). However, due to the complicated nature of the random variable $R(n,\epsilon)$, deriving an exact expression for this PDF is highly intractable. As an alternative, we use approximations to obtain a closed-form representation of the said PDF. Specifically, we approximate the dispersion term $V(\gamma)$ as unity for cases with high values of γ, which can result from operations with large sizes of the metasurface and/or high transmit SNRs. Accordingly, by utilizing the simplified form of the dispersion $V(\gamma)$, we can statistically characterize the achievable rate as

$$\mathrm{E}[R] = \mathrm{E}[C] - \beta, \qquad \mathrm{Var}[R] = \mathrm{Var}[C], \tag{9}$$

where β is a constant given by $\frac{Q^{-1}(\epsilon)}{\sqrt{n}}$.

Based on the above, we observe a change in the mean of the random variable R with respect to the random variable C. Thus, the rate R can be well–approximated as a Gaussian distribution in terms of C,[2] with mean and variance given by

$$\mathrm{E}[R] = \log(1+\rho\nu) - \beta, \tag{10a}$$

$$\sqrt{\mathrm{Var}[R]} = \frac{\rho\log_2 \mathrm{e}}{1+\rho\nu}\sqrt{\varrho(\varpi+1)}, \tag{10b}$$

in which $\nu = \frac{1}{1+\zeta}A_N^2\left(\zeta N^2 + \sum_{i=1}^N \alpha_{g_i}^2\right)$, $\varrho = \frac{1}{1+\zeta}A_N^2\sum_{i=1}^N\alpha_{g_i}^2$, $\varpi = \frac{1}{1+\zeta}A_N^2\left(2\kappa_{\mathrm{r}}N^2 + \sum_{i=1}^N\alpha_{g_i}^2\right)$, and $\zeta = \frac{\sqrt{m-1}}{\sqrt{m}-\sqrt{m-1}}$, with $A_N = d_y d_z$ representing the area of a RIS element.

B. Hardening Metric

Inspired by the definition of channel hardening in a RIS-aided SISO communication with infinite blocklength [9], we introduce a metric for the rate hardening in RIS-assisted SPC subjected to Nakagami fading, namely,

$$\iota = \frac{\mathrm{E}[R]}{\mathrm{Var}[R]}, \tag{11}$$

where $\mathrm{E}[R]$ represents the mean and $\mathrm{Var}[R]$ is the variance of the achievable rate. The rate of change/growth of this metric with various system parameters can provide conclusive evidence of the hardening, which can be further quantified. Given the analytical intractability of the rate term, Monte-Carlo simulations will be employed in section IV to investigate the impact of the finite blocklength on the hardening property of the system under study.

Recalling the expressions in (10a) in (10b), we observe that the mean scales as $\mathcal{O}(\log N)$ while the variance scales as $\mathcal{O}(N^{-1})$.[3] Consequently, the ratio of $\mathrm{E}[R]/\mathrm{Var}[R]$ tends to a large number as $N \to \infty$ with diminished growth rate, providing preliminary evidence of channel hardening and proving the usefulness of the metric proposed metric shown in 11. Here, it must be emphasized that these results are attributed to the high values of γ, which may not hold true for all cases. Next, we revert to the use of the scaling theory of RIS to assess the trend and draw insights for general cases.

[1]The blocklength n is measured in channel use, the achievable rate R and the capacity C are measured in bits per channel use.

[2]This implies that the term $\frac{R-\mathrm{E}(R)}{\mathrm{Var}(R)}$ converges to a Gaussian distribution with zero mean and unit variance asymptotically, i.e., $\mathcal{N}(0,1)$, [10].

[3]The proof is a straightforward substitution of parameter values in terms of N in (10a) and (10b) to obtain the scaling factors with respect to the RIS size.

C. SNR Scaling with the RIS Size

In the RIS-assisted communication system employing short packets for the delivery of information in an isotropic scattering environment with i.n.i.d. Nakagami fading channels, when $N \rightarrow \infty$ the end-to-end SNR γ will scale with the square of the number of RIS elements, i.e., N^2. Thus, the dispersion term being a function of the random variable γ, it will also be affected as does the achievable rate R. As such, the formulae (7) and (8) can be re-written as

$$C(\gamma) = \log(1 + N^2 k), \tag{12}$$

$$V(\gamma) = 1 - \frac{1}{(1 + N^2 k)^2}, \tag{13}$$

where k is a constant given by $k = \frac{\rho A_N \Omega_{h_i} \Omega_{g_i} \Gamma^2(m+1/2)}{m\Gamma^2(m)}$.

The above proposition will be verified using Monte-Carlo simulations, as detailed in section IV.

IV. Validation and Discussion

In this section, we validate the channel hardening property using Monte-Carlo simulations with the ensemble average technique over 10^7 channel realizations, and analyze the effects of the transmit power, the RIS size, the Nakagami shape parameter, and the blocklength.

A. Effects of the Transmit Power

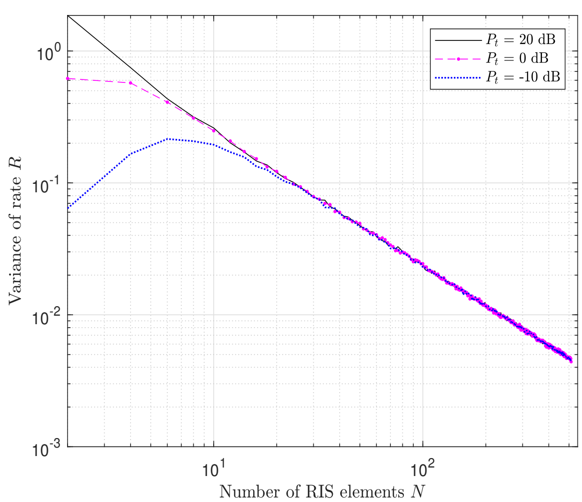

Fig. 2: Variance of the rate vs. the number of RIS elements, in logarithmic scale.

In Fig. 2, the variance of the achievable rate of the RIS-aided SPC system is depicted to examine the deterministic nature of the channel. For a fixed level of transmit power, we observe that with an increase in the number of RIS elements that assist in the information transfer, the channel becomes more and more deterministic, which is established by the fact that the achievable rate obtained by the system shows negligible variation with an increase in the RIS size. Here, it is important to highlight that while perturbations are noticeable at low values of P_{t} and N, they diminish as N increases.

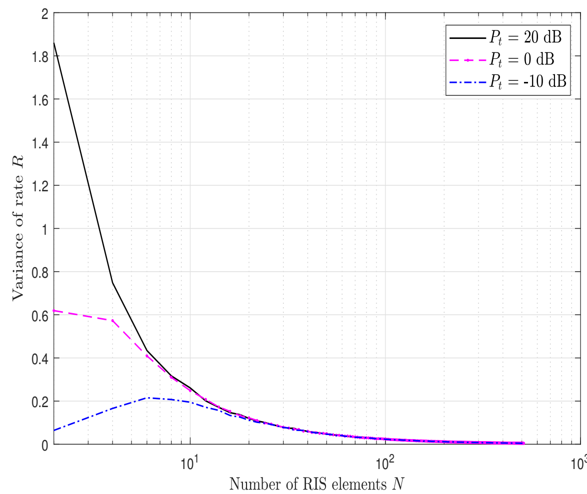

Fig. 3: Variance of the rate vs. the number of RIS elements.

Figure 3 shows the variance of the achievable rate of the SPC system in linear scale. As observed, the slope of the variance of the achievable rate decreases with an increase in the number of RIS elements N. In other words, in a SPC system with a small RIS size and low transmit power, the channel becomes less deterministic and rather more probabilistic. Conversely, for larger vales of N, the channel becomes more deterministic regardless of the transmit power, thus corroborating the proposition stated in section III-C with regard to the SNR scaling with the RIS size.

B. Effects of the Blocklength

The impact of the blocklength is depicted in Fig. 4, where minimal to no effect is observed on the variance of the achievable rate. In the figure, 'cu' means channel use. Consequently, it can be inferred that the influence of the blocklength in the SPC system under study is constrained, as the achieved rate hardens analogous to the hardening behavior of the channel observed in [4] for MIMO systems.

Figure 5 shows the changes in the variance of the achievable rate R as the transmit power P_{t}, or,

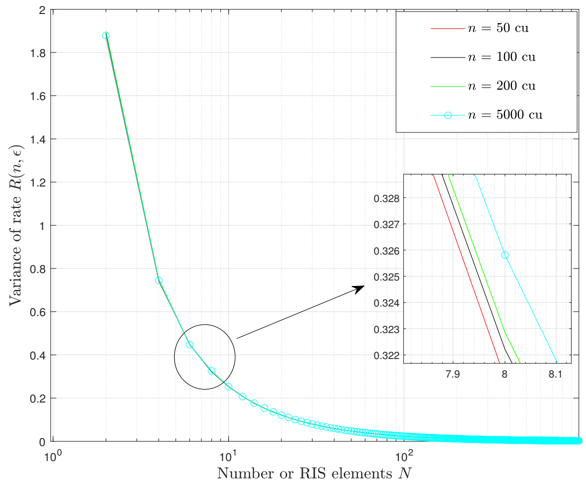

Fig. 4: Variance of the rate vs. the number of RIS elements for different values of the blocklength n.

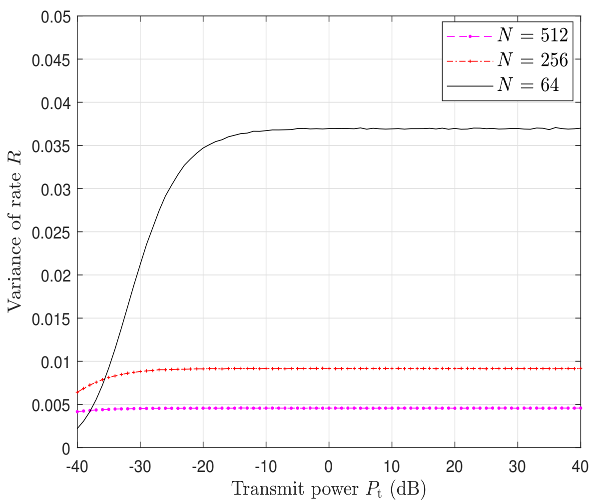

Fig. 5: Variation of the rate as a function of the transmit power. Here, $\sigma^2 = 1$.

equivalently, the transmit SNR ρ (given that the noise variance $\sigma^2 = 1$), is varied, while maintaining other system parameters, such as N, fixed. From the figure, noticeable fluctuations in the variance are observed at low SNR levels, predominantly for smaller values of N, which tend to stabilize at high SNRs. Notably, within the typical range of SNR values, a deterministic behavior of the rate term is observed, which is characterized by a low variance.

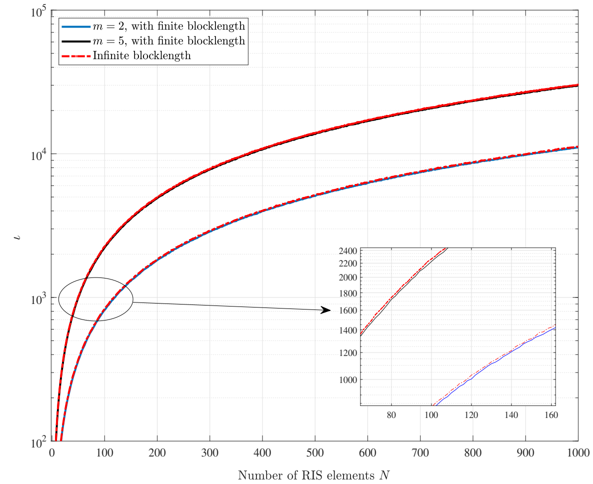

Fig. 6: Hardening metric vs. the number of RIS elements for $n = 100$.

C. Metric Evaluation

Figure 6 illustrates the hardening metric ι, cf. Eq. (11), versus the RIS size. Here, the effect of the Nakagami shape parameter m on the channel hardening process is depicted. As observed, the channel becomes more deterministic with an increase in the shape parameter, which can be attributed to the fact that the randomness of the channel is correlated with the severity of the fading.

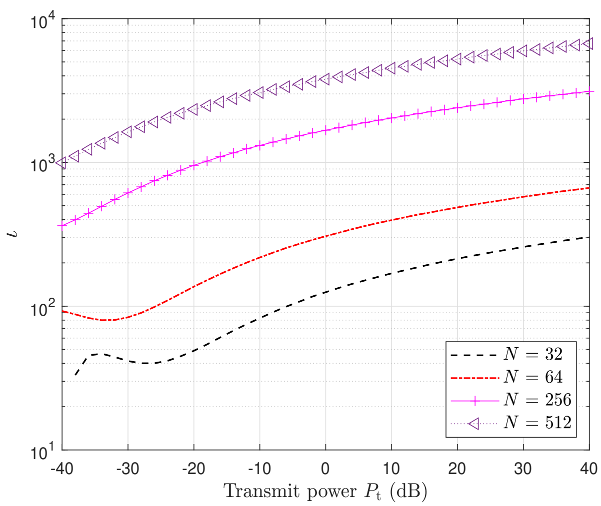

Fig. 7: Hardening metric vs. the transmit power for different RIS sizes.

Figures 7 and 8 illustrate the hardening metric versus the transmit power and the RIS size, respectively.

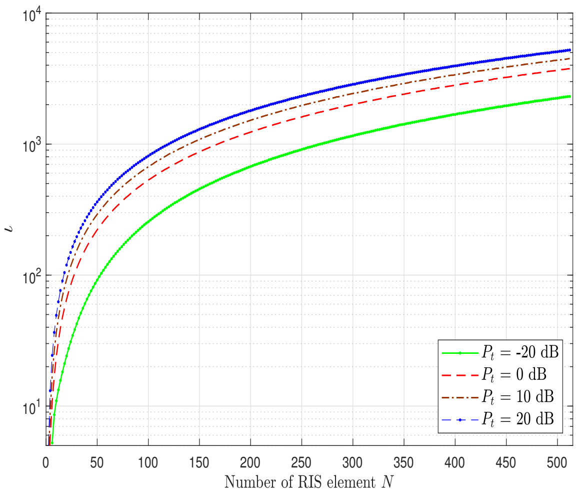

Fig. 8: Hardening metric vs. the RIS size for different transmit powers.

In this experiment, the blocklength n is set to 100. As observed, the metric is more stable at high values of N, and the effect of the transmit power P_{t} is only limited to lower values of N. Conversely, it can be inferred that ι is more suitable for observing the rate hardening with respect to N compared to P_{t}, though the effect of the variation in P_{t} can be observed. Furthermore, the channel becomes more deterministic with an increase in N compared to an increase in P_{t} in the RIS-aided SPC system.

V. Conclusion

In this paper, we investigated the channel hardening in RIS-assisted SPC systems. The results showed that as the number of RIS elements increases, the channel becomes more deterministic and the variance of the achievable rate decreases. The analysis also showed that the Nakagami fading parameter plays a significant role in the hardening process, with higher values leading to a more deterministic channel. The effects of the transmit power and the RIS size show the dominance of the latter, highlighting the importance of the transmit power for SPC systems employing smaller RISs. These findings highlight the potential of RIS-assisted systems in achieving high reliability and low latency by operating close to the Shannon rate with SPC. Particularly, the channel hardening property confirmed in this study can be exploited to simplify the design of RIS-assisted SPC systems and improve the resource allocation mechanisms for achieving the stringent requirements of URLLC applications. Ongoing work includes the extension of the study to MIMO configurations.

References

[1] J. Sachs *et al.*, "5G radio network design for ultra-reliable low-latency communication," *IEEE Netw.*, vol. 32, no. 2, pp. 24–31, 2018.

[2] G. T. 22.104, "Service requirements for cyber-physical control applications in vertical domains," 2019.

[3] Y. Polyanskiy *et al.*, "Channel coding rate in the finite blocklength regime," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2307–2359, 2010.

[4] J. Östman *et al.*, "URLLC with massive MIMO: Analysis and design at finite blocklength," *IEEE Trans. Wirel. Commun.*, vol. 20, no. 10, pp. 6387–6401, 2021.

[5] M. Almekhlafi *et al.*, "Joint scheduling of eMBB and URLLC services in RIS-aided downlink cellular networks," in *Proc. Intnl. Conf. Comp. Commun. Netw. (ICCCN)*, 2021, pp. 1–9.

[6] P. Harris *et al.*, "Performance characterization of a real-time massive MIMO system with LOS mobile channels," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1244–1253, 2017.

[7] C. Liaskos *et al.*, "A new wireless communication paradigm through software-controlled metasurfaces," *IEEE Commun. Mag.*, vol. 56, no. 9, pp. 162–169, 2018.

[8] E. Björnson and L. Sanguinetti, "Rayleigh fading modeling and channel hardening for reconfigurable intelligent surfaces," *IEEE Wireless Commun. Lett.*, vol. 10, no. 4, pp. 830–834, 2021.

[9] H. Ibrahim *et al.*, "Exact coverage analysis of intelligent reflecting surfaces with Nakagami-m channels," *IEEE Trans. Veh. Technol.*, vol. 70, no. 1, pp. 1072–1076, 2021.

[10] A. Bereyhi *et al.*, "Channel hardening of IRS-aided multi-antenna systems: How should IRSs scale?" *IEEE J. Sel. Areas Commun.*, vol. 41, no. 8, pp. 2321–2335, 2023.

[11] D.-D. Tran *et al.*, "Short-packet communications for MIMO NOMA systems over Nakagami-m fading: BLER and minimum blocklength analysis," *IEEE Trans. Veh. Technol.*, vol. 70, no. 4, pp. 3583–3598, 2021.

[12] N. H. Tu and K. Lee, "Performance analysis and optimization of multihop MIMO relay networks in short-packet communications," *IEEE Trans. Wirel. Commun.*, vol. 21, no. 6, pp. 4549–4562, 2022.

[13] J. Zheng *et al.*, "Average achievable rate and average BLER analyses for MIMO short-packet communication systems," *IEEE Trans. Veh. Technol.*, vol. 70, no. 11, pp. 12 238–12 242, 2021.

[14] X. Sun *et al.*, "Short-packet downlink transmission with non-orthogonal multiple access," *IEEE Trans. Wirel. Commun.*, vol. 17, no. 7, pp. 4550–4564, 2018.

[15] B. Makki *et al.*, "Finite block-length analysis of large-but-finite MIMO systems," *IEEE Wireless Commun. Lett.*, vol. 8, no. 1, pp. 113–116, 2019.

[16] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. Hoboken, NJ, USA: John Wiley & Sons, 2005.