Martingale Theory Based Definition and Analysis of Energy Self-Sustainability in Batteryless Internet of Things

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***Abstract*—Radio-frequency (RF) based Wireless Energy Transfer (WET) enables devices to avoid wired power-supply and battery replacements. The ubiquitous availability of RF energy elevates "energy self-sustainability" as a pivotal goal for wireless sensor networks. However, for RF energy harvesting networks, the term "energy self-sustainability" lacks a precise mathematical characterization. This paper presents a robust mathematical definition for energy self-sustainability within integrated data and energy networks. By utilizing Martingale theory, we develop a mathematical framework that determines the energy self-sustainability, by acquiring the stochastic properties of energy harvesting and consumption processes. The fundamental paradigm of utilizing this framework to derive energy self-sustainability is demonstrated. In-depth explorations have been conducted on the diverse stochastic characteristics of energy harvesting and consuming processes. Additionally, this study delves into the anticipated uninterrupted operating duration and associated energy expectation. Monte-Carlo simulations confirm the precision of our theoretical evaluations. By analyzing the correlation between harvested and consumed energy in the context of varied energy self-sustainability requirements, this paper provides design guidance for energy transmitters and batteryless wireless devices.**

***Index Terms*—Energy self-sustainability, batteryless internet of things, wireless energy transfer, martingale theory, energy queue, access control analysis.**

I. Introduction

A. Background and Motivation

Recent advancements in Wireless Energy Transfer (WET) technology [1] extend the operational duration of Batteryless Wireless Devices (BWD) [1]. Contrasting ambient energy harvesting [2], RF-based wireless energy transfer, relying solely on a receiving antenna and a rectifier, can be economical and compact. Additionally, RF-based wireless energy transfer offers greater control than ambient techniques. By managing RF energy stations, BWD energy can be modulated on demand. Merging Wireless Data Transfer (WDT) with wireless energy transfer results in the Data and Energy Integrated Network (DEIN) [3], a potential cornerstone for energy self-sustainability in 6G. RF energy harvesting benefits scenarios with dense BWD deployment, such as wireless body area networks [4] and wireless sensor networks [1]. The energy efficiency and energy self-sustainability of BWDs have garnered attention across various societal domains [5].

The hardware architecture of BWDs can be categorized based on energy supply: direct-consumption and storage-consumption modes [6]. In the former, BWDs utilize energy directly from RF, dependent on a consistent RF source. Interruptions in RF supply cause BWD energy outages. Conversely, in storage-consumption mode, RF-harvested energy is initially stored in an energy storage equipment before consumption. This mode decouples harvesting and consuming processes. A BWD experiences an energy gap when stored energy falls below its consumption threshold. With the assistance of power management chips, storage-consumption BWDs' energy storage module can acquire energy from RF energy harvesting while powering data access operations for the BWDs.

Since RF energy transfer's inception, energy self-sustainability has been a wireless network aspiration [7], [8]. It is envisioned that networks operating consistently without the need for manual battery replacements, ultimately achieving energy-neutrality [9] for the BWDs. As a consequence of the unstable RF energy source, BWDs often experience power interruptions during sustained energy harvesting process and data transmission process, leading to their operation in intermittent computing mode [10]. The energy self-sustainability of BWDs should reflect the device's ability to sustain operation during such intermittent computing processes. However, the absence of a quantitative definition for energy self-sustainability, in contrast to the abundance of metrics for wireless networks, poses a significant obstacle to optimization efforts. Few works detail the definition or analytical techniques of this concept. The objective of this research is to establish a mathematical framework for quantifying the energy self-sustainability of DEIN. This framework will provide a theoretical basis for evaluating and optimizing the performance of DEIN.

Central to our analysis, energy self-sustainability mirrors the device's energy buffer status. Existing DEIN research primarily amplifies physical layer efficiency [11], [12] to bolster the energy buffer. Moreover, BWD access protocols

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[1]Devices devoid of wired power or manual battery replacement, but may contain energy storage equipment.

impact the energy buffer. Our work concentrates on the energy buffer from this access behavior perspective. Beyond energy self-sustainability, the energy buffer provides insights into other important metrics: anticipated uninterrupted operational duration and energy expectancy, which is pivotal for energy station transmission strategies.

B. Related Works

Energy self-sustainability denotes a BWD's stable functioning over time. For direct-consumption mode BWDs, a potential metric is the comprehensive outage probability that takes into account both energy supply and communication performance. Lu *et al*. [13] introduced a hybrid mode BWD that toggles between storage-consumption mode and direct-consumption mode based on RF energy availability. Their metric was the weighted probability of energy below a threshold. Shi *et al*. [14], [15] utilized joint probabilities involving harvested energy and SINR thresholds to investigate the outage performance of direct-consumption mode BWDs in cellular networks or ambient backscatter networks. Yang *et al*. [16] defined the energy efficiency of BWD as the ratio of the throughput and the consumed energy. This approach suits direct-consumption mode BWDs, but less so for storage-consumption mode BWDs that can tap into stored energy.

Sparse work discusses storage-consumption mode BWDs' energy self-sustainability. For storage-consumption mode BWDs, it is important to consider the dynamic variations in energy buffers caused by the energy harvesting and consuming processes [17], [18]. Delgado [18] conducted a Markov analysis to characterize the communication process of RF energy harvesting enhanced LoRaWAN devices. The authors proposed that by selecting appropriate configurations based on different application behaviors and environmental conditions, it is possible to achieve energy-neutral operations for LoRaWAN terminals through the utilization of cached energy in the energy buffer. Unfortunately, the author did not delve into the performance of energy self-sustainability after successfully achieving energy-neutral operation in terminal devices, such as the duration of device functionality within the selected configurations. Silva [19] set out an abstract optimization model for BWDs to achieve and prolong energy neutral operation in internet of things networks. Data aggregation is exploited to extend the energy neutral operation of BWDs by adjusting their energy consuming processes. Energy neutral operation reveals the ideal state of BWD, in which the BWD relies entirely on the harvested energy to perform its functions. However, it cannot demonstrate the BWD's performance of maintaining energy, thus preventing analysis of BWD's energy self-sustainability. Despite the absence of explicit mention or analysis, the following research touches upon the definition of energy self-sustainability of storage-consumption mode BWDs. Lu *et al*. [20] explored the point-to-point downlink transmission between an access point and storage-consumption mode BWDs in cellular networks. They considered both time-switching and power-splitting architectures, with a specific emphasis on average energy outage likelihood. They thought that the energy outage occurred when the BWD could not harvest sufficient RF energy from the ambiance to operate the circuit, which ignored the energy storage of the energy buffer. Liu *et al*. [21] studied the cell-load impact on the energy efficiency of storage-consumption mode BWDs in a heterogeneous cellular network with different types of base stations. The authors argued that the system was self-sustainable if the total energy harvested by the system was larger than the energy required for consumption. A similar definition was adopted in [22], which investigated how to adjust the cycle of beacons in a WiFi network in order to make the BWDs self-sustainable. They thought the system was long-term energy sustainable if the average harvested energy was larger than the average consumed power. However, this kind of definition only considered the relationship between the average value of the energy harvesting process and the energy consuming process, which ignored both the energy storage in batteries and the stochastic nature of those two processes. It is more like a necessary condition. Guruacharya *et al*. [23] suggested that an energy system's self-sustainability reflected the non-occurrence likelihood of an energy outage. The authors highlight the significant impact of energy storage on energy self-sustainability. However, this rigorous requirement only suited high-reliability contexts, such as those harnessing solar or magnetic induction energy. However, in a DEIN system, BWDs should be allowed to operate in an intermittent computing mode, wherein BWDs experience recurrent power interruptions and subsequent reboots after harvesting an adequate amount of energy. RF-powered BWDs, with RF energy's low efficiency, typically were not deployed in ultra-reliable settings [6].

C. Novel Contributions

In light of the above, this paper delves into RF-powered BWDs' energy self-sustainability through energy queue analysis. Pioneering in its approach, the primary contributions include:

- Proposing a mathematical energy self-sustainability definition for DEIN BWDs and articulating it using Martingale theory. The energy self-sustainability is characterized as the percentage of time during which the BWD operates effectively. Its mathematical expression has been demonstrated to be dependent on certain parameters intricately associated with the energy harvesting and consuming processes.
- Providing basic methods for determining the values of parameters under several fundamental stochastic characteristics that can be used to model energy harvesting and consuming processes.
- Presenting the expected uninterrupted operation duration and energy queue expectancy of the BWD by analyzing the BWD's energy queue.
- Validating our theoretical insights against Monte Carlo simulation outcomes. The relationship between the average harvested energy and the average consumed energy under different energy self-sustainability requirements is investigated.

The rest of this paper is organized as follows: Section II introduces Martingale theory basics. Section III details the system model and our energy self-sustainability definition. Sections IV and V, drawing on Martingale theory, delve into energy self-sustainability, expected uninterrupted operation, and energy expectancy. Section VI presents numerical analysis, and Section VII concludes.

II. Preliminary: Martingale Theory

Martingale theory provides a methodology for discrete stochastic process analysis. This section offers a concise overview of Martingale theory and its applications.

Definition 1 (Martingale process). For $\forall n \geqslant 0$, stochastic process $\{X_n : n \in N\}$ is a Martingale process if

$$\begin{cases} \mathbb{E}[|X_n|] < \infty, \\ \mathbb{E}[X_{n+1}|X_0, X_1, ..., X_n] = X_n. \end{cases} \tag{1}$$

A Martingale process, at any point in time, expects its future values to equal its current value. It holds several key properties:

(1) $\mathbb{E}[X_{n+1}] = \mathbb{E}[X_n] = \mathbb{E}[X_0]$.
(2) If X_n is a function of $Y_0, Y_1, ..., Y_n$, then $\mathbb{E}[X_{n+k}|Y_0, Y_1, ..., Y_n] = \mathbb{E}[X_n|Y_0, Y_1, ..., Y_n] = X_n$.
(3) If $g(Y_0, Y_1, ..., Y_n)$ is a bounded function of $Y_0, Y_1, ..., Y_n$, then

$$\mathbb{E}[g(Y_0, Y_1, ..., Y_n)X_{n+k}|Y_0, Y_1, ..., Y_n] \\ = g(Y_0, Y_1, ..., Y_n)\mathbb{E}[X_{n+k}|Y_0, Y_1, ..., Y_n].$$

(4) The sum and difference of several Martingale processes is still a Martingale process. ■

Definition 2 (Supermartingale). For $\forall n \geqslant 0$, stochastic process $\{X_n : n \in N\}$ is a supermartingale if

$$\begin{cases} \mathbb{E}[|X_n|] < \infty, \\ \mathbb{E}[X_{n+1}|X_0, X_1, ..., X_n] \leqslant X_n. \end{cases} \tag{2}$$

A supermartingale expands on the Martingale process. It has the following primary attributes:

(1) For $0 \leqslant k \leqslant n$, $\mathbb{E}[X_n] \leqslant \mathbb{E}[X_k] \leqslant \mathbb{E}[X_0]$.
(2) If X_n is a function of $Y_0, Y_1, ..., Y_n$, then $\mathbb{E}[X_{n+k}|Y_0, Y_1, ..., Y_n] \leqslant X_n$.
(3) If $g(Y_0, Y_1, ..., Y_n)$ is a bounded function of $Y_0, Y_1, ..., Y_n$, then

$$\mathbb{E}[g(Y_0, Y_1, ..., Y_n)X_{n+k}|Y_0, Y_1, ..., Y_n] \leqslant g(Y_0, Y_1, ..., Y_n)X_n.$$

(4) The product of two independent martingales is still a supermartingale. ■

Building a Martingale or supermartingale process based on the intended stochastic process enables the use of the Martingale/supermartingale's advantageous properties for investigation. We introduce the stopping theory of Martingale for further study.

Definition 3 (Stopping time). $\{Y_n, n \geqslant 0\}$ is a random sequence. Let F_{Y_n} represent all the information supplied by $Y_0, Y_1, ..., Y_n$. If T is a non-negative random variable (r.v.), if $\{T = n\} \in F_{Y_n}$, then we say that T is a stopping time of $\{Y_n, n \geqslant 0\}$. In other words, the event $\{T = n\}$ is decided by $Y_0, Y_1, ..., Y_n$. ■

Theorem 1 (Stopping theory of Martingale). Let $\{X_n\}$ be a Martingale process and t be a stopping time of $\{X_n\}$. If the following conditions are satisfied, namely

(1) $\mathbb{P}(t < \infty) = 1$
(2) $\mathbb{E}[X_t] < \infty$
(3) $\lim_{n\to\infty} \mathbb{E}[X_n I_{t>n}] = 0$

then we have

$$\mathbb{E}[X_t] = \mathbb{E}[X_0]. \tag{3}$$

Theorem 2 (Optional stopping theory of supermartingale). If $\{M_n\}$ is a supermartingale related to $\{X_n\}$ and t is a stopping time of $\{X_n\}$, then we have

$$\mathbb{E}[M_0] \geqslant \mathbb{E}[M_t] \tag{4}$$

III. System Model and Definition of Energy Self-Sustainability

A. System Model

One of the key advantages of RF energy harvesting technology is that energy transmitters are various and controllable. From the perspective of BWDs, all energy transmitters transmit RF signals that may contain different energy levels. Since we focus on analyzing the energy buffer of BWDs, all these downlink RF signals are considered as discrete energy packets. Even the continuous energy transmission by dedicated RF energy transmitters can be converted into discrete energy packets by appropriate sampling. Without loss of generality, we consider an RF-powered batteryless device network as shown in Fig. 1. A Hybrid Access Point (HAP) that integrates wireless data transfer function and wireless energy transfer function serves as the coordinator of the network. BWDs are capable of harvesting RF energy from the HAP's downlink energy packets. With the wireless energy transfer function, HAP transmits energy packets with an average rate of $\mu = 1/\lambda$, where λ means average amounts of energy packets arrive in a single time-slot. After receiving the energy packets, storage-consumption mode BWDs store the energy in an energy storage unit by the BWD's energy management module. Powered by the energy stored in the energy storage unit, the BWD accesses the network and delivers its data to the HAP. With different access control protocols, the energy consumption of BWD will exhibit variable characteristics. With the wireless data transfer function, HAP harvests the data from all active BWDs.

The size of the energy packet is assumed to be constant. However, after passing through a wireless channel, the harvested energy varies over time. Therefore, the energy harvested during the n-th time-slot can be modeled as

$$a_n = \lambda P_{HAP} T_{ep} h_t \eta, \tag{5}$$

where P_{HAP} is the transmit power of the HAP, T_{ep} is the time used for transmitting an energy packet, η represents the energy harvesting efficiency of the BWD, and h_t means the instantaneous channel gain between the HAP and BWD. We consider a block fading channel that the channel gain remains unchanged in a single time-slot while randomly varying between different

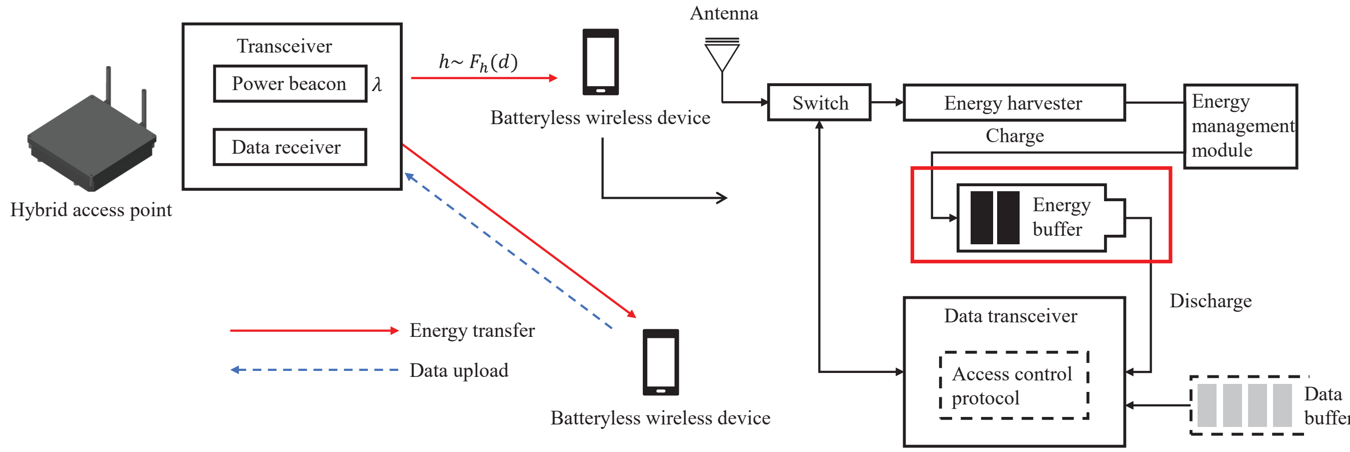

Fig. 1. Schematic of an RF-powered batteryless device network.

time slots. The channel gain is assumed to be independent and identically distributed (i.i.d) which can be expressed as [24]

$$h_t = G_R G_T \left(\frac{c}{4\pi f_0 d_0}\right)^2 \left(\frac{d_0}{d_\mu}\right)^\beta \zeta_\mu. \tag{6}$$

Here, G_R and G_T are the transmit and receive antenna gains respectively, c is the light speed, f_0 is the carrier frequency, d_0 is the near-field range of the transmitter and d_μ is the distance between the BWD and the HAP. Furthermore, $\zeta_\mu \sim Exp(1)$ accounts for small-scale Rayleigh fading. Various channel models exert an influence on the stochastic characteristics of energy harvesting. We will conduct a comprehensive analysis of these distinct stochastic properties in Section IV.

We represent the energy consumed during the n-th time-slot as $s(n)$. The randomness of $s(n)$ is induced by different behaviors of the BWD during the communication process, such as transmission, reception, sensing, or sleeping. The access control protocol determines the BWD's specific behavior at different time-slot. The energy harvesting process and energy consuming process are independent of each other since the energy harvesting process is only related to channel states, while the energy consuming process is only related to the BWD's access control protocol. Therefore, the energy reserve of the energy buffer can be modeled as a queuing process as shown in Fig. 2. The cumulative energy consumed in the first N time slots can be expressed as

$$S(N) = \sum_{n=1}^{N} s(n). \tag{7}$$

Similarly, the cumulative energy harvested in the first N time slots can be expressed as

$$A(N) = \sum_{n=1}^{N} a(n). \tag{8}$$

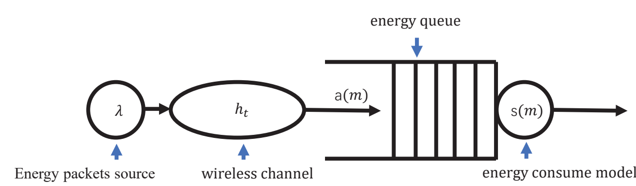

Fig. 2. Energy buffer's queuing process in BWD

The instantaneous queue length of the energy buffer in the n-th time-slot can be accordingly derived as

$$Q(n) = max\{Q(n-1) + a(n) - s(n), 0\} = (l_0 + A(n) - S(n))^+, \tag{9}$$

where $(x)^+$ means $max\{0, x\}$ and l_0 means the initial energy in the BWD's energy buffer. $Q(n) > 0$ represents the energy at time-slot n. In fact, after the energy in the energy buffer is lower than a threshold which is related to the circuit characteristics of BWD, the BWD cannot be activated anymore. $Q(n) = 0$ means that the energy is lower than the active threshold. For the convenience of explanation, we use $Q(n) = 0$ to represent the fact that the BWD runs out of its energy storage.

B. Definition of Energy Self-Sustainability

Let $Q(t)$ denote the energy stored in the energy buffer at the time-slot t. Due to the ongoing processes of energy consuming and harvesting, the energy level of the BWD continuously experiences variations. If the harvested energy cannot satisfy consumption demands, the BWD eventually depletes its energy reserve, while halting its energy consuming process. When sufficient energy is harvested, the BWD recommences its operations, and the energy storage resumes its dynamic state.

The BWD possesses a distinctive performance metric due to its inherent variability in energy levels, which is further enhanced by RF energy harvesting. This sets it apart from traditional communication devices that are not faced with energy storage challenges. This metric is termed as "energy self-sustainability". Drawing from the Merriam-Webster definition, self-sustainability implies: 1) the capacity of the device to sustain operations without external intervention, and 2) the device's ability to continue functioning once activated. This leads us to the subsequent formal definition:

Definition 4 (energy self-sustainability). The energy self-sustainability φ of an RF energy harvesting enhanced BWD is characterized as the percentage of time during which the device operates effectively. Specifically, it corresponds to the time fraction where $Q(t)$ exceeds a pre-established threshold W_{th}, which is essential for the proper functioning of the BWD. ■

This definition aptly captures the dual facets of energy self-sustainability for the BWD: the capability to self-sustainability and the capability to reactivated after harvesting enough energy. Given the BWD's susceptibility to energy outages, it becomes pertinent to evaluate the duration that it can sustain stable operations. Furthermore, this definition considers devices' energy buffer and characterizes the stochastic nature of energy arrival and consumption processes. Compared to the existing discussions on energy self-sustainability, the proposed metric is more applicable to RF-powered networks as it accommodates intermittent operations [25] caused by frequent power outages. The formal expression for energy self-sustainability is then articulated as

$$\varphi = \frac{\int_0^T \varepsilon\left(Q(t) - W_{th}\right) dt}{T}, \tag{10}$$

where $\varepsilon(x)$ is a binary function that $\varepsilon(x) = 1$ when $x > 0$ and $\varepsilon(x) = 0$ in other situations.

According to (10), $\varphi = 1$ represents that the device never suffers from an energy outage, it can function as a common device. When T takes a finite value, $\varphi \in (0, 1)$ represents that the device can function properly for φT time. Let $T \to \infty$, then φ becomes a probability of functioning, and it can be expressed as

$$\varphi = \lim_{T\to\infty} \frac{\int_0^T \varepsilon\left(Q(t) - W_{th}\right) dt}{T} = \lim_{T\to\infty} \frac{\sum_{n=0}^{\infty} \varepsilon\left(Q(n) - W_{th}\right)}{T}. \tag{11}$$

According to the definition of probability, φ should be defined as

$$\varphi = \mathbb{P}[Q(n) > W_{th}]. \tag{12}$$

We use $\mathbb{E}[Q(n)]$ to represent the energy expectancy, and we define the time that BWD experiences its first energy interruption as τ, which is then expressed as

$$\tau = \underset{0\leqslant n\leqslant T}{inf} \{n|Q(n) = 0, Q(0) = l_0\}. \tag{13}$$

In subsequent sections, we investigate the mathematical relationship between these metrics (φ, τ and $\mathbb{E}[Q(n)]$) and two major factors that impact the energy queue: the energy harvesting process and energy consuming process.

IV. Analysis of Energy Self-Sustainability

In this section, our focus centers on scrutinizing the energy self-sustainability of BWD from the perspective of the MAC layer. Drawing from insights in Section III-B, the procedures of energy accumulation and expenditure are represented as discrete stochastic sequences, while the energy reserve of the BWD is modeled as a queuing process. Employing the discrete Martingale theory emerges is beneficial for interpreting such queuing phenomena, primarily due to its capability to encapsulate the BWD's access behavior. Martingale theory is an effective tool for analyzing the limitation of discrete stochastic sequences. It provides an efficient method for establishing stochastic process models of queueing systems, thereby enhancing our understanding of system behavior characteristics.

A. General Mathematical Expression

By echoing the discussions in Section III-B, the metric φ of the energy self-sustainability is the likelihood that BWD's energy reserve surpasses a threshold W_{th}. This likelihood is further dissected by formulating Martingale sequences pertinent to both energy harvesting and consuming processes.

Initially, we introduced two supermartingale sequences proposed in [26], which are termed as exponential arrival-martingale and exponential service-martingale, respectively.

Definition 5 (Exponential arrival-martingale). For every $\theta > 0$, if there is a parameter $K_a(\theta) \geqslant 0$ and a function h_a to make the process

$$M_a(n) = h_a(a_n)\, e^{\theta(A(n) - nK_a(\theta))}, n \geqslant 0, \tag{14}$$

is a supermartingale process. Among them, K_a is parameter related to θ, and h_a is a function of energy arrival process $A(n)$. Then, the $M_a(n)$ is arrival process $A(n)$'s exponential arrival-martingale. ■

Definition 6 (Exponential service-martingale). For every $\theta > 0$, if there is a parameter $K_s(\theta) \geqslant 0$ and a function h_s to make the process

$$M_s(n) = h_s(s_n)\, e^{\theta(nK_s(\theta) - S(n))}, n \geqslant 0, \tag{15}$$

is a supermartingale process. Among them, K_s is parameter related to θ, and h_s is a function of energy consuming process $S(n)$. Then, the $M_s(n)$ is service process $S(n)$'s exponential service-martingale. ■

Note that the format of exponential arrival- and service-martingale is similar to that of effective bandwidth [27]. However, the effective bandwidth is defined by the Laplace transform of $A(n)$ or $S(n)$, which cannot retain the randomness of these discrete sequences. By contrast, exponential arrival- and service-martingales are still random processes, which can properly characterize the random changes of energy storage caused by BWD's MAC layer protocol. Note that K_a, K_s, h_a and h_s should be chosen based on the characteristics of the specific energy arrival process and energy consuming processes to make $M_a(n)$ and $M_s(n)$ supermartingales. We will analyze the different access behaviors of BWDs later. Now, suppose that we already have A_n's exponential arrive-martingale $M_a(n)$ and S_n's exponential service-martingale

$M_s(n)$. According to the fourth property of supermartingale in *Definition* 2, we can construct a supermartingale process related to the energy queue length by multiplying $M_a(n)$ with $M_s(n)$, which derived as

$$M_Q(n) = M_a(n) * M_s(n) = h_a(a_n) h_s(s_n) e^{\theta(A(n)-S(n)+nK_s(\theta)-nK_a(\theta))}. \tag{16}$$

Since K_a, K_s, h_a and h_s all depends on θ, let

$$\theta^* := \sup\{\theta > 0 : K_a = K_s\}. \tag{17}$$

Therefore, $M_Q(n)$ can be further derived as

$$M_Q(n) = h_a(a_n) h_s(s_n) e^{\theta^*(A(n)-S(n))}. \tag{18}$$

According to the stopping theory of supermartingale, as explained in *Theorem* 2, we can obtain the bound of energy self-sustainability φ.

Lemma 3. For any $W_{th} > 0$, the energy self-sustainability φ, the energy harvesting process' function h_a, the energy consuming process' function h_s and the parameter θ^* satisfy the following relationship,

$$\varphi(W_{th}) \leqslant \frac{\mathbb{E}[h_a]\mathbb{E}[h_s]}{H} e^{-\theta^* W_{th}}, \tag{19}$$

where we have $H := \min\{h_a(a(n)) h_s(s(n)) : a(n) > s(n)\}$. ■

Proof. See Appendix A. ■

Given the values of K_a, K_s, h_a and h_s, we can derive the energy self-sustainability φ. In the subsequent subsections, we explain the methodologies to determine these parameters based on various stochastic processes.

B. Parameter Values for Energy Harvesting Process

As discussed in Section III-A, the BWD's energy harvesting process can be modeled as a series of energy packets passing through a wireless channel and arriving at the energy buffer. By considering an i.i.d wireless channel, the energy harvesting process $a(n)$ is also i.i.d. h_a is a constant (without loss of generality, $h_a = 1$), and K_a can be calculated by solving the following equation, which is expressed as

$$\mathbb{E}\left[e^{\theta a(n)}\right] = e^{\theta K_a}. \tag{20}$$

Based on *Definition* 5, the chosen of h_a and K_a should make the exponential arrival-martingale $M_a(n)$ be a supermartingale process. According to the definition of supermartaingale, we have the following derivation

$$\begin{aligned}
&\mathbb{E}[M_a(n+1)|a(1), a(2), \ldots, a(n)] \\
&= \mathbb{E}\left[h_a(a(n+1))e^{\theta(A(n+1)-(n+1)K_a(\theta))}|a(1), a(2), \ldots, a(n)\right] \\
&= h_a(a(n+1))e^{\theta(A(n)-nK_a(\theta))}\mathbb{E}\left[e^{\theta(a(n+1)-K_a(\theta))}|a(1), a(2), \ldots, a(n)\right]
\end{aligned} \tag{21}$$

With the i.i.d. property of $a(n)$ and $h_a = 1, \mathbb{E}\left[e^{\theta a(n)}\right] = e^{\theta K_a}$, we can further simplify (21) as

$$\begin{aligned}
&\mathbb{E}[M_a(n+1)|a(1), a(2), \ldots, a(n)] \\
&= h_a(a(n))e^{\theta(A(n)-nK_a(\theta))}\mathbb{E}\left[e^{\theta(a(n+1))}\right]e^{-\theta K_a(\theta)} = M_a(n).
\end{aligned} \tag{22}$$

C. Parameter Values for Energy consuming process

The energy consuming process $s(n)$, given its stochastic behavior, can be understood under various scenarios. In the elementary scenario, $s(n)$ emerges as a constant sequence, denoted as $s(n) = e$. Such a situation arises when the BWD operates consistently in a single mode or when it functions in multiple modes with similar energy consumption. In this scenario, the BWD's mean energy consumption can be deduced by temporal averaging, with e representing this average. For the specific case of $s(n) = e$, one can assign values $K_s = e$ and $h_s = 1$. The validation of this follows a similar trajectory to the energy harvesting process by setting $\mathbb{E}\left[e^{-\theta(s(n+1))}\right]e^{\theta K_s(\theta)} = 1$. A more intricate scenario envisages $s(n)$ possessing the Markov property, which are commonly employed for modeling the access behavior of BWD. This is observed when the BWD's access behavior follows a Markov-based model. Within this discourse, we spotlight two exemplary access behaviors, namely a duty cycle (DuC)-based protocol and a CSMA/CA-based protocol. Due to the limited energy storage, efficient energy utilization is a crucial consideration for BWDs, while DuC and CSMA/CA are two protocols that appropriately address this concern [22], [28]. Note that this does not refer to any specific protocol, but rather to two categories of protocols that follow the basic characteristics of CSMA/CA or DuC.

1) DuC-based access protocols: According to [28], DuC protocols predominantly cater to wireless-powered sensor networks, which offer the advantage of modulating energy consumption by transitioning between the BWD's sleep and active modes. Considering the operational stance of the BWD, the state space is denoted as $w \in \{0, 1\}$, with $w = 0$ symbolizing a sleep mode and $w = 1$ representing an active mode. Furthermore, the transition probabilities from the sleep to the active mode and vice-versa are represented as p_a and q_a, respectively. Notably, for a specific DuC protocol, these probabilities, p_a and q_a, can be derived from the relative operational durations in both modes. The transition matrix can be easily derived as

$$T_{i,j} = \begin{pmatrix} 1-p_a & p_a \\ q_a & 1-q_a \end{pmatrix}. \tag{23}$$

We define an output function $f(w)$, which represents the energy consumption of the BWD in different states. Specifically, the BWD may consume e_0 energy in the sleep mode and consume e_1 energy in the active mode.

2) CSMA/CA based access protocols: CSMA/CA-based access mechanisms represent another prevalent protocol paradigm that is extensively utilized in RF-powered BWDs due to its distributed nature and low collision probabilities [22], [29]. Figure 3 illustrates a generic Markov model encapsulating CSMA/CA-based protocols. Within this framework, a network comprising of L BWDs competing for a shared channel is considered. The state space $w \in \{1, 2, .., L\}$ denotes that the channel is under the utilization of the l-th BWD's successful transmission. Observations are made from the standpoint of the L-th BWD. Specifically, $w = L$ implies that the channel is occupied by the L-th BWD's

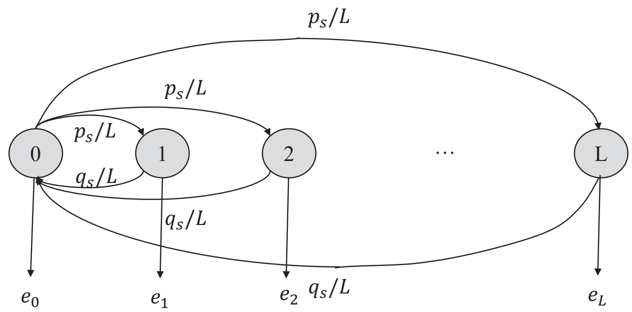

Fig. 3. A general Markov model of a CSMA based protocol.

successful transmission. Furthermore, $f(L) = e_L$ implies the L-th BWD may consume e_L energy for transmission. This model is deemed to be appropriate as energy consumption remains consistent across different states [30]. When $w = [1, 2, .., L-1]$, the channel is occupied by other BWD's successful transmission, and the L-th BWD suspends its backoff counter waiting for the channel to be free. When $w = 0$, the channel is free and all BWDs are in the state of backoff. The L-th BWD consumes e_0 energy in this state. p_s and q_s represent the probability of the channel generating a successful transmission and collision, respectively. Actually, p_s and q_s of a specific CSMA-based protocol can be calculated by a Markov chain-based method [31]. The transition matrix can be derived as

$$T_{i,j} = \begin{pmatrix} 1-p_s & p_s/L & \cdots & p_s/L \\ q_s & 1-q_s & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ q_s & 0 & \cdots & 1-q_s \end{pmatrix}. \tag{24}$$

3) Martingale analysis: When the access behavior can be modeled as a Markov chain, we first obtain the transition matrix $T_{i,j}$, as

$$T_{i,j} = \mathbb{P}\left(s(n+1) = j | s(n) = i\right). \tag{25}$$

We then perform an exponential column transformation on $T_{i,j}$.

$$T^{\theta}_{i,j} = T_{i,j} e^{\theta f(w(j))}, \tag{26}$$

where $f(w)$ is an output function, which represents the energy consumption at state w. Let $\rho(T^{\theta}_{i,j})$ be the spectral radius of $T^{\theta}_{i,j}$, and h be its corresponding right-eigenvector. With the Markov property of $s(n)$, we have the following derivation:

$$\begin{aligned} &\mathbb{E}\left[M_s(n+1)|s(1), s(2), \ldots, s(n)\right] = \mathbb{E}\left[M_s(n+1)|s(n)\right] \\ &= \mathbb{E}\left[h_s(s(n+1))e^{\theta((n+1)K_s(\theta) - S(n+1))}|s(n)\right] \\ &= e^{\theta(nK_s(\theta) - S(n))} h_s(s(n))\rho(T^{\theta}_{i,j})e^{\theta K_s} \\ &= M_s(n)\rho(T^{\theta}_{i,j})e^{\theta K_s}, \end{aligned} \tag{27}$$

which implies that we can choose $K_s = \frac{\log \rho(T^{\theta}_{i,j})}{\theta}$ to make the $M_s(n)$ is a supermartinagle process for every θ.

To recapitulate, when long-term estimations of the BWD's average energy consumption are feasible, we may characterize the term $s(n)$ as a constant sequence by setting $K_s = e$ and $h_s = 1$. When the access behavior of the BWD can be modeled as a Markov chain, $K_s = \frac{\log \rho(T^{\theta}_{i,j})}{\theta}$ can be adopted, while h_s is defined as the right-eigenvector associated with the transition matrix's spectral radius. After establishing the stochastic properties of $a(n)$ and $s(n)$, the suitable values for h_a, h_s, K_a and K_s can be deduced using the methods previously described. By incorporating all these determined values, the metric for energy self-sustainability, denoted as φ, can be derived as indicated by Eq. (19). The approach to parameters selection under diverse stochastic characteristics has been collated in TABLE I.

V. Analysis of Other Performance Metrics

A. Expected Uninterrupted Operating Duration

Due to the inherent stochastic characteristics of energy harvesting and energy consumption processes, BWDs may encounter disruptions in their power supply, which may negatively impact their communication performance. The concept of "expected uninterrupted operating duration" aims to estimate the time at which the first power interruption is likely to occur by taking into account the stochastic characteristics of energy harvesting and energy consumption processes. We analyze the relationship between the initial energy l_0 and the BWD's uninterrupted operating duration. Let us define sequences $\{d_i = a(n) - s(n), \forall i \geqslant 0\}$ and $\left\{S_i = \sum_{i=1}^{n-1} d_i, \forall i \geqslant 0\right\}$, where $a(n)$ and $s(n)$ are the harvested and consumed energy during the n-th time-slot, respectively, which are defined in Section III-A. Obviously, the energy reserve in the n-th time-slot can be defined as

$$Q(n) = (l_0 + S_n)^+, \tag{28}$$

where S_n can be either a positive or a negative value. According to the random walk theory, we can analyze the relationship between $Q(n)$ and S_n. For any random walk process, only the following three conditions may occur [32]:

(1) (Oscillating) If $\mathbb{E}[d_i] = 0$, then $\mathbb{P}(\lim sup_{n\to\infty} S_n = +\infty) = 1$, $\mathbb{P}(\lim inf_{n\to\infty} S_n = -\infty) = 1$. In this case, $Q(n)$ may keep oscillating in a limited range.

(2) (Drift to $+\infty$) If $\mathbb{E}[d_i] > 0$, then $\mathbb{P}(\lim_{n\to\infty} S_n = +\infty) = 1$. In this case, $Q(n)$ may oscillate up to infinity.

(3) (Drift to 0) If $\mathbb{E}[d_i] < 0$, then $\mathbb{P}(\lim n \to \infty S_n = -\infty) = 1$. In this case, $Q(n)$ may oscillate down to zero.

In these three conditions, when $n \to \infty$, $Q(n)$ is only related to d_i, while it is independent of l_0. It indicates that the BWD's energy self-sustainability is independent of its initial energy storage. This conclusion is quite intuitive. Because the definition of energy self-sustainability is based on probability, its value depends on the random variables in the definition rather than the constant term. Although the BWD's initial energy is independent of energy self-sustainability, it may directly affect the BWD's expected uninterrupted operating duration.

According to Theorem 1, we can investigate the mathematical relationship between the initial energy and the BWD's expected uninterrupted operating duration. We first construct a Martingale process $M(n) = M_1(n) - M_2(n) + M_3(n)$, where $M_1(n) = A(n) - n\mathbb{E}[a(n)]$ is the linear arrival-martingale related to the energy harvesting process, $M_2(n) = S(n) -$

TABLE I
Parameter Values in Several Common Scenarios.

	Randomness	Scenario	Parameter values
Energy harvesting process	i.i.d	Energy packets pass through wireless channels	$h_a = 1, \mathbb{E}\left[e^{\theta a(n)}\right] = e^{\theta K_a}$
Energy consumption process	constant sequence	Predictable average energy consumption	$h_s = 1, K_s = e$
	Markov property	DuC-based/CSMA-based protocols	$h_s = \rho(T^{\theta}_{i,j}), K_s = \frac{\log\rho(T^{\theta}_{i,j})}{\theta}$

$n\mathbb{E}[s(n)]$ is the linear service-martingale related to the energy consuming process, and $M_3(n) = l_0$ is a constant sequence related to the initial energy.

Lemma 4. $M(n) = M_1(n) - M_2(n) + M_3(n)$ is a Martingale process. ■

Proof. See Appendix B. ■

We define the stopping time of $M(n)$ as the time when the BWD experiences its first energy interruption. In other words, when the stopping time happens, we have $Q(n) = 0$. Therefore, the stopping time is expressed as

$$\tau = \inf_{0\leqslant n\leqslant T} \{n|Q(n) = (l_0 + A(n) - S(n))^+ = 0\}. \tag{29}$$

Based on the definition of the stopping time τ, $Q(n)$ always be positive in its definitional domain. Thus, we further simplify τ as

$$\tau = \inf_{0\leqslant n\leqslant T} \{n|Q(n) = l_0 + A(n) - S(n) = 0\}. \tag{30}$$

According to the stopping theory of Martingale, we have

$$\mathbb{E}[X_\tau] = \mathbb{E}[X_0] = l_0. \tag{31}$$

When $Q(\tau) = 0$, $\mathbb{E}[X_\tau]$ can be derived as

$$\begin{aligned}\mathbb{E}[X_\tau] &= \mathbb{E}[l_0 + A(\tau) - S(\tau) - \tau\mathbb{E}[a(n)] \\ &\quad + \tau\mathbb{E}[s(n)]\,|l_0 + A(\tau) - S(\tau) = 0] \\ &= \mathbb{E}[\tau\mathbb{E}[s(n)] - \tau\mathbb{E}[a(n)]] = \tau(\mathbb{E}[s(n)] - \mathbb{E}[a(n)]).\end{aligned} \tag{32}$$

Therefore, by combining (31) and (32), we further derive the stopping time as

$$\tau = \frac{l_0}{\mathbb{E}[s(n)] - \mathbb{E}[a(n)]}. \tag{33}$$

Observe from Eq. (33) that the expected uninterrupted operating duration τ is related to the initial energy l_0, expectation of the energy harvesting process $a(n)$ and expectation of the energy consuming process $s(n)$. The energy consumption can be analyzed by modeling and analyzing the communication behavior of the BWD [33]. By adjusting the rate of energy packets and by knowing the channel condition, the energy harvesting process is also predictable. Therefore, we can estimate a BWD's next energy outage time by obtaining its current energy value.

B. The Expectation of Energy Queue

"Expectation of Energy Queue" refers to the average value of the energy queue in a device. It is used to estimate the amount of remaining energy stored in the device. The expression of φ is actually the Complementary Cumulative Distribution Function (CCDF) of the energy queue $Q(n)$. Intuitively, we can obtain the expectation of the energy queue length as

$$\begin{aligned}\mathbb{E}[Q(n)] &= \int_0^\infty x\mathbb{P}\{Q(n) = x\}\,dx = \int_0^\infty \mathbb{P}\{Q(n) > x\}\,dx \\ &= \int_0^\infty \varphi(x)\,dx = \frac{\mathbb{E}[h_a]\,\mathbb{E}[h_s]}{H}\int_0^\infty e^{-\theta^* x}\,dx \\ &= \frac{\mathbb{E}[h_a]\,\mathbb{E}[h_s]}{H\theta^*}.\end{aligned} \tag{34}$$

VI. Numerical Results

The theoretical predictions are validated by Monte Carlo simulations in this section. The key physical layer parameters are captured in TABLE II, which is in line with our experimental setup [34]. We validate our mathematical framework by using the CSMA/CA protocol and the DuC protocol as examples. We conduct our simulations in OMNet++, where the HAP transmits energy packets periodically and the BWDs continuously report information with the CSMA/DuC protocol unless it has no energy. The positions of all nodes are randomly generated within a 2 m × 2 m area. The experimental parameters are listed in Table. II, which are from the IEEE 802.11 standard [35]. The validation experiment outcomes are depicted with box diagrams. The Monte Carlo simulation is conducted for a total of 50 runs, with each run consisting of 200,000 time slots. The parameters used in the simulation remain constant throughout all iterations.

Fig. 4 portrays a juxtaposition between the simulation outcomes and the theoretical scrutiny concerning the expected uninterrupted operating duration of the BWD. We examine three diverse energy consumption paradigms. Here, with the parameters in Table. II, the successful transmission probability p_s in CSMA/CA protocol is 0.807 and the awaken probability p_a in DuC is 0.5, which are used as the input to our mathematical framework. When a CSMA-based protocol is adopted, the BWD expends 50 mW during successful transmissions and 1 mW during backoff maneuvers, analogous to typical Wi-Fi equipment. Conversely, when a DuC-based protocol is adopted, the BWD expends 30 mW and 1 mW in active

TABLE II
PARAMETER SETTING

Parameter	Value	Parameter	Value
Operation frequency f_0	915 MHz	Energy packet rate μ	0.6 $packets/s$
Distance between the HAP and the BWD	2 m	Transmit power of the HAP	10 W
Antenna gains G_R G_T	5 dBi	Duration of an energy packet	100 ms
Pass loss exponent β	2	Energy threshold W_{th}	10 $mW \cdot s$
SIFS Time in CSMA/CA	28 us	Backoff slot time	20 us
DIFS Time in CSMA/CA	128 us	Minimum contention window	16
Retry limit in CSMA/CA	8	Maximum contention window	1024
Message interval	Exp (30 ms)	Message length	1 kB
Awaken period in DuC	50 ms	Sleep period in DuC	20 ms
Number of BWDs	18	Supercapacitor's maximum capacity	5V 80 mF

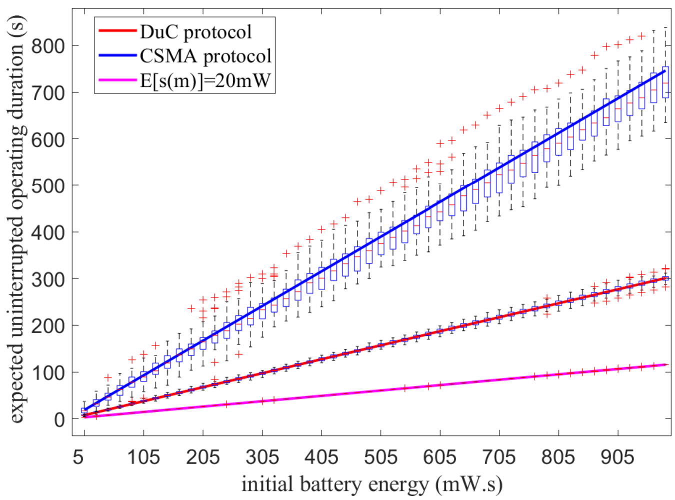

Fig. 4. Comparison of simulation and numerical results for the variation of expected uninterrupted operating duration with initial energy, where $p_s = 0.8$, $q_s = 0.2$, $p_a = 0.5$ and $q_a = 0.2$.

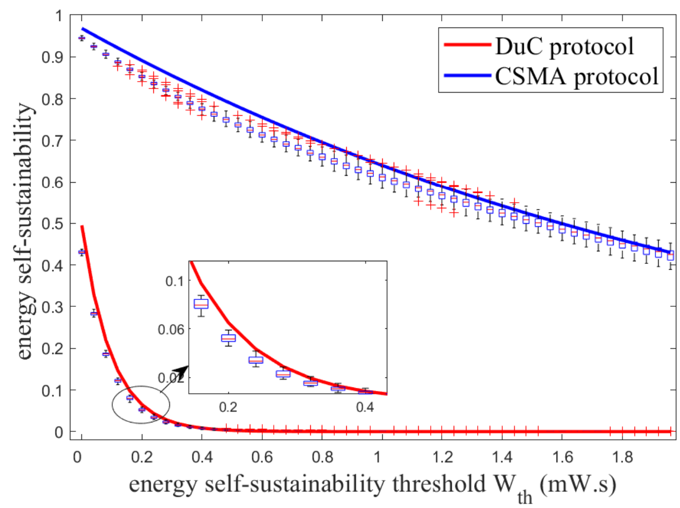

Fig. 5. Comparison of simulation and numerical results for the variation of energy self-sustainability with W_{th}

mode and hibernation mode, respectively, which resembles a standard ZigBee apparatus. An analysis of $\mathbb{E}[s(m)] = 20\ mW$ is incorporated in Fig. 4. The linear interrelationship between the anticipated continuous operation duration and the initial energy, given specific energy harvesting consumption models, is discerned from (33). This assertion is further corroborated by Monte Carlo simulation outcomes.

Fig. 5 elucidates the precision of the theoretical assessment pertaining to energy self-sustainability. With a stable energy packet rate of 0.6 $packets/s$, the figure delineates the correlation between Energy Self-Sustainability (ESS) and the energy threshold W_{th}. The ESS and W_{th} association approximates an exponential function. As W_{th} amplifies, the ESS converges to zero, which implies that with fixed energy harvesting and consumption models, the likelihood of the BWD retaining energy surpassing the energy threshold diminishes with increasing energy thresholds. In the context of our parameter setup, the BWD, which utilizes the DuC protocol, exhibits a longer duration in an active state and consumes a greater amount of energy. Consequently, this leads to a reduced level of energy self-sustainability when compared to the BWD that employs the CSMA protocol, under the same W_{th}.

In Fig. 6, we investigate the expectation of the energy queue length with different energy packet rates. As the energy packet rate increases, the expectation of the energy queue length increases accordingly, and the comparison of the theoretical results with the simulation results also proves the accuracy of the theoretical analysis. This shows that when the system reaches a saturation state, the expectation of the energy queue length is predictable. The BWD's energy can be maintained at a higher level by increasing the energy packet rate.

In Fig. 7, the impact of energy packet rate on ESS is analyzed under different energy consumption models. The analyzed BWD with CSMA protocol and DuC protocol utilizes identical parameters to the preceding experimentation. Additionally, an energy consumption model with a constant 50 mW is also analyzed. We set W_{th} as $10mW.s$, which means that the BWD can work normally for at least 100 timeslots with the minimum power if its energy is greater than W_{th}. When the energy packet rate is small, the energy harvesting process cannot match the energy consuming process. In this situation, the BWD's energy self-sustainability is nearly zero, which means the BWD's energy is hardly ever greater than W_{th} over a long operation time. As the energy packet rate increases, the BWD harvests more energy and the energy self-sustainability increases accordingly. For example, when $\lambda = 0.5$, the BWD's ESS is 0.84 when BWD adopts a CSMA-based protocol, and the BWD's ESS is 0.25 when BWD

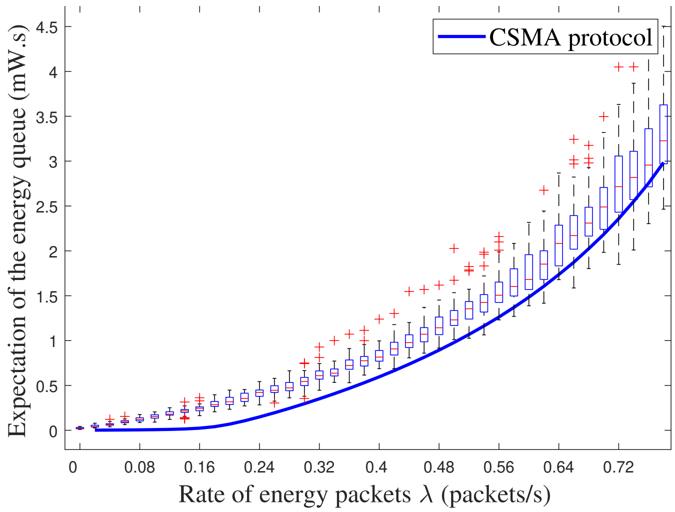

Fig. 6. Comparison of simulation and numerical results for the variation of expectation of the energy queue length with energy packet rate, where $p_s = 0.8$, $q_s = 0.2$, $p_a = 0.5$ and $q_a = 0.2$.

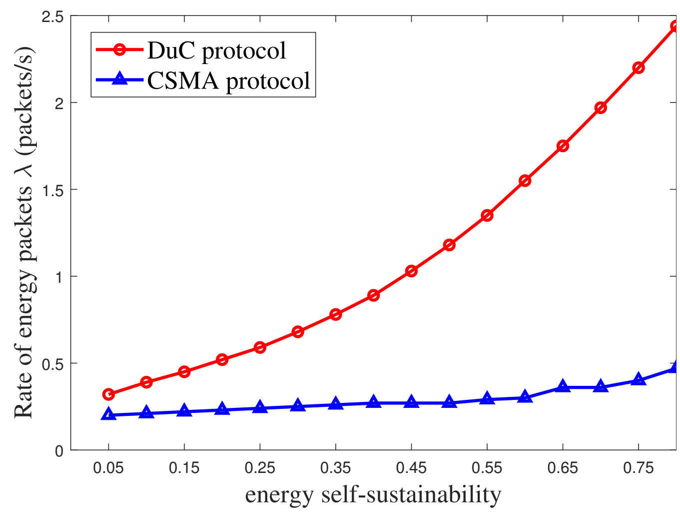

Fig. 8. Minimum energy packet rate required under different ESS requirements.

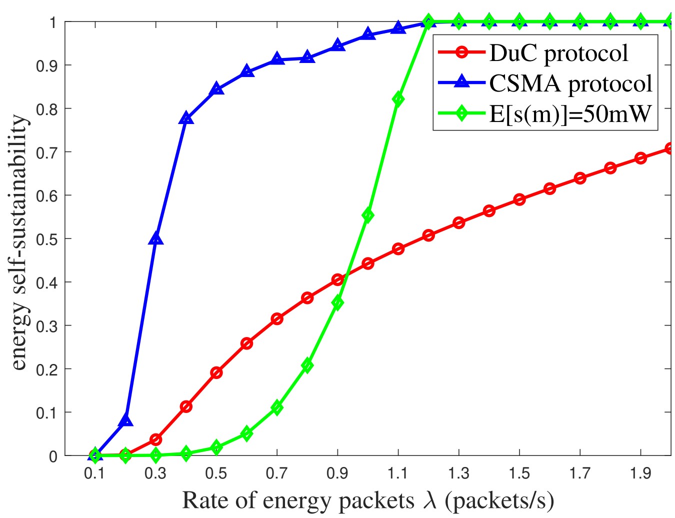

Fig. 7. The impact of energy packet rate on energy self-sustainability.

adopts a DuC-based protocol. When BWD adopts a CSMA-based protocol, it may take backoff operations when collisions happen, and consume less energy accordingly. Therefore, the ESS of a CSMA-BWD is higher than that of a DuC-BWD, when the energy packet rates are same. As the energy packet rate gets larger, the ESS may gradually approach 1, which means the BWD's energy is always greater than W_{th}.

We fixed W_{th} to 10 $mW \cdot s$, and then we investigated the minimum energy packet rate required by the BWD under different ESS requirements. As shown in Fig. 8, when the energy consumption model is fixed, the required minimum energy packet rate increases as the ESS requirement increases. It is intuitive that in order to meet the higher ESS requirement, the BWD needs to harvest more energy from the energy packets. Furthermore, as analyzed before, the CSMA-based protocols have greater randomness and the CSMA-BWD may consume less energy than a DuC-BWD because it is often in a backoff state. Therefore, under the same ESS requirement, a DuC-BWD needs a larger energy packet rate to meet its larger energy consumption. When designing the HAP's slot allocation, Fig. 8 shows the minimum number of time slots that need to be allocated to energy transmission in order to satisfy a certain ESS requirement of the BWD.

In Fig. 9, we investigate the maximum average energy consumption of the BWD under different ESS requirements. When the ESS requirement is fixed, the maximum average energy consumption increases as the energy packet rate increases. As the energy packet rate increases, the BWD harvests more energy, this allows the BWD to consume more energy. As shown in Fig. 9, when we preset the ESS requirement as 0.3 and the energy packet rate is 1 $packet/s$, the allowed BWD's maximum average energy consumption is 51.4 mW. Once the BWD's average energy consumption exceeds this threshold, it cannot meet the predetermined ESS requirement. Furthermore, as the ESS requirement becomes more stringent, the allowed BWD's maximum average energy consumption at the same energy packet rate will decrease.

VII. Conclusion

In this paper, we presented a rigorous mathematical characterization of energy self-sustainability within a DEIN, premised on the dynamic variations observed in the energy queue of the BWD. By applying the Martingale theory, we derived a close upper bound for the BWD's energy self-sustainability, its inaugural outage time, and the expected value of the energy queue. The values of the key parameters of the proposed mathematical expression are directly associated with the stochastic characteristics of the energy harvesting and energy consumption processes. This paper presents the values of key parameters for several fundamental scenarios. Each of them corresponds to different stochastic characteristics. Our expectation is that researchers can apply this method to derive the energy self-sustainability of devices in their studied protocols, enabling a comprehensive analysis of their research. By conducting an analysis of the energy arrival and energy consumption behavior in the deployed network,

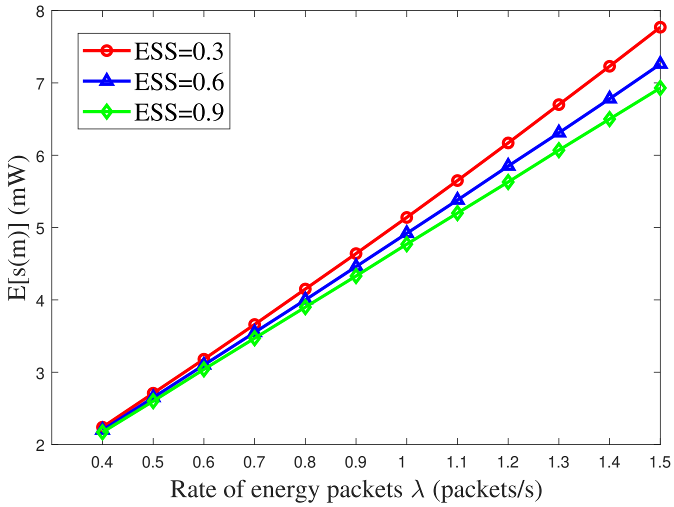

Fig. 9. Maximum average energy consumption under different ESS requirements.

system designers can explore the minimum energy harvesting requirement or the maximum tolerable energy consumption of devices for achieving different energy self-sustainability demands. For example, in our experimental setup, for a BWD employing a DuC-based access protocol to achieve an energy self-sustainability of 0.6, the energy transmission rate of the energy packets from the energy transmitter must exceed 1.5 $packets/s$. Similarly, the mean energy consumption must remain below 51.4 mW given an energy packet transmission rate of 1 $packets/s$. These research findings hold active implications for the long-term stability and sustainable development of the network.

The benefits and limitations of the energy self-sustainability we proposed are summarized here. By leveraging this metric, researchers can effectively illustrate the long-term energy sustain capability of the investigated devices in a more intuitive manner. Furthermore, by adopting this metric as a research objective, researchers can design protocols that enable devices to achieve energy self-sustainability. However, there are still some aspects that require further exploration. For example, this paper provides the foundational framework for addressing the energy self-sustainability we proposed. However, the analysis regarding specific protocols requires further exploration by researchers. In addition, the definition presented in this paper is defined from the perspective of device. Further research is needed to better describe the energy self-sustainability of a network. In practical deployments, it is crucial for devices to possess accurate awareness of their residual energy and the stochastic nature of their energy harvesting and consumption processes.

Appendix A
Proof of Lemmas

A. Proof of Lemma3

Let $M_Q(n)$'s stopping time N means the first time when $A(n) - S(n)$ exceeds W_{th},

$$N := \min\{n : A(n) - S(n) \geqslant W_{th}\} \tag{35}$$

As discussed in Section V-A, the initial energy l_0 cannot affect the BWD's energy self-sustainability. Therefore, we can let $l_0 = 0$, and accordingly $A(n) - S(n) = Q(n)$. N should be a limit value once the stopping time happens. In other words, $\varphi(W_{th}) = \mathbb{P}(Q(n) \geqslant W_{th}) = \mathbb{P}(N < \infty)$. For proof convenience, we bound the stopping time by k, and $N \wedge k := \min\{N, k\}$. According to the optional stopping theory,

$$\begin{aligned}
&\mathbb{E}[h_a(a(0))]\,\mathbb{E}[h_s(s(0))] = \\
&\mathbb{E}[M_Q(0)] \geqslant \mathbb{E}[M_Q(N \wedge k)\, I_{N<k}] \\
&= \mathbb{E}\left[h_a(a(N))\, h_s(s(N))\, e^{\theta^*(A(n)-S(n))} I_{N<k}\right] \geqslant He^{\theta^* W_{th}}\mathbb{P}(N < k).
\end{aligned} \tag{36}$$

By letting $k \to \infty$, we can obtain (19).

B. Proof of Lemma4

Let us first prove that the linear arrive-martingale $M_1(n)$ is a Martingale process.

$$\begin{aligned}
\mathbb{E}[|M_1(n)|] &= \mathbb{E}[|A(n) - n\mathbb{E}[a(n)]|] \leqslant \mathbb{E}[|A(n)|] + \mathbb{E}[|n\mathbb{E}[a(n)]|] \\
&= n\mathbb{E}[a(n)] + n\mathbb{E}[a(n)] = 2n\mathbb{E}[a(n)] < \infty
\end{aligned} \tag{37}$$

According to Definition 1, $M_1(n)$ satisfies the first condition.

$$\begin{aligned}
&\mathbb{E}[M_1(n+1)\,|M_1(0), M_1(1), \ldots M_1(n)] \\
&= \mathbb{E}[M_1(n) + a(n+1) - \mathbb{E}[a(m)]\,|M_1(0), M_1(1), \ldots M_1(n)] \\
&= \mathbb{E}[M_1(n)\,|M_1(0), M_1(1), \ldots M_1(n)] \\
&\quad + \mathbb{E}[a(n+1)\,|M_1(0), M_1(1), \ldots M_1(n)] \\
&\quad - \mathbb{E}[\mathbb{E}[a(n)]\,|M_1(0), M_1(1), \ldots M_1(n)] \\
&= \mathbb{E}[M_1(n)\,|M_1(n)] + \mathbb{E}[a(n+1)] - \mathbb{E}[a(n)] \\
&= M_1(n) + \mathbb{E}[a(n)] - \mathbb{E}[a(n)] = M_1(n)
\end{aligned} \tag{38}$$

Therefore, $M_1(n)$ also satisfies the second condition, and it is accordingly a Martingale process. Similar to $M_1(n)$, the linear service-martingale $M_2(n)$ is also a Martingale process. Then we prove that $M_3(n)$ is a Martingale process.

$$\mathbb{E}[|M_3(n)|] = \mathbb{E}[|l_0|] = l_0 < \infty \tag{39}$$

$$\begin{aligned}
&\mathbb{E}[M_3(n+1)\,|M_3(0), M_3(1), \ldots M_3(n)] \\
&= \mathbb{E}[M_3(n+1)] = l_0 = M_3(n)
\end{aligned} \tag{40}$$

$M_3(n)$ is a Martingale process since it satisfies the conditions in Definition 1. According to the fourth property of Martingale process we listed in Section II, $M(n) = M_1(n) - M_2(n) + M_3(n)$ is a Martingale process.

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