THE ROLE OF PUBLIC SECTOR EMPLOYMENT IN MACROECONOMICS: THEORY AND EVIDENCE

By

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Author's declaration

I, Pavlos Balamatsias, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

I declare that the work in this thesis was carried out in accordance with the requirements of the University's Regulations and that it has not been submitted for any other academic award.

All Chapters of this thesis are sole-authored. All errors remain mine.

Signed by: Pavlos Balamatsias
Date: September 30, 2024

Abstract

This thesis contains three chapters, studying the role of public sector employment in the macroeconomy.

Chapter 1 examines how public sector employment affects the labour market for the UK. I use a two-sector, random search and matching model but, based on countries' data and recent advances in the labour economics literature, assume a frictional labour market with persistent increases in unemployment, due to inelastic investment in private sector vacancies. I find that, unlike frictionless models, increasing hirings of public sector employees lowers unemployment with limited crowding out and is a useful countercyclical and welfare-improving tool.

Chapter 2 studies how public sector employment and public sector output affect aggregate output and employment, using a Heterogeneous Agent New Keynesian model with public sector employment, public sector firms and my Chapter 1 labour market. This model focuses on an important but mostly overlooked component of fiscal policy, offers a new way of thinking about the role of the public sector in recessions, and a comprehensive, realistic setup capturing all the propagation mechanisms and effects while simulating the behaviour of key variables. US results show that these policies raise aggregate output and employment and crowd in private sector employment, as inelastic investment in private sector vacancies results in limited crowding out and a small, positive aggregate demand channel.

Chapter 3 extends the research question in Chapter 2 for France, UK and the US under different tax and monetary policies. I first establish some key facts about these countries' public sector, public sector employment and labour markets indicating large country variations, and distinct results that are also affected by the tax mix: Public sector employment lowers unemployment in all countries, with multipliers above unity for France and the US, and the effect is larger in the US under lump-sum taxes and in France under a mix of lump-sum and labour taxation. Similarly, increasing public sector firms' output in France leads to large increases in aggregate output, with multipliers above unity for a mix of taxes, while effects are bigger in the US when only lump-sum taxes are used; conversely the effect is negative in the UK. Finally, a countercyclical policy mitigates the effects of business cycles in France, particularly when the ZLB binds.

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Data and Funding Statement

This thesis makes use of a number of data sources. More specifically, I make use of national accounting identities data from Eurostat, the office for National Statistics, the Bureau of Economic Analysis and the Organisation for Economic Co-operation and Development. I also make use of labour force survey data from the French Labour Force Survey, the UK Labour Force Survey and the US Current Population Survey. The help provided by the staff of the National Institute of Statistics and Economic Studies (INSEE) in accessing the French Labour Force Survey Data is gratefully acknowledged.

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Table of Contents

Page

List of Tables

List of Figures

Figure **Page**

CHAPTER 0

Introduction

This thesis is composed of three chapters, each of which can be read as a stand-alone academic paper.

The first chapter focuses on the effects of public sector employment in a frictional labour market. Public sector employment and its effects have not been adequately studied, despite its prevalence in most economies since the early 20th century. Furthermore, recent advances in the labour economics literature indicate significant frictions in labour markets, limiting their ability to adjust to business cycles, a position further substantiated by data from many countries showing that recessions lead to large, prolonged reductions of employment and job vacancies, carrying on years after an economy has re-entered an expansionary phase. It is important for researchers to combine these facts and examine how public sector employment can mitigate the impact of recessions and study the optimal policy mix and welfare implications of these policies. The first chapter is trying to add to this research agenda.

The remaining two chapters build on the premise of the first, but expand their scope on the whole economy and also study public sector output, another important and prevalent component of public sector policy in most economies since the start of the 20th century. To this end, I combine my Chapter 1 model with a Heterogeneous Agent New Keynesian (HANK) model, the workhorse model of modern macroeconomics, building a comprehensive, realistic setup allowing me to study all the effects and propagation mechanisms that increasing public sector employment and public sector output has on an economy. I then use this model on different countries and examine how their labour markets and public sectors impact on the effects of public sector employment and public sector output.

More specifically, in **Chapter 1** I study the role of public sector employment and how it can affect unemployment over the business cycle in the UK, using a random search and matching model (Mortensen & Pissarides, 1994) with public sector employment (Quadrini & Trigari, 2007, Navarro et al., 2017, Albrecht et al., 2019). However, I assume that investment in private sector vacancies is inelastic (Coles & Moghaddasi Kelishomi, 2018, Broer et al., 2021), unlike standard models of free entry where vacancies quickly adjust to bring unemployment back to its long-run value, an assumption backed by empirical data for many countries showing that business cycles lead to large, persistent reductions in vacancies and employment (Shimer, 2005; 2012). The rationale behind my model is that in a frictional labour market, public sector employment can reduce unemployment with limited crowding out effects. I also make my model more realistic by having endogenous private sector job destruction rates and hirings of public sector employees depending on public sector employment and unemployment, so I can use data on public sector job destruction rates and hirings of public sector employees on my analysis.

I find that increasing public sector employment leads to a large and persistent unemployment reduction with limited crowding out, as inelastic investment in private sector vacancies ensures private sector employment stays mostly unchanged. Policies focusing on changing the public sector job destruction rate can lower unemployment, but result in larger, longer lasting crowding out effects whereas increasing hirings of public sector employees leads to bigger unemployment reduction and minimises crowding out. Conversely, increasing public sector employment under free entry increases unemployment as the private sector freely reduces investment in private sector vacancies. Additionally, public sector employment can serve as an effective countercyclical policy tool and positively affect social welfare.

Chapter 2 and **Chapter 3** focus on how increasing public sector employment and public sector output affect aggregate output and employment, analyzing and quantitatively evaluating their effects and propagation mechanisms. To this end,

Chapter 2 combines Chapter 1 with a HANK model of heterogeneous households, incomplete asset markets and frictional goods market and asset market (Broer et al., 2021, Ravn & Sterk, 2017, 2021), and public sector firms producing goods (Pappa, 2009, Forni et al., 2010, Economides et al., 2013, 2017).

My model builds on the most novel Two-Agent New Keynesian models of sticky prices, monopolistic competition in the goods market and heterogeneous households, where workers' households supply labour and invest subject to frictions and capitalists' households do not work but own assets[1], and HANK models of sticky prices and monopolistic competition in the goods market, heterogenenous households varying in skills, labour supply and asset market participation and incomplete asset markets [2]. I assume a continuum of ex-ante heterogeneous households in productivity, split into homogeneous capitalists' households that own firms and invest in public sector bonds and workers' households that are also ex-post heterogeneous, working in public sector firms, private sector firms or being unemployed, and facing different (un)employment probabilities. Asset markets are incomplete, as workers' households invest in a zero-net supply household bond and face no-borrowing constraints, so both the sum and the individual asset holdings of workers' households are zero, resulting in uninsurable unemployment risk. Finally, the goods market features monopolistic competition and sticky prices.

This setup results in a rich model, where increasing public sector firms' output and public sector employment increases aggregate output and employment, and creates dynamic variable effects and redistribution channels. Additionally my model is one of the few combining heterogeneous households, uninsurable risk and incomplete asset markets, monopolistic competition and sticky prices with a frictional labour market, and the first to study public sector employment and public sector output, so it helps cover a significant literature gap in macroeconomics and fiscal policy, by focusing on an important but largely overlooked component of fiscal policy, and offers a new innovative way of thinking about the role of the public sector during recessions. Finally my results are both novel, as public sector employment and public sector output have not been adequately studied,

[1] Auclert et al. (2018), Bilbiie (2020), Cantore & Freund (2021),Courtoy (2022), Klein et al. (2022)

[2] Auclert et al. (2018; 2020), Hagedorn et al. (2019), Kaplan & Violante (2014), McKay & Wolf (2022)

but also realistic, as I use a comprehensive model, capturing all the propagation mechanisms and effects generated by the labour market, unemployment risk and aggregate demand elements I use, and simulating the behaviour of key variables.

Using this model in **Chapter 3** I study how public sector employment and public sector output affect aggregate output and employment under different tax and monetary policies in France, UK and the US. First, I provide a systematic study on a number of key facts about these countries' public sector, public sector employment and labour markets as in Fontaine et al. (2020). I find that European countries have larger public sectors, as they are more interventionist (Colli & Nevalainen, 2019, De Lange & Merlevede, 2020, Christiansen, 2011, OECD, 2017, Putniņš, 2015) and have more extensive welfare systems (Blanchard, 2014, Nickell, 1997, Petrongolo & Pissarides, 2008, Salvanes, 1997), but this trend is slowly declining in the UK (Cumbers, 2019, Schmidt, 2003). In addition, public sector employment is a large, relatively stable part of the labour force, especially in Europe. In the labour market unemployment increases are very large and persistent, particularly in the US and the UK, while the French labour market is more rigid.

These differences also carry in my simulation. France has smaller, more elastic investment in private sector vacancies and private sector job destruction rates, sticky wages and longer-lived policy shocks, the US has bigger but more inelastic investment in private sector vacancies and private sector job destruction rates, very elastic wages and shorter-lived shocks, while the UK exhibits much smaller elasticity in all labour market variables. Consequently, the effects of policy vary significantly between countries, with the tax mix altering their effectiveness. When capitalists' households pay higher taxes, increasing public sector employment lowers unemployment in France and the US, with multipliers above unity (in absolute values) and a stronger effect in the US. However, when both households' taxes rise the effect is stronger in France, and the multiplier is larger, as the positive effects are amplified. Similarly when both households' taxes rise, increasing public sector firms' output in France leads to large increases in aggregate output, with a multiplier above unity, and smaller effects in the US. Conversely the UK's very inelastic labour market makes the policy effects negative. Finally, a countercyclical policy can mitigate the adverse effects of business cycles in France, particularly in the ZLB, but has no effect in the US.

CHAPTER 1

THE ROLE OF PUBLIC SECTOR EMPLOYMENT UNDER PERSISTENT UNEMPLOYMENT

1.1 Introduction

It is an underappreciated fact in economics that governments employ large fractions of the workforce in many developed economies. On average, over 20 % of the labour force in the UK consists of public sector employees. In European countries, characterized by extensive welfare states and numerous state-owned enterprises, public sector employment is around 25% of the labour force and in many cases even exceeds 30% of the labour force (OECD. 2019,2021). Also as shown in Table 1.1, even when using a narrower definition, without accounting for state-owned enterprises, public sector employment is well above 20% of total employment and represents 20 to 25% of total government expenditure in most OECD countries. In addition, as seen in column 4, public sector employment actually goes up in most countries during recessions. These two facts indicate that public sector employment is not only an important aspect of fiscal policy, the labour market and the economy in general, but that it is also important in explaining the business cycles fluctuations of unemployment. Therefore, changes in public sector employment can potentially have large and important effects on an economy, particularly during recessions: If governments fire public sector employees, this can raise unemployment; on the other hand if they hire additional public sector employees, this can lower unemployment and get an economy out of recession.

Table 1.1: Public Sector and the Labour Market

Country	General Government Employment (% of Total Employment)	Government Expenditure on Wages (% of Total Expenditure)	Unemployment Rate	CrossCorrelation (N^G, u)
Belgium	18.63	23.23	7.25	0.86
Denmark	28.80	29.87	6.03	0.87
Estonia	22.39	28.20	8.01	0.64
Finland	24.66	24.79	7.89	0.18
France	21.98	22.60	8.92	0.19
Germany	10.94	17.22	5.25	0.85
Greece	17.16	24.06	18.16	-0.35
Hungary	20.39	21.81	7.19	0.31
Ireland	16.22	25.33	9.52	-0.32
Italy	13.89	20.47	9.78	-0.61
Latvia	20.64	22.38	10.83	0.82
Lithuania	23.59	28.15	9.66	0.91
Netherlands	12.50	18.78	5.04	0.36
Norway	29.90	29.26	3.62	0.90
Spain	15.66	25.04	18.18	0.73
Sweden	29.37	25.41	7.50	-0.32
United Kingdom	17.54	23.15	5.87	0.73
United States	15.73	25.79	6.36	0.91
Average	20.01	23.88	8.61	0.74

Note: Data on General Governemnt Employment (% of Total Employmnent), Government Wage Expenditure (% of Total Expenditure), Unemployment Rate for selected OECD countries from 2007-2021. (Source: OECD).

Despite the fact that this “direct” effect of public sector employment on the labour market is well documented in terms of data, so far most research on the cyclical role of the government abstracts from public sector employment; instead most authors focus instead on the age-old question of how large the "government expenditure multiplier" is. This measures how an increase in government expenditure, usually in the form of an increase in public sector consumption, transfer payments, or public sector investment, affects the economy. An important implication of this analysis is that government expenditure only affects the labour market "indirectly" and not "directly".

In this paper, the first chapter of my PhD thesis, I aim to fill this gap in the literature, by investigating both empirically and theoretically the role of public sector employment over the business cycle and seeing how changes in public sector employment "directly" contribute to unemployment over the business cycle.

More specifically, I build a random search and matching model based on Mortensen & Pissarides (1994) but I augment it by including both public sector employment and private sector employment (Quadrini & Trigari, 2007, Navarro et al., 2017, Albrecht et al., 2019). However, and unlike previous research incorporating exogenous public sector employment in simple, frictionless search and matching models, I instead build a realistic labour market by adding two important features. First, I assume an inelastic process of investment in private sector vacancies, using recent advances in the search and matching literature from Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021). More specifically I argue that, unlike standard search and matching models with free entry, where private sector firms can immediately respond to an increase in unemployment by creating as many new vacancies as needed to bring unemployment back to its long-run value, the creation of new vacancies by private sector firms tends to be sluggish and inelastic. This assumption is also backed by empirical data, showing that in many countries business cycles result in large reductions in vacancies and in large and persistent increases in unemployment (Shimer, 2005, 2012). My key insight is that if investment in private sector vacancies is inelastic, then increasing public sector employment has powerful positive effects on unemployment, and crowding out is limited, as there are already many "crowded out" workers.

The second feature I add to this model is that, following Broer et al. (2021) I assume that the job destruction rate in the public sector depends on the stock of public sector employment and the private sector job destruction rate is endogenous. I also assume that the process governing the hirings of new public sector employees is not fully exogenous, but also depends on the level of public sector employment and unemployment in the economy, in contrast to the literature on search and matching models described above. This adds more realism to my framework, allowing me to incorporate data on public sector job destruction rates and hirings of public sector employees and is in contrast to earlier papers, where job destruction rates are equal in both sectors for simplicity, follow an exogenous process or are just calibrated to fit the model (Quadrini & Trigari, 2007, Navarro et al., 2017, Albrecht et al., 2019).

After specifying my model, I compute the steady state equilibrium and calibrate the model parameters using MATLAB and Dynare; I then use the two-quarter, quarterly Labour Force Survey data on employment and unemployment taken from the UK Data Service for a period of 19 years (2003Q1 - 2021Q4) to estimate the values of job destruction rates in the private sector and the public sector and the steady-state values of unemployment, public sector employment and private sector employment. I then study how a public sector employment policy that increases hirings of public sector employees affects unemployment.

Results indicate that increasing hirings of public sector employees leads to a large and persistent unemployment reduction but with some limited crowding out. This is a result of the inelastic investment in private sector vacancies, which means that the stock of private sector vacancies does not decrease in response to the policy shock, so private sector employment remains relatively unchanged. The type of public sector employment policy can also have significant results: A policy based on decreasing the public sector job destruction rate leads to a more prolonged drop in unemployment, but also creates a much larger and longer lasting crowding out effect. When the shock in the hirings of public sector employees lasts longer, unemployment decreases more than the baseline case, but with a larger crowding out, while for a shorter-lived policy the reduction in unemployment is half the one in the baseline case, while crowding out is also smaller. These results indicate that policies based on increasing hirings of public sector employees reduce unemployment more and minimise crowding out, especially if they are short-lived.

I also look at the effects of increasing public sector employment for the standard case of free entry, where investment in private sector vacancies is perfectly elastic, and find that unemployment now increases in response to the policy shock as the private sector freely reduces investment in private sector vacancies and private sector employment. Additionally a countercyclical public sector employment policy, where the government increases hirings of public employees if unemployment goes up, mitigates the negative effect with no significant difference in crowding out. Finally, increasing public sector employment also appears to have large and positive effects on social welfare, which increases in all the cases I study, with the exception of the free entry case and when the public sector is not as productive as the private sector. These results are in agreement with the results of Michaillat (2014), Gomes (2018), Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021) and similar to statistical data (Shimer, 2005, 2012)

The rest of the Chapter is organised as follows. In Section 2, I present the relevant literature. Section 3 focuses on model building and the quantitative analysis. Section 4 analyses the results. Section 5 concludes.

1.2 Related Literature

The fundamental question I research is how public sector employment affects unemployment and the labour market, particularly in recessions, using a search and matching model. This places my paper in the tradition of papers on labour macroeconomics, and in particular search and matching models. This paper also focuses on macroeconomics and fiscal policy, and more specifically on the effect of public sector employment. However, as I discuss below, this literature abstracts almost completely from public sector employment, so filling this gap is one my main planned contributions.

Looking at the labour macroeconomics literature, one of the seminal papers is that of Mortensen & Pissarides (1994), describing a labour market where jobs are destroyed and created each period, while workers and firms "search" the market to create matches. This search and matching model has become one of the most widely used labour market models; however, despite its success, it cannot explain why unemployment exhibits big and persistent increases, while vacancies decrease a lot during recessions, as seen in data for many countries, despite large job destruction shocks (Shimer, 2005, 2012). The reason seems to be the free entry condition in this model. Following this assumption, when unemployment increases, firms immediately create enough vacancies to bring unemployment back to its original level. Based on this critique, Hall (2005) and Hagedorn & Manovskii (2008) assume that there exist no large job destruction shocks, which is not supported empirically.

One reason for the large, persistent unemployment increases and the large decrease in vacancies during recessions is given by Ljungqvist & Sargent (2017, 2021). They develop the idea of the fundamental surplus, which is the quantity deducted from output going to wages, capital payments, production costs and taxes, and not used for creating new vacancies. If this fundamental surplus is a large fraction of output, shocks result in large decreases in vacancies and in large, persistent unemployment increases, as the fraction of output used for creating new vacancies decreases a lot.

Coles & Moghaddasi Kelishomi (2018), build a search and matching model with a more realistic labour market exhibiting large, persistent unemployment increases, large decreases in vacancies and strong job destruction shocks, by relaxing the free vacancy creation process and assuming inelastic vacancy creation, due to investment costs associated with creating vacancies. The implication is that when there is a job destruction shock, there is a large increase in unemployment, which remains above its steady-state value for a long time period, and a large, prolonged reduction in vacancies as firms cannot produce the number of vacancies needed quickly enough.

Looking at public sector employment, a number of researchers have studied its role in the labour market, usually by extending frictionless search and matching models. However their results are mixed: Some (Quadrini & Trigari, 2007, Navarro et al., 2017, Albrecht et al., 2019) find that public sector employment negatively affects aggregate employment, while others (Holmlund & Linden, 1993, Boeing-Reicher & Caponi, 2024) suggest that increasing public sector employment can in fact lower unemployment and stabilize economic turbulence.

In addition, there are many New Keynesian (NK) models incorporating public sector output and public sector employment from a macroeconomic viewpoint (Forni et al., 2010, Economides et al., 2013, 2017, Papageorgiou & Vourvachaki, 2017), but mainly do so for accounting purposes in Representative Agent (RA) or simple Two-Agent New Keynesian (TANK) models of Ricardian and rule-of-thumb households. Among the few NK models studying public sector employment from a macroeconomic viewpoint, Pappa (2009) and Michaillat (2014) find it can positively affect aggregate employment despite some crowding out, especially during recessions. Also, Gomes (2018) shows that governments increasing hirings of public sector employees by lowering wage premia raise aggregate employment with limited crowding out. Conversely Hörner et al. (2007) uses a two-sector model and finds that negative shocks raise unemployment and crowd out the private sector the bigger the public sector is, as workers gravitate to more secure public sector jobs; however Algan et al. (2002) indicates that it is not the size of public sector employment that is responsible for the crowding out, by rather high wage premia, and high substitutability of private and public sector goods, which forces wages to rise and prices to drop in the private sector.

Broer et al. (2021), combines the fundamental surplus, inelastic investment in vacancies and endogenous job destruction rates with imperfectly competitive goods market, sticky prices due to firms' market power, workers' households and capitalists' households, in a Heterogeneous Agent New Keynesian (HANK) model. This combination leads to a large, negative feedback loop between unemployment and output due to the frictional labour market and the features of the HANK block.

I build on the literature studying public sector employment from a macroeconomic viewpoint described above; however, instead of a perfectly elastic job creation process, I assume that investment in private sector vacancies is in fact inelastic as in Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021). This allows me to build a more realistic labour market exhibiting large, persistent unemployment increases and large decreases in vacancies, which is what we see in statistical data on unemployment and vacancies for many countries (Shimer, 2005, 2012), helping me study the effects of increasing public sector employment in a more realistic setup.

Another important novelty of my model is that I directly estimate job destruction rates using quarterly Labour Force Survey data taken from the UK Data Service from 2003Q1 to 2021Q4. Also, I assume that the job destruction rate in the private sector is endogenous, (Broer et al., 2021) and that job destruction rates between private sector and public sector are different, with public sector job destruction rates being much smaller, as these jobs are much more secure. This is in contrast to earlier papers, where job destruction rates are set equal in both sectors for simplicity, follow an exogenous process or are just calibrated to fit the model.

1.3 Model

In this Section I present my search and matching model. I consider a two-sector matching model with a public sector and private sector, similar to Quadrini & Trigari (2007), Navarro et al. (2017) and Albrecht et al. (2019). The hiring process in this model takes place though "random" search and matching. This means that in each period, unemployed workers search for a job in both the public sector and the private sector, randomly "meeting" either one and accept the first job they are offered. Also, following Fontaine et al. (2020) job destruction rates are different in the two sectors, with public sector job destruction rates being much smaller. This is meant to capture the fact that in most countries public sector employment is much safer, as many public sector employees have permanent jobs and fewer public sector employees are fired compared to private sector employees. Finally, private sector vacancies are created inelastically as in Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021) [1].

Regarding public sector employment, the number of hirings of public sector employees at time t, $\mathcal{H}_t^G$, equals posted public sector vacancies at time t, v_t^G, times the probability that a vacancy is filled at time t, q_t, so $\mathcal{H}_t^G = v_t^G q_t$. However, rather than having a fully exogenous process, as seen in other papers, I assume that the number of hirings of public sector employees at time t $\mathcal{H}_t^G$, equals the steady state value of hirings of public sector employees $\mathcal{H}^G$, multiplied by a term capturing the effect that deviations of public sector employment from its steady state value have on the number of hirings of public sector employees, a term capturing the effect of deviations of unemployment from its steady state and an exogenous shock:

$$\mathcal{H}_t^G = \mathcal{H}^G \left[\frac{N_t^G}{N^G}\right]^{\varepsilon_{\mathcal{H},G}} \left[\frac{u_t}{u}\right]^{\varepsilon_u} \chi_t^{\mathcal{H},G}. \tag{1.1}$$

where N_t^G is public sector employment at time t and N^G the steady state public sector employment, u_t is unemployment at time t and u the steady state unemployment. The parameter $\varepsilon_{\mathcal{H},G}$ is a stability parameter ensuring that public sector employment returns to its steady state value after a shock in the hirings of public sector employees[2] and ε_u is a parameter capturing how hirings of public sector employees are affected by changes in unemployment. For my baseline estimation I assume that $\varepsilon_u = 0$ so hirings of public sector employees are completely acyclical,

[1] In this chapter, I do not examine how the public sector finances increases in public sector employment. This is obviously a simplification as tax changes affect investment in private sector vacancies and the whole economy. In Chapters 2 and 3 I also examine the effects of taxation as I use a general equilibrium model, but for now I assume the government levies a lump-sum tax which does not affect the labour market

[2] $\varepsilon_{\mathcal{H},G}$ ensures that shocks in the hirings of public sector employees do not lead to permanent changes in public sector employment. This parameter is negative $\varepsilon_{\mathcal{H},G} < 0$; otherwise public sector employment would keep increasing (decreasing) after an initial increase (reduction) to hirings of public sector employees and completely crowd out (in) the private sector

based on empirical data which show that hirings of public sector employees are relatively acyclical; however I will also be running a counterfactual policy experiment where $\varepsilon_u > 0$ to study the effects of a countercyclical policy. Finally, $\chi_t^{\mathcal{H},G}$ is the exogenous shock on the hirings of public sector employees, with mean equal to unity that follows a stochastic process:

$$\log\left(\chi_t^{\mathcal{H},G}\right) = \rho_{\mathcal{H},G}\log\left(\chi_{t-1}^{\mathcal{H},G}\right) + \left(1-\rho_{\mathcal{H},G}\right)\log\left(\chi^{\mathcal{H},G}\right) + \nu_t^{\mathcal{H},G}. \tag{1.2}$$

where $\rho_{\mathcal{H},G}$ is the autocorrelation parameter and $\nu_t^{\mathcal{H},G}$ a white noise innovation, drawn from a normal distribution with mean zero.

For simplicity and following Michaillat (2014), I assume the public sector wage policy rule is exogenous, and equal to private sector wages. Obviously this assumption is not realistic, however in this chapter it does not matter at all, since I use a random search and matching model with exogenous hiring of public sector employees. As a result, wages play no part in my analysis as unemployed workers do not choose a job vacancy based on the wage, they just randomly "meet" and fill a job vacancy, private or public. Also, as we will see later in this model, private sector wages are not determined by bargaining but are a fixed percentage of output, based on the theory of the Fundamental Surplus (Ljungqvist & Sargent, 2017, 2021), so there are no wage changes which affect my results. As a result, I have:

$$w_t^G = w_t^P, \tag{1.3}$$

where w_t^G is the public sector wage and w_t^P is the private sector wage at time t.

I assume there is a unit mass of individuals who are either employed by the public sector, the private sector or are unemployed. Public sector employment at time t, N_t^G, is the sum of last period's public sector employment stock which was not destroyed at the beginning of the current time period plus the new labour inflows in the public sector at time t:

$$N_t^G = (1-\delta_t^G)N_{t-1}^G + \mathcal{H}_t^G, \tag{1.4}$$

where δ_t^G is the job destruction rate in the public sector at time t.

The evolution of private sector employment is similar. Private sector employment at time t, N_t^P, is the sum of last period's private sector employment stock which was not destroyed at the beginning of the current time period plus the new hirings of private sector employees at time t $\mathscr{H}_t^P$ which is, similarly to hirings of public sector employees, equal to posted vacancies at time t, v_t^P, times the probability that a vacancy is filled at time t, q_t so $\mathscr{H}_t^P = v_t^P q_t$. As a result:

$$N_t^P = (1-\delta_t^P)N_{t-1}^P + \mathscr{H}_t^P, \tag{1.5}$$

where δ_t^P is the job destruction rate in the private sector, which I assume to be endogenous as in Broer et al. (2021). More specifically, the job destruction rate in the private sector at time t, δ_t^P, is equal to the steady state value of private sector job destruction rate δ^P times a term capturing how changes in the value of filled private sector vacancies affect the private sector job destruction rate:

$$\delta_t^P = \delta^P \left[\frac{J_t^F}{J^F}\right]^{-\varepsilon_{\delta,P}}, \tag{1.6}$$

where J_t^F is the value of a filled private sector vacancy at time t and J^F the steady state value of a filled private sector vacancy.

The job destruction rate in the public sector at time t, δ_t^G is also acyclical, similarly to equation 1.1, and equal to the steady state value of public sector job destruction rates multiplied with a term capturing the effect that changes in public sector employment have on job destruction rates in the public sector and an exogenous shock:

$$\delta_t^G = \delta^G \left[\frac{N_t^G}{N^G}\right]^{\varepsilon_{\delta,G}} \chi_t^{\delta,G}. \tag{1.7}$$

As in equation 1.1 the parameter $\varepsilon_{\delta,G}$ is a stability parameter ensuring that public sector employment returns to its steady state value after a shock in the public sector job destruction rate. $\chi_t^{\delta,G}$ is the exogenous shock on the public sector job destruction rate, with mean equal to unity that follows a stochastic process:

$$\log\left(\chi_t^{\delta,G}\right) = \rho_{\delta,G}\log\left(\chi_{t-1}^{\delta,G}\right) + \left(1-\rho_{\delta,G}\right)\log\left(\chi^{\delta,G}\right) + v_t^{\delta,G}, \tag{1.8}$$

where $\rho_{\delta,G}$ is the autocorrelation parameter and $v_t^{\delta,G}$ is a white noise innovation, drawn from a normal distribution with mean zero. For my baseline results, I assume that public sector job destruction rates are completely deterministic, by having $\varepsilon_{\delta,G} = 0$ and $v_t^{\delta,G} = 0$. I then relax this assumption in my counterfactual exercises by giving a numerical value to $\varepsilon_{\delta,G}$ and making $v_t^{\delta,G} \neq 0$ to compare the effects of decreasing public sector job destruction rates to increasing the hirings of public sector employees.

I keep the production side of the economy simple in this chapter, as I focus only on the mechanics of the labour market and how public sector employment affects it, by using the simplest setup available in the literature. All firms are homogeneous and use only labour as an input, which they hire by posting new vacancies, and their productivity (or output) at time t, p_t evolves exogenously according to:

$$logp_t = \rho_p logp_{t-1} + (1-\rho_p) logp + \eta_t, \tag{1.9}$$

where ρ_p is the autocorrelation parameter and η_t a white noise innovation with mean zero.

Unemployment at time t, u_t is defined as all the individuals not employed in any of the two sectors:

$$u_t = 1 - (1-\delta_t^G) N_{t-1}^G - (1-\delta_t^P) N_{t-1}^P. \tag{1.10}$$

Unemployed workers randomly match with open vacancies from either the public or the private sector. The matching technology m_t is a simple Cobb-Douglas equation, similar to Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021):

$$m_t = A u_t^{\gamma} (v_t)^{1-\gamma}, \tag{1.11}$$

where γ is the matching elasticity parameter and A the scale parameter of the matching function.

The total number of vacancies in the economy at time t, is just the sum of private sector vacancies and public sector vacancies, i.e. $v_t = v_t^P + v_t^G$. Following Coles & Moghaddasi Kelishomi (2018) vacancies in the private sector at time t v_t^P, are equal to the intermediate stock of private sector vacancies of the previous time period $v_{o_{t-1}}^P$ plus investment in private sector vacancies at time t, i_t:

$$v_t^P = v_{o_{t-1}}^P + i_t. \tag{1.12}$$

The intermediate stock of private sector vacancies at time t, $v^P_{o_t}$ is equal to the number of vacancies not destroyed in period t minus the number of vacancies filled by unemployed workers at time t:

$$v^P_{o_t} = (1-\delta^P)\left[v^P_t - \zeta^P_t q_t v^P_t\right]. \tag{1.13}$$

where $(1-\delta^P)v^P_t$ is the number of private sector vacancies which are not destroyed and remain open at time t and $(1-\delta^P)\zeta^P_t q_t v^P_t$ the number of vacancies not destroyed that get filled at time t. Following Broer et al. (2021) private sector vacancies at time t are destroyed at a constant rate equal to the steady state value of private sector job destruction rate δ^P.

In a one sector model the number of vacancies that get filled is simply equal to $(1-\delta^P)q_t v^P_t$ as it only depends on the probability a vacancy is filled at time t, q_t times the number of private sector vacancies v^P_t. But in a two-sector random search and matching model the number of private sector vacancies that get filled also depends on the probability that an unemployed worker "meets" and fills a private sector open vacancy at time t, ζ^P_t, which is equal to the ratio of private sector vacancies to total vacancies and decreases as public sector vacancies increase. Therefore, I also add the probability that an unemployed worker "meets" and fills a private sector open vacancy at time t ζ^P_t in equation (13) ($(1-\zeta^P_t)$ being the probability of "meeting" a public sector vacancy at time t) which is equal to the ratio of private sector vacancies to total vacancies:

$$\zeta^P_t = \frac{v^P_t}{v^P_t + v^G_t}. \tag{1.14}$$

As in Coles & Moghaddasi Kelishomi (2018) the aggregate level of investment in private sector vacancies at time t i_t, depends on the steady state value of investment in private sector vacancies and the cost associated with creating new private sector vacancies and is equal to:

$$i_t = i + \xi J_t, \tag{1.15}$$

where ξ is the elasticity of investment in private sector vacancies, i the steady state value of investment in private sector vacancies and J_t denotes the value of an open vacancy in the private sector at time t. The value of an open private sector vacancy J_t at time t is:

$$J_t = -c + \zeta_t^P q_t J_t^F + \beta(1-\delta^P)\left[1-\zeta_t^P q_t\right]\mathbb{E}_t\{J_{t+1}\}, \tag{1.16}$$

where c is the cost of an unfilled vacancy and β the discount factor. $\zeta_t^P q_t$ and $(1-\zeta_t^P)q_t$ are the probabilities that a vacancy in the private sector or the public sector is filled at time t respectively, and J_t^F the value of a filled private sector vacancy at time t.

The parameter ξ is crucial in determining how inelastic investment in vacancies is in the private sector. More specifically as $\xi \to 0$ investment in vacancies in the private sector becomes more inelastic and sluggish and adjusts very slowly, as the free entry condition does not hold anymore. Consequently, negative shocks in the economy create large, persistent increases in unemployment and, large persistent reductions in private sector vacancies. In addition, as $\xi \to 0$, the effect that an increase in public sector employment has on investment in private sector vacancies is mitigated by making the importance of the cost of open vacancies in the private sector smaller. On the other hand, as $\xi \to \infty$ investment in private sector vacancies becomes elastic. This means that the value of an open vacancy at time t J_t is equal to zero at all times and as a result the model becomes a standard search and matching model with free entry.

The probability of "meeting" a private sector vacancy at time t, ζ_t^P is very important in this model because the assumption of "random" search and matching means that this variable creates crowding out in the private sector when public sector employment increases. More specifically, when public sector vacancies at time t, v_t^G increase, this decreases the value of an open vacancy for private sector firms, J_t. The reason is that the government increases the hirings of public sector employees at time t, $\mathcal{H}_t^G$; so it is now less probable a vacancy in the private sector is matched with an unemployed worker and in addition unmatched vacancies have a cost to remain open. As a result the value of an open vacancy, J_t decreases and reduces investment in private sector vacancies i_t and the stock of private sector vacancies v_t^P. However, as explained before, this effect is mitigated by the parameter governing the elasticity of investment in private sector vacancies ξ.

Market tightness at time t, θ_t, determines how the job finding rates of workers change over the business cycles and is equal to:

$$\theta_t = \frac{v_t}{u_t}. \tag{1.17}$$

The probability of a filled vacancy at time t q_t and the probability of a worker finding a job at time t κ_t are respectively $q_t = \frac{m_t}{v_t}$ and $\kappa_t = \frac{m_t}{u_t}$, but following Broer et al. (2021) I use equations 1.11 and 1.17 to write them in terms of market tightness:

$$q_t = A\theta_t^{-\gamma}, \tag{1.18}$$

$$\kappa_t = A\theta_t^{1-\gamma}. \tag{1.19}$$

1.3.1 Value Functions

The value of being unemployed at time t, V_t^U is:

$$V_t^U = z + \beta \mathbb{E}_t \left\{ V_{t+1}^U + \zeta_t^P \kappa_t \left[V_{t+1}^{E,P} - V_{t+1}^U \right] + (1-\zeta_t^P)\kappa_t \left[V_{t+1}^{E,G} - V_{t+1}^U \right] \right\}, \tag{1.20}$$

where z is the value of home production and $\zeta_t^P \kappa_t$ and $(1-\zeta_t^P)\kappa_t$ are the probabilities that a worker finds a job in the private sector and public sector at time t, respectively.

The values of employment in the private sector $V_t^{E,P}$ and in the public sector $V_t^{E,G}$ at time t are respectively:

$$V_t^{E,P} = w_t^P + \beta \mathbb{E}_t \left\{ V_{t+1}^{E,P} + \delta_t^P \left[V_{t+1}^U - V_{t+1}^{E,P} \right] \right\}, \tag{1.21}$$

$$V_t^{E,G} = w_t^G + \beta \mathbb{E}_t \left\{ V_{t+1}^{E,G} + \delta_t^G \left[V_{t+1}^U - V_{t+1}^{E,G} \right] \right\}. \tag{1.22}$$

The value of a filled vacancy in the private sector J_t^F at time t is:

$$J_t^F = p_t - w_t^P + \beta(1-\delta_t^P)\mathbb{E}_t \left\{ J_{t+1}^F \right\}. \tag{1.23}$$

Finally, I follow Ljungqvist & Sargent (2017, 2021) and assume that wages in the private sector are fixed and, as we will see in the Quantitative Analysis subsection, represent a very large percentage of output. This ensures, as the theory of the Fundamental Surplus posits, that the part of output that firms keep and use to invest in vacancies is a very small fraction of overall output, so a negative productivity shock creates very large drops of that fraction, ensuring that job destruction shocks can last for a long time, by greatly limiting the part of output available for investment in private sector vacancies:

$$w_t^P = w^P \tag{1.24}$$

1.3.2 Quantitative Analysis

In this subsection, I simulate the model described above around the certainty-equivalent steady state using Dynare and Matlab. The model is calibrated and estimated in two steps. First, I calibrate the steady state values of the variables in my model using UK data; I also externally calibrate some parameter values by targeting steady state values of the model or using common values from the relevant literature. The parameters which are the most important ones for my analysis, are calibrated using the Simulated Method of Moments (SMM). In this method, the model parameters in question are calibrated to ensure that the real empirical moments that I estimate in the data, match the relevant simulated empirical moments in my model.

1.3.2.1 Calibration

The first step is to set the steady state targets for my model, presented in Table 1.2. I begin by calculating the steady state values of unemployment, public sector employment, job destruction rate in the private sector and job destruction rate in the public sector, as in Fontaine et al. (2020), using the two-quarter, quarterly UK Labour Force Survey data (UKLFS) from the UK Data Service for a time period of nineteen years (2003Q1 to 2021Q4). For the moment I only make a brief discussion of the data; I will make a complete analysis in Chapter 3 where I use data for France, the UK and the US.

The data sample is a rotating panel of five waves of households. Each quarter, one fifth of the sample is renewed as one wave leaves the sample and another wave replaces it. The survey provides information on individual and household characteristics, economic activity and labour market status. Using this data, I construct the stocks of public sector employment, private sector employment and unemployment and the flows between employment and unemployment. All my data are seasonally adjusted, because removing seasonal components allows me to study only the underlying trends and non-seasonal economic fluctuations in the labour market, and detrended using an HP filter with a smoothing parameter of 100000 to separate the cyclical component from the raw data of the time series.

The distinction between public sector employment and private sector employment comes from the survey data where individuals are classified according to their employer. I define public sector employment as individuals working in the Central government, Local government, University or other grant-funded educational establishment, Health authorities/NHS, Armed forces and Nationalised industries/State corporations. My methodology differs from Fontaine et al. (2020) who do not include Nationalised Industries/State Corporations in public sector employment, because in Chapters 2 and 3 I add public sector output in my model, and an important component of it is public sector output produced by these type of firms.

I assume that I have a unit mass of individuals working in either the public sector, the private sector or being unemployed. Starting with the steady state unemployment rate u, I match it with the average unemployment rate in my dataset which equals 5.66%; I also match the steady state public sector employment rate N^G to the average public sector employment as a percentage of the labour force which is 22.73%. As a result, I set the steady state value of private sector employment as N^P at 71.61% targeting a labour force equal to unity. I use the methodology of Fontaine et al. (2020) to calculate the job destruction rates for the public sector and private sector but as I mentioned I also include employees in state owned enterprises in my definition of public sector employment. I then match the steady state job destruction rates with the average values in my data; so I set the public sector job destruction rate δ^G at 2.07% and the private sector job destruction rate δ^P at 3.65%.

Table 1.2: Steady State Target

Parameter	Parameter Name	Value	Source/Target
u	Unemployment	0.0566	Data
N^G	Public Sector Employment	0.2273	Data
N^P	Private Sector Employment	0.7161	Data
δ^P	Job Destruction Rate (Private Sector)	0.0365	Data
δ^G	Job Destruction Rate (Public Sector)	0.0207	Data
θ	Market Tightness	1	Normalise
p	Productivity	1	Normalise
v	Vacancies	0.0566	θ=1
w^P	Private Sector Wage	0.9900	fundamental surplus of 1%

Following the theory of the fundamental surplus of Ljungqvist & Sargent (2017, 2021), I set the steady state private sector wage $w^P = p(1 - fs)$. The fundamental surplus $(1 - fs)$ is the part of output used to pay taxes, wages, production costs that the firm cannot use for investment in vacancies. The steady state values of productivity p and market tightness θ are both normalized to unity, values which are common in the literature. This means that the steady state value of vacancies $v = 0.0566$ is set equal to the unemployment level, targeting a market tightness θ equal to unity (Shimer, 2005, Coles & Moghaddasi Kelishomi, 2018).

The externally calibrated parameters are presented in Table 1.3. Regarding the elasticity parameter of the matching function γ, I follow Coles & Moghaddasi Kelishomi (2018) and set the value of $\gamma = 0.6$. I set $\beta = 0.9984$ targeting an annual discount rate of 4% and the value of home production $z = 0.4$ as in Shimer (2005). Finally I set the autocorrelation of the shock to hirings of public sector employees $\rho_{\mathscr{H},G} = 0.9000$ as in Cantore & Freund (2021), a common value for this type of fiscal policy parameter .

Table 1.3: Externally Calibrated parameters

Parameter	Parameter Name	Value	Source/Target
γ	Matching Function Elasticity	0.6	Coles & Moghaddasi Kelishomi (2018)
β	Discount Factor	0.9984	Annual discount rate 4%
z	Home Production	0.4	Shimer (2005)
$\rho_{\mathscr{H},G}$	Public Sector Hirings Shock Autocorrelation	0.900	Cantore & Freund (2021)

1.3.2.2 Simulated Method of Moments (SMM)

I this subsection, I analyse the Simulated Method of Moments I use to calibrate the main parameter values for my model. First, using the data of the two-quarter, quarterly UK Labour Force Survey data from 2003Q1 to 2021Q4, I estimate the standard deviation of unemployment σ_u, the standard deviation of private sector job destruction rates σ_{δ^P}, the autocorrelation of unemployment $autocorr(u)$, the standard deviation of public sector employment σ_{N^G} and the autocorrelation of public sector employment $autocorr(N^G)$. I also measure the standard deviation of productivity σ_p and the autocorrelation of productivity, $autocorr(p)$ using data on real average output per worker from the Organisation for Economic Co-operation and Development (OECD) as in Shimer (2005). I then simulate the same empirical moments for my model and present the results in Table 1.4.

Table 1.4: Simulation Results

Empirical Moment	Data	Calibration	Source
σ_u	0.1761	0.1727	Directly Estimated
σ_{δ^P}	0.0870	0.0884	Directly Estimated
$autocorr(u)$	0.9670	0.8916	Directly Estimated
σ_{N^G}	0.0305	0.0304	Directly Estimated
$autocorr(N^G)$	0.9006	0.9096	Directly Estimated
σ_p	0.0314	0.0314	Directly Estimated
$autocorr(p)$	0.4504	0.4504	Directly Estimated

As we can see in Table 1.4, the real empirical moments from the dataset and the estimated empirical moments from the model are a very close match, meaning that my model with its simulated empirical moments and its calibrated parameters is a very good approximation of the UK economy and the real empirical moments that come from the data.

In Table 1.5, I present the internally calibrated parameters that I get using the Simulated Method of Moments. I internally calibrate the parameters ξ, fs, $\varepsilon_{\delta,P}$, $\varepsilon_{\mathscr{H},G}$, $\sigma_{\mathscr{H},G}$, ε_p and ρ_p so that the simulated empirical moments in my model match the real empirical moments that I estimated in the data.

Table 1.5: Calibrated Parameters

Parameter	Parameter Name	Value	Source/Target
ξ	Coefficient of Investment in Private Sector Vacancies	1.0000e-06	σ_u
fs	Fundamental Surplus	0.0100	$\sigma_{\delta,P}$
$\varepsilon_{\delta,P}$	Elasticity of Private Sector Job Destruction Rate	-0.2425	$autocorr(u)$
$\varepsilon_{\mathscr{H},G}$	Feedback Parameter	-6.7871	$autocorr(N^G)$
$\sigma_{\mathscr{H},G}$	i.i.d Shock parameter	0.1249	σ_{N^G}
ε_p	i.i.d Shock Parameter	0.0265	σ_p
ρ_p	TFP Shock Autocorrelation Parameter	0.6899	$autocorr(p)$

The two crucial parameters are the elasticity of investment in private sector vacancies ξ as this parameter affects the response of investment in private sector vacancies, and consequently the effect on unemployment, to an increase in public sector employment and business cycles and the fundamental surplus fs which also captures how much investment in private sector vacancies is affected by business cycle shocks. The results of the calibration show that for the UK economy investment in private sector vacancies is extremely inelastic and also that business cycles can have huge impacts on the surplus available to private sector firms for investing in private sector vacancies. In addition, the value of the elasticity of the private sector job destruction rate $\varepsilon_{\delta,P}$ indicates that the private sector in the UK follows a relatively procyclical policy with regards to private sector employment, with the private sector job destruction rate increasing during a recession. Also the value of the feedback parameter $\varepsilon_{\mathscr{H},G}$ is a relatively small number (in absolute terms), which shows that shocks in public sector employment are relatively long-lasting.

1.4 Results

In this section, I look more closely at the effect of an increase in public sector employment. I study a 10-year time period, at monthly frequency, and assume a one-period positive shock in public sector employment, in the form of a one standard deviation increase in the hirings of public sector employees $\mathscr{H}_t^G$, and compare this policy to a positive productivity shock in order to examine the different mechanisms behind each shock and evaluate their effectiveness. I also study the impact of different public sector employment policies, using different values for the elasticity of hirings of public sector employees, $\varepsilon_{\mathscr{H},G}$, and the elasticity of public sector job destruction rate, $\varepsilon_{\delta,G}$. This analysis will help identify how different types of public sector employment policies affect the labour market, and find which policy is the best in reducing unemployment with the smallest crowding out.

Furthermore, I look more closely in the effect of the elasticity of investment in private sector vacancies, by changing the value of ξ. This analysis is particularly interesting as ξ is one of the main driving forces behind my results, governing the response of the private sector to changes in public sector employment, so analyzing its effects can help better understand the underlying forces at work in the labour market. I also examine how the negative effects of business cycles in employment can be mitigated if the rule governing the hirings of public sector employees has a cyclical component, by having it also respond to changes in unemployment. Finally, I analyse the welfare effects of increasing public sector employment when investment in private sector vacancies in inelastic or when it operates under the free entry condition, for shocks differing in size and duration and for different levels of public sector productivity.

1.4.1 Baseline Results

I start my analysis with the effects of the estimated policy response, where the government increases the hirings of public sector employees. This policy experiment is based on the calibration results; I therefore set the feedback parameter, $\varepsilon_{\mathscr{H},G} = -6.7871$. I also set the elasticity of public sector job destruction rate, $\varepsilon_{\delta,G}$, and the exogenous shock term $v_t^{\delta,G}$ equal to zero, in order to focus just on the effects from increasing the hirings of public sector employees. I then compare these results to a positive shock in productivity, in the form of a one standard deviation increase in productivity p_t. This analysis allows me to closely examine the different underlying mechanisms behind each shock and how these affect the labour market and make a first evaluation on the effectiveness of increasing public sector employment.

As we can see in Figure 1.1 the results between the two shocks are qualitatively the same; however quantitatively there are differences in both the size and the duration of the effects. The first difference is seen in Figure 1.1 Panel C, where we find that when hirings of public sector employees increase, the result is a large reduction in unemployment u_t, which peaks at 1.98% and is also very persistent, as the effect last for about 50 months. However for a positive productivity shock the effect is much larger, as the reduction in unemployment is around two times the one I find when increasing public sector employment. The persistence of the positive effect is also larger now, as the effect of the productivity shock is active for 120 months. More importantly, while increasing hirings of public sector employees crowds out of private sector employment N_t^P, as seen in Figure 1.1 Panel D, in the case of the productivity shock private sector employment actually goes up. What is also important is that in both cases, inelastic investment in private sector vacancies leads to large, persistent changes in private sector vacancies and unemployment, mirroring both the behaviour of these variables in the data (Shimer, 2005, 2012) and the reuslts of Coles & Moghaddasi Kelishomi (2018), and Broer et al. (2021), indicating that my model and its calibration is a very good match for the behaviour of these variables seen in the data.

The mechanism behind this result is seen in Panels E to I. To increase the hirings of public sector employees the government raises public sector vacancies v_t^G. This increases market tightness θ in Panel E (which in turn lowers the probability of a vacancy being filled q_t, since $q_t = A\theta^{-\gamma}$) both because unemployment u_t decreases but also because the number of total vacancies v_t goes up. The increase in total vacancies v_t happens because public sector vacancies have increased and because there are now more private sector vacancies v_t^P, which have a cost to maintain, that remain unfilled. Consequently, the increase in total vacancies v_t makes the probability an unemployed worker "meets" a private sector vacancy ζ_t^P in Panel F and investment in private sector vacancies i_t in Panel G decrease and reduces private sector employment; however the coefficient of investment in private sector vacancies ξ is so small that this reduction is practically zero. As a result, private sector employment is crowded out but by a very small amount, as firms just stock up on unfilled vacancies which they cannot destroy for the duration of the policy shock.

A positive productivity shock creates two opposing effects. First it increases the number of private sector vacancies v_t^P and reduces unemployment u_t. However, the increase in vacancies raises market tightness θ_t while the stock of public sector

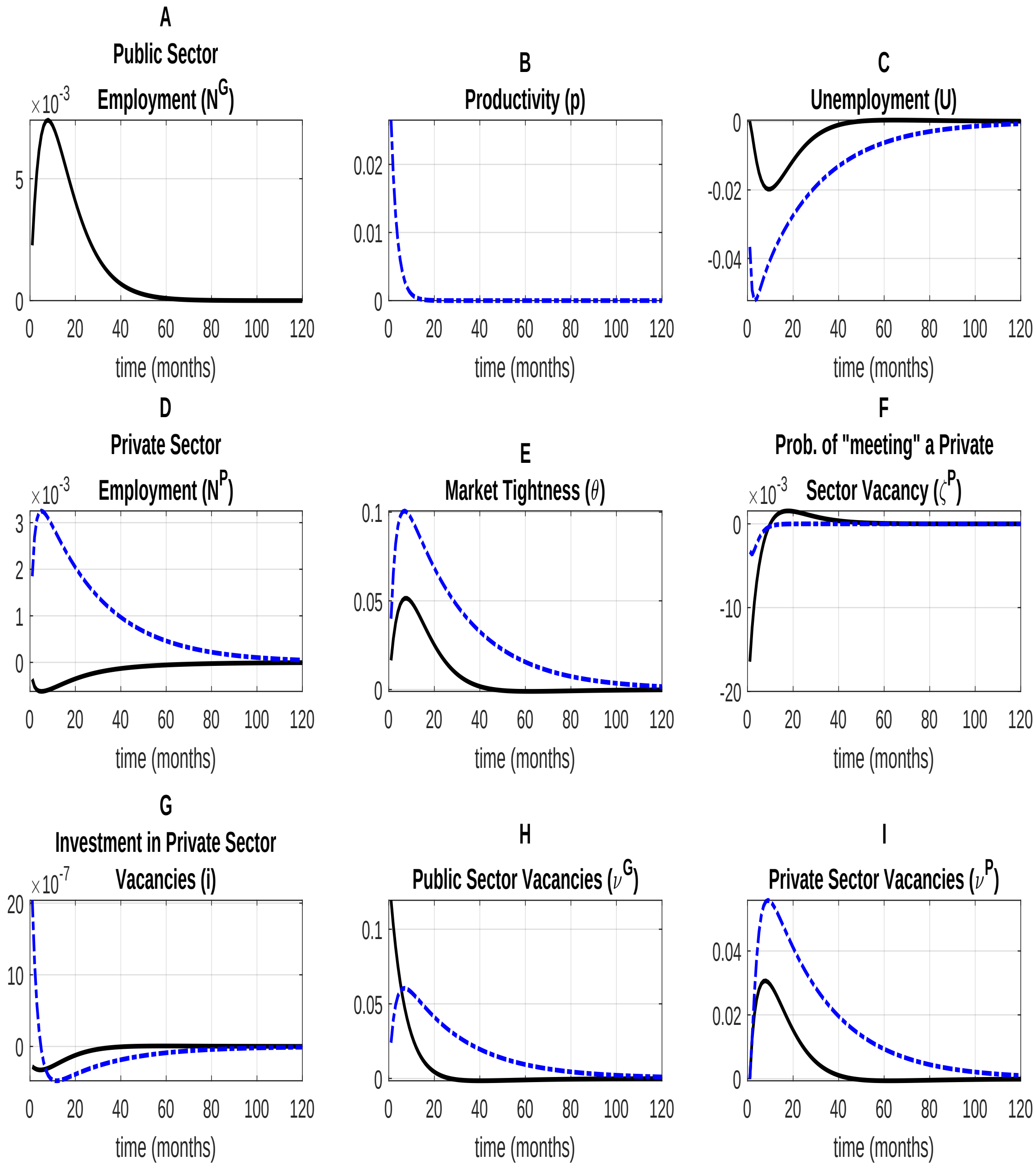

Figure 1.1: Effect of an increase in Public Sector Employment and of an increase in Productivity.

vacancies v_t^G also increases, raising the stock of total vacancies v_t and reducing the probability an unemployed worker "meets" an open vacancy ζ_t^P. This would normally have a negative impact in the labour market by reducing investment in private sector vacancies i_t but inelastic investment in private sector vacancies, together with the strong positive effects by the productivity shock, ensure that private sector vacancies do not reduce, and that the overall effect remains positive. Essentially the inelastic behaviour of the private sector and the very big reduction in unemployment make both sectors stock up on vacancies which they cannot destroy.

1.4.2 Counterfactual Experiments

I now focus on the effects of different public sector employment policies, comparing four distinct cases. I first use the estimated public sector employment policy. I then control for a passive public sector employment policy, where the government only decreases the public sector job destruction rate, a public sector employment policy where the shock on the hirings of public sector employees lasts longer and one where it ends faster. These results are of particular interest as they can help identify how different public sector employment policies, namely increases in the hirings of public sector employees or reductions in the public sector job destruction rate, affect the labour market and which one is optimal in terms of reducing unemployment with the minimum amount of crowding out. Furthermore, this subsection provides information on how these policies should be implemented to achieve the best results in the economy.

As we can see in Figure 1.2 Panel B, the estimated public sector employment policy and a public sector employment policy where the shock ends faster ($\varepsilon_{\mathscr{H},G} = 17.5$) have very similar effects on unemployment. More specifically, and as we have also seen in Figure 1.1, unemployment in the estimated case decreases by 1.98% at its peak while for $\varepsilon_{\mathscr{H},G} = 17.5$ the maximum reduction is around half the one in the baseline case; furthermore the duration of the shock is very close between these two cases, lasting approximately 50 months. Looking at the effects on private sector employment we find that the level of crowding out is also very close here, being around 0.06% and 0.04% respectively, and also lasts around 70 months.

Results are very different for the other two cases. Starting with the passive public sector employment case I find that the reduction in unemployment is larger than the baseline case, as unemployment now decreases by 2.51% at its peak. This effect is also extremely persistent, and in fact much more than the baseline case or any of the other counterfactual cases that I study, as it goes on well beyond the 10-year period. This result also comes at the cost of a much larger crowding out of private sector employment, which decreases by 0.218% at its peak and also seems to carry on well after the 10-year period. Finally the effects of a policy where the increase in the hirings of public sector employees is longer lasting follow a similar path to the passive policy case, as unemployment decreases by 3.43% at its peak, with the effect dying out after 80 months; also the crowding out of private sector employment is smaller compared to the passive policy case, although still considerably bigger than the estimated case and for $\varepsilon_{\mathcal{H},G} = 17.5$, at 0.11%.

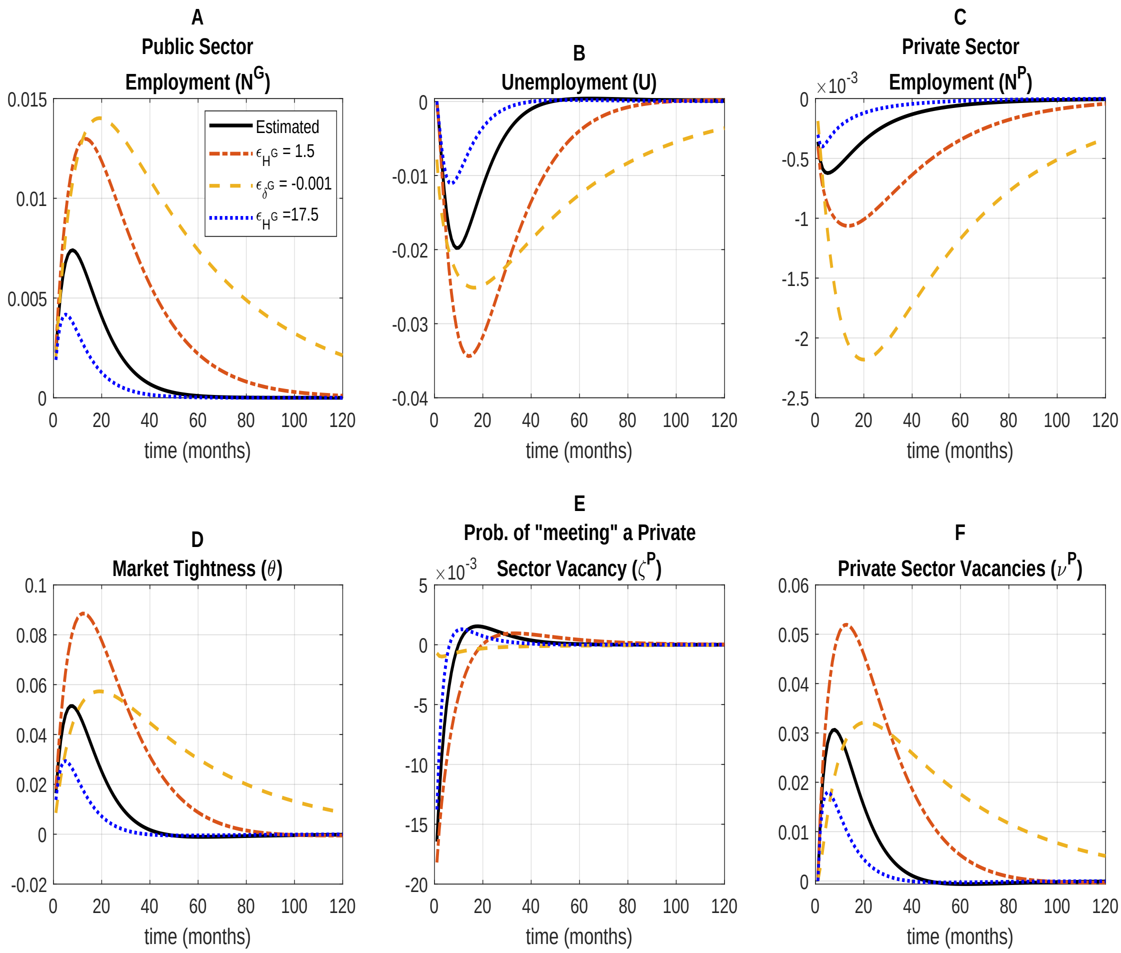

Figure 1.2: Different Public Sector Employment Policies: Estimated Case ($\varepsilon_{\mathcal{H},G}$ = -8.6855, $\varepsilon_{\delta,G}$ = 0), Shorter lasting shock Case ($\varepsilon_{\mathcal{H},G}$ = -17.5, $\varepsilon_{\delta,G}$ = 0), Passive Case($\varepsilon_{\mathcal{H},G}$ = 0, $\varepsilon_{\delta,G}$ = -0.001), Longer lasting shock Case ($\varepsilon_{\mathcal{H},G}$ = -1.5, $\varepsilon_{\delta,G}$ = 0).

These results become clearer in Panels D to F. Starting with Panel D, we can see that the increase in market tightness θ is the smallest for the case of a short lasting increase in hirings of public sector employees, followed by the baseline case. This effect becomes much larger when the public sector job destruction rate decreases and achieves it maximum increase for the case of a longer-lasting increase in the hirings of public sector employees. Also the effects of these policies last much longer compared to the baseline case and a short lasting increase in the hirings of public sector employees. What is also important to note is that when the job destruction rate decreases, the change in market tightness is small and gradual each period compared to the other three policies. At the same time the probability an unemployed worker "meets" an open private sector vacancy ζ_t^P in Panel E behaves differently when the public sector job destruction rate decreases, as it only shows a small reduction.

The differences are explained by the way this policy works. When the public sector job destruction rate decreases fewer jobs in the public sector are destroyed each period and very few public sector vacancies v_t^G are destroyed so they number stays almost unchanged. As a result, workers employed by the public sector who would lose their jobs and would then find a job in the private sector remain in the public sector; however once again private sector vacancies do not change as investment in private sector vacancies i_t is very inelastic. This explains why the probability an unemployed worker "meets" an open private sector vacancy ζ_t^P decreases by a very small amount and why private sector vacancies v_t^P in Panel F are slowly increasing each month, compared to the cases where hirings of public sector employees increase, where the changes are much faster.

These results, although taken with a grain of salt as I only focus on the labour market, provide us with some important insights about how governments should model their public sector employment policies. As the results indicate for the UK, the government should focus on increasing the hirings of public sector employees and not on reducing layoffs as these policies result in smaller crowding out effects in the private sector. Furthermore, these policy changes should be relatively short in duration, as prolonged continuation of such programs also contributes to larger private sector employment crowding out.

1.4.3 Different Vacancy Investment Elasticity

In this subsection, I focus on how the elasticity of investment in private sector vacancies affects unemployment and the labour market. The parameter ξ is one of the main driving forces behind the results in my model, as it governs the response of the private sector to changes in public sector employment, so analyzing its effects can help better understand the underlying forces at work in the labour market. To this end, I look an inelastic case, where $\xi = 0.09e-08$, the estimated case of $\xi = 1.0000e-06$ and the free entry case of $\xi = 100000$.

As we can see in Figure 1.3, results are markedly different in the free entry case when compared to the other two cases. More specifically, the estimated and inelastic cases are practically unchanged, as unemployment decreases by 1.98% below its steady state 4 months after the shock happens, while private sector employment drops by about 0.06%. This implies that in the UK economy investment in private sector vacancies is extremely inelastic, although again one should take into account that this model only focuses in the labour market.

For the free entry case results are completely different. Increasing hirings of public sector employees now increases unemployment, which goes up by 0.79% and also leads to a very quick and large crowding out of private sector employment, which decreases by 0.22% only five months after the shock starts. This happens because, as we can see in Panel F, private sector vacancies now actually decrease. As in the other two cases, to increase the hirings of public sector employees the government creates more public sector vacancies v_t^G, which changes market tightness θ and the probability of "meeting" a private sector vacancy ζ_t^P. Since there are now more private sector vacancies that remain unfilled, and because private sector vacancies costs money to maintain, the value of an open vacancy J_t decreases. Now however, because the coefficient of investment in private sector vacancies $\xi = 100000$, the private sector can actually reduce investment in private sector vacancies i_t. As a result, this lowers private sector employment much more than the other two cases.

Concluding, these results indicate that increasing public sector employment can decrease unemployment but with some limited crowding out, provided that investment in private sector vacancies is relatively inelastic in the economy, the government focuses on increasing the hirings of public sector employees instead of reducing public sector job destruction rates, and that such policies are active for a limited time period. These conditions are necessary for the government policy to be successful; otherwise the recovery of the private sector is impeded as private sector employment is crowded out.

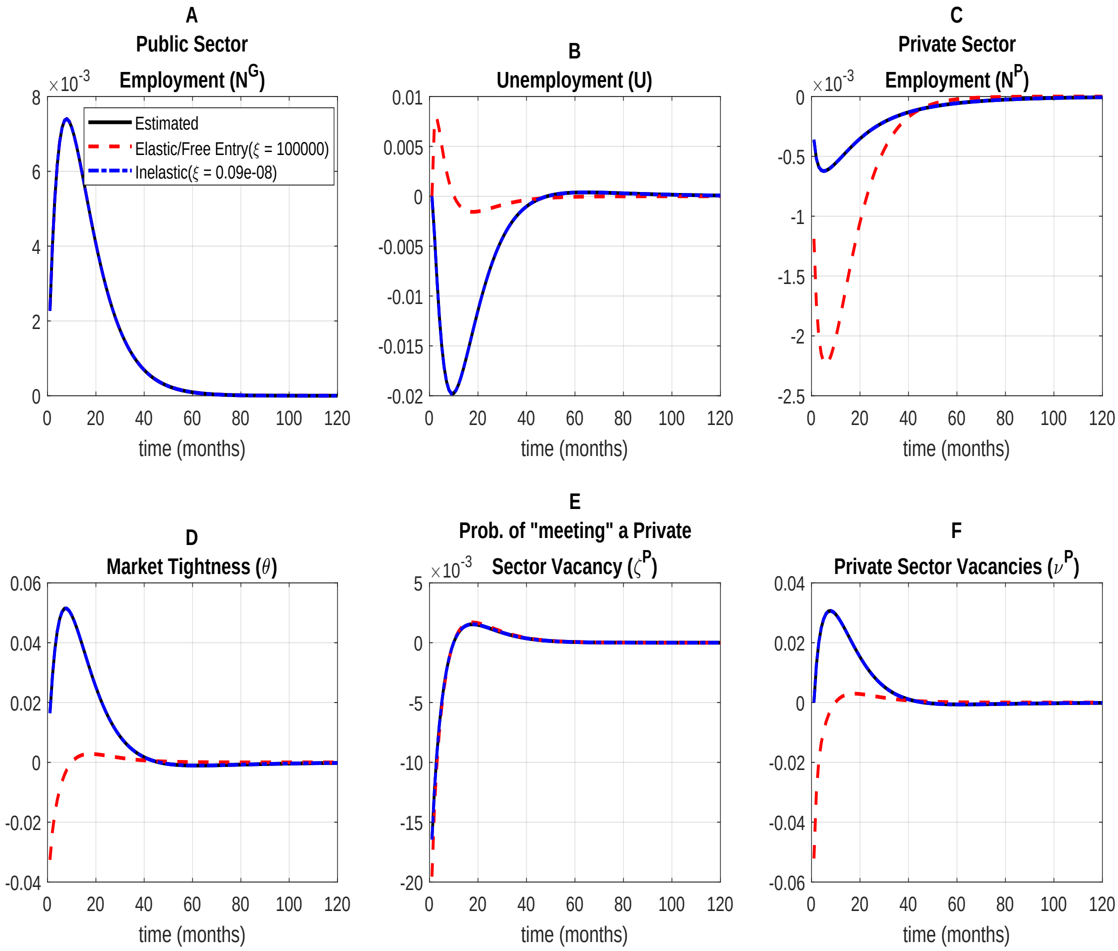

Figure 1.3: Different elasticity of investment in private sector vacancies: Estimated Case ($\xi = 1.0016e-06$), Inelastic Case ($\xi = 0.09e-08$), Free Entry Case ($\xi = 100000$).

1.4.4 Countercyclical Public Sector Employment

Finally, I look at the effect of increasing public sector employment when the economy is hit by a negative productivity shock. In this subsection I study the effect of a negative shock in productivity, in the form of a one standard deviation decrease in productivity p_t for the baseline estimated case and for a counterfactual case where hirings of public sector employees also respond to changes in unemployment.

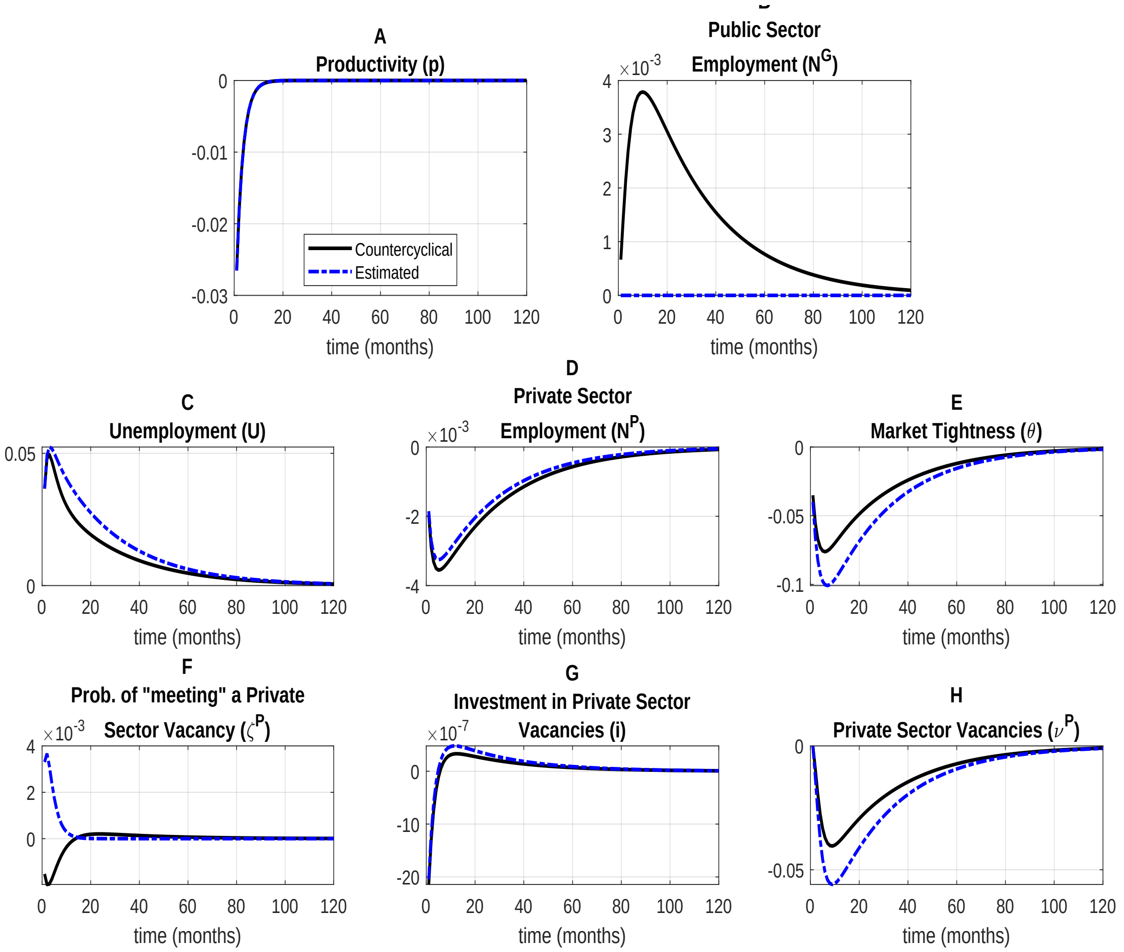

Figure 1.4: Policy response to a productivity shock: Estimated Case, Countercyclical Public Sector Employment Case.

As we can see in Figure 1.4 Panel C, a negative productivity shock in the estimated case, leads to a large, persistent increase in unemployment, which increases by 5.18% at its peak and lasts for nearly the whole 10-year time period. However, a countercyclical public sector employment policy can significantly mitigate this negative effect, decreasing the rise in unemployment by 0.8 percentage points in the first few months. 12 months after the shock, increasing public sector employment by increasing the hirings of public sector employees achieves the maximum reduction in the rise in unemployment by 1.02 percentage points. What is also important is that the difference in private sector crowding out is very small, being only 0.03 percentage points at its peak. These results are also similar to Michaillat (2014), and Gomes (2018) who find that public sector employment can help reduce unemployment with limited crowding out, particularly during recessions.

The reason for the larger crowding out in the countercyclical case is is evident in Panels E to H where we see that private sector vacancies at time t v_t^P, decrease more in the estimated case compared to the countercyclical case. What this implies is that for a countercyclical policy the private sector stocks up more vacancies. More specifically, what happens in the baseline case is that the probability that a private sector vacancy is filled, which is the product of the probability that a vacancy is filled at time t q_t times the probability of "meeting" a private sector vacancy at time t ζ_t^P, actually increases. The reason for that is that there are more unemployed people available to fill private sector vacancies in the estimated case than in the case where public sector employment also increases, so the intermediate stock of private sector vacancies $v_{o_t}^P$ private sector vacancies at time t v_t are depleted more quickly.

1.4.5 Welfare Effects of Public Sector Employment

My analysis so far has indicated that public sector employment has a positive effect on the labour market, reducing unemployment with very small crowding out effects. However, to fully study the effects of increasing public sector employment it is necessary to examine how this policy affects the whole economy, not just the labour market. To this end, I now turn my attention to the welfare implications of public sector employment. More specifically I look how output in the economy and the costs associated with private sector vacancies change in response to public sector employment and determine if increasing public sector employment can indeed improve welfare or not.

Before moving on with the welfare analysis I first need to make a few assumptions. So far my model assumes that the productivity process in equation 1.9 only applies to the private sector and that public sector employees are simply hired and do not produce anything[3]. I now change this assumption and say that the public

[3] This is similar to idea of Keynes (1936) that "The government should pay people to dig holes in

sector is also producing goods (which have no price and are consumed freely by everyone) and that the productivity in the two sectors is the same so $p_t^G = p_t^P = p_t$. I will then relax this assumption and examine the welfare implication of a less productive public sector where $p_t^G = 0.5$, allowing me to examine the impact that different productivity levels in the public sector have on my results.

I use the simplest setup possible for the production of goods in the two sectors. The production functions in the public sector and in the private sector are $Y_t^G = p_t N_t^G$ and $Y_t^P = p_t N_t^P$ respectively, while production by unemployed workers is $Y_t^U = zu_t$. Therefore aggregate output in the economy at time t Y_t, is just the sum of production by the public sector and the private sector plus home production:

$$Y_t = Y_t^G + Y_t^P + Y_t^U. \tag{1.25}$$

The costs in this economy are the sum of the cost maintaining open private sector vacancies, which comes from equation 1.16 and is equal to the cost of a vacancy plus the number of private sector vacancies, cv_t^P, plus the cost of investment in private sector vacancies which follows a quadratic adjustment process $\frac{1}{2\xi}(i_t - i)^2$. As a result welfare in the economy is just the sum of aggregate output minus all the costs associated with private sector vacancies:

$$Welfare_t = Y_t - cv_t^P - \frac{1}{2\xi}(i_t - i)^2. \tag{1.26}$$

In Figure 1.5, I present the effects that increasing public sector employment has on welfare. I focus on 5 cases: I start with the estimated case where I use the values of my baseline calibration and assume equal productivity between the two sectors, and I then examine the case where the productivity of the public sector is half that of the private sector, to see how this affects the results. I also examine the results for the counterfactual case where the shock on the hirings of public sector employees lasts longer ($\varepsilon_{\mathcal{H},G} = 1.5$) and for a larger shock, namely a 2-standard deviation increase in the hirings of public sector employees, to indicate if there any changes resulting from differences in either the size or the duration of the policy shock. Finally, I look at the effects under the assumption of free entry by setting the coefficient of investment in private sector vacancies $\xi = 100000$.

the ground and then fill them up" during recessions.

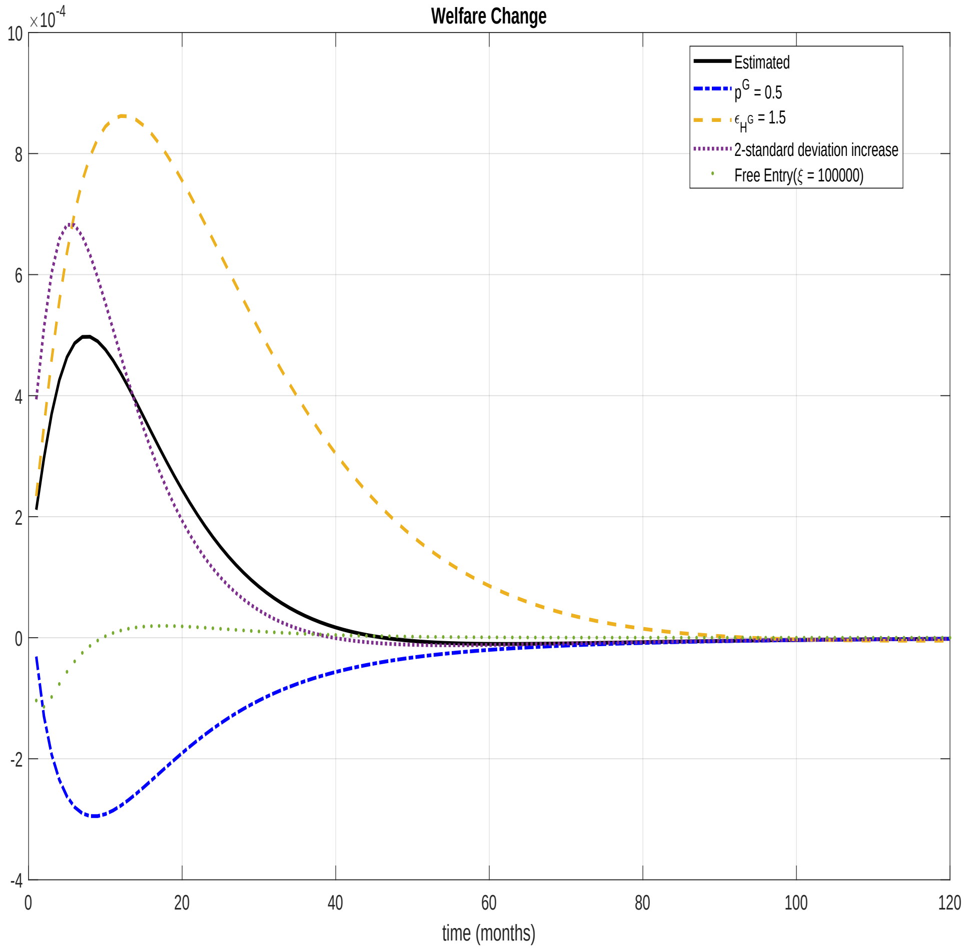

Figure 1.5: Welfare Effects of Public Sector Employment.

Results in Figure 1.5 indicate that under inelastic investment in private sector vacancies, and assuming equal productivity between the two sectors, increasing public sector employment by increasing hirings of public sector employees has positive and persistent effects on social welfare, that become larger and more persistent when the duration or the size of the shock increases. This result depends on the fact that, as seen in the previous subsections, investment in number of private sector vacancies is highly inelastic and does not change so private sector firms essentially stockpile empty vacancies. The result on social welfare turns negative for the case of free entry, however it should be noted that the reduction is very small and only lasts for a few months. Only when the public sector is half as productive as the private sector, does increasing public sector employment reduce social welfare.

Building on these findings and on the results in the previous subsections, it seems safe to argue that temporary increases in hirings of public sector employees during recessions can have powerful effects on unemployment and improve social welfare, as long as public sector employment is not too unproductive. However, permanent increases in public sector employment can be more costly, due to greater crowding out, so public sector employment increases during recessions should truly be temporary to avoid crowding out later. Also, it should be pointed out again that this model only covers the labour market and that I will be studying the full effects of increasing public sector employment in the next two chapters so my findings here, while indicative of the effects that public sector employment has, should be taken with a grain of salt as they do not cover the whole economy.

1.5 Conclusion

In this paper I investigated how increasing hirings of public sector employment affects the labour market when unemployment is particularly persistent. I built a random search and matching model with two novel features: First, inelastic investment in private sector vacancies. Second, hirings of public sector employees are not fully exogenous, and the private sector job destruction rate is endogenous and greater than the public sector job destruction rate. I simulate my model using the two-quarter, quarterly UK Labour Force Survey data from 2003Q1 to 2021Q4, and estimate how increasing hirings of public sector employees affects unemployment and private sector employment under different policies, different degrees of elasticity in investment in private sector vacancies and when the economy is hit by negative shocks in productivity.

I find that increasing hirings of public sector employees leads to a large, persistent unemployment reduction, with limited private sector employment crowding out, due to the inelastic investment in private sector vacancies which ensures private sector vacancies and private sector employment stay mostly unchanged. The type of public sector employment policy also affects my results as reducing public sector job destruction rates leads to more prolonged unemployment drops but also to larger, longer lasting crowding out of private sector employment. I find that the biggest reduction in unemployment comes when the shock in the hirings of public sector employees lasts longer while private sector employment crowding out does not significantly change. Conversely for a shorter-lived policy both the maximum reduction in unemployment and crowding out are smaller. This shows that policies based on increasing public sector employees achieve the best results as they decrease unemployment more and minimise crowding out, especially if they are short-lived, compared to policies targerting the job destruction rate.

I also look at the effects of increasing public sector employment for the standard case of free entry, where investment in private sector vacancies is perfectly elastic, and find that unemployment now rises as the private sector freely reduces investment in private sector vacancies and private sector employment. Additionally, a countercyclical public sector employment policy, where the governments increases hirings of public employees if unemployment goes up, significantly mitigates the negative effect with no significant difference in crowding out. Finally, increasing public sector employment also positively affects social welfare, with the exception of the free entry case and when the public sector is significantly less productive.

A limitation of my model is that it focuses only on the labour market and cannot analyze the effects in the rest of the macroeconomy. Therefore, in my second chapter I combine it with a Heterogeneous Agent New Keynesian (HANK) model (Broer et al., 2021, Ravn & Sterk, 2017, 2021), where public sector firms produce goods using government expenditure on consumption and public sector employment[4]. The resulting model is one of the very few combining heterogeneous households and incomplete markets, monopolistic competition and sticky prices with a frictional labour market, and the first to study public sector employment and public sector output, ensuring results that are both novel, as public sector employment has not been adequately studied, and realistic, as it can capture all the propagation mechanisms and effects in an economy, unlike earlier, simpler models and my Chapter 1 model.

[4]Pappa (2009), Forni et al. (2010), Economides et al. (2013; 2017)

CHAPTER 2

A Heterogeneous Agent New Keynesian Model of Search Frictions and Public Sector Employment

2.1 Introduction

A major macroeconomic trend in most countries over the last century is that government expenditure is a large part of the economy ranging between 45% and 55% and has several components: The public sector buys goods and services, redistributes income, invests in infrastructure, and provides education, healthcare and social welfare. It is however an underappreciated fact in business cycle research that a large part of government expenditure is composed of public sector employment and public sector output; in fact one could safely argue that the public sector is the single biggest employer in most countries as it hires numerous workers to produce goods and services. However, despite their prevalence, public sector employment and public sector output are mostly overlooked as research focuses on government expenditure on consumption, investment and transfers.

Given this lack of research, it is unclear how increasing public sector employment and public sector output affects the economy and whether they can crowd in labour or crowd it out. In this paper I aim to fill this gap in the literature, studying how increasing public sector employment and public sector output affect aggregate output and employment by analyzing the effects and propagation mechanisms in a model economy and quantitatively evaluating these policies. To this end, I build a HANK model of heterogeneous households, incomplete asset markets and frictional goods market, labour market and asset market (Broer et al., 2021, Ravn & Sterk, 2017, 2021), where public sector firms produce goods using government expenditure on consumption and public sector employment[1].

The HANK block features a continuum of ex-ante heterogeneous households in productivity, split into workers households and capitalists households. Capitalists' households are homogeneous, own firms and invest in public sector bonds. Workers households are ex-post heterogeneous working in public sector firms, private sector firms or being unemployed, and as a result face different employment/unemployment probabilities. Asset markets are incomplete, as workers households invest in a zero-net supply household bond, so the sum of asset holdings of workers households zero, and face a no-borrowing constraint, so individual asset holdings of workers households are also zero. Combining heterogeneous workers households and incomplete asset markets means that the workers households face uninsurable unemployment risk as they cannot insure themselves against changes in their employment condition. Finally the goods market features monopolistic competition and sticky prices due to adjustment costs.

This block is akin to the most novel models: Two-Agent New Keynesian (TANK) models of sticky prices, monopolistic competition in the goods market and heterogeneous households, where workers households supply labour and invest subject to frictions and capitalists households do not work but own assets,[2] and HANK models of sticky prices and monopolistic competition in the goods market, heterogenenous households varying in skills, labour supply and asset market participation and incomplete asset markets[3]. The framework I use creates a rich model of heterogeneous households and incomplete asset markets, uninsurable risk and inequality dynamics, goods market and asset market frictions, where increasing public sec-

[1] Pappa (2009), Forni et al. (2010), Economides et al. (2013; 2017)

[2] Auclert et al. (2018), Bilbiie (2020), Cantore & Freund (2021),Courtoy (2022), Klein et al. (2022)

[3] Auclert et al. (2018; 2020), Hagedorn et al. (2019), Kaplan & Violante (2014), McKay & Wolf (2022)

tor firms' output and public sector employment increases aggregate output and employment and creates dynamic variable effects and redistribution channels.

In the labour market block I use the model I built in Chapter 1 (Coles & Moghaddasi Kelishomi, 2018, Broer et al., 2021): A random SAM model with no on the job search where instead of free entry I assume inelastic investment in private sector vacancies determined by the value of open vacancies. This friction is crucial in my model, as it ensures that increasing public sector employment reduces unemployment with only small crowding out, and also makes it more realistic as recessions result in large, persistent increases in unemployment and large reductions in vacancies matching the pattern of these variables seen in many countries' data (Shimer, 2005, 2012). Also, in addition to the vacancy filling and job finding probability, I use the probability an unemployed workers household finds a private sector job, which affects the value functions of workers households and private sector firms and crowds out private employment if public sector employment increases (Navarro et al., 2017). Finally, I use two job destruction rates with public sector job destruction rates being lower, as public sector employment is more secure (Fontaine et al., 2020).

These blocks help fully identify all the effects and propagation mechanisms of the policies I study. In fact, my model is one of the few combining heterogeneous households uninsurable risk and incomplete asset markets, monopolistic competition and sticky prices with a frictional labour market, and the first to study public sector employment and public sector output. Both these variables have not been adequately studied, so my research helps cover a significant gap in macroeconomics and fiscal policy literature, by focusing on an important but largely overlooked component of fiscal policy, and offers a new innovative way of thinking about the role of the public sector during recessions, which has both academic but also policy implications. In addition my results are both novel and realistic, as I use a comprehensive and realistic model capturing all the propagation mechanisms and effects in an economy, unlike earlier, simpler models while the frictions in my model allow it to simulate the behaviour of labour market variables such as unemployment and job vacancies but also public sector variables such as public debt and government expenditure.

Initially, as public sector employment increases, it increases public sector firms' output. This directly increases aggregate output, creating a *Direct Effect*. What makes my analysis unique however, are the *Indirect Effects* which show how the private sector is affected. As public sector employment rises, the probability an unemployed workers household fills a private sector vacancy drops. This *Labour Market Effect* operates through labour market frictions, and specifically the elasticity of vacancy investment, and is novel in my model, reducing the value of open vacancies and investment in private sector vacancies. This effect then propagates to the whole economy, reducing aggregate demand and inflation, and raises private sector job destruction rates. As a result, private sector vacancies, private sector employment private sector firms' output, and aggregate output decrease.

The HANK block features a two-prong *Aggregate Demand Effect*, whose strength depends on price stickiness. First a *Redistribution Effect* makes unemployed workers households hired by public sector firms consume more raising aggregate demand, inflation, private sector firms' output and wages, and indirectly increasing aggregate output. In the next periods, additional *Indirect Aggregate Demand Effects* propagate in the labour market, as higher inflation raises the value of filled vacancies and investment in private sector vacancies and lowers private sector job destruction rates. This raises private sector employment, private sector firms' output, wages and inflation further and indirectly increases aggregate output even more.

Finally an *Unemployment Risk Effect* is created by heterogeneous workers households and incomplete asset markets. The *Direct Effect* and *Aggregate Demand Effect* lower unemployment risk of workers households and precautionary savings, raising the interest rate of household bonds and aggregate demand. This further increases inflation, investment in private sector vacancies, private sector employment, private sector firms' output and wages, and increases aggregate output even more. However the *Labour Market Effect* also negatively affects this channel so the overall result is ambiguous.

Applying this model and its effects in the US economy, I find that increasing public sector firms' output by raising public sector employment, lowers unemployment and, more importantly, increases private sector employment. This is a result of the relatively small size of US public sector and its short lived policy shocks, which combined with the inelastic investment in private sector vacancies and the private sector job destruction rate and flexible wages, limit crowding out from the *Labour Market Effect*. Furthermore aggregate output also increases, although this effect is a bit smaller as the very same features of the US economy that limit crowding out also make the *Direct Effect Aggregate Demand Effect* and the *Unemployment Risk Effect*, and the effects on the goods market smaller.

2.2 Related Literature

My paper relates to three strands of the macroeconomics literature, the first being HANK models. In early Real Business Cycle models of households intertemporally smoothing out consumption, perfectly competitive goods market and flexible prices fiscal policy is largely ineffective, while New Keynesian (NK) models of financially constrained households, monopolistic competition in the goods market and sticky prices find multipliers above one. However, these models cannot replicate the dynamic variable responses in data(Auclert et al., 2018, Hagedorn & Manovskii, 2008, Cantore & Freund, 2021), so TANK models with both household types, sticky prices, and monopolistic competition in the goods market were built(Galí et al., 2007, Bilbiie, 2008). Now increasing government expenditure increases output, with multipliers above one, but most variables still only replicate empirical findings on impact.

This led to more complex models: TANK models of heterogeneous households, where workers households supply labour and save, subject to labour and asset market frictions, capitalists households do not work but own assets, sticky prices and monopolistic competition in the goods market(Auclert et al., 2018, Bilbiie, 2020, Cantore & Freund, 2021, Courtoy, 2022, Klein et al., 2022) and HANK models of sticky prices and monopolistic competition in the goods market, heterogenenous households varying in skill, labour supply and asset market participation and incomplete asset markets (Auclert et al., 2018, 2020, Hagedorn et al., 2019, Kaplan & Violante, 2014, McKay & Wolf, 2022). Increasing government expenditure still increases output with multipliers above one but also creates dynamic, realistic variable effects, as heterogeneous households and incomplete asset markets, monopolistic competition in the goods market and sticky prices create unisnurable risk, households' heterogeneity, goods market, asset market, and labour market frictions.

My model also relates to the macro/labour literature, starting with the SAM model of Mortensen & Pissarides (1994), where jobs are destroyed and created and workers and firms "search" the market, creating matches. This model is one of the most widely used, but cannot capture the big, persistent unemployment increases and the drop in vacancies during recessions found in data (Shimer, 2005, 2012). Based on this critique, Hall (2005) and Hagedorn & Manovskii (2008) assume no large job destruction shocks exist, which is unsupported empirically. Ljungqvist & Sargent (2017, 2021) put forth the idea of the fundamental surplus, which is the part of output used for wages, production costs and taxes and not for investment in vacancies, and posit that if it is a large part of output, then shocks decrease investment in vacancies, creating large, persistent unemployment increases. Coles & Moghaddasi Kelishomi (2018), build a realistic labour market by assuming inelastic vacancy creation, and find that job destruction shocks now create both large, persistent increases in unemployment and large drops in vacancies.

Finally, my paper builds on the fiscal policy literature, namely frictionless SAM models where public sector employment negatively affects aggregate employment (Quadrini & Trigari, 2007, Navarro et al., 2017, Albrecht et al., 2019), and NK models adding public sector output and public sector employment for accounting purposes (Forni et al., 2010, Economides et al., 2017, Papageorgiou & Vourvachaki, 2017). Among the few NK models studying public sector employment and public sector output Pappa (2009) and Michaillat (2014) find that they raise aggregate output and employment, despite some crowding out, especially in recessions; Gomes (2018) also finds similar results in a heterogeneous agent setup.

In my model, I combine the frictional SAM labour market, public sector employment and public sector firms and a HANK framework. Very few models combine a HANK model with a frictional labour market, and this is the first model of this type studying public sector firms' output and public sector employment. This makes my results both novel, but also realistic, as I can study all possible effects of this policy by combining the effects from the HANK block with those from the SAM block.

The rest of the chapter is organised as follows. Section 3 focuses on model building and calibration. In Section 4 I analyse my results. Section 5 concludes.

2.3 Model

2.3.1 Households

My model is a HANK model of heterogeneous households, incomplete asset markets and frictions in the goods market, labour market and asset market (Broer et al., 2021, Ravn & Sterk, 2017, 2021). The economy is populated by a continuum of infinitely lived heterogeneous households, featuring two forms of heterogeneity. Households are ex-ante heterogeneous in terms of productivity with $z_i \in \{0,1\}$ being the ex-ante determined, constant and household-specific productivity. Households with full productivity $z_i = 1$ are workers households and households with zero productivity $z_i = 0$ are capitalists households.

Workers households are a continuum of mass 1 indexed by $i \in [0,1]$, and are also ex-post heterogeneous, supplying labour to private sector firms, public sector firms, or being unemployed. I denote the employment state of each individual workers household by $S = \left\{G, P, U\right\}$ where G means working in public sector firms, P working in private sector firms and U being unemployed. Asset markets are incomplete as workers households can save but not borrow in a zero net supply household bond, meaning that the sum of asset holdings for all workers households is zero. Additionally workers households face a no-borrowing constraint, so the individual asset holdings of workers households are also zero. Combining heterogeneous workers households and incomplete asset markets means workers households face uninsurable unemployment risk as they cannot insure themselves against changes in their employment condition; additionally, changes in the unemployment risk affect demand for savings and demand for goods by workers households.

Capitalists' households indexed by $i \in [1, 1 + pop_C]$, with $pop_C << 1$ are homogeneous and do not supply labour, own the economy's private sector firms and invest in public sector bonds, but not in household bonds.

2.3.1.1 Workers Households

The representative workers household i has preferences over consumption, represented by the following lifetime utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t^S \left(C_{t,i}^S, N_{t,i}^{S,W} \right), \tag{2.1}$$

where $C_{t,i}^S$ is consumption of workers household i at time t and $N_{t,i}^{S,W}$ the labour supply of workers household i at time t. The period utility function is:

$$U_{t,i}^S = \frac{(C_{t,i}^S)^{(1-\eta)}}{1-\eta} - \zeta N_{t,i}^{S,W}, \tag{2.2}$$

where η is the risk aversion parameter and ζ the disutility of labour.

Employed workers households receive a wage at time t which is taxed at a rate τ_t^N while unemployed workers households receive an unemployment benefit and do not pay taxes. I assume that each workers household inelastically supplies one unit of labour (so $N_{t,i}^{G,W} = 1$ for workers households employed by public sector firms, $N_{t,i}^{P,W} = 1$ for workers households employed by private sector firms and $N_{t,i}^{U,W} = 0$ for unemployed workers households). After-tax wages and unemployment benefits are either invested or used for consumption. Workers households have the following budget constraint:

$$C_{t,i}^S + B_{t,i}^S \leq (1-\tau_t^N) w_t^S N_{t,i}^{S,W} + b_t (1 - N_{t,i}^{S,W}) + B_{t-1,i}^S \frac{R_{t-1}}{\pi_t}. \tag{2.3}$$

$B_{t,i}^S$ is the quantity of bonds employed workers households invest in at time t (the value of bonds at time t is $B_{t,i}^S P_t$). Wages at time t are w_t^S and b_t is the unemployment benefit, while $\frac{R_{t-1}}{\pi_t}$ is the rate of return adjusted for inflation at time t, π_t. The Bellman equation for workers households is:

$$V_{t,i}^S(\Omega_{t,i}^S) = \max_{C_{t,i}^S, B_{t,i}^S} \left\{ \frac{(C_{t,i}^S)^{(1-\eta)}}{1-\eta} - \zeta N_{t,i}^{S,W} + \beta \mathscr{W}_{t,i}^S(\Omega_{t+1,i}^S) \right\}$$

$$s.t.\ C_{t,i}^S + B_{t,i}^S \leq (1-\tau_t^N) w_t^S + b_t (1 - N_{t,i}^{S,W}) + B_{t-1,i}^S \frac{R_{t-1}}{\pi_t}, \tag{2.4}$$

$$C_{t,i}^S \geq 0.$$

$$B_{t,i}^S \geq 0.$$

Where $\Omega^S_{t,i} = \left[B^S_{t-1,i}, N^{S,W}_{t,i}\right]$ is the idiosyncratic state variable and $\mathscr{W}^S_{t,i}(\Omega^S_{t+1,i})$ is the post decision value functions for the representative workers household i at time t. Combining Equations, I get the intertemporal maximization problem for workers households:

$$V^S_{t,i}(\Omega^S_{t,i}) = \max_{C^S_{t,i}, B^S_{t,i}} \left\{ \left\{ \frac{(C^S_{t,i})^{(1-\eta)}}{1-\eta} - \zeta N^{S,W}_{t,i} + \beta \mathscr{W}^S_{t,i}(\Omega^S_{t+1,i}) \right\} + \lambda^S_{t,i} \left[(1-\tau^N_t) w^S_t N^{S,W}_{t,i} \right.\right.$$

$$\left.\left. + b_t(1-N^{S,W}_{t,i}) + B^S_{t-1,i}\frac{R_{t-1}}{\pi_t} - C^S_{t,i} - B^S_{t,i} \right] + \mu^S_{t,i}\left[B^S_{t,i} - B^S_i\right] \right\}. \tag{2.5}$$

The post decision value functions for workers households $\mathscr{W}^S_{t,i}(\Omega^S_{t+1,i})$ depends on their future idiosyncratic state and future labour market condition. More specifically workers households at time t face different future employment conditions, which depend on the labour market conditions they will face at time $t+1$: Employed workers households can lose their job and stay unemployed, remain employed in the sector they work, lose their job but get hired in the different sector, or lose their job but get rehired in the same sector. At the same time unemployed workers households can remain unemployed or get hired in one of the two sectors. The post decision value functions for workers households at time t are respectively:

$$\mathscr{W}^G_{t,i}(\Omega^S_{t+1,i}) = \mathbb{E}_t \left\{ \delta^G_t (1-\kappa_{t+1}) V^U_{t+1,i} + \left[1 - \delta^G_t\left(1-\kappa_{t+1}\left(1-\zeta^P_{t+1}\right)\right)\right] V^G_{t+1,i} + \delta^G_t \zeta^P_{t+1}\kappa_{t+1} V^P_{t+1,i} \right\}, \tag{2.6}$$

$$\mathscr{W}^P_{t,i}(\Omega^S_{t+1,i}) = \mathbb{E}_t \left\{ \delta^P_t (1-\kappa_{t+1}) V^U_{t+1,i} + \left[1 - \delta^P_t\left(1-\zeta^P_{t+1}\kappa_{t+1}\right)\right] V^P_{t+1,i} + \delta^P_t \left(1-\zeta^P_{t+1}\right)\kappa_{t+1} V^G_{t+1,i} \right\}. \tag{2.7}$$

$$\mathscr{W}^U_{t,i}(\Omega^S_{t+1,i}) = \mathbb{E}_t \left\{ (1-\kappa_{t+1}) V^U_{t+1,i} + \zeta^P_{t+1}\kappa_{t+1} V^P_{t+1,i} + (1-\zeta^P_{t+1})\kappa_{t+1} V^G_{t+1,i} \right\}, \tag{2.8}$$

where κ_{t+1} is the probability that a workers household finds a job, at time $t+1$, ζ^P_{t+1} the probability that the job a workers household finds at time $t+1$ is at a private sector firm and $(1-\zeta^P_{t+1})$ the probability that the job a workers household finds at time $t+1$ is at a public sector firm. δ_t^G is the job destruction rate in the public sector at time t and δ_t^P the job destruction rate in the private sector at time t.

As seen in Equations 2.6 to 2.8, workers households have different employment and unemployment probabilities, and consequently different degrees of unemployment risk, depending on whether they work for public sector firms, private sector firms or being unemployed, and it is these differences that make for heterogeneous workers households in my model. The result of combining heterogeneous workers households and incomplete asset markets is that workers households face uninsurable unemployment risk, investing in a one-period household bond (Schmitt-Grohé & Uribe, 2003, Neumeyer & Perri, 2005). However as in Ravn & Sterk (2017,2021) and Broer et al. (2021) these bonds are in zero net supply, so the sum of asset holdings for all workers households is zero. For simplicity, workers households also face a no-borrowing constraint so the individual asset holdings of workers households also equal zero. As a result, the borrowing constraint must bind for all but one type of workers households and I prove that when I compute the steady state equilibrium, where I find that the Euler equation is satisfied with inequality for all but for one type of workers households. These constraints ensure that I can include the precautionary savings motive of workers households in my model without including any wealth effects which could make solving it computationally much more difficult (see Appendix B.1.1 for further analysis).

The first order conditions with respect to consumption and bond holdings for workers households employed by public sector firms are:

$$\frac{\partial V_{t,i}^G}{\partial C_{t,i}^G} = 0 \Rightarrow (C_{t,i}^G)^{-\eta} = \lambda_{t,i}^G, \tag{2.9}$$

$$\begin{aligned}\frac{\partial V_{t,i}^G}{\partial B_{t,i}^G} = 0 \Rightarrow (C_{t,i}^G)^{-\eta} = \beta \mathbb{E}_t \Bigg\{ \frac{R_t}{\pi_{t+1}} \Big[\delta_t^G (1-\kappa_{t+1})(C_{t+1,i}^U)^{-\eta} + \delta_t^G \zeta_{t+1}^P \kappa_{t+1} (C_{t+1,i}^P)^{-\eta} \\ + \Big[1-\delta_t^G \Big(1-\kappa_{t+1}\Big(1-\zeta_{t+1}^P\Big)\Big)\Big](C_{t+1,i}^G)^{-\eta}\Big]\Bigg\} + \mu_{t,i}^G. \end{aligned} \tag{2.10}$$

The first order conditions with respect to consumption and bond holdings for workers households employed by private sector firms are:

$$\frac{\partial V^P_{t,i}}{\partial C^P_{t,i}} = 0 \Rightarrow (C^P_{t,i})^{-\eta} = \lambda^P_{t,i}, \tag{2.11}$$

$$\frac{\partial V^P_{t,i}}{\partial B^P_{t,i}} = 0 \Rightarrow (C^P_{t,i})^{-\eta} = \beta \mathbb{E}_t \left\{ \frac{R_t}{\pi_{t+1}} \left[\delta^P_t (1-\kappa_{t+1})(C^U_{t+1,i})^{-\eta} + \left[1-\delta^P_t \left(1-\zeta^P_{t+1}\kappa_{t+1}\right)\right](C^P_{t+1,i})^{-\eta} \right.\right.$$

$$\left.\left. +\delta^P_t \left(1-\zeta^P_{t+1}\right)\kappa_{t+1}(C^G_{t+1,i})^{-\eta}\right]\right\} + \mu^P_{t,i}. \tag{2.12}$$

Finally, the first order conditions with respect to consumption and savings are for unemployed workers households are:

$$\frac{\partial V^U_{t,i}}{\partial C^U_{t,i}} = 0 \Rightarrow (C^U_{t,i})^{-\eta} = \lambda^U_{t,i}, \tag{2.13}$$

$$\frac{\partial V^U_{t,i}}{\partial B^U_{t,i}} = 0 \Rightarrow (C^U_{t,i})^{-\eta} = \beta \mathbb{E}_t \left\{ \frac{R_t}{\pi_{t+1}} \left[(1-\kappa_{t+1})(C^U_{t+1,i})^{-\eta} + \zeta^P_{t+1}\kappa_{t+1}(C^P_{t+1,i})^{-\eta} \right.\right.$$

$$\left.\left. +(1-\zeta^P_{t+1})\kappa_{t+1}(C^G_{t+1,i})^{-\eta}\right]\right\} + \mu^U_{t,i}. \tag{2.14}$$

2.3.1.2 Capitalists' Households

Capitalists' households do not supply labour. The preferences of a representative capitalists household i, over consumption are given by the following lifetime utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U^C_{t,i}\left(C^C_{t,i}\right), \tag{2.15}$$

where $C^C_{t,i}$ is the consumption of capitalists household at time t. The period utility function is of a linear form:

$$U^C_{t,i} = \frac{(C^C_{t,i})^{(1-\eta^C)}}{(1-\eta^C)}. \tag{2.16}$$

η^C is the risk aversion parameter of capitalists households. For analytical simplicity capitalists households are risk neutral, so $\eta^C = 0$ (Broer et al., 2021, Ravn &

Sterk, 2017, 2021). Capitalists' households have complete access to asset markets, investing in risk-free public sector bonds, which are less liquid. They also own the economy's private sector firms, pay lump-sum taxes and receive all profits, which are untaxed for simplicity. The capitalists households budget constraint is:

$$C^C_{t,i} + B^C_{t,i} \leq Div^C_{t,i} + B^C_{t-1,i}\frac{R^C_{t-1}}{\pi_t} - \tau_t. \tag{2.17}$$

where $B^C_{t,i}$ are the government bonds bought by capitalists households at time t and τ_t are lump-sum taxes at time t. $B^C_{t-1,i}$ are last period's government bonds owned by capitalists households while $\frac{R^C_{t-1}}{\pi_t}$ is the rate of return of government bonds, adjusted for the inflation rate at time t, π_t. Firms' dividends received by capitalists households are equal to profits minus the cost of maintaining private sector vacancies and investing in private sector vacancies, so $Div^C_{t,i} = \frac{d_t - cv^P_t - \frac{1}{2\xi}(i_t - i)^2}{pop_C}$

The Bellman equation for capitalists households is:

$$V^C_{t,i} = \max_{C^C_{t,i}B^c_{t,i}} \left\{ C^C_{t,i} + \beta \mathbb{E}_t V^C_{t+1,i} \right\}$$

$$s.t.\ C^C_{t,i} + B^C_{t,i} \leq Div^C_{t,i} + B^C_{t-1,i}\frac{R^C_{t-1}}{\pi_t} - \tau_t, \tag{2.18}$$

$$B^C_{t,i} \geq 0.$$

Capitalists households have the following intertemporal maximization problem:

$$V^C_{t,i}(B^C_{t-1,i}) = \max_{C^C_{t,i}, B^C_{t,i}} \Big\{ C^C_t + \beta \mathbb{E}_t V^C_{t+1} + \lambda^C_{t,i}\Big[Div^C_{t,i}$$

$$+ B^C_{t-1,i}\frac{R^C_{t-1}}{\pi_t} - \tau_t - C^C_{t,i} - B^C_{t,i}\Big] + \mu^C_{t,i}\Big[B^C_{t,i} - B^C_i\Big]\Big\}. \tag{2.19}$$

The first order conditions with respect to consumption and public sector bonds are respectively:

$$\frac{\partial V^C_{t,i}}{\partial C^C_{t,i}} = 0 \Rightarrow 1 = \lambda^C_{t,i}, \tag{2.20}$$

$$\frac{\partial V_{t,i}^C}{\partial B_{t,i}^C} = 0 \Rightarrow 1 = \beta \mathbb{E}_t \left[\frac{R_t^C}{\pi_{t+1}} \right] + \mu_{t,i}^C, \tag{2.21}$$

2.3.2 The Labour Market

The labour market is based on the Chapter 1 model. I assume a two-sector matching model with public sector firms and private sector firms with "random" search and matching as in Navarro et al. (2017), Quadrini & Trigari (2007) and Albrecht et al. (2019) where unemployed workers households search for a job in both sectors randomly "meeting" and accepting a vacancy, regardless of the sector.

Following Fontaine et al. (2020) job destruction rates differ between sectors, with public sector job destruction rates being much smaller, as in most countries public sector employment is safer, with fewer public sector employees getting fired compared to private sector employees. Finally, investment in private sector vacancies is inelastic and depends on the value of unfilled vacancies as in Coles & Moghaddasi Kelishomi (2018) and Broer et al. (2021).

I assume there is a unit mass of workers households who are either employed or unemployed. Public sector employment at time t, $N_t^{G,W}$, equals last period's public sector employment stock which was not destroyed at the beginning of the current time period plus the hirings of public sector employees at time t:

$$N_t^{G,W} = (1 - \delta_t^G) N_{t-1}^{G,W} + \mathcal{H}_t^G, \tag{2.22}$$

where δ_t^G is the public sector job destruction rate at time t and $\mathcal{H}_t^G = q_t v_t^G$ are the hirings of public sector employees at time t which equals the probability of a filled vacancy at time t, q_t times public sector vacancies at time t, v_t^G.

Private sector employment at time t, $N_t^{P,W}$, is equal to last period's private sector employment stock which was not destroyed at the beginning of the current time period plus new hirings of private sector employees at time t, $\mathcal{H}^P$ which, similarly to hirings of public sector employees, are equal to posted vacancies at time t, v_t^P, times the probability that a vacancy is filled at time t, q_t so $\mathcal{H}_t^P = v_t^P q_t$. As a result we have:

$$N_t^{P,W} = (1-\delta_t^P)N_{t-1}^{P,W} + \mathcal{H}_t^P. \tag{2.23}$$

Private sector job destruction rate at time t, δ_t^P is endogenous as in Broer et al. (2021) and equal to the steady state value of the private sector job destruction rate δ^P times a term capturing how changes in the value of filled private sector vacancies affect the private sector job destruction rate:

$$\delta_t^P = \delta^P \left[\frac{J_t^F}{J^F}\right]^{-\varepsilon_{\delta,P}}, \tag{2.24}$$

where J_t^F is the value of a filled private sector vacancy at time t, J^F the steady state value of a filled private sector vacancy and $\varepsilon_{\delta,P}$ a parameter capturing the degree of procyclicality (for $\varepsilon_{\delta,P} > 0$) or countercyclicality (for $\varepsilon_{\delta,P} < 0$) governing private sector job destruction rates.

Unemployment at time t, u_t is the sum of individuals not employed in any sector:

$$u_t = 1-(1-\delta_t^G)N_{t-1}^{G,W} - (1-\delta_t^P)N_{t-1}^{P,W}. \tag{2.25}$$

Unemployed workers households randomly match with open vacancies from either the public or the private sector. The matching technology m_t is a simple Cobb-Douglas equation(Coles & Moghaddasi Kelishomi, 2018, Broer et al., 2021):

$$m_t = \mathcal{M} u_t^{\gamma}(v_t)^{1-\gamma}, \tag{2.26}$$

where γ is the elasticity parameter of the matching function and $\mathcal{M}$ the scale parameter of the matching function.

The total number of vacancies in the economy at time t, equals private sector vacancies and public sector vacancies, so $v_t = v_t^P + v_t^G$. Following Coles & Moghaddasi Kelishomi (2018) private sector vacancies at time t, v_t^P are equal to the intermediate stock of private sector vacancies of the previous time period $v_{o_{t-1}}^P$ plus investments in private sector vacancies at time t, i_t:

$$v_t^P = v_{o_{t-1}}^P + i_t. \tag{2.27}$$

The intermediate stock of private sector vacancies at time t, $v^P_{o_t}$ is equal to the number of unfilled vacancies that were not destroyed:

$$v^P_{o_t} = (1-\delta^P)[\left(v^P_t - \zeta^P_t q_t v^P_t\right)\Big], \tag{2.28}$$

where $(1-\delta^P)v^P_t$ is the number of private sector vacancies which are not destroyed and remain open at time t and $(1-\delta^P)\zeta^P_t q_t v^P_t$ is the number of vacancies not destroyed that get filled at time t. Following Broer et al. (2021) private sector vacancies at time t are destroyed at a constant rate equal to the steady state value of private sector job destruction rate δ^P. In a one sector model the number of vacancies that get filled is simply equal to $(1-\delta^P)q_t v^P_t$ as it only depends on the probability a vacancy is filled at time t, q_t times the number of private sector vacancies v^P_t. But in a two-sector random search and matching model the number of private sector vacancies that get filled also depends on the probability that an unemployed workers household "meets" and fills a private sector open vacancy at time t, ζ^P_t, which is equal to the ratio of private sector vacancies to total vacancies and decreases as public sector vacancies increase. Therefore I also add the probability that an unemployed workers household "meets" and fills a private sector open vacancy at time t ζ^P_t in equation (13) ($(1-\zeta^P_t)$ is the probability of "meeting" a public sector vacancy at time t):

$$\zeta^P_t = \frac{v^P_t}{v^P_t + v^G_t}. \tag{2.29}$$

As in Coles & Moghaddasi Kelishomi (2018) investment in private sector vacancies at time t, i_t depends on the cost associated with creating private sector vacancies and the value of unfilled vacancies:

$$i_t = i + \xi J_t, \tag{2.30}$$

where ξ is the elasticity of investment in private sector vacancies at time t. As $\xi \to 0$, investment in private sector vacancies becomes more inelastic meaning the free entry condition for vacancies does not hold anymore. As a result, investment in private sector vacancies, the stock of private sector vacancies and private sector employment adjust very slowly which means that negative shocks in the economy create high and persistent increases in unemployment and large persistent reductions in private sector vacancies. However, as $\xi \to 0$, increases in public sector employment have smaller effects as changes in the value of open vacancies in the private sector, have a smaller effect in investment in private sector vacancies, the stock of private sector vacancies and private sector employment. When instead $\xi \to \infty$ investment in private sector vacancies becomes elastic and we get the standard search and matching model with free entry.

Market tightness at time t, θ_t, which determines the job finding rate of workers over the business cycle is just the ratio of total vacancies at time t v_t over unemployment at time t u_t.

$$\theta_t = \frac{v_t}{u_t}. \tag{2.31}$$

The probability of a filled vacancy at time t q_t and the probability a workers household finding a job at time t κ_t are respectively $q_t = \frac{m_t}{v_t}$ and $\kappa_t = \frac{m_t}{u_t}$, but following Broer et al. (2021) I use Equations 2.26 and 2.31 to write them in terms of market tightness:

$$q_t = A\theta_t^{-\gamma}, \tag{2.32}$$

$$\kappa_t = A\theta_t^{1-\gamma}. \tag{2.33}$$

2.3.3 Firms

Production in the private sector consists of three parts. First, a continuum of intermediate goods firms produce a homogeneous good, using only private sector employment as an input, and sell it in a competitive market. Second, a continuum of wholesale goods firms indexed by $j \in [0,1]$ produce a continuum of differentiated goods $j \in [0,1]$, using the intermediate good as an input. Wholesale goods are sold to final goods firms and used to produce the final good. The final good is bought by workers households and capitalists households to consume, and by public sector firms to use as an input alongside public sector employment. Intermediate goods firms and final goods firms behave competitively, while wholesale goods firms have market power, operating under monopolistic competition, and their prices are sticky.

2.3.3.1 Intermediate good sector

Intermediate goods firms produce a homogeneous good, using only labour which they hire by posting vacancies and sell this good in a competitive market at a price $P_t^{\mathscr{X}}$. One unit of labour produces A_t units of the intermediate good. The total production of intermediate goods firms is:

$$\mathscr{X}_t = A_t N_t^P. \tag{2.34}$$

$\mathscr{X}_t$ is the quantity of the intermediate good produced by the intermediate goods firms at time t and N_t^P is labour demand by intermediate goods firms at time t.

Following Pissarides (2009) and Broer et al. (2021), I assume wages are not fully sticky, but there is some degree of cyclicality so they adjust to changes in private sector firms' output. As a result I assume a wage rule of the form:

$$w_t^P = w^P \left(\frac{Y_t^P}{Y^P} \right)^{\epsilon_w}, \tag{2.35}$$

where w^P is the steady state private sector wage and ϵ_w the elasticity of private sector wages to deviations of private sector firms' output from its steady state value.

Total factor productivity at time t, A_t evolves exogenously according to:

$$A_t = (1-\rho_A)A + \rho_A A_{t-1} + \eta_t^A. \tag{2.36}$$

Where A is the steady state total factor productivity, $0 < \rho_A < 1$ the autocorrelation parameter and η_t^A a white noise innovation drawn from a normal distribution with mean zero.

To hire labour the intermediate goods firms posts vacancies. The value of an open vacancy in the private sector at time t, J_t is equal to:

$$J_t = -c + \zeta_t^P q_t J_t^F + \beta(1-\delta^P)\left[1-\zeta_t^P q_t\right] \mathbb{E}_t \{J_{t+1}\}, \tag{2.37}$$

where c is cost of an unfilled vacancy; $\zeta_t^P q_t$ and $1-\zeta_t^P q_t$ are the probabilities that a vacancy in the private sector gets filled by an unemployed workers households or stays open at time t, respectively.

As mentioned in Chapter 1 the probability that an unemployed workers household fills an open private sector vacancy ζ_t^P is a very important variable in this model. Under random search and matching, when public sector employment $N_t^{G,W}$ increases it means that the hirings of public sector employees $\mathcal{H}_t^G$ increase by increasing public sector vacancies v_t^G. This lowers the probability that a private sector vacancy is matched with an unemployed workers households, and since unfilled private sector vacancies have a cost, the value of an open vacancy in the private sector J_t decreases. As a result investment in private sector vacancies i_t, private sector vacancies v_t^P and private sector employment $N_t^{P,W}$ decrease; however this effect is mitigated by the elasticity of investment in private sector vacancies ξ: As $\xi \to 0$, investment in private sector vacancies i_t becomes more inelastic, and less affected by changes in the value of an open vacancy in the private sector, J_t.

Finally the value of a filled vacancy in the private sector at time t, J_t^F is:

$$J_t^F = A_t P_t^{\mathcal{X}} - w_t^P + \mathbb{E}_t \beta \left\{ \left[\frac{C_{t+1}^C}{C_t^C} \right]^{-\eta^C} (1-\delta_t^P) J_{t+1}^F \right\}. \tag{2.38}$$

2.3.3.2 Final good sector

There is a single homogeneous final good in the economy, produced by a large number of final goods firms. The production function of the representative final goods firm exhibits constant returns to scale and is a constant elasticity of substitution (CES) bundler of wholesale goods, so no other factors are used to produce the final good. The production function of the final good for a representative final goods firm is:

$$Y_t^P = \left[\int_0^1 (y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.39}$$

Y_t^P is the quantity of the final good produced by a representative final goods firm at time t, y_t^j the quantity of the wholesale good of type j produced by the wholesale goods firm j at time t and ε the elasticity of substitution between different wholesale goods. I assume $1 < \varepsilon < \infty$, so wholesale goods are imperfect substitutes of each other, and this gives wholesale goods firms market power. The representative final goods firm maximizes its profits, using the following profit function:

$$d_t = P_t Y_t^P - \int_0^1 p_t^j y_t^j dj \tag{2.40}$$

d_t is the profit of the representative final goods firm at time t, P_t the price of the final good at time t and p_t^j the price of the wholesale good of type j produced by the wholesale goods firm j at time t. Profits are total revenue, which is the quantity of final good produced and sold times its price, minus total cost, which is the sum of all the quantities of wholesale goods bought times their respective prices. Solving the firms' maximization problem (see Appendix B.2.1) gives me demand for each wholesale good of type j produced by wholesale goods firm j at time t, y_t^j:

$$y_t^j = \left(\frac{p_t^j}{P_t}\right)^{-\varepsilon} Y_t^P \tag{2.41}$$

Equation 2.41 shows that demand for each wholesale good depends negatively on its relative price and positively on private sector firms' total output. Since the final goods sector operates under perfect competition, profits must equal zero, so $d_t = 0$ and Equation 2.40 becomes:

$$P_t Y_t^P = \int_0^1 p_t^j y_t^j dj \tag{2.42}$$

Substituting Equation 2.41 on Equation 2.42 and taking out of the integral all variables without an index j gives us the aggregate price level at time t, P_t (see Appendix B.2.1):

$$P_t = \left[\int_0^1 p_t^{j^{1-\varepsilon}} dj\right]^{\frac{1}{1-\varepsilon}} \tag{2.43}$$

2.3.3.3 Wholesale good sector

Each wholesale goods firm j, uses the intermediate good to produce a wholesale good of type j at time t, y_t^j using a linear production function:

$$y_t^j = \mathscr{X}_t^j, \tag{2.44}$$

where X_t^j is the intermediate good bought by wholesale goods firm j at time t. The total cost function is:

$$TC_{t,j} = P_t^{\mathscr{X}} \mathscr{X}_t^j. \tag{2.45}$$

Where $TC_{t,j}$ is the total cost of intermediate goods firm j at time t. To solve the cost minimization problem I combine Equations 2.44 and 2.45. I then differentiate Equation 2.45 with respect to y_t^j and get the marginal cost of the wholesale goods firm j at time t, $MC_{t,j}$:

$$\frac{\partial TC_{t,j}}{\partial y_t^j} \Rightarrow MC_{t,j} = P_t^{\mathscr{X}} \tag{2.46}$$

Real profits of the wholesale goods firm j at time t d_t^j, equal revenue (output of the wholesale good of type j produced by wholesale goods firm j at time t, y_t^j, times its relative price at time t, $\frac{p_t^j}{P_t}$), minus total cost at time t

$$d_t^j = \frac{p_t^j}{P_t} y_t^j - P_t^{\mathscr{X}} y_t^j. \tag{2.47}$$

Substituting Equation 2.41 on Equation 2.47 gives us the final expression:

$$\Rightarrow d_t^j = \left[\frac{p_t^j}{P_t} - P_t^{\mathscr{X}}\right]\left(\frac{p_t^j}{P_t}\right)^{-\varepsilon} Y_t^P. \tag{2.48}$$

2.3.3.4 Prices and Inflation

Each wholesale good of type j produced by wholesale goods firm j at time t, y_t^j, is differentiated from the others and thus an imperfect substitute, and this gives wholesale goods firms market power. More specifically, wholesale goods firms operate under monopolistic competition and have some power over prices and as a result nominal prices are sticky (Rotemberg, 1982a,b). Wholesale goods firms face quadratic price adjustment costs, given by $\mathscr{R}(\cdot) = \frac{\xi_\rho}{2}\left[\frac{p_t^j}{p_{t-1}^j} - \pi\right]^2$ where $\xi_\rho \geq 0$ is an adjustment cost parameter measuring the degree of price rigidity in the goods market and π is steady state inflation. When $\xi_\rho = 0$ firms pay no price adjustment costs and prices are flexible. A representative wholesale goods firm j producing a wholesale good of type j at time t, y_t^j, adjusts its price to maximize the discounted sum of future profits subject to demand for y_t^j. Using Equation 2.48 and adding a stochastic discount factor the maximization problem becomes:

$$\max_{p_{t+i}^j} d_t^j = \max_{p_{t+i}^j} \mathbb{E}_t \sum_{i=0}^{\infty} \left\{ X_{t,t+i} \left[\left(\frac{p_{t+i}^j}{P_{t+i}} \right) y_{t+i}^j - P_{t+i}^{\mathscr{X}} y_t^j - \frac{\xi_\rho}{2} \left(\frac{p_{t+i}^j}{p_{t+i-1}^j} - \pi \right)^2 Y_{t+i}^P \right] \right\} \quad (2.49)$$

Where $X_{t,t+i} = \beta^{t+i} \left[\frac{C_{t+1}^C}{C_t^C} \right]^{-\eta^C}$ is the stochastic discount factor. Some Algebra (see Appendix B.2.2) yields the New Keynesian Phillips Curve:

$$\frac{\xi_\rho}{(\varepsilon - 1)} [\pi_t - \pi] \pi_t = \mu P_{t+i}^{\mathscr{X}} - 1 + \frac{\xi_\rho}{(\varepsilon - 1)} \mathbb{E}_t X_{t,t+i} [\pi_{t+1} - \pi] \pi_{t+1} \frac{Y_{t+1}^P}{Y_t^P}. \quad (2.50)$$

2.3.3.5 An alternative specification - Flexible prices

In this variation I simplify my analysis by assuming wholesale goods firms can reset their prices each period, so the price adjustment cost parameter $\xi_\rho = 0$ in Equation 2.50. This gives me the optimal price p_t^j of the wholesale good of type j, produced by wholesale goods firm j at time t, y_t^j for flexible prices:

$$1 = \mu P_{t+i}^{\mathscr{X}} \quad (2.51)$$

2.3.4 Government

2.3.4.1 Government budget constraint and fiscal policy

Government at time t levies labour income taxes on workers households τ_t^N, lump-sum taxes on capitalists households τ_t and issues one-period bonds B_t, to finance government expenditure on buying final goods at time t, G_t^C, wages of workers households employed by public sector firms at time t N_t^G, unemployment benefits b_t and interest rates on past public debt, $B_{t-1}\frac{R_{t-1}^C}{\pi_t}$. The government budget constraint is:

$$G_t^C + w_t^G N_t^G + b_t u_t + B_{t-1}\frac{R_{t-1}^C}{\pi_t} = \tau_t^N (w_t^P N_t^P + w_t^G N_t^G) + pop_C \tau_t + B_t. \tag{2.52}$$

Hirings of public sector employees by public sector firms at time t, $\mathscr{H}_t^G$, equal the number of public sector vacancies at time t, v_t^G times the probability a vacancy is filled at time t q_t so $\mathscr{H}_t^G = q_t v_t^G$. However rather than having an exogenous process, I assume that hirings of public sector employees at time t $\mathscr{H}_t^G$ is equal to its steady state value $\mathscr{H}^G$, multiplied by a term capturing the effect that deviations of public sector employment from its steady state value have on hirings of public sector employees and an exogenous shock with mean equal to unity that follows a stochastic process:

$$\mathscr{H}_t^G = \mathscr{H}^G \left[\frac{N_t^G}{N^G}\right]^{\varepsilon_{\mathscr{H},G}} \left[\frac{u_t}{u}\right]^{\varepsilon_u} \chi_t^{\mathscr{H},G}, \tag{2.53}$$

$\varepsilon_{\mathscr{H}^G}$ is a stability parameter which I use to make sure that public sector employment returns back to its steady state value after a shock in the hirings of public sector employees while ε_u captures how hirings of public sector employees react to changes in unemployment. I assume that $\varepsilon_u = 0$ meaning that hirings of public sector employees are acyclical, based on empirical data which indicate that public sector employment is relatively acyclical, but I will change this assumption in Chapter 3 when I look at the effects of countercyclical public sector employment policies. Finally, $\chi_t^{\mathscr{H},G}$ is the exogenous shock on the hirings of public sector employees which follows a stochastic process:

$$\log\left(\chi_t^{\mathscr{H},G}\right) = \rho_{\mathscr{H},G}\log\left(\chi_{t-1}^{\mathscr{H},G}\right) + \left(1-\rho_{\mathscr{H},G}\right)\log\left(\chi^{\mathscr{H},G}\right) + \nu_t^{\mathscr{H},G}, \tag{2.54}$$

where $\rho_{\mathscr{H},G}$ is the autocorrelation parameter and $\nu_t^{\mathscr{H},G}$ is a white noise innovation, drawn from a normal distribution with mean zero.

For the job destruction rate in the public sector at time t, δ_t^G, I simplify my analysis a bit by assuming that it is equal to its steady state value. This is meant to capture the fact that the number of public sector employees which leave the public sector every time period is a relatively stable number, so we have:

$$\delta_t^G = \delta^G. \tag{2.55}$$

The lump-sum tax on capitalists households at time t τ_t, adjusts in order to repay the public debt. For the labour income tax on workers households at time t τ_t^N, I assume for now that it remains fixed; I will then relax this assumption in Chapter 3 by having workers households repay some of the debt with higher taxes:

$$\tau_t^N = \tau^N + \kappa_{\tau,N}(B_t - B), \tag{2.56}$$

where $\kappa_{\tau,N}$ is the labour income tax response to public debt which for now I set equal to zero. Government expenditure on consumption at time t, is equal to its steady state value so $G_t^C = G^C$. Public debt at time t B_t, reacts positively to changes in government expenditure, while following a fiscal rule preventing an explosive debt path:

$$B_t = B_{t-1} - \rho_B (B_{t-1} - B) + \kappa_B (G_t - G). \tag{2.57}$$

The government's wage policy rule is exogenous, and based on Michaillat (2014) I assume that public sector wages equal to private sector wages. Obviously this assumption is somewhat unrealistic, however it does not affect my results, given that I use a random search and matching framework, with exogenous hiring of public sector employees. Consequently the wage level has no effect as unemployed workers households do not choose a vacancy based on the level of wages, but rather randomly "meet" and fill a job vacancy. Also private sector wages are not determined by any form of bargaining but are instead equal to a fixed percentage of private sector firms' output based on the theory of the Fundamental Surplus (Ljungqvist & Sargent, 2017, 2021):

$$w_t^G = w_t^P. \tag{2.58}$$

2.3.4.2 Production of goods by Public Sector Firms

Following Forni et al. (2010), Economides et al. (2013; 2017) and Papageorgiou & Vourvachaki (2017) there is a large number of public sector firms producing goods, using final goods purchased by the government G_t^C and public sector employees N_t^G. The production technology of the representative public sector firm is:

$$Y_t^G = A_t^{(1-z)}(G_t^C)^z(N_t^G)^{(1-z)}, \tag{2.59}$$

where Y_t^G is the output of the representative public sector firm at time t, $z > 0$ the output elasticity of the final good, $(1-z)$ the output elasticity of labour and A_t total factor productivity at time t.

2.3.4.3 Monetary policy

As seen in the sections on workers households and capitalists households there are two different interest rates in this economy: The interest rate on household bonds traded between workers households R_t and the interest rate on public sector bonds which capitalists households buy, R_t^C. The reason for this setup is because, as mentioned earlier, I assume heterogeneous workers households and incomplete asset markets in my model; consequently, heterogeneous workers households face uninsurable unemployment risk and cannot invest in assets to insure themselves. However, adding a public sector means I need a separate market for public sector debt where workers households cannot trade. As a result I have two different asset markets with two different interest rates and the monetary authority sets the nominal interest rate for households bonds only, according to the following Taylor rule:

$$R_t = R\left[\frac{\pi_t}{\pi}\right]^{\varphi_\pi}, \tag{2.60}$$

$\varphi_\pi > 1$ is the interest rate response to inflation, R the steady state nominal interest rate and π steady state inflation.

2.3.5 Aggregation

I restrict attention to the case where all private sector firms and all public sector firms behave identically. This means they have the same demand elasticity, face the same marginal costs and demand, set the same prices and produce the same quantity of output. I begin with the aggregate production function of the final good at time t, Y_t^P in Equation 2.39:

$$Y_t^P = \left[\int_0^1 (y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where y_t^j is the output of the wholesale good of type j produced by wholesale goods firm j at time t. y_t^j is the same across all wholesale goods firms, so I take it out of the integral:

$$Y_t^P = y_t^j \left[\int_0^1 dj\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

$$\Rightarrow Y_t^P = y_t^j.$$

Using the production function of the wholesale goods firm j in Equation 2.44 I get:

$$y_t^j = \mathscr{X}_t^j.$$

Since I am integrating over the unit integral and every intermediate goods firm produces the same output:

$$y_t^j = \int_0^1 \mathscr{X}_t^j dj.$$

Applying this to the expression above I get:

$$Y_t^P = \int_0^1 \mathscr{X}_t^j dj.$$

The intermediate good is split among private sector firms along the unit interval, so the aggregate amount of intermediate good supplied to wholesale goods firms equals the sum of intermediate good supplied to all wholesale goods firms:

$$\mathscr{X}_t = \int_0^1 \mathscr{X}_t^j dj.$$

Consequently, aggregate output of the private sector final goods firms at time t, Y_t^P equals the sum of output of all wholesale good firms j:

$$Y_t^P = \mathscr{X}_t.$$

Since $\mathscr{X}_t = A_t N_t^P$, this gives us the aggregate output of private sector firms, through a very standard production function:

$$Y_t^P = A_t N_t^P. \tag{2.61}$$

Similarly, the aggregate output of public sector firms at time t, Y_t^G equals the sum of output of all public sector firms and is given by Equation 2.59:

$$Y_t^G = A^{(1-z)}(G_t^C)^z(N_t^G)^{(1-z)}. \tag{2.62}$$

Following Pappa (2009) and Economides et al. (2013, 2017), aggregate output in the economy, at time t, Y_t equals demand for goods by the public and private sectors.

$$Y_t = C_t + cv_t^P + \frac{1}{2\xi}(i_t - i)^2 + G_t^C + \frac{\xi_\rho}{2}[\pi_t - \pi]^2 Y_t^P, \tag{2.63}$$

where $\frac{\xi_\rho}{2}[\pi_t - \pi]Y_t^P$ captures the demand for the intermediate good to meet price adjustment costs.

I close my model with aggregate consumption C_t, dividends Div_t, bond holdings and labour supply at time t:

$$C_t = u_t C_t^U + N_t^{P,W} C_t^P + N_t^{G,W} C_t^G + pop_C C_t^C \tag{2.64}$$

$$Div_t = pop_C Div_t^C \tag{2.65}$$

$$N_t^G = N_t^{G,W} \tag{2.66}$$

$$N_t^P = N_t^{P,W} \tag{2.67}$$

$$B_t^W = B_t^G + B_t^P + B_t^U \tag{2.68}$$

$$B_t = pop_C B_t^C \tag{2.69}$$

2.3.6 The Propagation mechanism

I now analyze the mechanism through which increasing public sector employment propagates through the model and the feedback loops and interactions generated between the building blocks of my model. This will help me illustrate a number of novel interactions and effects which do not exist in other models, and therefore have not been studied enough.

When the shock first happens and public sector employment N_t^G increases, it increases public sector firms' output Y_t^G, which directly increases aggregate output Y_t. This *Direct Effect* is simple and straightforward with its size and duration being positively affected by the size of public sector employment and public sector firms' output but also by the feedback parameter $\varepsilon_{\mathscr{H}^G}$. In addition to this however, there are a number of *Indirect Effects*, which show how public sector employment affects the private sector. These *Indirect Effects* are the most important in my analysis, as they are responsible for creating propagation channels and interactions between the different parts of the economy. To fully understand these effects, I turn my attention to the building blocks of the model.

Starting with the labour market block, when public sector employment N_t^G increases, the probability that an unemployed workers household fills a private sector open vacancy ζ_t^P in Equation 2.29 drops as we have random search and matching and there are now more public sector vacancies. Consequently, the value of an open vacancy J_t and investment in private sector vacancies i_t in Equation 2.30 decrease, reducing private sector vacancies v_t^P and private sector employment N_t^P. This *Labour Market Effect* operates in a completely novel way in my model due to labour market frictions, in this case the elasticity of investment in private sector vacancies: As $\xi \to \infty$, my model becomes a frictionless SAM model with free entry where increasing public sector employment leads to a large reduction in private sector vacancies and large crowding out effects seen in other papers[4]. Conversely when $\xi \to 0$ investment in private sector vacancies becomes more inelastic, so increasing public sector employment reduces unemployment with limited crowding out, or even has crowding in effects. In later periods the *Labour Market Effect* spreads to the whole economy: Aggregate demand drops as workers households

[4]e.g. Quadrini & Trigari (2007), Navarro et al. (2017) and Albrecht et al. (2019)

who have lost their jobs consume less. As a result inflation π_t, and through it the value of a filled vacancy J_t^F decrease, which raises the private sector job destruction rate δ_t^P in Equation 2.24. Consequently investment in private sector vacancies i_t private sector vacancies v_t^P private sector employment N_t^P decrease further and, private sector firms' output Y_t^P wages and aggregate output Y_t also decrease.

In the HANK block we have the *Aggregate Demand Effect*, standard in NK models, which as seen in Equation 2.50 depends on inflation π_t and price stickiness, which in turn is determined by the Rotemberg price adjustment cost parameter ξ_ρ, and breaks down in two parts. First, the government redistributes income to unemployed workers households hired by public sector firms, creating a *Redistribution Effect*. Now unemployed workers households who find a job in public sector firms consume more as the public sector firms wage w_t^G is larger than the unemployment benefit b_t and additionally if capitalists households are taxed to finance the increase in public sector employment, we have a income transfer from capitalists households, who have a smaller marginal propensity to consume (MPC), to workers households who have a larger MPC. As a result, aggregate consumption also increases, raising aggregate demand and inflation π_t, private sector firms' output Y_t^P and wages, and indirectly increasing aggregate output Y_t. Then in the next periods we have additional *Indirect Aggregate Demand Effects*, which propagate in the labour market: Higher inflation raises the value of a filled vacancy J_t^F and investment in private sector vacancies i_t, lowering private sector job destruction rates δ_t^P in Equation 2.24, with larger values of the elasticity of the private sector job destruction rate $\varepsilon_{\delta,P}$ making this effect larger. This raises private sector employment N_t^P, private sector firms' output Y_t^P, wages and inflation π_t further and indirectly increases aggregate output Y_t even more, creating a multiplier effect.

Finally, there is an *Unemployment Risk Effect*, created by heterogeneous workers households and incomplete asset markets. The *Direct Effect* and the *Aggregate Demand Effect* lower unemployment risk of workers households and their demand for precautionary savings, and further increase aggregate demand. To clear the asset market the interest rate for household bonds R_t in the Euler equation in 2.12 must increase. To be consistent with the monetary policy in the Taylor rule in Equation 2.60, and with the further increase in aggregate demand, there must be an additional increase in inflation π_t. This raises investment in private sector vacancies and reduces the private sector job destruction rate even more, further raises private sector employment N_t^P, private sector firms' output Y_t^P and wages and increases aggregate output Y_t further, reinforcing the multiplier effect. However, the overall result of the *Unemployment Risk Effect* is ambiguous as it is also influenced by the *Labour Market Effect*: The reduction of private sector vacancies and private sector employment created in this channel makes demand for precautionary savings increase. This lowers interest rates for household bonds R_t and inflation π_t and, by decreasing investment in private sector vacancies even more, further reduces private sector employment N_t^P, private sector firms' output Y_t^P, wages and aggregate output Y_t.

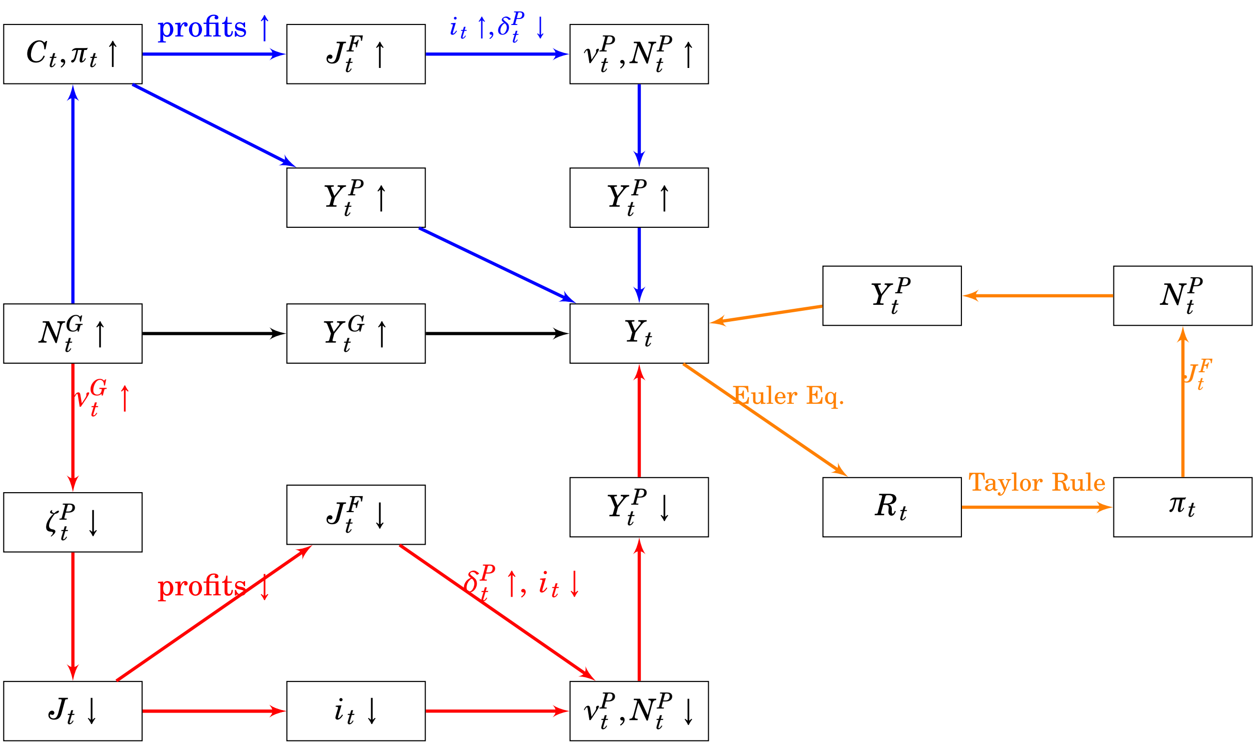

Figure 2.1: The Propagation Mechanism

2.3.7 Quantitative Analysis

In this subsection, I simulate the model described above around the certainty-equivalent steady state using Dynare and Matlab. The model is calibrated and estimated in two steps, using the same method as in Chapter 1. First, I use US data to calibrate the steady state values of the variables in my model. I also externally calibrate some parameter values by targeting values in the non-stochastic steady state of the model or by using common values from the relevant literature. Then for the parameters of interest in my model, I calibrate them using the Simulated Method of Moments (SMM), where the parameters in question are calibrated to ensure that the real empirical moments in the data, match the relevant simulated empirical moments in my model.

2.3.7.1 Calibration

I begin by setting the steady state target values, presented in Table 2.1. I focus on the USA, using monthly data from 2003 to 2021 from the Current Population Survey (CPS), and follow the methodology of Fontaine et al. (2020) to calculate the job destruction rates in the private and public sector. Similarly to Chapter 1 I only make a brief overview of the data now, with a complete analysis in Chapter 3.

The American CPS is a monthly Labour Force Survey where each survey sample of households is interviewed for four consecutive months, dropped out of the sample for eight months and interviewed again for another four months. Using these datasets I construct the stocks of public sector employment, private sector employment, unemployment and the flows between employment and unemployment. All my data are seasonally adjusted, because removing seasonal components allows me to study only the underlying trends and non-seasonal economic fluctuations in the labour market, and detrended using an HP filter with a smoothing parameter of 100000 to separate the cyclical component from the raw data of a time series.

I classify individuals as public sector employees or private sector employees based on their answers about their employer. In the US public sector employment includes all individuals working for the federal, state or local governments. My methodology here differs from Fontaine et al. (2020) who categorize employees in State Owned Enterprises as private sector employees. The reason for this different categorisation is because I also add public sector output in my model and, although State Owned Enterprises are a very small part of the US economy, as we will see in Chapter 3 an important component of public sector output in Europe is produced in State Owned Enterprises, while the number of people working in these parts of the public sector are a large part of the labour force in Europe.

Table 2.1: Steady State Target

Parameter	Parameter Name	Value	Source/Target
u	Unemployment	0.0628	Data
N^G	Public sector employment	0.1487	Data
N^P	Private sector employment	0.7885	Data
δ^P	Job destruction rate (Private sector)	0.0363	Data
δ^G	Job destruction rate (Public sector)	0.0244	Data
G^C	Government Expenditure on Consumption	0.1362	Data
θ	Market Tightness	1	Normalise
Y	Aggregate Output	1	Normalise
Y^P	Private Sector Firms' Output	0.8204	Data
Y^G	Public Sector Firms' Output	0.1796	Data
π	Inflation	0.0022	Annual Steady State Inflation 2.5%
v	Vacancies	0.0628	θ=1
w^P	Private Sector Wage	0.8141	Fundamental Surplus
w^G	Public Sector Wage	0.8141	No wage premium
τ^N	Tax Rate Labour Income	0.4500	Data
τ	Lump-sum Tax Rate	0.3000	Model Fit

Note: Data are in quarterly frequency from 2003Q1-2021Q4 and seasonally adjusted. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources:BEA, CPS, OECD).

Starting with the steady state target values, I match the steady state unemployment rate u with the average unemployment rate in the data which equals 6.28%; I also match the steady state public sector employment rate N^G with the average public sector employment as a percentage of the labour force at 14.87%. Consequently, I set the steady state value of private sector employment as N^P at 78.85%. I set market tightness θ equal to unity, a common value in the literature. This means that the steady state value of vacancies v is set equal to the unemployment level, targeting a market tightness θ equal to unity (Shimer, 2005, Coles & Moghaddasi Kelishomi, 2018). Finally, following Fontaine et al. (2020) I calculate the job destruction rates in the public and private sector and match the average values from the data with my steady state values, setting the private sector job destruction rate δ^P at 3.63% and the public sector job destruction rate δ^G at 2.44%.

The steady state value for inflation π is set targeting an annual inflation rate of 2.5%. I set the steady state values of aggregate output $Y = 1$ and use government finance statistics from the Bureau of Economic Analysis (BEA) for the steady state public firms' output Y^G which equals 0.1796%; I then set the steady state value of private sector sector firms' output Y^P targeting a steady state aggregate output Y equal to unity. For the steady state value of government expenditure on consumption G^C I also use government finance statistics and set it at 13.62%. Private sector wages are not fully sticky, but following Pissarides (2009) and Broer et al. (2021), respond to the business cycle; for the steady state private sector wage w^P, I follow the theory of the fundamental surplus (Ljungqvist & Sargent, 2017, 2021). The fundamental surplus is the part of output used for wages, production costs and taxes and not for investment in private sector vacancies in the economy. As a result, the steady state private sector wage is $w^P = P^{\mathscr{X}} A(1 - fs)$ as in Broer et al. (2021). Regrading public sector wages, I assume they are equal to private sector wages, so $w^G = w^P$. Finally I set labour income taxes τ^N targeting values on labour taxation, including social security contributions, from OECD labour taxation data and lump-sum taxes τ are set to a sensible value to help solve the model.

Table 2.2: Externally Calibrated parameters

Parameter	Parameter Name	Value	Source/Target
γ	Matching function elasticity	0.6	Coles & Moghaddasi Kelishomi (2018)
β	Discount factor	0.9967	Annual discount rate 4%
pop_C	Number of Capitalists' Households	0.100	Broer et al. (2021)
η	CRRA Coeffcient	2	Broer et al. (2021)
η^C	Capitalists' Households CRRA Coeffcient	0	Broer et al. (2021)
ϵ	Price Elasticity	10	Standard
μ	Price Mark-up	1.1	Standard
ξ_p	Rotemberg Price Adjustment Cost	600	Standard
z	Consumption Good Share in Public Sector Firms' Output	0.3000	Standard
$\rho_{\mathscr{H}^G}$	Public Sector Hirings Shock Autocorrelation	0.9000	Cantore & Freund (2021)
ϕ_π	Taylor Response to Inflation	1.5	Broer et al. (2021)

I present my externally calibrated parameters in Table 2.2. Regarding preferences, I set the discount factor $\beta = 0.9967$ implying a steady state real interest rate of 4%. The number of capitalists households $pop_C = 0.1$ as in Broer et al. (2021). For workers households the risk aversion parameter $\eta = 2$ while capitalists households are risk neutral so their risk aversion parameter $\eta^C = 0$. The elasticity of substitution $\epsilon = 10$ and the price mark-up $\mu = 1.1$, common values in the literature. For public sector firms, the output elasticity of the consumption good $z = 0.3000$ and the output elasticity of public employment is $(1 - z) = 0.7000$ (Pappa, 2009, Papageorgiou & Vourvachaki, 2017) and the Rotemberg price adjustment cost index $\xi_\rho = 600$.

2.3.7.2 Simulated Method of Moments

I this subsection, I use the Simulated Method of Moments to calibrate my model's main parameter values. I use CPS data and estimate the standard deviation of unemployment σ_u, the standard deviation of private sector job destruction rates σ_{δ^P} the cross correlation of vacancies and unemployment $corr(v,u)$, the autocorrelation of unemployment $autocorr(u)$, the standard deviation of public sector employment σ_{N^G} and the autocorrelation of public sector employment $autocorr(N^G)$. I also estimate the standard deviation of productivity σ_A and the autocorrelation of productivity, $autocorr(A)$ using data on real average output per worker from OECD (Shimer, 2005) and the cross correlation of public debt and government expenditure $corr(B,G)$ and the autocorrelation of public debt $autocorr(B)$ using data from BEA. I then simulate the same empirical moments for my model.

Table 2.3: Simulation Results

Empirical Moment	Data	Calibration	Source
σ_u	0.2891	0.2194	Directly Estimated
σ_{δ^P}	0.1297	0.1120	Directly Estimated
$corr(v,u)$	-0.8268	-0.8779	Directly Estimated
$autocorr(u)$	0.8292	0.8094	Directly Estimated
σ_{N_G}	0.0166	0.0165	Directly Estimated
$autocorr(N^G)$	0.5996	0.6056	Directly Estimated
σ_A	0.0130	0.0130	Directly Estimated
$autocorr(A)$	0.7941	0.7940	Directly Estimated
$corr(B,G)$	0.5798	0.5900	Directly Estimated
$autocorr(B)$	0.9086	0.8361	Directly Estimated

Note: Data are in quarterly frequency from 2003Q1-2021Q4. Series are seasonally adjusted then taken in log form and detrended with an HP filter with smoothing parameter of 100000. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: BEA, CPS, OECD).

As we can see in Table 2.3, the real empirical moments from the dataset and the estimated empirical moments from the model are a close match, the only exceptions bring the autocorrelation of public debt and the standard deviation of unemployment. Despite these exceptions the close fit for the rest of the empirical moments indicates that my model with its simulated empirical moments and calibrated parameters is a good approximation of the US and the real empirical moments that come from the data.

Table 2.4: "Internally" Calibrated Parameters

Parameter	Parameter Name	Value	Source/Target
ξ	Elasticity of Investment in private sector vacancies	0.5857	σ_u
fs	Fundamental Surplus	0.0100	σ_{δ^P}
$\varepsilon_{\delta,P}$	Elasticity of Private Sector Job Destruction Rate	-0.4598	$corr(v,u)$
ϵ_w	Elasticity of Private Sector wages to Private Sector Firms' Output	0.9900	$autocorr(u)$
$\varepsilon_{\mathscr{H}^G}$	Feedback Parameter	-67.7722	$autocorr(N^G)$
$\sigma_{\mathscr{H}^G}$	i.i.d. shock parameter	0.5246	σ_{N_G}
ε_A	i.i.d. shock parameter	0.0059	σ_A
ρ_A	TFP Shock Autocorrealtion Parameter	0.9114	$autocorr(A)$
ρ_B	Public Debt Autocorrelation Parameter	0.0486	$autocorr(B)$
κ_B	Public Debt response to Government Expenditure	1.7134	$corr(B,G)$

Finally in Table 2.4 I have my "internally" calibrated parameters, which I estimate using Simulated Method of Moments. Starting with the coefficient of investment in private sector vacancies ξ, we see that investment in private sector vacancies is quite inelastic, with $\xi = 0.5857$. The fundamental surplus ratio fs is also very small at 0.0100 which is the lower bound for my calibration. Looking at the elasticity of private sector wages I find that $\epsilon_w = 0.9900$ while the elasticity of private sector job destruction rates is much smaller, with $\varepsilon_{\delta,P} = -0.4598$. These values imply that business cycles in the US lead to massive wage changes but changes in the private sector job destruction rate are much smaller, while the low values of ξ and fs mean that investment in private sector vacancies is quite inelastic but that business cycles also have very large effects in the fundamental surplus ratio, and in the resources available to private sector firms for investment in private sector vacancies. Finally the feedback parameter on the hirings of public sector employees, is extremely large in the US, ($\varepsilon_{\mathscr{H}^G} = -67.7722$), so when hirings of public sector employees increase in the US the effect is relatively brief.

2.4 Results

I now investigate the effect of an increase in public sector firms' output, caused by an increase in public sector employment. The increase is debt-financed and repaid by raising lump-sum taxes on capitalists households. I study a 10-year time period, at monthly frequency, assuming a one-period positive shock, in the form of a one standard deviation increase in hirings of public sector employees $\mathscr{H}_t^G$.

I start by comparing the effects of positive shocks in total factor productivity and public sector employment, in order to determine the different way these shocks affect both the goods market and the labour market. I will also be running a number of counterfactual experiments comparing my baseline results with results under more elastic investment in private sector vacancies, and under flexible prices to ascertain the impact of the *Labour Marker Effect* and *Aggregate Demand Effect*. Additionally, I will examine how my results change if I do away with heterogeneous workers households and incomplete asset markets and instead assume complete markets and a risk averse representative agent household whose members can fully insure themselves against risk, which will help determine the effect of the *Unemployment Risk Effect*. Finally, I will study how my results change under a more accommodate monetary policy, by bringing my model closer to the ZLB.

2.4.1 Results - TFP and Public Sector Employment shocks

I start my analysis by comparing the effects of increasing public sector employment N_t^G and total factor productivity A_t, assuming a one standard deviation increase for each variable. The results for these shocks, given in Figure 2.2, are markedly different from the findings in Chapter 1. More specifically in Figure 2.2, Panel C increasing productivity A_t now leads to a large, persistent rise in unemployment u_t, which increases by as much as 8.28% with the effect lasting 4-5 years. More importantly this change comes solely from private sector employment. Conversely, increasing public sector employment N_t^G decreases unemployment u_t by about 2% with this effect also lasting around 60 months and leading to a marginal increase in private sector employment N_t^P. These results are similar to a number of papers (Gali, 1999, Francis & Ramey, 2005, Gregory et al., 2016, Furlanetto et al., 2020)

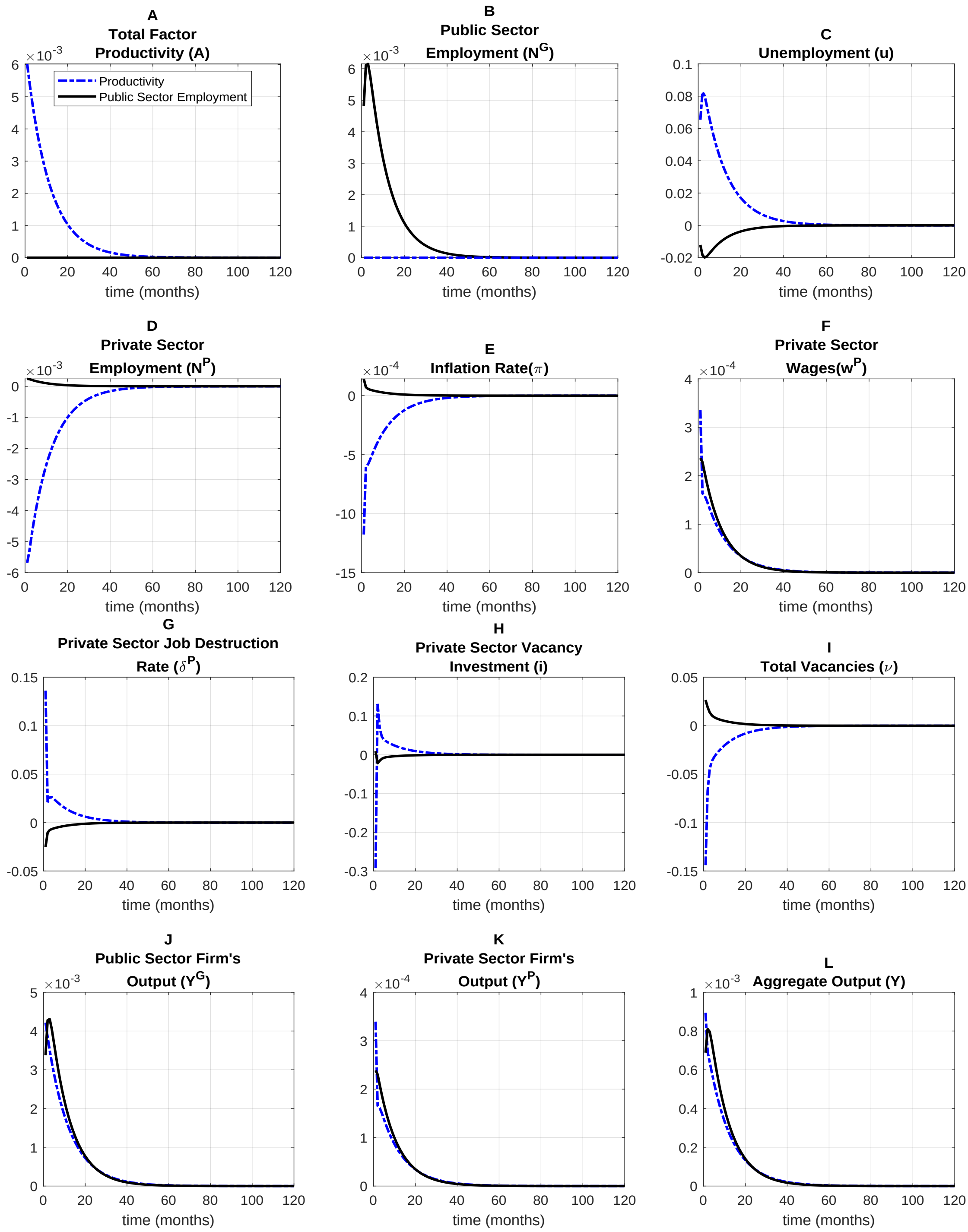

Figure 2.2: Effect of an increase in Productivity and in Public Sector Employment.

indicating that productivity shocks raise output but lower employment, while aggregate demand shocks increase both output and employment.

The different effects are a result of the different responses of prices and wages shocks, as seen in Panels E to J. When total factor productivity increases, marginal costs also go down, which lowers prices and inflation π_t, as seen in Panel E. At the same time the increase in private sector firms' output Y_t^P raises private sector wages w_t^P in Panels F and K since private sector wages are procyclical, as seen in Equation 2.35, but $\epsilon_w = 0.9900$ means that the rise in wages is almost one for one with the increase private sector firms' output. As a result, the value of filled vacancies J_t^F goes down as does the value of open vacancies J_t initially, which in turn increases the private sector job destruction rate δ_t^P and reduces investment in private sector vacancies i_t. These two effects combined reduce vacancies v_t, and private sector employment N_t^P.

The effect of public sector employment works though the model's propagation mechanism. As public sector employment N_t^G goes up, it increases public sector firms' output Y_t^G, which increases aggregate output Y_t resulting in the *Direct Effect*. At the same time in the *Labour Market Effect* the probability an unemployed workers household fills a private sector open vacancy ζ_t^P, and investment in private sector vacancies i_t decrease. As this effect propagates aggregate demand, inflation π_t and the value of filled vacancies J_t^F decrease, raising the private sector job destruction rate δ_t^P and reducing private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t.

In the following periods, the *Aggregate Demand Effect* begins working. In the *Redistribution Effect* income is transferred from low-MPC capitalists households who pay higher lump-sum taxes to repay the public debt to high-MPC unemployed workers households hired by public sector firms. This raises consumption, inflation π_t and private sector firms' output Y_t^P, indirectly increasing aggregate output Y_t. Then the *Indirect Aggregate Demand Effects* come into play as higher inflation raises the value of filled vacancies J_t^F and investment in private sector vacancies i_t and reduces the private sector job destruction rate δ_t^P. As a result, private sector employment N_t^P, inflation π_t and private sector firms' output Y_t^P, increase again, further increasing aggregate output Y_t and creating a multiplier effect.

Finally, the *Unemployment Risk Effect* lowers unemployment risk of workers households and demand for precautionary savings, raising interest rates for household bonds R_t, aggregate demand and inflation π_t. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P, and increases aggregate output Y_t even more, reinforcing the multiplier.

Looking at this shock quantitatively, we can see that it is not very large. The reason is that the *Direct Effect* is rather short-lived as $\varepsilon_{\mathscr{H}G} = -67.7722$. The *Labour Market Effect* is also small as $\xi = 0.5857$, and $fs = 1\%$ make investment in vacancies i_t relatively inelastic, and although $\varepsilon_{\delta^P} = -0.4598$ and $\epsilon_w = 0.9900$ make the increase in the private sector job destruction rate δ_t^P relatively small and the change in wages much larger, these same characteristics also make *Aggregate Demand Effect* relatively small. However, the *Direct Effect* and *Aggregate Demand Effect* seems to be stronger than the *Labour Market Effect* and the overall effect is positive, so inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t increase. Consequently, the *Unemployment Risk Effect* is positive, so investment in private sector vacancies, private sector employment N_t^P, private sector firms' output Y_t^P, wages rise further. As a result, aggregate output Y_t increases by 0.08% and private sector firms' output Y_t^P sees a marginal rise.

2.4.2 Results - Elastic Vacancy Investment

I now turn my attention to the effect that the elasticity of investment in private sector vacancies has on my results. To this end, I compare the results of the baseline case with the case of elastic investment in private sector vacancies, by setting $\xi = 100000$. What is clear from Figures 2.3 and 2.4 is that results are practically the same between the two cases. More specifically in Figure 2.3 Panel B, increasing public sector employment N_t^G has the same effect in unemployment u_t in both cases, although it is initially slightly smaller in the elastic case. Similarly the effect on the rest of the labour market variables is also the same, the only exception being investment in private sector vacancies i_t in Panel E, which is more volatile in the elastic case for the first 2-3 periods, and the private sector job destruction rate δ_t^P in Panel F where the reduction is initially larger in the baseline case. The effects on the rest of the economy are also unchanged as we can see in Figure 2.4

where private sector firms' output Y_t^P and aggregate output Y_t have no discernible differences between these two cases, the only exception being inflation π_t, which initially increases more in the baseline case.

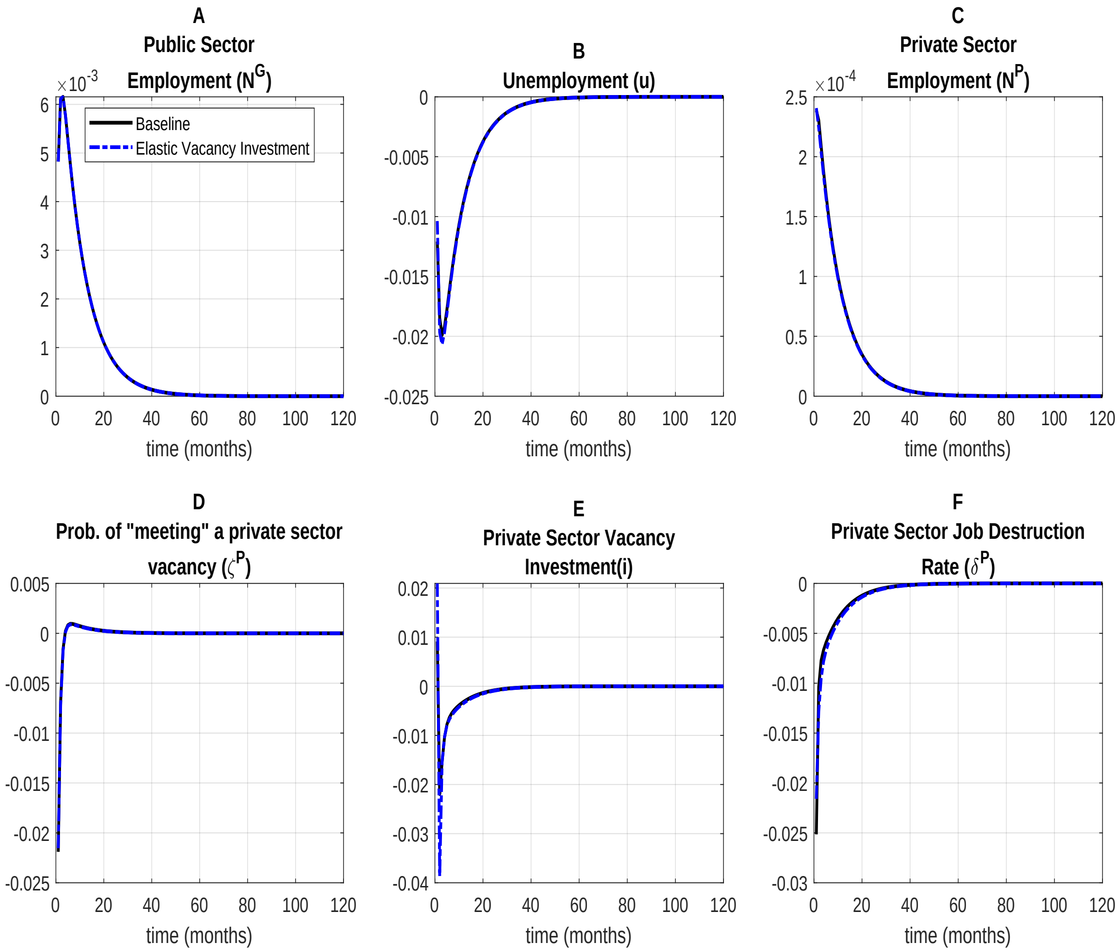

Figure 2.3: Public Sector Employment shock: Baseline Case ($\xi = 0.7447$); Elastic Vacancy Creation Case ($\xi = 100000$).

The reason for this result is that the feedback parameter $\varepsilon_{\mathcal{H}^G} = -67.7722$ is a relatively small number; as a result, shocks in the hirings of public sector employees have a very small but also short-lasting effect. Consequently increasing public sector employment, although effective in reducing unemployment and even creating a small crowding in of private sector employment, has more limited effects in the rest of the economy. This means that the *Labour Market Effect* is very small, as the drop in the value of an open vacancy J_t is very small, so any reduction in investment in private sector vacancies i_t, is limited. Because of that, any propagation of this effect through the economy in later periods is also small so changes in aggregate demand, inflation π_t, the value of filled vacancies J_t^F and the private sector job destruction rate δ_t^P are so very small. Consequently, the reduction in private sector vacancies v_t^P and private sector employment N_t^P, private sector firms' output Y_t^P, wages and aggregate output Y_t through the *Labour Market Effect* is limited even for elastic investment in private sector vacancies.

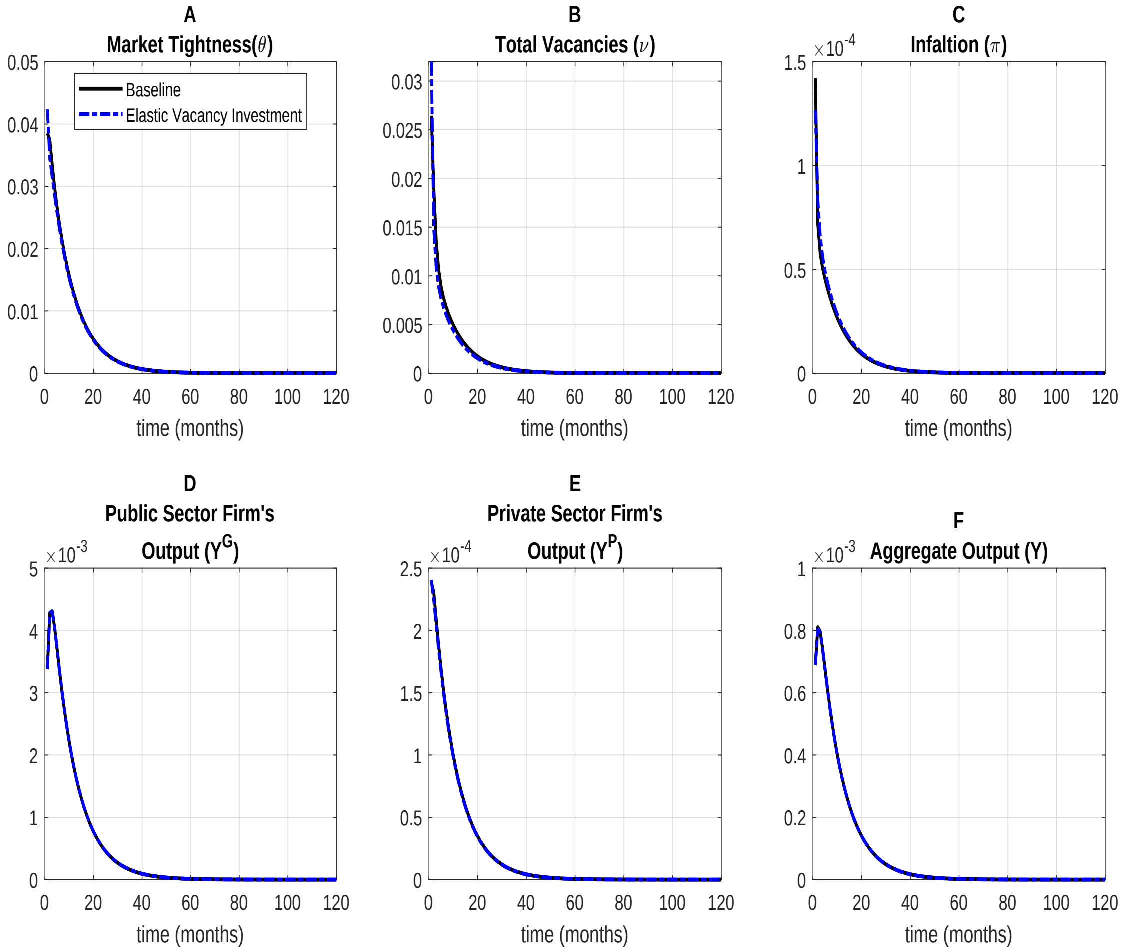

Figure 2.4: Public Sector Employment shock: Baseline Case ($\xi = 0.7447$); Elastic Vacancy Creation Case ($\xi = 100000$).

Similarly the *Aggregate Demand Effect* is also limited and does not significantly change between the two cases. First, the value of a filled vacancy J_t^F increases as aggregate demand expands. This means that investment in private sector vacancies i_t increases, while at the same time the private sector job destruction rate δ_t^P decreases. As a result the number of private sector vacancies v_t^P and private sector employment N_t^P increase. This in turn increases private sector firms' output Y_t^P and inflation π_t more, and makes the increase in aggregate output Y_t greater, reinforcing the multiplier effect. Finally, the *Unemployment Risk Effect* also behaves identically in both cases. As unemployment risk of workers households and demand for precautionary savings decrease, the interest rate for household bonds R_t aggregate demand and inflation π_t increase. Combined with the *Aggregate Demand Effect* this raises investment in private sector vacancies even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P, wages and increases aggregate output Y_t further.

2.4.3 Results - Flexible Prices

In this part of my analysis I look how changes in price stickiness affect my results, by comparing a fully flexible prices case where I set $\xi_\rho = 0$ to the baseline case of sticky prices where $\xi_\rho = 600$. The results I find in this subsection are particularly important as they will help me further examine how the *Aggregate demand Effect* impacts on the effectiveness of public sector employment and public sector firms' output.

Starting in Figure 2.5 Panel B I still find that in both cases increasing public sector employment N_t^G leads to a large and persistent decrease in unemployment u_t, which decreases by 2% in the baseline case while for flexible prices the reduction is less than half that value, indicating that flexible prices dramatically reduce the effectiveness of increasing public sector employment. More importantly, flexible prices also result in crowding out of private sector employment N_t^P compared to a small crowding in effect in the baseline case.

Initially the *Labour Market Effect* is the same in both the baseline case and the flexible prices case, which is evident in Panel D where the probability that an unemployed workers household fills a private sector open vacancy ζ_t^P decreases by the same amount in both cases. As public sector employment N_t^G increases, the probability that an unemployed workers household fills a private sector open vacancy ζ_t^P drops, which decreases the value of an open vacancy J_t and investment in private sector vacancies i_t. As this effect propagates to the rest of the economy in later periods it lowers inflation π_t and the value of filled vacancies J_t^F, increasing the private sector job destruction rate δ_t^P. These factors reduce private sector vacancies v_t^P and private sector employment N_t^P which in turn decrease private sector firms' output Y_t^P, wages and aggregate output Y_t.

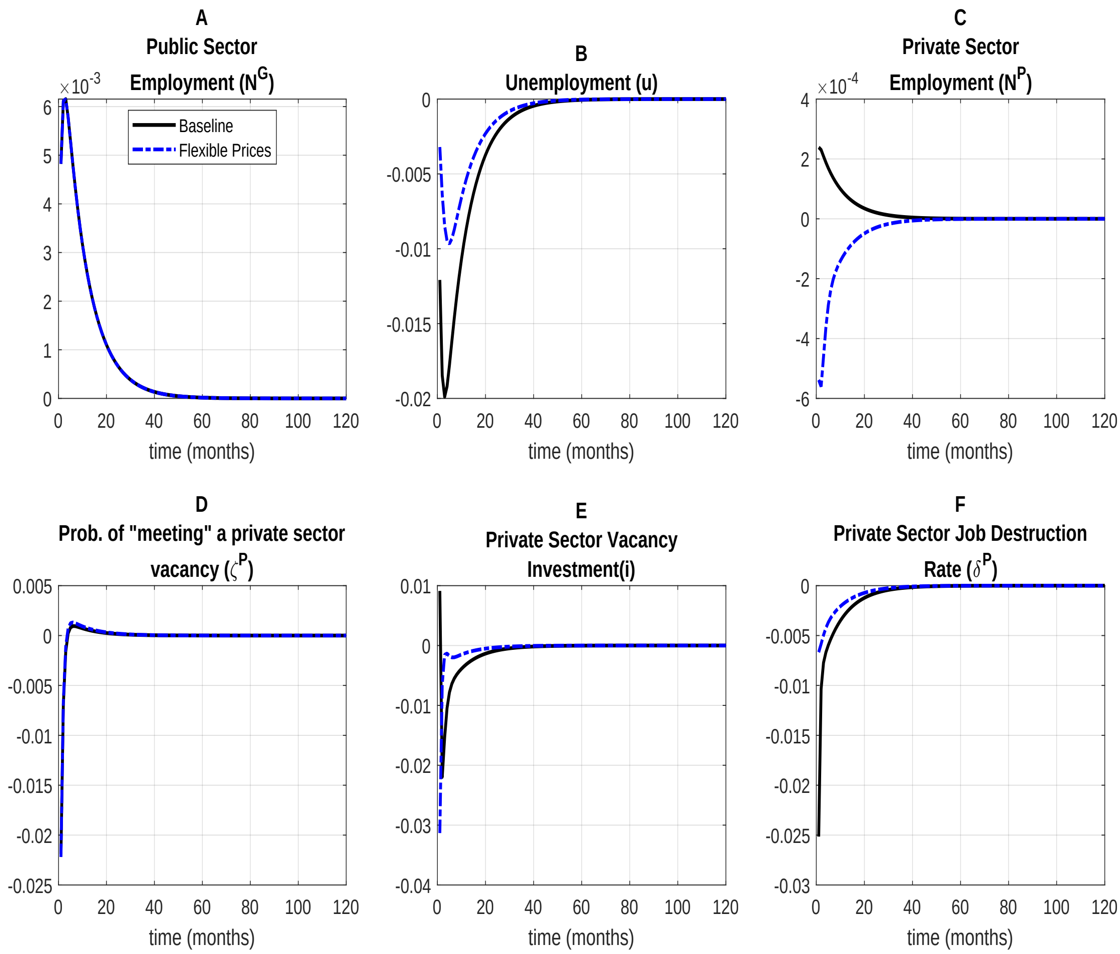

Figure 2.5: Public Sector Employment shock, Sticky Prices (ξ_ρ = 600); Flexible Prices (ξ_ρ = 0).

However, now because of price flexibility there is no *Aggregate Demand Effect*. In the *Redistribution Effect* because prices are fully flexible, unemployed workers households who are hired by public sector firms do not increase their consumption as prices have increased too much. Consequently, aggregate demand does not change and only inflation π_t sees a massive surge which means that investment in private sector vacancies i_t, the private sector job destruction rate δ_t^P, the number of private sector vacancies v_t^P and private sector employment N_t^P are unaffected. As a result private sector firms' output Y_t^P and aggregate output Y_t also do not change. The result of the *Aggregate Demand Effect* is captured by Panel E and F in Figure 2.5, where under sticky prices investment in private sector vacancies i_t initially increases, and the reduction in the private sector job destruction rate δ_t^P is greater, which in turn raises the labour market tightness θ and total vacancies v_t in Panels G and H in Figure 2.6. Conversely for flexible prices, investment in private sector vacancies actually decreases and the drop in the private sector job

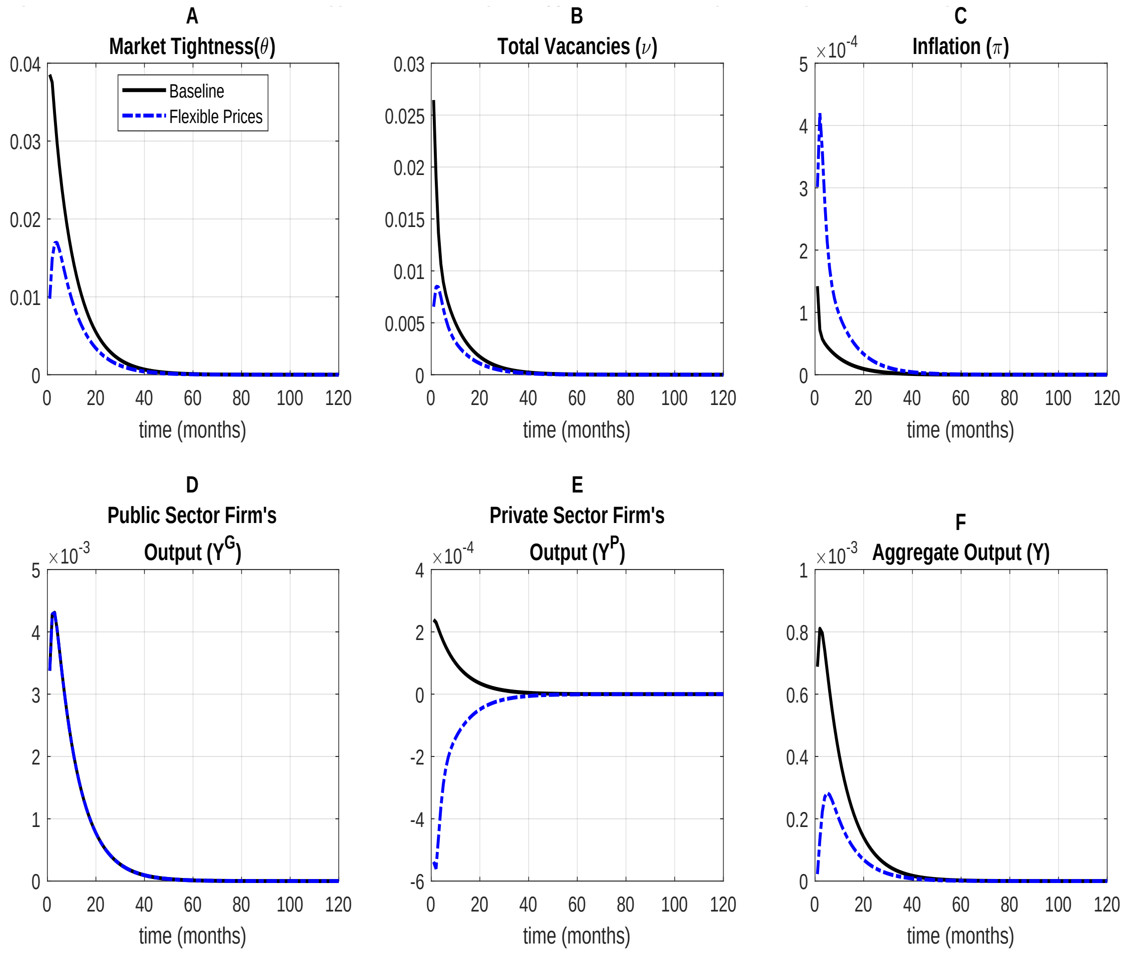

Figure 2.6: Public Sector Employment shock, Sticky Prices (ξ_ρ = 600); Flexible Prices (ξ_ρ = 0).

Finally, the *Unemployment Risk Effect* is, similarly to the baseline case, negatively affected by the *Labour Market Effect* but at a much larger scale so any subsequent positive effects the reduction in the unemployment risk of workers households has are much smaller under flexible prices. Conversely in the baseline case, the combination of the *Direct Effect* with a stronger *Aggregate Demand Effect* makes for a larger positive *Unemployment Risk Effect*, because prices are sticky.

Closing my analysis with the effects in the goods market in Figure 2.6, I find that the effects of increasing public sector firms' output are also smaller as aggregate output Y_t increases less (0.028%) while similarly to private sector employment, private sector firms' output is crowded out and stays below its steady state value for approximately 50 months. In short, flexible prices reinforce the *Labour Market Effect*, and make the *Aggregate Demand Effect* and *Unemployment Risk Effect* weaker through the propagation mechanism.

2.4.4 Results - Heterogeneous Households and Homogeneous Household

I now turn my attention to the effects that heterogeneous households, incomplete asset markets and unemployment risk have on my results. I do away with these assumptions and instead assume complete markets and a risk averse representative agent household whose members work or are unemployed, own private sector firms and invest in public sector bonds. Essentially the heterogeneous workers households and the homogeneous capitalists households are now a single household whose members pool their income from all sources - labour income, unemployment benefits, dividends from private sector firms and returns from public sector bonds - together. More importantly there are no incomplete asset markets and no unemployment risk as this household insures itself against risk. The changes in the model equations are presented in Appendix B.3.

What is clear is that altering these assumptions completely changes the results of my model. More specifically in Figure 2.7 Panel B increasing public sector employment now has an even stronger effect, as unemployment decreases by 3.05% and more importantly, as seen in Panel C, the crowding in effect is now much larger while also lasting longer than the baseline case.

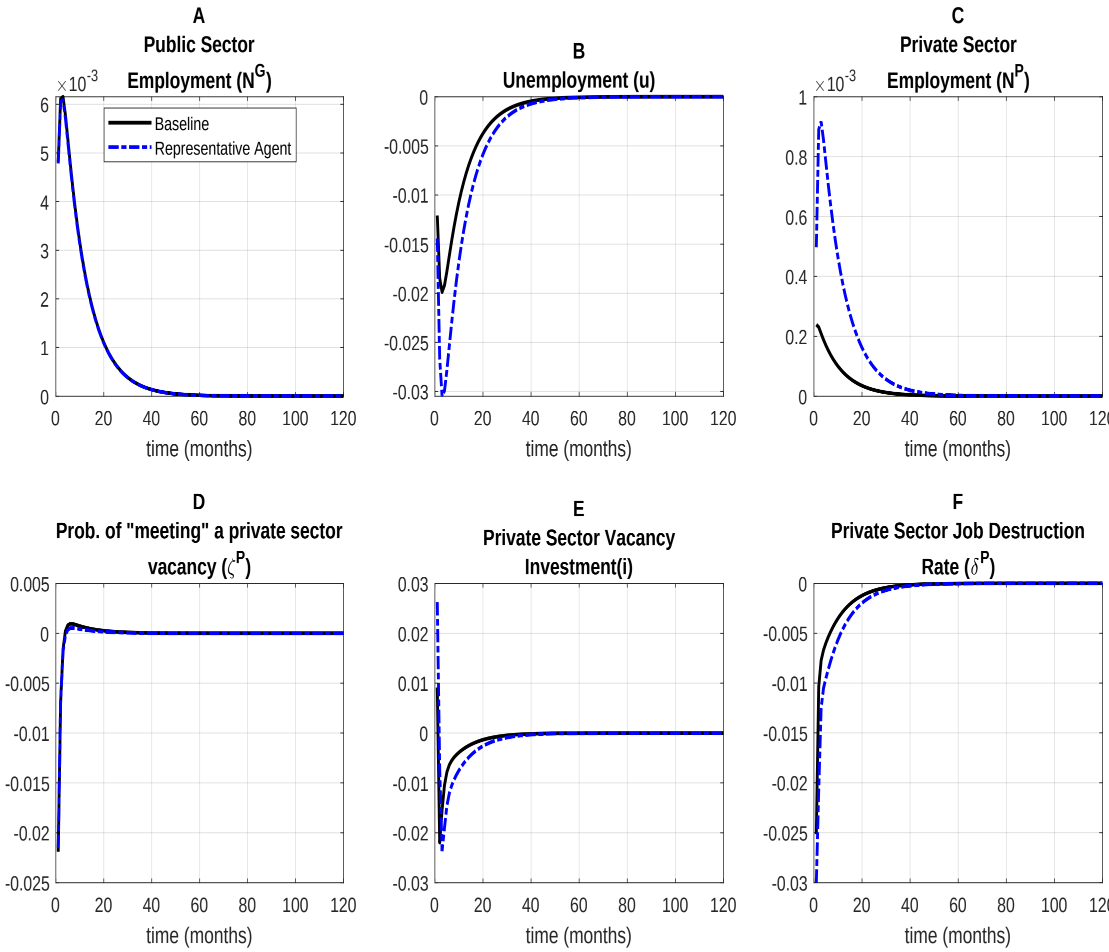

Figure 2.7: Public Sector Employment shock: Heterogenenous Households; Homogeneous Household

The analysis is unchanged: In the *Direct Effect*, rising public sector employment N_t^G increases public sector firms' output Y_t^G, which directly increases aggregate output Y_t. In the *Labour Market Effect*, as public sector employment N_t^G increases, the probability an unemployed workers household fills a private sector open vacancy ζ_t^P drops, decreasing the value of an open vacancy J_t and investment in private sector vacancies i_t. However this effect is now weaker and does not propagate as household members employed in private sector firms who lose their jobs now have access to other income sources, so aggregate demand and inflation π_t do not decrease and the private sector job destruction rate δ_t^P does not increase. As a result private sector vacancies v_t^P, private sector employment N_t^P private sector firms' output Y_t^P and aggregate output Y_t do not decrease as much as in the baseline case.

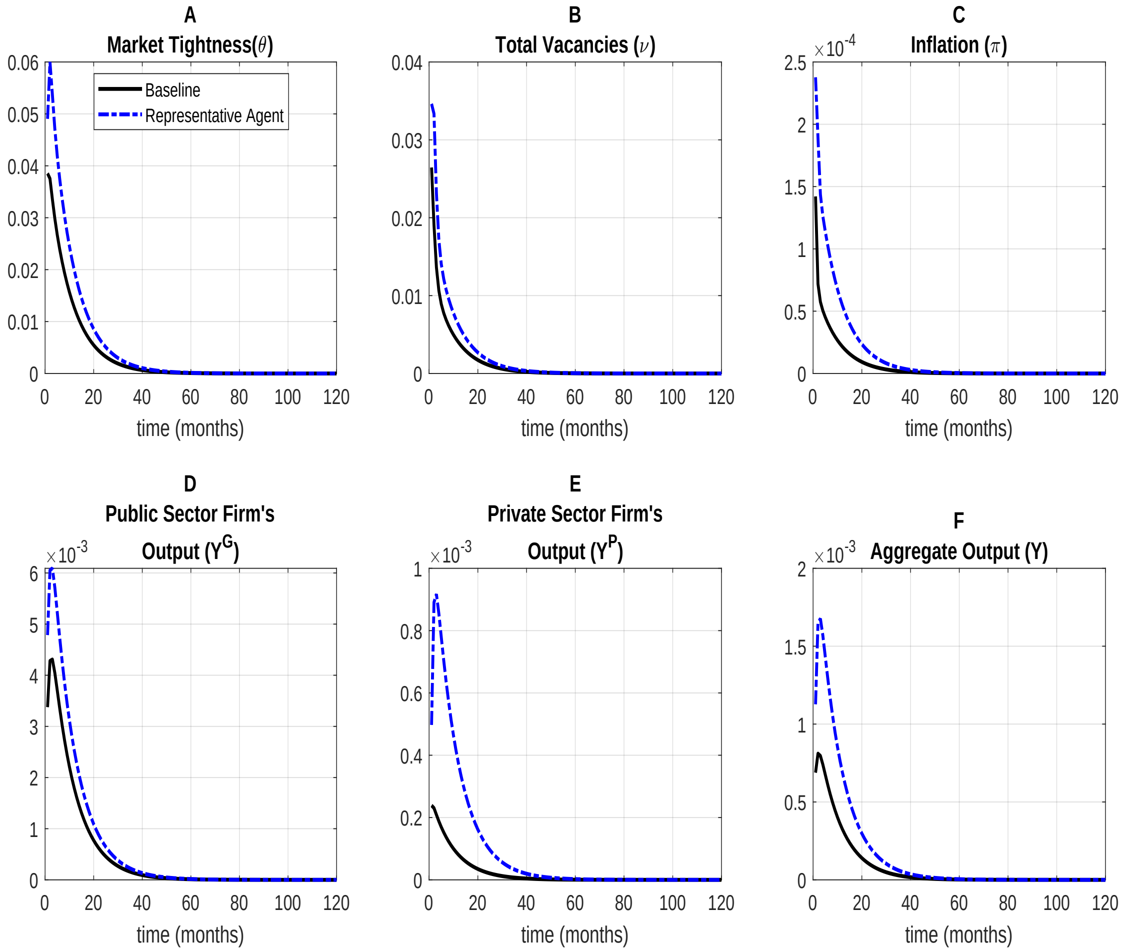

Figure 2.8: Public Sector Employment shock: Heterogenenous Households; Homogeneous Household

Meanwhile the *Aggregate Demand Effect*, is strengthened. First, in the *Redistribution Effect* the income taken away by debt returns to the household as labour income given to unemployed household members hired in public sector firms. As a result the increase in consumption, aggregate demand, inflation π_t and private sector firms' output Y_t^P is larger, which also makes the indirect increase in aggregate output Y_t larger. Then in the the next periods, the *Indirect Aggregate Demand Effects* and the overall *Aggregate Demand Effect* are also larger: Higher inflation raises the value of a filled vacancy J_t^F and investment in private sector vacancies i_t, lowering the private sector job destruction rate δ_t^P. This raises private sector employment N_t^P, private sector firms' output Y_t^P, wages and inflation π_t further and increases aggregate output Y_t even more, creating a multiplier effect.

Therefore, even though there is no *Unemployment Risk Effect* the homogeneous household mitigates its consumption losses. This makes the positive effect in the economy bigger as seen in Panels G to I in Figure 8 where inflation π_t, market tightness θ_t and vacancies v_t now increase more while the private sector job destruction rate δ_t^P in Panel F Figure 7 decreases more. As a result, increasing public sector firms' output Y_t^G now leads to an increase in aggregate output Y_t which is more than two times that in the baseline case (0.167% compared to 0.08%) and to a bigger, longer lasting increase in private sector firms' output.

2.4.5 Results - Monetary Policy Responsiveness

I close my analysis by looking at how differences in monetary policy responsiveness affect my results. I assume that instead of a normal Taylor Rule central banks follow a less responsive monetary policy by setting $\phi_\pi = 1.45$. This case is of particular importance as there are times like the Great Depression, or even normal economic times, when monetary policy is particularly unresponsive to increases in government expenditure, which can in principle make fiscal policy more effective as in Albertini et al. (2014), Christiano et al. (2011) and Woodford (2011). I will study the interaction of fiscal and monetary policy more formally and in greater detail in Chapter 3, using the Occasionaly Binding Constaint (OccBin) Dynare tool package developed by Guerrieri & Iacoviello (2015) to study the effects of public sector employment when the economy operates in the Zero Lower Bound.

As we can see in Figure 2.9 there are no discernible changes in the way increasing public sector employment affects the labour market. More specifically, in Panel B the reduction in unemployment u_t is the same in the baseline case and the Zero Lower Bound and the effect also lasts for 50-60 months in both cases. The only difference is that the crowding in of private sector employment N_t^P, is slightly larger for a less responsive monetary policy.

The reason for the similar results is that the policy shock is very small and relatively short lasting, as $\varepsilon_{\mathscr{H}^G} = -67.7722$. The mechanism is unchanged: As public sector employment N_t^G increases, it increases public sector firms' output Y_t^G which directly increases aggregate output Y_t creating a *Direct Effect*. Higher public sector employment N_t^G lowers the probability an unemployed workers household fills a private sector open vacancy ζ_t^P, reducing the value of open vacancies J_t and investment in private sector vacancies i_t; this *Labour Market Effect* then propagates through the model, reducing aggregate demand and inflation π_t and increasing the private sector job destruction rate δ_t^P. As a result, private sector vacancies v_t^P and private sector employment N_t^P drop, which in turn decreases private sector firms' output Y_t^P, wages and aggregate output Y_t.

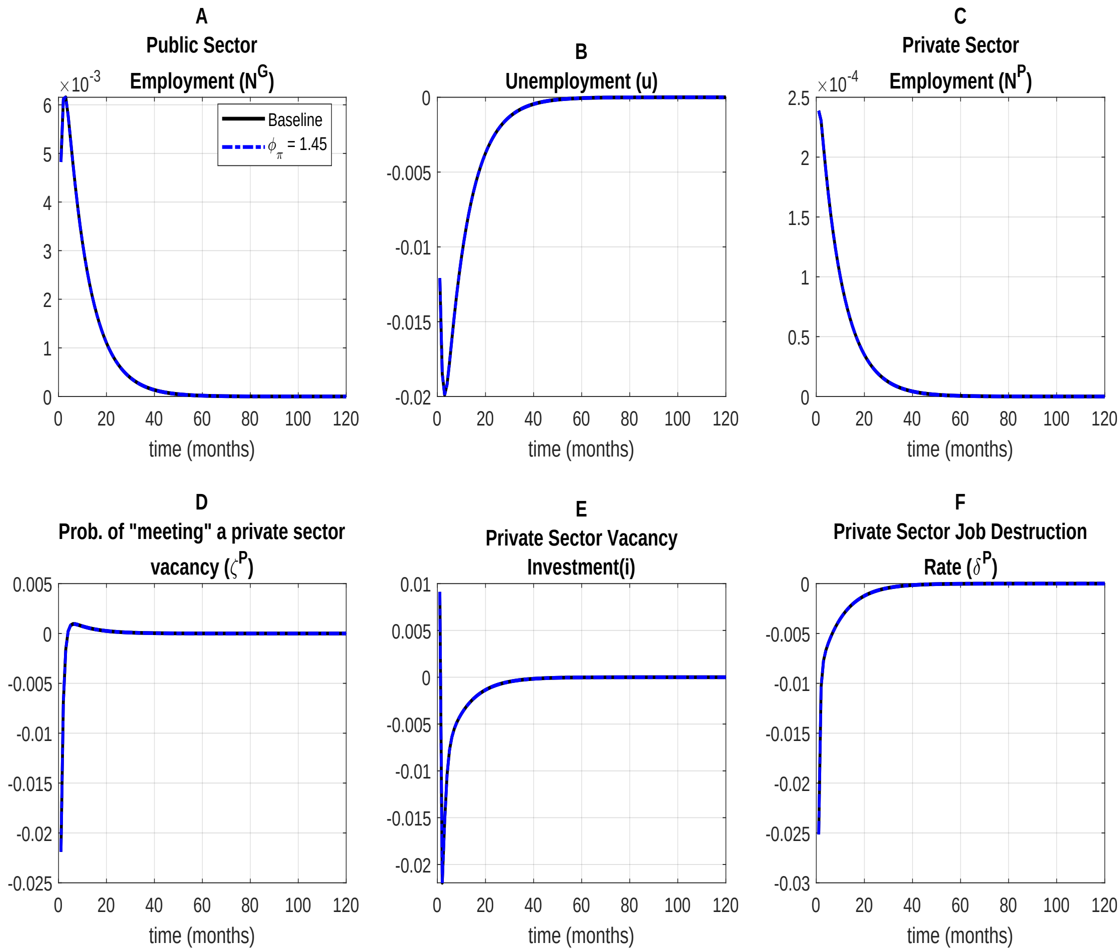

Figure 2.9: Public Sector Employment shock: Less Responsive Monetary Policy (ϕ_π = 1.45); Normal Taylor Rule (ϕ_π = 1.50).

The *Aggregate Demand Effect*, works as before. In the *Redistribution Effect*, the government redistributes income to unemployed workers households hired in public sector firms. This raises aggregate demand, inflation π_t, private sector firms' output Y_t^P and wages, indirectly increasing aggregate output Y_t further. In the next periods the *Indirect Aggregate Demand Effects*, that propagate in the labour market are unchanged: Higher inflation raises the value of filled vacancies J_t^F and investment in private sector vacancies i_t, lowering private sector job destruction rates δ_t^P. This raises private sector employment N_t^P, private sector firms' output Y_t^P, wages and inflation π_t further and increases aggregate output Y_t even more.

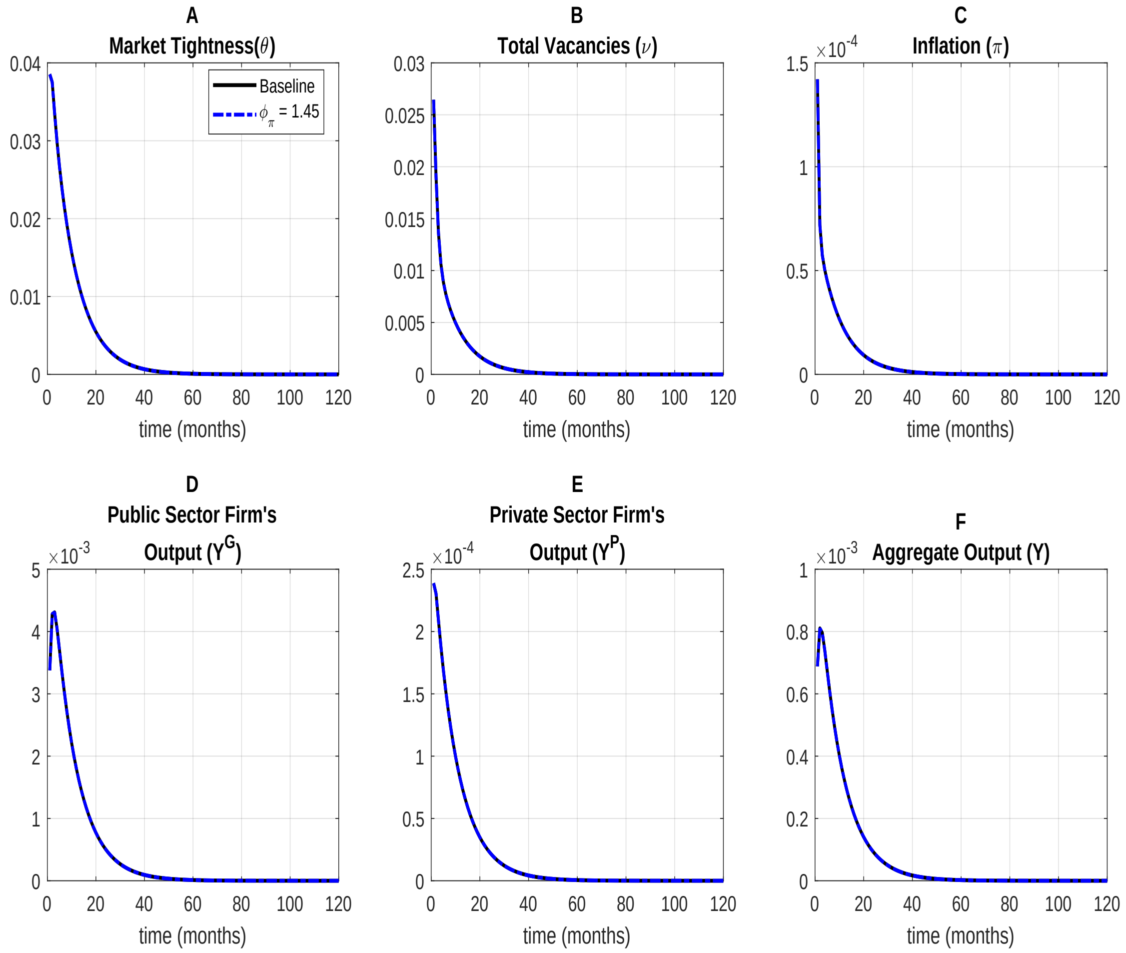

Figure 2.10: Public Sector Employment shock: Less Responsive Monetary Policy (ϕ_π = 1.45); Normal Taylor Rule (ϕ_π = 1.50).

Finally in the *Unemployment Risk Effect* the *Direct Effect* and the *Aggregate Demand Effect* lower unemployment risk of workers households and their demand for precautionary savings, increasing the interest rate for household bonds R_t, aggregate demand and inflation π_t. This raises investment in private sector vacancies, private sector employment N_t^P, private sector firms' output Y_t^P and wages and increases aggregate output Y_t. Similarly to the labour market, we see in Figure 2.10 that there is no difference in increasing public sector firms' output Y_t^G between the two cases as both the increase in aggregate output Y_t in private sector firms' output is unchanged.

2.4.6 Multiplier Analysis

I conclude this section by focusing on the multipliers for each of the policy experiments I study. Following Hagedorn et al. (2019) and Monacelli et al. (2010), I start by calculating the Unemployment Multiplier $\frac{du_t}{dN_t^G}$, which measures the reduction in unemployment when public sector employment increases. I then use the change in public sector firms' output, caused by the public sector employment shock, to compute the Aggregate Output Multiplier $\frac{dY_t}{dY_t^G}$, which is the increase in aggregate output when public sector firms' output increases. Calculating both these multipliers allows me to see how increasing public sector employment and public sector firms' output affects not just output but also the labour market; additionally this analysis makes sense from a policy viewpoint as it helps see how fiscal policy can smooth out the effects of business cycles not just in output but also in employment. I focus on the cumulative multiplier, representing the discounted percentage change in aggregate output (unemployment) relative to the discounted percentage change in public sector firms' output (public sector employment), which allows me to examine the full effect of fiscal policy, not just the immediate impact a policy might be having.

$$\frac{\sum dY_t}{\sum dY_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t - Y}{Y} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t^G - Y^G}{Y^G} \right\}} \frac{Y}{Y^G} \tag{2.70}$$

$$\frac{\sum du_t}{\sum dN_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{u_t - u}{u} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{N_t^G - N^G}{N^G} \right\}} \frac{u}{N^G} \tag{2.71}$$

Table 2.5: Public Sector Employment Multipliers

Policy Experiment	Baseline Case	Elastic Vacancy Investment	Flexible Prices	Representative Agent	Zero Lower Bound
Aggregate Output Multiplier	1.0259	1.0224	0.4051	1.3974	1.0259
Unemployment Multiplier	-1.4792	-1.5001	-0.8069	-2.3004	-1.4793

My results are presented in Table 2.5. The first important result is that in all the cases I study the multipliers are positive numbers above unity with the exception of the flexible price case; more importantly the cumulative Unemployment Multiplier is above unity (in absolute terms) in all cases. This result implies that increasing public sector employment in the US is very useful at reducing unemployment and can help smooth out unemployment fluctuations created by business cycles and also crowd in the private sector.

More specifically in the baseline case, elastic vacancy investment case and under a more accommodative policy I find a large and positive effect on aggregate output when increasing public sector firms' output with a multiplier of 1.0259 for the baseline case and for $\phi_\pi = 1.45$, and 1.0224 for the elastic vacancy investment case. Similarly, increasing public sector employment leads to a large reduction in unemployment with the multiplier of -1.4792 for the baseline case, -1.4793 when $\phi_\pi = 1.45$ and -1.5001 for the elastic vacancy investment case. These findings as well as the ones in the previous subsections show that for the case of the US economy, increasing public sector employment can be very useful in reducing unemployment, however the size of the public sector and the relatively short duration of policy shocks, together with the inelastic behaviour of investment in private sector vacancies and the private sector job destruction rate make the effects on the goods market more limited, by making *Aggregate Demand Effect* and *Unemployment Risk Effect* smaller.

Under flexible prices the Aggregate Output Multiplier equals 0.4051 and the Unemployment Multiplier −0.8069, so flexible prices fiscal policy is far less effective, as the *Labour Market Effect* is magnified and both the *Aggregate Demand Effect* and *Unemployment Risk Effect* are mitigated, as seen in the previous subsection. Finally for the representative agent case both multipliers are much improved, being equal to 1.3974 and −2.3004, which is indicative of the important role heterogeneous workers households, incomplete asset market and uninsurable unemployment risk play in my model.

To sum up, all my results indicate that increasing public sector employment is particularly effective at reducing unemployment and, more importantly also leads to a small crowding in of private sector employment. The effect on the goods market is more mitigated however, as the nature of the public sector and the labour market in the US economy make any positive effects in aggregate demand smaller.

2.5 Conclusion

In this paper I investigated the effect of increasing public sector employment in aggregate output and unemployment. I built a HANK model with a frictional search and matching labour market, public sector employment and public sector firms. I argued that my model's results are both novel and realistic, being one of the few models combining heterogeneous households and incomplete asset markets, sticky prices and a frictional labour market, and the first to study public sector employment and public sector firms' output, allowing it to capture all the propagation mechanisms and effects in an economy, unlike earlier, simpler models that omit some parts, and consequently some effects and propagation mechanisms.

Using this model for the US I find that these policies can significantly lower unemployment and even lead to an increase of private sector employment. Furthermore they increase in aggregate output, however the effect here is more muted as the relatively small size of US public sector and the short duration of policy shocks, combined with the inelastic behaviour of investment in private sector vacancies and the private sector job destruction rate make the effects on aggregate demand and consequently on the goods market more limited.

These results speak to the importance of explicitly modelling public sector employment and public sector firms' output in HANK models. They also show the importance of monopolistic competition, sticky prices, heterogeneous households and incomplete asset markets as well as a realistic, frictional labour market in macroeconomic models.

A natural limitation of my approach is that I do not have physical capital in my model so adding physical capital, owned by capitalists households is a natural next step as it would create a even more realistic setup, and also allow me to study the effects of the policies studied on investment. Additionally studying different types of financing policies such as public debt, capital taxes or labour taxes could provide some interesting comparisons.

CHAPTER 3

Assessing the Effects of Public Sector Employment in Different Countries

3.1 Introduction

The size and behaviour of the public sector, particularly public sector employment, and the flows between employment conditions are critical to our understanding of both labour market dynamics and business cycles. Furthermore, they can be important additions in state-of-the-art models, such as frictional SAM models (Coles & Moghaddasi Kelishomi, 2018, Ljungqvist & Sargent, 2017, 2021), TANK[1] and HANK [2] models, and help us get a more complete, realistic depiction of an economy. However, as seen in Chapter 2, both the effects of public sector employment, as well as models combining HANK with SAM to study these effects, have not been adequately developed.

The objective of this chapter is two fold: The first is to establish a number of key facts about the public sector, public sector employment and labour market stocks and flows in France, the UK and the US, using national accounting and Labour

[1] Auclert et al. (2018), Bilbiie (2020), Cantore & Freund (2021),Courtoy (2022), Klein et al. (2022)

[2] Auclert et al. (2018; 2020), Hagedorn et al. (2019), Kaplan & Violante (2014), McKay & Wolf (2022)

Force Survey data covering a period of nineteen years (2003 to 2021), providing a systematic study of these countries' public sectors and their labour markets similar to Fontaine et al. (2020), which influence the effects of public sector employment.

The second objective is to find how increasing public sector output and public sector employment affects aggregate output and employment via a debt financed increase repaid by raising taxes on capitalists households or taxes on capitalists households and workers households. To do so, I use my Chapter 2 model where I combine a HANK model (Ravn & Sterk, 2017, 2021, Broer et al., 2021) and a frictional labour market (Coles & Moghaddasi Kelishomi, 2018) with public sector employment and public sector firms producing goods using government expenditure on consumption and public sector employment[3].

This model, one of the few to combine heterogeneous households and incomplete asset markets, monopolistic competition, sticky prices and a frictional labour market and the first to study public sector employment and public sector output, makes my results both novel, as these policies have not been adequately studied, but also realistic, capturing all the propagation mechanisms and effects in an economy, unlike earlier, simpler models. Also, it allows me to see how such policies affect European countries, exemplified here by France, which have large public sectors, greater degrees of employment protection and labour market rigidities and countries with smaller, less interventionist governments and flexible labour markets like the US and the UK. Additionally the frictions I incorporate in my model allow it to simulate the behaviour of labour market variables such as unemployment and job vacancies but also public sector variables such as public debt and government expenditure, matching the behaviour these variables exhibit in many different countries.

Both the stylised facts and the results are of particular interest to policymakers and to the macroeconomics literature. For policymakers they can help improve the monitoring of business cycles and the design of efficient fiscal responses. For macroeconomics, this paper can be a reference point for models combining HANK and SAM with public sector employment and public sector output and a guideline for studying these fiscal tools. Both models combining HANK and SAM as well as the effects of public sector employment and public sector output have not been

[3] Pappa (2009), Forni et al. (2010), Economides et al. (2013; 2017)

adequately studied so this model provides a new framework that macroeconomic models should ideally incorporate but also answers important policy questions.

Data seem to validate the conventional wisdom that European countries have larger public sectors, compared to the US, a result of more interventionist public sectors (Colli & Nevalainen, 2019, De Lange & Merlevede, 2020, Christiansen, 2011, OECD, 2017, Putniņš, 2015) and more extensive welfare states in Europe (Blanchard, 2014, Nickell, 1997, Petrongolo & Pissarides, 2008, Salvanes, 1997). The UK values are also larger than the US but steadily declining, due to the austerity policies followed after 2012 and the emphasis in free market, and greater liberalisation after 1980 (Cumbers, 2019, Schmidt, 2003).

Furthermore public sector employment is a large, relatively stable part of the labour force, especially in Europe. During the Great Recession France followed a procyclical policy and the UK a countercyclical one, but both countries increased public sector employment during Covid; US policy on the other hand is acyclical. Additionally all countries exhibit large, persistent unemployment increases during economic downturns, particularly the US and the UK. Finally labour flows are smaller in France and the public sector reduces inflows and outflows during recessions while labour flows are larger in the UK and the US (with the exceptions of US public sector inflows), and their public sectors increase outflows during recessions.

My model simulation also indicates important differences in private sector reactions and overall policy effects between countries. Investment in private sector vacancies and private sector job destruction rates are smaller but more elastic in France, wages are more sticky and policy shocks are longer-lived and these traits amplify both the crowding out via the *Labour Market Effect* and the positive effects of the *Direct Effect*, *Aggregate Demand Effect* and *Unemployment risk Effect*. Conversely, the US has bigger but more inelastic investment in private sector vacancies and private sector job destruction rates, highly elastic wages and shorter-lived shocks, so the overall effect on the economy is smaller, while the UK exhibits much smaller elasticity in all its labour market variables.

These labour market and public sector differences make the results of policy shocks vary significantly between countries; in addition changes in the tax mix financing policy shocks alters their effectiveness by interacting with countries' labour market and public sector characteristics, amplifying or mitigating the model's effects. When capitalists households pay higher taxes, increasing public sector employment reduces unemployment in France and the US, with multipliers above unity (in absolute values), and a stronger effect in the US (-1.4792) compared to France (-1.0292) as the *Labour Market Effect* is smaller in the US due to its more inelastic labour market. However, when both households' taxes rise the effect is stronger in France with a multiplier of -1.8872 as the *Aggregate Demand Effect* is now amplified, especially in France with its more elastic labour market. Similarly when both households' taxes rise, increasing public sector firms' output in France leads to large increases in aggregate output, with a multiplier above unity at 1.8728, and smaller effects in the US (1.3806). Conversely, the effects of this policy are negative in the UK, a result of its particularly inelastic labour market.

Finally I examine the effects of a countercyclical public sector employment policy. I find that once again the different labour market and public sector traits between countries lead to different results as increasing public sector employment and public sector firms' output in France mitigates the increase in unemployment and reduces the drop in aggregate output, with this effect becoming quantitatively large when the economy operates in the ZLB. For the case of the US the same policy does not have any significant effects on unemployment or aggregate output both for the case of a standard Taylor rule and in the ZLB.

The rest of this Chapter is organised as follows. Section 2 focuses on analysing data on public sector and the labour market for the three countries. In Section 3 I present my calibration strategy. In Section 4 I analyse my results. Section 5 concludes.

3.2 Data on Public Sector and Labour Markets

I start my analysis by establishing some key facts about public sector size, public sector employment and labour market differences in Europe and the US.

3.2.1 Government Expenditure and Public Sector Output

Starting with the size of the public sector, I use quarterly data from 2003Q1-2021Q4 from Eurostat for France, the Office for National Statistics (ONS) for the

UK and the Bureau of Economic Analysis (BEA) for the US, focusing on Total Government Expenditure, Government Expenditure on Wages and Public Sector Output. I seasonally adjust all data, removing any seasonal components so I can study only the underlying trends and non-seasonal economic fluctuations.

Following National Accounting Identities definitions, Total Government expenditure at time t, G_t is the sum of all types of expenditure in all levels of the public sector - namely Central Government, Local Governments/State Governments, Social Security Funds and State Owned Enterprises - and is made up of Consumption Expenditures G_t^C, Investment Expenditures G_t^I, Government Expenditure on Wages G_t^W and Social Benefits and Transfers G_t^S.

$$G_t = G_t^C + G_t^I + G_t^W + G_t^S \tag{3.1}$$

Government Expenditure on Wages G_t^W is the total remuneration, in cash or kind, payable to public sector employees; I will be using this variable to see what part of Total Government Expenditure goes to public sector employment and how important it is in driving the cyclicality of Total Government Expenditure.

The third variable I use, Public Sector Output Y_t^G, is the sum of all output the public sector produces and is composed of Non-Market Output, which is output not sold or sold in economically not significant prices and is used in the private or public sector, Market Output which is output sold in economically significant prices in either sector and Output for Own Final Use which is only used by the public sector.

The sum of Public Sector Output Y_t^G and Private Sector Output Y_t^P equals Aggregate Output Y_t which, following the expenditure approach of GDP, is equal to consumption expenditure by households, investment expenditure by the public and private sector and consumption by the public sector:

$$Y_t^G + Y_t^P \equiv Y_t = C_t + I_t + G_t^I + G_t^C, \tag{3.2}$$

where consumption expenditure by households C_t includes compensation of employees in the private and public sector and Social Benefits and Transfers.

3.2.2 Labour Market Stocks and Flows.

Information about labour markets comes from the French Labour Force Survey (FLFS), the UK Labour Force Survey (UKLFS) and the US Current Population Survey (CPS). The FLFS and the UKLFS are both conducted quarterly. Their sample is a rotating panel of households, composed of six waves in the case of the FLFS and five waves in the UKLFS. Each quarter one sixth (fifth in the UKLFS) is renewed as the oldest wave leaves and a new wave replaces it. The CPS is instead conducted at a monthly frequency. Households are surveyed for four months consecutively, dropped out of the sample for eight months and interviewed again for another four months. The surveys provide information on individual and household characteristics, economic activity and labour market status.

Using these labour force statistics from 2003Q1 to 2021Q4, and following Fontaine et al. (2020) I construct the stocks of public sector employment, private sector employment and unemployment as well as inflows and outflows for public sector employment and private sector employment. All my data are seasonally adjusted, because removing any seasonal components allows me to study only the underlying trends and non-seasonal economic fluctuations in the labour market.

The distinction between public sector employment and private sector employment comes from the survey data where individuals are classified according to their employer. In France, I include the following categories in public sector employment: Central Government, Local Government, State Owned Enterprises, Public Hospital and Social Security. Private Company and Self-Employed are considered private sector employment. Similarly public sector employment in the UK is made up of Central government, Local government, University of other grant-funded educational establishment, Health authorities/NHS, Armed forces and Nationalised industries/State corporations. In the US public sector employment includes all individuals working for the federal, state or local governments. My methodology differs from Fontaine et al. (2020) who exclude employees in State Owned Enterprises in France and employees in Nationalised Industries/State Corporations in the UK from their definition of public sector employment. The reason I am making this different categorisation is because I add public sector output in my model, and an important component of it is public sector output (5 – 7% of GDP in Europe) produced by these type of firms; also the number of people working in these parts of the public sector are a large part of the labour force in France (6% of the labour force) and the UK (2%).

To analyse the labour market dynamics in my dataset I follow Fontaine et al. (2020) and use some fundamental equations to describe how the stocks of unemployment, public sector employment and private sector employment evolve, and the flows between unemployment and different employment states. However one difference in my analysis is that do not report the stock of inactive workers; furthermore I do not report flows between public sector employment and private sector employment. Based on the distinction I make between employees I measure public sector employment L_t^G as the sum off all individuals working in the public sector and private sector employees L_t^P as the sum of people working in the private sector. I also calculate the stock of unemployment U_t as the sum of all unemployed people. Adding these three stocks together gives me the labour force in the economy at time t, L_t. I then calculate public sector employment as a percentage of the labour force at time t, N_t^G and private sector employment as a percentage of the labour force at time t, N_t^P as the ratio of public sector employees to the labour force and the ratio of private sector employees to the labour force, respectively:

$$N_t^G = \frac{L_t^G}{L_t}, \tag{3.3}$$

$$N_t^P = \frac{L_t^P}{L_t}. \tag{3.4}$$

Similarly, I calculate the unemployment rate at time t, u_t:

$$u_t = \frac{U_t}{L_t}. \tag{3.5}$$

I now turn my attention to the flows between different employment states. Starting with labour market inflows, I calculate public sector employment inflows, or hirings of public sector employees as a percentage of unemployed people at time t $\mathcal{H}_t^G$ as the number of people the public sector hires at time t, F_t^G over the number of unemployed people:

$$\mathscr{H}_t^G = \frac{F_t^G}{U_t}. \tag{3.6}$$

Similarly, I measure the hirings of private sector employees as a percentage of unemployed people at time t, $\mathscr{H}_t^P$:

$$\mathscr{H}_t^P = \frac{F_t^P}{U_t}. \tag{3.7}$$

Finally, I measure labour market outflows as percentage of their relevant labour market stocks. The public sector job destruction rate at time t δ_t^G is the number of job separations in the public sector at time t, S_t^G as a percentage of the number of public sector employees:

$$\delta_t^G = \frac{S_t^G}{L_t^G}, \tag{3.8}$$

The private sector job destruction rate at time t, δ_t^P is:

$$\delta_t^p = \frac{S_t^P}{L_t^P}, \tag{3.9}$$

3.2.3 Public Sector in Europe and the United States

Figure 3.1 depicts the behaviour of Total Government Expenditure, Government Expenditure on Wages and Public Sector Output. The first important finding is that all variables are large, relatively stable components of GDP, particularly Government Expenditure, and Public Sector Output. Additionally their response to business cycles is similar as they follow a countercyclical policy, especially during Covid (2020Q1 to 2021Q4) when most variables increase as much as 25%, or even 60% in some cases.

In France Total Government Expenditure and Public Sector Output are much larger, averaging 55% and 21.5% of GDP respectively, and also more stable. Total Government Expenditure is the most stable of the three averaging 53% between 2003 and 2008, then rising to 58% as a result of the Great Recession and remaining stable afterwards. Public Sector Output decreased from 22% to 20.5% between 2005 and 2008, but bounced back during the Great Recession converging back to its average. Government expenditure on Wages is on average around 13% of GDP, or 24% of Total Government Expenditure, and behaves similarly to Public Sector Output, decreasing by 0.5 percentage points between 2005 and 2008, but returning to its average during the Great Recession and remaining stable afterwards.

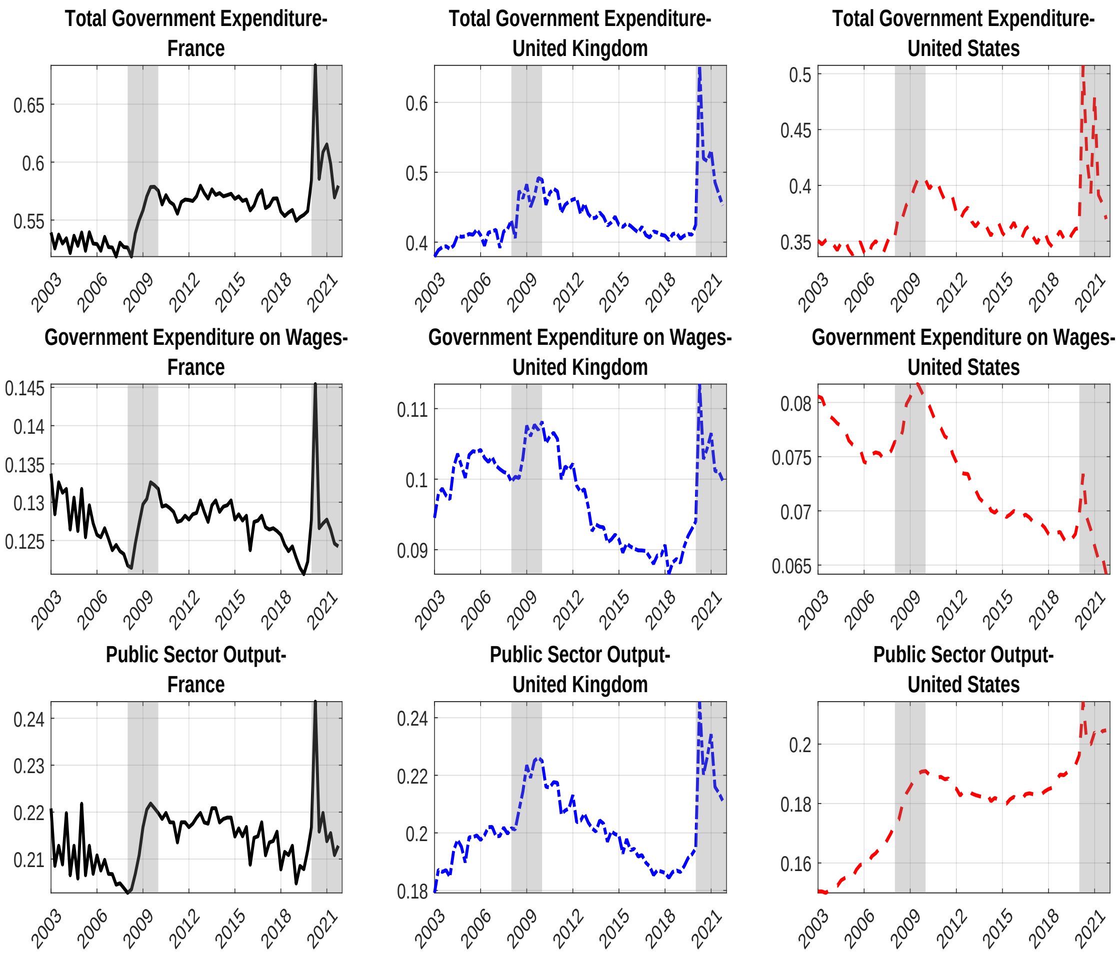

Figure 3.1: Total Government Expenditure, Government Expenditure on Wages & Public Sector Output (% of GDP), quarterly, seasonally adjusted data 2003Q1-2021Q4. (Sources: Eurostat, ONS, BEA)

In the UK Government Expenditure equals 44% of GDP and Public Sector Output 20% of GDP on average, but both variables are in a slow, steady decline after 2010, a result of austerity programmes followed by successive UK governments. Government Expenditure on Wages is smaller in the UK (9.8%) and follows a strong downwards trend after 2010, averaging 10.2% until 2010 but decreasing afterwards, falling to 8.6%, before Covid. Finally Total Government Expenditure and Public Sector Output in the US are smaller, but still significant, at 37% and 18% of GDP. Furthermore both variables, especially Public Sector Output have steadily increased after 2006 with Public Sector Output being slightly larger from the UK value in 2018. Conversely, Government Expenditure on Wages averages at 7.3% and follows a completely different path being in a steady decline since 2003 (apart from the Great Recession), and falling to 6.7% by 2019.

3.2.4 Labour Markets in Europe and the United States

3.2.4.1 Stocks

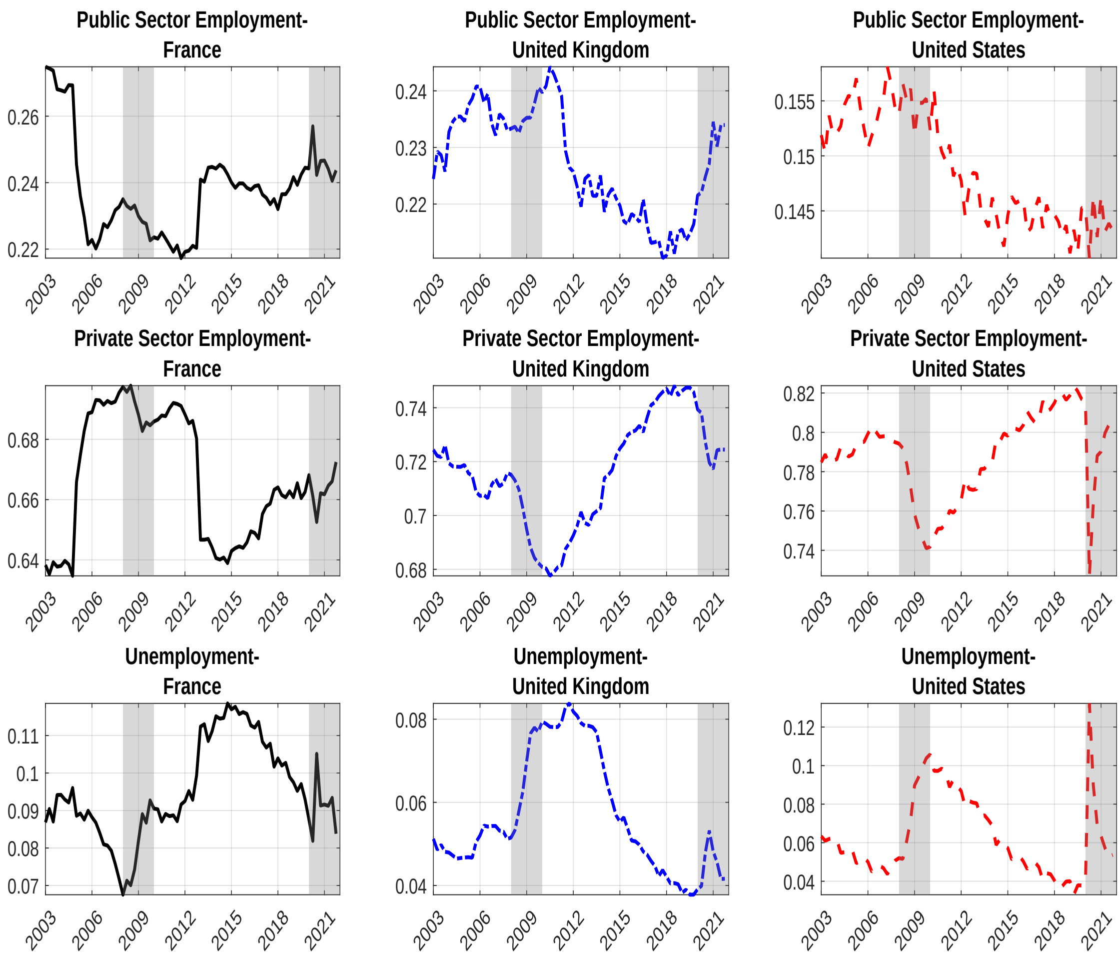

Figure 3.2: Labour Market Stocks (as % of labour force) quarterly, seasonally adjusted data 2003Q1-2021Q4. (Sources: FLFS, UKLFS, CPS).

In Figure 3.2 I present the stocks of Public Sector Employment, Private Sector Employment and Unemployment. As it happens with the other public sector variables, Public Sector Employment is a large, relatively stable, part of the labour force. Public Sector Employment is larger in France, being on average 23.79%. It decreased in 2004 from 26.92% to 22.14%, but in 2012 increased to 24.42% and remained stable thereafter. UK Public Sector Employment is on average 22.73% of the labour force, but is also declining like the rest of the public sector, going from 24% in 2011 to 21.4% in 2019. Meanwhile Public Sector Employment in the US is almost unchanged with only marginal changes, even during the Great Recession and Covid. Public Sector Employment also behaves differently between countries in recessions as France reduced Public Sector Employment from 23.31% to 22.40% while the UK followed a countercyclical policy, increasing it from 23.33% to 23.97%; however both countries increased Public Sector Employment during Covid. Meanwhile the US follows an acyclical policy.

Looking at unemployment, we see that it is larger in France, averaging 9.52%[4], compared to 5.66% and 6.28% in the UK and the US. Furthermore, in all three countries unemployment increases are very persistent. The Great Recession caused unemployment to increase in the UK and the US, only returning to its pre-crisis value after 8 years. France also experienced a prolonged increase in unemployment from 2013 until mid-2018, but the rise was smaller. Finally, business cycles cause much larger increases in unemployment in the UK and the US.

[4]Unemployment in France is on average 0.48 percentage points higher than ILO values and after 2014 this difference rises to 1.2 percentage points on average. This "Unemployment Halo" is defined by the FLFS as people ILO counts as outside the labour force and not as unemployed but whose situation is very similar to unemployment i.e. unemployed people wanting to work and available for work, but who are for some reason unable to seek work. Depending on the individuals added in this "halo", unemployment figures are between 1 and 5 percentage points above the more "strict" ILO definiotion. For more information see INSEE (2020)

3.2.4.2 Flows

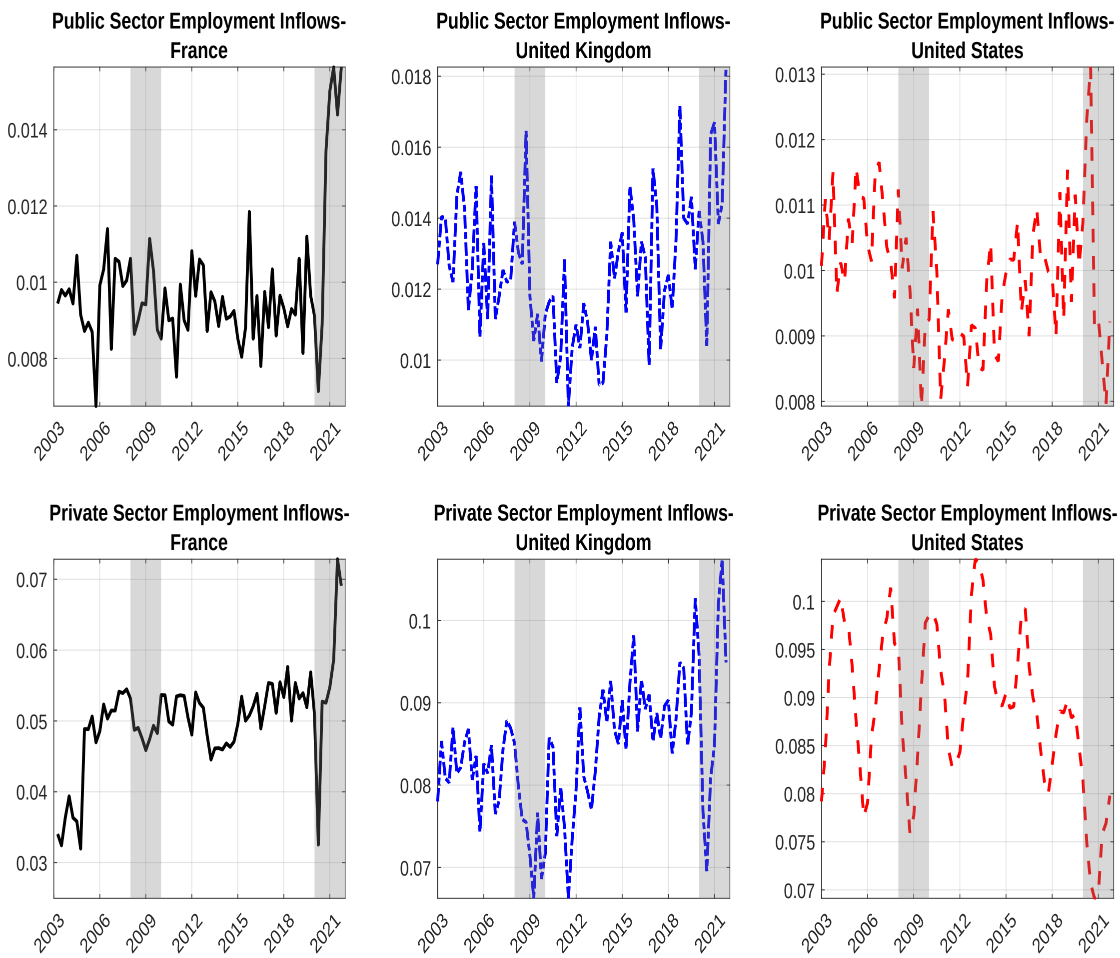

Figure 3.3: Labour Market Inflows (as % of relevant stocks); quarterly, seasonally adjusted data 2003Q1-2021Q4. (Sources: FLFS, UKLFS, CPS).

In Figure 3.3 I show the Public Sector Employment Inflows and Private Sector Employment Inflows. France actually has the smallest number of Public Sector Employment Inflows, with an average value of 0.98%, which contracts both during the Great Recession and Covid. The UK on the other hand has the biggest number of inflows, averaging 1.26%, which is usually 30% higher from the other two countries and in some cases even 80% higher. Inflows converged between the three countries from 2011 to 2014, as UK inflows sharply dropped due to austerity, but quickly bounced back. US inflows are almost similar in size to France with an average value of 0.99% and exhibit marginal differences over time.

When it comes to Private Sector Employment Inflows France has an average of 4.96%, which is nearly half the size of inflows in the other countries. The UK has an average of 8.41%, very close to the US value of 8.90%, so the two countries have about 50% more Private Sector Employment inflows compared to France.

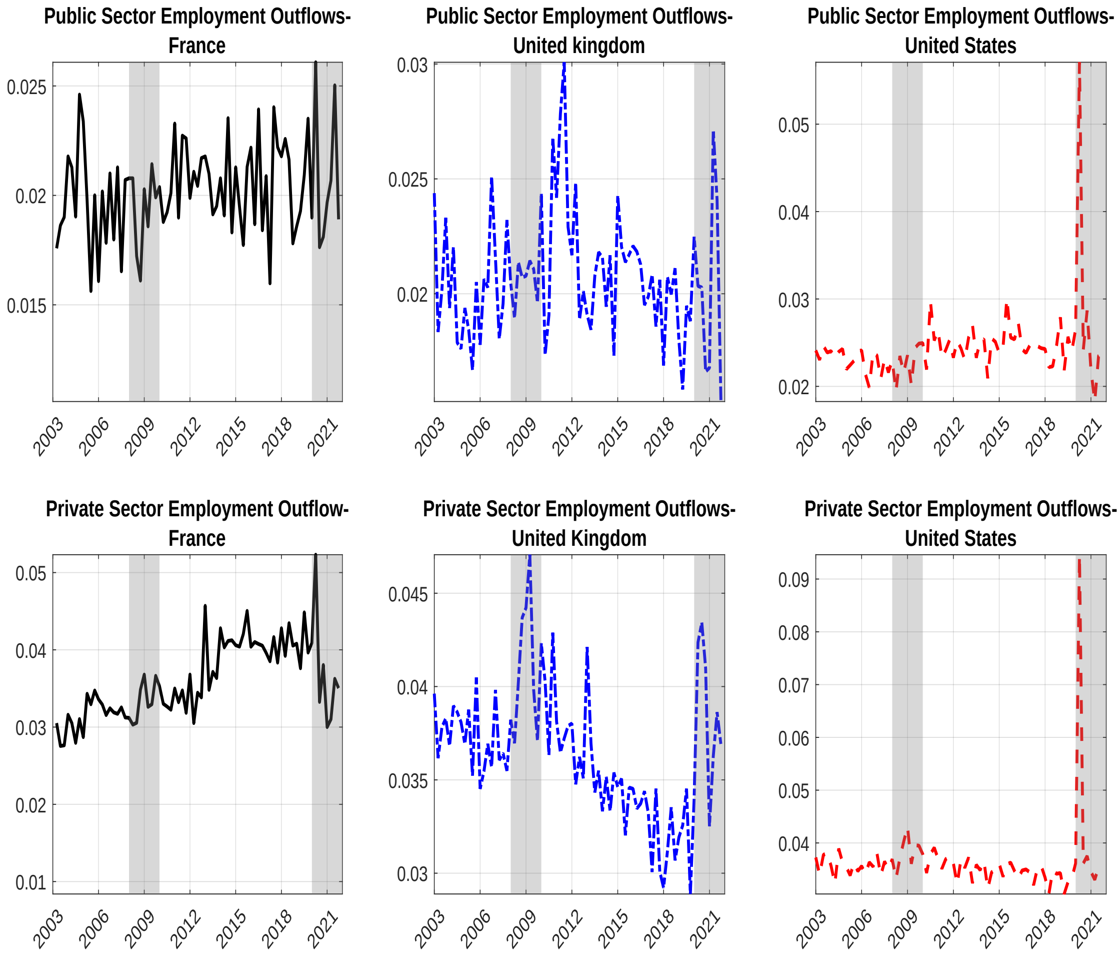

Figure 3.4: Labour Market Outflows(as % of unemployment); quarterly, seasonally adjusted data 2003Q1-2021Q4. (Sources: FLFS, UKLFS, CPS).

I close my analysis with Public Sector Employment Outflows and Private Sector Employment Outflows in Figure 3.4. France has relatively smaller Public Sector Employment Outflows, with an average of 2.01%, which decrease when the economy contracts. Private Sector Employment outflows are also smaller, averaging 3.56%, but after 2014 they have become higher than the UK and US ones. The UK and the

US have on average greater Public Sector Employment outflows (2.07% and 2.44%) and Private Sector Employment outflows (3.65% and 3.63%) which increased during the Great Recession. During Covid, outflows in the US skyrocketed, at 5.70% and 9.46% respectively, but remained stable in France and the UK, a result of the furlough schemes that prevented massive layoffs in Europe.

3.2.5 What drives the differences in public sector size and labour market behaviour?

The differences in the size and behavior of public sectors between countries are the result of various economic, political and historical factors, starting with the degree of economic interventionism. In Europe a policy called *Dirigisme* (French *diriger*: to direct) has been followed since the early 20th century, consisting of directed public sector investment and the establishment of state owned enterprises, in an otherwise free market economy. The reasons behind such a policy vary: The public sector owns state owned enterprises in nascent industries unable to attract investors, industries the government tries to (re)develop, natural monopolies or strategically important sectors. *Dirigisme* has somewhat fallen out of favour after 1980, but European countries still follow it to an extent, as their public sectors still own and invest in state owned enterprises in the utilities and energy sectors, but also in economically (chemicals, metallurgy, machinery, electronics) or politically important (shipbuilding, aerospace, vehicles, arms industry) sectors. [5].

The second difference is in how the public sector protects the well-being of all its citizens, promotes equitable wealth and income distribution, and provides the minimal provisions for a good life, or in other words the welfare state. In Europe the public sector is more generous compared to the US, as in addition to contribution-financed social security, it also provides tax-financed universal healthcare and education, pensions, unemployment, childcare and income benefits and social housing(Alesina et al., 2001, 2018). In the US, similar programs do exist but are smaller in size and coverage, while a larger part of their financing comes from the private sector, either as charities or as individual spending on

[5] OECD.(2016; 2017) Christiansen (2011); Putniņš (2015),Colli & Nevalainen (2019); De Lange & Merlevede (2020)

private healthcare providers and private pension plans (Adema et al.(2011); Rank et al.(2021)).

The literature explaining the differences in the labour market between Europe and the US is quite exhaustive. Many European countries have consistently exhibited larger unemployment levels compared to the US since the late 70s and this phenomenon has carried on until today. Salvanes (1997) finds this is the result of product market and labour market rigidities although in some cases rigidities can boost job creation and reduce unemployment. Additionally, the degree of labour protection, union coverage and collective bargaining structure as well as the size and duration of welfare and unemployment benefits can potentially explain higher unemployment rates in Europe as in Petrongolo & Pissarides (2008); however Nickell (1997) and Blanchard (2014) do point out that many European countries who follow such policies can in fact combine them with low unemployment rates.

In Tables 3.1 and 3.2 I summarize my data by presenting the average values and the correlation matrix for the public sector and labour market stocks I use for the three countries. Results in Table 3.1 are similar to the ones in Figures 3.1 and 3.2, where Total Government Expenditure G, Government Expenditure in Wages G^W, Public Sector Output Y^G and Public Sector Employment N^G are very large in France while being a lot smaller in the US and standing somewhere in-between these two extremes in the UK. Also Unemployment u is bigger in France and smaller in the other two countries while Private Sector Employment N^P is larger in UK and the US.

Table 3.1: Average values of public sector and labour market variables

		G	G^W	Y^G	N^G	N^P	u
	France	0.5589	0.1276	0.2141	0.2379	0.6669	0.0952
Average Values	United Kingdom	0.4354	0.0980	0.2017	0.2273	0.7161	0.0566
	United States	0.3686	0.0733	0.1796	0.1487	0.7885	0.0628

Note: Data are in quarterly frequency, seasonally adjusted from 2003Q1-2021Q4. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

The correlation matrix in Table 3.2 highlights how public sector and labour markets differ between France UK and the US, but also how they respond to business cycles. The series are in log form and in quarterly frequency. All values are seasonally adjusted removing any seasonal components so I can study only the underlying trends and non-seasonal economic fluctuations, and detrended using an HP filter with a smoothing parameter of 100000 to separate the cyclical component from the raw data of a time series.

Table 3.2: Correlation Matrix

		G	G^W	Y^G	N^G	N^P	u
Cross Correlation (France)	G	1	0.9120	0.8944	0.1235	−0.0882	0.0262
	G^W	-	1	0.9270	0.2698	−0.2927	0.2266
	Y^G	-	-	1	0.1662	−0.1992	0.1883
	N^G	-	-	-	1	−0.8551	0.3672
	N^P	-	-	-	-	1	−0.7941
	u	-	-	-	-	-	1
Cross Correlation (United Kingdom)	G	1	0.8649	0.9302	0.6086	−0.6599	0.5308
	G^W	-	1	0.9443	0.8105	−0.7077	0.4394
	Y^G	-	-	1	0.7885	−0.7672	0.5794
	N^G	-	-	-	1	−0.7400	0.3774
	N^P	-	-	-	-	1	−0.8948
	u	-	-	-	-	-	1
Cross Correlation (United States)	G	1	0.7676	0.7873	0.2832	−0.8941	0.8597
	G^W	-	1	0.8384	0.3398	−0.8433	0.7576
	Y^G	-	-	1	0.4541	−0.8445	0.7723
	N^G	-	-	-	1	−0.2150	0.0660
	N^P	-	-	-	-	1	−0.9723
	u	-	-	-	-	-	1

Note: Data are in quarterly frequency from 2003Q1-2021Q4. The series are seasonally adjusted, taken in log form and then detrended with an HP filter with smoothing parameter of 100000. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

As evident from these numbers and from Figures 3.1-3.4 there are significant differences between each country. Starting with France, the cross correlation between the different public sector variables is very large, with the exception of public sector employment where the values are much smaller, indicating that public sector employment is relatively unaffected by changes in the rest of the public sector. Also the cross-correlation of public sector variables with private sector employment is very small, with the exception of public sector employment.

Finally public sector variables appear to be relatively acyclical as their correlation with unemployment is not very large.

In the UK public sector variables, including public sector employment, are more closely correlated with each other and in addition they appear to have a bigger negative correlation with private sector employment. When it comes to the effect of business cycles the public sector appears to be following a countercyclical policy, altough this effect is smaller when it comes to public sector employment.

Finally in the US, the cross correlation between the different public sector variables and the cross correlation of public sector employment with the rest of the public sector variables is similar to that in France. Additionally I find a very strong negative cross-correlation between private sector employment and all the public sector variables except public sector employment. Also public sector variables appear to be countercyclical, even more that in the UK, as their correlation with unemployment is very large; however public sector employment is acyclical as the value there is very close to zero.

3.3 Quantitative Analysis

In this Section, I simulate the model from Chapter 2 around the certainty-equivalent steady state using Dynare and Matlab for France, UK and the US. First I calibrate a number of parameters to target values in the non-stochastic steady state of the model, using common values in the literature. Then, I use Simulated Method of Moments (SMM) which is the same method I used in Chapter 2, to calibrate the remaining parameters, which are the most important ones for my analysis. Using this method, I calibrate the parameters to ensure that the real empirical moments in the data match the simulated empirical moments in my model.

3.3.1 Calibration

First, I set up the steady state target values, presented in Table 3.3, using data from 2003 to 2021 from the FLFS, the UKLFS and the CPS. Using these datasets, I construct the stocks of public sector employment, private sector employment and unemployment, and the flows between unemployment and private sector employment, and between unemployment and public sector employment.

Table 3.3: Steady State Target

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
u	Unemployment	0.0952	0.0566	0.0628	Data
N^G	Public Sector Employment	0.2379	0.2273	0.1487	Data
N^P	Private Sector Employment	0.6669	0.7161	0.7885	Data
δ^P	Job Destruction Rate (Private Sector)	0.0356	0.0365	0.0363	Data
δ^G	Job Destruction Rate (Public Sector)	0.0201	0.0207	0.0244	Data
G^C	Government Expenditure in Consumption	0.1436	0.1544	0.1362	Data
θ	Market Tightness	1	1	1	Normalise
Y	Aggregate Output	1	1	1	Normalise
Y^P	Private Sector Firms' Output	0.7859	0.7980	0.8204	Data
Y^G	Public Sector Firms' Output	0.2141	0.2020	0.1796	Data
π	Inflation	0.0022	0.0022	0.0022	Annual Steady State Inflation of 2.5%
v	Vacancies	0.0952	0.0566	0.0628	θ=1
w^P	Private Sector Wage	0.8820	0.8447	0.8141	Fundamental Surplus
w^G	Public Sector Wage	0.8820	0.8447	0.8141	No Wage Premium
τ_N	Labour Income Tax Rate	0.5200	0.5000	0.4500	Data
τ	Lump-Sum Tax Rate	0.4000	0.3500	0.3000	Normalise

Note: Data used to calculate steady state target values are in quarterly frequency from 2003Q1-2021Q4 and seasonally adjusted. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

Starting with steady state unemployment u , I match the average unemployment rates in the sample for each of my countries. This gives an unemployment rate of 9.52% in France, 5.66% in the UK and 6.28% in the US. Similarly for the steady public sector employment N^G, I match the average values of public sector employment as a percentage of the labour force in my data, giving me public sector employment equal to 23.79% in France, 22.73% in the UK and 14.87% in the US. I then set the steady state value of private sector employmnent N^P targeting a labour force equal to unity. I set market tightness θ normalized to unity, a common value in the literature. As a result I set each country's steady state value of vacancies v equal to the unemployment level (Shimer, 2005, Coles & Moghaddasi Kelishomi, 2018). Finally, following Fontaine et al. (2020), I calculate the job destruction rates for public sector firms and private sector firms; however as I mentioned when analysing my data sources, I include employees in state owned enterprises in public sector employment, not private sector employment. This gives me a public sector job destruction rate δ^G equal to 2.01% for France, 2.07% for the UK and 2.44% for the US. Using the same methodology for the private sector job destruction rate δ^P I get a value of 3.56% for France, 3.65% for the UK and 3.63% for the US.

I set the steady state value of aggregate output $Y = 1$ and the steady state value of the inflation rate π, targeting an annual inflation rate of 2.5%. I use government finance statistics from Eurostat, the ONS and the BEA for the steady state value of public sector firms' output Y^G as percentage of GDP. This gives 21.41% in France, 20.20% in the UK and 17.96% in the US. I then set the steady state value of private sector firms' output Y^P, targeting the steady state value of aggregate output Y. I also use government finance statistics for the steady state value of government expenditure on consumption G^C as a percentage of GDP and get 14.36% in France, 15.44% in the UK and 13.62% in the US. I set the tax rate on labour income τ^N targeting values on labour taxation, including social security contributions, using the Organisation for Economic Co-operation and Development (OECD) data on labour taxes, and the lump-sum tax τ is set to solve the government's budget constraint.

For the steady state private sector wage w^P I use the theory of the fundamental surplus of Ljungqvist & Sargent (2017,2021) which is the part of output used for paying wages, production costs and taxes and not for investing in private sector vacancies. To this end the steady state private sector wage $w^P = P^{\mathcal{X}}(1 - fs)$, where fs is the fundamental surplus ratio, which I calibrate internally. To simplify my analysis, public sector wages equal private sector wages, so $w^G = w^P$.

Table 3.4: Parameter Values

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
γ	Matching function elasticity	0.6000	0.6000	0.6000	Coles & Moghaddasi Kelishomi (2018)
β	Discount factor	0.9967	0.9967	0.9967	Annual discount rate 4%
pop_c	Number of Capitalists Households	0.1000	0.1000	0.1000	Broer et al. (2021)
η	CRRA Coeffcient	2	2	2	Broer et al. (2021)
η^C	Capitalists Households CRRA coefficient	0	0	0	Risk Neutral
ϵ	Price Elasticity	10	10	10	Standard
μ	Price mark-up	1.1	1.1	1.1	Standard
ξ_ρ	Rotemberg Price Adjustment Cost	600	600	600	Standard
z	Consumption Good Share in Public Sector Firms' Output	0.3000	0.3000	0.3000	Standard
$\rho_{\mathcal{H},G}$	Public Sector Hirings Shock Autocorrelation Parameter	0.9000	0.9000	0.9000	Cantore & Freund (2021)
ϕ_π	Taylor Response to Inflation	1.5000	1.5000	1.5000	Broer et al. (2021)
κ_{τ^N}	Response of Labour Income Taxes to Public Debt	(0;0.0500)	$(0;3.0000e-07)$	(0;0.0500)	Cantore & Freund (2021)

The externally calibrated parameters in Table 3.4 are the same for the three countries I study and I use standard values from the literature. Starting with the elasticity of the matching function γ I set it equal to 0.6 as in Coles & Moghaddasi Kelishomi (2018). Regarding preferences I set the discount factor $\beta = 0.9967$, implying a steady state interest rate of 4%. The number of capitalists households

$pop_c = 0.1$ as in Broer et al. (2021). For workers households the risk aversion parameter $\eta = 2$, while for the risk neutral capitalists households their risk aversion parameter $\eta^C = 0$. The elasticity of substitution $\epsilon = 10$ the price mark-up $\mu = 1.1$, and the Rotemberg price adjustment cost parameter $\xi_\rho = 600$, all common values in the literature. For public sector firms, the output elasticity of the consumption good $z = 0.3000$, which makes the output elasticity of public sector employment $(1 - z) = 0.7000$ (Pappa, 2009, Papageorgiou & Vourvachaki, 2017) and I set the autocorrerlation parameter on hirings of public sector employees $\rho_{\mathscr{H},G} = 0.9000$, a standard value for this fiscal policy parameter (Cantore & Freund, 2021).

This leaves me with two parameters, the Taylor Rule response to inflation parameter ϕ_π and the response of labour income to public debt $\kappa_{\tau,N}$, which I change as part of my counterfactual exercises. More specifically I set $\phi_\pi = 1.5000$ which is the standard value in the literature. I will then study how changes in monetary policy responsiveness change the effects of policy; however, instead of simply changing the value of ϕ_π, I will be using the Occasionally Binding Constaint (OccBin) toolkit developed by Guerrieri & Iacoviello (2015), which uses a first-order perturbation approach to solve dynamic models with occasionally binding constraints. Finally I set $\kappa_{\tau,N} = 0$ in my basic calibration so I assume that workers households do not repay public debt increases. I then run a counterfactual where I keep all my calibrated parameters the same but set $\kappa_{\tau,N} = 0.0500$ to study if there are any differences in the effects of increasing public sector employment if workers households also partly contribute to the debt repayment[6].

3.3.2 Simulated Method of Moments

I now analyse the SMM I use to calibrate my main parameter values. Using the labour market datasets from FLFS, UKLFS and CPS I estimate the standard deviation of unemployment σ_u, the standard deviation of the private sector job destruction rate $\sigma_{\delta,P}$, the cross correlation between vacancies and unemployment $corr(v,u)$, the auto-correlation of unemployment $autocorr(u)$, the standard deviation of public sector employment $\sigma_{N,G}$ and the auto-correlation of public

[6] For the UK, the data and empirical moments, result in estimated parameter values very close to explosive. As a result I cannot make big changes to $\kappa_{\tau,N}$ as the model has no rational expectations solution and becomes explosive which means the Blanchard-Kahn conditions cannot be satisfied.

sector employment $autocorr(N^G)$. I also estimate the standard deviation of Total Factor Productivity σ_A and the auto-correlation of Total Factor Productivity $autocorr(A)$ using data on real average output per worker from the OECD (Shimer, 2005). Finally I use data from Eurostat, the ONS and the BEA to estimate the cross-correlation of Public Debt with Government Expenditure $corr(B,G)$, and the autocorrelation of Public Debt $autocorr(B)$.

In Table 3.5, I present my simulation results. As we can see, the values of the estimated empirical moments from the model are very close to the values of the real empirical moments from the dataset, the only exceptions being the autocorrelation of unemployment in the UK, the autocorrelation of public debt for the UK and the US and particularly the standard deviation of unemployment in the US for which the fit is very poor. Despite these exceptions my model is still a very good approximation of the real economy for the three countries.

Table 3.5: Simulation Results

Country/ Empirical Moment	France		United Kingdom		United States		Source
	Data	Calibration	Data	Calibration	Data	Calibration	
σ_u	0.1113	0.1226	0.1761	0.1739	0.2891	0.2194	Directly Estimated
$\sigma_{\delta,P}$	0.0967	0.1023	0.0870	0.0870	0.1297	0.1120	Directly Estimated
$corr(v,u)$	-0.3548	-0.3656	-0.6739	-0.7395	-0.8268	-0.8779	Directly Estimated
$autocorr(u)$	0.8882	0.8594	0.9670	0.6094	0.8292	0.8094	Directly Estimated
$\sigma_{N,G}$	0.0486	0.0485	0.0305	0.0304	0.0166	0.0165	Directly Estimated
$autocorr(N^G)$	0.9095	0.9186	0.9006	0.9096	0.5996	0.6065	Directly Estimated
σ_A	0.0220	0.0220	0.0314	0.0314	0.0130	0.0130	Directly Estimated
$autocorr(A)$	0.5467	0.5467	0.4504	0.4504	0.7941	0.7940	Directly Estimated
$corr(B,G)$	0.6417	0.6417	0.3613	0.3890	0.5798	0.5900	Directly Estimated
$autocorr(B)$	0.9331	0.9173	0.9737	0.8459	0.9086	0.8361	Directly Estimated

Note: Data used to calculate empirical moments values are in quarterly frequency from 2003Q1-2021Q4. Series are seasonally adjusted then taken in log form and detrended with an HP filter with smoothing parameter of 100000. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

In Table 3.6 I have my internally calibrated parameters. Starting with the coefficient of investment in private sector vacancies ξ, we see that investment in private sector vacancies is quite inelastic in all countries, with the UK being the most inelastic, followed by the US and France. The fundamental surplus ratio fs is also small being 5.50% in France, 5.84% in the UK, 1% in the US, suggesting that business cycles significantly affect the fundamental surplus. France also has a large elasticity of private sector job destruction rates $\varepsilon_{\delta,P} = -0.8499$ and a small elasticity of private sector wages $\epsilon_w = 0.0933$ while in the US the elasticity of private sector wages is extremely big with $\epsilon_w = 0.9900$ and the elasticity of private sector job destruction rates much smaller, with $\varepsilon_{\delta,P} = -0.4598$.

Table 3.6: Calibrated Parameters

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
ξ	Coefficient of Investment in private sector vacancies	0.6852	0.0984	0.5857	σ_u
fs	Fundamental Surplus	0.0550	0.0584	0.0100	$\sigma_{\delta,P}$
$\varepsilon_{\delta,P}$	Elasticity of Private Sector Job Destruction Rate	-0.8499	-0.3523	-0.4598	$corr(v,u)$
ϵ_w	Elasticity of Private Sector wages to Private Sector Firms' Output	0.0933	0.1686	0.9900	$autocorr(u)$
$\varepsilon_{\mathcal{H},G}$	Feedback Parameter	-5.4920	-6.2943	-67.7722	$autocorr(N^G)$
$\sigma_{\mathcal{H},G}$	i.i.d Shock parameter	0.2311	0.1544	0.5246	$\sigma_{N,G}$
ε_A	i.i.d Shock Parameter	0.0163	0.0265	0.0059	σ_A
ρ_A	TFP Shock Autocorrelation Parameter	0.7573	0.6899	0.9114	$autocorr(A)$
ρ_B	Public Debt Autocorrelation Parameter	0.0617	0.0520	0.0486	$autocorr(B)$
κ_B	Public Debt response to Government Expenditure	0.5812	0.5181	1.7134	$corr(B,G)$

As a result, business cycles in France create large changes in private sector job destruction rates while wages are relatively sticky, resulting in large changes in unemployment via this channel, but in the US the same shocks lead to massive wage but relatively small employment changes. In the UK both wages ($\epsilon_w = 0.1686$) and the private sector job destruction rate are inelastic ($\varepsilon_{\delta,P} = -0.3523$), which means that changes in unemployment from that channel are also smaller, as in the US. Finally the feedback parameter on the hirings of public sector employees, $\varepsilon_{\mathcal{H},G}$ is small in France ($\varepsilon_{\mathcal{H},G} = -5.4920$) and the UK ($\varepsilon_{\mathcal{H},G} = -6.2943$) and extremely large in the US, ($\varepsilon_{\mathcal{H},G} = -67.7722$). So when hirings of public sector employees increase in Europe, public sector employment stays above its steady state value for a longer period of time while in the US this effect is more brief, indicating the public sector differences between countries.

3.4 Results

I now investigate the effect of an increase in public sector firms' output, caused by an increase in public sector employment. I consider two different financing schemes: In the first case, public debt increases and only lump-sum taxes on capitalists households increase to repay it. In the second case, both lump-sum taxes on capitalists households and labour income taxes on workers households increase. I study a 10-year time period, at monthly frequency, assuming a one-period positive shock, in the form on an increase in the hirings of public sector employees $\mathscr{H}^G$ resulting in a 1% increase in public sector employment N_t^G. [7]

3.4.1 Results-Lump Sum Taxes

I start my analysis with the labour market. The first important finding comes in Figure 3.5 Panel B, where increasing public sector employment N_t^G creates a large reduction in unemployment u_t, decreasing by 3.2% in the US and by 2.3% in France. Conversely in the UK unemployment initially goes up, peaking at 0.64% but this effect quickly changes and unemployment decreases by 1.5%, 14 months after the policy shock. The effect is also persistent in all countries, the small values of ξ and fs making investment in private sector vacancies particularly inelastic (Coles & Moghaddasi Kelishomi, 2018, Broer et al., 2021). However this policy results in crowding out of private sector employment N_t^P in France and the UK, in Panel C. Conversely in the US, there is a small, short-lasting crowding in.

To explain these results, I study each country separately. In France, the policy shock lasts longer as $\varepsilon_{\mathscr{H}^G} = -5.4920$ and public sector employment N_t^G is larger, so when it rises it leads to a large increase in public sector firms' output Y_t^G, which is also larger in France, resulting in a large increase of aggregate output Y_t, and a larger *Direct Effect*. However the *Labour Market Effect* is also larger: The probability of "meeting" a private sector vacancy ζ_t^P decreases in Panel D, which reduces investment in private sector vacancies i_t in Panel F, and this effect is larger in France as $\xi = 0.6852$ and $fs = 5.50\%$ make investment in private sector vacancies more elastic. As this effect propagates, aggregate demand, inflation π_t and the value of filled vacancies J_t^F decrease, leading to large increases in the private sector job destruction rate δ_t^P, due to the sticky wages ($\epsilon_w = 0.0933$) and elastic private sector job destruction rates ($\varepsilon_{\delta^P} = -0.8499$), and an even larger reduction in investment in private sector vacancies i_t. These factors lower private sector vacancies v_t^P and market tightness θ_t, and make the reduction in private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t larger.

[7] In Appendix C I present my results when I exclude the Covid period, examining how the labour market operated in more "normal" economic times and what changes the pandemic brought to it.

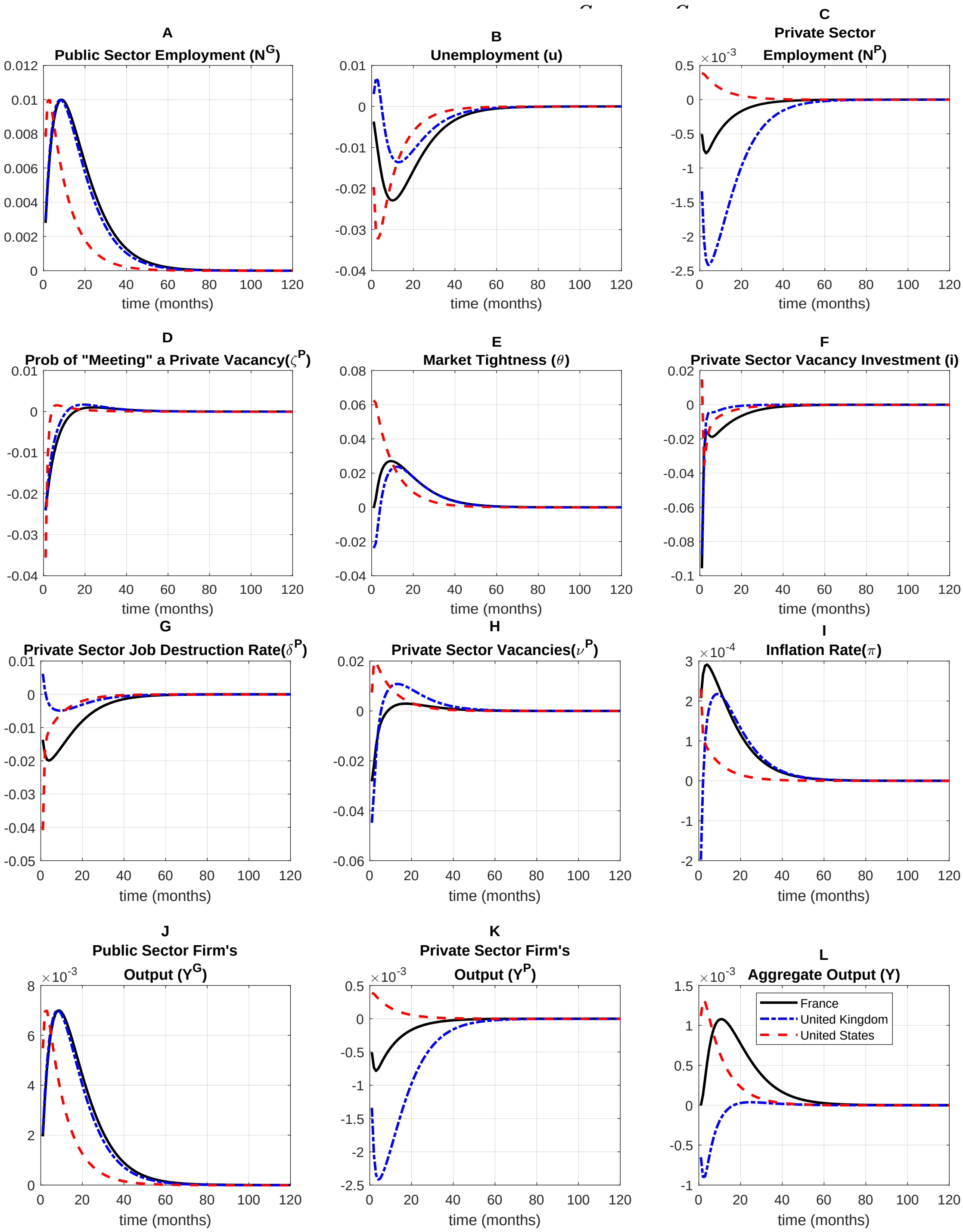

Figure 3.5: Public Sector Employment shock - Debt financed via lump-sum taxes.

lived as $\varepsilon_{\mathscr{H}G} = -67.7722$, but the *Labour Market Effect* is also smaller: $\xi = 0.5857$, and $fs = 1\%$ make investment in vacancies i_t more inelastic, while $\varepsilon_{\delta^P} = -0.4598$ and $\epsilon_w = 0.9900$ make the increase in the private sector job destruction rate δ_t^P smaller. As a result market tightness θ_t and private sector vacancies v_t^P decrease less, as do private sector employment N_t^P, inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t.

In the following periods, the *Aggregate Demand Effect* begins working. In the *Redistribution Effect* lump-sum taxes on capitalists households rise to repay the public debt, so income is redistributed from low-MPC capitalists households to unemployed workers households hired by public sector firms. This raises consumption, inflation π_t and private sector firms' output Y_t^P, indirectly increasing aggregate output Y_t. Then the *Indirect Aggregate Demand Effects* come into play: Higher inflation raises the value of filled vacancies J_t^F so investment in private sector vacancies i_t increases and the private sector job destruction rate δ_t^P decreases. As a result, private sector vacancies v_t^P go up, private sector employment N_t^P, inflation π_t and private sector firms' output Y_t^P, increase again, further increasing aggregate output Y_t and creating a multiplier effect. Finally, the *Unemployment Risk Effect* lowers unemployment risk of workers households and demand for precautionary savings, raising interest rates for household bonds R_t, aggregate demand and inflation π_t. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P and increases aggregate output Y_t even more, reinforcing the multiplier.

Results now change between countries, as the *Aggregate Demand Effect* is larger in France. Investment in private sector vacancies i_t is more elastic and increases more, quickly bouncing back. In addition the rise in inflation π_t leads to a large increase in the value of filled vacancies J_t^F and a large reduction in the private sector job destruction rate δ_t^P, as it is more elastic and wages are more sticky in France, helping investment in private sector vacancies i_t increase even more in later periods. These effects combined are stronger than the *Labour Market Effect* as the overall effect in Panel G is negative, and market tightness θ_t and private sector vacancies v_t^P quickly turn positive. As a result, I find a large increase in aggregate demand, inflation π_t, private sector employment N_t^P and private sector firms' output Y_t^P, and a large increase in aggregate output Y_t.

The *Unemployment Risk Effect* in Panel I is also positive, meaning the *Direct Effect* and *Aggregate Demand Effect* are stronger than the *Labour Market Effect*: As the unemployment risk of workers households and demand for precautionary savings decrease, the interest rate for household bonds R_t aggregate demand and inflation π_t increase. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further raising private sector employment N_t^P and private sector firms' output Y_t^P and increasing aggregate output Y_t even more. The overall effect in the goods market is summed up in Panels J to L where increasing public sector firms' output Y_t^G in France leads to a large increase in aggregate output Y_t by 0.108%, but again at the cost of crowding out.

In the US, the *Aggregate Demand Effect* is smaller. The increase in investment in vacancies i_t and the reduction in private sector job destruction rate are smaller, but the *Labour Market Effect* is also small, so the overall effect on market tightness θ_t and private sector vacancies, v_t^P is positive; consequently inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t also increase. The *Unemployment Risk Effect* is positive, if somewhat smaller than France, so investment in private sector vacancies, private sector employment N_t^P, private sector firms' output Y_t^P, wages rise further. This in turn increases aggregate output Y_t even more, with the overall effect being larger than in France at 0.13% but shorter lasting; however private sector firms' output Y_t^P now responds differently and increases.

In the UK the *Direct Effect* is almost as large, and the policy shock lasts as much as in France, since $\varepsilon_{\mathscr{H}^G} = -6.2943$. However, the *Labour Market Effect* is smaller because $\xi = 0.0984$ and $fs = 5.84\%$, so the drop in investment in private sector vacancies i_t is smaller. As the effect then propagates aggregate demand, inflation π_t and the value of filled vacancies J_t^F go down, which raises the private sector job destruction rate δ_t^P but both UK wages ($\epsilon_w = 0.1686$) and private sector job destruction rates are sticky ($\varepsilon_{\delta^P} = -0.3523$), so this effect is also small. As a result the reduction in private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P, wages and aggregate output Y_t is smaller compared to France and the US. However, the values of the coefficient of investment in private sector vacancies ξ, the fundamental surplus fs and the elasticities of wages ϵ_w and private sector job destruction rates ε_{δ^P} make the *Aggregate Demand Effect* so small that it turns out to be smaller from the *Labour Market Effect*, resulting in a negative *Unemployment Risk Effect* in the first periods after the shock. Consequently, both aggregate output Y_t and private sector firms' output Y_t^P decrease for the duration of the shock.

3.4.2 Results - Lump Sum Taxes & Labour Income Taxes

I now focus at the case where both taxes increase. Starting with Panels B and C in Figure 3.6, increasing public sector employment N_t^G in France now leads to a larger, more prolonged reduction in unemployment u_t which decreases by 3.96%. Also, while initially there is crowding out of private sector employment N_t^P this quickly changes and approximately 12 months after the shock there is a large persistent crowding in effect. In the US both the reduction in unemployment and the crowding in effect are slightly smaller but last a few more months.

As in the lump-sum tax case, the policy shock is longer-lasting (as $\varepsilon_{\mathcal{H}^G} = -5.4920$) and public sector employment N_t^G is bigger in France, so when it rises, it causes a large increase in public sector firms' output Y_t^G, which is also larger in France, leading to a large increase of aggregate output Y_t and a larger *Direct Effect*. In the *Labour Market Effect* investment in private sector vacancies i_t decreases more in Panel F, as $\xi = 0.6852$ and $fs = 5.50\%$. As the effect propagates it lowers aggregate demand, inflation π_t and the value of filled vacancies J_t^F, leading to a large increase in the private sector job destruction rate δ_t^P in Panel G, due to its elasticity ($\varepsilon_{\delta^P} = -0.8499$) and sticky wages ($\epsilon_w = 0.0933$), and even larger reduction in investment in private sector vacancies i_t. This results in a larger reduction in private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t.

The *Direct Effect* is again smaller and short-lived in the US as $\varepsilon_{\mathcal{H}^G} = -67.7722$. In the *Labour Market Effect* $\xi = 0.5857$, and $fs = 1\%$ so the reduction in investment in vacancies i_t is smaller while $\varepsilon_{\delta^P} = -0.4598$ and $\epsilon_w = 0.9900$ make the rise in the private sector job destruction rate δ_t^P smaller. As a result the drop in private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t is smaller.

The difference now is that both labour income taxes on workers households and lump-sum taxes on capitalists households rise to repay the public debt. This means

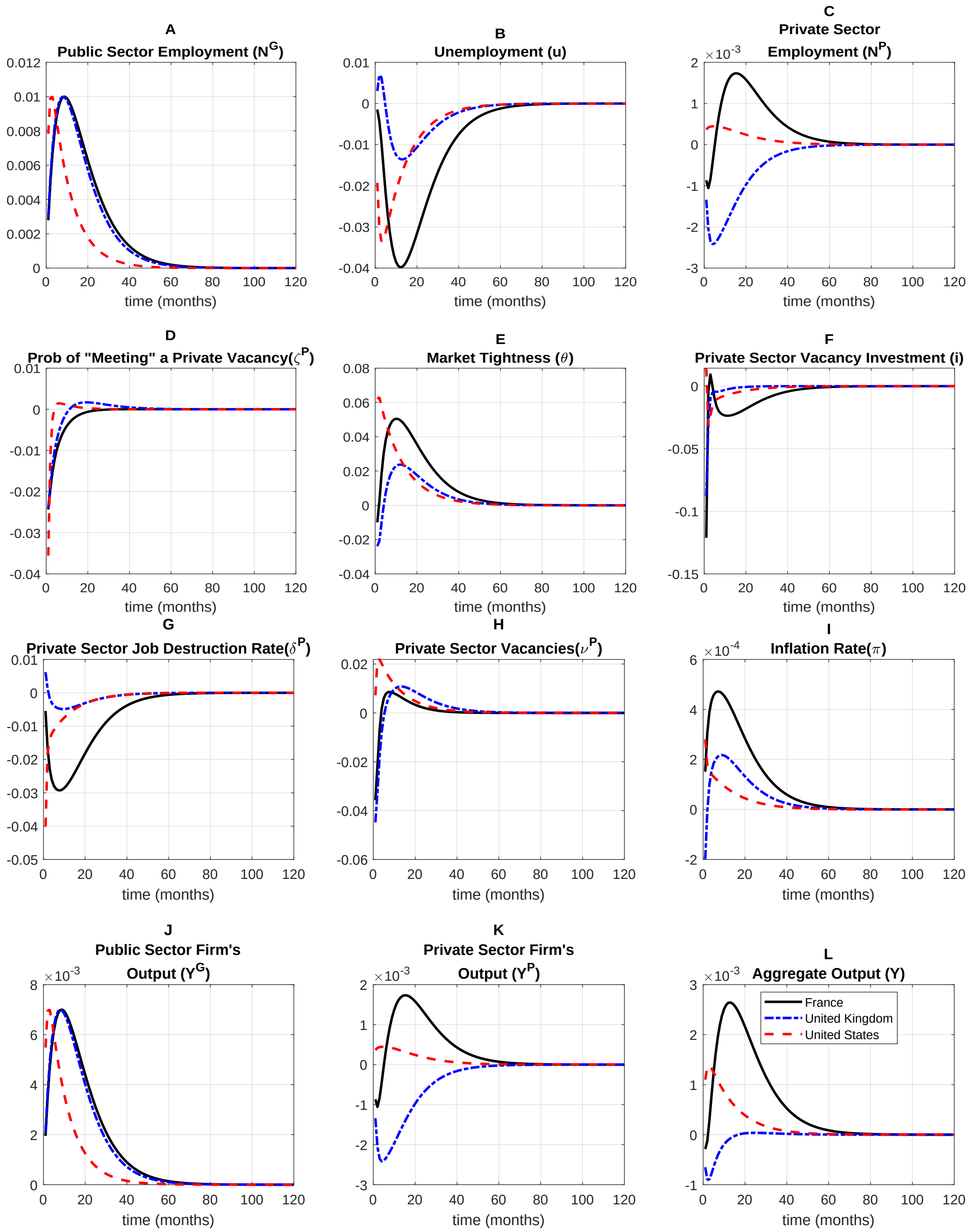

Figure 3.6: Public Sector Employment shock - Debt financed by lump-sum taxes and labour income taxes.

the *Redistribution Effect* is weaker and the *Aggregate Demand Effect* initially smaller, so consumption, inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t increase less at first. However, capitalists households pay smaller taxes so investment in private sector vacancies i_t increases more, but in later periods. As a result the *Indirect Aggregate Demand Effects* are larger and this makes the rise in aggregate demand, inflation π_t and private sector firms' output Y_t^P bigger; consequently the indirect increase in aggregate output Y_t is now larger and the *Aggregate Demand Effect* stronger but in later periods. Finally, the *Unemployment Risk Effect* lowers unemployment risk of workers households and demand for precautionary savings, raising interest rates for household bonds R_t, aggregate demand and inflation π_t. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P and wages, and increases aggregate output Y_t even more, reinforcing the multiplier.

In France, the *Redistribution Effect* is weaker, as investment in private sector vacancies i_t does not increase much initially, which explains the bigger impact of the *Labour Market Effect* in Panels D to H. However, in later periods the rise in inflation π_t leads to a large increase in the value of filled vacancies J_t^F and a large drop in the private sector job destruction rate δ_t^P, as it is more elastic while wages are sticky in France. As a result investment in private sector vacancies i_t increases more, being more elastic in France, and also to a larger degree than when only lump-sum taxes increase but in later periods. So initially the *Aggregate Demand Effect* is weaker but quickly becomes bigger than the lump-sum tax case leading to a larger increase in private sector vacancies v_t^P and market tightness θ_t. In turn, private sector employment N_t^P, inflation π_t and private sector firms' output Y_t^P increase more, resulting in a large increase in aggregate output Y_t, larger than under lump-sum taxes.

The *Unemployment Risk Effect* in Panel I is also larger now: The unemployment risk of workers households and demand for precautionary savings go down, so the interest rate for household bonds R_t aggregate demand and inflation π_t rise. As the *Aggregate Demand Effect* and *Direct Effect* are bigger now, private sector vacancy investment goes up and the private sector job destruction rate decreases even more. This further raises private sector employment N_t^P, private sector firms' output Y_t^P and wages and increases aggregate output Y_t even more. Consequently the overall effect is larger because we see in Panels J to L that increasing public sector firms' output Y_t^G in France now leads to a much larger increase of aggregate output Y_t at 0.26% with crowding in of private sector firms' output Y_t^P.

The *Aggregate Demand Effect* in the US is less responsive. The *Redistribution Effect* is smaller now so the *Labour Market Effect* has a bigger impact at first, and the *Indirect Aggregate Demand Effects* are larger and longer lasting as in France. However private sector vacancy investment i_t is more inelastic in the US, so the difference in its increase is very small; similarly the reduction in the private sector job destruction rate is practically the same, because it is very inelastic while wage elasticity is almost unitary. As a result the increase in private sector vacancies v_t^P, aggregate demand, inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t is marginally larger now.

Consequently the *Unemployment Risk Effect* is only slightly bigger: Unemployment risk of workers households and demand for precautionary savings decreases so the interest rate for household bonds R_t and inflation π_t increase, but given the *Aggregate Demand Effect* the change is very small. As a result investment in private sector vacancies private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t increase by a very small amount. Therefore the overall effect in the goods market is mostly unchanged: Aggregate output Y_t increases by 0.137% and private sector firms' output Y_t^P increases slightly less, but both effects now last longer.

Finally, and as mentioned in the calibration subsection, UK data and empirical moments result in nearly explosive estimated parameter values. As a result I can only make very small changes to the value of κ_τ^N otherwise the model ends up having no rational expectations solutions and becomes explosive, meaning that the Blanchard-Kahn conditions cannot be satisfied. Therefore, I cannot find any differences between the lump-sum tax case and this case.

3.4.3 Multiplier Analysis

I now turn my attention to analysing the fiscal multipliers of my policy experiment. Following Hagedorn et al. (2019) and Monacelli et al. (2010), I focus on the unemployment multiplier, which measures the reduction in unemployment when public sector employment increases. I then use the change in public sector firms' output created by the public sector employment shock to calculate the aggregate output multiplier, which is the increase in aggregate output when public sector firms' output increases. I focus on the cumulative multiplier, which represents the discounted percentage change in aggregate output (unemployment) relative to the discounted percentage change in public sector firms' output (public sector employment):

$$\frac{\sum dY_t}{\sum dY_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t - Y}{Y} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t^G - Y^G}{Y^G} \right\}} \frac{Y}{Y^G} \tag{3.10}$$

$$\frac{\sum du_t}{\sum dN_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{u_t - u}{u} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{N_t^G - N^G}{N^G} \right\}} \frac{u}{N^G} \tag{3.11}$$

Table 3.7: Public Sector Employment Multipliers

Country	Debt Financing Scheme	Aggregate Output Multiplier	Unemployment Multiplier
France	Lump-Sum Taxes	0.7274	-1.0292
	Labour Income & Lump-Sum Taxes	1.8728	-1.8872
United Kingdom	Lump-Sum Taxes	-0.1632	-0.3820
	Labour Income & Lump-Sum Taxes	-0.1632	-0.3820
United States	Lump-Sum Taxes	1.0259	-1.4792
	Labour Income & Lump-Sum Taxes	1.3806	-1.8579

Results are in Table 3.7. What is important to note is that in the case of France and the US both the aggregate output multipliers and unemployment multipliers are quite large numbers. Given that the policy I study is financed by debt re-payed by raising taxes, these values suggest that increasing public sector employment and public sector firms' output can be quite effective. A closer look at the results also indicates that, similarly to the impulse response functions, differences in public sector size and labour market characteristics play a big part in the effectiveness of fiscal policy.

Starting with the unemployment multiplier, I find that it is a large number above unity, indicating that increasing public sector employment can be useful in reducing unemployment and in maintaining employment stability, by lowering unemployment volatility and smoothing out the effects of business cycles in the labour market. Specific country results show that when only lump-sum taxes on capitalists households increase the effect is stronger in the US (-1.4809), while the multiplier in France is also above unity but much smaller at -1.0292. This result mirrors the findings in my main analysis where the labour market traits of the US, namely inelastic private sector vacancy investment, inelastic private sector job destruction rates and elastic wages, make increasing public sector employment more effective because, although the positive aggregate demand effects are smaller any negative effects are also mitigated, leading to a reduction of unemployment and crowding in of private sector employment.

When both taxes increase the effect is bigger, and now the multiplier for France is larger than the US one, and is in fact the largest unemployment multiplier I find at -1.8872. This indicates how tax policy can impact on the effects and propagation mechanisms of increasing public sector employment. This is more evident in France where increasing public sector employment results in large positive aggregate demand effects but the crowding out effect is also very large due to the more elastic investment in private sector vacancies and private sector job destruction rates and the more rigid wages. However when both taxes rise, these very same characteristics make the aggregate demand, and overall policy effect, larger and crowd in private sector employment in France.

Results are similar in the goods market. France has the smallest multiplier when debt is repaid only with lump-sum taxes at 0.7274. However, when both taxes increase results change as increasing public sector firms' output in France now leads to a much larger increase in aggregate output with a multiplier larger than unity at 1.8728 and a strong crowding in effect. US multipliers are smaller, but above unity for both tax plans at 1.0259 and 1.3806 respectively. The findings once again highlighting how public sector size and labour market differences lead to different results between countries and create different outcomes under different tax plans.

Conversely, the effect of increasing public sector employment in the UK is negative for the goods market (−0.1632) while the effect in the labour market is much smaller (−0.3820) as the values of the elasticity of vacancy investment, job destruction rates and wages make the *Aggregate Demand Effect* much smaller than France and the US. Also as explained in my quantitative analysis and my results I cannot actually find any differences between the case where only lump-sum tax case increase and the case where both taxes increase as the UK data and empirical moments result in nearly explosive estimated parameter values. As a result I can only make small changes to the tax policy I study because my model has no rational expectations solutions and becomes explosive, meaning that the Blanchard-Kahn conditions cannot be satisfied.

To sum up, both my baseline results and my multiplier analysis indicate that in the US, increasing public sector employment and public sector firms' output can be very effective in raising aggregate output and lead to big unemployment drops as the country's labour market and public sector traits cause the positive aggregate demand effects to be smaller but at the same time mitigate crowding out enough so the policy is more effective. Conversely in countries with more elastic investment in private sector vacancies, more elastic private sector job destruction rates but higher wage rigidity like France, positive aggregate demand effects are larger but crowding out is even stronger, so the policy effect is weaker; however if the right tax mix is used, increasing public sector employment and public sector firms' output in a country like France leads to much larger increases in aggregate output with a multiplier larger than unity, bigger unemployment reductions and crowding in effects, and an overall stronger result than the US.

3.4.4 Increasing Public Sector Employment in the ZLB

I finish my analysis examining the effect of increasing public sector employment, as a response to an increase in the private sector job destruction rate. More specifically, I use Equation 2.24 in Chapter 2 but now I assume that the private sector job destruction rate at time t, δ_t^P is also subject to an exogenous shock:

$$\delta_t^P = \delta^P \left[\frac{J_t^F}{J^F}\right]^{-\varepsilon_{\delta,P}} \chi_t^{\delta,P}. \tag{3.12}$$

$\chi_t^{\delta,P}$ is the exogenous shock on the private sector job destruction rate with mean equal to unity that follows a stochastic process:

$$\log\left(\chi_t^{\delta,P}\right) = \rho_{\delta,P} \log\left(\chi_{t-1}^{\delta,P}\right) + \left(1-\rho_{\delta,P}\right)\log\left(\chi^{\delta,P}\right) + v_t^{\delta,P}, \tag{3.13}$$

where $\rho_{\delta,P}$ is the autocorrelation parameter and $v_t^{\delta,P}$ is a white noise innovation, drawn from a normal distribution with mean zero. For this policy experiment, I assume a one standard deviation increase in the private sector job destruction rate, and examine the effects on unemployment u_t and aggregate output Y_t when public sector employment stays the same and when it follows a countercyclical policy, responding to this shock by increasing hirings of public sector employees $\mathscr{H}_t^G$, similarly to Equation 2.53:

$$\mathscr{H}_t^G = \mathscr{H}^G \left[\frac{N_t^G}{N^G}\right]^{\varepsilon_{\mathscr{H},G}} \left[\frac{u_t}{u}\right]^{\varepsilon_u} \chi_t^{\mathscr{H},G}. \tag{3.14}$$

In this equation, similarly to Chapter 2, when $\varepsilon_u = 0$ public sector employment does not change in response to deviations of unemployment from its steady state, while for $\varepsilon_u > 0$, when unemployment increases, hirings of public sector employees, and consequently public sector employment, also increase. For each of these two cases I am using the Occbin toolkit (Guerrieri & Iacoviello, 2015) to study the effects on unemployment and aggregate output when the monetary policy follows a normal Taylor Rule i.e. the interest rate for household bonds R_t in equations 2.12 and 2.60 follows a non-binding rule, and when the economy operates in the ZLB and the interest rate does not decrease below a certain value (in this case below one).

I start my analysis with the results for France. In Figure 3.7 I present the effects of an increase in the private sector job destruction rate when the monetary policy operates under a normal Taylor Rule, and in Figure 3.8 I run the same shock at the ZLB. In both cases I examine the results in unemployment and aggregate output when public sector employment follows an acyclical rule and a countercyclical rule with regards to changes in unemployment.

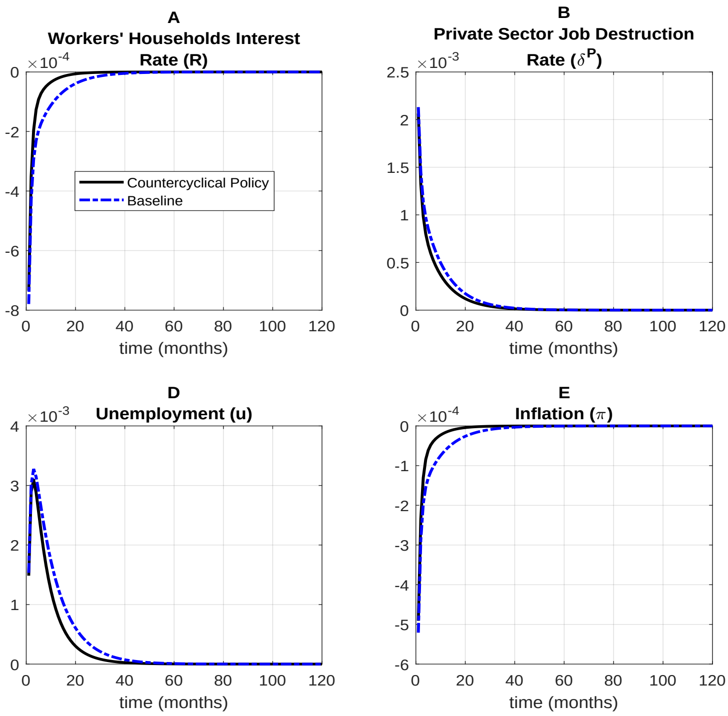

Figure 3.7: Private Sector Job Destruction Rate Shock Under Taylor Rule - France.

A rise in the private sector job destruction rate in Figure 3.7, increases unemployment u_t, while inflation π_t, the interest rate of workers households R_t and aggregate output Y_t decrease. A countercyclical public sector employment policy mitigates this effect, however the result is quantitatively small. More specifically in Panel D, we see that unemployment u_t goes up by 0.35% in the baseline case but this effect is smaller and goes down faster for the countercyclical case, as increasing public sector employment N_t^G reduces the increase in unemployment u_t by as much as 0.05 percentage points 7 months after the shock, with this effect being very persistent. Similarly the effect in the goods market is also significant, but small, as increasing public sector firms' output Y_t^G by increasing public sector employment reduces the decrease in aggregate output Y_t in Panel F by about 0.03 percentage points, while inflation π_t and the interest rate of workers households R_t also see smaller reductions with this policy.

The effect of a countercyclical policy is more evident in the ZLB. In Figure 3.8, the negative shock in the private sector is now larger, leading to a bigger increase in unemployment u_t by 1.8%, larger drops in inflation π_t and the interest rate of workers households R_t, while aggregate output Y_t goes down by 2%. Similarly, the effect of increasing public sector employment N_t^G is larger as it reduces the increase in unemployment by as much as 1.2 percentage points, with this effect remaining very persistent. The effect in the goods market is also larger, as increasing public sector firms' output Y_t^G by increasing public sector employment reduces the decrease in aggregate output Y_t by as much as 1.37 percentage points. The effect on inflation π_t and the interest rate of workers households is also bigger now as both variables slightly rise two months after the shock, before returning to their steady state.

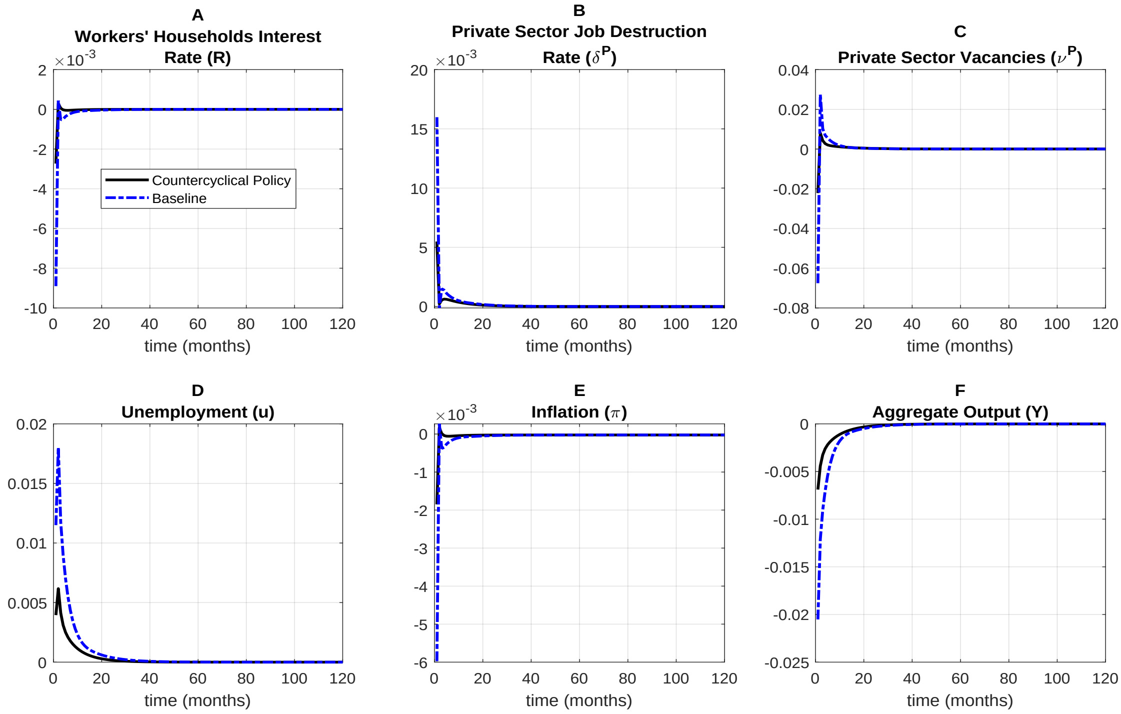

Figure 3.8: Private Sector Job Destruction Rate Shock in ZLB - France.

These results are based on the propagation mechanisms and effects outlined in the previous subsections. Public sector employment N_t^G is bigger in France, so when it rises it causes a large increase in public sector firms' output Y_t^G, which is also larger in France, leading to a large increase in aggregate output Y_t and a larger *Direct Effect*. In the *Redistribution Effect*, unemployed workers households hired by public sector firms now consume more, raising aggregate demand, inflation π_t and private sector firms' output Y_t^P, indirectly increasing aggregate output Y_t. Then in the *Indirect Aggregate Demand Effects*, higher inflation raises the value of a filled vacancy J_t^F and investment in private sector vacancies i_t, and lowers private sector job destruction rates δ_t^P, raising private sector employment N_t^P, private sector firms' output Y_t^P and inflation π_t further and indirectly increasing aggregate output Y_t even more. These effects are larger in France due to its relatively more elastic investment in private sector vacancies and private sector job destruction rates, and highly inelastic wages. This results in a positive *Unemployment Risk Effect*, as unemployment risk and demand for precautionary savings of workers households goes down, increasing the interest rate for household bonds R_t and aggregate demand. Consequently inflation π_t and investment in private sector vacancies rise even more, further raising private sector employment N_t^P, private sector firms' output Y_t^P and increasing aggregate output Y_t further.

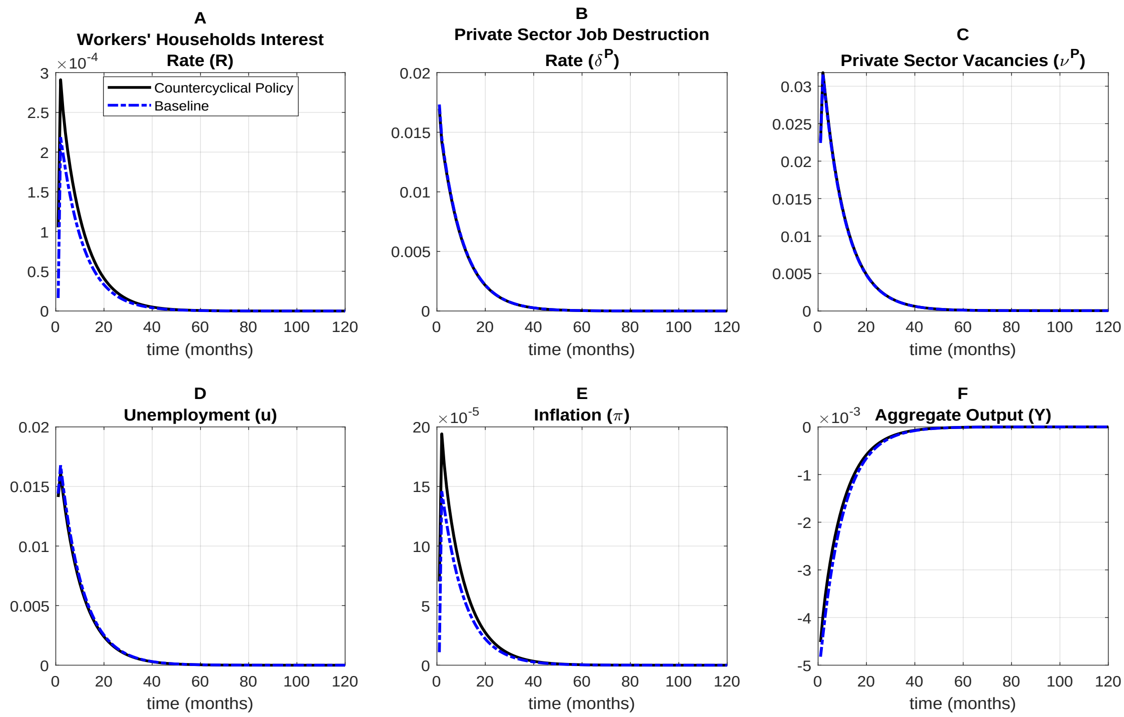

Figure 3.9: Private Sector Job Destruction Rate Shock Under Taylor Rule - US.

Conversely, the effects for the case of the US economy are very different. Starting with the results under a normal monetary policy in Figure 3.9 we see in Panel

D that the increase in unemployment u_t is bigger compared to France being approximately 1.6% and the effect on aggregate output Y_t nearly double the one in France. Additionally there is no significant difference between the case of an acyclical and a countercyclical public sector employment policy, as unemployment u_t and aggregate output Y_t are practically the same between the two cases. Results remain the same when the economy operates in the ZLB in Figure 3.10 as there is no change between the countercyclical policy case and the baseline case, which mirrors the behaviour of the US economy in the previous subsections as well as in Chapter 2, where there are no significant differences between the different counterfactual policy experiments I run.

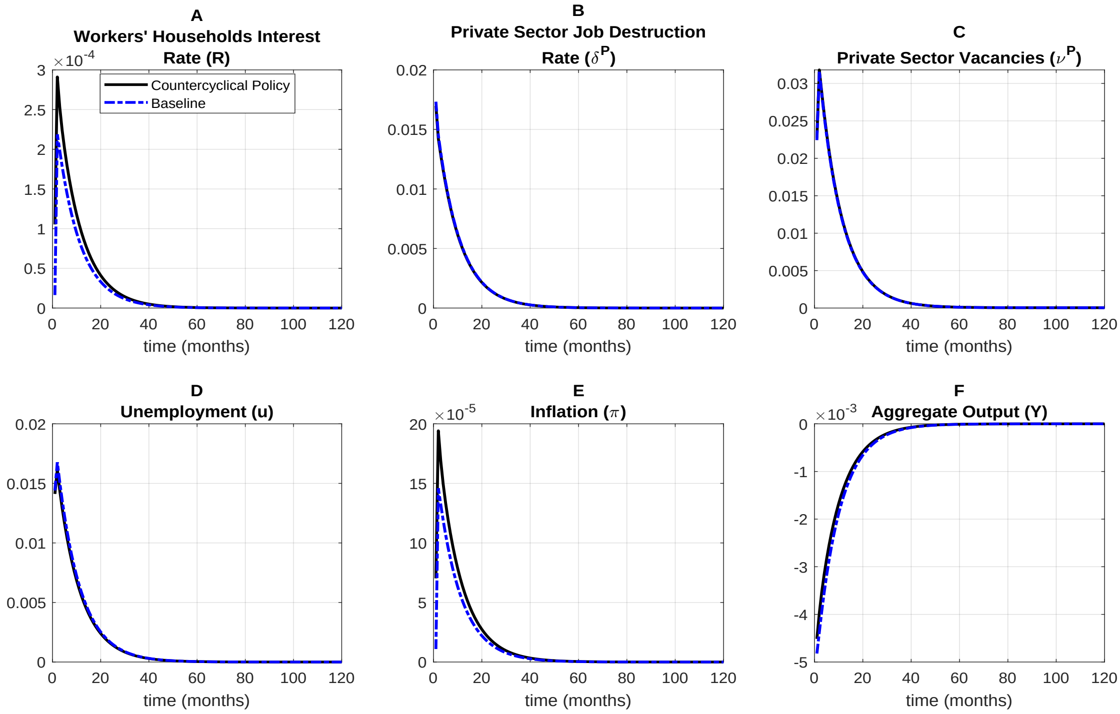

Figure 3.10: Private Sector Job Destruction Rate Shock in ZLB - US.

The most interesting finding is that the increase in the private sector job destruction rate δ_t^P leads to a slight rise in inflation π_t by 0.015%, which in turn increases workers households interest rate R_t and private sector vacancies v_t^P. The reason for this seemingly strange result, lies in the way the US public sector and labour market are structured. Investment in private sector vacancies and the private sector job destruction rate are relatively inelastic, while the elasticity of wages is almost unitary, as $\epsilon_w = 0.9900$; consequently the rise in the private sector job destruction rate δ_t^P reduces private sector firms' output, leading to a almost one of one reduction of private sector wages. This, coupled with sticky prices, means that the value of a filled vacancy J_t^F in Equation 2.38 actually increases and leads to this very small rise in inflation. Finally the baseline and countercyclical policy have no differences, because as seen in the previous subsections, the *Direct Effect* is smaller as public sector employment N_t^G and public sector firms' output Y_t^G are smaller in the US, and any policy shocks are relatively short lasting. In addition both the *Labour Market effect*, *Aggregate Demand Effects* and *Unemployment Risk Effect*, are much smaller, resulting in a weaker overall effect in the economy.

To sum up, both the results for a countercyclical policy as well as the baseline results and the multiplier analysis indicate significant variation in the effects that increasing public sector employment and public sector firms' output has on the countries I study. This is a result of the differences in the labour market and the public sector which make both the crowding out and the amplification effects of fiscal policy different between countries. The tax mix used to finance fiscal expansions is also important as it interacts with each country's individual labour market and public sector characteristics and changes the magnitude of the crowding out and amplification effects generated by policy shocks.

3.5 Conclusion

The objective of this Chapter was two-fold: The first was to establish a number of key facts and highlight the differences in the size and scope of the public sector, public sector employment and the labour market stocks and flows between countries in Europe, exemplified here by France, the UK and the US. The second objective was to study the effect that increasing public sector employment and public sector output via a debt financed increase repaid by raising taxes on capitalists households or taxes on capitalists households and workers households, by combining a HANK model and a frictional labour market with public sector employment and public sector firms producing goods using government expenditure on consumption and public sector employment.

The data indicate that European countries, due to their more interventionist public sectors and more extensive welfare states, have larger public sectors compared to the US and the UK. All countries also exhibit large and persistent unemployment increases during economic downturns, particularly the US and the UK. Finally, labour flows are quite smaller in France and the public sector reduces hirings but also firings during recessions. In the UK and the US labour flows are much larger (with the exceptions of US public sector inflows), and the public sectors increase their outflows during the Great Recession and Covid.

Simulating my model also reveals important differences in the way the private sector and the whole economy reacts to policy shocks between countries. In France private sector vacancy investment and private sector job destruction rates are smaller but more elastic, while wages are very sticky and policy shocks are longer lasting; as a result both the crowding out via the *Labour Market Effect* and the positive effects of the *Direct Effect*, *Aggregate Demand Effect* and *Unemployment risk Effect* are bigger. Conversely investment in private sector vacancies and private sector job destruction rates are larger but more inelastic in the US, wages are highly elastic and policy shocks rather shorter-lived so the overall policy effects are smaller. Finally the UK exhibits much smaller elasticity in all labour market variables.

These differences in the labour market and public sector characteristics of each country result in very distinct policy effects between countries, but also in differing results when the tax policy changes. When only capitalists households pay higher taxes, increasing public sector employment leads to large reductions in unemployment, with multipliers above unity for France and the US, with the effect being larger in the US; however when both households pay higher taxes, the effect becomes stronger in France. Similarly when both households' taxes go up, increasing public sector firms' output in France leads to large increases in aggregate output, with an aggregate output multiplier above unity, but the result is reversed when only taxes on capitalists households go up. On the other hand the effects of this fiscal policy in the UK reduce aggregate output and result in a much smaller drop in unemployment due to its very inelastic labour market.

Finally I examine the effects of a countercyclical public sector employment policy. I find that the more elastic private sector vacancy investment and private sector job destruction rates, sticky wages and longer-lived policy shocks in France mean that increasing public sector employment and public sector firms' reduce the increase in unemployment and decrease the drop in aggregate output, with this effect being quantitatively much stronger when the economy operates in the ZLB. On the other hand this policy does not have any significant effect on unemployment or aggregate output in the US both for the case of a standard Taylor and in the ZLB, a result of the more inelastic investment in private sector vacancies and private sector job rates, highly elastic wages and shorter-lived shocks that the US exhibits.

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Appendix A

Appendix to Chapter 1

A.1 Steady State Equations

To calculate the steady state value of private sector employment I make use of Equation 1.5:

$$N^P = \frac{1 - u - (1-\delta^G)N^G}{(1-\delta^P)}. \tag{A1}$$

Following Shimer (2005) and Coles & Moghaddasi Kelishomi (2018), I set the market tightness equal to unity i.e. $\theta = 1$, a common value in the literature. The steady state value of vacancies in the economy is then calculated using the steady state version of Equation 1.17:

$$\theta = \frac{v}{u},$$

$$\Rightarrow v = \theta u. \tag{A2}$$

Following the methodology of Fontaine et al. (2020) and using data from 2003Q1 - 2021Q4 for the UK, I set the separation rates of public sector employees $\delta^G = 0.0365$ and private sector employees $\delta^P = 0.0207$. This allows me to calculate the steady state vacancy filling rate q and the steady state job finding rate κ. I begin by calculating the steady state value of the evolution of public sector employment given by the steady state of Equation 1.4:

$$N^G = \left(1 - \delta^G\right) N^G + \mathscr{H}^G,$$

$$\Rightarrow \delta^G N^G = \mathscr{H}^G. \tag{A3}$$

Similarly, the steady state value of the evolution of private sector employment is given by the steady state of Equation 1.5:

$$N^P = \left(1 - \delta^P\right) N^P + \mathscr{H}^P$$

$$\Rightarrow \delta^P N^P = \mathscr{H}^P. \tag{A4}$$

I then add A4 to A3 and make use of $v^G + v^P = v$, $\mathscr{H}^G = q v^G$ and $\mathscr{H}^G = q v^P$:

$$\delta^G N^G + \delta^P N^P = q v^G + q v^P$$

$$\Rightarrow \delta^G N^G + \delta^P N^P = q v. \tag{A5}$$

To calculate qv, I make use of the steady state versions of Equations 1.18 and 1.19 where I get respectively:

$$q = \frac{m}{v} \Rightarrow m = q v \tag{A6}$$

$$\kappa = \frac{m}{u} \Rightarrow m = \kappa u. \tag{A7}$$

A6 and A7 are equal, so I can write A5 by using A7 instead of qv and solve for κ:

$$\delta^G N^G + \delta^P N^P = \kappa u$$

$$\Rightarrow \kappa = \frac{\delta^G N^G + \delta^P N^P}{u}. \tag{A8}$$

The steady state value of matches m is given from Equation A7 using the steady state values of κ in A8 and u. Setting $\gamma = 0.6$ (Coles & Moghaddasi Kelishomi, 2018), I can calculate the scale parameter A using the steady state version of Equation 1.11:

$$A = \frac{m}{u^\gamma v^{1-\gamma}}. \tag{A9}$$

Finally q is given by the steady state version of Equation 1.18:

$$q = \frac{m}{v}. \tag{A10}$$

Using the results above I can now calculate the steady state values of public sector vacancies using Equation A3, private sector vacancies using Equation A4 and the probability of meeting a private sector employer with the steady state version of Equation 1.14:

$$v^G = \frac{\delta^G N^G}{q}, \tag{A11}$$

$$v^P = \frac{\delta^P N^P}{q}, \tag{A12}$$

$$\zeta^P = \frac{v^P}{v^P + v^G}. \tag{A13}$$

The steady state intermediate stock of vacancies v_o^P is simply the steady state of Equation 1.13:

$$v_o^P = \left(1 - \delta^P\right)\left[\left(v^P - \zeta^P q v^P\right)\right]. \tag{A14}$$

I then calculate the steady state investments in vacancies by combining A14 and the steady state of Equation 1.15:

$$v^P = v_o^P + i$$

$$\Rightarrow v^P = \left(1-\delta^P\right)\left[\left(v^P - \zeta^P q v^P\right)\right] + i$$

$$\Rightarrow i = v^P\left[\delta^P + \left(1-\delta^P\right)\zeta^P q\right]. \tag{A15}$$

The public sector wage is assumed to be equal to the private sector wage:

$$w^G = w^P. \tag{A16}$$

The steady state value of a filled private sector vacancy J^F is given by the steady state of Equation 1.23:

$$J^F = \frac{p - w^P}{1-\beta\left(1-\delta^P\right)}. \tag{A17}$$

Assuming that the steady state value of a private sector vacancy in Equation 1.16 is zero i.e. $J = 0$ the cost of posting a vacancy c is:

$$c = \beta(1-\delta^P)\zeta^P q J^F. \tag{A18}$$

Finally I solve for the steady state values of the values of employment in the public sector $V^{E,G}$, in the private sector ($V^{E,P}$) and the value of unemployment V^U. I start by calculating the difference between the value of public sector employees and unemployment in the steady state given by Equations 1.22 and 1.20:

$$V^{E,G} - V^U = w^G + \beta V^{E,G} + \beta\delta^G V^U - \beta\delta^G V^{E,G} - z - \beta V^U - \beta\zeta^P\kappa(V^{E,P} - V^U) - \\ \beta(1-\zeta^P)\kappa V^{E,G} + \beta(1-\zeta^P)\kappa V^U$$

$$\Rightarrow (V^{E,G} - V^U)[1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa] = w^G - z - \beta\zeta^P\kappa(V^{E,P} - V^U)$$

$$M = V^{E,G} - V^U = \frac{w^G - z - \beta\zeta^P\kappa(V^{E,P} - V^U)}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}. \tag{A19}$$

I then plug A19 to find the difference between the value of the private sector employment and unemployment in the steady state given by Equations 1.21 and 1.20:

$$V^{E,P} - V^U = w^P + \beta V^{E,P} + \beta\delta^P\left[V^U - V^{E,P}\right] - z - \beta V^U - \beta\zeta^P\kappa\left[V^{E,P} - V^U\right] - \beta\left(1-\zeta^P\right)\kappa\left[V^{E,G} - V^U\right]$$

$$\Rightarrow V^{E,P} - V^U = w^P + \beta\left(V^{E,P} - V^U\right) + \beta\delta^P\left[V^U - V^{E,P}\right] - z - \beta\zeta^P\kappa\left[V^{E,P} - V^U\right] - \beta(1-\zeta^P)\kappa\left[\frac{w^G - z - \beta\zeta^P\kappa\left(V^{E,P} - V^U\right)}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}\right]$$

$$\Rightarrow \left[V^{E,P} - V^U\right]\left[1 - \beta + \beta\delta^P + \beta\zeta^P\kappa - \frac{\beta(1-\zeta^P)\kappa\beta\zeta^P\kappa}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}\right] = w^P - z - \beta(1-\zeta^P)\kappa\left[\frac{w^G - z}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}\right].$$

For simplicity I name the second term in the LHS as O and I get the difference between the value of the private sector employment and unemployment in the steady state:

$$[V^{E,P} - V^U]O = w^P - z - \beta(1-\zeta^P)\kappa\left[\frac{w^G - z}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}\right]$$

$$\Rightarrow \Omega \equiv V^{E,P} - V^U = \frac{w^P - z}{O} - \frac{\beta(1-\zeta^P)\kappa}{O}\left[\frac{w^G - z}{1 - \beta + \beta\delta^G + \beta(1-\zeta^P)\kappa}\right]. \tag{A20}$$

I then plug A20 in A19 and find the difference between the value of the public sector employment and unemployment in the steady state:

$$M \equiv V^{E,G} - V^U = \frac{w^G - z - \beta\zeta^P \kappa\Omega}{1-\beta+\beta\delta^G+\beta(1-\zeta^P)\kappa}. \tag{A21}$$

I then plug A21 and A20 in 1.20 to find the steady state value of unemployment:

$$V^U = \frac{z}{1-\beta} + \frac{\beta}{1-\beta}\zeta^P\kappa\Omega + \frac{\beta}{1-\beta}(1-\zeta^P)\kappa M. \tag{A22}$$

A.2 Testing the bounds of the elasticity of vacancy investment

In this Section of the Appendix, I examine how the results of my model change if I change the calibration for the parameter that governs the elasticity of vacancy investment, ξ. As ξ has a very low value in my baseline calibration, this can be perceived as unrealistic given that it essentially means that investment in private sector vacancies is completely inelastic. So I now examine how my simulation results, the calibrated parameter values and the baseline results change if I use a different lower bound in the simulated method of moments for ξ. Essentially I try to see whether my model with its empirical moments and calibrated parameters are still a good enough approximation of the real empirical moments and the data of the UK economy.

Table A.1: Simulation Results

Empirical Moment	Data	Calibration ($\xi = 0.0100$)	Calibration ($\xi = 1.0400$)	Source
σ_u	0.1761	0.1747	0.2029	Directly Estimated
σ_{δ^P}	0.0870	0.0874	0.0826	Directly Estimated
$autocorr(u)$	0.9670	0.8787	0.6184	Directly Estimated
σ_{N^G}	0.0305	0.0304	0.0304	Directly Estimated
$autocorr(N^G)$	0.9006	0.9096	0.9096	Directly Estimated
σ_p	0.0314	0.0314	0.0314	Directly Estimated
$autocorr(p)$	0.4504	0.4504	0.4504	Directly Estimated

More specifically, I try two alternative calibrations, where in the SMM I set higher lower bounds for the parameter ξ. I therefore choose a lower bound for $\xi = 0.0100$ in the first case and a lower bound for $\xi = 1.0400$ in the second

Results are given in Table A.1. As we can see here, when $\xi = 0.0100$ the real and the simulated empirical moments are still a very good fit for the standard deviation of unemployment σ_u and the standard deviation of the private sector job destruction rate σ_{δ^P}; conversely when $\xi = 1.0400$ the fit for the standard deviation of unemployment σ_u is not as good. Additionaly the fit for the autocorrelation of unemployment $autocorr(u)$ is not a good as it was in the baseline case, being 0.8787 when $\xi = 0.0100$ and 0.6184 when $\xi = 1.0400$. The rest of the simulated empirical moments are unchanged.

Table A.2: Calibrated Parameters

Parameter	Parameter Name	Value ($\xi = 0.0100$)	Value ($\xi = 1.0400$)	Source/Target
ξ	Coefficient of Investment in private sector vacancies	0.0100	1.0400	σ_u
fs	Fundamental Surplus	0.0100	0.0139	σ_{δ^P}
ε_{δ^P}	Elasticity of Private Sector Job Destruction Rate	-0.2400	-0.3127	$corr(v,u)$
$\varepsilon_{\mathscr{H}^G}$	Feedback Parameter	-6.7871	-6.7871	$autocorr(N^G)$
$\sigma_{\mathscr{H}^G}$	i.i.d Shock parameter	0.1249	0.1249	σ_{N^G}
ε_p	i.i.d Shock Parameter	0.0265	0.0265	σ_p
ρ_p	TFP Shock Autocorrelation Parameter	0.6899	0.6899	$autocorr(p)$

In Table A.2 I present the new calibrated parameter values where I have changed the lower bound in my simulation for ξ. As we can see now $\xi = 0.0100$ in the first counterfactual case and $\xi = 1.0400$ in the second which are the new lower bounds I have set[1]. This change does not appear to affect any of the other parameters with the exception of ε_{δ^P} which goes from -0.2425 to -0.2400 and -0.31127 for each counterfactual respectively, and the fundamental surplus fs which goes to 0.0139 when $\xi = 1.0400$.

[1]Additional tests I ran for lower bounds closer to the original one show that ξ always "pushes" towards the lower bound, and that the baseline value is in fact the minimum value this parameter can have.

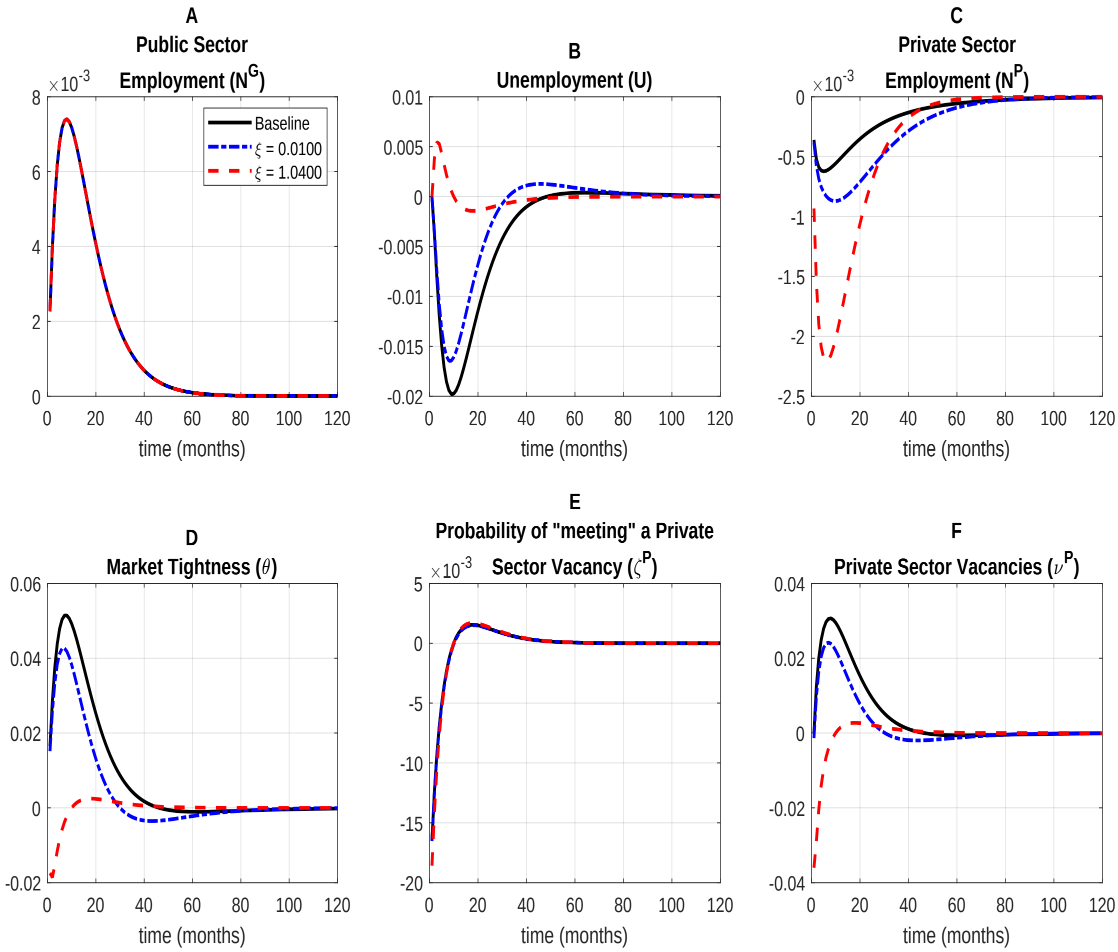

Figure A.1: Effect of an increase in Public Sector Employment: Baseline Case ($\xi = 1.0016e-06$), Counterfactual Cases ($\xi = 0.0100$; $\xi = 1.0400$).

Looking at the effects of increasing public sector employment by increasing the hirings of public sector employees in Figure A.1, we find that qualitatively the result are unchanged in the baseline case and when $\xi = 0.0100$ as unemployment decreases with some private sector crowding out. Quantitatively results are different as the effect is larger in the baseline case, with unemployment decreasing by 2.02% at its peak; for $\xi = 0.0100$ unemployment decreases 1.65 at its peak so there is a difference of 0.32 percentage points, but more importantly in this case there is also a small increase in unemployment in later periods lasting approximately 40 months and peaking at 0.13%. The crowding out of private sector employment is also larger for $\xi = 0.0100$ by about 0.02 percentage points (0.06% compared to

0.08%) while lasting a longer time period as well. Alternatively when $\xi = 1.0400$ results change significantly, as unemployment now initially increases, peaking at 0.55% and then very quickly returns to its steady state, with only a small reduction of 0.14% which lasts for a few months. The crowding out effect is also much larger as private sector employment decreases by 0.22%.

The reason for this change is again evident in Panel D to F. As ξ becomes bigger, the private sector is able to reduce investment in private sector vacancies more effectively that in the baseline case and when $\xi = 1.0400$ there is actually a large crowding out effect of private sector vacancies and private sector employment, which increases unemployment. Conversely when $\xi = 0.0100$ the private sector keeps stocking up on vacancies but the number is smaller than the baseline case, so the crowding out remains small and unemployment still decreases but to a smaller decree than the baseline case.

APPENDIX B

Appendix to Chapter 2

B.1 Steady State Equations

To calculate the steady state value of private sector employment I make use of Equation 2.23:

$$N^{P,W} = \frac{1 - u - (1-\delta^G)N^{G,W}}{(1-\delta^P)}. \tag{A23}$$

Following Shimer (2005) and Coles & Moghaddasi Kelishomi (2018), I set the market tightness equal to unity i.e. $\theta = 1$, a common value in the search and matching literature. Steady state vacancies are then calculated using the steady state version of Equation 2.31:

$$\theta = \frac{v}{u},$$

$$\Rightarrow v = \theta u. \tag{A24}$$

Following the methodology of Fontaine et al. (2020) and using US data from 2003Q1 - 2021Q4, I set the separation rates of public sector employees δ^G and private sector employees δ^P for the two countries. This will allow me to calculate the steady state vacancy filling rate q and the steady state job finding rate κ. I begin by calculating the steady state value of the evolution of public sector employment given by the steady state of Equation 2.22:

$$N^{G,W} = \left(1-\delta^G\right)N^{G,W} + qv^G,$$

$$\Rightarrow \delta^G N^{G,W} = qv^G. \tag{A25}$$

Similarly, the steady state value of the evolution of private sector employment is given by the steady state of Equation 2.23:

$$N^{P,W} = \left(1-\delta^P\right)N^{P,W} + qv^P$$

$$\Rightarrow \delta^P N^{P,W} = qv^P. \tag{A26}$$

I then add A25 to A26 and make use of $v^G + v^P = v$

$$\delta^G N^{G,W} + \delta^P N^{P,W} = qv^G + qv^P$$

$$\Rightarrow \delta^G N^{G,W} + \delta^P N^{P,W} = qv. \tag{A27}$$

To calculate qv, I make use of the steady state versions of Equations 2.32 and 2.33 where I get respectively:

$$q = \frac{m}{v} \Rightarrow m = qv \tag{A28}$$

$$\kappa = \frac{m}{u} \Rightarrow m = \kappa u. \tag{A29}$$

Since A28 and A29 are equal, I can write A27 by using A29 instead of qv and solve for κ:

$$\delta^G N^{G,W} + \delta^P N^{P,W} = \kappa u$$

$$\Rightarrow \kappa = \frac{\delta^G N^{G,W} + \delta^P N^{P,W}}{u}. \tag{A30}$$

The steady state value of matches m is given from Equation A28 using the steady state values of κ in A29 and u. Setting $\gamma = 0.6$ (Coles & Moghaddasi Kelishomi (2018)), I can calculate the scale parameter $\mathscr{M}$ using the steady state version of Equation 2.26:

$$\mathscr{M} = \frac{m}{u^\gamma v^{1-\gamma}}. \tag{A31}$$

Finally q is given by the steady state version of Equation 2.33:

$$q = \frac{m}{v}. \tag{A32}$$

Using the results above I calculate the steady state values of public sector vacancies using Equation A25, private sector vacancies using Equation A26 and the probability of meeting a private sector employer with the steady state version of Equation 2.29:

$$v^G = \frac{\delta^G N^{G,W}}{q}, \tag{A33}$$

$$v^P = \frac{\delta^P N^{P,W}}{q}, \tag{A34}$$

$$\zeta^P = \frac{v^P}{v^P + v^G}. \tag{A35}$$

The steady intermediate stock of vacancies v_o^P is simply the steady state of Equation 2.28:

$$v_o^P = \left(1 - \delta^P\right)\left[\left(v^P - \zeta^P q v^P\right)\right]. \tag{A36}$$

I then calculate the steady state investments in vacancies by combining A36 and the steady state of Equation 2.30:

$$v^P = v_o^P + i$$

$$\Rightarrow v^P = \left(1-\delta^P\right)\left[\left(v^P - \zeta^P q v^P\right)\right] + i$$

$$\Rightarrow i = v^P\left[\delta^P + \left(1-\delta^P\right)\zeta^P q\right]. \tag{A37}$$

I then calculate the steady state values of labour demand by private sector firms and labour demand by public sector firms by using the steady state version of Equations 2.67 and 2.66:

$$N^P = N^{P,W}, \tag{A38}$$

$$N^G = N^{G,W}. \tag{A39}$$

The public sector wage is assumed to be equal to the private sector wage :

$$w^G = w^P. \tag{A40}$$

B.1.1 Workers Households

The steady state consumption of workers households employed by public sector firms, private sector firms or being unemployed is found by using the steady state versions of their budget constraints in Equation 2.3:

$$C^S + B^S \leq (1-\tau^N)w^S + b(1-N^S) + B^S\frac{R}{\pi}$$

Workers households invest in a household bond in zero net supply making the sum of asset holdings for all workers households is zero; workers households also face a no-borrowing constraint which also equates the individual asset holdings of workers households to zero. Therefore $B^G = 0$, $B^P = 0$, $B^U = 0$ and the consumption of workers households employed by public sector firms, workers households employed by private sector firms and unemployed workers households is:

$$C^G = (1-\tau^N)w^G, \tag{A41}$$

$$C^P = (1-\tau^N)w^P, \tag{A42}$$

$$C^U = b, \tag{A43}$$

$$C^W = N^{G,W}C^G + N^{P,W}C^P + uC^U, \tag{A44}$$

The Euler Equations for the three types of workers households are all satisfied with inequality save for one type of workers households where it is satisfied with equality. I use the steady state version of Equations 2.10, 2.12 and 2.14 and divide both sides with consumption:

$$(C_i^G)^{-\eta} = \beta\frac{R}{\pi}\left[\delta^G(1-\kappa)(C_i^U)^{-\eta} + \delta^G\zeta^P\kappa(C_i^P)^{-\eta} + \left[1-\delta^G\left(1-\kappa\left(1-\zeta^P\right)\right)\right](C_i^G)^{-\eta}\right] + \mu_i^G,$$

$$(C_i^P)^{-\eta} = \beta\frac{R}{\pi}\left[\delta^P(1-\kappa)(C_i^U)^{-\eta} + \left[1-\delta^P\left(1-\zeta^P\kappa\right)\right](C_i^P)^{-\eta} + \delta^P\left(1-\zeta^P\right)\kappa(C_i^G)^{-\eta}\right] + \mu_i^P,$$

$$(C_i^U)^{-\eta} = \beta\frac{R}{\pi}\left[(1-\kappa)(C_i^U)^{-\eta} + \zeta^P\kappa(C_i^P)^{-\eta} + (1-\zeta^P)\kappa(C_i^G)^{-\eta}\right] + \mu_i^U.$$

Wages, and therefore consumption, of employed workers households are equal if they work for public sector firms or private sector firms, so I substitute C_i^G with C_i^P:

$$1 = \beta\frac{R}{\pi}\left[\delta^G(1-\kappa)\frac{(C_i^U)^{-\eta}}{(C_i^P)^{-\eta}} + \delta^G\zeta^P\kappa + 1 - \delta^G\left(1-\kappa\left(1-\zeta^P\right)\right)\right] + \frac{\mu_i^G}{(C_i^P)^{-\eta}},$$

$$1 = \beta\frac{R}{\pi}\left[\delta^P(1-\kappa)\frac{(C_i^U)^{-\eta}}{(C_i^P)^{-\eta}} + 1 - \delta^P\left(1-\zeta^P\kappa\right) + \delta^P\left(1-\zeta^P\right)\kappa\right] + \frac{\mu_i^P}{(C_i^P)^{-\eta}},$$

$$1 = \beta\frac{R}{\pi}\left[(1-\kappa)+\zeta^P\kappa\frac{(C_i^P)^{-\eta}}{(C_i^U)^{-\eta}}+(1-\zeta^P)\kappa\frac{(C_i^P)^{-\eta}}{(C_i^U)^{-\eta}}\right]+\frac{\mu_i^U}{(C_i^U)^{-\eta}}.$$

Dividing both sides with β and simplifying gives me:

$$\frac{1}{\beta}=\frac{R}{\pi}\left[\delta^G(1-\kappa)\frac{(C_i^U)^{-\eta}}{(C_i^P)^{-\eta}}+1-\delta^G(1-\kappa)\right]+\frac{\mu_i^G}{\beta(C_i^P)^{-\eta}}, \tag{A45}$$

$$\frac{1}{\beta}=\frac{R}{\pi}\left[\delta^P(1-\kappa)\frac{(C_i^U)^{-\eta}}{(C_i^P)^{-\eta}}+1\right]+\frac{\mu_i^P}{\beta(C_i^P)^{-\eta}}, \tag{A46}$$

$$\frac{1}{\beta}=\frac{R}{\pi}\left[(1-\kappa)+\kappa\frac{(C_i^P)^{-\eta}}{(C_i^U)^{-\eta}}\right]+\frac{\mu_i^U}{(C_i^U)^{-\eta}}. \tag{A47}$$

These conditions hold with equality if and only if households are liquidity constrained. In the steady state, the real interest rate for households' bond is smaller that the discount rate $\left(\frac{R}{\pi}<\frac{1}{\beta}\right)$. The Euler equation of workers households employed by public sector firms A45 and the Euler equation of unemployed workers households A47 are consistent with this conditions if $\mu_i^G>0$ and $\mu_i^U>0$. The Euler equation of workers households employed by private sector firms A46 is also consistent if $\mu_i^P=0$. This is a necessary condition; if not workers households employed by private sector firms would face a binding borrowing constrain meaning that they hold positive amounts of debt. But this would violate market clearing for household bonds as unemployed workers households and workers households employed by public sector firms hold zero bonds.

Finally I use the steady state version of Equation 2.12 to calculate the steady state value of the interest rate for household bonds:

$$R=\frac{(C^P)^{-\eta}\pi}{\beta\left\{\left[\delta^P(1-\kappa)(C^U)^{-\eta}+\left[1-\delta^P\left(1-\zeta^P\kappa\right)\right](C^P)^{-\eta}+\delta^P\left(1-\zeta^P\right)\kappa(C^G)^{-\eta}\right]\right\}} \tag{A48}$$

B.1.2 Capitalists' Households & Private Sector Firms

The steady state return rate of government bonds $\frac{R^C}{\pi}$, is computed by solving the capitalists' household Euler condition in Equation 2.21:

$$1 = \beta \frac{R^C}{\pi}$$

$$\Rightarrow \frac{R^C}{\pi} = \frac{1}{\beta} \tag{A49}$$

Similarly, to find the steady state price of the intermediate good $P^{\mathscr{X}}$, and total factor productivity A, I use the steady state values of Equations 2.50 and 2.61:

$$1 = \mu P^{\mathscr{X}} \quad , \quad AN^P = Y^P$$

Simply rearranging terms gives me the results:

$$P^{\mathscr{X}} = \frac{1}{\mu} \tag{A50}$$

$$A = \frac{Y^P}{N^P} \tag{A51}$$

The steady state level of government bonds held by capitalists households is:

$$\Rightarrow B^C = \frac{B}{pop_C} \tag{A52}$$

Solving for C^C in Equation 2.17 and using Equation (A49) gives us the steady state value of capitalists' households consumption:

$$C^C + B^C = Div^C + B^C \frac{R}{\pi}$$

$$\Rightarrow C^C = Div^C + B^C \left(\frac{R}{\pi} - 1 \right) \tag{A53}$$

The steady state value of a filled private sector vacancy J^F is given by the steady state of Equation 2.38:

$$J^F = \frac{A_t P_t^{\mathscr{X}} - w^P}{1 - \beta \left(1 - \delta^P \right)}. \tag{A54}$$

Assuming the steady state value of a private sector vacancy in Equation 2.37 is zero i.e. $J = 0$ the cost of posting a vacancy c is:

$$c = \beta (1 - \delta^P) \zeta^P q J^F. \tag{A55}$$

B.1.3 Prices and Inflation

The steady state version of the New Keynesian Phillips curve for the sticky prices case is given by Equation 2.50:

$$\frac{\xi_\varrho}{(\varepsilon-1)}[\pi-\pi]\pi = \mu MC_j - 1 + \frac{\xi_\varrho}{(\varepsilon-1)}\mathbb{E}_t X[\pi-\pi]\pi\frac{Y^P}{Y^P} \tag{A56}$$

In the steady state this simply becomes the New Keynesian Phillips curve for the flexible price case:

$$1 = \mu MC_{t,j} \tag{A57}$$

B.1.4 Government

Steady state output of public sector firms, Y^G equals the steady value of Equation 2.59:

$$Y^G = A\left(G^C\right)^z\left(N^G\right)^{1-z} \tag{A58}$$

The steady state public debt level B, is found by solving the steady state version of Equation 2.52:

$$G^C + w^G N^G + bu + B\frac{R^C}{\pi} = \tau^N\left(w^P N^P + w^G N^G\right) + pop_c\tau + B$$

$$\Rightarrow B = \frac{\tau^N\left(w^P N^P + w^G N^G\right) + pop_c\tau - G^C - w^G N^G - bu}{\left(\frac{R^C}{\pi} - 1\right)} \tag{A59}$$

B.1.5 Aggregation

To find the steady state values of the aggregate variables, I simply drop the time subscript from Equations 2.63 to 2.69:

$$Y^P = AN^P \tag{A60}$$

$$Y^G = A^{(1-z)}\left(G^C\right)^z\left(N^G\right)^{1-z} \tag{A61}$$

$$Y = C + G^C + cv_t^P \tag{A62}$$

$$C = uC^U + N^{P,W}C^P + N^{G,W}C^G + pop_C C^C \tag{A63}$$

$$Div = pop_C Div^C \tag{A64}$$

$$N = N^{P,W} + N^{G,W} \tag{A65}$$

B.2 Complete Derivations

B.2.1 Derivation of Final Goods Sector

The firm's maximization problem is:

$$\max_{y_t^j} d_t = \max_{y_t^j} \left\{ P_t Y_t^P - \int_0^1 p_t^j y_t^j dj \right\} \tag{A66}$$

Substituting 2.39 on 2.40 I get:

$$\max_{y_t^j} d_t = \max_{y_t^j} P_t \left[\int_0^1 (y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_t^j y_t^j dj \tag{A67}$$

The first order condition with respect to the quantity of wholesale good is:

$$\frac{\partial d_t}{\partial y_t^j} = 0 \Rightarrow P_t \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left[\int_0^1 (y_t^j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}-1} \left(\frac{\varepsilon - 1}{\varepsilon} \right) y_t^{j\,\frac{\varepsilon-1}{\varepsilon}-1} = p_t^j \tag{A68}$$

Playing around with terms and simplifying gives us the demand for each wholesale good of type j produced by wholesale goods firm j at time t, y_t^j:

$$y_t^j = \left(\frac{p_t^j}{P_t} \right)^{-\varepsilon} Y_t^P \tag{A69}$$

Equation A69 shows that demand for each wholesale good depends negatively on its relative price and positively on private sector firms' total output. Since the final goods sector operates under perfect competition, profits must equal zero, so $d_t = 0$ and I get:

$$P_t Y_t^P = \int_0^1 p_t^j y_t^j dj \tag{A70}$$

Substituting A69 on A70 and taking out of the integral all variables without an index j gives us the aggregate price level at time t, P_t:

$$P_t Y_t^P = \int_0^1 p_t^j \left(\frac{p_t^j}{P_t}\right)^{-\varepsilon} Y_t^P dj$$

$$P_t Y_t^P = P_t^{\varepsilon} Y_t^P \int_0^1 p_t^{j^{1-\varepsilon}} dj$$

$$P_t^{1-\varepsilon} = \int_0^1 p_t^{j^{1-\varepsilon}} dj$$

$$P_t = \left[\int_0^1 p_t^{j^{1-\varepsilon}} dj\right]^{\frac{1}{1-\varepsilon}} \tag{A71}$$

B.2.2 Derivation of New Keynesian Phillips Curve

A representative wholesale goods firm j producing a wholesale good of type j at time t, y_t^j, adjusts its price to maximize the discounted sum of future profits subject to demand for y_t^j, as in Equation 2.48. Using Equation 2.48 and adding a stochastic discount factor the maximization problem becomes:

$$\max_{\{p_{t+i}^j\}} d_t^j = \max_{\{p_{t+i}^j\}} \mathbb{E}_t \sum_{i=0}^{\infty} \left\{ X_{t,t+i} \left[\left(\frac{p_{t+i}^j}{P_{t+i}}\right) y_{t+i}^j - P_{t+i}^{\mathscr{X}} y_t^j - \frac{\xi_\rho}{2} \left(\frac{p_{t+i}^j}{p_{t+i-1}^j} - \pi\right)^2 Y_{t+i}^P \right] \right\} \tag{A72}$$

Where $X_{t,t+i} = \beta^{t+i} \left[\frac{C_{t+1}^C}{C_t^C}\right]^{-\eta^C}$ is the stochastic discount factor. Using Equation 2.41 to substitute y_{t+i}^j:

$$\max_{\{p^j_{t+i}\}} d^j_t = \max_{\{p^j_{t+i}\}} \mathbb{E}_t \sum_{i=0}^{\infty} \left\{ X_{t,t+i} \left[\left(\frac{p^j_{t+i}}{P_{t+i}} \right)^{(1-\varepsilon)} - P^{\mathscr{X}}_{t+i} \left(\frac{p^j_{t+i}}{P_{t+i}} \right)^{-\varepsilon} - \frac{\xi_\rho}{2} \left[\frac{p^j_{t+i}}{p^j_{t+i-1}} - \pi \right]^2 \right] Y^P_{t+i} \right\} \tag{A73}$$

To find the optimality condition for price setting, I solve the profit maximization problem of the representative wholesale goods firm j at time $t+i$:

$$\frac{\partial d^j_t}{\partial p^j_{t+i}} = 0 \Rightarrow (1-\varepsilon) \left(\frac{p^j_{t+i}}{P_{t+i}} \right)^{-\varepsilon} \frac{Y^P_{t+i}}{P_{t+i}} + \varepsilon P^{\mathscr{X}}_{t+i} \left(\frac{p^j_{t+i}}{P_{t+i}} \right)^{-\varepsilon-1} \frac{Y^P_{t+i}}{P_{t+i}} - \xi_\rho \left[\frac{p^j_{t+i}}{p^j_{t-1+i}} - \pi \right] \frac{Y^P_{t+i}}{p^j_{t-1+i}}$$

$$+\xi_\rho \mathbb{E}_t X_{t,t+i} \left[\frac{p^j_{t+1+i}}{p^j_{t+i}} - \pi \right] \frac{p^j_{t+1+i}}{(p_{t+i})^2} Y^P_{t+1+i} = 0$$

$$\Rightarrow (\varepsilon-1) \frac{(p^j_{t+i})^{-\varepsilon}}{(P_{t+i})^{1-\varepsilon}} Y^P_{t+i} = \varepsilon P^{\mathscr{X}}_{t+i} \left(\frac{p^j_{t+i}}{P_{t+i}} \right)^{-\varepsilon-1} \frac{Y^P_{t+i}}{P_{t+i}} - \xi_\rho \left[\frac{p^j_{t+i}}{p^j_{t-1+i}} - \pi \right] \frac{Y^P_{t+i}}{p^j_{t-1+i}}$$

$$+\xi_\rho \mathbb{E}_t X_{t,t+i} \left[\frac{p^j_{t+1+i}}{p^j_{t+i}} - \pi \right] \frac{p^j_{t+1+i}}{(p_{t+i})^2} Y^P_{t+1+i}$$

Setting $i = 0$, we get the optimal price of the wholesale good of type j, y^j_t produced by firm j at time t, p^j_t:

$$\frac{(p^j_t)^{-\varepsilon}}{(P_t)^{-\varepsilon}} \frac{Y^P_t}{P_t} = \mu P^{\mathscr{X}}_t \left(\frac{p^j_t}{P_t} \right)^{-\varepsilon-1} \frac{Y^P_t}{P_t} - \frac{\xi_\rho}{(\varepsilon-1)} \left[\frac{p^j_t}{p^j_{t-1}} - \pi \right] \frac{Y^P_t}{p^j_{t-1}}$$

$$+\frac{\xi_\rho}{(\varepsilon-1)} \mathbb{E}_t X_{t,t+i} \left[\frac{p^j_{t+1}}{p^j_t} - \pi \right] \frac{p^j_{t+1}}{(p_t)^2} Y^P_{t+1} \tag{A74}$$

Where $\mu = \frac{\varepsilon}{\varepsilon-1}$ is the price mark-up. Since all wholesale goods firms face the same marginal costs and the same demand elasticity, they also set the same price, face the same demand and produce the same amount of output; therefore I have $p^j_t = P_t$. This implies that the optimal price p^j_t of the wholesale good of type j produced by firm j at time t, y^j_t equals the aggregate price level at time t, P_t. Rewriting the optimality condition in Equation A74 in terms of inflation at time t $\pi_t = \frac{P_t}{P_{t-1}}$ gives us the New Keynesian Phillips curve:

$$\frac{\xi_\rho}{(\varepsilon-1)}[\pi_t - \pi]\pi_t = \mu P_t^{\mathscr{X}} - 1 + \frac{\xi_\rho}{(\varepsilon-1)}\mathbb{E}_t X_{t,t+i}[\pi_{t+1} - \pi]\pi_{t+1}\frac{Y_{t+1}^P}{Y_t^P}. \tag{A75}$$

B.3 Complete Markets Case

In the case of the representative agents' household the only things that change is the households' maximization problem and the fact that now there are no risk neutral agents in the economy so we do not have η^C, only η. The Equations that change are the budget constraint, the Bellman equation and the intertemporal maximization problem for the household, the value of an open vacancy, the value of a filled vacancy and the Taylor Rule which now sets the nominal interest rate for public sector bonds, as household bonds do not exist now. All other Equations remain the same.

B.3.1 The Representative Agent's Household

$$V_t(B_{t-1}) = \max_{C_t,B_t}\left\{\frac{(C_t)^{(1-\eta)}}{1-\eta} + \lambda_t\Big[Div_t + (1-\tau_t^N)\Big(w_t^P N_t^{P,W} + w_t^G N_t^{G,W}\Big)\right.$$

$$\left. + b_t u_t + B_{t-1}^C \frac{R_{t-1}^C}{\pi_t} - \tau_t - C_t - B_t^C\Big]\right\}. \tag{A76}$$

$$\frac{\partial V_t}{\partial C_t} = 0 \Rightarrow (C_t)^{-\eta} = \lambda_t. \tag{A77}$$

$$\frac{\partial V_t}{\partial B_t^C} = 0 \Rightarrow (C_t)^{-\eta} = \beta\mathbb{E}_t\left[(C_{t+1})^{-\eta}\frac{R_t^C}{\pi_{t+1}}\right] \tag{A78}$$

B.3.2 Firms

$$J_t = -c + \zeta_t^P q_t J_t^F + \beta(1-\delta^P)\left[1-\zeta_t^P q_t\right]\mathbb{E}_t\left\{\left[\frac{C_{t+1}^C}{C_t^C}\right]^{-\eta} J_{t+1}\right\}. \tag{A79}$$

$$J_t^F = A_t P_t^{\mathscr{X}} - w_t^P + \mathbb{E}_t\,\beta\left\{\left[\frac{C_{t+1}^C}{C_t^C}\right]^{-\eta}(1-\delta_t^P)J_{t+1}^F\right\}. \tag{A80}$$

B.3.3 Government

$$R_t^C = R^C \left[\frac{\pi_t}{\pi}\right]^{\varphi_\pi}. \tag{A81}$$

APPENDIX

C

APPENDIX TO CHAPTER 3

C.1 Results in the Pre-Covid Period

In this Appendix, I change the period my model covers, dropping data for the Covid time period (2020Q1-2021Q4). The reason is that the pandemic, and the reactions to it, damaged the global economy, leading to dramatic drops in employment and GDP around the world. Furthermore the pandemic itself and the subsequent lockdowns are a unique event from an economic perspective, with nothing similar in recent history. Removing this period allows me to study the effects of public sector employment during more normal economic periods.

C.1.1 Calibration

Similarly to my main results, I set up the steady state target values, in Table C.1. Using FLFS, UKLFS and CPS data from 2003Q1 to 2019Q4, I construct the stocks of public sector employment, private sector employment and unemployment, and the flows between unemployment and employment.

As in my main results, I match each country's average unemployment rate in the data with the steady state unemployment rate u and set it at 9.57% in France, 5.81% in the UK and 6.20% in the US. Steady state public sector employment N^G

is set at 23.70% in France, 22.72% in the UK and 14.93% in the US matching the average values of public sector employment as a percentage of the labour force, while the steady state private sector employment N^P is set targeting a labour force normalised to unity. Market tightness θ remains equal to unity and the steady state value of vacancies v equal to the unemployment level (Shimer, 2005; Coles & Moghaddasi Kelishomi, 2018). Finally, I calculate the job destruction rates for public sector firms and private sector firms as in Fontaine et al. (2020), but with the inclusion of employees in state owned enterprises in public sector employment, resulting in a public sector job destruction rate δ^G of 2.00% for France, 2.08% for the UK and 2.40% for the US, with the private sector job destruction rate δ^P set at 3.55% for France, 3.63% for the UK and 3.55% for the US.

Table C.1: Steady State Target

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
u	Unemployment	0.0957	0.0581	0.0620	Data
N^G	Public Sector Employment	0.2370	0.2272	0.1493	Data
N^P	Private Sector Employment	0.6673	0.7147	0.7887	Data
δ^P	Job Destruction Rate (Private Sector)	0.0355	0.0363	0.0355	Data
δ^G	Job Destruction Rate (Public Sector)	0.0200	0.0208	0.0240	Data
G^C	Government Expenditure in Consumption	0.1417	0.1533	0.1386	Data
θ	Market Tightness	1	1	1	Normalise
Y	Aggregate Output	1	1	1	Normalise
Y^P	Private Sector Firms' Output	0.7865	0.8004	0.8232	Data
Y^G	Public Sector Firms' Output	0.2135	0.1996	0.1768	Data
π	Inflation	0.0022	0.0022	0.0022	Annual Steady State Inflation of 2.5%
v	Vacancies	0.0957	0.0581	0.0628	θ=1
w^P	Private Sector Wage	0.8650	0.8447	0.8174	Fundamental Surplus
w^G	Public Sector Wage	0.8650	0.8447	0.8174	No Wage Premium
τ_N	Labour Income Tax Rate	0.5200	0.5000	0.4500	Normalise
τ	Lump-Sum Tax Rate	0.4000	0.3500	0.3000	Normalise

Note: Data used to calculate steady state target values are in quarterly frequency from 2003Q1-2019Q4 and seasonally adjusted. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

The steady state aggregate output Y is set equal to unity and the steady state inflation π, is set targeting an annual inflation rate of 2.5%, so they remain the same pre and post-Covid. I use government finance statistics from Eurostat, the ONS and the BEA for the steady state value of public sector firms' output Y^G as percentage of GDP and find they are mostly unchanged being equal to 21.35% in France, 19.96% in the UK and 17.68% in the US; similarly the steady state value of government expenditure on consumption G^C as a percentage of GDP is 14.17% in France, 15.33% in the UK and 13.86% in the US. I then set the steady state value of private sector firms' output Y^P, targeting the steady state value of aggregate output Y. I also keep the labour income tax rate τ^N and the lump-sum tax τ the same as in my main sample. Finally the steady state private sector wage w^P is calculated using the theory of the fundamental surplus as in Ljungqvist & Sargent (2017;2021), so $w^P = P^{\mathcal{X}}(1 - fs)$ where fs is the fundamental surplus ratio, which I calibrate internally, and public sector wages equal private sector wages so $w^G = w^P$.

Table C.2: Parameter Values

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
γ	Matching function elasticity	0.6000	0.6000	0.6000	Coles & Moghaddasi Kelishomi (2018)
β	Discount factor	0.9967	0.9967	0.9967	Annual discount rate 4%
pop_c	Number of Capitalists Households	0.1000	0.1000	0.1000	Broer et al. (2021)
η	CRRA Coeffcient	2	2	2	Broer et al. (2021)
η^C	Capitalists Households CRRA coefficient	0	0	0	Risk Neutral
ϵ	Price Elasticity	10	10	10	Standard
μ	Price mark-up	1.1	1.1	1.1	Standard
ξ_ρ	Rotemberg Price Adjustment Cost	600	600	600	Standard
z	Consumption Good Share in Public Sector Firms' Output	0.3000	0.3000	0.3000	Standard
$\rho_{\mathcal{H},G}$	Public Sector Hirings Shock Autocorrelation Parameter	0.9000	0.9000	0.9000	Cantore & Freund (2021)
ϕ_π	Taylor Response to Inflation	(1.5000; 1.2500)	(1.5000; 1.4999)	(1.5000; 1.2500)	Broer et al. (2021)
κ_{τ^N}	Response of Labour Income Taxes to Public Debt	(0;0.0100)	$(0;3.0000e-07)$	(0;0.0100)	Cantore & Freund (2021)

The externally calibrated parameters in Table C.2 are the same for the three countries in the pre and post-Covid periods so there are no changes there. Also similarly to the post-Covid period, data and empirical moments for the UK result in estimated parameter values very close to explosive, so I cannot make big changes to $\kappa_{\tau,N}$ as the model has no rational expectations solution and becomes explosive, meaning the Blanchard-Kahn conditions cannot be satisfied.

C.1.2 Simulated Method of Moments

I now use the SMM to calibrate my main parameter values. First I estimate the standard deviation of unemployment σ_u, the standard deviation of the private sector job destruction rate $\sigma_{\delta,P}$, the cross correlation between vacancies and unemployment $corr(v,u)$, the auto-correlation of unemployment $autocorr(u)$, the standard deviation of public sector employment $\sigma_{N,G}$ and the auto-correlation of public sector employment $autocorr(N^G)$ using the data from the FLFS, UKLFS and the CPS. Standard deviation of Total Factor Productivity σ_A and the auto-correlation of Total Factor Productivity $autocorr(A)$ are estimated using real average output per worker from the OECD (Shimer, 2005), while the cross-correlation of Public

Debt with Government Expenditure $corr(B,G)$, and the autocorrelation of Public Debt $autocorr(B)$ are estimated using data from Eurostat, the ONS and the BEA.

My simulation results in Table C.3, show that the estimated empirical moments from the model are still a close fit for the real empirical moments from the data while the fit for the autocorrelation of unemployment in the UK has improved. However the fit for the autocorrelation of public sector debt is now worse for all three countries and, more importantly, I now find a very big difference in the real and simulated standard deviation of unemployment for the US. Despite these changes my model and its calibrated parameters are a good approximation of the real economy for the three countries.

Table C.3: Simulation Results

Country/ Empirical Moment	France		United Kingdom		United States		Source
	Data	Calibration	Data	Calibration	Data	Calibration	
σ_u	0.1053	0.1246	0.1820	0.1816	0.2403	0.1222	Directly Estimated
$\sigma_{\delta,P}$	0.0758	0.0857	0.0687	0.0689	0.0392	0.0411	Directly Estimated
$corr(v,u)$	-0.7479	-0.8774	-0.9201	-0.9275	-0.9443	-0.8580	Directly Estimated
$autocorr(u)$	0.9367	0.9401	0.9811	0.9109	0.9786	0.7995	Directly Estimated
$\sigma_{N,G}$	0.0510	0.0509	0.0233	0.0232	0.0163	0.0162	Directly Estimated
$autocorr(N^G)$	0.9290	0.9389	0.8479	0.8564	0.6636	0.6702	Directly Estimated
σ_A	0.0095	0.0095	0.0125	0.0125	0.0084	0.0084	Directly Estimated
$autocorr(A)$	0.8655	0.8657	0.8992	0.8994	0.7889	0.7888	Directly Estimated
$corr(B,G)$	0.8357	0.7627	0.5679	0.5133	0.6445	0.6571	Directly Estimated
$autocorr(B)$	0.9614	0.8892	0.9833	0.8396	0.9761	0.8801	Directly Estimated

Note: Data used to calculate empirical moments values are in quarterly frequency from 2003Q1-2019Q4. Series are seasonally adjusted then taken in log form and detrended with an HP filter with smoothing parameter of 100000. Public Sector variables are calculated as % of GDP, Labour Market data as % of the labour force. (Sources: Eurostat, ONS, BEA, FLFS, UKLFS, CPS).

My internally calibrated parameters are given in Table C.4. Comparing pre and post-Covid values I find that the coefficient of investment in private sector vacancies ξ saw a dramatic increase in France due to Covid, going from 0.4850 to 0.6850, so investment in private sector vacancies became more elastic. A similar pattern is evident in the US, although the effect is much smaller (from 0.5511 to 0.5857). The effect was the opposite in the UK as ξ dropped from 0.2088 to less than half at 0.0984. The fundamental surplus ratio fs followed similar trajectories in France and the UK increasing from 5.00% to 5.50% and from 3.61% to 5.84% repsectively, but remained unchanged in the US at 1%.

Table C.4: Calibrated Parameters

Parameter	Parameter Name	France	United Kingdom	United States	Source/Target
ξ	Coefficient of Investment in private sector vacancies	0.4850	0.2088	0.5511	σ_u
fs	Fundamental Surplus	0.0500	0.0361	0.0100	$\sigma_{\delta,P}$
$\varepsilon_{\delta,P}$	Elasticity of Private Sector Job Destruction Rate	-0.7260	-0.3242	-0.2114	$corr(v,u)$
ϵ_w	Elasticity of Private Sector wages to Private Sector Firms' Output	0.2714	0.2349	0.9900	$autocorr(u)$
$\varepsilon_{\mathscr{H},G}$	Feedback Parameter	-3.2855	-12.9979	-58.8383	$autocorr(N^G)$
$\sigma_{\mathscr{H},G}$	i.i.d Shock parameter	0.1880	0.1920	0.4715	$\sigma_{N,G}$
ε_A	i.i.d Shock Parameter	0.0034	0.0039	0.0039	σ_A
ρ_A	TFP Shock Autocorrelation Parameter	0.9548	0.9764	0.9083	$autocorr(A)$
ρ_B	Public Debt Autocorrelation Parameter	0.0522	0.0520	0.0295	$autocorr(B)$
κ_B	Public Debt response to Government Expenditure	3.2569	1.3144	1.6937	$corr(B,G)$

The elasticity of private sector job destruction rates $\varepsilon_{\delta,P}$ also rose due to Covid, especially in France and the US, while the elasticity of private sector wages ϵ_w was bigger in France and the UK but did not change in the US. These parameter values indicate that prior to Covid, business cycles in France created large changes in private sector job destruction rates, so unemployment saw big changes thought this channel, while wages and investment in private sector vacancies were relatively more inelastic, and changes in unemployment via this channel were smaller. Conversely in the US the same shocks lead to massive wage and larger changes in investment in private sector vacancies, but affected the private sector job destruction rate to a smaller degree. Finally labour market variables in the UK were relatively more inelastic before Covid and the pandemic reinforced this effect.

Looking at the feedback parameter on the hirings of public sector employees, $\varepsilon_{\mathscr{H},G}$ is small in France ($\varepsilon_{\mathscr{H},G} = -3.2855$) but much larger in the UK ($\varepsilon_{\mathscr{H},G} = -12.9979$) and especially in the US, ($\varepsilon_{\mathscr{H},G} = -58.8383$). So when hirings of public sector employees increase in Europe, public sector employment stays above its steady state value for a longer period of time while in the US this effect is more brief. Compared to the pre-Covid period these policy shocks became shorter lasting in France and the US, while the opposite holds for the UK.

C.2 Results

I now re-run the same policy experiment as in my main results, studying a one-period increase in the hirings of public sector employees $\mathscr{H}^G$ that increases public sector employment N_t^G by 1% for a 10-year time period, at monthly frequency.

C.2.1 Results-Lump Sum Taxes

Starting with the labour market in Figure C.1 Panel B, increasing public sector employment N_t^G now significantly lowers unemployment u_t in all countries, unlike the post-Covid case it initially increased for the UK, the small values of ξ and fs making investment in private sector vacancies inelastic and the effect on unemployment persistent (Coles & Moghaddasi Kelishomi, 2018, Broer et al., 2021). Quantitatively the effect is bigger in the UK (2.96%), followed by the US (2.80%) and France (2.34%). Also this policy again crowds out private sector employment N_t^P in France and the UK in Panel C and crowds it in, to a small degree, in the US.

This result is again due countries' labour market and public sector traits. In France, the policy shock lasts longer as $\varepsilon_{\mathscr{H}^G} = -3.2855$ and public sector employment N_t^G is larger, so its rise leads to a large increase in public sector firms' output Y_t^G, which is also larger in France, resulting in a large increase of aggregate output Y_t, and a larger *Direct Effect*. The *Labour Market Effect* is again large in France as the drop in the probability of "meeting" a private sector vacancy ζ_t^P in Panel D reduces investment in private sector vacancies i_t in Panel F, but smaller than the main results, as $\xi = 0.4850$ and $fs = 5\%$ make investment in private sector vacancies more inelastic. Still as the effect propagates, aggregate demand, inflation π_t and the value of filled vacancies J_t^F decrease. This result, combined with sticky wages ($\epsilon_w = 0.2714$) and elastic private sector job destruction rates ($\varepsilon_{\delta^P} = -0.7260$), sharply increases the private sector job destruction rate δ_t^P and leads to a big drop in investment in private sector vacancies i_t. As a result private sector vacancies v_t^P and market tightness θ_t decrease, leading to a large reduction in private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t.

The *Direct Effect* in the US is also smaller in the pre-Covid period, as N_t^G and Y_t^G are smaller, and more brief as $\varepsilon_{\mathscr{H}^G} = -58.2816$. However now there are some differences in the *Labour Market Effect* as $\xi = 0.5589$, and $fs = 1\%$ so vacancy investment i_t is slightly more elastic, but this effect remains limited, while $\varepsilon_{\delta^P} = -0.2109$ and $\epsilon_w = 0.9900$ make the increase in the private sector job destruction rate δ_t^P, and the overall effect on investment in private sector vacancies smaller than in the post-Covid case. In the end market tightness θ_t and private sector vacancies v_t^P, and consequently private sector employment N_t^P, inflation π_t, private

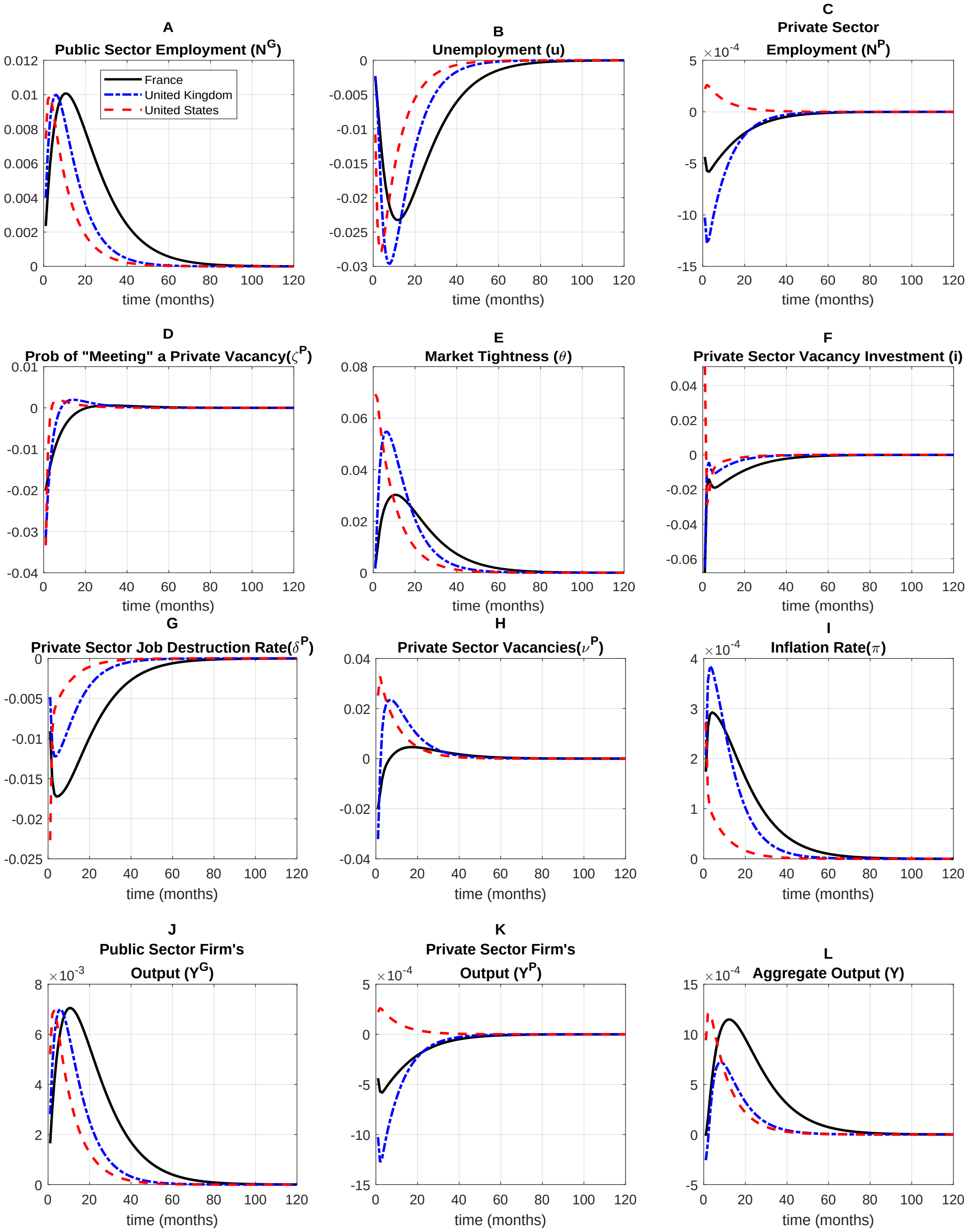

Figure C.1: Public Sector Employment shock - Debt financed via lump-sum taxes.

sector firms' output Y_t^P and aggregate output Y_t all decrease but less than in than France and the post-Covid sample.

In the following periods, the *Aggregate Demand Effect* begins working. First in the *Redistribution Effect* consumption, inflation π_t and private sector firms' output Y_t^P rise, indirectly increasing aggregate output Y_t as income from capitalists households is redistributed to unemployed workers households hired by public sector firms. Then in the *Indirect Aggregate Demand Effects* inflation goes up pushing the value of filled vacancies J_t^F and investment in private sector vacancies i_t the same way, and lowering the private sector job destruction rate δ_t^P. As a result, private sector vacancies v_t^P, private sector employment N_t^P, inflation π_t and private sector firms' output Y_t^P, go up again, further increasing aggregate output Y_t. Finally in the *Unemployment Risk Effect* unemployment risk of workers households and demand for precautionary savings go down, making interest rates for household bonds R_t, aggregate demand and inflation π_t rise. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P, increasing aggregate output Y_t even more.

The *Aggregate Demand Effect* remains larger in France, but smaller than the full sample. Investment in private sector vacancies i_t is more inelastic and initially increases less now, but quickly bounces back as higher inflation π_t , combined with sticky wages, lead to a large increase in the value of filled vacancies J_t^F; as a result the highly elastic private sector job destruction rate δ_t^P, sees a large reduction. These two effects more than exceed the *Labour Market Effect* as the overall effect in Panel G is negative, so market tightness θ_t and private sector vacancies v_t^P quickly rebound and turn positive. As a result I find a large increase in aggregate demand, inflation π_t, private sector employment N_t^P and private sector firms' output Y_t^P, and a large increase in aggregate output Y_t.

The *Unemployment Risk Effect* in Panel I is also positive, as the *Direct Effect* and *Aggregate Demand Effect* are stronger than the *Labour Market Effect*: Lower unemployment risk of workers households and demand for precautionary savings push the interest rate for household bonds R_t, aggregate demand and inflation π_t to go up. This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further raising private sector employment N_t^P and private sector firms' output Y_t^P and increasing aggregate output Y_t even more. The effect in the goods market is summed up in Panels J to L where increasing public sector firms' output Y_t^G in France does crowd out private sector firms' output to an extent, but results in a large increase (0.115%) in aggregate output Y_t.

In the US, investment in private sector vacancies i_t rises and the private sector job destruction rate goes down but the *Aggregate Demand Effect* is still smaller. Still the overall effect on market tightness θ_t and private sector vacancies, v_t^P remains positive, as the *Labour Market Effect* is also small, so inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t still rise. The *Unemployment Risk Effect* also remains positive, reinforcing the positive effect on investment in private sector vacancies, private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t even more. The overall effect in the goods market is still slightly bigger that in France at 0.121% but smaller than in the main results.

What changes between the pre and post-Covid case is the UK results. The *Direct Effect* is similar to France, but the *Labour Market Effect* has changed: $\xi = 0.2088$ and $fs = 3.61\%$ still limit the drop in investment in private sector vacancies i_t but the effect is not as extreme as the post-Covid one. Similarly the effect on the value of filled vacancies J_t^F and the private sector job destruction rate δ_t^P is bigger as both UK wages ($\epsilon_w = 0.2349$) and private sector job destruction rates ($\varepsilon_{\delta^P} = -0.3242$) are more elastic so this effect, and the reduction in private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P, and aggregate output Y_t, is now bigger. Then in the following periods these labour market traits reinforce the *Aggregate Demand Effect*, making it bigger than the *Labour Market Effect*, leading to an *Unemployment Risk Effect* in Panel I that is now positive, but smaller than the other two countries. As a result aggregate output Y_t now increases (0.07%) but private sector firms' output Y_t^P is still crowded out.

C.2.2 Results - Lump Sum Taxes & Labour Income Taxes

I now study the case where both taxes increase. As in my main results, I focus only on France and the US as I can only make very small changes to κ_τ^N for the UK.

Increasing public sector employment N_t^G in France now leads to a larger, more prolonged reduction in unemployment u_t which decreases by 3% as seen in Panels

B and C in Figure C.2. Also, while initially there is crowding out of private sector employment N_t^P this quickly changes and 10 months after the shock there is a large persistent crowding in effect. In the US both the reduction in unemployment and the crowding in effect are slightly smaller but also last a few more months.

The mechanism remains unchanged: In France $\varepsilon_{\mathscr{H}G} = -3.2855$ so the policy shock lasts longer and public sector employment N_t^G is bigger, so when it rises, it causes a large increase in public sector firms' output Y_t^G, which is also larger in France, leading to a large increase of aggregate output Y_t and a larger *Direct Effect*. In the *Labour Market Effect* investment in private sector vacancy i_t is more inelastic as $\xi = 0.4850$ and $fs = 5\%$ so it initially decreases less. As the effect propagates it lowers aggregate demand, inflation π_t and the value of filled vacancies J_t^F, leading to a large increase in the private sector job destruction rate δ_t^P in Panel G, spurred by the elasticity of the private sector job destruction rate ($\varepsilon_{\delta^P} = -0.7260$) and sticky wages ($\epsilon_w = 0.2714$). This results in a large reduction in investment in private sector vacancies i_t, private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t.

The *Direct Effect* is again smaller and short-lived in the US as $\varepsilon_{\mathscr{H}G} = -58.2816$. In the *Labour Market Effect* $\xi = 0.5589$, and $fs = 1\%$ so the reduction in investment in vacancies i_t is smaller while $\varepsilon_{\delta^P} = -0.2109$ and $\epsilon_w = 0.9900$ make the rise in the private sector job destruction rate δ_t^P smaller. As a result the drop in private sector vacancies v_t^P, private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t is smaller.

Now that both workers households and capitalists households repay the public debt the *Redistribution Effect* is weaker and the *Aggregate Demand Effect* initially smaller, so consumption, inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t increase less at first. But capitalists households pay smaller taxes, so investment in private sector vacancies i_t increases more in future periods, making the *Indirect Aggregate Demand Effects* larger and the rise in aggregate demand, inflation π_t and private sector firms' output Y_t^P bigger. As a result the indirect increase in aggregate output Y_t is now larger and the *Aggregate Demand Effect* stronger in later periods. Finally, the *Unemployment Risk Effect* lowers unemployment risk of workers households and demand for precautionary savings, raising interest rates for household bonds R_t, aggregate demand and inflation π_t.

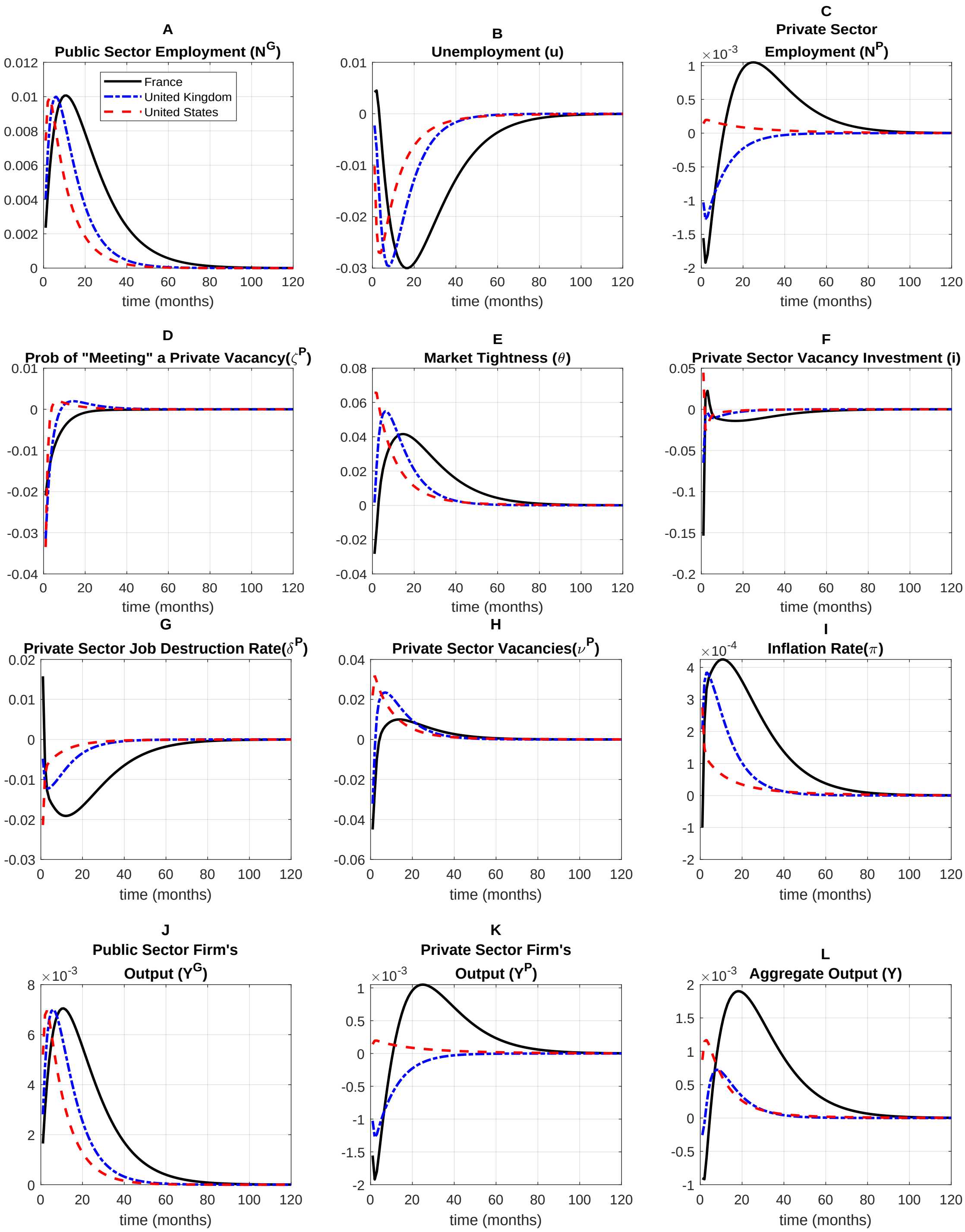

Figure C.2: Public Sector Employment shock - Debt financed by lump-sum taxes and labour income taxes.

This raises private sector vacancy investment and reduces the private sector job destruction rate even more, further increases private sector employment N_t^P, private sector firms' output Y_t^P and wages, and increases aggregate output Y_t.

In France, the *Redistribution Effect* is weaker, so the *Labour Market Effect* in Panels D to H is bigger. Also, similarly to the case where only lump-sum taxes rise, investment in private sector vacancies i_t is more inelastic and initially increases less; however it greatly increases in later periods, and to a larger degree than when only lump-sum taxes increase: The rise in inflation π_t, together with sticky wages, leads to a large increase in the value of filled vacancies J_t^F and a large reduction private sector job destruction rate δ_t^P, due to its more elastic nature in France, which also exceeds the one seen when only lump-sum taxes increase. So initially the *Aggregate Demand Effect* is weaker but quickly becomes bigger than the lump-sum tax case leading to a larger increase in private sector vacancies v_t^P and market tightness θ_t. In turn, private sector employment N_t^P, inflation π_t and private sector firms' output Y_t^P increase more, resulting in a large increase in aggregate output Y_t, larger than under lump-sum taxes.

The *Unemployment Risk Effect* in Panel I is also larger now: The unemployment risk of workers households and demand for precautionary savings go down, so the interest rate for household bonds R_t aggregate demand and inflation π_t rise. As the *Aggregate Demand Effect* and *Direct Effect* are bigger now, private sector vacancy investment goes up and the private sector job destruction rate decreases even more. This further raises private sector employment N_t^P, private sector firms' output Y_t^P and wages and increases aggregate output Y_t even more. The overall effect is larger because we see in Panels J to L that increasing public sector firms' output Y_t^G in France now leads to a much larger increase of aggregate output Y_t at 0.190% with crowding in of private sector firms' output Y_t^P.

The *Aggregate Demand Effect* in the US is less responsive. The *Redistribution Effect* is smaller now so the *Labour Market Effect* has a bigger impact at first, and the *Indirect Aggregate Demand Effects* are larger and longer lasting as in France. However private sector vacancy investment i_t is more elastic in the US, so the difference in its increase is very small; similarly the reduction in the private sector job destruction rate is practically the same, because it is very inelastic while wages are extremely elastic. As a result the increase in private sector vacancies

v_t^P, aggregate demand, inflation π_t, private sector firms' output Y_t^P and aggregate output Y_t is marginally larger now, but is still longer-lasting.

The *Unemployment Risk Effect* is only slightly bigger now: Unemployment risk of workers households and demand for precautionary savings decreases so the interest rate for household bonds R_t and inflation π_t increase, but given the *Aggregate Demand Effect* the change is very small. As a result investment in private sector vacancies private sector employment N_t^P, private sector firms' output Y_t^P and aggregate output Y_t increase by a very small amount. Therefore the overall effect in the goods market is mostly unchanged: Aggregate output Y_t increases as much as in the case of lump-sum taxes 0.118% and private sector firms' output Y_t^P increases slightly less, but both effects now last longer.

C.2.3 Multiplier Analysis

I finish my analysis with the fiscal multipliers. First I calculate the unemployment multiplier, which measures the reduction in unemployment when public sector employment increases and then use the change in public sector firms' output created by the public sector employment shock to calculate the aggregate output multiplier, which is the increase in aggregate output when public sector firms' output increases. As in Hagedorn et al. (2019) and Monacelli et al. (2010) I calculate the cumulative multiplier, which gives me discounted percentage change in aggregate output (unemployment) relative to the discounted percentage change in public sector firms' output (public sector employment):

$$\frac{\sum dY_t}{\sum dY_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t - Y}{Y} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{Y_t^G - Y^G}{Y^G} \right\}} \frac{Y}{Y^G} \tag{C.1}$$

$$\frac{\sum du_t}{\sum dN_t^G} = \frac{\sum_{t=0}^{T} \beta^t \left\{ \frac{u_t - u}{u} \right\}}{\sum_{t=0}^{T} \beta^t \left\{ \frac{N_t^G - N^G}{N^G} \right\}} \frac{u}{N^G} \tag{C.2}$$

Table C.5: Public Sector Employment Multipliers

Country	Debt Financing Scheme	Aggregate Output Multiplier	Unemployment Multiplier
France	Lump-Sum Taxes	0.7635	-1.0374
	Labour Income & Lump-Sum Taxes	1.4226	-1.5482
United Kingdom	Lump-Sum Taxes	0.5033	-0.8174
	Labour Income & Lump-Sum Taxes	0.5033	-0.8174
United States	Lump-Sum Taxes	0.9801	-1.2718
	Labour Income & Lump-Sum Taxes	1.0903	-1.3755

Results are in Table C.5. The unemployment multipliers remain large numbers above unity in France and the US so increasing public sector employment can reduce unemployment and maintain employment stability, even in the pre-Covid case. The effect remains stronger in the US (-1.2718) when only lump-sum taxes on capitalists households increase, followed by France at -1.0374 as the labour market and public sector traits in the US reinforce the positive aggregate demand effects while mitigating any negative effects. Comparing these numbers with the post-Covid case, the multiplier is now slightly larger in France while for the US it is considerably smaller.

The effect is again strengthened when both taxes increase and the multiplier is larger for France (-1.5482), compared to the US (-1.3755), so again the tax policy impacts on the effects and propagation mechanisms of increasing public sector employment, especially in France where the private sector job destruction rate is more elastic and private and private sector wages more sticky. However the effect is quantitatively smaller than the post-Covid case in both countries because investment in private sector vacancies is more inelastic in the pre-Covid case.

Results are similar in the goods market for France which has a multiplier smaller than unity when debt is repaid only with lump-sum taxes. However, when both taxes rise, increasing public sector employment and public sector firms' output in France leads to a much larger increase in aggregate output with a multiplier larger than unity at 1.4226 and results in a strong crowding in effect. On the other hand the US has a multiplier above unity when both taxes rise, but now its a smaller value of 0.9801 when only lump-sum taxes rise. The findings again indicate changes in tax policy lead to different results, and that each country's public sector size and labour market traits interact with the tax policy in different ways. In addition these findings indicate that the effects of increasing public sector firms' output are quantitatively larger for the post-Covid sample.

Contrary to my baseline results, the UK economy now operates in a sort of middle ground in terms of multiplier size between France and the US, unlike the post-Covid case where the aggregate output multiplier was a negative number. This change occurs because the pandemic altered the labour market by making investment in private sector vacancies and private sector wages a lot more inelastic.