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On-the-job wage dynamics

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ABSTRACT

This paper assesses wage setting and wage dynamics in a search and matching framework where (i) workers and firms on occasion can meet multilaterally; (ii) workers can recall previous encounters with firms; and (iii) firms cannot commit to future wages and workers cannot commit to not searching in the future. The resulting progression of wages (from firms paying just enough to keep their workers) yields a compensation structure consistent with well established but difficult to reconcile observations on pay dynamics within jobs at firms. Along with wage tenure effects, serial correlation in wage changes and wage growth are negatively correlated with initial wages.

1. Introduction

How do firms and employees agree wages? Evidence from internal labour markets and from worker-firm matched data reveals that the job alone does not determine compensation. Instead, a rich and dynamic picture of pay emerges. In particular,

- similar workers in the same position are not paid the same wage[2];
- job tenure generally has a positive impact on wages although nominal wage cuts occur with regularity[3];
- serial correlation occurs in wage changes so that there are predictable winners and losers[4];
- initial labour market conditions matter such that cohorts who earn more on entry maintain their advantage through time — after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage.[5]

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2 See Mortensen (2005) for an overview on wage dispersion. Baker et al. (1994a) find a strong individual component to pay determination. Job levels or positions are important to compensation, but there is also substantial individual variation in pay within levels as well as in the growth rate of pay. There are likewise large overlaps in pay across levels. Wage jumps at promotions are much smaller than differences in mean pay across levels.

3 Elsby and Solon (2019) review the evidence from worker-firm administrative data across multiple countries and find that between 10% and 25% of job stayers experience a year-on-year wage cut. See also Baker et al. (1994a), Baker et al. (1994b); McLaughlin (1994); and Card and Hyslop (1997).

4 See Baker et al. (1994b); Lillard and Weiss (1979); and Hause (1980).

5 Baker et al. (1994a), Baker et al. (1994b) find that after controlling for composition differences, the progression of a cohort's wage depends in part on the average starting wage. See also Kahn (2010); Oyer (2006); and Oreopoulos et al. (2004), Martins et al. (2012). Although their findings are somewhat different, Beaudry and DiNardo (1991) also report that cohorts matter.

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These observations are challenging to collectively obtain in competitive labour models, in models of internal labour markets, and in standard job search models.

To account for these findings on wage dynamics within an employment spell, this paper uses job search frictions in a market without worker and firm commitments.[6] Search as specified in this paper differs, however, from conventional 'black-box' random matching as well as from directed search frictions. Although the labour search literature claims numerous insights and successes (such as generating equilibrium wage dispersion among similar workers), it does not readily yield a sufficiently rich pattern of compensation over time for a particular worker-job match. This incomplete picture may stem from the underlying specification of search frictions rather than from the general search and matching approach. This paper therefore adopts an alternative matching specification, the stock-flow specification, which offers a plausible, empirically valid microfoundation for search frictions and matching dynamics.[7]

The stock-flow matching framework posits two natural as well as relatively novel features of job search. In particular, workers and firms

i. on occasion (but not every time) encounter each other multilaterally during job search[8]
ii. can remember past encounters.[9]

In a labour market with stock-flow matching, when a seller, i.e. a worker, goes on the market in search of a partner, he or she immediately becomes fully informed about the number of suitable buyers in the stock, i.e. the stock of job vacancies. If lucky, the worker finds several viable options and matches quickly. If the worker is unlucky, the market turns up few (more precisely only one) or possibly no viable opportunities. In the event that no acceptable vacancies exist in the marketplace, the worker must wait (possibly alongside other workers seeking similar work) to match with the flow of new jobs.

Recall of past encounters follows from the full revelation of all opportunities and competition in the marketplace. Matched and unmatched traders remember their past marketplace experiences. As time proceeds, the gradual flows in and out govern the number of traders active in the marketplace so that unmatched workers who constantly visit the market anticipate and find only negligible changes, if any, between one day and the next.[10]

Employed workers can likewise recall their most recent visit to the job market and who was there at the time. On the other hand, they are unaware of the intervening turnover since that last visit occurred. As job opportunities and competition turn over, the worker and the employer will not directly observe this turnover unless the worker actively engages in job search. If they have not visited the market since matching, the market may have changed, at random, more profoundly over a substantial period of continuous employment.

Consider wage determination under stock-flow job search with complete recall when firms cannot commit to future wages and workers cannot commit to not search while employed.[11] After job search reveals the number of currently available jobs, all suitable firms bid for the worker's services. If the worker finds that only one job option is currently available, the firm offers a monopsony payoff that claims all of the gains to trade for the firm. On the other hand, with more than one firm involved, the firms engage in competitive Bertrand bidding. This time, the worker extracts the gains to trade. At the outset of the employment relationship, wage dispersion obtains and depends on the number of competitive bidders found at that time.[12]

Now suppose that at any time after a firm and worker pair up, the firm can update its offer. In other words, as the firm cannot commit to future wages, a new wage is offered in each instant. The worker can either accept the latest offer or go again to the market to elicit bids. Although the pair perfectly remember their last visit, they do not know what has happened in the market since that visit. The firm updates its wage offer knowing that as time proceeds, firms and workers come and go and the number of prospective bidders in the market evolves randomly. The worker must physically visit the market to learn the actual number of bidders currently in the market.

This process provides a new source of wage progression with job tenure at a firm. Employers will want to avoid bidding with the (anticipated) firms in the market and keep the worker away from the market with a sufficiently high wage offer. Such an offer

6 Waldman (2012) reviews the literature on internal labour markets and considers a variety of explanations for wage dynamics based on imperfect information linked to human capital acquisition, job assignment, learning and tournaments. These explanations offer insights but abstract from some broad market considerations of competition.

7 The matching framework used here is most closely related to the matching models of Taylor (1995), Coles (1999) and Lagos (2000). Emerging empirical evidence indicates this framework has more validity than random matching. See Coles and Smith (1998), Petrongolo and Pissarides (2001), Andrews et al. (2013), Gregg and Petrongolo (2005), Coles and Petrongolo (2008), and Kuo and Smith (2009).

8 Looking at hires from unemployment, Guo (2022) finds that "over 30% of [workers] had multiple offers simultaneously just before starting ... and relative to workers with one offer, comparable workers with multiple offers enjoy a persistent wage premium of over 10% for about nine years".

9 Carrillo-Tudela and Smith (2017) find evidence that displaced workers recall previous contacts.

10 In practice, recall of even recent experiences may be limited to some extent as some potential traders may intermittently, not constantly, visit the marketplace. Like the full revelation of opportunities and competition, complete recall is a strong but useful abstraction.

11 Taylor (1995) and Coles and Muthoo (1998) examine wages in this set-up without on-the-job search. More recently, Bradley and Gottfries (2021) embed exogenous job search while employed into job search model the embeds stock-flow matching. In their model, firms commit to a wage schedule thereby complementing the results here.

12 Faberman and Menzio (2018) document not only that the arrival of applications is consistent with the duration dependence matching pattern from stock-flow mechanics but also that starting wages are in line with the determination process adopted here. In particular they find that "vacancies that are filled within 1 week receive a higher number of applications per week than vacancies that are filled in 2 weeks, which, in turn, receive a higher number of applications per week than vacancies that are filled after a month. "In addition, the "starting wage ... is positively related to the duration of the vacancy, negatively related to the number of applicants to the vacancy, and negatively related to the number of candidates interviewed for the vacancy."

outbids the evolving threat of the worker sampling the market, not the actual firms at the job centre. The firm's wage offer thus inhibits potential job-to-job transitions. The resulting progression of wages from firms paying just enough (in a specific time period) to keep their workers from searching while employed yields a compensation structure consistent with the above findings.

No-search wages face two countervailing forces from turnover in the market. Previous bidders gradually leave the market and new options enter the market. Outside options therefore can rise or fall depending on this birth and death process. Wages not only differ at the outset, they also evolve in different patterns. For monopsony wages, the unfortunate history (from the worker's perspective) fades and the outside option improves. Low initial wages rise over time. For competitively bid wages, the more favourable history that led to high initial wages fades and eventually a less attractive expectation of the number of new firms matters more. Although wages start at different points and evolve in different patterns, they ultimately converge with sufficiently long tenures.

Job availability and (unobserved but anticipated) turnover in the marketplace jointly determine wage dynamics and wage dispersion. Initial wages and their subsequent progression at a job within a firm combine to create a distribution of wages at a point in time. Although it is difficult to formulate an explicit expression for the distribution, numerical methods reveal sensible shapes for a range of parameters. In a homogeneous environment, the cross section of wages is dispersed around an interior mode with prominent tails on both sides. The model can also generate reasonable mean-min ratios and thus overcome the lack of frictional wage dispersion found in standard search models by Hornstein et al. (2011).

The key determinant of compensation is the expected payoff from the threat of search. As an employment spell progresses, the search option evolves thereby driving the results without accompanying job-to-job mobility. Potential competition drives wages but frictions limit its full scope. As in Yamaguchi (2010) and Bagger et al. (2014) the outside option evolves as potential partners come and go but unlike those papers, firms react to the threat of search rather than the trigger of an actual job offer for renegotiation. Because stock-flow matching in effect builds in duration dependence, the evolving threat of search while employed and not its realization determines wages. As a result, turnover is less pronounced.

The next section describes the general framework and the process governing vacancy turnover. Section 3 describes the worker's and firm's decisions in this economy. Section 4 derives the payoffs in the job centre as workers and firms search. Section 5 derives payoffs for existing worker-firm pairings, and Section 6 derives wage dynamics. Section 7 describes a numerical example of the wage progression, wage dispersion, and the impact of job tenure. The last section concludes.

2. Economic environment

Building on Taylor (1995) and Coles and Muthoo (1998), this section develops a stock-flow matching model of the labour market in which workers cannot commit to not search in the future and firms cannot commit to wages in the future. To begin suppose homogeneous workers and homogeneous firms populate an economy with a small, highly specialized labour market. Both agents are risk neutral, discount the future at rate $r > 0$, and maximize expected lifetime payoffs.

The economy operates over an infinite sequence of discrete time periods of length $dt > 0$. Each time period consists of two sub-periods - an internal labour market stage and a job search stage. At the start of time ($t = 0$) the economy is empty with entry occurring randomly over time.

At any point in time, a worker is said to be "attached" to a particular firm's job if the worker produced output for that firm in the previous period. If the worker did not produce output for a firm in the previous period, the worker is unattached or equivalently unemployed and actively looking for a job. A firm without an attached worker is a vacancy that is also actively looking to recruit a worker. Unemployed workers receive flow payoff $b\,dt$ per period. When a worker agrees to produce for a firm, the worker generates output $x\,dt > b\,dt$. To keep the exposition and notation uncluttered, workers and jobs live forever.[13]

2.1. Sub-period activity

In the first sub-period (the internal labour market stage), an attached firm offers its worker a wage $w\,dt$ in the current period. The worker then either accepts or rejects this wage. A worker who accepts the offer receives the wage payment, generates the per period output, and remains attached to the firm as they both move on to the next time period.

New firms and new workers then enter the economy at the start of the second sub-period. As time progresses, new workers individually enter the economy (between the first and second sub-period) at the constant, exogenous Poisson rate $\alpha > 0$. For dt small, $\alpha\,dt$ is the approximate probability that a new worker enters in period t. Likewise, new firms each with a single, indivisible job vacancy enter in the same manner and at the same rate but independently of workers.[14] Over time the population in the economy is therefore balanced with equal expected numbers but at any given point in time there may be either more workers or more firms.

These new entrants enter the job search stage and visit the labour market looking for partners. For simplicity, unemployed workers pay no search costs to visit the market. A worker who rejected the internal wage offer likewise enters the labour market in this second stage of the time period. The rejected firm goes to the market as well.

[13] Job destruction shocks can be incorporated (death and discounting are related) but some caution would be needed. A familiar approach specifies that workers become unemployed whereas firms leave the market following a job destruction shock. Given equal and exogenous arrival rates, this specification would lead the number of workers growing unboundedly higher than the number of firms. Endogenous firm entry would, of course, remedy this difficulty. This paper abstracts from worker-firm separations and from an endogenous number of vacancy/firms as in Pissarides, 2000.

[14] As firm entry is exogenous, job vacancy costs do not mediate the firm's participation decision. Without this role, flow search costs for vacant jobs are set to zero.

During this stage, an attached worker pays a search cost $\xi \in (0, b/r)$ to participate and solicit alternative offers in the labour market. By rejecting an offer, the attached worker in effect chooses to enter the job market and check the posted list of vacancies, if any. The worker is said to search by sampling the market (as detailed below) while the attached firm remains a feasible employment option. Moreover, because an attached worker who accepts a first stage offer does not visit the labour market, the worker is unable to search while employed without the firm becoming aware of this activity. In effect, if a worker in any period would like to try their luck in the labour market, the paired firm becomes aware of this activity before per period production takes place. The firm fully observes the search activity of an attached worker and can modify its first stage pay offer in the second stage.

2.2. Stock-flow matching

Following the stock-flow matching approach (see Smith, 2020 for an overview) information about the availability of firms and workers in the job search stage is centralized. Unemployed/unattached workers including any entrants as well as offer-rejecting workers all register their availability at a marketplace such as a job centre, a website, or some other established platform as soon as they enter the market looking for partners. Vacant and new entrant jobs along with any rejected firms similarly post the availability of their employment opportunities. The firm maintains their listing until the job hires a worker.

Agents in this centralized marketplace are perfectly informed about all available trading opportunities that have registered there. When any worker enters the marketplace, he or she immediately observes the number of vacancies in the market as well as any other workers in the job centre. After the worker checks the list of posted vacancies, there are no frictions or delays in processing the information. All information regarding the viability of a position is immediately made clear and common knowledge at the job centre.

Given the number workers and jobs in the marketplace, a complete information, competitive auction occurs. Each worker in the marketplace auction pool observes a proposed offer from each firm in the auction pool, including all of the just rejected firms who can update their internal first sub-period offers. Firms post their offer knowing the number of workers and number of competing firms in the market in the second stage of the current period. Although all decision making is based on expectations of future behaviour, offers are for only the current period. Firms cannot commit to future wage payments in their offers. Workers likewise cannot commit to withholding future search for other employers. For entrant workers, the acceptance of an offer thus corresponds to their initial compensation.

In this auction, the process of pairing workers and jobs occurs within the second stage of the period. One by one, nature arbitrarily chooses a worker from the auction pool who either selects and accepts one of the posted offers or passes on all offers. All of the other agents in the auction pool observe if a worker and firm pair together. A matched pair both leave to immediately produce output and transfer the agreed payment. The process continues until all workers have had a chance to pair up or there are no more offers available. Unmatched workers and firms remain behind as unemployed workers and vacant firms who both wait for further trading opportunities.

Fig. 1 illustrates the events of the two sub-periods. The sequential selection of workers in the second sub-period coordinates trade and avoids the complications associated with two workers selecting the same job as in the directed search models of Burdett et al. (2001) and Albrecht et al. (2006). In the second sub-period stage, there are no impediments to trade such as coordination frictions from either new born entrants, unattached agents, or attached agents. As will be seen the entry process of workers and firms in this economy coupled with stock-flow matching undermines the simultaneous build-up of traders on both sides. As such the sequential pairing in artificial time within a period addresses events off the equilibrium path.[15]

The stock-flow trading structure of the auction pool in the second sub-period implies contacts and agreements between workers and firms occur within a single time period. Given these mechanics of search, the accompanying search costs ξ differ from the flow costs and benefits associated with the on-going activities of production x, payment w, and unemployment b that contribute nonzero payoffs over a measure of time. These job search costs (which may be small) are paid and shift payoffs at only a (measure zero) moment of time. Attached worker search activity becomes compressed as dt becomes small so that the search costs could reasonably reflect a mass point cost on the order of values of payoff functions. Although the notation and derivations adopt this Dirac delta type specification, the arguments and proofs hold if attached worker search costs were a flow. The cost would become $\xi\, dt$ in Proposition 4. In Proposition 6 as well as in equation (5), ξ would vanish to zero and have no effect on equilibrium wages. Proposition and 7 would be redundant. Otherwise the results remain intact.

2.3. Beliefs

When an attached firm makes its internal market stage wage offer and when the worker subsequently decides to accept or reject the attached firm's offer, they are both unaware of the entry of any workers and of jobs since they last visited the market place. In particular, since the date the two first became attached or since the last time the worker searched on the job, whichever is shortest, the Poisson arrival processes govern their beliefs about what other agents they expect to encounter in the job centre. Thus, the attached firm and worker beliefs in the stages before search are based on the workers and firms that were there at the last visit and the duration since its last visit.

[15] See Peters and Severinov (1997) and Albrecht et al. (2006) for more on competitive auctions.

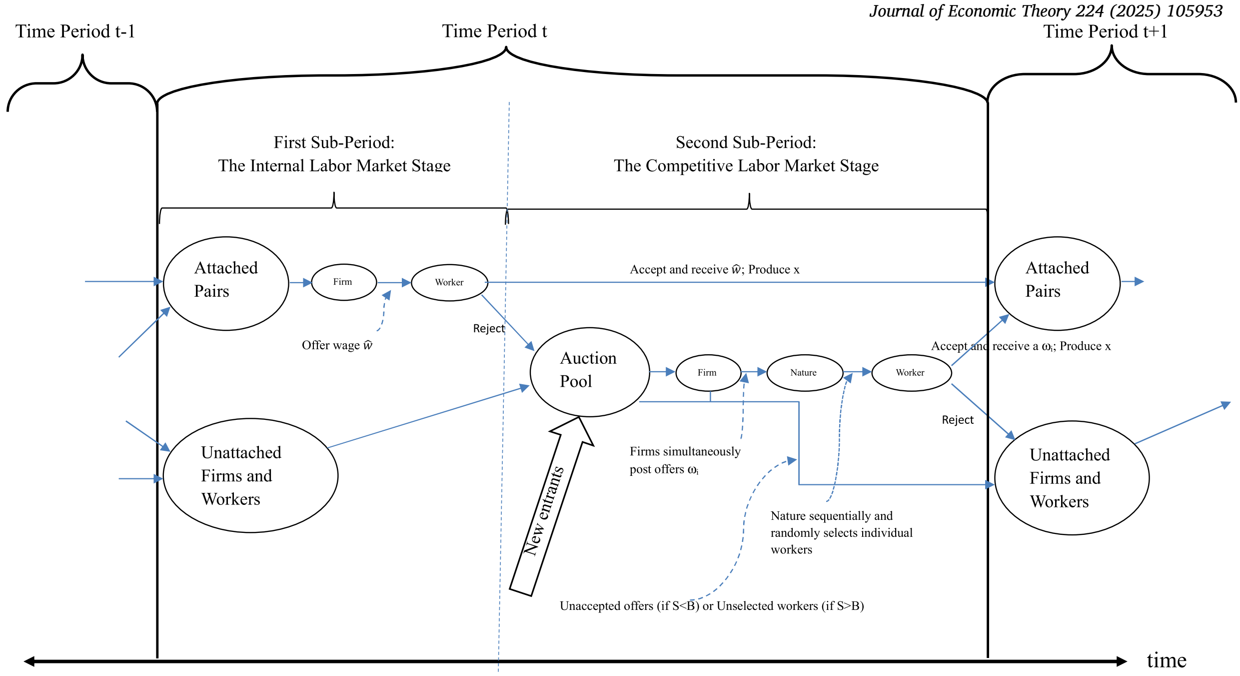

Fig. 1. Time Line.

Since workers enter at Poisson rate α, workers and firms share the belief that the probability of i new entrants over a duration τ since the last auction is given by

$$\frac{e^{-\alpha\tau}(\alpha\tau)^i}{i!} \tag{1}$$

A symmetric belief applies to the number of new jobs that entered. Visiting the market reveals all past entry information completely and the worker as well as the firm update their beliefs accordingly.

3. Decisions within periods

Focusing on symmetric, pure Markov strategies, this section describes (in recursive order) sub-period strategic behaviour and establishes that at any point in time, immediate trade occurs so that at the end of the period unattached workers and unattached firms do not both simultaneously exist in the market.

3.1. Second stage job search

Suppose there are $B \geq 1$ firms bidding for $S \geq 1$ workers in the open labour market auction. Since agents are homogeneous, the outcome is well understood. The optimal strategy for a worker on the short side contemplating a bid involves a reservation wage strategy - the worker will accept the highest offer (or will randomly select among the highest offers) provided that the offer yields a discounted expected payoff greater than or equal to the continuation value of waiting as an unemployed, unattached worker in the job market. Similarly, when there are more workers than firms in the auction, a firm will likewise have a threshold payoff to having a worker accept its offer. This payoff must be less than or equal to the value of being a vacancy. These continuation payoffs depend on the expected number of traders in future periods.

Given these strategies, the short side of the market determines the outcome of the auction in a second stage of activity. If $B > S$, firms bid up to their threshold bids thereby making them indifferent between hiring and having an open position at the start of next period. Provided there are gains to trade, these bids exceed the worker's reservation payoff and workers are willing to accept these bids. All workers are hired leaving $B - S$ vacancies indifferent between waiting and hiring. On the other hand, if $S \geq B$, firms offer the worker's reservation payoff. The worker accepts leaving $S - B$ unemployed.

Immediate trade emerges so that unattached workers and unattached workers do not coexist entering the next period. Moreover, the outcome is the familiar Bertrand result leaving the long side indifferent between being attached or unattached. Basic induction demonstrates that this indifference generates payoff equivalence to adding a buyer-seller pair. The following proposition summarizes these observations.

Proposition 1 *(Immediate trade). Given $B \geq 1$ jobs and $S \geq 1$ workers in the second stage auction pool,*

i. *if* $S \geq B$*, all* B *firms offer the workers' their reservation payoff. A subset of* B *workers (who are indifferent between employment and waiting unattached) accept this offer.*
ii. *if* $S < B$*, at least* $S + 1$ *firms competitively bid such that all firms are indifferent between hiring and waiting unattached with an open job vacancy. All other firms offer the same or lower payoff. All* S *workers accept one of these competitively bid offers.*
iii. *immediate trade occurs –* $\min\{B, S\}$ *worker-firm pairs form leaving* $\max\{B - S, 0\}$ *buyers and* $\max\{S - B, 0\}$ *sellers waiting to find partners.*
iv. *payoff equivalence occurs – an auction with* B *jobs and* S *workers yields the same worker and firm payoffs as an auction with* $B + 1$ *jobs and* $S + 1$ *workers.*

Proof. See Appendix. □

It is important to note that the payoffs to going forward are contingent on net agent entry, in this case the difference between B and S, not the actual levels of buyer and seller entry.[16]

3.2. First stage internal labour market

In the first stage, an attached worker again adopts a reservation wage strategy for accepting the associated firm's internal offer and forgoing job sampling in the next sub-period. Given this reservation wage in the first stage, the attached firm's choice is effectively whether to offer this first stage reservation wage. Anything above this threshold lowers profit and anything below results in moving to a second stage auction. As the worker rejects any offer below its no-search threshold wage, any offer below this reservation wage triggers a second stage visit to the market.

4. Payoffs in the job centre

Proposition 1 describes (without completely characterizing) the outcome of the second sub-period labour market auction. This section completes the characterization of the job centre outcomes by deriving the lifetime discounted expected payoffs for workers and firms as they enter (or if they already exist unattached in) the auction pool market.

From Proposition 1, immediate trade occurs in the second sub-period so that unattached jobs and unemployed workers do not coexist over time. From point [iii.] only unattached workers or only unattached firms carry over into the next period. Moreover, point [iv.] implies that the job search decisions of other attached worker-firm pairs do not affect the payoffs in the second sub-period. Adding another worker along with another firm is payoff equivalent.

Given payoff equivalence, the payoff relevant active agents in the second sub-period are those who entered in previous periods but are without an attached partner along with any new entrants this period. Moreover, payoffs can be derived from auctions with only one lone trader matching with one or more potential partners on the other side of the market. Therefore, let

$$N_t = Stock\ of\ workers\ who\ entered - Stock\ of\ vacancies\ that\ entered$$

denote the history of net agent entry up through date t. N_t represents <u>either</u>

- the number of traders in competition on the same side of the market

<u>or</u>

- the number of potential partners on the other side.

Bids and accepted offers in both sub-periods will depend on this history of market entry. If $N_t \in \mathbb{N}^+ = \{1, 2, 3, \ldots\}$, there is at least one unemployed worker on the long side of the market waiting for a firm to enter the market with a vacancy. Moreover, immediate trade in the second period implies there are exactly $\max\{0, N_t\}$ workers and $-\min\{0, N_t\}$ vacant jobs waiting in the marketplace carried over across time periods.

When a new firm enters the market at date t, history changes so that $N_t = N_{t-dt} - 1$. If at date $t - dt$ there are at least two available workers waiting, then at date t when the second stage auction bidding takes place, the entering vacancy has at least $N_{t-dt} = N_t + 1$ potential workers. Thus, for this case, accounting for the change in history of firm entry, we have $N_t \in \mathbb{N}^+ = \{1, 2, 3, \ldots\}$.

Since immediate trade occurs in the second sub-period, two relevant cases arise for a new worker entering in the job centre:

- Long-side Case: No viable jobs are waiting in which case the worker becomes unemployed after entry.
- Short-side Case: The market has one or more excess firms available, in which case the firm or firms compete against each other (if more than one firm) for the new worker resulting in immediate employment.

[16] Coles and Muthoo (1998) demonstrate that in the stock-flow framework without on-the-job search there is a unique Markov equilibrium in which exchange occurs immediately.

Similar cases apply when a firm with a new open vacancy enters the job centre.

4.1. The long-side of the market

To derive the reservation payoffs in the second sub-period job search stage, let $V(N_t)$ denote the expected payoff to a worker waiting on the long side of the market who has $N_t - 1$ other workers competing for employment. Since there are no jobs currently available as this worker waits for jobs to appear, standard dynamic techniques imply that

$$V(N_t) = b\,dt + \frac{1}{1+rdt}[\alpha dt\, V(N_t+1) + \alpha dt\, V(N_t-1) + (1-2\alpha dt)\, V(N_t)] + O(dt^2) \qquad N_t \in N^+ \tag{2}$$

The worker receives net flow payments b while waiting. During a short interval of duration dt, a competing worker arrives with probability $\alpha\, dt$ and increases the number of available workers by one. With the same probability a firm arrives during this interval and the workers all pursue this job. Bertrand-like competition makes the worker indifferent between employment and waiting with one less competitor. If there are no other workers when a firm arrives, the worker receives the boundary payoff $V(0)$ as one firm bids for one worker.

To find the respective employer shares with two or more firms bidding, now consider the case in which firms with a vacancy are waiting on the long side of the market for a worker to enter, that is $N_t \in \mathbb{N}^- = \{-1,-2,-3,\ldots\}$. Applying the same logic, the payoff for these firms while they wait is given by

$$\Pi(N_t) = \frac{1}{1+rdt}[\alpha dt\, \Pi(N_t+1) + \alpha dt\, \Pi(N_t-1) + (1-2\alpha)\, \Pi(N_t)] + O(dt^2) \quad N_t \in N^- \tag{3}$$

where $\Pi(0)$ is the associated boundary condition with one firm bidding for one worker.

Proposition 2 *(Payoffs while waiting). The discounted expected payoffs for unattached workers and unattached firms are given by*

$$V(N_t) = \lambda^{N_t}(V(0) - b/r) + \frac{b}{r} \qquad N_t \in \mathbb{N}^+$$

and

$$\Pi(N_t) = \lambda^{-N_t}\Pi(0) \qquad N_t \in \mathbb{N}^-$$

where

$$\lambda = \frac{r + 2\alpha - (r^2 + 4r\alpha)^{1/2}}{2\alpha}$$

The boundary conditions are

$$V(0) = \frac{\alpha(1-\lambda)(x+b) + rb}{r(r + 2\alpha(1-\lambda))}$$

and

$$\Pi(0) = x/r - V(0)$$

Proof. See Appendix. □

4.2. The short-side of the market

If an attached worker-firm pair visit the market (that is, if the worker samples the market at some point in the relationship), the pair can re-attach with a renegotiated wage reflecting the newly updated circumstances revealed at the job centre. Re-attachment is not only feasible, it is also as good as any other available attachment. Re-attachment can thus constantly recur making the match indefinite. The joint match payoff for an infinitely-lived pairing equals total discounted production x/r. There is no job creation margin so the wage simply divides match rents.[17]

Proposition 3 *(Auction payoffs from competing agents). The firm's share in auctions with more than one worker is given by*

$$\Pi(N_t) = \frac{x}{r} - V(N_t) \qquad N_t \in \mathbb{N}^+$$

[17] The critical point is that the match payoff is constant over time. It is readily seen that Proposition 5 holds for any constant match payoff. An alternative approach for establishing the results is to impose no job sampling and then demonstrate that this choice is optimal.

Workers payoff from multiple competing firms in an auction is given by

$$V(N_t) = \frac{x}{r} - \Pi(N_t) \qquad N_t \in \mathbb{N}^-$$

Proof. Follows from the splitting of the match payoff x/r and the long side payoffs in Proposition 2. □

5. Attached workers and firms

This section turns to the strategies and payoffs of workers and firms who enter the first sub-period of time period t attached to each other. The section first establishes the joint worker-firm payoff to remaining in this match forever. The next step derives Proposition 4 which describes the worker's expected payoff to job search (i.e. not accepting the internal wage offer and visiting the market) given the state of the job market when the matched formed and the time elapsed since that formation. The third step establishes in Proposition 5 that the attached firm's internal wage offer makes the worker indifferent between searching and not searching.

Once a worker and firm join together in employment, the joint payoff to a match is x/r. The firm's wage payments over time (offered and accepted during the first sub-period) determine the allocation of this joint payoff as they share the value of the match. A wide variety of compensation schemes - that is promised, committed payments over time paid in the first sub-period coupled with a commitment not to search - can deliver these shares so wages are indeterminate when commitment is feasible. This section establishes that the constant threat of job search during the match along with the inability to commit to future wage payments pins down wages in the internal labour market of the first sub-period.

In the first stage of each period, an attached firm simply chooses between offering their attached worker's (first sub-period) reservation wage at the time or inducing job search with a lower offer. If search occurs, the outcome is common knowledge - both the worker and the firm become informed about the number of available employment opportunities for the worker in the job centre.[18] Given the number of viable opportunities found in the job centre, a new auction results and the new re-negotiated wage depends on the number of searching workers and vacancies in the job centre at that moment of job search.

5.1. Job search

An attached pair does not observe entry or turnover in the job centre until they visit the job centre. Expectations of finding potential partners and the payoff to job search therefore depend on whom they last saw there (the known participants from the auction that last attached the worker to the firm), and the duration (determining the expected flows in and out of participants) since that auction.

More specifically, at date t, the expected payoff to job search for an attached worker who

- last visited the job centre a duration $\tau > 0$ ago
- was hired at the job centre when the history of net entry at that time was $N_{t-\tau}$

is given by

$$W(t; N_{t-\tau}, \tau) = -\xi + \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t-\tau}, \tau)$$

where $f(k; N_{t-\tau}, \tau)$ is the probability of observing history k given a duration τ since initial history $N_{t-\tau}$. Let $F(k; N_{t-\tau}, \tau)$ denote the cumulative distribution for f.

The Poisson arrival processes govern turnover (unobserved during attachment duration τ) in the job centre. The (Poisson) number of unobserved workers less the (Poisson) number of unobserved vacancies follows a Skellam distribution. See Irwin, 1937, Skellam, 1946.[19] For $N_{t-\tau} = 0$, the probability of observing a history of new net entry of k after duration τ at date t is given by

$$\Pr(N_{t+\tau} = k \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha t} \sum_{j=\max\{0,-k\}}^{\infty} \frac{(\alpha t)^{2j+k}}{j!(j+k)!}$$

[18] Common knowledge rules out the possibility that a worker visits the job centre and calls for an auction only if conditions are favourable. As demonstrated below the firm can infer worker behaviour from its wage offer.

[19] This probability can also be expressed for any k using a modified Bessel function of the first kind

$$\Pr(N_{t+\tau} = k \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha\tau} I_k(2\alpha\tau)$$

where $I_k(z) = I_{|k|}(z)$. Alternatively,

$$\Pr(N_{t+\tau} = k, k > 0 \mid N_t = 0) = f(k; 0, \tau) = e^{-2\alpha t} \sum_{j=k}^{\infty} \frac{(\alpha t)^{2j-k}}{j!(j-k)!}$$

with negative values for k found by the symmetry of Skellam distribution.

The law of motion for f

$$f(N_t;N_{t-\tau},\tau+d\tau)=(1-2\alpha d\tau)f(N_t;N_{t-\tau},\tau)$$
$$+\alpha d\tau\, f(N_t+1;N_{t-\tau},\tau)-\alpha d\tau\, f(N_t-1;N_{t-\tau},\tau)$$

implies

$$\dot f(N_t;N_{t-\tau},\tau+d\tau)=\alpha f(N_t+1;N_{t-\tau},\tau)-2\alpha f(N_t;N_{t-\tau},\tau)+\alpha f(N_t-1;N_{t-\tau},\tau)$$

Proposition 4 *(Attached worker's expected payoff from search). The worker payoff to search while employed is*

$$W(t;N_{t-\tau},\tau)=-\Pi(0)\sum_{i=1}^{\infty}\lambda^i f(-i;N_{t-\tau},\tau)$$
$$+(V(0)-b/r)\sum_{j=0}^{\infty}\lambda^j f(j;N_{t-\tau},\tau)$$
$$+F(-1;N_{t-\tau},\tau)(x-b)/r+b/r-\xi$$

which evolves according to

$$\dot W(t;N_{t-\tau},\tau)=\frac{\alpha(1-\lambda)^2}{\lambda}\left[-\Pi(0)\sum_{i=1}^{\infty}\lambda^i f(-i;N_{t-\tau},\tau)\right.$$
$$\left.+(V(0)-b/r)\sum_{j=0}^{\infty}\lambda^j f(j;N_{t-\tau},\tau)\right]$$

Proof. See Appendix. □

5.2. Accepting an internal offer

The worker's payoff from not going to the market at any point in time t depends on the first sub-period internal wage offer at the time. Suppose a firm offers the instantaneous wage $\hat w(t;N_{t-\tau},\tau)\,dt$ (for the current interval of duration dt) to its worker from the last period. This worker has a duration $\tau>0$ of continuous employment without an intervening visit to the job market since attachment began at date $t-\tau$, at which time it had a history of net entry $N_{t-\tau}$.

Let $E(t;N_{t-\tau},\tau)$ denote the expected payoff at date t to employment in the first sub-period given the relevant history and duration. If the worker accepts the current internal wage offer and decides not to search at this point in time, it follows that the worker's expected payoff is this period's wage offer plus the discounted payoff of either search next period or continued attachment next period, whichever is larger:

$$E(t;N_{t-\tau},\tau)=\hat w(N_{t-\tau},\tau)dt$$
$$+\frac{1}{1+rdt}\max\{E(t;N_{t-\tau},\tau+dt),W(t;N_{t-\tau},\tau+dt)\}+O(dt^2).$$

Manipulating and letting dt become small gives

$$rE(t;N_{t-\tau},\tau)=\hat w(t;N_{t-\tau},\tau)+\max\{\dot E(t;N_{t-\tau},\tau),\dot W(t;N_{t-\tau},\tau).\tag{4}$$

A firm can clearly offer a sufficiently high wage such that $E(t;N_{t-\tau},\tau)\geq W(t;N_{t-\tau},\tau)$. In this case, because search activity is observable, the firm effectively bribes the worker to not to visit the job centre after duration τ. Moreover, if the firm chooses to offer such a no-search wage, the firm would optimally offer the lowest possible wage so that the no search condition holds with equality

$$E(t;N_{t-\tau},\tau)=W(t;N_{t-\tau},\tau).$$

Proposition 5 *(Wage offers to attached workers). In the first sub-period, an attached firm offers its worker the worker's reservation wage $w(t;N_{t-\tau},\tau)$ for all t which the worker always accepts.*

Proof. See Appendix. □

Given there is a positive cost of visiting the job centre, it is efficient for the worker and the firm to save the search cost and split the match payoff within the current employment match at any given τ. In this economy, job search is a wasteful, rent seeking activity. Once a match is formed, search does not generate any further gains to trade (such as finding a better match) or match specific rents. The relationship does not fundamentally change when the worker visits the job centre. There are no new opportunities generated by a visit - existing opportunities are merely realized. Search does not change the expected gains to trade at any given point in time, it just reallocates the division of these benefits. Since workers and firms share the same risk neutral, intratemporal preferences, and since all firms are identical, there is no potential role for meaningful job search.

6. Per period wages

This section first defines an equilibrium. Propositions 6, 7, and 8 then completely describe the first and second sub-period accepted equilibrium wage offers as functions of parameters and thereby characterize the unique employment oriented equilibrium.

Let $w(t; N_{t-\tau}, \tau)$ denote the lowest wage that makes an attached worker willing to forgo visiting the job centre. As time progresses, Proposition 5 reveals that subsequent first sub-period offers will continuously make the worker indifferent between accepting the internal first sub-period wage offer and search while employed. The (employment oriented) worker accepts the internal offer and remains attached to the firm so that

$$rE(t; N_{t-\tau}, \tau) = w(t; N_{t-\tau}, \tau) + \dot{E}(t, N_{t-\tau}, \tau)$$

Definition. An Employment Oriented, Markov Perfect Bayesian Equilibrium consists of attached worker and attached firm beliefs on new entrants, firm wage offers strategies, and worker acceptance strategies such that for all dates $t \geq 0$, for all states $N_t \in \mathbb{Z}$, and for all durations $\tau \geq 0$:

- *A*ttached workers and firms expect symmetric entry of new workers and jobs over the duration τ given in equation (1)
- *I*n the first sub-period,
  - firms offer wages $w(t; N_{t-\tau}, \tau)$ to attached workers such that workers are indifferent between accepting the offer and visiting the second sub-period auction pool
  - workers accept any wage offer equal to or above their reservation wage threshold
- In the second sub-period
  - for $N \geq 0$, offers from firm to unattached workers $w(t; N_t \geq 0, \tau = 0)$ equal the workers' second sub-period reservation threshold
  - for $N < 0$, offers from firms to unattached workers $w(t; N_t < 0, \tau = 0)$ make the firm indifferent between hiring the worker and waiting as an unattached job vacancy
  - workers accept the highest offer equal to or above their reservation payoff in the second sub-period.

The process of offering either reservation payoffs or of competitive bidding leads to the following characterization of wages that firms pay their workers after duration $\tau > 0$ given that market history $N_{t-\tau}$ at the date of hiring.

Proposition 6 *(Attached worker equilibrium wages). For $\tau > 0$, equilibrium wages are given by*

$$w(t; N_{t-\tau}, \tau) = b - r\xi + F(-1; N_{t-\tau}, \tau)(x - b)$$

Proof. See Appendix. □

The payment b is the familiar outside option often present in search models with wage bargaining. The search cost parameter ξ reflects the firm's monopsony power. If these search costs were flow costs that become vanishingly small as $dt \to 0$, at every moment the firm's internal first sub-period wage would equal the expected wage from visiting the job market. A non-vanishing search cost, $\xi > 0$, creates a wedge between the internal offer and the expected market wage which yields a simple translation down of the wage schedule.[20]

At the time of matching, the distribution $f(k; N_t, \tau = 0)$ is degenerate at the realized value of N_t so that $\lim_{\tau \to 0} F(-1; N_{t-\tau}, \tau)$ is either zero or one, depending on whether or not more than one firm is bidding. As such, initial wages depend on the short side of the market:

$$w(t; N_t < 0, \tau = 0) = \lim_{\tau \to 0} w(t; N_t < 0, \tau) = x - r\xi$$
$$w(t; N_t \geq 0, \tau = 0) = \lim_{\tau \to 0} w(t; N_t \geq 0, \tau) = b - r\xi$$

In addition, firms pay a hiring bonus. When there is a lone firm bidding (that is, for $N_t \geq 0$), the switch (at implicit duration $\tau = 0$) from costless job search as an unattached worker to costly job sampling while employed requires additional compensation in the second sub-period pay offer at the job centre. As noted, when such bidding in the second sub-period job market takes place, workers will accept offers where they are indifferent between working and waiting, implying that the payoff at the start of employment equals the payoff to waiting. To satisfy indifference and equate the payoff of visiting the market in the next instant for an attached worker with the payoff of an unattached worker requires a compensating signing on bonus equal to the search cost ξ.

[20] The payoff to employment must outweigh the payoff to non-participation. Suppose workers could quit the market. In the worst case where workers are paid $b - r\xi$ forever, workers would quit. The parameter restriction $\xi < b/r$ rules out this outcome.

Unattached workers who have more than one suitor ($N_t < 0$) receive the same initial bonus payment. In this case, the firms, not workers, become indifferent between hiring and continued search. Although the worker strictly prefers employment from Bertrand bidding over continued search, the firm who hires the worker subsequently obtains monopsony power benefits $r\xi$ per period. Firms bid for these rents and are willing to pay upto ξ in their initial hiring pay. Proposition 7 describes payments when workers are hired.

Proposition 7 *(Hiring bonus). For all N_t, firms pay newly hired workers from the auction pool an additional ξ to accept their offer.*

Proof. See Appendix. □

From Proposition 6, wages at date t, are positively related to the probability $F(-1; N_{t-\tau}, \tau)$ that the current employer will find competition for the worker's services. The worker is prepared to accept a lower wage and avoid re-negotiating the terms of employment when there is a higher probability that the employer can become a monopsonist. This probability of competition evolves over time with the duration of employment, rising or falling depending on the state when attachment first occurred.

After the first payment, wages can increase or decrease depending on the initial market conditions. The symmetry of the Skellam distribution (given equal arrival rates), implies that wages for $N_{t-\tau} < 0$ are decreasing over time ($\dot{w} < 0$) and are increasing for $N_{t-\tau} \geq 0$. If $N_{t-\tau} \geq 0$, the firm was initially in a monopsonistic position on the short side of the market. As turnover occurs in the job centre, the probability that the current employer remains a monopsonist decreases over time. The outside option of the worker therefore improves and the firm increases its wage offer to avoid the worker visiting the job centre. On the other hand, starting with multiple initial bidders, i.e. for $N_{t-\tau} < 0$, the same turnover increases the likelihood over time that the current employer could become a monopsonist. The worker's outside option and hence $w(t; N_{t-\tau}, \tau)$ decreases with τ, as the threat of any potential loss brought about by an induced visit to the job centre increases over time.

The following proposition formalizes this reasoning.

Proposition 8 *(Wage progression). Wage progression satisfies*

$$\dot{w}(t; N_{t-\tau}, \tau) = \alpha[f(0; N_{t-\tau}, \tau) - f(-1; N_{t-\tau}, \tau)](x - b)$$

Proof. See Appendix. □

Although wages start and evolve very differently from different states N_t, all wages limit to the same value as the employment spell becomes long:

$$\lim_{t\to\infty} w(t; N_{t-\tau}, \tau) = (x + b)/2 - r\xi \tag{5}$$

The Skellam distribution flattens out over time with variance $2\alpha\tau$. In the limit as $\tau \to \infty$, the monotonic cumulative distribution $F(i;; N_{t-\tau}, \tau)$ at the critical $i = -1$ converges to one half

$$\lim_{\tau\to\infty} F(-1; N_{t-\tau}, \tau) = 0.5$$

for all initial N_t. As a result, all wages converge as the history of initial conditions recedes. The Skellam process governing vacancy turnover implies that the distribution of N_t converges to a distribution with a mean zero and variance equal to ∞ as $t \to \infty$. The history of the initial state fades over time so that eventually all workers face the same prospects in the job centre. Since the effect of N_t is only transitory, wages converge to a unique wage.

7. Numerical example

The section presents a numerical exercise to explore the empirical relevance for this stock-flow approach. The first subsection illustrates the impact from altering turnover - the entry rate of workers and firms - on wages over time. The second subsection documents the accompanying wage dispersion. The final subsection regression analysis then reveals that the wage generation process aligns with the observation that wages depend on job tenure and hiring conditions. Taken together these results conform with the motivating evidence discussed in the Introduction.

7.1. Wages and turnover

First consider the determinants of the way in which wages evolve over time. Initial wages (excluding the hiring bonus) take on two possible values tied to either (i) unemployment benefits b or (ii) productivity x less the (monopsony-related) flow value of workers search costs ξ. The evolving likelihood of being on the short side of the market then governs wage progression as described by $w(t; N_{t-\tau}, \tau)$ for duration τ and initial hiring history $N_{t-\tau}$. The principle parameter controlling the likelihood of being on the short side in the market is α. This Poisson arrival rate of agents on both sides of the market determines the evolution of the Skellam distribution at the core of the wage determination process described in Proposition 6.

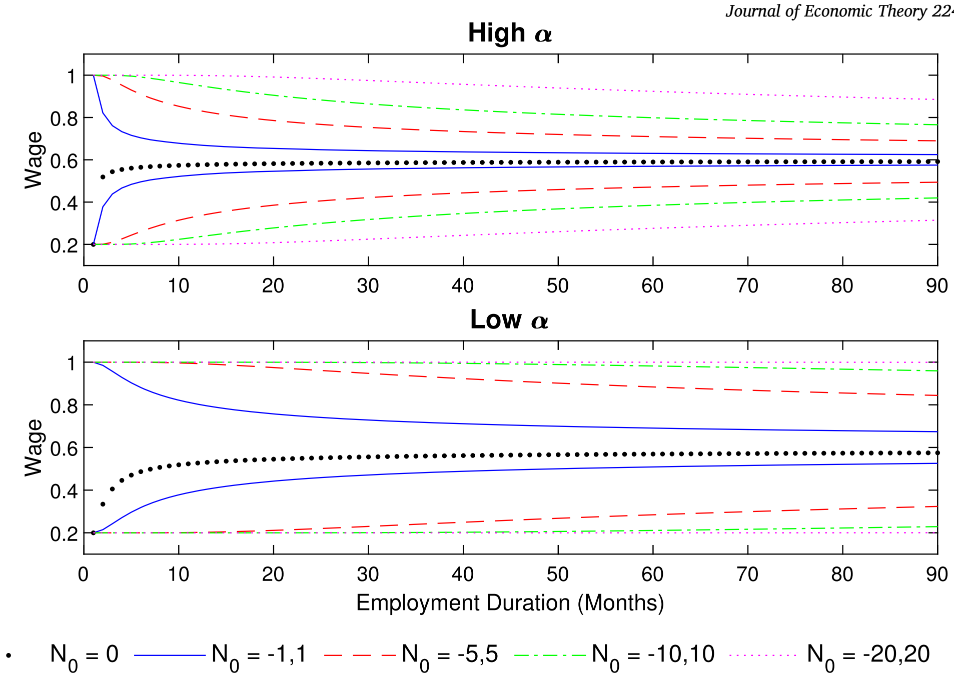

Fig. 2. Wages by Duration.

To gauge the impact of α, Fig. 2 plots wage progressions for two values of α and various initial conditions for $N_{t-\tau}$. The high α used for the top panel in Fig. 2 is three times the baseline case; the low value used in the bottom panel of Fig. 2 is one third the baseline value. The baseline specification is that the probability of an arrival of at least one worker in a given month equals one half so that $\alpha = 0.6931$. The remaining parameters are standard values.

Parameter	Value
x	1
b	.20
ξ	0.05
r	0.0042

A comparison of the two panels in Fig. 2 reveals that wage persistence increases as turnover in the marketplace falls, that is, as the economy moves further away from a competitive setting. The high α turnover plot in the top panel of Fig. 2 converges faster than the low α plot in Fig. 2. However, persistence lasts even for the high value α. For $N_{t-\tau} = -1$, after ten years the wage remains more than 11% above the long run wage whereas the wage for an $N_t = 0$ is 4% below the long run wage which aligns with Guo (2022) who finds that new hires from unemployment with multiple job offers just before starting employment enjoy a persistent wage premium (above hires with one offer) of over 10% for about nine years. The impact of the initial conditions fades over time but the convergence is most pronounced early on in the employment spell. Nonetheless, wage changes are predictable, both positive and negative, serially correlated and persistent, which all conform with the evidence noted above.

7.2. *Wage dispersion*

Now consider wage dispersion resulting from workers facing the same market turnover but hired at different points in time and under different market conditions who are thus experiencing a variety of wage paths. In the model, workers and firms inhabit a small, particular market to highlight the essential mechanics of wage determination with (the threat of future) worker search. An observed statistical market will typically contain numerous such particular entities. As observed statistics are broader than the model, the numerical approach here replicates and aggregates the model across a number of small markets to mimic familiar statistics.

Simulated entry of workers and firms occurs over 120 periods of ten years. Repeating the exercise over 100 markets all with the same initial $N_0 = 0$ yields a panel of wages for employed workers as well as information on unemployment and vacancies. Cross section wages from the last period are computed using all of the employer-employee pairs that formed during the ten year long period. These wages are conditional on the initial N_t at hiring and the subsequent duration of employment.

There is no ergodic steady state in the model. The variance of the Skellam distribution linearly increases with duration τ. As time progresses the likelihood of long queues of either workers or of firms increases which anchors some wages at more extreme values of the long side of the market as measured by N_t. A worker's participation decision and a zero profit firm entry condition are not

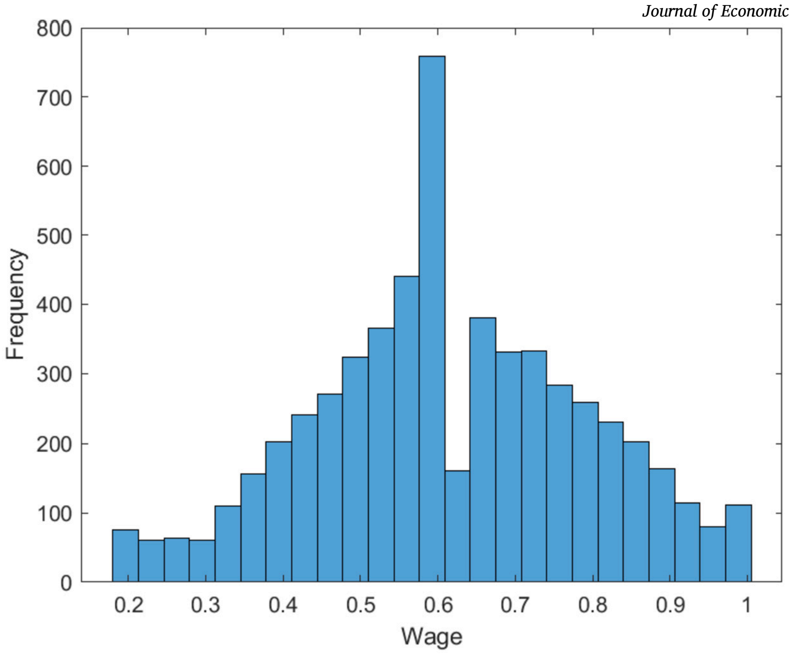

Fig. 3. Wage Dispersion after 10 Years with Bounded N_t.

present to moderate entry. In practice, however, at some point workers and firms will want to find preferable alternatives to markets with extreme values for N_t. Although the associated value matching and smooth pasting tools associated with potential bounds are understood, the impact on the evolution of the Skellam distribution of agents F is less so.

Fig. 3 presents the wage distribution with a simple correction – it includes only workers who matched with $-10 < N_t < 10$. The truncated distribution is roughly symmetric and approximately unimodal around the long run wage. Other familiar labour market statistics are also reasonable. The ratio of the mean wage to the minimum wage is 3.01. The associated U-V ratio equals roughly 1.02 and the unemployment rate is just below three percent at 2.61%.[21]

7.3. Job tenure

Do the wages occurring in the simulated market yield the same returns to tenure and initial conditions observed in the data as outlined in the Introduction? Regressing the cross sectional wage in the last period on tenure and initial hiring conditionals N_t yields the tenure effect. In particular, regressing logged wages on logged tenure and (unlogged) N_t at the time of hiring yields

$$\ln(w) = -0.797\,[0.007] + 0.057\,[0.002] * ln(Tenure) - 0.071\,[0.0004] * N_t$$
$$R^2 = 0.863$$

with standard errors in square brackets. Tenure on average raises wages. Initial wages also affect wages as expected.[22]

8. Conclusion

Search models provide an elegant and powerful framework for understanding why and how wages increase with job tenure. Firms that face a moral hazard problem of workers who are unable to commit to not pursuing attractive outside job offers have an incentive to backload wages in order to retain their workforce. Burdett and Coles (2003) show that with full firm commitment (and with risk-averse workers), backloading generates smoothly increasing wages. Postel-Vinay and Robin (2002) assume that firms do not commit to future wages but instead reset the piece rate they pay a worker each time the worker receives an attractive outside

[21] Without truncating the extreme values, the simulated distribution of wages exhibits symmetry with three local peaks aligning with the two initial wages and the long run limiting value of wages. The labour market statistics are less affected by the truncation. The ratio of the mean wage to the minimum wage is 2.96. The associated U-V ratio equals 1.01 and the unemployment rate rises to just below eight percent at 7.92%.

[22] The regression parameter estimates for both the truncated by N_t and the non-truncated wages are very similar. Note that in this regression the measure for initial condition N_t at hiring is very exact and precise for the individual. Although the empirical literature documents that initial conditions matter and are persistent, the measures of job competition used are far more general than in the above regression. Local unemployment rates for example are broad measures whereas N_t is very particular to the individuals circumstances. Finding the appropriate benchmark in the model is unclear but rerunning the regression without any such measure does not substantially alter the tenure coefficient estimate or its significance.

offer, implying that the worker's piece rate also increases stochastically and in discrete steps with tenure in response to competitors' attempts to poach the worker.

This paper likewise assesses wage setting when firms cannot commit to future wages and when workers cannot commit to not search while employed. The stock-flow job search frictions in which (i) workers and firms on occasion meet multilaterally and (ii) workers can recall previous encounters with firms generates a progression of wages with firms paying just enough to keep their workers, as in the familiar competitive labour market. The emerging compensation structure is consistent with well established but difficult to reconcile observations on pay dynamics within jobs at firms. In particular, this framework delivers

- wage dispersion and wage growth dispersion including wage cuts
- persistence - serial correlation in wages that creates predictable winners and losers
- initial conditions matter - which can be broadly viewed as cohort effects

These results here contribute an alternative perspective to the debate on the impact of job tenure on wage growth that allows both wage increases or decreases over time. Some papers find large and significant tenure effects, while others estimate small or insignificant effects (see Abraham and Farber, 1987; Altonji and Shakotko (1987); Topel, 1991; Dustmann and Meghir, 2005; Beffy et al., 2006; Buchinsky et al., 2010). The mixed empirical evidence may tie in with the initial conditions.

In the formulation of this paper, homogeneous workers search for identical jobs. Job search while employed does not lead to better matches. Because such search is costly, it is inefficient but workers are tempted to use it to increase their wages. In equilibrium, firms and workers do not engage in this wasteful rent-seeking behaviour and agree to the efficient outcome. Nonetheless, the ever evolving threat of job search coupled with the absence of multi-period commitments not only overcomes the well known Diamond paradox but also empirically relevant wage profiles over time.

The avoidance of the option of job search aligns with the observation that only a small fraction (less than 5 percent) of employed workers are actively searching. Fallick and Fleischman (2004), Nagypál (2005), Nagypál (2008). Jolivet et al. (2006) find that “relative to involuntary mobility (reallocation shocks and lay-offs), voluntary mobility is a rather rare event” in many European countries and in the US. Workers may not search when they are not at risk of job loss but the threat to do so disciplines wages in a way that is consistent with a variety of challenging wage setting observations.

CRediT authorship contribution statement

Eric Smith: Conceptualization, Formal analysis, Funding acquisition, Investigation, Methodology, Software, Writing – original draft, Writing – review & editing.

Declaration of competing interest

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Appendix

Proof of Proposition 1.

Reservation thresholds: Let $\omega_f^{S<B} \leq x$ be the offer that makes the firm indifferent between hiring and waiting another period when $S < B$. Let $\omega_e^{S \geq B}$ be the offer that makes the worker indifferent between accepting employment and waiting another period when $S \leq B$.

Worker strategies: A worker selected from the auction pool will choose the highest available offer provided that this payoff is greater than or equal to continuation payoff to waiting next period, i.e. $\omega_e^{S \geq B}$.

Bidding notation: Let $w_i^{S<B} \geq 0$ denote the bid of firm $i = 1, ...B$ when $S < B$. Let $w_i^{S \geq B} \geq 0$ denote firm i's bid for $S \geq B$. Let $w_{-i}^{S<B}$ and $w_{-i}^{S \geq B}$ denote the vectors of bids of the other firms for $S < B$ and $S \geq B$ respectively. Order $\{w_{-i}^{S<B}\}$ in descending order and let w_{-B} be the B^{th} element in this ordering.

(i) Since there are at least as many workers as jobs in the auction pool ($S \geq B$), for any $w_{-i}^{S \geq B}$ a worker will eventually emerge from the auction pool who is willing to accept a reservation wage bid of $\omega_e^{S \geq B}$.

As the firm can readily become unattached next period, it is suboptimal to bid lower bid and forgo employment this period. Any bid above the worker's reservation wage generates lower profit. It follows that for any $w_{-i}^{S \geq B}$, a firm's strictly dominant strategy is to bid as a monopolist and offer worker's reservation payoff as in Diamond (1971)

$$w_i^{S \geq B} = \omega_e^{S \geq B}$$

All workers accept this offer resulting in B hires.

(ii) For $w_{-B} < w_e^{S<B}$ (low offers from other firms), firm i's best response w_{BR} to $w_{-i}^{S<B}$ is to offer the lowest accepted payment that just outbids the highest accepted bid w_{-B}. For $w_B = w_f^{S>B}$, firm i is willing to also bid $w_f^{S<B}$ (and be indifferent between hiring and waiting) or to bid any lower offer and wait. For $w_B > w_f^{S>B}$, the best response is to offer any payment below this highest accepted bid thereby not hiring. A worker will not choose this offer as there are higher payments on offer. Taken together, the best response is thus

$$\begin{aligned} w_{BR} &= \max\{w_{-B} + \epsilon, \omega_e^{S>B}\} && w_{-B} < w_f^{S<B} \\ &= w \in [0, w_f^{S<B}] && w_{-B} = w_f^{S<B} \\ &= w \in [0, w_f^{S<B}) && w_{-B} > w_f^{S<B} \end{aligned}$$

A fixed point $w_{BR} = w_{-B}$ uniquely occurs at $w_f^{S<B}$. The subgame equilibrium outcome exhibits S such fixed points and hence $S+1$ bids at this level. All other bids are less than or equal to this fixed point $w_f^{S<B}$. The workers accept some set of S of the $w_f^{S<B}$. This outcome of the competitive auction corresponds the Bertrand outcome.

(iii) The strategies in (i) and (ii) enact all feasible trades as determined by the number of traders on the short side of the market - $\min\{B, S\}$. The $\mid B - S \mid$ traders from the long side remain and are left waiting unattached.

(iv) Immediate trade leaves either $S - B$ or $B - S$ to wait and receive their continuation payoffs. Adding an additional pair does not alter the number of remaining agents and hence the continuation payoffs that determine bids. The bids and acceptance decisions remain unchanged but with one additional pair trading. The worker and firm payoffs are unaffected. □

Proof of Proposition 2.
Manipulating terms in (2) and letting $dt \to 0$ gives

$$\alpha V(N_t + 1) - (r + 2\alpha)V(N_t) + \alpha V(N_t - 1) = -b$$

Following the same approach for (3) likewise produces

$$\alpha\Pi(N_t - 1) - (r + 2\alpha)\Pi(N_t) + \alpha\Pi(N_t + 1) = 0$$

The characteristic equations for the homogeneous version of the two second order, linear difference equations have the same two distinct roots. $\lambda < 1$ is the stable root for both. Adding the particular solutions gives general solutions. Chapter 11 of Sydsæter et al. (2008) reviews the solution methods for difference equations. The literature review Smith (2020) outlines applications of these methods to stock-flow matching models.

To find the boundary conditions, note that

$$\Pi(0) + V(0) = x/r$$

At the boundary $N_t = 0$, the firm again makes an offer to the lone worker that makes the worker indifferent between accepting and rejecting. Now, however, the waiting payoff accounts for the possibility that a worker or a firm may arrive next period. If a firm arrives, the worker would get $x/r - \Pi(1)$.

$$V(0) = \frac{1}{1 + rdt} b\,dt + \alpha dt\, V(1) + \alpha dt\, x/r - \Pi(1) + (1 - 2\alpha)\, V(0)$$

Plugging in

$$V(1) = \lambda(V(0) - b/r) + b/r$$

as well as

$$\Pi(1) = \lambda\Pi(0) = \lambda(x/r - V(0))$$

and solving give the boundary condition for $V(0)$ The boundary for $\Pi(0)$ follows from the joint payoff. □

Proof of Proposition 4.
Using $V(i) = x/r - \Pi(i)$ for $i < 0$ yields

$$W(t; N_{t-\tau}, \tau) = -\xi + \sum_{i=1}^{\infty}[x/r - \Pi(-i)]f(-i; N_{t-\tau}, \tau) + \sum_{j=0}^{\infty} V(j) f(j; N_{t-\tau}, \tau)$$

Plugging in from Proposition 2 produces

$$W(t; N_{t-\tau}, \tau) = \sum_{i=1}^{\infty}[x/r - \lambda^i \Pi(0)]f(-i; N_{t-\tau}, \tau)$$

$$+\sum_{j=0}^{\infty}[\lambda^j(V(0)+b/r]f(j;N_{t-\tau},\tau)-\xi$$

Dropping the history and duration notation $(N_{t-\tau},\tau)$, the law of motion for f yields

$$\begin{aligned}\frac{d}{dt}\sum_{i=1}^{\infty}\lambda^i f(-i) &= \sum_{i=1}^{\infty}\lambda^i[\alpha f(-i-1)-2\alpha f(-i)+\alpha f(-i+1)]\\ &=\frac{\alpha}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i)-\frac{\alpha}{\lambda}\lambda f(-1)-2\alpha\sum_{i=1}^{\infty}\lambda^i f(-i)\\ &+\alpha\lambda f(0)+\alpha\lambda\sum_{i=1}^{\infty}\lambda^i f(-i)\\ &=\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i)-\alpha f(-i)+\alpha\lambda f(0)\end{aligned}$$

Similar derivations yield

$$\begin{aligned}\frac{d}{dt}\sum_{j=0}^{\infty}\lambda^j f(-j) &= \sum_{j=0}^{\infty}\lambda^j[\alpha f(j+1)-2\alpha f(j)+\alpha f(j-1)]\\ &=\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=0}^{\infty}\lambda^j f(j)-\frac{\alpha}{\lambda}f(0)+\alpha f(-1)\end{aligned}$$

and

$$\begin{aligned}\frac{d}{dt}F(-1)=\frac{d}{dt}\sum_{i=1}^{\infty}f(-i) &= \sum_{i=1}^{\infty}[\alpha f(-i-1)-2\alpha f(-i)+\alpha f(-i+1]\\ &=\alpha[f(0)-f(-1)]\end{aligned}$$

Differentiation and substituting in the above relationships gives

$$\begin{aligned}\dot{W}(t;N_{t-\tau},\tau) &= -\Pi(0)\left[\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i)-\alpha f(-1)+\alpha\lambda f(0)\right]\\ &+(V(0)-b/r)\left[\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j)-\frac{\alpha}{\lambda}f(0)+\alpha f(-1)\right]\\ &+\frac{x-b}{r}\alpha\left[f(0)-f(1)\right]\\ &=-\Pi(0)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i)\\ &+(V(0)-b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j)\\ &+\alpha f(0)\left[-\lambda\Pi(0)-(V(0)-b/r)/\lambda+\frac{x-b}{r}\right]\\ &=-\Pi(0)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i)\\ &+(V(0)-b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j)\\ &+\alpha f(0)(1-\lambda)\left[x(0)-(V(0)-b/r)/\lambda\right]\end{aligned}$$

Note that

$$V(0)-b/r=\frac{\alpha(1-\lambda)(x-b)}{r(r+2\alpha(1-\lambda))}$$

and

$$\Pi(0)=\frac{(r+\alpha(1-\lambda)(x-b)}{r(r+2\alpha(1-\lambda))}$$

so that

$$\Pi(0) - (V(0) - b/r)/\lambda = \frac{(r\lambda - \alpha(1-\lambda)^2)(x-b)}{r\lambda(r + 2\alpha(1-\lambda))}$$

It is straightforward to establish that $r\lambda = \alpha(1-\lambda)^2$. Hence with the history and duration $(N_{t-\tau}, \tau)$ notation restored in f

$$\dot{W}(t; N_{t-\tau}, \tau) = -\Pi(0)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{i=1}^{\infty}\lambda^i f(-i; N_{t-\tau}, \tau)$$
$$+ (V(0) - b/r)\frac{\alpha(1-\lambda)^2}{\lambda}\sum_{j=1}^{\infty}\lambda^j f(j; N_{t-\tau}, \tau) \quad \square$$

Proof of Proposition 5.
The payoff to the firm of paying the lowest no-search wage is

$$\frac{x}{r} - E(t; N_{t-\tau}, \tau) = \frac{x}{r} - W(t; N_{t-\tau}, \tau).$$

In contrast, any wage offer below the no-search threshold $w(N_{t-\tau}, \tau)$ triggers a visit to the job centre where all information is revealed. The payoff to a firm inducing the worker to search is determined after the worker pays the search cost. Hence this search payoff to the firm equals

$$\frac{x}{r} - W(t; N_{t-\tau}, \tau) - \xi$$

It is more profitable for the firm to avoid the outcome of worker job sampling. The argument applies for any τ hence the jointly optimal outcome is a relationship that avoids incurring search costs. $\square$

Proof of Proposition 6.
As noted, accepting the no-search reservation wages implies

$$E(t; N_{t-\tau}, \tau) = W(t; N_{t-\tau}, \tau).$$

From equation (4), the accepted wage satisfies

$$w(t; N_{t-\tau}, \tau) = rW(t; N_{t-\tau}, \tau) - \dot{W}(t; N_{t-\tau}, \tau)$$

Plugging in for W and for $\dot{W}$ from Proposition 4 and collecting terms gives

$$w(t; N_{t-\tau}, \tau) = -\Pi(0)[r - \alpha(1-\lambda)^2/\lambda]\sum_{i=1}^{\infty}\lambda^i f(-i; N_{t-\tau}, \tau)$$
$$+ [V(0) - b/r][r - \alpha(1-\lambda)^2/\lambda]\sum_{j=0}^{\infty}\lambda^j f(j; N_{t-\tau}, \tau)$$
$$+ F(-1)(x-b) + b - r\xi$$

Again, it is straightforward to establish that $r\lambda = \alpha(1-\lambda)^2$. As a result the first two terms in the above equation drop giving the stated wage equation. $\square$

Proof of Proposition 7.
(i) Suppose $N_t < 0$. Firms bid such that they are indifferent between hiring and continued search. The payoff to having an attached worker in the next instant is given by $x/r - E(t; N_{t+dt}, dt)$. It follows that

$$\Pi(N_t < 0) = -D - [x - w(t; N_t, 0)]dt + \frac{1}{1 + rdt}\left[x/r - E(t; N_{t+dt}, dt)\right]$$

where D is an initial bonus payment paid to the worker in the second sub-period at the instant of employment. Plugging in $\Pi(N_t) = x/r - V(N_t)$ gives

$$x/r - V(N_t) = -D + [x - w(t; N_t, 0)]dt$$
$$+ \frac{1}{1 + rdt}\left[x/r + \xi - \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt)\right]$$

As

$$\lim_{\tau \to 0}\sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt) = V(N_t)$$

$D = \xi$ as $dt \to 0$.

(ii) Now suppose $N_t \geq 0$. The firm's offer in the job centre makes the worker indifferent between employment and unattached search. The payoff to employment conditional on the history of net entry is given by

$$V(N_t \geq 0) = D + w(t; N_t, 0)dt + \frac{1}{1+rdt} E(t; N_{t+dt}, dt)$$
$$= D + w(t; N_t, 0)dt + \frac{1}{1+rdt} \left[-\xi + \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt) \right]$$

where again D is an initial bonus payment paid at the instant of employment. Once again, as

$$\lim_{\tau \to 0} \sum_{k=-\infty}^{\infty} V(k) f(k; N_{t+dt}, dt) = V(N_t)$$

$D = \xi$ as $dt \to 0$. □

Proof of Proposition 8.
Differentiation establishes

$$\dot{w}(t; N_{t-\tau}, \tau) = \dot{F}(-1; N_{t-\tau}, \tau)(x - b + c)$$

Again the law of motion over time for f gives

$$\dot{F}(-1; N_{t-\tau}, \tau) = \frac{d}{dt} \sum_{-\infty}^{i=-1} f(i; N_{t-\tau}, \tau)$$
$$= \sum_{-\infty}^{i=-1} [\alpha f(i-1; N_{t-\tau}, \tau) - 2\alpha f(i; N_{t-\tau}, \tau) + \alpha f(i+1; N_{t-\tau}, \tau)]$$
$$= \alpha [f(0; N_{t-\tau}, \tau) - f(1; N_{t-\tau}, \tau)]$$

Plugging in gives the desired result. □

Data availability

No data was used for the research described in the article.

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