Bank Liquidity, Interbank-Rate Setting and Heterogeneous Lending Responses

Theory of Central Banking Driven by Bank Liquidity Constraint

Tianxi Wang*

Abstract: Central banks move the interbank borrowing rate through nominal operations that alter the private sector's portfolio of fiat money and government bonds. This paper elucidates a mechanism by which such operations move the interbank rate, thereby affecting bank lending and thence the wide economy. At the core is a liquidity constraint to which bank lending is subject. By affecting the maximal tightness of banks' liquidity constraints, the aggregate portfolio of fiat money and bonds determines the interbank rate. Accordingly, an operation that alters the portfolio moves the interbank rate. The tightness of the liquidity constraint depends on a bank attribute related to money circulation and affects banks' responses to monetary policy. When the central bank (say) decreases the interbank rate, liquidity-unconstrained banks decrease their lending rates, but contrary to received wisdom, maximally constrained banks increase theirs.

Keywords: Bank liquidity, interbank borrowing rate, aggregate portfolio of fiat money and government bonds, liquidity constraint on bank lending, heterogeneous policy response, money circulation

Classification: JEL: E40, E50, G21

***Correspondence address:** Department of Economics, University of Essex, Colchester, CO4 3SQ, U.K. wangt@essex.ac.uk

I thank for their comments Wouter den Haan, Chao He, Michael Kumhof, Mohammed Lahcen, Ricardo Reis, Xuan Wang and seminar particupants at Academia Sinica, National Taipei University, Fudan University, SHUFE and Economeric Society North American Summer Meeting 2019.

1 Introduction

The interbank reserve borrowing rate is a basic policy target of central banks. To shift this rate, they conduct nominal operations that change the aggregate portfolio of fiat money and government bonds that the private sector holds, exemplified by open market operations. When such an operation is conducted to shift the interbank rate, one tends to think that the rate shift is all that matters for bank lending and the portfolio change matters only because it is instrumental for shifting the rate, like scaffolding for building a house; when the house is built, the scaffolding can be put aside. However, this paper demonstrates that the portfolio change has an impact on bank lending by itself. And this impact is important for two reasons. First, it is through this impact that a portfolio changing nominal operation shifts the interbank rate. Hence, the direct impact of the portfolio change on bank lending is an intrinsic part of the policy operation' effects.

Second, taking into account this impact overturns received wisdom regarding banks' responses to monetary policy. When a central bank shifts the interbank rate, received wisdom predicts that all banks' lending rates should shift in the same direction as the interbank rate : They should *all* rise when it rises and *all* fall when it falls. The reasoning is straightforward. The interbank rate is the cost of banks obtaining liquidity. If the rate falls, then banks' funding costs all fall, and so should all their lending rates. This reasoning, however, overlooks the direct impact of the portfolio change that causes the interbank-rate shift in the first place. When this impact is accounted for, the picture changes. The impact is exerted through a *liquidity constraint* to which bank lending is subject. In response to an interbank-rate shift, liquidity unconstrained banks shift their lending rates in the same direction as the interbank rate, but banks that are maximally liquidity-constrained shift theirs in the opposite direction: Their lending rates *fall* when the interbank rate rises and *rise* when it falls.

The liquidity constraint results from the following facts.[1] Banks issue nominal deposit liabilities to finance lending. These liabilities need fiat money to service: E.g. depositors might demand to withdraw and to meet this demand fiat money is needed. Therefore, *lending demands fiat-money liquidity to service*. If a bank's position of fiat-money reserves is inadequate to meet this liquidity demand, the bank has to borrow fiat money on the interbank reserve market. Placing government bonds as collateral substantially facilitates this borrowing and reduces its costs.[2] A bank's capacity for secured borrowing is thus limited by its bond positions. This capacity plus its reserve position constitutes the quantity of liquidity that the bank can obtain without resorting to unsecured borrowing, which is costly. Hence the *liquidity constraint*: To avoid costly unsecured borrowing, banks must limit the lending scale below a threshold in order that they can gather liquidity to service lending from their reserve and bond positions.

The liquidity constraint has two implications. First, it engenders a component of the marginal lending cost. To enlarge lending by one unit, the bank needs to issue one more unit of liability and find extra liquidity to service it, which strains the liquidity constraint and incurs a cost that is proportional to the tightness of the constraint represented by the Lagrange multiplier. Second, holding fiat money now delivers two benefits: It earns interest on the interbank reserve market; and it relaxes the liquidity constraint. The scale of the former is equal to the interbank rate, the latter the tightness of the constraint. Banks that face the tightest constraints, therefore, obtain the maximal total sum of the benefits. In equilibrium, these banks outbid other banks in acquiring fiat money and their marginal benefit from holding it is equalised to the marginal cost of holding it due

[1] More details are given in Appendix A.

[2] See Heider and Hoerova (2009), Figure 1. In general, the importance of public debt as collateral for obtaining liquidity is well established in economics literature. See Woodford (1990), Aiyagari and McGrattan (1998), Holmström and Tirole (1998), and recently Angeletos *et al.* (2016). Some empirical evidence is provided by Grobéty (2018).

to time preference. This cost is a constant. Hence, the sum of the interbank rate and the maximal tightness (i.e. the largest Lagrange multiplier) is a constant.

A change to the private sector's portfolio of fiat money and government bonds affects the tightness of banks' liquidity constraints. For intuition, consider an open market operation of buying bonds. It injects into the economy fiat money, which, in steady states, tends to reduce its unit real value, while leaving its aggregate real value largely unchanged. A unit of the bonds is defined by a fixed stream of nominal dividend; its real value hence falls when the unit real value of fiat money reduces. Furthermore, the operation retires bonds from the economy. The aggregate real value of bonds, therefore, falls. Altogether, the operation changes little fiat money's aggregate real value and lowers bonds', and hence lowers the real value of the portfolio of fiat money and bonds. Because fiat money and bonds are liquid assets, this value represents the aggregate liquidity supply. By changing this supply, a change to the portfolio alters the tightness of liquidity constraints and, in particular, the maximal tightness. This constitutes the direct impact of portfolio changes referred to at the beginning of the Introduction.

It is via this direct impact that a portfolio change shifts the interbank rate in steady states. The sum of the maximal tightness and the interbank rate, I have argued, is a constant. Whenever the portfolio change increases (decreases) the maximal tightness, the interbank rate must accordingly decrease (increase).

This mechanism implies the aforesaid heterogeneous responses of bank lending to monetary policy: Whenever a portfolio changing operation shifts the interbank rate, liquidity unconstrained banks' lending rates shift in the same direction as the interbank rate, but maximally constrained banks' shift in the opposite direction. For intuition, suppose the interbank rate falls. Then the cost of obtaining liquidity falls for *all* banks. This change, however, is not the only show in town. By the mechanism, the portfolio change causes the interbank-rate to fall exactly because it increases the maximal tightness of the liquidity constraint. This impact does not concern banks whose liquidity constraints are nonbinding; their lending rates duly fall. However, it concerns banks whose liquidity constraints are of the maximal tightness: The impact is exactly that their liquidity constraints are tightened. This increases their liquidity-constraint costs. The increase, it turns out, outweighs the decrease in the cost of obtaining liquidity. In net, these banks' lending costs rise. Consequently, they *raise* lending rates, and *contract* lending scales, in response to rate cutting operations.

Evidence for the heterogeneous response to monetary policy is presented in Section 4. In fact, the paper's theoretical analysis leads to a novel empirical approach to evaluate banks' liquidity constraints. It is applied to test the prediction of the opposite responses to monetary policy of those two types of banks, using the same data as Kashyap and Stein (2000), and the results are consistent. Furthermore, other supportive evidence is found in existing empirical studies such as Wang *et al.* (2022).

Given the importance of liquidity constraint tightness, a key question arises: what drives it? I identify a determining factor – a bank attribute related to money circulation. After a bank's money is lent out, it is used by the borrowers for multitudes of trades, whereby it circulates into the counterparties' accounts, some of which are with other banks. That is, for each bank, a fraction of the money issued from its lending flows out into other banks, the rest flowing back to itself, as illustrated in Figure 1 below. In the model economy, banks have heterogeneous outflow fractions.[3] I find that the higher the outflow fraction, the tighter the liquidity constraint.

Literature

[3]The outflow fraction depends on how the bank connects with the real economy and with other banks, as Wang (2024) demonstrates.

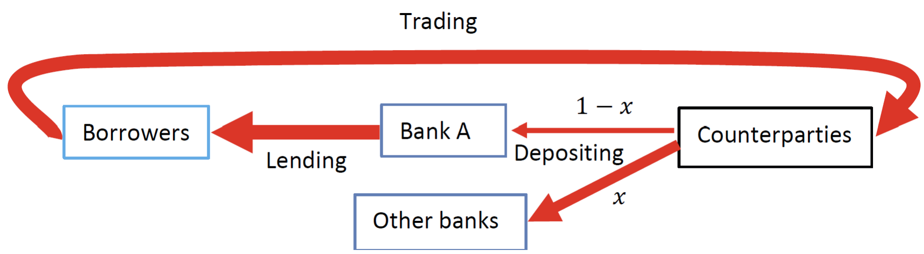

Figure 1: The arrows represent money flowing directions. Fraction x of money that Bank A lends out circulates into other banks, so its outflow fraction is x.

This paper joins the long theoretical discussion on how central banks' nominal operations affect the real economy. In this paper, the real sector borrows bank money – banks' promises to pay fiat money – as a medium of exchange, while fiat money is for meeting banks' liquidity demand; and this demand imposes a liquidity constraint on bank lending. Traditionally, the literature focuses on fiat money's role as a medium of exchange and is unconcerned with the liquidity constraint, although some studies of this literature incorporate the Diamond-Dybvig style banking (e.g. Williamson, 2012). In the traditional literature, nominal non-neutrality is driven by either a nominal rigidity (for a survey see Gali, 2008; Walsh, 2010), or incomplete information on monetary shocks (Lucas, 1972) (Lucas 1972), or the search friction (Kiyotaki and Wright, 1989; Lagos and Wright, 2005), or by exogenous rules on banks' holding of excess reserves (Mishkin, 2016). On none of these the present paper depends.

More closely is this paper related to recent studies that aim to understand monetary policy through bank liquidity management; see Bianchi and Bigio (2022), De Fiore *et al.* (2018) and Piazzesi and Schneider (2018).[4] These three papers and the present one all assume that banks face liquidity risk and trade reserves on an interbank market. Bianchi and Bigio (2022) and the present paper share an interest in nominal non-neutrality and find different mechanisms for it. They

[4]The problem of banks' reserve holding is also considered by studies unrelated to the nominal non-neutrality, e.g. Cavalcanti *et al.* (1999) and Ennis (2018).

underline the search friction on the interbank reserve market, while the present paper underlines the liquidity constraint, which is essentially a collateral constraint. A similar collateral constraint is considered by De Fiore *et al.* (2018), who, furthermore, consider a leverage constraint and unsecured borrowing, which I do not. Unlike the present paper, however, their model does not account for a nominal side or interbank liabilities. Piazzesi and Schneider (2018) underline the importance of the institutional features of the payment system for the interplay between security prices, inflation, and policy transmission. In their paper, the key friction is the leverage cost, which varies with the collateral ratio. Overall, these four papers consider different facets and are complementary to one another.

To this literature, the present paper contributes two findings. First, the tightness of a bank's liquidity constraint matters for its response to policy-rate shifts. Indeed, unconstrained banks and maximally constrained banks change their lending behaviour in opposite directions. Second, the tightness of the liquidity constraint is determined by the outflow fraction, a bank attribute that has received little attention thus far.

This paper builds on a general equilibrium analysis of bank money creation. Analysis of this kind can be found in early literature, e.g. Geanakoplos and Dubey (2003); Bloise and Polemarchakis (2006) present a survey. Recent developments include Donaldson *et al.* (2018), Jakab and Kumhof (2015), Wang (2019), Kumhof and Wang (2021), Morrison and Wang (2020) among others. These studies are not concerned with economic implications of the aggregate nominal portfolio of fiat money and government bonds.

2 The Model

The cornerstone of the model is a constraint that bank liquidity management imposes on bank lending. This constraint results from three facts. (i) What banks lend to real sectors is not fiat money but bank money, which is banks' promises to pay fiat money, mainly in the form of demand deposit nowadays. (ii) Banks need fiat-money liquidity to service demand deposits, not only because of depositor withdrawals, but also because of fact (iii). (iii) If one unit of money flows from a Bank A's deposit account into a Bank B's (e.g. because the Bank A's depositor buys from the Bank B's), then Bank A owes a unit of fiat money to Bank B. These facts, detailed in Appendix A, the model represents.

Time is discrete and infinite; I consider steady states only. The model economy is populated by banks, entrepreneurs and workers. To focus on banks, I let entrepreneurs and workers live for one period, while banks live forever, with a discount factor of $\beta < 1$. Entrepreneurs are endowed with ξ units of human capital, workers with one unit of labour; banks' endowment will be made clear soon. If an entrepreneur employs l workers, then he produces $A\xi^{1-\alpha}l^{\alpha}$ units of the consumption good, corn, within the period, where $0 < \alpha < 1$. Corn delivers a linear utility to all agents and is perishable. Without loss of generality, I normalise $\xi = 1$. Workers not employed by entrepreneurs each produce w units of corn in autarky within the period. I assume that there are more workers than can be employed by entrepreneurs, which has two implications. First, entrepreneurs hire workers at a real wage of w. Second, the opportunity cost of labour employed by entrepreneurs is w. The first-best quantity l^{FB} of labour that each entrepreneur employs is hence:

$$\begin{aligned} l^{FB} &= \operatorname{argmax}_l Al^{\alpha} - wl \\ &= \left(\frac{A\alpha}{w}\right)^{\frac{1}{1-\alpha}}. \end{aligned} \tag{1}$$

The economy has H units of fiat money and B units of a nominal government bond. Each unit of the bond is a Lucas tree, paying out dividend of d units of "money" every period. Money, in the model economy, means both fiat money and bank money. Bank money is a bank's promise to pay fiat money. Banks are endowed with a franchise whereby such a promise is equally accepted as a means of payment as fiat money. Because banks do not default in the model economy, one unit of bank money – i.e. a bank's promise to pay one unit of fiat money – has the same real value as one unit of fiat money. The bond dividends are financed with a lump sum tax T on entrepreneurs. Besides collecting this tax revenue to pay bond dividends, the fiscal authority has no other activity. Its budget constraint is simply:

$$T = Bd. \tag{2}$$

At the beginning of each period, all the assets (H, B) are held by banks, because workers and entrepreneurs, living for one period, are not involved with inter-temporal saving.

Entrepreneurs' borrowing of bank money

Should an entrepreneur be able to hire workers with her promise to pay, she would promise to pay them w units of corn when the project matures and the first-best allocation l^{FB} would be attained. I assume this is *not* the case. To hire workers, entrepreneurs have to borrow bank money as a means of wage payment. Borrowing is in nominal terms: If the gross borrowing rate is R, then a loan of 1 unit of bank money is to be settled with a repayment of R units of money (either bank money or fiat money). Entrepreneurs obtain money for loan repayment (and paying the tax) when they produce corn and sell it for money at price p on a market. On this market, workers use their nominal wage incomes to buy corn. More on the corn market will be said later.

Because entrepreneurs hire workers at real wage w, the nominal wage is wp. If an entrepreneur borrows M units of money, then she hire l workers, where

$$wp \cdot l \leq M. \tag{3}$$

Entrepreneurs' decision problem is hence:

$$\max_{M,l} p \cdot Al^{\alpha} - R \cdot M, \text{ s.t. } (3). \tag{4}$$

Because $R > 1$ – which will be shown latter – entrepreneurs' budget constraint (3) binds: They spend all borrowed money paying wages. It follows that

$$M = M(R) := p\left(\frac{A\alpha}{w^{\alpha}R}\right)^{\frac{1}{1-\alpha}} \tag{5}$$

$$l = l^{FB} \cdot R^{-\frac{1}{1-\alpha}} < l^{FB}. \tag{6}$$

In reality, there is bank lending specialisation: Each firm can borrow from only a limited number of banks.[5] To model this fact in a simple manner, I assume that each entrepreneur can borrow from one bank only. Specifically, I assume there is a continuum $[0, 1]$ of banks, each monopolising lending to a continuum $[0, 1]$ of entrepreneurs.

Bank-money circulation and bank types

Banks are heterogeneous in an attribute related to the circulation of bank money, which consists of three stages. First, Banks lend bank money to entrepreneurs. Second, entrepreneurs pass *all*

[5] Bank specialisation has been well documented in empirical research. See among others Jonghe *et al.* (2016), Liu and Pogach (2016), Ongena and Yu (2017) and Paravisini *et al.* (2014).

of it to workers that they employ. And third, workers deposit it back with banks. As a result of this circulation, a bank sees a fraction of its money issued from lending circulates into the deposit accounts of other banks; see Figure 1. This fraction is referred to as the bank's *outflow fraction*, denoted by x (representing "exodus"). I assume that a bank's outflow fraction is a constant over time and hence defines the bank's permanent type; this assumption will be discussed in Subsection 3.4. Banks have heterogeneous outflow fractions. The mass of banks with an outflow fraction no greater x is $F(x)$; function $F(\cdot)$ is thus the c.d.f. of $x \in [0, 1]$. Hereafter, subscription x is used to denote the value pertaining to type x; e.g. M_x denotes the lending scale of type x.

To simplify the exposition, I assume that the outflow of each bank's money circulates into all the other banks evenly. Given that any single bank is infinitesimal, each bank receives an equal inflow Υ_I of other banks' money, where

$$\Upsilon_I = \mathbf{E}_x\{xM_x\} := \int_0^1 xM_x \, dF(x). \tag{7}$$

Consider a typical bank, which is of type x and enters each period with h units of fiat money and b units of the bond. If the bank sets the lending rate R, then it lends out $M = M(R)$ units of its bank money, where $M(\cdot)$ is given by (5). Of this money, $(1-x)M$ units are deposited back with the bank itself, but xM units are deposited into other banks, whereby it owes liabilities of xM to these banks.[6] Meanwhile, it receives deposits of Υ_I units of other banks' money, whereby those banks owe Υ_I to the bank. Its balance sheet is as follows.

[6]This is because of fact (iii) above: If one unit of Bank A's money circulates into Bank B, then Bank A owes a unit of fiat-money reserves to Bank B. See Fact 3 in Appendix A for more details.

Assets	Liabilities
Loans to entrepreneurs ($M \cdot R$)	Deposits of its own money ($(1 - x)M$)
	Liabilities to the other banks into which its money is deposited (xM)
Credit to those other banks whose money is deposited into this bank (Υ_I)	Deposits of other banks' money (Υ_I)
Fiat money and the government bond (h, b)	Equity

Table 1: The balance-sheet of a typical bank after the money circulation

The size of the bank's deposits is thus

$$D = \Upsilon_I + (1 - x)\, M. \tag{8}$$

Its net interbank credit position is thus

$$\Upsilon_n = \Upsilon_I - xM. \tag{9}$$

With these notations, the bank's balance sheet can be consolidated into Table 2.

Assets	Liabilities
Loans to entrepreneurs ($M \cdot R$)	Deposits (D)
Net interbank credit (Υ_n)	
Fiat money and the government bond (h, b)	Equity

Table 2: The balance-sheet consolidated from Table 1

Aggregating across banks,

$$\mathbf{E}_x\{\Upsilon_{n,x}\} = 0 \tag{10}$$

$$\mathbf{E}_x\{D_x\} = \mathbf{E}_x\{M_x\}. \tag{11}$$

Equation (10) says that the net interbank positions are cancelled out in aggregate; this is because one bank's credit position is the counterparty's liability. Equation (11) says that the aggregate deposit is equal to the aggregate lending; this is because all the bank money that workers deposit into banks comes from banks' lending in the first place.

Bank liquidity management and the interbank reserve market

Lending of bank money needs fiat-money liquidity to service. Demands for liquidity come from both depositors and other banks. I assume that bank deposits are all *demand deposits*, whereby depositors can demand to withdraw fiat money anytime. Consequently, banks face a liquidity risk. Each bank sees a fraction $\widetilde{\omega}$ of its depositors demand withdrawals. Ex ante, $\widetilde{\omega} = \omega > 0$ with probability $\mu > 0$ and $\widetilde{\omega} = 0$ with probability $1 - \mu > 0$, and its realisation is independent across banks and over time. To meet the withdrawal demand $\widetilde{\omega}D$, the typical bank, whose balance sheet is given in Table 2, can gather fiat money from two sources. First, it holds a fiat-money stock h. Second, it can obtain fiat money by demanding its debtor banks to settle what they owe with fiat-money reserves. In this way, liquidity demands are passed along the interbank liability links. Meanwhile, the bank is demanded to settle its liabilities to other banks. Once all interbank positions are settled, the bank receives fiat money equal to its net interbank credit position Υ_n. The two sources put together, the bank gathers $h + \Upsilon_n$ units of fiat-money reserves to meet the withdrawal demand $\widetilde{\omega}D$. *The net reserve position* of the bank, denoted by Λ, is hence:

$$\Lambda = \Lambda(\widetilde{\omega}, x) := h + \Upsilon_n - \widetilde{\omega}D, \tag{12}$$

where the dependence of Λ on the outflow fraction x is due to (8) and (9).

After the bank meets all the depositor withdrawal and interbank settlement demands, its balance sheet changes from Table 2 to Table 3.

Assets	Liabilities
Loans to entrepreneurs ($M \cdot R$)	Deposits ($(1-\widetilde{\omega})D$)
The net reserve position (Λ)	
The government bond (b)	Equity

Table 3: The balance-sheet of the typical bank after meeting liquidity demands

If $\Lambda > 0$, the bank has a surplus of reserves which it can lend to other banks on the interbank reserve market. Let the interbank rate be ρ. Note that if $\rho = 0$, the bank is indifferent to supplying reserves to the market or withholding them. Hence, the aggregate supply of reserves on the interbank market is

$$S_R(\rho) = \begin{cases} \mathbf{E}_{\widetilde{\omega},x}\{\max(\Lambda(\widetilde{\omega}, x), 0)\}, & \text{if } \rho > 0; \\ \text{any value} \in [0, \mathbf{E}_{\widetilde{\omega},x}\{\max(\Lambda(\widetilde{\omega}, x), 0)\}], & \text{if } \rho = 0. \end{cases} \tag{13}$$

If $\Lambda < 0$, the bank is short of $-\Lambda$ units of reserves to fully meet its liquidity demands. Failure to do so would cause a prohibitively costly liquidity crisis and is not an option. To cover the shortfall, the bank needs to borrow $-\Lambda$ units of fiat money. The aggregate demand of reserves on the interbank market is hence

$$D_R(\rho) = \mathbf{E}_{\widetilde{\omega},x}\{\max(-\Lambda(\widetilde{\omega}, x), 0)\}. \tag{14}$$

The interbank reserve market clears if and only if

$$S_R(\rho) = D_R(\rho). \tag{15}$$

I assume that reserve borrowing must be collateralised with the government bond.[7] If a bank holds b units of the bond, then its borrowing capacity is $(q + d)\, b/\,(1 + \rho)$, where q is the ex-dividend price of the bond. The bank is thus subject to the following *liquidity constraint*:

$$-\Lambda(\widetilde{\omega}, x) \leq \frac{q + d}{1 + \rho}\, b, \quad \forall \widetilde{\omega} \in \{0, \omega\}. \tag{16}$$

The goods market

After the interbank reserve market clears, entrepreneurs produce corn and the corn market opens where they sell corn for money at price p. At this stage, entrepreneurs want money only to repay their loans and to pay the lump sum tax T. In aggregate, the former is equal to $\int_0^1 R_x M_x \, dF(x)$, and the latter, by (2), to Bd. Hence, their aggregate demand of money is $\int_0^1 R_x M_x \, dF(x) + Bd$, which is thus the nominal value of corn that they sell.

Buyers of corn are of two groups. First, entrepreneur-employed workers are going to exit the economy, so they spend all their wage incomes buying corn. Because entrepreneurs' budget constraint (3) is binding, the aggregate wage income equals the aggregate lending $\int_0^1 M_x \, dF(x)$. Those

[7] I thus assume away unsecured interbank borrowing, which simplifies the exposition. However, so long as unsecured borrowing is more costly or more frictional (e.g. with a probability it might not be available), the paper's results will qualitatively hold.

workers who have withdrawn use fiat money; let its quantity be $H_w \leq H$. Those who have not withdrawn use bank money in their deposit accounts, its quantity being $\int_0^1 M_x \, dF(x) - H_w$. Observe that because $\int_0^1 M_x \, dF(x) < \int_0^1 R_x M_x \, dF(x)$, workers' spending alone is inadequate to meet entrepreneurs' demand for money. The market clearing hence commands the existence of a second group of buyers, which are banks. In the model economy (as in reality), bank borrowers use money rather than real goods to settle loans, so banks obtain money rather than real goods for profit. If a bank wants to spend part of its profit on the consumption of its shareholders, it pays them dividends with its own bank money, which they then use to buy corn on the market. Recall that bank money is the bank's promise to pay fiat money. Hence, issuance of bank money appears on the bank's liability side. If the typical bank issues z units of its bank money for dividend payout, then its balance sheet changes from Table 3 to Table 4.

Assets	Liabilities
Loans to entrepreneurs ($M \cdot R$)	Deposits ($(1 - \widetilde{\omega})D$)
The net reserve position (Λ)	Dividend payout (z)
The government bond (b)	Equity

Table 4: The balance-sheet of the typical bank after paying dividend z

Let $Z = \mathbf{E}_{\widetilde{\omega},x}\{z(\widetilde{\omega}, x)\}$ be the aggregate bank dividend payout. Then the aggregate supply of money on the corn market is hence $\int_0^1 M_x \, dF(x) + Z$. The corn market clearing commands:

$$\underbrace{\int_0^1 R_x M_x dF(x) + Bd}_{\text{Entrepreneurs' sales revenue}} = \underbrace{H_w}_{\text{Fiat money withdrawn}} + \underbrace{\int_0^1 M_x dF(x) - H_w}_{\text{Bank money issued in lending}} + \underbrace{Z}_{\text{In dividends}} . \tag{17}$$

As the nominal price level p is determined on the corn market, it is ultimately determined by banks' dividend decisions in the model economy.

Loan repayment and interbank clearing

Before exiting the economy, entrepreneurs use the money from corn sales to repay their loans and to pay the lump sum tax, the revenue of which is then distributed to banks as bond dividends. As a result, by Equation (17), all the fiat money withdrawn returns to banks. Also all the outstanding bank money returns to banks and is redeemed, be it issued in lending or in dividend payout. While it returns to banks as a whole, one bank's money might flow into another.[8] As a result, new interbank liabilities are created, in addition to those resulting from reserve trading. All the interbank liabilities are cleared, first through netting and then with fiat-money reserves. As a result, each bank's liabilities are fully redeemed, leaving only equity on the liability side, while the asset side comprises a position of fiat money and the same bond position as at the start of the period. The typical bank's balance sheet changes thus from Table 4 to Table 5:

Assets	Liabilities
Fiat-money reserves (F)	
The government bond (b)	Equity

Table 5: The balance-sheet of the typical bank after the interbank clearance

To find F for the typical bank, I return to its balance sheet in Table 4. From the asset side, it receives MR units of money through the loan repayment; with the interbank interest accrued, it receives $\Lambda(1+\rho)$ units of money through the net reserve position; and it receives bd units of money as the bond dividend. On the liability side of Table 4, $(1-\widetilde{\omega})D+z$ units of its bank money flow

[8] E.g. entrepreneurs might obtain one bank's money and use it to repay another.

out, which the bank needs to redeem. The bank's fiat money position after the interbank clearance equals the inflows minus the outflows: $F = [MR + \Lambda(1+\rho) + bd] - [(1-\widetilde{\omega})D + z]$. Using (8) and (12),

$$F = h + M(R-1) + \Lambda\rho + bd - z. \tag{18}$$

The bond market

After the interbank clearance, the market opens where banks trade the bond for fiat money at price q, thereby adjust their asset positions for the next period. The typical bank enters the bond market with b units of the bond and F units of fiat money. Its next period positions (h', b') satisfy the following equation:

$$F + qb = h' + qb'. \tag{19}$$

The bond market clears if

$$\mathbf{E}_x\{b'_x\} = B. \tag{20}$$

Note that when the bond market opens, all the fiat money and the bond are held by banks: $\mathbf{E}_x\{F_x\} = H$ and $\mathbf{E}_x\{b_x\} = B$. Aggregating the budget constraint (19), I find that the bond-market clearing condition (20) is equivalent to:

$$\mathbf{E}_x\{h'_x\} = H. \tag{21}$$

Banks' decision problem and equilibrium definition

Now consider the decision problem of the typical bank. To begin with, Equations (18) and (19) lead to the following *budget constraint* of the bank:

$$M(R-1) + \Lambda(\widetilde{\omega}, x)\rho + db = z + \left[(h' + qb') - (h + qb)\right]. \tag{22}$$

On the left side of (22) is the bank's earning v in the period, which consists of three incomes: The lending profit $M(R-1)$, the interbank interest $\Lambda\rho$ and the bond dividend bd. This earning is spent on either paying out dividend z (i.e. consumption) or changing the bank's portfolio (i.e. saving), as exhibited on the right hand side of (22).

Altogether, the typical type x bank enters a period with asset portfolio (h, b), consumes z/p units of corn and leaves the period with portfolio (h', b'). The bank faces loan demand $M(\cdot)$ given by (5), the liquidity constraint (16), and the budget constraint (22). The bank's decision problem is hence as follows:

$$\Pi(h, b; x) := \max_{R, z, (h', b') \geq \mathbf{0}} \mathbf{E}_{\widetilde{\omega}} \left\{ \frac{z}{p} + \beta \Pi\left(h', b'; x\right) \right\}, \quad s.t.\ M = M(R),\ (16),\ (22). \tag{23}$$

Now I can formally define stationary equilibrium. For this purpose, the following observation regarding (h', b') helps. The bank's earning depends on the liquidity shock $\widetilde{\omega}$, but the asset holding (h', b') does not; the variation to the earning caused by $\widetilde{\omega}$ is entirely absorbed by consumption z. The reason is that both the marginal cost and benefit of holding assets are independent of $\widetilde{\omega}$. The cost consists in consumption sacrifice and the marginal value of consumption is independent of $\widetilde{\omega}$

due to linear utility. The benefit depends only on the future liquidity shocks, which are independent of the present one.[9]

As banks' asset holdings are independent of $\widetilde{\omega}$, they can be stationary: $(h', b') = (h, b)$. As a result, I can focus on a stationary equilibrium simply defined as follows.

DEFINITION 1. *An equilibrium consists of decision policies* $(R, z(\widetilde{\omega}, \cdot), h', b') : [0,1] \rightarrow \mathbf{R}^+ \times \mathbf{R}^+ \times \mathbf{R}^+ \times \mathbf{R}^+$ *and macroeconomic conditions* (ρ, p, q, Υ_I)*, such that:*

1. *Given* (ρ, p, q, Υ_I)*, the policies* $(R(x), z(\widetilde{\omega}, x), h'(x), b'(x))$ *constitute a solution of the type* x *bank's Problem* (23) *if* $(h, b) = (h'(x), b'(x))$*;*

2. (ρ, p, q) *clears the interbank reserve market, the corn market, and the bond market, in the form of Equations* (15)*,* (17)*, and* (20)*.*

3. Υ_I *satisfies* (7)*, with* $M_x = M(R(x))$*.*

The model economy is described by the aggregate nominal portfolio (H, B) that the private sector holds and the distribution $F(\cdot)$ of bank type x, which is the outflow fraction. The paper's core interest is to analyse how the steady-state interbank rate ρ varies with the aggregate portfolio (H, B). This analysis is done in Section 3 and its empirical relevance is considered in Section 4.

Passing on to the analysis, I make the following technical assumptions.

$$\frac{\omega}{1-\omega} < \frac{(\beta - \alpha)\,\beta}{(1-\mu)\,(1-\beta)} \tag{A1}$$

$$\frac{\omega}{1-\omega} < \mathbf{E}\,(x)\,\beta^{\frac{1}{1-\alpha}}. \tag{A2}$$

[9]In case of general concave utility, to keep the asset holding independent of $\widetilde{\omega}$, the device of Lucas (1990) can be used. That is, type-x banks form a family and they pool their earnings together when they decide the dividend payout z and the asset holding (h', b').

Both assumptions require the withdrawal fraction ω under the liquidity shock be small. Their purposes will be explained later.

3 The Interbank Rate and Aggregate Nominal Portfolio

In this section, I will first solve banks' decision problem (23) and then move on to the market clearings. Hereafter, the real counterpart of a nominal variable y is denoted by $\widetilde{y}$; that is, $\widetilde{y} = y/p$ and $y = p\widetilde{y}$. By (5), entrepreneurs' real demand $\widetilde{M}$ for bank money

$$\widetilde{M}(R) = \left(\frac{A\alpha}{w^{\alpha}R}\right)^{\frac{1}{1-\alpha}}. \tag{24}$$

3.1 Banks' decision: Heterogeneous response to interbank-rate shifts

Of the typical bank's decision problem (23), the liquidity constraint (16) is of utmost importance; indeed, as will be shown, it is by fiddling with this constraint, a change to the aggregate portfolio (H, B) move the interbank rate ρ. Intuitively, this constraint arises because bank lending engenders a demand for fiat-money liquidity. By Equations (8), (9) and (12), the net reserve position

$$\Lambda(\widetilde{\omega}, x) = \left[h + (1-\widetilde{\omega})\Upsilon_I\right] - \left[x + \widetilde{\omega}(1-x)\right]M. \tag{25}$$

By Equation (25), each unit of lending M reduces the net reserve position by $x + \widetilde{\omega}(1-x)$. That is because lending each unit of bank money demands $x + \widetilde{\omega}(1-x)$ unit of fiat-money liquidity to service. This *marginal liquidity demand of lending* comprises two parts. First, of each unit of bank money being lent out, x fraction flows out into other banks and becomes an interbank liability, to settle which the bank needs x unit of fiat money. Second, of the rest $1-x$ unit that is deposited back into the bank, $\widetilde{\omega}$ fraction is in a withdrawal demand, to meet which the bank needs $\widetilde{\omega}(1-x)$ unit of fiat money.

Let $\omega_e := \mu\omega$ denote the average probability of withdrawal. Then,

$$\tau_x^e := x + \omega_e(1-x). \tag{26}$$

$$\overline{\tau}_x := x + \omega(1-x) \tag{27}$$

are respectively the marginal liquidity demand of lending ex ante and that ex post under the liquidity shock (i.e. $\widetilde{\omega} = \omega$). Both of them increase with the outflow fraction x.

Obviously, Constraint (16) is tighter if the liquidity shock hits and $\widetilde{\omega} = \omega$. With $\Lambda(\omega, x)$ substituted by (25), the liquidity constraint (16) is equivalent to:

$$\overline{\tau}_x M \leq V + (1-\omega)\Upsilon_I, \tag{28}$$

where

$$V := h + \frac{q+d}{1+\rho} b \tag{29}$$

is the the quantity of liquidity that the bank stores with asset portfolio (h, b). On the left hand side of (28) is the quantity of reserve liquidity that the bank demands to service its lending M under the liquidity shock. The right hand side represents two sources whence the bank obtains liquidity to meet the demand. One is its liquidity stock V. The other is its interbank credit position Υ_I created by deposits of Υ_I units of other banks' money. Upon settlement, the bank receives Υ_I units of fiat money, but, under the liquidity shock, a fraction ω of these is withdrawn by the depositors of Υ_I. The bank hence retains $(1-\omega)\Upsilon_I$ units of reserves to service its lending.

The real counterpart of Constraint (28) is often more convenient and given below:

$$\overline{\tau}_x \widetilde{M} \leq \widetilde{V} + (1-\omega)\,\widetilde{\Upsilon}_I. \tag{30}$$

Let the Lagrange multiplier of Constraint (30) be λ. Then, λ represents the tightness of the liquidity constraint and is equal to the marginal value of liquidity in relaxing the constraint and also to the marginal cost of liquidity demand in tightening it.

Dividend z substituted using the budget constraint (22), Λ substituted using (25), the Lagrangian of Problem (23) *in real terms* is

$$\begin{aligned}\mathcal{L} = \widetilde{M}(R(\widetilde{M})-1) + (\widetilde{h} + (1-\omega_e)\widetilde{\Upsilon}_I - \tau_x^e \widetilde{M})\rho + d\widetilde{b} - \left[\widetilde{h}' + q\widetilde{b}' - \left(\widetilde{h} + q\widetilde{b}\right)\right] \\ +\beta\Pi\left(\widetilde{h}', \widetilde{b}'; x\right) + \lambda\left[\widetilde{V} + (1-\omega)\,\widetilde{\Upsilon}_I - \overline{\tau}_x \widetilde{M}\right],\end{aligned} \tag{31}$$

where $R(\cdot)$ is the inverse function $\widetilde{M}(\cdot)$ given by (24). The first order condition with respect to $\widetilde{M}$ immediately leads to the following lemma.

LEMMA 1. *The optimal lending rate depends on the outflow fraction x, the constraint tightness λ and the interbank rate ρ as follows.*

$$R = R(x, \lambda, \rho) := \frac{1}{\alpha} \times (1 + \rho\tau_x^e + \lambda\overline{\tau}_x). \tag{32}$$

Hence $R > 1$ indeed. Of equation (32), the term $1/\alpha$ is the mark-up factor due to the monopolistic power that the bank has over its borrower entrepreneurs, and

$$C := 1 + \rho\tau_x^e + \lambda\overline{\tau}_x \tag{33}$$

is the bank's marginal lending cost, which comprises three components. First, one unit of bank money is the bank's promise to pay one unit of fiat money, that is, a liability of 1. Second, ex ante, lending one unit of bank money needs τ_x^e unit of liquidity to service and the cost of liquidity is ρ per unit. Hence the *liquidity-service cost* $\rho\tau_x^e$. Third, ex post, when the liquidity shock hits, one more unit of lending increases the bank's liquidity demand by $\overline{\tau}_x$ unit, which tightens liquidity constraint (30) and imposes a *liquidity-constraint cost* of $\lambda\overline{\tau}_x$.

The tightness λ of the liquidity constraint is determined by the following Kuhn-Tucker condition:

$$\lambda[\widetilde{V} + (1-\omega)\,\widetilde{\Upsilon}_I - \overline{\tau}_x \widetilde{M}(R(x,\lambda,\rho))] = 0, \tag{34}$$

which determines λ as a function of the bank's liquidity stock $\widetilde{V}$, its outflow fraction x, its interbank credit position $\widetilde{\Upsilon}_I$, and the interbank rate ρ.

As for the bank's portfolio choice (h', b'), the value of holding a unit of fiat money as reserves and the value of using it to buy the bond are respectively:

$$\frac{\partial \mathcal{L}}{\partial \widetilde{h}'} = \beta(1+\rho)(1+\frac{\lambda}{1+\rho}) - 1 \tag{35}$$

$$\frac{1}{q}\frac{\partial \mathcal{L}}{\partial \widetilde{b}'} = \beta\frac{q+d}{q}(1+\frac{\lambda}{1+\rho}) - 1. \tag{36}$$

If $1+\rho > (q+d)/q$, reserves dominates the bond and no banks demand the bond, violating equilibrium Condition (20). If $1+\rho < (q+d)/q$, similarly, no banks demand fiat money, violating Condition (21). The bond market clearing, therefore, commands that $1+\rho = (q+d)/q$, which leads to:

Proposition 1. *The rate of return generated by the bond is equal to that by reserves:*

$$\frac{d}{q} = \rho. \tag{37}$$

Intuition for Proposition 1 is as follows. As seen from equation (29), the bank can use both a fiat-money position h and a bond position b to stock liquidity. In the model economy, the bond has zero haircut when it is used as collateral for borrowing fiat money. Hence, the bond is as liquid as fiat money. Consequently, the bond bears no illiquidity premium relative to fiat money.

Proposition 1 has two implications. First, because a bond position can be fully converted into reserves, the quantity V of liquidity that is obtained from a portfolio (h, b) equals its market value: By (37),

$$V = h + qb. \tag{38}$$

Second, individual banks are indifferent between fiat money and the bond. Hence, the composition of the bank's portfolio choice (h', b') is indeterminate; what is determined is its market value $V' = h' + qb'$, or equivalently by (38), the quantity of liquidity that the bank saves for the future. Note that banks' decision on V' is made when they choose the consumption (i.e. dividend) z on the corn market, because their earning is split between saving and consumption according to the budget constraint (22).

Using (35), the Kuhn-Tucker conditions regarding $\widetilde{V}'$ are:

$$\rho + \lambda \leq \frac{1}{\beta} - 1; \quad \widetilde{V}' \geq 0; \quad \widetilde{V}'(\frac{1}{\beta} - 1 - \rho - \lambda) = 0. \tag{39}$$

Intuitively, the marginal cost of holding liquid assets (fiat money and the bond) is due to time preference and equals $1/\beta - 1$. The marginal benefit comprises two components. One, liquid assets earn return at rate ρ in the next period by Proposition 1. The other, one more unit of liquidity relaxes the bank's liquidity constraint (30), the benefit of which is λ. The Kuhn-Tucker conditions (39) thus say that the bank holds liquid assets only if the marginal benefit $\rho + \lambda$ suffices to offset the marginal cost $1/\beta - 1$.

Of the two components of the marginal benefit, the interbank rate ρ is the same across banks, but the tightness λ of their liquidity constraints varies. Let $\overline{\lambda}$ denote the *maximal tightness* of banks' liquidity constraints:

$$\overline{\lambda} := \max_{x \in [0,1]} \lambda(x).$$

By (39), $\overline{\lambda} \leq (1-\beta)/\beta - \rho$. Should this inequality hold strictly, then $\widetilde{V}' = 0$ for all banks, which is not the case in equilibrium. Hence,

$$\overline{\lambda} = \frac{1-\beta}{\beta} - \rho; \tag{40}$$

and $\widetilde{V}' > 0$ only if $\lambda = \overline{\lambda}$. That is, only maximally liquidity-constrained banks hold liquid assets, because they obtain the maximal benefit from doing so. Equation (40) implies that

$$\overline{\lambda} \in \left[0, \frac{1-\beta}{\beta}\right]. \tag{41}$$

Moreover, Equation (40) leads to the following corollary.

Corollary 1. *The interbank rate ρ moves in one direction if and only if the maximal tightness $\overline{\lambda}$ moves in the opposite direction by the same magnitude.*

Thus far, Equation (34) shows that the tightness of the liquidity constraint λ depends on the present liquidity stock $\widetilde{V}$, and Equation (39) shows that this liquidity stock, when it was chosen in the last period, depends on the present λ. This interdependence of λ and $\widetilde{V}$ leads both to be a function of $(x, \rho, \widetilde{\Upsilon_I})$. Then, by (7), the interbank credit $\widetilde{\Upsilon_I}$ is the fixed point of:

$$\widetilde{\Upsilon_I} = \mathbf{E}_x\{x\widetilde{M}(R(x, \lambda(x, \rho, \widetilde{\Upsilon}_I), \rho))\}. \tag{42}$$

This equation determines $\widetilde{\Upsilon_I}$ as a function of ρ.[10] With this knowledge, λ and $\widetilde{V}$ are functions of (x, ρ). As demonstrated above, ρ depends on $\overline{\lambda}$ via (40) and, more importantly, banks demand liquid assets only if $\lambda = \overline{\lambda}$. It is hence more convenient to write $(\lambda, \widetilde{V})$ as functions of $\overline{\lambda}$ instead of ρ, which are given by Proposition 2 below.

Proposition 2. *Under Assumptions* (A1) *and* (A2)*, there exist two thresholds* $x^L\left(\overline{\lambda}\right)$ *and* $x^H\left(\overline{\lambda}\right)$ *of the outflow fraction* x *such that* $0 < x^L \leq x^H < 1$; $x^L = x^H \Leftrightarrow \overline{\lambda} = 0$; *and the following is true.*

(i) The tightness of a bank's liquidity constraint $\lambda = 0$ *if its outflow fraction* $x \leq x^L(\overline{\lambda})$, λ *increases with* x *over* $\left[x^L(\overline{\lambda}), x^H(\overline{\lambda})\right]$ *to* $\overline{\lambda}$, *and* $\lambda = \overline{\lambda}$ *if* $x \geq x^H(\overline{\lambda})$.

(ii) The bank's liquid-asset holding $\widetilde{V}$ *depends on* x *and* $\overline{\lambda}$ *as follows.*

(a) If $\overline{\lambda} > 0$, *then* $\widetilde{V} = V_N(x, \overline{\lambda})$, *where* V_N *is the minimum value of* $\widetilde{V}$ *to meet the liquidity constraint* (30) *at* $\lambda = \overline{\lambda}$. *For any* $(x, \overline{\lambda}) \in [0, 1] \times [0, \frac{1-\beta}{\beta}]$,

$$V_N(x, \overline{\lambda}) = \max\left(0, \overline{\tau}_x \widetilde{M}\left(R(x, \overline{\lambda}, \frac{1-\beta}{\beta} - \overline{\lambda}\right) - (1-\omega)\,\widetilde{\Upsilon}_I(\overline{\lambda})\right). \tag{43}$$

Moreover, $V_N(x, \overline{\lambda}) > 0 \Leftrightarrow x > x^H(\overline{\lambda})$ *for any* $\overline{\lambda} \in [0, \frac{1-\beta}{\beta}]$.

[10]The existence of a unique solution for (42) is shown in the proof of Proposition 2.

(b) *If* $\overline{\lambda} = 0$, *then* $\widetilde{V}$ *is indeterminate so long as it meets the liquidity constraint:* $\widetilde{V}(x, 0) \in [V_N(x, 0), \infty)$.

The proof of this proposition is relegated in Appendix B; so are the proofs that are not given in the main text hereafter.

Result (i) says that the tightness λ of a bank's liquidity constraint increases with its outflow fraction x. The result depends on Assumption (A1). The reason is that x has two offsetting effects on the liquidity demand $\overline{\tau}_x M(R(x, \lambda, \rho))$, and thereby on λ. The first is a positive effect through the marginal liquidity demand $\overline{\tau}_x$. If one unit of the bank's money flows out into other banks it requires one unit of reserves to settle, whereas if it is deposited with the bank, it is being withdrawn with probability ω. As a result, $\partial \overline{\tau}_x / \partial x = 1 - \omega$ (see 27). Second, a higher x decreases the liquidity demand by reducing the lending scale M, which, in turn, is by raising the lending rate R (see 32). As the positive effect is in the scale of $1 - \omega$, it dominates the negative one, so that the liquidity demand increases with x, if ω is sufficiently small, which Assumption (A1) ensures.

Result (ii) concerns bank liquidity management. To meet the liquidity demand, a bank obtains reserves from two sources: The liquidity stock V and the interbank credit position Υ_I (see 28). The former the bank actively chooses, the latter it passively receives. The liquidity demand, as demonstrated above, increases with the outflow fraction x. Consequently, as Proposition 2 shows, the outflow fraction demarcates three scenarios of liquidity management when $\overline{\lambda} > 0$ (i.e. some banks' liquidity constraints are binding), as illustrated in Figure 2 below.

First, if $x < x^L$, then the bank's liquidity demand is low. For such banks, the interbank credit position $\widetilde{\Upsilon_I}$ alone provides more than enough liquidity: They holds no liquid assets ($\widetilde{V} = 0$), yet still their liquidity constraints are non-binding ($\lambda = 0$) and hence engender no cost on lending. The existence of this group of banks – i.e. $x^L > 0$ – is guaranteed by Assumption (A2). The reason

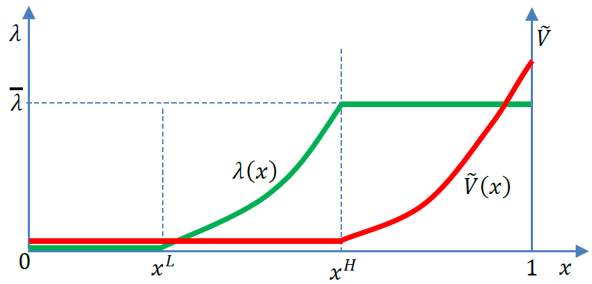

Figure 2: $(\lambda, \widetilde{V})$ as functions of the outflow fraction x if $\overline{\lambda} > 0$

is that at $x \approx 0$, the marginal liquidity demand $\overline{\tau}_x \approx \omega$ (see 27) and Assumption (A2) guarantees that it is small enough so that the liquidity demand $\overline{\tau}_x M_x$ can be met by $\widetilde{\Upsilon}_I$ alone and costlessly.

Second, if $x \in (x^L, x^H)$, then the bank's liquidity demand is intermediate. Such banks still meet liquidity demand entirely with $\widetilde{\Upsilon}_I$. However, now their liquidity demand is higher that this source is insufficient to render their liquidity constraints nonbinding ($\lambda > 0$), which hence impose a cost $\lambda\overline{\tau}_x$ on lending (see 33).

Third, if $x > x^H$, then the bank's liquidity demand is high. Meeting such high demand with $\widetilde{\Upsilon}_I$ only is too restraining and it is worth holding liquid assets ($\widetilde{V} > 0$). Such high demand induces a tightest liquidity constraint ($\lambda = \overline{\lambda}$) and a highest liquidity-constraint cost $\overline{\lambda}\overline{\tau}_x$.

Due to this effect on bank liquidity management, the outflow fraction x has a profound impact on how banks respond to policy-rate shifts. To present this impact, I introduce the following notations. Proposition 2 essentially expresses λ as a function of $(x, \rho) \in [0, 1] \times (0, (1-\beta)/\beta]$; write it as $\lambda(x, \rho)$. Then,

$$R^*(x, \rho) := R\left(x, \lambda\left(x, \rho\right), \rho\right) \tag{44}$$

$$\widetilde{M}^*(x, \rho) := \widetilde{M}(R^*(x, \rho)) \tag{45}$$

are respectively the lending rate and real lending scale of a type-x bank when the interbank rate is ρ.

Proposition 3. *$\frac{\partial R^*(x,\rho)}{\partial \rho} > 0$ if $x \in [0, x^L(\rho))$ so $\lambda = 0$, and $\frac{\partial R^*(x,\rho)}{\partial \rho} < 0$ if $x \in (x^H(\rho), 1]$ so $\lambda = \overline{\lambda}$. That is, if the interbank rate ρ changes, liquidity-unconstrained banks change their lending rates in the same direction as ρ, but maximally constrained banks move theirs in the opposite direction.*

Proof. By (32), $\frac{\partial R^*}{\partial \rho} = \frac{1}{\alpha} \cdot \frac{\partial C}{\partial \rho}$, where

$$C = 1 + \rho \tau_x^e + \lambda \overline{\tau}_x. \tag{46}$$

By Proposition 2, if $x < x^L$, then $\lambda = 0$ and hence $\frac{\partial C}{\partial \rho} = \tau_x^e > 0$. And if $x > x^H$, then $\lambda = \overline{\lambda} = \frac{1-\beta}{\beta} - \rho$ (see 40) and hence $\frac{\partial C}{\partial \rho} = -\left(\overline{\tau}_x - \tau_x^e\right) = -\left(1-\mu\right)\omega(1-x) < 0$, where the last equality uses Equations (26) and (27). □

Proposition 3 is counter-intuitive. When a central bank changes the interbank reserve borrowing rate, one tends to believe that in response, banks' lending rates should *all* change in the same direction as the interbank rate. The argument is straightforward: A rise (fall) in the interbank rate increases (decreases) the cost of liquidity for all banks; consequently, all their lending costs and rates should rise (fall). By Proposition 3, however, this true only for liquidity-unconstrained banks; maximally constrained banks change their lending rates in the opposite direction.

Intuition can be found in the proof of the proposition. By Equation (46), the marginal lending cost C comprises three components: The cost 1 of redeeming liability, the liquidity service cost $\rho \tau_x^e$, and the liquidity-constraint cost $\lambda \overline{\tau}_x$ (see the discussion of Equation 33). Now suppose the interbank rate ρ falls by one unit. Then, indeed, the liquidity service cost $\rho \tau_x^e$, *of any bank*, decreases by τ_x^e unit. Should this be the only change to the lending cost, all banks' lending costs and rates would fall, as one tends to believe. However, it is not; the liquidity-constraint cost $\lambda\overline{\tau}_x$ also changes for some banks. Not among them, though, are liquidity unconstrained banks, for which $\lambda = 0$ and hence this cost remains zero. These banks' lending rates, therefore, duly fall. But for maximally constrained banks, the liquidity-constraint cost changes. For them $\lambda = \overline{\lambda}$, and by Corollary 1, when ρ falls by one unit, $\overline{\lambda}$ *necessarily* rises by one unit. As a result, the liquidity-constraint cost increases by $\overline{\tau}_x$ unit. Moreover, this increase is more than offsets the τ_x^e unit decrease in the liquidity service cost: $\overline{\tau}_x > \tau_x^e$ (see the proof above). Consequently, the lending costs and rates of maximally constrained banks *rise* when interbank rate ρ *falls*.

Regardless the distribution of the outflow fraction $F(x)$, there always exist such "anomalous" banks, namely banks whose lending rates move in the opposite direction to the interbank-rate. The reason is that these are maximally liquidity constrained banks and in equilibrium only such banks hold liquid assets (see the discussion of the Kuhn-Tucker conditions 39) and hence they must exist.

Proposition 3 has implications for the aggregate real impact of monetary policy. In the model economy, by (6), the equilibrium employment is lower than the first-best level. A rise in the the aggregate employment therefore increases efficiency. The aggregate employment equals $\widetilde{\mathbf{M}}(\rho)/w$, where $\widetilde{\mathbf{M}} := \mathbf{E}_x\{\widetilde{M}^*(x,\rho)\}$ is the aggregate real lending scale. To how $\widetilde{\mathbf{M}}(\rho)$ changes with the interbank rate ρ, Corollary 2 is relevant; because $\widetilde{M}'(R^*) < 0$ by (24), the corollary immediately follows from Proposition 3.

Corollary 2. *$\frac{\partial \widetilde{M}^*(x,\rho)}{\partial \rho} < 0$ if $x < x^L$ and $\frac{\partial \widetilde{M}^*(x,\rho)}{\partial \rho} > 0$ if $x > x^H$.*

That is, a cut in ρ increases the real lending scales of types $x \in [0, x^L)$, but decreases those of types $x \in (x^H, 1]$, while having unknown effects for $x \in [x^L, x^H]$. In the general case, therefore, the aggregate effect depends on the distribution $F(\cdot)$ of bank type x. However, if x^L is very small, there are few liquidity unconstrained banks and the aggregate effect should be dominated by liquidity constrained banks. Moreover, if the interbank rateρ is very low, by (40) the maximal tightness $\overline{\lambda}$ is very high, which would suggest very few banks be liquidity unconstrained – i.e. x^L be low. This intuition is confirmed by the lemma below.

LEMMA 2. *(i)* $\widetilde{\mathbf{M}}'(\rho) > 0$ *if* x^L *is sufficiently small. (ii)* $\lim_{\rho\to 0} x^L = 0$ *if* $\frac{\omega}{1-\omega} \geq \mathbf{E}(x)$.

The lemma suggests that if the interbank rate ρ is already very low, cutting it further could actually *decrease* the aggregate bank lending, counter-intuitively.

Proposition 2 states how banks' lending behaviour changes *if* the interbank rate ρ shifts. In the next subsection, I consider how ρ varies with the the aggregate portfolio (H, B) of fiat money and the government bond. In reality, to shift the interbank rate, central banks use operations that change the nominal portfolio of fiat money and government bonds that the private sector holds. Such operations can be thus modelled as a change (Δ_H, Δ_B) to the aggregate portfolio (H, B). Here are examples.

1. Open market operations. Assume that the central bank of the model economy holds a stock B' of the government bond. The dividend $B'd$, ultimately paid by the lump-sum tax payers, would drain reserves out of the economy each period. To avoid this complication, I assume that this dividend $B'd$ is rebated, via the fiscal authority, to the taxpayers. An open market operation whereby the central bank exchanges N units of the government bond for qN units of fiat money, with $N > 0$ represents buying and $N < 0$ selling, decreases the private sector's bond position by N and increases its fiat money position by qN. This operation is hence represented by $(qN, -N)$. It changes the central bank's balance sheet as follows.

2. Repos/Reverse Repos: Suppose the central bank trades N units of reserves with banks at a policy rate ρ^p, using the government bond as collateral, with $N > 0$ representing the central bank

Assets	**Liabilities**
Other assets	Other liabilities
The bond $(q \cdot (B' + N))$	Banks' reserves $(H + qN)$
	Equity

Table 6: The central bank's balance sheet change with open market operation $(qN, -N)$

lending (i.e. repo) and $N < 0$ borrowing (i.e. reverse repo). Then after the positions are settled and the collateral returned, the private sector's fiat money position is reduced by $N \cdot \rho^p$ and its bond position is unchanged. The operation is hence represented by $(-N\rho^s, 0)$ and changes the central bank's balance sheet as follows.

Assets	**Liabilities**
Other assets	Other liabilities
The bond $(q \cdot B'))$	Banks' reserves $(H - N\rho^p)$
	Equity $(+N\rho^p)$

Table 7: The balance sheet change with Repo/Reverse Repo $(-N\rho^p, 0)$

Below I ignore the transition and focus on a comparative static analysis of how the steady-state interbank rate ρ depends on the aggregate nominal portfolio (H, B). I demonstrate that what matters is the portfolio's composition. The size of the permanent bond is defined by the amount Bd of dividends it pays out each period. Hence, the composition of the portfolio is represented by the following *bond-to-fiat money ratio* δ:

$$\delta := \frac{Bd}{H}. \tag{47}$$

I will show that by adjusting δ, the central bank can shift the interbank rate ρ to any level feasible in equilibrium.

3.2 Equilibrium: The mechanism of setting the interbank rate

In each period, three markets open sequentially in the model economy: The interbank reserve market, the corn market, and the bond market. Clearing of these markets determine the three price variables: The interbank rate ρ, the corn price p and the bond price q. I have considered the bond market clearing and found the bond price q as a function of ρ given by (37). In what follows, I will consider the clearing of the other two markets, starting with the interbank reserve market.

Lemma 3. *The interbank reserve market clears if and only if*

$$\omega_e \int_0^1 \widetilde{M}^* (x, \rho) \, dF(x) = \widetilde{H}. \tag{48}$$

Equation (48) is driven by the fact that in the model economy fiat money serves as bank reserves rather than as a means of payment for real goods; the latter role is served by bank money. For intuition of (48), recall that a single bank's reserve demand is due to both depositor withdrawals and interbank settlements. In aggregate, the interbank positions cancel each other (see 10). Hence, the aggregate reserve demand of banks is equal to the aggregate depositor withdrawal, which is the product of the average withdrawal probability ω_e and the aggregate deposit $\mathbf{E}_x\{D_x\}$, the latter equal to the aggregate lending $\mathbf{E}_x\{M_x\}$ by Equation (11). Therefore, the left hand side of Equation (48) is the aggregate (real) reserves demand of banks and the right hand side is the aggregate supply of reserves.

Because $\widetilde{H} = H/p$, Equation (48) determines the steady-state price level p as a function of ρ. The equation has the following implication immediately.

Proposition 4. $\lim_{\omega_e \to 0} p^{-1} = 0$ *and* $\partial p^{-1}/\partial \omega_e > 0$ *if* ω_e *is sufficiently small.*

That is, if the depositor withdrawal probability ω_e decreases from a low enough level, fiat money's real value p^{-1} falls, and the nominal price level p rises. Moreover, if the withdrawal probability goes to nil, fiat money becomes worthless. Intuition for them follows that for condition (48): In aggregate, the use of fiat money is to meet depositor withdrawal demands. Consequently, if depositor withdrawals are less probable to happen, fiat money is less useful and less valuable. If they never happen, it is useless and hence worthless; that is, it will cease to circulate.

In the model economy, because banks do not default, depositors withdraw for reasons unrelated to bank default. In real life, one such reason is that cash is the only accepted means of payment. The incidence ω_e of this type of withdrawal has been incessantly reduced in recent decades by the advance of digital payment technologies. According to Proposition 4, this technological advance has impacts on the long-term price level, which, therefore, is not purely a monetary phenomenon. Furthermore, technological advance might eliminate this type of withdrawals altogether in the future. Should that happen, *fiat money would cease to circulate*; and bank money would instead be a promise to pay real goods, e.g. gold or silver as in historical times.

The corn market, as shown by Condition (17), depends on banks' dividend payout z to clear. Banks' payout decision, as shown in their budget constraint (22), is driven by the typical consumption-saving trade-off. Naturally, the corn market clears if and only if the asset market clears, as shown by Lemma 4.

LEMMA 4. *The corn market clears if and only if the aggregate demand of assets is equal to the aggregate supply:*

$$\mathbf{E}_x\{V_x'\} = H + qB. \tag{49}$$

A bank's demand for assets depends on their return rates, both of which are equal to the interbank rate ρ by Proposition 1. By Equation (40), ρ is rigidly connected with the maximal tightness $\overline{\lambda}$ of the liquidity constraint. Therefore, the asset market clearing determines ρ, in a way that $\overline{\lambda}$ plays a crucial role, as detailed below.

Banks' asset demand is presented in Proposition 2. In a steady state, $V' = V$ and Proposition 2 shows how the real asset demand $\widetilde{V}$ depends on the outflow fraction x and the maximal tightness $\overline{\lambda}$; write this dependence as $\widetilde{V}(x, \overline{\lambda})$. The real aggregate asset demand is thus

$$\widetilde{\mathbf{D_L}}(\overline{\lambda}) := \int_0^1 \widetilde{V}(x, \overline{\lambda})\, dF(x).$$

The real aggregate supply of assets $\widetilde{\mathbf{S_L}} = \widetilde{H} + q\widetilde{B}$. By Equation (48), the aggregate real value $\widetilde{H}$ of fiat money is a function of the interbank rate ρ; write it as $\widetilde{H}(\rho)$. Using Equations (37) and (47), the real aggregate asset supply is

$$\widetilde{\mathbf{S_L}}(\rho) = \widetilde{H}(\rho)\left(1 + \frac{\delta}{\rho}\right). \tag{50}$$

Equation (49) is hence equivalent to

$$\int_0^1 \widetilde{V}(x, \overline{\lambda}) dF(x) = \widetilde{H}(\rho)\left(1 + \frac{\delta}{\rho}\right), \tag{51}$$

which together with Equation (40) determines the steady-state interbank rate ρ.

Equation 51 clearly shows that the bond-to-fiat money ratio δ determines the steady-state interbank rate ρ. The logic chain for this determination consists of three links.

Link 1: δ determines the aggregate liquidity supply $\widetilde{\mathbf{S_L}}$ by Equation (50). Indeed, by the equation, given ρ, the aggregate liquidity supply linearly increases with the bond-to-fiat money ratio δ. Intuition is as follows. The bond's nominal dividend per period is Bd, which is δ times of the total quantity H of fiat money. As the fiat money's real value is $\widetilde{H}(\rho)$, so the real value of the bond dividend is $\delta \cdot \widetilde{H}(\rho)$. By Proposition 1, the price-to-earning ratio of the bond is $1/\rho$. Hence, the aggregate real value of the bond is $\delta\widetilde{H}(\rho) \cdot 1/\rho$ and linearly increases with δ if ρ were fixed.

Link 2: The aggregate liquidity supply $\widetilde{\mathbf{S_L}}$ determines the maximal tightness $\overline{\lambda}$ of the liquidity constraint through the market clearing condition:

$$\widetilde{\mathbf{S_L}} = \widetilde{\mathbf{D_L}}(\overline{\lambda}) = \int_0^1 \widetilde{V}(x, \overline{\lambda}) dF(x), \tag{52}$$

where $\overline{\lambda} \in [0, (1-\beta)/\beta]$ by Equation (41). By Proposition 2, if $\overline{\lambda} > 0$, $\widetilde{V}(x, \overline{\lambda})$ is determinate, equal to $V_N(x, \overline{\lambda})$. Hence the aggregate demand $\widetilde{\mathbf{D_L}}$ equals $\mathbf{D}_N(\overline{\lambda})$, where

$$\mathbf{D}_N(\overline{\lambda}) := \int_0^1 V_N(x, \overline{\lambda})\, dF(x). \tag{53}$$

If $\overline{\lambda} = 0$, then $\widetilde{V}(x, \overline{\lambda}) \geq V_N(x, 0)$ is subject to no upper bound. As a result, the aggregate demand $\widetilde{\mathbf{D_L}}$ can be anywhere above $\mathbf{D}_N(0)$. Therefore, the clearing condition (52) leads to two scenarios. First, if

$$\widetilde{\mathbf{S_L}} > \max_{z \in [0, (1-\beta/\beta]} \mathbf{D}_N(z), \tag{54}$$

then $\overline{\lambda} = 0$ and hence $\lambda = 0$ for any bank; that is, if the liquidity supply abounds, then no bank's liquidity constraint binds. Second, if Inequality (54) does not hold, then $\overline{\lambda}$ is positive and determined by the liquidity supply $\widetilde{\mathbf{S_L}}$ through $\widetilde{\mathbf{S_L}} = \mathbf{D}_N(\overline{\lambda})$.

Link 3: The maximal tightness $\overline{\lambda}$ determines the steady-state interbank rate ρ through Equation (40), whereby $\rho = (1-\beta)/\beta - \overline{\lambda}$.

Through this logic chain, a shift to the composition δ of the aggregate portfolio (H, B) moves the interbank rate ρ. An open market operation $(qN, -N)$, where $N > 0$ represents the central bank buying the bond, $N < 0$ selling, shifts the composition from Bd/H to $(B-N)d/(H+qN)$. By choosing a proper N, the central bank can shift the composition to any $\delta \in (0, \infty)$ and thereby set the steady-state interbank rate to any feasible value, which, by Equations (37) and (40), is within $(0, (1-\beta)/\beta]$.

Proposition 5. *To set the steady-state interbank rate at $\rho \in (0, (1-\beta)/\beta]$, the central bank elects the bond-to-fiat money ratio to be:*

$$\delta = \begin{cases} S_r(\rho), & \text{if } \rho < \frac{1-\beta}{\beta}; \\ \textbf{\textit{any value}} \geq S_r\left(\frac{1-\beta}{\beta}\right), & \text{if } \rho = \frac{1-\beta}{\beta}. \end{cases} \tag{55}$$

Here the rate setting function $S_r(\rho)$ is defined over $\rho \in \left(0, \frac{1-\beta}{\beta}\right]$ by

$$S_r(\rho) := \rho\left(\frac{\mathbf{D}_N(\frac{1-\beta}{\beta} - \rho)}{\widetilde{H}(\rho)} - 1\right), \tag{56}$$

where $\mathbf{D}_N(\overline{\lambda})$, given by Equation (53), is the aggregate real value of liquid assets necessary for banks to meet their liquidity constraints and $\widetilde{H}(\cdot)$, given by Equation (48), is the aggregate real value of fiat money. Moreover, $S_r(\rho) < S_r\left(\frac{1-\beta}{\beta}\right)$ if $\rho < \frac{1-\beta}{\beta}$ and $S_r\left(\frac{1-\beta}{\beta}\right) \in \left(0, \frac{1-\beta}{\beta} \cdot \frac{1-\omega_e}{\omega_e}\right)$.

By Equation (40), $\rho < (1-\beta)/\beta$ if and only if $\overline{\lambda} > 0$, that is, some banks' liquidity constraints are binding. Hence, the proposition has the following corollaries.

1. The aggregate fiat money-bond portfolio is neutral in steady states: A change of the portfolio from (H, B) to $z \cdot (H, B)$ for any $z > 0$ produces no real effects.

2. Fiat money is non-neutral in steady states if banks are liquidity-constrained: Any variation to H changes δ, and hence moves ρ, if $\rho < (1-\beta)/\beta$.

If presently $\rho < (1-\beta)/\beta$ and the central bank wants to marginally adjusts it, Proposition 5 specifies the position to which the bond-to-fiat money ratio δ should travel. However, it says nothing about the direction of this travelling, that is, about the sign of S_r'. Based on the logic chain expounded above, one might think $S_r' > 0$: By Link 1, an increase in δ tends to raise the liquidity supply $\widetilde{\mathbf{S_L}}$; a higher liquidity supply tends to relax the liquidity constraints and reduce $\overline{\lambda}$; and if $\overline{\lambda}$ decreases, then ρ increases. This is indeed the direct effect that a change in δ has on ρ. However, whenever ρ starts moving, its movement produces two indirect effects that muddies the water.

First, when ρ moves, the aggregate real value $\widetilde{H}(\rho)$ of fiat money changes, which affects the liquidity supply $\widetilde{\mathbf{S_L}}$ through Equation (50). In the general case, the direction of $\widetilde{H}(\rho)$ changing is unclear, as explained blow.

Second, when ρ moves, each type-x of banks changes their lending scales $\widetilde{M}^*(x, \rho)$, and hence the interbank credit position $\widetilde{\Upsilon_I} = \mathbf{E}_x\{x\widetilde{M}^*(x, \rho)\}$ starts moving. This movement affects the tightness of banks' liquidity constraints because $\widetilde{\Upsilon_I}$ is one of the sources where banks obtain liquidity to meet the constraint (see 30). As explained below, the direction of $\widetilde{\Upsilon_I}$ moving is unclear too.

The reason why the directions of $\widetilde{H}$ or $\widetilde{\Upsilon}_I$ changing with ρ are unclear lies in Corollary 2, whereby $\widetilde{M}_\rho^{*'}(x, \rho) < 0$ if $x < x^L$ and $\widetilde{M}_\rho^{*'}(x, \rho) > 0$ if $x > x^H$. As a result, the sign of $\widetilde{H}'(\rho) = \mathbf{E}_x\{\widetilde{M}_\rho^{*'}(x, \rho)\}$ and that of $\widetilde{\Upsilon_I}'(\rho) = \mathbf{E}_x\{x\widetilde{M}_\rho^{*'}(x, \rho)\}$ are unclear. Consequently, the sign of S_r' cannot be determined in the general case.

This sign, however, can be determined in a special case where there are only two types of banks, as examined below.

3.3 The special case with two types

In this subsection, I assume that the distribution of types x is as follows:

$$x = \begin{cases} 0, & \text{with probability } f_0 > 0 \\ \kappa, & \text{with probability } f_\kappa > 0, \end{cases}$$

where $\kappa \in (0, 1)$ and $f_0 + f_\kappa = 1$.

Proposition 6. *In the two-type case,* $S_r'(\rho) > 0$ *for* $\rho \in \left(0, \frac{1-\beta}{\beta}\right)$ *and* $\lim_{\rho \to 0} S_r(\rho) = 0$.

According to this proposition, if the central bank wants to raise the interbank rate ρ, it should increase the bond-to-fiat money ratio $\delta = S_r(\rho)$ of the portfolio held by the private sector, with an open market operation of selling the bond to retire fiat money. If the central bank wants to reduce ρ, it should decrease δ, with an open market operation of buying the bond to inject fiat money. This is agreement with the conventional argument regarding the effects of open market operations. However, there are two differences.

First, the mechanism is different. The conventional argument focuses on the supply of reserves: An operation of selling the bond decreases the supply and thereby raises the cost (i.e. the interest rate) of borrowing them, and vice versa. In contrast, what matters in the paper is not the supply H of reserves, but the composition of the reserves-bond portfolio (H, B). In particular, if both H and B increase while the composition is unchanged, then the reserve borrowing rate would not move. Moreover, the liquidity constraint plays no explicit role in the conventional argument, whereas in this paper, the reserve borrowing rate is shifted if and only if the tightness of the constraint is altered.

Second, the predicted real effects are different, in two ways. (1) Open market operations cause only a *transitory* real effect in the conventional argument, whereas they shift the interbank rate *in the steady state* according to this paper. (2) By Proposition 3 and Corollary 2, a shift in the reserve borrowing rate produces opposite impacts on the lending behaviour of liquidity-unconstrained and maximally constrained banks, whereas the conventional argument predicts no such heterogeneity.

The heterogeneity presented by Proposition 3 and Corollary 2 is indeed a core result of this paper's. Empirical evidence for it is presented in Section 4. Passing on to that, I shall first discuss an assumption of the paper, using the special case of this subsection.

3.4 Discussion: Fluctuating outflow fractions

The paper has demonstrated the importance of the outflow fraction x for bank responses to monetary policy. Thus far, I have assumed it is a permanent attribute of banks, invariant over time. In this subsection, I will first discuss the empirical relevance of the assumption and then demonstrate the robustness of the paper's core results if the outflow fraction fluctuates over time in a Markov process.

The assumption is made to simplify the exposition and sharpen the paper's messages, but it also has a ground on empirical facts. While the outflow fraction has not been reliably measured (as it has received little attention thus far), there is evidence that it tends to be stable over time. A bank's outflow fraction is highly correlated with its deposit-market share: The greater the share, the greater the chance that the bank's money circulates back to itself and hence the smaller the outflow fraction. The deposit market shares of banks, especially those of big banks, are stable over time. E.g. the market shares of the largest five U.K. banks over the last decade are illustrated in Figure 3 below.[11]

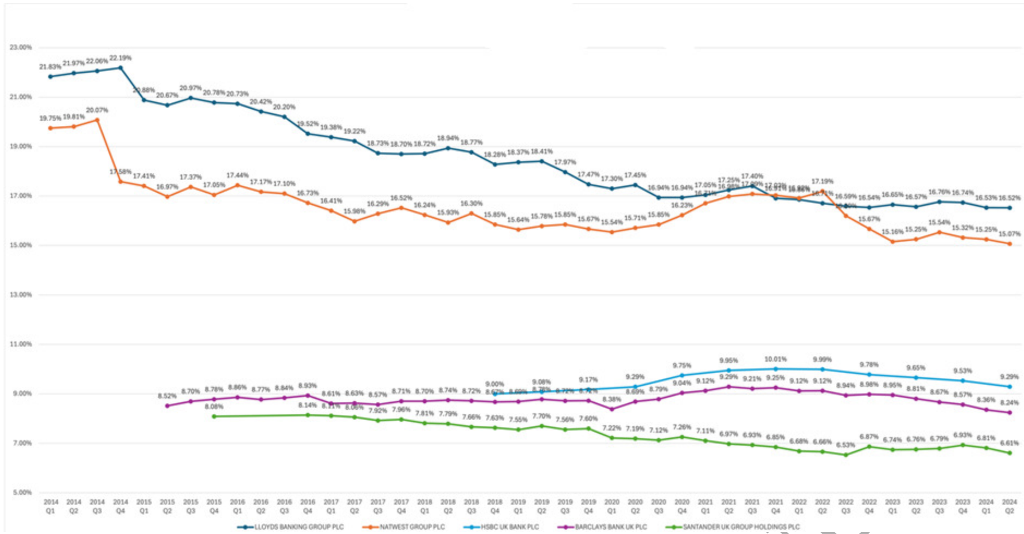

Figure 3: The shares of the U.K. deposits for the five largest banks

Stable the outflow fraction might be, in reality we would still expect it fluctuating over time as other bank attributes do. Here I demonstrate that if the model is extended to allow banks' outflow fractions to fluctuate in a Markov process, the following two core results of the paper still hold.

(i) Different types of banks respond to reserve-borrowing rate shifts in opposite ways.

(ii) It is by altering the tightness of the liquidity constraint that a change to the composition δ of the aggregate nominal portfolio shifts the reserve borrowing rate.

To simplify the exposition, I use the special case of Subsection 3.3. Now suppose that a bank's outflow fraction, instead of being permanent, changes over one period according to the following

[11]Data on pound sterling deposits of individual banks are obtained from BankForce (Moody's, 2025) and the quarterly reports of the banks. HSBC reports only biannually for the UK market (while quarterly for global markets). The data on the total deposits are from the Bank of England (BoE, 2025).

Markov process:

$$Pr(x' = 0|x = 0) = Pr(x' = \kappa|x = \kappa) = \sigma \in (\frac{1}{2}, 1];$$

and that the uncertainty resolves before any decision is made. Note that the model in Section 2 is a special case where $\sigma = 1$. The stability of the outflow fraction means $\sigma \approx 1$, which I assume in this subsection. In a stationary equilibrium, the distribution of the outflow fraction is

$$x = \begin{cases} 0, & \text{with probability } \frac{1}{2}; \\ \kappa, & \text{with probability } \frac{1}{2}. \end{cases} \tag{57}$$

The typical bank's liquidity and budget constraints, which depend only on its present type, are unchanged. Its decision problem is modified from (23) to:

$$\Pi(h, b; x) := \max_{R,z,(h',b')\geq \mathbf{0}} \mathbf{E}_{\widetilde{\omega}} \left\{ \frac{z}{p} + \beta \mathbf{E}_{x'} \{\Pi(h', b'; x')\,|x\} \right\}, \quad s.t.\ M = M(R),\ (16),\ (22).$$

This modification has no effect on banks' lending decision, which is unrelated to the next period type x'. Hence, the lending rate is still given by (32). Similarly, the bond is still as liquid as fiat money so that Equation (37) holds. Hence, as before, the composition of a single bank's portfolio (h', b') is indeterminate and what is determined is the market value $V' = h' + qb'$.

The modification affects banks' asset demand $\widetilde{V}'$. The intuition behind the original Kuhn-Tucker conditions (39) still holds: The demand is driven by the two benefits from holding liquid assets: They deliver return at the rate of the interbank rate ρ; and they relax the liquidity constraint in the next period. However, because the type fluctuates, the latter benefit for type x becomes

$\phi_x(\widetilde{V}') := \mathbf{E}_{x'}\{\lambda(x', \widetilde{V}')|x\}$, where the dependence of λ on $(x', \widetilde{V}')$ is given by (34). The Kuhn-Tucker conditions change from (39) to:

$$\rho + \phi_x(\widetilde{V}') \leq \frac{1}{\beta} - 1; \quad \widetilde{V}' \geq 0; \quad \widetilde{V}'(\frac{1}{\beta} - 1 - \rho - \phi_x(\widetilde{V}')) = 0. \tag{58}$$

As in Subsection 3.3, type 0 is liquidity unconstrained: $\lambda(0, \widetilde{V}') = 0$ for any $\widetilde{V}'$. Hence,

$$\phi_0(\widetilde{V}'_0) = (1-\sigma)\lambda(\kappa, \widetilde{V}'_0); \quad \phi_\kappa(\widetilde{V}'_\kappa) = \sigma\lambda(\kappa, \widetilde{V}'_\kappa).$$

If $1 - \sigma$ is sufficiently small, then $\phi_0 < \phi_\kappa$ for any $(\widetilde{V}'_0, \widetilde{V}'_\kappa)$. Then, as before, type κ outbids type 0 in acquiring assets, namely, $\widetilde{V}'_0 = 0$ and $f_\kappa \widetilde{V}'_\kappa = \widetilde{\mathbf{S_L}}$; using (50) and (57),

$$\widetilde{V}'_\kappa = 2\widetilde{H}(\rho)(1 + \frac{\delta}{\rho}). \tag{59}$$

And the third part of Conditions (58) implies

$$\lambda(\kappa, \widetilde{V}'_\kappa) = \frac{1}{\sigma}(\frac{1-\beta}{\beta} - \rho), \tag{60}$$

which is the extension of (40) and implies that $\rho \in [0, (1-\beta)/\beta]$.

The robustness of core result (ii) can be shown with Equations (59) and (60). A change to δ shifts the real asset value $\widetilde{V}'_\kappa$ by (59); this shift changes the tightness λ through function $\lambda(\kappa, \widetilde{V}'_\kappa)$; and this change moves the interbank rate ρ through Equation (60).

Now I establish the robustness of core result (i), namely Proposition 3. A complication induced by the extension is that because the type fluctuates, in steady states the same type of banks holds different quantities of assets: Fraction σ of present type κ banks comes from a type κ bank of the last period and holds now assets worth $\widetilde{V}'_\kappa$, but $1-\sigma$ fraction comes from a previous type 0 and holds no assets; and similarly for present type 0 banks. As a result, there are four groups of banks, each consisting of a type $x \in \{0, \kappa\}$ holding a $j \in \{0, 1\}$ times of $\widetilde{V}'_\kappa$ assets. Let $R^*_j(x, \rho)$ denote the lending rate of group (x, j) when the interbank rate is ρ, which is an extension of $R^*(x, \rho)$ in Proposition 3. The counterpart of the proposition is as follows.

Proposition 7. *Regardless of their asset holding, type* 0 *banks change their lending rates in the same direction as* ρ, *but type* κ *banks change theirs in the opposite direction: For* $\rho < \frac{1-\beta}{\beta}$,

$$\frac{\partial R^*_j(0,\rho)}{\partial \rho} > 0 \quad \textit{and} \quad \frac{\partial R^*_j(\kappa,\rho)}{\partial \rho} < 0, \quad \forall j \in \{0,1\}. \tag{61}$$

A reduction in the persistency σ of the outflow fraction, however, does have consequences. In each period, a mass $\frac{1}{2}(1-\sigma)$ of previous type-0 banks finds that their outflow fraction jumps from 0 to κ and they hold no liquid assets to help with the high liquidity demand, so that they will be severely liquidity constrained. Note that this mass is larger when the persistency σ is lower. That is, if the outflow fraction is more transitory, the liquidity constraint causes more efficiency losses. This is in a similar vein to the insight of Moll (2014) that "If productivity shocks are relatively transitory, financial frictions result in large long-run productivity losses..." However the present paper differs to Moll (2014) in an important aspect. In Moll (2014), in aggregate, wealth (in form of capital) can be accumulated to alleviate the collateral constraint and this undoes capital misallocation caused by the constraint if the productivity shock is permanent. In contrast, in the present paper, the liquid assets – fiat money and the government bond – are in a fixed supply, and hence the liquidity constraints of banks with a high outflow fraction still bind even if it is permanent.[12]

[12]In reality, majority of collateral that banks use for reserve borrowing is a government debt, directly or indirectly owed. E.g. Copeland *et al.* (2010) documents that in the U.S. Tri-Party Repo Market, 82.7% of the collateral is of this kind during the sample period; see Table 2 as well as Figure 6.

4 Evidence for the Predicted Heterogeneity

A key policy rate of central banks is the interest rate at which banks borrow reserves. Proposition 3 predicts that different types of banks respond to the policy-rate shifts in opposite ways. This section present empirical evidence for this prediction. It consists of two parts. First, subsection 4.1 performs a simple test of Proposition 3 using a novel approach based on this paper's theory. Then, Subsection 4.2 collects evidence from existing empirical studies.

4.1 A test based on a novel empirical approach

To test Proposition 3, it is necessary to empirically identify which banks are liquidity unconstrained and which maximally liquidity-constrained. The liquidity constraint (28) reveals two ways whence a bank obtains liquidity to meet its liquidity demand $\overline{\tau}_x M$. One is to *actively* acquire liquid assets (fiat money and government bonds) whereby the bank stores V units of liquidity. The other, the bank *passively* receives reserves from the inflow of Υ_I units of other banks' money. The existing studies focus on the *active* way. They postulate that a bank with certain traits is of greater easiness to acquire liquid assets and hence less liquidity constrained. For example, one such trait is large size according to Kashyap and Stein (2000) (KS hereafter). In contrast, this paper focuses on the *passive* way. Inflows of other banks' money create interbank credit positions for the bank; outflows of its own money create interbank liability positions. The net credit position so passively formed, Υ_n, is informative of the bank's liquidity constraint, as shown below.

PROPOSITION 8. *If a bank is liquidity unconstrained, then* $\Upsilon_n > 0$*. If* $\Upsilon_n < 0$*, then the bank is maximally liquidity-constrained.*

Interbank positions are recorded in data as reserve positions because they are concerned with one bank owing reserves to another. E.g. in the U.S., interbank credit positions are recorded as

Fed funds sold, interbank liability positions as Fed funds purchased. I define a bank's *net funds position* as the difference of the Fed funds sold minus the funds purchased. This position is a natural proxy for Υ_n in Proposition 8. However, the two are not identical. The interbank-position data encompass not only positions passively created through bank-money circulation, but also positions generated by banks actively borrowing or lending reserves. However, Υ_I is concerned only with the former.

The idea, based on Proposition 8, is that while the net funds position in a single period is subject to a varieties of disturbances, the position being persistently positive (negative) indicates the bank is likely to be liquidity unconstrained (maximally constrained).

This approach is applied to the U.S. data that KS used.[13] Following KS, I use RCFD1350 for the Fed funds sold. I use RCFD2800 for the Fed funds purchased (KS do not use Fed funds purchased data). Therefore, $NFS_{i,t} := \text{RCFD1350}_{i,t} - \text{RCFD2800}_{i,t}$ is the net funds position of bank i in quarter t. Then, for a given $\alpha \in (0,1)$, define Positive $\geq \alpha$ as the set of banks whose net funds positions are positive for no less than α fraction of quarters in which they appear in the data; and define Negative $\geq \alpha$ in a parallel way. That is,

$$\begin{aligned}\text{Positive} \geq \alpha \quad &:= \quad \{i,\ s.t.\ |\{t|NFS_{i,t} > 0\}| \geq \alpha T_i\}\,;\\ \text{Negative} \geq \alpha \quad &:= \quad \{i,\ s.t.\ |\{t|NFS_{i,t} < 0\}| \geq \alpha T_i\}\,,\end{aligned}$$

where T_i is the number of quarters in which bank i appears in the data. If α is close to 1, banks in group Positive $\geq \alpha$ are likely to be liquidity unconstrained, and those in group Negative $\geq \alpha$ maximally constrained. Following KS, I use the inverse of the effective Fed funds rate $1/\rho$ to

[13] The Fed Funds rates data is available at U.S. Fed (Board of Governors of the Federal Reserve System, 2025), and the bank attributes data at Wharton Research Data Services (WRDS, 2025). I thank Guohua He for his excellent research assistance.

represents the monetary stance. Proposition 3 implies (in Corollary 2) that the lending scales of banks in Positive $\geq \alpha$ increase with the monetary stance $1/\rho$, but the lending scales of banks in Negative $\geq \alpha$ decrease with it. To test this prediction, I run regression (62) for each group of banks.

$$\Delta \log L_{i,t} = C + \sum_{j=1}^{4} \alpha_j \Delta \log L_{i,t-j} + \sum_{j=0}^{5} F_j \frac{1}{\rho_{t-j}} + \delta_t + \varepsilon, \tag{62}$$

where $L_{i,t}$ represents the total loans of Bank i in quarter t and ρ_t is the effective Fed funds rate in quarter t.[14] As KS, I am interested in the sum of coefficients $F_{sum} := \sum_{j=0}^{5} F_j$. Proposition 3 predicts that $F_{sum} > 0$ *for group Positive* $\geq \alpha$ *and* $F_{sum} < 0$ *for group Negative* $\geq \alpha$ if α is sufficiently close to 1.

For $\alpha = 90\%$, the results of $\{F_j\}_{j=0,\ldots,5}$ for the two groups and also for the full sample are reported in Table 8 below. For other values of α, the results of F_{sum} for the two groups of banks are reported in Table 9 below.

These results are consistent with Proposition 3 : F_{sum} is significantly positive for all the Positive $\geq \alpha$ groups and significantly negative for all but one of the Negative $\geq \alpha$ groups.

4.2 Connections with existing empirical work

This paper's theory is consistent with many existing empirical studies on banks' responses to monetary policy changes, as detailed below.

A. KS (i.e. Kashyap and Stein, 2000). Their core finding is that $\partial^2 L_{it}/\partial B_{it}\partial\rho_t > 0$ for small banks (i.e. banks whose size is below 95 percentile of the distribution), but $\partial^2 L_{it}/\partial B_{it}\partial\rho_t < 0$ for

[14]Following KS, $L_{i,t}$ is measured with RCFD 1400. The monthly effective Fed funds rates are from: https://fred.stlouisfed.org/series/FEDFUNDS. The quarterly interbank rate is defined as the average of the three monthly Fed funds rates of the quarter.

	(1)	(2)	(3)
	Full sample	Positive $\geq$ 90%	Negative $\geq$ 90%
F0	0.61***	0.33*	-3.44***
	(0.168)	(0.166)	(0.998)
F1	0.88***	0.84***	4.08***
	(0.163)	(0.160)	(0.968)
F2	-0.72***	-0.47***	-2.06**
	(0.151)	(0.148)	(0.898)
F3	-0.12	-0.63***	-0.53
	(0.171)	(0.168)	(1.017)
F4	2.03***	2.27***	-0.79
	(0.164)	(0.161)	(0.977)
F5	-1.10***	-1.14***	1.15*
	(0.106)	(0.104)	(0.631)
F_{sum}	1.57***	1.19***	-1.58***
	(0.099)	(0.098)	(0.584)
N	629331	180644	9936
R^2	0.8988	0.9805	0.0329

Table 8: Results of regression (62) for the full sample, Positive ≥ 0.9 and Negative ≥ 0.9 groups; standard errors in parentheses, * for $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

	(1) Positive $\geq$ 80%	(2) Positive $\geq$ 85%	(3) Positive $\geq$ 90%	(4) Positive $\geq$ 95%	(5) Positive $\geq$ 100%
F_{sum}	1.39***	1.32***	1.19***	1.04***	0.56***
	(0.077)	(0.084)	(0.098)	(0.144)	(0.243)
N	317160	246267	180644	99634	40138
R^2	0.9730	0.9775	0.9805	0.9793	0.9820

	(1) Negative $\geq$ 80%	(2) Negative $\geq$ 85%	(3) Negative $\geq$ 90%	(4) Negative $\geq$ 95%	(5) Negative $\geq$ 100%
F_{sum}	-1.35***	-1.38***	-1.58***	-1.28***	-0.95
	(0.421)	(0.504)	(0.584)	(0.478)	(0.649)
N	18258	13325	9936	5979	3079
R^2	0.0286	0.0316	0.0329	0.0850	0.0888

Table 9: Results of regression (62) for the two groups with a variety of α; standard errors in parentheses, * for $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

big banks (i.e. those above 99 percentile),[15] where L_{it} represents Bank i's loan growth at time t and B_{it} represents what they label the "balance-sheet liquidity" , defined as the sum of securities

[15] In their notations, the finding is $\partial^2 L_{it}/\partial B_{it}\partial M_t < 0$ for small banks and $\partial^2 L_{it}/\partial B_{it}\partial M_t > 0$ for big banks, where M_t represents the monetary stance. An increase in M_t always represents monetary expansion, which implies $\partial M/\partial \rho < 0$ (see KS, page 413). Hence $\partial^2 L_{it}/\partial B_{it}\partial M_t > 0$ (< 0) if and only if $\partial^2 L_{it}/\partial B_{it}\partial \rho_t < 0$ (> 0).

plus Fed funds lent relative to the total assets. While KS's narratives are consistent mostly with $\partial^2 L_{it}/\partial B_{it}\partial\rho_t > 0$, the present paper can explain why *both* $\partial^2 L_{it}/\partial B_{it}\partial\rho_t > 0$ for small banks *and* $\partial^2 L_{it}/\partial B_{it}\partial\rho_t < 0$ for big banks.

To begin with, I use the approach based on Proposition 8 to assess the liquidity constraints of these two groups of banks. By Table 1 of KS, small banks are overall net Fed funds lenders, big banks borrowers.[16] Then, Proposition 8 suggests that *small banks are likely to be liquidity unconstrained and big banks maximally constrained.* This finding is at odds with the view of KS, who postulate that big banks are less liquidity-constrained than small banks because the former can acquire liquidity more easily and cheaply than the latter. This discrepancy arises because I use a different approach to assess the tightness of banks' liquidity constraints, as expounded preceding Proposition 8.

I now demonstrate that the present paper implies $\partial^2 L_{it}/\partial B_{it}\partial\rho_t > 0$ for liquidity unconstrained banks and $\partial^2 L_{it}/\partial B_{it}\partial\rho_t < 0$ for banks maximally constrained. In the present paper, the banks are heterogeneous only in the outflow fraction x. Note that

$$\frac{\partial^2 L}{\partial B\partial\rho} = \frac{\partial^2 L}{\partial x\partial\rho}\cdot\left(\frac{\partial B}{\partial x}\right)^{-1}. \tag{63}$$

The signs of both terms on the right hand side are to be determined, starting with $\partial^2 L/\partial x\partial\rho$. In reality, a bank's loan growth L is the net outcome of new loan issuance and the retiring of existing loans. In the present paper, the latter is unconsidered and the former is $M(R)$, given by (5). Hence,

[16] The net Fed funds lent relative to the total assets is 3% for the 8404 banks below 75th percentile, 2% for the 1681 banks between 75th and 90th, −1% for the 560 banks between 90th and 95th, and −5% for bank above 99th percentile.

$L = M(R)$, where $R = R^*(x, \rho)$ in equilibrium. It follows that:

$$\frac{\partial^2 L}{\partial x \partial \rho} = M'(R) \frac{\partial^2 R}{\partial x \partial \rho} + M''(R) \frac{\partial R}{\partial x} \frac{\partial R}{\partial \rho}. \quad (64)$$

Lemma 5. $\frac{\partial^2 L}{\partial x \partial \rho} < 0$ *for both groups of banks if*

$$\rho < \frac{1-\alpha}{2-\alpha} \frac{1}{\omega_e + x(1-\omega_e)}. \quad (65)$$

$1/\alpha$ is the mark-up factor in the bank's lending rate; probably for most banks $1/\alpha \geq 1.2$,[17] so $\alpha \leq 0.83$ and $(1-\alpha)/(2-\alpha) > 0.14$. For the whole sample period of KS, the effective Fed funds rate is no larger than 20% per annum,[18] and hence the quarterly rate $\rho \leq 5\%$. Therefore, inequality (65) holds and $\frac{\partial^2 L}{\partial x \partial \rho} < 0$ for both groups of banks.

Now consider the sign of $\partial B/\partial x$, first for liquidity unconstrained banks. By Proposition 2, these banks do not need to hold liquid assets to meet their liquidity needs. Hence, the liquid assets in their holding result mainly from investment of liquidity that they obtain from the net interbank credit position. It brings in $\Upsilon_n = \Upsilon_I - xM$ (see 9) units of Fed funds, a fraction of which the bank invests in the assets that compose the *balance-sheet liquidity* defined by KS (i.e. securities and Fed funds lent). Assume this fraction is homogeneous across banks in the group, denoted by γ. Then, $B = \gamma(\Upsilon_I - xM)$. It follows that $\partial B/\partial x < 0$ *for liquidity unconstrained banks.*

Second, consider maximally liquidity-constrained banks. By Proposition 8, unlike an unconstrained bank, their net interbank credit positions are likely to be negative and thus shown on the liability side rather than on the asset side. The liquid assets on their asset side are acquired

[17]Loecker and Eeckhout (2017) reports that during the sample period of KS, the unweighted average markup of the US publicly traded firms is between 1.4 and 1.7 (see their Figure 2) and that smaller firms tend to have a higher markup.
[18]See `https://fred.stlouisfed.org/series/FEDFUNDS`.

to meet their liquidity constraints. For this purpose, they need to hold liquid assets of value $V = \overline{\tau}_x M - (1-\omega)\Upsilon$ by Proposition 2. A fraction of these liquid assets consists in the balance-sheet liquidity. Assume this fraction is homogeneous across banks in the group, denoted by $\gamma' > 0$. Then, $B = \gamma' (\overline{\tau}_x M - (1-\omega)\Upsilon_I)$. Hence $\partial B/\partial x > 0$ *for maximally liquidity-constrained banks.*

Altogether, by (63), $\partial^2 L_{it}/\partial B_{it}\partial\rho_t = (-)\cdot(-) > 0$ for liquidity unconstrained banks and $\partial^2 L_{it}/\partial B_{it}\partial\rho_t = (-)\cdot(+) < 0$ for banks maximally constrained.

B. Empirical studies on the pass-through of the policy rate to banks' lending rates. The pass-through is shown to be negatively correlated with banks' government-bond positions by Altavilla *et al.* (2020) in Figure 4 and with their cash positions by Bluedorn *et al.* (2017) in Table 4. Moreover, Kishan and Opiela (2000) show that smaller banks exhibit a higher pass-through.

These findings are consistent with Proposition 3, whereby if the interbank rate moves, the lending rates of liquidity unconstrained banks move in the same direction, but those of the maximally constrained move in the opposite direction. Therefore, the pass-through is positively correlated with bank attributes that indicate the bank is liquidity unconstrained, negatively with attributes that indicate it is maximally constrained. The latter attributes include the positions of fiat money and government bonds; Proposition 3 is hence consistent with Bluedorn *et al.* (2017) and Altavilla *et al.* (2020). Moreover, the present paper has argued in Part A above that small banks are likely to be liquidity unconstrained, hence the consistency with Kishan and Opiela (2000).

C. Wang *et al.* (2022). They make two counter-intuitive empirical findings. (i) One tends to expect that a lower Fed funds rate ρ enlarges bank lending, but they find that the aggregate lending scale falls if ρ decreases from 0.9% (see their Figure 5). (ii) Because ρ is the cost for banks to obtain liquidity, one tends to expect that a higher ρ diminishes the return of banks' equity capital, but they find that this return increases with ρ if $\rho < 2\%$ (see Figures 5 and 7 and Table VII).

The present paper is consistent with both of these empirical findings. Lemma 2 is consistent with Finding (i). Regarding finding (ii), let $v\left(x,\rho,\widetilde{\omega}\right)$ be the earning of type x banks if the interbank rate is ρ, for a given realisation of $\widetilde{\omega}$; v is given on the left-hand side of Equation (22). Finding (ii) is that the average bank earning $\mathbf{E}_{\widetilde{\omega},x}(v(x,\rho,\widetilde{\omega}))$ increases with ρ when $\rho < 2\%$ and is consistent with the following lemma.

LEMMA 6. *If* $\mathbf{E}(x) < \frac{\omega}{1-\omega}$, *then* $\frac{d\mathbf{E}_{\widetilde{\omega},x}(v(x,\rho,\widetilde{\omega}))}{d\rho} > 0$ *for sufficiently small* ρ.

Intuition for the lemma is parallel to that for Lemma 2. I have shown in the proof and discussion of Proposition 3 that the lending costs of maximally constrained types $x > x^H$ actually decrease with ρ. As a result, when ρ rises, their costs fall and earnings rise, whereas the earnings of liquidity unconstrained types $x < x^L$ fall. Between these two offsetting effects on the average earning, the former dominates if ρ is very small, because that indicates that the liquidity constraint is overall very tight.

5 Conclusion

A key policy tool of central banks is the interbank reserve borrowing rate and they shift this rate with nominal operations that alter the composition of the aggregate portfolio of fiat money and government bonds that the private sector holds. This paper elucidates a full logic chain through which such an operation shifts the interbank rate and thence impacts the wide real economy. At the core is a constraint that bank liquidity management imposes upon bank lending, which unfolds as follows. Banks perform lending by issuing liabilities. These liabilities need fiat-money liquidity to service. Ex unsecured borrowing, the quantity of liquidity that a bank can gather is bounded by its positions of fiat money and government bonds. Hence the liquidity constraint: To avoid costly unsecured reserve borrowing, banks must limit the lending scale below a threshold determined by their reserve and government bonds positions.

A change to the composition of the aggregate portfolio of fiat money and government bonds shifts the interbank rate in steady states through a causal chain of three links. First, the change to the composition shifts the real value of the portfolio in steady states. Second, the shift to the portfolio's values moves the maximal tightness of the liquidity constraint. Third, in equilibrium, the sum of the maximal tightness and the interbank rate, being the total benefit of holding fiat-money reserves, is equalised to the constant cost of holding them. When the maximal tightness moves to one direction, the interbank rate must move to the opposite direction.

Banks respond in opposite ways to an interbank-rate shift: Liquidity-unconstrained banks adjust their lending rates in the same direction as the interbank rate, while maximally constrained banks adjust theirs in the opposite direction. Suppose the interbank rate falls. This decreases the cost of obtaining liquidity for *all* banks. Liquidity unconstrained banks receive only this impact and accordingly lower their lending rates. However, maximally liquidity constrained banks receive, in addition, a countervailing impact: The interbank rate is reduced precisely because the tightness of their liquidity constraints is raised. This increases their lending costs by an amount more than offsetting the decrease in the liquidity cost. Consequently, their overall lending costs, and hence their lending rates, increase.

Bank-money circulation affects banks' liquidity constraints. As borrowers use borrowed money for multitudes of trades, each bank sees a fraction of money from its lending flow out into other banks. The higher the outflow fraction, the tighter the liquidity constraint.

The paper underlines fiat money's role to serve banks' liquidity needs rather than as a medium of exchange. Counterparties demand a bank to redeem its liability with fiat money for concerns about its solvency or other reasons; e.g. depositors withdraw because cash is the only accepted means of payment. The advance of digital payment technologies, by reducing bank liquidity needs unrelated to insolvency, has an impact on the steady-state price level. Should it eliminate this type of liquidity needs, fiat money would cease to circulate and bullion standards might return.

Supplementary data

The data and codes for this paper are available on the Journal repository. They were checked for their ability to reproduce the results presented in the paper. The replication package for this paper is available at the following address: https://doi.org/10.5281/zenodo.16756610.

Affiliations

[1]Department of Economics, University of Essex, Colchester, CO4 3SQ, U.K.

References

Aiyagari, S.R. and McGrattan, E. (1998). ‘The optimum quantity of debt’, *Journal of Monetary Economics*, vol. 42(3), pp. 447–469.

Altavilla, C., Canova, F. and Ciccarelli, M. (2020). ‘Mending the broken link: Heterogeneous bank lending rates and monetary policy pass-through’, *Journal of Monetary Economics*, vol. 110, pp. 81–98.

Angeletos, G., Collard, F. and Dellas, H. (2016). ‘Public debt as private liquidity: Optimal policy’, NBER.

Bianchi, J. and Bigio, S. (2022). ‘Banks, liquidity management and monetary policy’, *Econometrica*, vol. 90(1), pp. 391–454.

Bloise, G. and Polemarchakis, H.M. (2006). ‘Theory and practice of monetary policy: Guest editors’ introduction’, *Economic Theory*, pp. 1–23.

Bluedorn, J., Bowdler, C. and Koch, C. (2017). ‘Heterogeneous bank lending responses to monetary policy: New evidence from a real-time identification’, *International Journal of Central Banking*, vol. 13, pp. 95–149.

Board of Governors of the Federal Reserve System (2025). ‘Federal funds effective rates data’, `https://fred.stlouisfed.org/series/FEDFUNDS`.

BoE (2025). ‘Bank of england database’, `https://www.bankofengland.co.uk/boeapps/database/fromshowcolumns.asp?Travel=NIxSUx&FromSeries=1&ToSeries=50&DAT=RNG&FD=1&FM=Jan&FY=2014&TD=19&TM=Aug&TY=2024&FNY=&CSVF=TT&html.x=151&html.y=55&C=KDW&Filter=N`.

Cavalcanti, R., Erosa, A. and Temzelides, T. (1999). 'Private money and reserve management in a random-matching model', *Journal of Political Economy*, vol. 107, pp. 929–945.

Copeland, A., Martin, A. and Walker, M. (2010). 'The tri-party repo market before the 2010 reforms', FRB of New York.

De Fiore, F., Hoerova, M. and Uhlig, H. (2018). 'Money markets, collateral and monetary policy', University of Chicago, Becker Friedman Institute for Economics.

Donaldson, J.D., Piacentino, G. and Thakor, A. (2018). 'Warehouse banking', *Journal of Financial Economics*, vol. 129, pp. 250–267.

Ennis, H. (2018). 'A simple general equilibrium model of large excess reserves', *Journal of Monetary Economics*, vol. 98, pp. 50–65.

Gali, J. (2008). *Monetary Policy, Inflation, and the Business Cycle*, NJ, U.S.A.: Princeton University Press.

Geanakoplos, J. and Dubey, P. (2003). 'Inside and outside fiat money, gains to trade, and is-lm', *Economic Theory*, vol. 21, pp. 347–397.

Gennaiolia, N., Martin, A. and Rossi, S. (2018). 'Banks, government bonds, and default: What do the data say?', *Journal of Monetary Economics*, vol. 98, pp. 98–113.

Grobéty, M. (2018). 'Government debt and growth: The role of liquidity', *Journal of International Money and Finance*, vol. 83, pp. 1–22.

Heider, F. and Hoerova, M. (2009). 'Interbank lending, credit risk premia and collateral', ECB.

Holmström, B. and Tirole, J. (1998). 'Private and public supply of liquidity', *Journal of Political Economy*, vol. 106(1), pp. 1–40.

Jakab, Z. and Kumhof, M. (2015). 'Banks are not intermediaries of loanable funds - and why this matters', Bank of England.

Jonghe, O.D., Dewachterz, H., Mulier, K., Ongena, S. and Schepens, G. (2016). 'Some borrowers are more equal than others: Bank funding shocks and credit reallocation', *Review of Finance*, vol. 24(1), pp. 1–43, doi:10.1093/rof/rfy040.

Kashyap, A. and Stein, J. (2000). 'What do a million observations on banks say about the transmission of monetary policy?', *American Economic Review*, vol. 90, pp. 407–428.

Keynes, J.M. (1914). *A Treatise on Money*, London, UK: Macmillan and Co. Limited.

Kishan, R. and Opiela, T. (2000). 'Bank size, bank capital, and the bank lending channel', *Journal of Money, Credit and Banking*, vol. 32, pp. 121–141.

Kiyotaki, N. and Moore, J. (2001). 'Evil is the root of all money', Clarendon Lecture No. 1, Oxford University.

Kiyotaki, N. and Wright, R. (1989). 'On money as a medium of exchange', *Journal of Political Economy*, vol. 97, pp. 927–954.

Kumhof, M. and Wang, X. (2021). 'Banks, money, and the zero lower bound on deposit rates', *Journal of Economic Dynamics and Control*, vol. 132.

Lagos, R. and Wright, R. (2005). 'A unified framework for monetary theory and policy analysis', *Journal of Political Economy*, vol. 113, pp. 463–484.

Liu, E.X. and Pogach, J. (2016). 'Global banks and syndicated loan spreads: Evidence from u.s. banks', Working paper.

Loecker, J. and Eeckhout, J. (2017). 'The rise of market power and the macroeconomic implications', http://www.nber.org/papers/w23687.

Lucas, R. E., J. (1972). 'Expectations and the neutrality of money', *Journal of Economic Theory*, vol. 4(2), pp. 103–124.

Lucas, R. E., J. (1990). 'Liquidity and interest rates', *Journal of Economic Theory*, vol. 50, pp. 237–264.

Mishkin, F.S. (2016). *Economics of Money, Banking, and Financial Markets 11th Ed.*, London, U.K.: Pearson Education.

Moll, B. (2014). 'Productivity losses from financial frictions: Can self-financing undo capital misallocation?', *American Economic Review*, vol. 104(10), pp. 3186–3221.

Moody's (2025). 'Bankfocus', https://login.bvdinfo.com/R1/BankFocus.

Morrison, A. and Wang, T. (2020). 'Bank liquidity, bank lending, and "bad bank" policies', https://repository.essex.ac.uk/29501/1/Liquidity12.pdf.

Ongena, S. and Yu, Y. (2017). 'Entrepreneur industry affiliation and multiple bank relationships', *Journal of Financial Services Research*, vol. 51(1), pp. 1–17.

Paravisini, D., Rappoport, V. and Schnabl, P. (2014). 'Comparative advantage and specialization in bank lending', Working paper.

Piazzesi, M. and Schneider, M. (2018). 'Payments, credit and asset prices', Mimeo.

Walsh, C.E. (2010). *Monetary Theory and Policy*, MIT Press, 3 edn.

Wang, T. (2019). 'Banks' wealth, banks creation of money, and central banking', *International Journal of Central Banking*, vol. 15(3), pp. 89–137.

Wang, T. (2024). 'Two approaches to modelling commercial banking', `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4948743`.

Wang, Y., Whited, T., Wu, Y. and Xiao, K. (2022). 'Bank market power and monetary policy transmission: Evidence from a structural estimation', *Journal of Finance*, vol. 77, pp. 2093–2141.

Williamson, S. (2012). 'Liquidity, monetary policy, and the financial crisis: A new monetarist approach', *American Economic Review*, vol. 102(6), pp. 2570–2605.

Woodford, M. (1990). 'Public debt as private liquidity', *American Economic Review*, vol. 80(2), pp. 382–388.

WRDS (2025). 'Wharton research data services', `https://wrds-www.wharton.upenn.edu/`.

Appendix A: Bank lending and liquidity management

The liquidity constraint, which is the cornerstone of this paper, results from the following facts pertaining to bank lending and liquidity management.

Fact 1: *A bank's money is the bank's promise to pay.* In everyday language, banks lend out "money" to real-sector borrowers. However, bank money is different in nature to fiat money that the central bank produces. A bank's money is the bank's promise to pay. This statement is self-evident in historical times when bank money often takes the form of banknote. A banknote typically reads "Bank X promises to pay the bearer value Y on demand",[19] and is therefore nothing but

[19]See Appendix C for samples.

a certificate of the bank's promise to pay. Banks perform lending by issuing banknotes, that is, by issuing their promises to pay. Nowadays, few banks print their own notes; the major form of bank money is bank deposit. A deposit, as a banknote, certifies the bank's promise to pay. E.g. a one-dollar deposit with JP Morgan is a contract wherein the bank promises to pay the depositor one dollar of greenback (on demand or at a specific time). As in historical times, banks perform lending by issuing their promise to pay, except that now this promise is recorded not on banknotes but with deposit accounts. E.g. suppose the HSBC lends to a firm £10 million at an interest rate of 8%. What the bank does is credit £10 million into the firm's deposit account on the liability side and add a loan of £10 $(1 + 8\%)$ on the asset side, that is, change its balance sheet as follows; this way of bank lending is noted by Keynes (1914).

Assets (in £million)	Liabilities (in £million)
Old assets (X_H)	Old liabilities (X_H)
Loan to the firm $(10 \times (1 + 8\%))$	Deposit of the firm (10)
	Equity $(+10 \times 8\%)$

Table 10: The double-ledger entries whereby the HSBC lends £10 million to the firm at 8%

The HSBC can credit £10 million of bank money into the firm's account with a click of a mouse because the money is just the bank's promise to pay £10 million. In principle, anyone is free to make a promise to pay; the specialty of banks is that only their promise to pay is widely accepted as a means of payment, others' not.[20]

Fact 2: *Bank liability needs liquidity to service.* The deposits that banks lend out (as in Table 10) are typically *demand deposits*, which are a promise to pay fiat money *on demand*. At any moment, depositors' demands to be paid with fiat money, that is, demands to withdraw cash, might arrive at the bank. Failure to meet the withdrawal demands means a costly liquidity crisis and is not an

[20] I can say to a coffee-shop keeper "I owe you two pound and give me a Latte", but he will say no. For an analysis of this specialty of banks, see Kiyotaki and Moore (2001).

option. Depositor withdrawal demands are thus one reason why bank deposits need liquidity (i.e. fiat money) to service. Another reason is explained in Fact 3.

Fact 3: *When one unit of Bank A's money circulates into Bank B, Bank A owes one unit of fiat-money reserves to Bank B.* Bank deposits are constantly used as a means of payment for multitudes of trading, whereby deposits of the buyer's bank flow into the seller's. When these two banks are different, an interbank liability is created. The example of Table 1 continued, suppose the firm spends £1 million buying a widget from someone that banks with Natwest. Then, Natwest will credit £1 million to the seller's account on the liability side. Moreover, because a bank's money is its promise to pay, £1 million of the HSBC's money flowing into Natwest means that Natwest now holds a promise of the HSBC to pay £1 million. Thus, Natwest adds a credit position of £1 million principal on the asset side. The change to Natwest's balance sheet is illustrated in Table 11.

Assets (in £million)	Liabilities (in £million)
Old Assets (X_N)	Old Liabilities (X_N)
Credit to the HSBC (1)	Deposit of the widget seller (+1)

Table 11: The balance sheet of Natwest when it receives £1 million of HSBC's money

To settle this interbank liability, HSBC needs to find £1 million of fiat-money reserves. In general, each bank sees a fraction of its money issued from lending flows out into other banks, becoming its liabilities to these banks, as illustrated in Figure 1. The bigger is this *outflow fraction*, the higher the interbank liabilities and the greater the liquidity demand.

Certainly, banks are also at the receiving ends of money circulation (like Natwest in Table 11). As a result, they hold credit positions to other banks and gain liquidity from them. These interbank positions are important for banks' liquidity management.

As a result of these three facts, *Bank lending is subject to a liquidity constraint.* Facts 1 to 3 together imply that lending of bank money needs fiat-money liquidity to service, through both depositor withdrawal and interbank settlement demands. If a bank's holding of fiat money is in adequate to meet the liquidity demand, the bank has to borrow fiat money on the interbank reserve market (or from the central bank). Placing collateral can substantially facilitate this borrowing and reduce its cost, as documented by Heider and Hoerova (2009).[21] Government bonds are preferred collateral.[22] A bank's capacity for secured borrowing is thus limited by its bond positions. Altogether, banks face a *liquidity constraint*: To avoid costly unsecured borrowing, they must restrict lending scale within an upper limit so that the liquidity demand engendered by lending can be met with their fiat money and bond positions.

[21]See Figure 1 of their paper. They also document the sheer size of secured interbank markets – e.g. $10 trillion in the United States in August 2007. Moreover, De Fiore *et al.* (2018) document that secured interbank borrowing is growing, while unsecured borrowing declining, in the U.S. over the past 15 years.

[22]Indeed, banks typically hold large positions of government bonds, on average 9% of their assets according to Gennaiolia *et al.* (2018).

Appendix C: Samples of Banknotes

The text in the cursive font reads: We promise to pay the Bearer on Demand the sum of Five pounds.

It reads: THE WALTHAM BANK Will pay to bearer on demand One Hundred Dollars.

It reads: North of Scotland Bank Limited promise to pay the bearer on Demand ONE POUND.