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Fragility under joint financing: The (moral) hazards of diversification ☆

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ABSTRACT

We study the effect of investment growth on firms' incentives under moral hazard. Adding value-increasing, but risky projects to a firm's portfolio can weaken incentives for safer ones, even when returns are independent. While the firm diversifies its sources of income, this risk contamination channel can increase its fragility. Such fragility is exacerbated in the presence of news about the value of investments. Firms can mitigate these effects by selecting safer new investments at the expense of value creation. Our model thus predicts that large firms or merged firms may be riskier or less productive than smaller firms.

1. Introduction

When firms expand and diversify, they bring different projects and technologies under a common fold. Rationales for firm expansion typically include the leveraging of acquired expertise, savings on fixed costs, or financial synergies. A vast theoretical literature documents indeed that financial synergies can lower a firm's cost of capital, thanks to diversification benefits (e.g. Boot and Thakor, 1993), or by reducing agency frictions (e.g. Laux, 2001). In this paper, we highlight instead the negative effect of expansion and diversification on agency frictions, via common financing. We show that common financing of investments with heterogeneous risk can exacerbate moral hazard problems. With moral hazard, riskier projects can undermine incentives for safer projects. When this risk contamination channel is active, firms become more fragile as they grow. Our analysis predicts that larger firms may become either riskier or less efficient.

We derive these insights in a stylized model with risk-neutral agents, in which a penniless firm borrows from deep-pocketed lenders to fund one or two risky projects. Each project is subject to moral hazard, as in Innes (1990) or Holmström and Tirole (1997): the firm can exert an unobservable effort to increase the project's likelihood of success. The effort on one project, however, has no impact on the success of the other project, and their payoffs are independent random variables. The firm has the option to raise a

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small loan to fund any single project, or a larger loan to finance both projects. The firm raises funds using one debt contract with a fixed repayment.[2] We analyze the firm's decision to expand and the impact of such expansion on the firm's financial fragility, captured by its probability of default. This stylized framework also allows us to analyze the consequences of merging two small firms running one investment each into a larger firm that combines these investments.

Under *single-project financing*, the firm chooses one project, and finances it by raising funds from competitive lenders. By design, a project has the highest Net Present Value (NPV) when the firm exerts effort instead of shirking. However, the firm chooses its effort level after the repayment of the loan is set, which implies that it shirks if this repayment is too high. As usual, a higher repayment reduces its stake in the project outcome, and thus disincentivizes effort. To get a clean benchmark, we focus on the case in which firm effort can be implemented in equilibrium. Hence, with single-project financing, the firm chooses the project with the highest NPV under effort.

Our main result is that the firm may shirk on both projects in equilibrium under *joint financing*, even if it prefers to exert effort on any project under single-project financing. Such a result arises if the new project (say B) is sufficiently riskier than the legacy project (say A). This risk contamination between projects arises under joint financing, even if the projects outcomes are independent and have unrelated fundamentals. We also show that the risk contamination may lead to an increase in the firm's probability of default, relative to single-project financing, even though the independence in project returns tends to decrease default risk. Moreover, even when the firm's overall default risk increases with joint financing, we show that it may still prefer to expand and fund two projects. The next paragraphs expose the intuition for the risk contamination channel.

To understand the forces leading to this result, consider the firm's incentives to exert effort on project A, when it is financed jointly with B. A standard insight from models with moral hazard is that a borrower's incentives to exert effort depend on the sensitivity of its payoff to its (unobservable) effort choice. Under joint-financing, the success or failure of project B changes the payoff sensitivity for project A. When project B succeeds, its cash flows can be used to repay the joint debt contracted to finance both projects. The revenue from project B acts as collateral to finance project A, which improves the firm's incentives to exert effort, similar to an increase in the firm's wealth. We call *net worth effect* this positive role of joint financing on incentives. On the other hand, when project B fails, it is now project A that should repay the entire joint debt. In this event, relative to the case of single-project financing, project A backs a larger debt. This reduces the firm's stake in this project and thus its incentives to exert effort. We call *net debt effect* this impact of joint financing on incentives, which is negative when the face value of debt increases.

Overall, incentives to exert effort on project A deteriorate under joint financing if the net debt effect dominates the net worth effect. As the discussion above suggests, this is the case when project B is sufficiently risky. When project B fails with a high probability, its collateral value for project A is limited, that is, the net worth effect is weak. It is rather project A that serves more often as collateral to repay the joint debt of the firm, which implies that the net debt effect is strong. When the net debt effect dominates, we say that the risk in one project (B in the case considered) contaminates the other, as its default risk endogenously makes the other project riskier, by undermining incentives to exert effort.

When the risk contamination operates in both directions – that is, the risk in project B contaminates project A and the risk in project A contaminates project B – the firm shirks on both projects under joint financing. Moreover, if the loss in NPV due to shirking is limited, the firm may still choose to expand by investing in both projects and shirk, rather than finance a single project, for which it exerts effort. If instead the NPV loss due to shirking is large enough, single financing occurs in equilibrium. In that latter case, our model can explain why firms may refrain from financing positive NPV projects, even when such projects could be financed on a standalone basis. When the firm chooses to finance two projects, we show that its probability of default can increase, relative to single-project financing. This is not obvious because in our model, under joint financing, a firm defaults only when both projects fail, while a single project failure is synonymous with firm default under single-project financing. The fact that success is independent across projects thus stacks the odds against fragility. Despite this benefit of diversification, our analysis shows that firms' probability of default can increase under joint financing due to risk contamination. Combining these two observations, we find that the firm may voluntarily increase its default risk when it chooses to expand and finance new projects.

We consider two extensions to our baseline model. First, we show that a firm becomes more conservative as it expands, precisely to avoid the risk contamination effect. In this extension, we let the firm choose between two types of B projects, to be financed jointly with its legacy project A. We show that the firm may not choose the project with the highest NPV if such a project is too risky. In this case, even if the safer of the two projects is not as valuable on a standalone basis than the riskier one, it adds more value under joint financing if it avoids the risk contamination effect. Hence, our model predicts that large firms may be more conservative than small firms by lowering the risk in new investments at the expense of NPV. Overall, when the risk contamination channel operates, large firms tend to be either riskier or less productive than small firms.

In the second extension, we show that joint financing can further increase default risk through an extra *news channel*. To this end, we extend the model to allow the firm to receive some news about the payoff of project B, before it chooses the level of effort for project A. The firm can then optimally adjust its effort on project A to this information. When project B succeeds, only the net worth effect plays out, and the firm's incentives improve. However, when the firm learns that project B will fail, it tends to lower its effort on project A, because only the net debt effect is present. However, there is more than correlation between news and effort: the expected probability of default across the possible news realizations increases. This result arises because, in our model, the firm

[2] We also show that our main results follow through even if the firm can use a loan contract with a repayment that is monotone in the total cash flows of the project(s). Our results are also robust if a firm with two projects can borrow via two different loan contracts, provided that each set of lenders have recourse to the cash flows of the other project (no ring-fencing).

defaults only if both projects fail. When project B succeeds, lenders get repaid in full, independently of the firm's effort choice for project A. In this event, stronger firm incentives, relative to the no-news case, do not reduce its expected default probability. In fact, lenders value project A only when project B fails. As the firm exerts less effort when it receives bad news than in the no-news case, the correlation in default risk translates into a larger expected probability of default.

Our model has several applications. As mentioned above, it speaks to the impact of M&A on fragility and firm growth more generally. Indeed the model implies that firms may become more fragile as they grow and diversify, even though the usual benefits from diversification are present in our model. Large firms who run multiple investments may thus be more fragile than small firms. As regard to M&A, the firm with two projects can be interpreted as a larger firm that combines two independent firms. In this interpretation, our model shows that both investments can become less profitable and riskier in the merged firm, relative to the pre-merger situation.[3] More generally, our theory applies to any situation in which moral hazard is present and firms finance assets with heterogeneous riskiness, either with joint debt or without ringfencing these assets.

Literature review

Several works have shown that joint financing of assets can mitigate frictions. In Subrahmanyam (1991), Boot and Thakor (1993), or Gorton and Pennacchi (1993), pooling assets with idiosyncratic risk can reduce or even eliminate information asymmetries.[4] When financing is subject to moral hazard, Diamond (1984) has shown that delegated monitoring of multiple projects by banks reduces monitoring costs. Building on a similar insight, Cerasi and Daltung (2000) and Laux (2001) show that diversification reduces moral hazard frictions, by highlighting the benefits of cross-pledging for a borrower's incentives (see also Tirole, 2010). In these works, borrowers have stronger incentives to exert effort under joint financing, for the other investments are also at risk when they fail on a given investment. We argue that this cross-pledging benefit of joint financing may disappear when assets are sufficiently heterogeneous. In this case, a risk contamination channel emerges, and joint financing exacerbates moral hazard frictions, instead of reducing them.

A closely related work is Banal-Estañol et al. (2013), who also identifies a flip side of financing synergies. This paper also shows that joint financing can generate contamination between otherwise unrelated projects. However, the mechanism in this paper is different from ours, because contamination is caused by exogenous deadweight costs of default, rather than moral hazard.[5] In a similar vein, Donaldson et al. (2020) emphasize a negative effect of asset pledgeability. In that paper, the authors show that an increase in pledgeability incentivizes firms to issue secured debt in order to dilute pre-existing unsecured debt. Our effect relies instead on the contamination of the investor's legacy assets by newly acquired assets, which serve as (poor) collateral under joint financing.

Securitization of loans can be seen as an engine for joint financing, as it entails pooling cash flows of different projects via Special Purpose Vehicles financed with debt. The great financial crisis triggered a debate about the effect of securitization on lenders' incentives to monitor loans in the mortgage market, mainly because securitization is seen as removing the skin-in-the-game component of incentives. Several works, including Keys et al. (2010), Purnanandam (2011), Piskorski et al. (2015) and Griffin and Maturana (2016), show that securitization led issuers to apply lax standards for subprime loans.[6] Plantin (2011) establishes, using a theoretical analysis, that a greater level of securitization needs not generate an inefficient outcome, even though it leads to less screening by lenders. While we obtain a similar result, we highlight the role of joint financing in reducing incentives to monitor. Relatedly, Chemla and Hennessy (2014) and Vanasco (2017) show that investors purchasing securitized loans face asymmetry of information as intermediaries acquire private information when screening borrowers before selling these loans (see also the empirical analysis by Downing et al., 2008). Our theoretical findings complement these works by showing that joint financing through securitization can reduce the quality of the loans granted.

Finally, we identify a news channel for fragility under joint financing. In our model, each investment serves as collateral for the other investments of the firm. Our results, suggesting that opacity about the payoffs of these assets may be optimal when they are used as collateral, are reminiscent of Dang et al. (2015), Gorton and Ordoñez (2014) and Monnet and Quintin (2017). Our news channel, however, is different from the Hirshleifer (1971) effect at play in these papers. In our model, the news received by the borrower induces it to correlate the effort choice with the payoff of the other investment, which serves as collateral, and this correlation exacerbates fragility.

The rest of the paper is structured as follows. Section 2 describes the environment. The benchmark case where the firm finances a single project is studied in Section 3. Our main results that joint financing weakens incentives and generates fragility are gathered in Section 4. Section 5 shows that fragility worsens in the presence of news about the payoff of some of the investments. We endogenize the quality of the investments in Section 6. Finally, Section 7 concludes. The Appendix contains some generalizations of the results, as well as proofs that are omitted in the main text.

2. Environment

The economy lasts for two dates $t = 0, 1$. There is one good, called cash. The economy is populated by a penniless firm and several deep-pocketed lenders. The firm has access to two projects, called A and B. Project $i \in \{A, B\}$ requires one unit of funding. It pays

[3] In this interpretation, the merger should be considered as an exogenous event, due to other considerations.

[4] Glode et al. (2022) show that these benefits depend on the market structure. If issuers have little market power, they prefer selling assets separately to preserve their information rent.

[5] See also Bahaj and Malherbe (2020).

[6] See Bubb and Kaufman (2014) for a criticism of these results.

X_i in case of success, and 0 otherwise. The probability of success depends on a binary effort choice $e_i \in \{0,1\}$ by the firm. With effort ($e_i = 1$), the project succeeds with probability $\pi_i(1) \equiv p_i$, while it succeeds with a lower probability $\pi_i(0) \equiv q_i < p_i$ if the firm shirks ($e_i = 0$). Effort costs $c_i > 0$ and is unobservable to third parties, hence the firm faces moral hazard.[7] Let

$$\Delta_i \equiv p_i - q_i \tag{1}$$

be the increase in the project's probability of success when the firm exerts effort. For any project, we assume that effort is preferable to shirking, and that each project has positive NPV under effort:

Assumption 1. $p_i X_i - c_i \geq \max\left\{q_i X_i, 1\right\},\ \forall\, i \in \{A, B\}$.

Assumption 1 leaves open the possibility that projects have positive NPV also when the firm shirks, that is, the sign of $q_i X_i - 1$ could be positive or negative.

The firm must choose whether to raise one unit of funds to finance only one of the projects available, or to raise two units from lenders to fund both projects A and B.

Assumption 2. Funding is raised by the firm using a single debt contract backed by the cash flows from all the projects operated.

Our focus on debt contracts rests on the wide popularity of such financing contracts in practice. This assumption has no bite when a firm finances a single project, because a project has binary payoffs. With two projects, we show in Appendix A that Assumption 2 can be replaced by a weaker requirement that the lenders' repayment is increasing with the total payoff of the projects, in the spirit of Innes (1990), at the cost of a minor extra assumption. Assumption 2 requires that the firm uses a single loan when it finances two projects. Alternatively, we could assume that the firm enters two one-unit loans, but that creditors of each loan have recourse to the cash flows of the other project (no ring-fencing). Without ring-fencing, if project $i \in \{A, B\}$ fails, its creditors can seize the remaining cash flows of project $j \neq i$, after the firm repays creditors of the loan funding project j. Then, the two debt contracts would collapse to a single standard debt contract, with a promised repayment equal to the sum of the repayments of the two one-unit loans.[8]

The firm will expand its operation and fund two projects whenever this increases its revenue, net of the borrowing costs. Our analysis aims to characterize the effect of this decision on the firm's incentives, and hence on its default risk, measured by its default probability. Our assumption that lenders compete among themselves implies that all the surplus goes to the firm, hence we are considering the best-case scenario for effort.

Remark 1. Our model characterizes a firm's decision to expand into a new business, and the consequences of this choice for fragility. As an alternative application of our analysis, consider two small firms, which finance one project each. We can characterize the consequences for firm fragility of merging the two small firms into one larger firm, that funds both projects.

3. Single-project financing

To set the stage for our analysis, we first analyze the benchmark in which the firm raises one unit of funds to operate one project. We follow the textbook analysis of financing under moral hazard to derive conditions for which the firm exerts effort in equilibrium (see e.g. Tirole, 2010).

For project $i \in \{A, B\}$, an equilibrium is characterized by an effort level $e_i^* \in \{0,1\}$ and a debt repayment $R_i^* > 1$. The presence of several competing lenders implies that the equilibrium value of (e_i^*, R_i^*) maximizes the firm's payoff

$$U^F = \pi(e_i)(X_i - R_i) - c_i \mathbf{1}_{e_i=1}, \tag{2}$$

subject to feasibility constraint $R_i \leq X_i$, the firm's incentive-compatibility constraint and the lenders' participation constraint. The lenders' participation constraint states that they make nonnegative profits on the loan granted: hence, given effort choice e_i^* by the firm, R_i^* must satisfy

$$\pi_i(e_i^*) R_i^* \geq 1. \tag{3}$$

The firm's incentive compatibility constraint states that the effort choice e_i^* must be optimal, given the repayment R_i^*. For instance, effort $e_i = 1$ is incentive-compatible if and only if:

$$p_i(X_i - R_i^*) - c_i \geq q_i(X_i - R_i^*). \tag{4}$$

The left-hand (right-hand) side of incentive constraint (4) is the firm's payoff when it exerts effort (shirks). If the firm shirks, it does not pay the cost of effort, c_i, but the project's success probability decreases from p_i to q_i.

[7] The firm may be viewed as a bank, in which case, the effort has a natural interpretation as a monitoring effort on loans extended by the bank. Since our model is general, however, we simply call the borrower a firm.

[8] In practice, we believe that ringfencing is difficult to implement, due to the presence of (unmodeled) reputation concerns or default costs. If such concerns are present, the borrower could prefer ex-post to repay one loan using the cash flow of the other project, even if this is not optimal from an ex-ante point of view.

Under Assumption 1, effort maximizes a project's NPV. However, the firm is unable to commit to an effort choice when there is moral hazard. Hence, the effort level $e_i = 1$ is chosen only if it is incentive compatible, that is, if constraint (4) holds for the repayment value $R_i^* = 1/p_i$.

Lemma 1. *With single-project financing of project $i \in \{A, B\}$, in equilibrium the firm exerts effort and $R_i^* = \frac{1}{p_i}$ if and only if*

$$c_i \leq \Delta_i \left[X_i - \frac{1}{p_i} \right] \equiv \bar{c}_i(p_i). \tag{5}$$

When the firm exerts effort, lenders break even if the loan repayment is $R_i^* = \frac{1}{p_i}$. Substituting this value into the incentive constraint (4) we obtain condition (5). When the cost of effort for a project is low enough, the firm financing that project exerts effort in equilibrium. The following assumption combines Assumption 1 with condition (5).

Assumption 3. $c_i \leq \min\left\{p_i X_i - 1, \bar{c}_i(p_i)\right\}, \ \forall\, i \in \{A, B\}$.

To conclude, under Assumption 3, when the firm chooses single-project financing, it exerts effort in equilibrium. We maintain this assumption in the rest of our analysis.

4. Joint-project financing

Next, we consider the case where the firm finances both projects. Since the returns of the two projects are uncorrelated, the probability that all projects fail decreases relative to single-project financing, for given effort choice levels. Despite this diversification benefit, we will show that joint financing of projects may weaken the firm's incentives and increase its default risk, and yet the firm may prefer joint financing to single financing.

Under joint financing, the firm raises 2 units from lenders. As with single-project financing, lenders break even in equilibrium.[9] For any possible profile of effort choices $(e_A, e_B) \in \{0,1\}^2$, denote R_{e_A,e_B} the face value of debt such that lenders break even when the firm chooses (e_A, e_B). The set $\mathcal{R} = \left\{ R_{e_A,e_B} \right\}_{(e_A,e_B)\in\{0,1\}^2}$ contains the four possible equilibrium debt repayments. Given $\left(e_A, e_B\right)$, we obtain loan repayment R_{e_A,e_B} from the lenders' participation constraint, taking the firm's limited liability constraint into account:

$$\begin{aligned} \pi_A(e_A)\pi_B(e_B)R_{e_A,e_B} + \pi_A(e_A)\left[1 - \pi_B(e_B)\right] \min\left\{ X_A, R_{e_A,e_B} \right\} \\ + \pi_B(e_B)\left[1 - \pi_A(e_A)\right] \min\left\{ X_B, R_{e_A,e_B} \right\} = 2. \end{aligned} \tag{6}$$

The left-hand side of equation (6) is the lenders' expected revenue from the debt contract. The right-hand side is the size of the loan granted to fund both projects. Equation (6) thus implicitly defines the debt repayment such that lenders break even, for any given profile of effort choices $(e_A, e_B) \in \{0,1\}^2$. To simplify the analysis, we assume that the payoff from any one successful project is sufficient to pay back the debt:

Assumption 4. $\min\left\{X_A, X_B\right\} \geq \frac{2}{1-(1-q_A)(1-q_B)}$

This assumption ensures that the firm defaults on its debt only if both projects fail. In this event, the firm defaults because it has zero cash flows. When at least one project succeeds, the lowest possible cash flow of the firm is $\min\left\{X_A, X_B\right\}$, which exceeds the maximum face value of the debt relevant for our analysis under Assumption 4. To see why, consider the face value of the debt such that lenders break even, provided that the firm repays in full when at least one project succeeds. For a given profile of effort choices $(e_A, e_B) \in \{0,1\}^2$, it is:

$$R_{e_A,e_B} = \frac{2}{1 - (1 - \pi_A(e_A)(1 - \pi_B(e_B))} \tag{7}$$

The highest value of R_{e_A,e_B} obtains when the firm shirks on both projects, that is, $(e_A, e_B) = (0,0)$. Assumption 4 is equivalent to $\min\left\{X_A, X_B\right\} \geq R_{00}$, and thus implies the stated property.

Next, we characterize the firm's incentive-compatible effort choices. As in the previous section, under moral hazard the firm's effort levels (e_A, e_B) are chosen after the loan repayment value $R \leq \min\left\{X_A, X_B\right\}$ has been set, that is, they solve the following problem:

$$\max_{(e_A,e_B)\in\{0,1\}^2} U^F_{joint}(e_A, e_B, R) = \pi_A(e_A)\pi_B(e_B)(X_A + X_B - R) + \pi_A(e_A)(1 - \pi_B(e_B))(X_A - R)$$

[9] The firm cannot gain from leaving rents to lenders. As expression (8) will show, its payoff is decreasing in the repayment value R and the incentive-compatibility constraint is tightened when R is increased. We can then focus on the case where the lenders' participation constraints hold as equality.

$$+\pi_B(e_B)(1-\pi_A(e_A))(X_B-R)-c_A\mathbf{1}_{e_A=1}-c_B\mathbf{1}_{e_B=1}. \tag{8}$$

We can now define an equilibrium as follows.

Definition 1. An equilibrium under joint financing is a pair of effort choices $(e_A^*, e_B^*) \in \{0,1\}^2$ and a debt repayment R^* such that (e_A^*, e_B^*, R^*) maximizes $U_{joint}^F(e_A, e_B, R)$ within the set of admissible values of e_A, e_B, R, such that: (i) the lenders' participation constraint is satisfied: $R = R_{e_A, e_B}$, and (ii) the firm's incentive compatibility constraint holds: (e_A, e_B) solves problem (8) given R.

Absent moral hazard, the firm would choose the effort profile (e_A^*, e_B^*), and the corresponding debt repayment value $R_{e_A^*, e_B^*}$, so as to maximize its ex-ante utility, subject to the lenders' participation constraint. Under Assumption 3, the firm would then exert effort on both projects, as this maximizes their NPV. With moral hazard, however, the effort profile must be incentive compatible, a constraint formalized by equation (8). In the next Section 4.1 we show that, under some conditions, the firm shirks on at least one project with joint financing even though it exerts effort with single-project financing. In the subsequent Section 4.2, we further refine these conditions to show that, in some cases, the firms prefer to shirk on both projects with joint financing.

4.1. *Joint financing can weaken incentives*

In this section, we provide conditions under which the firm shirks on at least one project in equilibrium. We proceed by contradiction and suppose that lenders expect the firm to exert effort on both projects. In this case, the face value of the two-unit loan is

$$R_{11} = \frac{2}{1-(1-p_A)(1-p_B)}, \tag{9}$$

which is obtained from equation (7), setting $(e_A, e_B) = (1, 1)$. The conjecture cannot be sustained in equilibrium if the firm prefers to shirk on at least one project, when faced with this repayment value. To fix ideas, we derive here the conditions under which it prefers to shirk on project A, and provide the complete set of conditions in the next result. Using equation (8), the firm prefers to shirk on project A if $U_{joint}^F(0, 1, R_{11})$ is strictly larger than $U_{joint}^F(1, 1, R_{11})$. We can rewrite this condition as follows:

$$\frac{c_A}{\Delta_A} > X_A - (1-p_B)R_{11}. \tag{10}$$

In the above inequality, the left-hand-side is the cost of effort for project A, normalized by the increase in the probability of success due to effort, $\Delta_A = p_A - q_A$. The right-hand-side is the sensitivity of the firm's payoff to the probability of success of project A (gross of the effort cost). To assess the effect of joint financing on the firm's incentives, we need to compare this payoff sensitivity to its counterpart under single-project financing, $X_A - \frac{1}{p_A}$, obtained from (2). Substituting in (10) the value of R_{11} obtained from equation (9), we get the following result.

Proposition 1. *In equilibrium, the firm shirks on at least one project under joint financing, while it exerts effort under single-project financing, if and only if*

$$c_A \in \left(\underline{c}_A(p_A, p_B), \bar{c}_A(p_A)\right), \quad \text{or} \quad c_B \in \left(\underline{c}_B(p_B, p_A), \bar{c}_B(p_B)\right), \tag{11}$$

where $\underline{c}_i(p_i, p_j) \equiv \Delta_i \left(X_i - \frac{2(1-p_j)}{1-(1-p_i)(1-p_j)}\right)$, *for* $i \neq j \in \{A, B\}$. *The interval* $\left(\underline{c}_i(p_i, p_j), \bar{c}_i(p_i)\right)$ *is nonempty if and only if*

$$p_j < \frac{p_i}{1+p_i}. \tag{12}$$

The lower bound $\underline{c}_A(p_A, p_B)$ for the cost of effort c_A is obtained from condition (10). A similar lower bound for c_B is derived using a symmetric argument. When condition (11) holds, the firm prefers to shirk on one project when faced with repayment R_{11}. In the proof of Proposition 1, we show that this condition is also necessary. That is, considering the possibility that the firm prefers to shirk on both projects when faced with repayment R_{11} does not enlarge the set of parameters under which an equilibrium with joint effort can be ruled out.

Condition (11) states that the effort costs c_i should not be too large, as otherwise the firm would shirk on project $i \in \{A, B\}$ already under single-project financing. But the costs should also not be too small, as otherwise the firm would exert effort under joint-project financing. Importantly, Proposition 1 establishes that these two requirements can both be satisfied for project $i \in \{A, B\}$ under condition (12), which requires project $j \neq i$ to be sufficiently riskier than project i. In what follows, we explain this result by comparing the firm's incentives across the two modes of project financing.

To fix ideas, we focus on the firm's incentives for project A and show why incentives deteriorate under joint-project financing when project B is sufficiently riskier than project A. To this end, consider the term on the right hand side of (10), which measures the sensitivity of the firm's expected payoff under joint financing, $U_{joint}^F(1, 1, R_{11})$, to effort on project A (probability of success). It can be decomposed as follows:

$$p_B X_A + (1 - p_B)(X_A - R_{11}) \tag{13}$$

Then, subtract from the above the corresponding expression in the case of single-project financing, given by $X_A - \frac{1}{p_A}$, and use (9) to substitute for R_{11}, to obtain:

$$\underbrace{p_B \frac{1}{p_A}}_{\text{net worth effect}} \underbrace{+ (1 - p_B)\left(\frac{1}{p_A} - \frac{2}{1-(1-p_A)(1-p_B)}\right)}_{\text{net debt effect}}. \tag{14}$$

The above expression measures the relative strength of the borrower's incentives with joint financing relative to single financing. Under joint financing, each project acts as collateral for the other project. This has two opposite effects on the firm's payoff sensitivity to effort, and hence on its incentives, which we call *net worth effect* and *net debt effect*.

The net worth effect captures the classic role of collateral, which mitigates moral hazard by increasing the borrower's net worth. If project B succeeds, its payoff X_B entirely covers the repayment R_{11} of the debt that finances both projects.[10] Thus, all the cash flows from project A accrue to the firm, unlike with single-project financing, when it gets only a fraction $X_A - \frac{1}{p_A}$. As extra wealth available to repay the loan, project B thus improves the firm's incentives on project A. This effect is described by the first term in (14), which is positive.

The *net debt effect* is the flip side of the net worth effect, When project B fails, it is now project A that acts as collateral for project B. In this event, we see from the expression of $U^F_{joint}(1, 1, R_{11})$ that the firm must repay the entire debt out of the cash flows of project A. In this case, the firm's payoff sensitivity to effort is reduced to $X_A - R_{11}$ (see (13)). This term is smaller than the corresponding term under single-project financing, $X_A - \frac{1}{p_A}$, if and only if $p_B < p_A/(1 - p_A)$, that is, if project B is sufficiently risky. Under this condition, the net debt effect is negative since, when project B fails, project A supports a larger debt under joint financing than under single-project financing.

Altogether, we can say that incentives deteriorate with joint financing when the net debt effect is so negative that it dominates the net worth effect. This happens when $p_B < p_A/(1 + p_A)$, as stated in condition (12) of the proposition. This condition is satisfied only when p_B is sufficiently smaller than p_A, that is, when B is sufficiently riskier than A. To understand this result, observe from equation (14) that the net worth effect (net debt effect) has a strong positive (negative) impact when the probability of success of project B is large (small). In addition when p_B decreases, the face value of the debt under joint financing, R_{11}, increases, which further increases the negative impact of the net debt effect, because the face value of the debt under single-project financing of A is independent of p_B. This explains why incentives deteriorate under joint-project financing when one project is sufficiently riskier than the other. In that case, joint financing generates risk contamination across projects, as the risk in one project increases the riskiness of the other project by weakening incentives.

Remark 2. Our main focus is on situations in which joint financing weakens incentives. However, from the claim in Proposition 1 it also follows that when projects are ex-ante identical ($p_A = p_B$), the opposite result arises: joint financing strengthens incentives, because the net worth effect prevails. This is true more generally when projects have similar levels of risk: condition (12) implies that if $p_B \in \left[\frac{p_A}{1+p_A}, \frac{p_A}{1-p_A}\right]$, we have $\underline{c}_i(p_i, p_j) > \bar{c}_i(p_i)$ for both projects $i \in \{A, B\}$, with $j \neq i$. When $c_i \in \left[\bar{c}_i(p_i), \underline{c}_i(p_i, p_j)\right]$ for both projects $i \in \{A, B\}$, the firm exerts effort under joint financing, even if it shirks under single-project financing. This benefit of cross-pledging with joint-financing under moral hazard has been discussed since, at least, Diamond (1984). We show that, when investments have different risk profiles, cross-pledging has a flip side due to the net debt effect.

In what follows, we impose the conditions stated in Proposition 1 and focus on the case in which project B is sufficiently riskier than A:

Assumption 5. $c_A \in \left[\underline{c}_A(p_A, p_B), \bar{c}_A(p_A)\right]$ and $p_B < \frac{p_A}{1+p_A}$.

Proposition 1 identifies conditions such that the firm shirks on at least one project in equilibrium. Furthermore, under Assumption 5, the firm prefers to shirk on project A when lenders expect it to exert effort on both projects. While these results do not yet rule out an equilibrium in which the firm shirks only on project B, we show next that these conditions imply in fact that the firm shirks on project A in equilibrium.

Corollary 1. *Under Assumptions 1-5, in equilibrium, the firm shirks on project A under joint financing.*

To prove Corollary 1, we need to show that shirking on project A is optimal also if lenders expect the firm to shirk on project B. This implies that, under the stated conditions, the firm must shirk on project A in equilibrium. The result obtains because effort levels are complement under joint financing, even though the projects' technologies are independent. To see this, consider the benefit from exerting effort on project A, given by the right-hand side of condition (10). As we explained above, this benefit increases with p_B, the probability of success of project B. This means that the firm is more likely to exert effort on project A when project B has a higher

[10] This is true under Assumption 4.

probability of success. When the firm shirks on project B, its probability of success falls to $q_B < p_B$, thus weakening the incentives for project A. Intuitively, if the firm shirks on project B, this project becomes worse collateral for project A, thereby undermining its incentives to exert effort.[11]

4.2. Joint financing can induce shirking on both projects

The analysis above shows that, under Assumptions 1 to 5, the firm shirks on project A in equilibrium with joint financing. In this situation, the risk in project B contaminates project A by weakening the firm's incentives on this project. As a result, the firm shirks on project A and hence lowers that project's probability of success. This in turn implies that project A becomes worse collateral when we now consider the firm's incentives for project B. We show in what follows that, due to cross-risk contamination between the two projects, the firm may shirk on both in equilibrium.

Before stating our main result, we consider first a situation where lenders expect the firm to shirk on project A and exert effort on B, thus charging a repayment of the loan equal to R_{01}. In such a situation, we identify conditions under which the firm shirks on project B. To do this, we proceed as in the proof of Proposition 1 in Section 4.1 and verify that, when faced with repayment R_{01}, the firm indeed prefers to shirk rather than to exert effort on project B. Consider the threshold $\underline{c}_B(p_B, q_A)$ obtained by substituting p_A with q_A in the expression of equation (11). We then have the following result.

Lemma 2. *Suppose that lenders expect the firm to shirk on project A under joint financing. Then, the firm chooses to shirk also on project B, while it exerts effort under single-project financing if and only if* $c_B \in (\underline{c}_B(p_B, q_A), \bar{c}_B(p_B))$. *This interval is nonempty if and only if*

$$q_A < \frac{p_B}{1 + p_B}. \tag{15}$$

The conditions in Lemma 2 mirror the ones in Proposition 1. In particular, for project A to contaminate B, the former must be sufficiently riskier than the latter, when the firm shirks on project A. Note that condition (15) is compatible with condition (12) in Proposition 1, which is needed for B to contaminate A and requires project B to be (sufficiently) riskier than project A if the firm exerts effort on A. When project B contaminates project A, the latter becomes riskier as the firm now shirks. If the default probability of project A drops enough due to shirking—that is, Δ_A is large enough—, this project without effort becomes in turn riskier than project B with effort, and so may contaminate it too. Condition (15) says that, for this to happen, q_A, the probability of success of project A without effort, must be low enough relative to p_B, which is compatible with Assumption 5 stating that p_B must be low enough relative to p_A, the probability of success of project A with effort.

Putting together the results obtained thus far yields one of our main findings: joint financing may generate a complete breakdown of incentives:

Proposition 2. *Suppose Assumptions 1 to 4 hold. Under the following conditions, in equilibrium, the firm exerts effort on any project under single financing, but shirks on both projects under joint financing:*

$$p_B \in \left(\frac{q_A}{1 - q_A}, \frac{p_A}{1 + p_A} \right), \tag{16}$$

$$c_A \in \left(\Delta_A \left(X_A - \frac{2(1 - p_B)}{1 - (1 - p_A)(1 - p_B)} \right), \Delta_A \left(X_A - \frac{1}{p_A} \right) \right), \tag{17}$$

$$c_B \in \left(\Delta_B \left(X_B - \frac{2(1 - q_A)}{1 - (1 - q_A)(1 - p_B)} \right), \Delta_B \left(X_B - \frac{1}{p_B} \right) \right). \tag{18}$$

Conditions (16) to (18) are obtained by combining Proposition 1 and Lemma 2. Under these conditions, there is no equilibrium in which the firm exerts effort on any of the two projects. In an equilibrium with joint financing, there is in fact cross-risk contamination between projects. Project B contaminates incentives for project A because the former is exogenously riskier than the latter. As a result, project A becomes endogenously riskier, and in turn contaminates project B. Overall, risk contamination weakens the firm's incentives for both project and leads it to shirk on both.[12]

4.3. Optimality of joint financing

In the previous section, we established that joint financing of projects may induce the firm to shirk in equilibrium. This clearly reduces the value of both projects, relative to single-project financing. In this section, we show that it is still possible that the firm prefers to fund both projects rather than a single one, even though it exerts effort only in the latter situation.

[11] Our claim that incentives for project A deteriorate when the firm shirks on project B relies on the fact that $(1 - q_B)R_{10} > (1 - p_B)R_{11}$, not only because $q_B < p_B$, but also because the repayment value increases, that is, $R_{10} > R_{11}$.

[12] We remind the reader that we have been focusing on the case in which $p_A > p_B$ for illustrative purpose. A symmetric analysis applies if project A is riskier under effort than project B.

To understand the tradeoff underlying this choice, consider a firm who contemplates financing either only project A, or both projects A and B. Adding project B lowers the revenue generated by project A, when the risk contamination effect weakens the firm's incentives, as we showed in Proposition 1. In addition, under the conditions identified in Proposition 2, project B also becomes less profitable under joint financing, than if it were financed on a standalone basis.[13] In spite of the reduction in revenue from each project, the overall payoff the firm receives from the two projects, with joint financing, may still be higher, under the conditions presented below.

Proposition 3. *Under the conditions of Proposition 2, the firm prefers financing projects A and B jointly, rather than financing only project $i \in \{A, B\}$, if and only if*

$$q_j X_j - 1 \geq (p_i - q_i) X_i - c_i, \quad \text{with } j \neq i \tag{19}$$

Condition (19) follows directly from the comparison between the firm's payoff when it finances project i only,

$$U_i^F = p_i X_i - 1 - c_i, \tag{20}$$

and its payoff under joint financing, given by

$$U_{joint}^F = q_A X_A + q_B X_B - 2. \tag{21}$$

Under the conditions of Proposition 2, the firm shirks on both projects under joint financing. Its payoff is thus equal to the sum of the NPV of the two projects without effort, given by (21). Intuitively, the firm prefers to finance both projects if the net present value from the second project (with shirking) more than compensates the loss in payoff due to the lower effort exerted on the first project. Observe that inequality (19) can hold only if $q_j X_j > 1$, that is, if the additional project j has positive NPV even when the firm shirks.[14]

When the firm funds two projects, and does not exert effort on any of them in equilibrium, as in the situation described in Proposition 3, we argue that joint financing with debt is equivalent to an outright sale of the projects. With a sale, the firm retains no stake in the project, and does not exert effort. Lenders, who anticipate this outcome, are willing to pay up to $q_A X_A + q_B X_B - 2$ for the two projects, leaving the selling firm with the same payoff as under joint financing. A sale of the projects' cash flows can be interpreted as a securitization of the underlying investments. When the risk contamination channel is active, our model thus predicts that firms securitize more assets as they expand, and reduce their monitoring of the underlying investments.

Since the firm captures all the gains from trade, its choice between single-project financing and joint financing always maximizes welfare, subject to the incentive constraints associated with the firm's moral hazard problem. However, the effect of joint financing on default risk is interesting in itself and we analyze this next.

4.4. Default risk

In this section, we study the effect of joint financing on the default risk of firms, measured by the probability that a firm defaults on its financial obligations. Under Assumption 4, with joint financing, this outcome obtains if and only if both projects fail. Financing two projects rather than one has two effects on default risk. First, because the two projects have independent returns, adding one project generates diversification benefits, which tends to reduce default risk. On the other hand, the risk contamination effect tends to weaken incentives and increase the riskiness of individual projects relative to single-project financing. The next result shows that the second effect may dominate, that is, joint financing may increase the firm's default risk.

Corollary 2. *Under Assumptions 1 to 5, joint financing increases the firm's probability of default relative to single-project financing of project $i \in \{A, B\}$ if and only if*

$$(1 - q_A)(1 - q_B) > 1 - p_i. \tag{22}$$

The firm's default probability increases relative to single-financing of the weakest project if and only if

$$(1 - q_A)(1 - q_B) \geq \max\left\{1 - p_A, 1 - p_B\right\}. \tag{23}$$

Corollary 2 states the result for two different reference points under single-project financing. In the baseline application of our model, the firm considers the possibility of expanding and funding jointly multiple lines of business. The relevant condition is then (22), because the firm chooses only one project under single-project financing. If instead we consider the impact of a merger of two firms, each financing different business, into a single, larger firm, the stronger condition (23) becomes relevant as well (see Remark 1). It states that the combined firm is more likely to default than the weakest of the two firms before the merger. Finally,

[13] Of course, similar effects arise if the firm considers instead project B as its standalone project.

[14] The firm faces a nontrivial choice between these options, even if the reverse risk contamination effect is not active, that is, if the firm exerts effort on the additional project j in equilibrium. In this case, the left-hand-side of condition (19) should be replaced by $p_j X_j - 1 - c_j$.

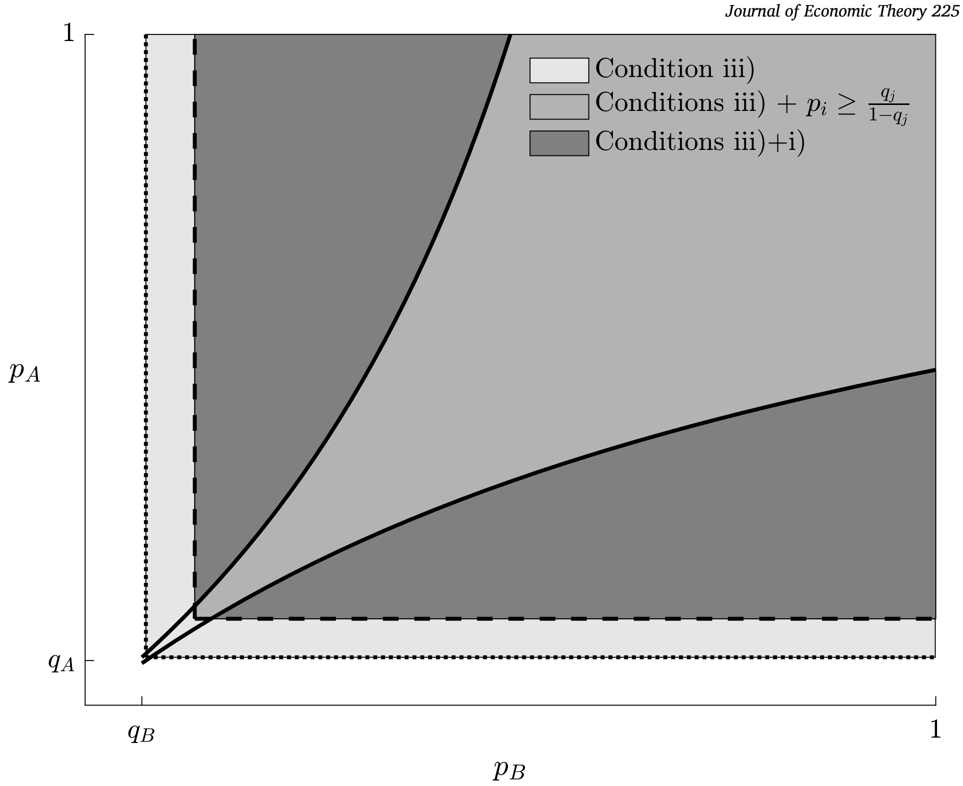

Fig. 1. Parameters range for which default risk increases with joint financing.

under the conditions of Corollary 2, joint financing not only increases the default probability, but the firm also defaults on a larger loan, relative to single-project financing.

4.5. Default risk in an example

In this section, we show that there exists an open set of parameters such that (i) the firm exerts no effort in equilibrium under joint financing, that is, Assumptions 1 to 4 and conditions (16)-(18) hold, (ii) the firm prefers borrowing 2 units rather than 1, that is, condition (19) holds for any $i \in \{A, B\}$, and (iii) the probability of default increases with joint financing (in the stronger sense that condition (23) holds). We establish the result via a numerical example.

We set $X_A = X_B = 30$, $q_A = q_B = 1/15$, and we let p_A vary in the interval $(q_A, 1]$ and p_B in $(q_B, 1]$. For each project $i \in \{A, B\}$, we set $c_i = \bar{c}_i(p_i)$ so that Assumption 3 holds as an equality, that is, we consider the highest value of the effort cost compatible with effort under single-project financing. This condition implies Assumption 1, which states that effort dominates shirking for both projects. Next, from equation (7), we obtain the face value of a two-unit loan when no effort is exerted: $R_{00} \approx 15.5 < 30$, so that Assumption 4 is satisfied. Finally, condition (ii), implying that the firm prefers to borrow 2 units rather than 1, holds by design when $c_i = \bar{c}_i(p_i)$, because then

$$q_i X_i - 1 = 2 > (p_i - q_i) X_i - c_i = \frac{p_i - q_i}{p_i}, \quad i \in \{A, B\}, \; j \neq i,$$

where the equality follows from the definition of $\bar{c}_i(p_i)$ in (5).

In Fig. 1, the remaining conditions in (i) and (iii) above hold jointly in the regions with the darkest shade of gray. There are two such regions in the (p_B, p_A) space, which are symmetric with respect to the 45 degree line, because projects are symmetric in this example. To see why the remaining conditions hold in those regions, observe first that the dotted boundaries of the lightest gray region correspond to the minimal admissible values of p_A and p_B for condition (iii) to hold. This implies that condition (iii) holds in the shaded regions above and to the right of these dotted boundaries. Next, the dashed boundaries of the intermediate gray region correspond to $p_i = \frac{q_j}{1-q_j}$ for $i \in \{A, B\}$ and $j \neq i$, which is the minimal value of p_i that satisfies condition (i). Finally, the solid boundary of the top (bottom) darkest gray region correspond to $p_B = \frac{p_A}{1+p_A}$ $(p_A = \frac{p_B}{1+p_B})$, the maximum value of p_B (p_A) that satisfies condition (i). Overall, condition (i), implying that the firm prefers to shirk on both projects in equilibrium holds in either darkest gray region. Given that condition (iii) holds there as well, these dark gray regions identify the parameter values for which the firm chooses to expand its operation and funds both projects, even though this undermines incentives and increases its default risk.

5. Risk contamination with news

So far, we assumed that under joint financing the firm chooses its effort simultaneously on both projects. Next we consider the case in which the firm first exerts effort on project B and then on project A. Before choosing its effort on project A, the firm receives a signal about the realization of project B's cash flow.[15] To characterize the effects of interim payoff news as clearly as possible, we focus on the case in which the signal about project B's cash flow is perfectly informative.

As we will see, with joint financing, the arrival of news about the payoff of project B affects the firm's incentives to exert effort on project A over and beyond the effects analyzed in Section 4. To make the analysis more transparent, we impose the following conditions:

Assumption 6. $c_B \le c(p_B, q_A)$, $c_A \le \underline{c}_A(p_A)$.

The first inequality in Assumption 6 implies that in equilibrium the firm always exerts effort on project B, even if it shirks on project A under joint financing. Relaxing that assumption would only make our result that default risk increases stronger, because reverse risk contamination from project A to B could then arise. When it comes to project A, Assumption 6 only implies that the firm would exert effort under single-project financing. As Proposition 1 shows, under that condition, whether the firm exerts effort or shirks with joint financing remains ambiguous, when it does not receive news about project B.

We show in what follows that, in the presence of news, the firm's default risk increases. For this analysis, we index variables with the superscript $n \in \{b, g, \emptyset\}$, where b (g) stands for bad (good) news, while $\emptyset$ stands for no news, as in our previous analysis. Let $R^{*,\emptyset} \in \{R_{11}, R_{01}\}$ be the equilibrium face value when the firm receives no news.

To establish the claim, we analyze first the effect of news on the firm's incentives to exert effort on project A when faced with repayment $R^{*,\emptyset}$. While $R^{*,\emptyset}$ may not be the equilibrium face value of the debt when the firm receives news, this analysis suffices to establish our claim. Let $p_B^n \in \{0,1\}$ be the realized value of p_B when news $n \in \{b, g\}$ arrive. When the firm faces repayment $R^{*,\emptyset}$ and receives news $n \in \{b, g\}$, it chooses to exert effort if and only if

$$c_A \le \Delta_A(X_A - (1 - p_B^n)R^{*,\emptyset}). \tag{24}$$

If the firm receives good news, that is, if $p_B^n = 1$, condition (24) holds, by Assumption 1. When the firm learns that project B will succeed with certainty, only the net worth effect is present, which improves the firm's incentives. As the sole claimant to project A, the firm finds it optimal to exert effort. If instead the firm receives bad news, that is, if $p_B^n = 0$, only the net debt effect is present, which undermines the firm's incentives. Thus, with bad news, the firm chooses a weakly lower effort than without news. The discussion above thus tells us that the effect of news on the firm's incentives for project A is ambiguous. In spite of this, overall default risk weakly increases, relative to the no-news case. The result follows from the asymmetry between news outcome. With news, the firm's expected probability of default is:

$$(1 - p_B)(1 - \pi_A(e_A^b)) + p_B \cdot 0, \tag{25}$$

where $e_A^b \in \{0,1\}$ denotes the effort exerted in project A after the arrival of bad news. The effort exerted after the arrival of good news does not appear in (25), since the firm never defaults when project B succeeds. In contrast, the firm's probability of default without news is:

$$(1 - p_B)(1 - \pi_A(e_A^\emptyset)), \tag{26}$$

where $e_A^\emptyset \in \{0,1\}$ is the effort exerted in A without news. We showed above that when faced with the equilibrium repayment $R^{*,\emptyset}$ of the no-news case, the firm chooses $e_A^b \le e_A^\emptyset$. As a result, the firm's default probability (weakly) increases in the presence of news. This comparison between expected probabilities of defaults reveals that the firm's default risk depends only on its effort choice for project A when project B fails. While the firm may exert more effort when it receives good news than in the no-news case, its default probability does not decrease, because project B alone is sufficient to repay the debt under Assumption 4. When instead bad news realizes, project A is the only source of income for lenders. The worsening of incentives leading to an increase in the probability of failure of project A thus translates into a (weakly) higher default probability for the firm overall.

We showed that the firm default risk weakly increases with news when faced with the equilibrium face value of debt of the no-news case, $R^{*,\emptyset}$. This in turn implies that, in an equilibrium with news, lenders require a (weakly) higher face value of the debt, which further undermines the firm's incentives, and increases its default risk.

Proposition 4. *Under Assumptions 1 to 4 and 6, the arrival of news weakly increases the firm's expected probability of default. It strictly increases it if*

$$c_A \in \left(\Delta_A(X_A - R_{11}), \underline{c}_A(p_A, p_B)\right), \tag{27}$$

which is a nonempty interval.

[15] Without the arrival of news at the interim stage, the sequentiality in effort decisions would play no role.

Proposition 4 establishes that the firm's default risk always weakly increases when the firm receives interim news about the payoff of one project. Condition (27) then characterizes the conditions under which the increase in default probability is strict. When $c_A \leq \underline{c}_A(p_A, p_B)$, we know from Proposition 1 that, in the absence of news, the firm exerts effort on project A in equilibrium under joint financing. In contrast, when $c_A > \Delta(X_A - R_{11})$, the incentive constraint in (24) in violated, hence the firm would shirk when it receives bad news, if faced with repayment R_{11}. Hence, when these two conditions hold, that is when c_A lies in the interval defined by condition (27), the firm's default risk strictly increases relative to the case without news.[16]

6. Optimal collateral choice

In this section we show that risk contamination between projects may also impact the firm's selection of new projects to finance, not just the scale of its operation. Under joint financing, the collateral value of a project matters, over and beyond its NPV when financed on a standalone basis. To formalize this insight, suppose that the firm has access to two types of B projects, $B1$ or $B2$. When it finances two projects, the firm can choose either $B1$ or $B2$, together with project A. Unlike the effort choice, the project choice is observable, and thus contractible.

We impose Assumption 3 for all projects, which implies that the firm would exert effort on any project under single-project financing. To make the analysis interesting, we assume that project $B1$ has a lower NPV than project $B2$:

$$NPV_{B1} \equiv p_{B1}X_{B1} - 1 - c_{B1} < p_{B2}X_{B2} - 1 - c_{B2} \equiv NPV_{B2}, \tag{28}$$

Under single-project financing, the above condition implies that the firm would choose project $B2$ over $B1$. Under joint financing, however, the firm also takes into account the potential risk contamination between projects. To focus on the risk contamination from B projects to project A, we assume that the firm would exert effort on any project B if it exerts effort on project A, that is,

Assumption 7. $c_{Bi} \leq \underline{c}_B(p_{Bi}, p_A)$ for $i \in \{1, 2\}$.

It then follows from Proposition 1 that, under Assumption 7, the firm exerts effort on project Bi for $i = 1, 2$ if lenders expect it to exert effort on both projects A and Bi. Hence, joint financing may only weaken incentives because risk contamination may arise from a B project to project A. We then obtain the following result:

Proposition 5. *Suppose that project $B2$ is riskier than project $B1$, that is, $p_{B2} < p_{B1}$ such that*

$$\underline{c}_A(p_A, p_{B2}) < c_A \leq \underline{c}_A(p_A, p_{B1}). \tag{29}$$

The firm then prefers to finance project $B1$ with project A, although project $B2$ has the highest NPV if

$$NPV_{B_2} - NPV_{B1} < \Delta_A X_A - c_A. \tag{30}$$

Condition (29) implies that only the higher-NPV, but riskier project $B2$ contaminates project A under joint financing. Hence, in equilibrium the firm would shirk on project A if it chose project $B2$. With project $B1$, instead, the firm sacrifices some NPV, but it can avoid this risk contamination. This generates a tradeoff in the choice of different B projects under joint financing, which is captured by condition (30): under joint financing the firm prefers the lower NPV but safer project $B1$, if the drop in NPV relative to project $B2$ it lower than the expected loss due to the induced reduction in effort on project A.

Observe that condition (30) is only a sufficient condition for the firm to prefer the safer project despite its lower NPV. Since project $B2$ contaminates project A and lowers effort on this project under joint financing, this could in turn lead to a cross-risk contamination between projects so that effort on B may also be lower (see Proposition 2). If this is the case, when choosing project $B2$, the firm would take into account both the drop in NPV from shirking on project A, as in equation (30), and the drop in NPV on project $B2$, relative to single-project financing.[17]

Overall, Proposition 5 shows that a firm might choose new investments conservatively to avoid the risk contamination effect. Relative to a firm who finances only one project, a firm with a legacy project tends to select safer new investments at the expense of pure value creation. Our model thus predicts that large firms may endogenously create less value than small firms, because risk contamination under joint financing can jeopardize their legacy investments.

[16] When condition (27) does not hold, news does not change the firm's default probability. As our discussion suggests, however, the result in Proposition 4 is weak only because the firm faces a binary effort choice. In a previous version of this article, we showed that with a continuous effort choice for project A, news always strictly increases the firm's default risk, because in that case $e^b_A < e^\emptyset_A$.

[17] Formally, if condition $c_{B2} > \underline{c}_B(p_{B2}, q_A)$ in Proposition 2 also holds, condition (30) becomes

$$q_{B2}X_{B2} - NPV_{B1} < \Delta_A X_A - c_A.$$

7. Conclusion

We study the effect of investment diversification on incentives and fragility. When multiple projects are jointly financed, each project acts as collateral for the other projects of a firm. We show that, if investments are subject to moral hazard, positive-NPV but risky projects may weaken incentives for safer projects under joint financing. The cross-pledging benefits under moral hazard identified by the literature thus have a flip side when investments have different risk profiles. As a result, while a firm expansion diversifies its sources of income, a risk contamination channel can ultimately increase its default risk. Such increase in default risk is exacerbated in the presence of news about the yields of investments.

Firms can mitigate the risk contamination arising with joint financing of multiple projects by modifying the type of projects they choose to operate. Large firms, anticipating potential risk contamination from new investments, act more conservatively in their project selection than small firms, who simply maximize net present value. Risk contamination under joint financing can thus explain why large firms are less productive than small firms, even if they have access to the same pool of investments. Similarly, due to this flip side of financial synergies, M&A activity may lower firm productivity relative to the pre-merger situation.

CRediT authorship contribution statement

Piero Gottardi: Writing – review & editing, Writing – original draft, Formal analysis. **Vincent Maurin:** Writing – review & editing, Writing – original draft, Formal analysis. **Cyril Monnet:** Writing – review & editing, Writing – original draft, Formal analysis.

Declaration of competing interest

None.

Appendix A. Monotone loan contracts

In this section, we relax Assumption 2. Now, the firm can raise funds using any loan contract with a repayment that is monotone in the total cash flows of the projects that are operated. We show that, under some minimal additional assumption, Proposition 1 continues to hold. The new assumption is as follows:

Assumption A.1. $X_A > X_B$.

Under Assumption A.1, the total cash flows from the projects can be ordered as follows: $0 < X_B < X_A < X_A + X_B$. We use here the notation R_i to indicate the repayment due when only project $i \in \{A, B\}$ succeeds, and R_{A+B} for the repayment when both projects succeed. We say that a loan contract has a monotone repayment schedule - in short, is monotone - if its repayment is increasing with the total cash flows, that is, if

$$0 \leq R_B \leq R_A \leq R_{A+B}. \tag{A.1}$$

A standard debt contract is monotone, because $R_A = R_B = R_{A+B}$. For a generic monotone loan contract, the lenders' participation constraint is

$$p_A p_B R_{A+B} + p_A(1 - p_B)R_A + p_B(1 - p_A)R_B \geq 2 \tag{A.2}$$

Before stating the result, we observe that no monotone contract can improve upon the standard debt contract with repayment value

$$R_A^0 = R_B^0 = R_{A+B}^0 = \frac{2}{1 - (1 - p_A)(1 - p_B)}, \tag{A.3}$$

when, for this repayment, the firm exerts effort on both projects. In this case, the firm extracts all the surplus from the two projects, as when there is no moral hazard. A more interesting question is whether the restriction to a standard debt contract binds when the conditions identified in Proposition 1 hold. That is, could the firm implement joint effort with a monotone loan contract, while it cannot with a standard debt contract? As the next result shows, the answer is negative.

Proposition A.1. *Under the same conditions of Proposition 1 and Assumption A.1, effort on project A cannot be implemented in equilibrium, even if monotone loan contracts are available.*

Proof. Similarly to the main text, we prove the result by contradiction. We suppose that lenders expect the firm to exert effort on both projects, and show that the firm prefers to shirk on project A under any monotone loan contract that satisfies the lenders' participation constraint (A.2), when they anticipate the firm to exert effort on both projects.

Consider a candidate loan contract with repayment schedule (R_A, R_B, R_{A+B}). We derive a necessary condition for this contract to implement joint effort in equilibrium. Let

$$r_A = R_A - R_A^0, \quad r_B = R_B - R_B^0, \quad r_{A+B} = R_{A+B} - R_{A+B}^0,$$

be the differences in repayment relative to the standard debt contract, specified in (A.3). Proposition 1 shows that, when the firm is faced with repayment schedule $(R_A^0, R_B^0, R_{A+B}^0)$, the incentive constraint to exert effort on project A is violated if $c_A > \underline{c}_A(p_A, p_B)$. A monotone loan contract can improve upon the standard debt contract only if it allows to relax this constraint. The expression of the incentive constraint for general monotone contracts is:

$$p_A p_B(X_A + X_B - R_{A+B}) + p_A(1-p_B)(X_A - R_A) + p_B(1-p_A)(X_B - R_B) \geq$$
$$q_A p_B(X_A + X_B - R_{A+B}) + q_A(1-p_B)(X_A - R_A) + p_B(1-q_A)(X_B - R_B) + c_A. \tag{A.4}$$

Using $\Delta_A = p_A - q_A$, the above inequality can be rewritten as follows:

$$\Delta_A(X_A - (1-p_B)R_A + p_B R_B - p_B R_{A+B}) \tag{A.5}$$
$$= \Delta_A \left(X_A - (1-p_B)(R_A^0 + r_A) + p_B(R_B^0 + r_B) - p_B(R_{A+B}^0 + r_{A+B})\right) \geq c_A \tag{A.6}$$

We then see that the incentive constraint is relaxed for a monotone loan contract relative to a standard debt contract only if

$$-(1-p_B)r_A + p_B r_B - p_B r_{A+B} > 0. \tag{A.7}$$

Next, we show that condition (A.7) cannot hold for a monotone loan contract satisfying (A.2). Given that the debt contract $(R_A^0, R_B^0, R_{A+B}^0)$ satisfies the lenders' participation constraint as equality (when they expect joint effort), a candidate monotone contract satisfies the lenders' participation constraint (A.2) if and only if

$$p_A(1-p_B)r_A + p_B(1-p_A)r_B + p_A p_B r_{A+B} \geq 0. \tag{A.8}$$

The above inequality can be equivalently rewritten as

$$-p_B r_{A+B} - (1-p_B)r_A \leq \frac{p_B}{p_A}(1-p_A)r_B. \tag{A.9}$$

Combining inequalities (A.7) and (A.9), we obtain

$$0 < -(1-p_B)r_A + p_B r_B - p_B r_{A+B} \leq p_B r_B + \frac{p_B}{p_A}(1-p_A)r_B = \frac{p_B}{p_A} r_B \tag{A.10}$$

Hence, we must have $r_B > 0$. Given that the standard debt contract satisfies the monotonicity constraints in (A.1) as equality, the contract $\left(r_A + R_A^0, \quad r_B + R_B^0, \quad r_{A+B} + R_{A+B}^0\right)$ satisfies (A.1) if and only if

$$r_{A+B} \geq r_A \geq r_B. \tag{A.11}$$

We saw that condition (A.7) requires $r_B > 0$. However, when $r_B \geq 0$, condition (A.7) is incompatible with the monotonicity constraints in condition (A.11), because then

$$-(1-p_B)r_A + p_B r_B - p_B r_{A+B} \leq -(1-p_B)r_A \leq 0.$$

This implies that there exists no monotone loan contract that simultaneously relax (A.4) relative to a standard debt contract and satisfies lenders' participation constraint (A.2). This proves the firm will shirk on project A in equilibrium when faced with the repayment schedule of any monotone loan contract that satisfies lenders' participation constraint (A.2). This concludes the proof. □

Appendix B. Proofs

Proof of Proposition 1. First, we derive the conditions in Proposition 1 ensuring that the firm prefers to shirk on project $i \in \{A, B\}$, when it exerts effort on project $j \neq i$. Next, we will show that these conditions are also necessary for the firm to prefer shirking on both projects.

Step 1: Conditions (11) and (12)

As argued in the main text, we obtain the lower bound $\underline{c}_i(p_i, p_j)$ for c_i, by substituting the value of R_{11} obtained from the lenders' participation constraint in condition (10). This latter condition ensures that the firm prefers to shirk on project i than exerting effort. The upper bound $\bar{c}_i(p_i)$ for c_i, follows from Assumption 3. Hence, we are left to derive conditions such that the interval for c_i given by these two bounds is nonempty. This is the case if and only if

$$\frac{2(1-p_j)}{1-(1-p_i)(1-p_j)} \geq \frac{1}{p_i} \tag{B.1}$$
$$\Leftrightarrow \quad 2(1-p_j)p_i \geq p_i + p_j - p_i p_j, \tag{B.2}$$
$$\Leftrightarrow \quad p_i \geq p_j + p_j p_i. \tag{B.3}$$

The last inequality is equivalent to condition (12).

Step 2: Joint Deviation

For the second part of the proof, we show that the firm prefers to exert effort on both projects instead of shirking on both, when faced with repayment R_{11}, if effort is implementable under single-project financing, and no single deviation is profitable. This proves that considering such a joint deviation does not enlarge the set of parameters that allow us to rule out an equilibrium with joint effort.

Given the same repayment value R_{11}, the firm prefers to exert effort on both projects, rather than shirking on both, if and only if:

$$p_A X_A + p_B X_B - \left[p_A + p_B - p_A p_B\right] R_{11} - c_A - c_B \geq q_A X_A + q_B X_B - [q_A + q_B - q_A q_B] R_{11},$$

or equivalently,

$$\begin{aligned} c_A + c_B &\leq \Delta_A X_A + \Delta_B X_B - \left[\Delta_A + \Delta_B - (p_A p_B - q_A q_B)\right] R_{11}, \\ &= \Delta_A X_A + \Delta_B X_B - 2\frac{\Delta_A + \Delta_B - (p_A p_B - q_A q_B)}{1-(1-p_A)(1-p_B)}. \end{aligned} \tag{B.4}$$

Our objective is to show that incentive-compatibility constraint (B.4) holds when effort is implementable under single-project financing, and no single deviation is profitable. Building on Step 1 of this proof, this property holds if and only if

$$\begin{aligned} &\min\left\{\underline{c}_A(p_A,p_B), \bar{c}_A(p_A)\right\} + \min\left\{\underline{c}_B(p_B,p_A), \bar{c}_B(p_B)\right\} \\ &\quad \leq \Delta_A X_A + \Delta_B X_B - 2\frac{\Delta_A + \Delta_B - (p_A p_B - q_A q_B)}{1-(1-p_A)(1-p_B)}. \end{aligned} \tag{B.5}$$

Observe that condition (B.5) is symmetric with respect to p_A and p_B. Hence, it is enough to consider the case $p_B \leq p_A$, because a symmetric analysis applies to the case $p_B \geq p_A$. When $p_B \leq p_A$, condition (12) implies that $\bar{c}_B(p_B) \leq \underline{c}_B(p_B, p_A)$. Since the value of the first term on the left-hand side of (B.5) depends on the value of p_B, we need to distinguish two cases.

Case 1: $p_B \in \left[\frac{p_A}{1+p_A}, p_A\right]$

In this case, condition (12), or equivalently 6, derived in Step 1 above, implies that $\bar{c}_A(p_A) \leq \underline{c}_A(p_A, p_B)$. To prove that (B.5) holds, observe that it is equivalent to

$$\begin{aligned} &0 \leq \Delta_A X_A + \Delta_B X_B - 2\frac{\Delta_A + \Delta_B - (p_A p_B - q_A q_B)}{1-(1-p_A)(1-p_B)} - \bar{c}_A(p_A) - \bar{c}_B(p_B) \\ &\Leftrightarrow 0 \leq \frac{\Delta_B}{p_B} + \frac{\Delta_A}{p_A} - 2\frac{\Delta_A + \Delta_B - p_A\Delta_B - p_B\Delta_A + \Delta_A\Delta_B}{p_A + p_B - p_A p_B} \end{aligned} \tag{B.6}$$

Cumbersome but simple manipulations show that the condition above is equivalent to

$$\Delta_B\left[-p_A\left(p_A p_B + p_A - p_B\right) + 2\Delta_A p_A p_B\right] \leq p_B \Delta_A \left(p_B p_A + p_B - p_A\right) \tag{B.7}$$

The term on the right-hand side of (B.7) is positive, because $p_B \geq \frac{p_A}{1+p_A}$ in Case 1. Hence, inequality (B.7) is satisfied if the term multiplying Δ_B on the left-hand side of (B.7) is negative. If instead it is positive, we can rewrite (B.7) as

$$\Delta_B \leq \frac{p_B \Delta_A \left(p_B p_A + p_B - p_A\right)}{2\Delta_A p_A p_B - p_A\left(p_A p_B + p_A - p_B\right)} \equiv G(\Delta_A) \tag{B.8}$$

The term on the right-hand side of (B.8) is decreasing with Δ_A. Hence, this inequality is satisfied for all values of Δ_A if it is satisfied for $\Delta_A = p_A$. We have

$$\begin{aligned} G(p_A) &= \frac{p_B p_A \left(p_B p_A + p_B - p_A\right)}{2p_A^2 p_B - p_A\left(p_A p_B + p_A - p_B\right)} \\ &= \frac{p_B\left(p_B p_A + p_B - p_A\right)}{p_A p_B - p_A + p_B} = p_B \geq \Delta_B \end{aligned} \tag{B.9}$$

Hence, the desired inequality (B.5) holds for all parameter configurations compatible with Case 1.

Case 2: $p_B \in \left[0, \frac{p_A}{1+p_A}\right]$

In this case, condition (12), implies that $\bar{c}_A(p_A) > \underline{c}_A(p_A, p_B)$. To prove that (B.5) holds, observe that it is now equivalent to

$$\begin{aligned} &0 \leq \Delta_A X_A + \Delta_B X_B - 2\frac{\Delta_A + \Delta_B - (p_A p_B - q_A q_B)}{1-(1-p_A)(1-p_B)} - \underline{c}_A(p_A,p_B) - \bar{c}_B(p_B) \\ &\Leftrightarrow 0 \leq \frac{\Delta_B}{p_B} + 2\frac{\Delta_A(1-p_B)}{1-(1-p_A)(1-p_B)} - 2\frac{\Delta_A + \Delta_B - p_A\Delta_B - p_B\Delta_A + \Delta_A\Delta_B}{1-(1-p_A)(1-p_B)} \\ &\Leftrightarrow 0 \leq \frac{\Delta_B}{p_B} - 2\frac{\Delta_B(1-p_A) + \Delta_B\Delta_A}{1-(1-p_A)(1-p_B)} \\ &\Leftrightarrow 0 \leq \Delta_B(p_A + p_B - p_A p_B) - 2p_B\Delta_B(1 - p_A + \Delta_A) \end{aligned}$$

$$\Leftrightarrow 0 \leq -2p_B\Delta_A - p_B + p_A p_B + p_A$$
$$\Leftrightarrow \Delta_A \leq \frac{1-p_A}{2} + \frac{p_A}{2p_B} \leq \frac{1-p_A}{2} + \frac{1+p_A}{2} = 1,$$

where the last inequality is implied by the upper bound for p_B in Case 2. Hence, since $\Delta_A = p_A - q_A < p_A \leq 1$, the desired inequality (B.5) also holds for Case 2, which concludes the proof. □

Data availability

No data was used for the research described in the article.

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