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Lobbying: influence under micro-targeting

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Abstract
Interest groups (IGs) can potentially influence policy making process or policy outcomes in their favor by using different lobbying tactics. In this paper, we study how and under what circumstances IGs exert influence on policy outcomes when they can use Micro-targeting (MT) as a lobbying strategy, i.e. the IGs can send group specific messages to a subset of voters. In the absence of IGs, the political candidate does not have means to privately commit a policy to a group of voters, who might vote for her after observing such policy commitment. Recognizing this, IGs can get policy favours from the political candidate in exchange for facilitating candidate's private commitment. We identify conditions in which MT is influential, in the sense of leading to a different policy outcome in the presence of IGs. The analysis fully characterizes the set of influential MT equilibria. The like minded IG does not have any direct influence, but its presence could severely impact the direct influence of unlike minded IGs given that the competing candidates are ideologically motivated. Moreover, this may also lead to polarisation between the two competing candidates.

1 Introduction

Political micro-targeting (MT), in a broad sense, means identifying the subset of voters so that voters can be delivered tailored messages based on their preferences and opinions. There is ample evidence that politics, to some extent, is driven by MT, which has become more effective and intensive over the years. The use of MT in politics has been around for a long time. However, the topic only became popular after the 2012 US presidential elections and more so after the unexpected outcome of the 2016 US presidential election. MT appears to be important for political parties and their campaigns, as it allows them to communicate to swing voters alone. However, political

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parties usually do not have the resources, infrastructure, or valuable data on voters to micro-target directly. This is where the intermediaries can come in and provide their services, either for monetary rewards or to further their political agendas. This paper aims to examine how political motives of interest groups (IGs) impact policies when these lobbying organisations can micro-target voters in exchange for policy favours.

In addition to communication/advertising agencies and consultancies, some IGs are also working towards building issue-specific databases. For instance, the National Rifle Association (NRA), a pro-gun lobbying organisation in the US, has built a massive database on gun owners. The NRA has been very successful in pushing for its pro-gun agenda, even though the majority of US citizens support strict gun laws.[1] Other examples of single-issue groups that utilize their members to exert influence include the AFL-CIO (the American Federation of Labour, and Congress of Industrial organisation), one of the largest lobby groups with 11 million members. The AFL-CIO endorses labor-friendly political candidates and mobilizes union members to vote, volunteer, and campaign. Similarly, the Sierra Club, an environmental organization with a large, eco-conscious membership, promotes conservation through litigation, public campaigns, and grassroots actions through members, demonstrating widespread public support for environmental causes. This prompts us to study the role of MT in the context of lobbying. In this paper, we study how and which IGs can influence policy outcomes when they help political parties to make use of sophisticated methods such as MT. We show that MT is an ineffective tool for IGs that have the same policy preference as the the political candidate; but interestingly, their presence in the political system itself could influence the policy outcome. In contrast, IGs who promote policies which are different from the candidate's ideal political position could potentially use MT to influence the policy outcomes in their favour. The underlying mechanism behind these results is that the candidate is able to send group-specific messages with the help of an IG, in return for which the candidate implements the IG's ideal policy.

In recent times, Facebook and Cambridge Analytica have combined data mining and data analysis on more than 71 million people and used it for strategic communication. It is claimed that strategic MT, based on this data, was first used in the British EU referendum and then in the 2016 US presidential election.[2] Even though these organisations are mainly businesses which want to sell this information for monetary rewards, there are other groups or organisations which make use of this information to exert influence on candidates. A review of the NRA legislation news page shows that it has had 230 victories at the state level from 2004 to 2013. Watkins (2013) found that in 2012, the NRA-ILA (NRA's institute for legislative actions) was very influential in promoting the legislation in Florida that would punish doctors who asked their patients about their gun possession.[3] Many argue that the power of the NRA comes from the money it spends on lobbying; but the NRA spends much less than and, arguably, is more influential than lobbying organisations like the National Association of Realtors

[1] Source Opensecret.org: 73% of the candidates the NRA directly supported at the federal level won in the 2016 US election. A recent poll conducted by Gallup shows that sixty-one percent of Americans favour stricter laws on the sale of firearms.

[2] Source: The Guardian (https://www.theguardian.com/news/2018/apr/10/cambridge-analytica-and-facebook-face-class-action-lawsuit.

[3] Source: nraila.org.

and the Koch Brothers.[4] The NRA has a huge membership and a powerful activist base. These members are very loyal to the organization and vote one way or the other based on the NRA's view of a particular candidate. Winkler (2013) argued that the real source of the NRA's influence comes from its members. This is somewhat in line with evidence that money does not play a major role in politics as supported by Azari (2015).

There is extensive research on IGs' influence on the policy making process, but the literature barely addresses MT. IGs are traditionally seen to influence policy in two ways: first, IG may provide campaign contributions in exchange (implicit or explicit) for policy favours (Grossman and Helpman 1994; Hillman and Riley 1989); second, IGs can also provide policy relevant information and thereby influence legislators' beliefs about the consequences of different policies (Milgrom and Roberts 1986; Austen-Smith andWright 1992). IGs may also engage in indirect lobbying, such as grass roots lobbying or issue advocacy advertising (Yu 2005; Kollman 1998; Bombardini and Trebbi 2011). While studying these lobbying tactics is undoubtedly important in understanding IGs' influence, studying MT as a lobbying strategy would shed some light on how MT is being used by IGs to their advantage.

In the literature of lobbying, influence is considered from different perspectives such as influence on the content of a bill or influence on the prospects of a given bill being enacted into law. In this paper, influence simply means an IG's ability to change the policy outcome in its favour, which would not be possible without IG's lobbying efforts. Influence might come about either because of the IG's direct lobbying effort or indirectly because of the IG's existence in the political system. We therefore further distinguish between direct and indirect influence.

We present a model of lobbying that does not include the traditional channels through which lobbying distorts policy making in favour of IGs. Instead, the IGs can use MT as a tool to lobby political candidates. Since we want to study the impact MT has on policy outcomes, we assume that information on some voters' preferences is private to some IGs. The analysis characterizes necessary and sufficient conditions for MT to be influential, i.e. to change policy outcomes compared to an alternative setting in which lobbying is not allowed.

In our simplified model, there are two political issues and a political ideological challenger aims to win an election against a passive incumbent by securing over half of the total votes. IGs attempt to influence the policy outcome by lobbying the challenger, focusing on a single policy issue. We assume that IGs can only lobby one of the candidates, who we call the 'challenger'. Voters, grouped based on their policy preferences, align with concerned like-minded IG. Lobbying involves the exchange of favors, with the challenger making policy promises to gain support. These promises serve as private commitment devices, influencing voters who compare the challenger's commitment and the incumbent's ideals during elections. The uncertainty about the challenger's post-election policy decisions arises due to the influence of IGs.

In the absence of IGs, the challenger publicly commits to a policy for all voters, with no cost incurred. For lobbying to be influential, the challenger must lose without

[4] According to Center for Responsive Politics, the NRA spent \$4.1 million and \$5.1 million on lobbying in 2018 and 2017 respectively, whereas National Association of Realtors spent \$53.7 million and \$54.5 million on lobbying in the same years.

it, compelling her to consider altering policies based on IG offers. Influence hinges on the challenger's incentive to change policies in response to IG offers, particularly when facing a close election and candidates are sufficiently opposed, leading to compromise on the challenger's ideal policy. The analysis identifies conditions for such behavior. Two crucial features of our model shape the analysis. Firstly, random access is employed, with only one IG gaining the opportunity to offer a deal to the challenger, highlighting the randomness and exclusivity of access.[5] Secondly, an IG with access can only negotiate on the policy issue it cares about, mirroring realistic scenarios where groups advocate for specific policy concerns, such as the NRA lobbying on gun policy rather than other issues like abortion.

The analysis examines influential MT across three ideological scenarios between candidates: (1) Polar case, where candidates are completely opposed; (2) Semi-alike case, with partial opposition; and (3) Alike case, where candidates are identical. In the semi-alike case, MT influences policy outcomes if the challenger prioritizes the issue on which she differs from the incumbent and the unlike-minded IG gets access. In the polar case, MT proves influential when challenger-opposed IGs gain access and the challenger accepts their offer, making the challenger better off despite compromising on an issue. In the alike case, there is no influence.[6] Necessary and sufficient conditions are identified across all scenarios for influential MT. In the semi-alike case, a crucial condition emerges, requiring the challenger to promise her ideal policy to the IG who aligns with her but opposes the incumbent. This ensures voter support without losing aligned voters in the presence of IGs. Notably, lobbying in the semi-alike case results in a more drastic policy shift compared to the polar case, where influential MT leads to partial differences in policy outcomes.

The remainder of the paper is organised as follow. Section 2 reviews the most relevant literature. Section 3 presents the model and a benchmark case. Section 4 presents the main results, which are discussed in Sect. 5. Section 6 concludes. All proofs are in the "Appendix".

2 Related literature

In lobbying literature, influence through campaign contributions and policy-relevant information have been extensively studied (Schlozman and Tierney 1986; Grossman and Helpman 2001). Research on the role of contributions in IGs' influence (Austen-Smith 1987; Grossman and Helpman 1994, 1996; Baron 1994) often treats them as bribes or means to gain access. Grossman and Helpman (2001) found contribution-based influence detrimental, while Austen-Smith andWright (1992) showed information-based influence to be welfare-enhancing. Bennedsen and Feldmann (2006) explored a setting where IGs use both contributions and information for influence. Despite comprehensive coverage of lobbying aspects, none of these studies incorporate MT in formal models. Our paper introduces a form of lobbying between

[5] Commonly, access is valuable and sought through campaign contributions, but in our study focusing on the role of MT, contributions are not considered a means to secure access.

[6] The challenger would never accept an offer necessitating deviation from her ideal policy, as she could always reject and commit to her preferred policy.

contributions and information, i.e. IGs delivering votes via private commitments in exchange for policy favors.

Numerous authors have explored electoral competition models to investigate IGs' influence on policy-makers (PM) through quid pro quo exchange (Bernheim and Whinston 1986; Besley and Coate 2001; Dixit et al. 1997; Grossman and Helpman 1994, 1996). Such quid pro quo exchange is detrimental to the social welfare when a strategic IG decides to offer a bribe to the PM, in exchange for which PM chooses a policy she would have not chosen otherwise. Most use a "menu-auction" framework where conditional contributions are binding, assuming all IGs play a role. We challenge this assumption by proposing a model where access is random, departing from the conventional understanding that politically experienced IGs always gain access. Moreover, empirical evidence by Wright (1996) indicates the coexistence of numerous IGs, prompting our model to capture the indirect influence of dormant IGs, subtly impacting policy-making without active lobbying.

The strand of literature on informational lobbying explores how the transmission of policy-relevant information by IGs may influence the policy choices of PMs. This information transmission can be costly (verifiable information) as discussed by Austen-Smith (1994); Austen-Smith andWright (1992); Dahm and Porteiro (2008); Schnakenberg (2017) or costless (cheap talk) as examined by Austen-Smith (1993); Austen-Smith and Banks (2002).[7] The consensus in this literature suggests that informational lobbying (in the absence of contributions) is generally welfare-enhancing, as it better informs PMs about the consequences of relevant policy issues. However, it can also be detrimental if the PM disproportionately prioritizes issues on which they are heavily lobbied, as noted by Cotton and Déllis (2016).

In all these studies on informational lobbying, the information of interest pertains to the consequences of different policies. In contrast, we consider a different kind of information, i.e. the information on voters' preferences, which facilitates MT. Unlike informational lobbying, which presents technical details to support specific policies, MT uses data analytics to deliver personalized political messages to voters. Both informational lobbying and MT can indirectly influence each other. For example, successful MT campaigns can shift public opinion, creating a more favourable environment for IGs to lobby PM. Conversely, informational lobbying can result in policies that align with the messages being pushed through MT campaigns. As a first step, we analyse influence where IGs have information on some voters' preferences and use that to influence PM. We leave it for the future research where both informational lobbying and MT can be incorporated to understand the nature of their combined influence.

Schelling (2006) explained that commitment serves no purpose if it is not communicated to its targeted audience. Commitments are designed to alter expectations of others' behavior, emphasizing the importance of targeted communication without leaks. We adopt a model that explicitly incorporates a mechanism allowing varied private commitments to be made to distinct sets of voters.

Closest to our paper is Grossman and Helpman (1999), exploring how endorsements by organized IGs convey information to voters. They focus on fixed and pliable policy issues, with voters uncertain about pliable issue preferences. They found public

[7] For a full survey of theoretical literature on IG's influence see Schnakenberg and Turner (2024).

endorsements ineffective, but private endorsements or non-perfectly complementary interests encourage parties to compete for endorsements. Our model significantly differs, especially in modelling interactions and preferences. The key contrast lies in our candidate's indirect office motivation through competition, while in their paper, parties lack specific preferences and pursue endorsements. We allow multiple non-competing IGs, isolating the endorsement's pure effect, and assign each voter to an IG for better message credibility. Additionally, the voters in our model know their preferences but may be uncertain about the challenger's policy, unlike their model where voters observe announced policies but remain uncertain about their impact.

In political endorsement literature, McKelvey and Ordeshook (1985) highlighted group endorsements as an effective means of communication with imperfectly informed voters. Their model features candidates who are unaware of voters' preferences, and a subset of voters who are uninformed about candidates' policy positions. These voters rely on pre-election polls and endorsements to gather information. In contrast, Grofman and Norrander (1990) explicitly models the endorsement process. In their setup, voters are uncertain about candidates' fixed policy stances on the real line. Two informed endorsers, with common knowledge of their preferences, influence voters by supporting candidates not too distant from their own ideals. Voters adjust beliefs about candidates' policies based on these non-strategic endorsements.

Lupia (1992) and Cameron and Jung (1995) examined the impact of endorsements on voting behavior in referendum scenarios with a status quo and an agenda setter proposing changes. The agenda setter aims to maximize her benefits through a proposal that must secure a majority vote to replace the status quo. Voters lack information on the proposed initiative's impact on their utilities but understand the agenda setter's utility function. Both studies include an endorser disseminating information about the initiative. In Lupia (1992), the non-strategic endorser informs voters whether the alternative leans left or right of the status quo, while Cameron and Jung (1995) introduces a strategic endorser with policy preferences, influencing voters through her decision. Our analysis extends the theoretical literature on electoral competition, challenging the assumption of public communication. Unlike studies like Laslier (2006), which explores ambiguity in mass communication, our model acknowledges modern election campaigns' ability to micro-target specific voters through IGs, allowing communication to be more targeted and private.

In Schipper and Woo (2019), political candidates use MT to influence a subset of voters, focusing on candidates with fixed ideological stances in a multi-dimensional policy space. The candidates compete for voters who are unaware of all political issues and are unsure about the exact policy position of the candidates. Candidates compete for uninformed voters, sending private messages about subsets of issues and policy positions. They find that election outcomes mirror those with fully informed voters but suggest breakdowns if candidates cannot utilize MT. Unlike their approach, our model assumes that voters are fully informed about candidates' ideal policies and allows IGs to privately reveal politicians' commitments, aiming to inform rather than persuade voters through MT.

In Prummer (2019), a micro-targeting model explores how political candidates strategically select advertising media outlets with different audiences to maximize their chances of winning. The study reveals that increasing media outlet fragmentation intensifies polarization between candidates. In our lobbying-focused MT context, we find that MT does not consistently lead to polarization; the outcome varies based on the candidates' ideologies, resulting in potential polarization to increase or decrease.

3 Model

There are 2 sets of players in the game: a challenger and voters. The challenger (c) is running for election where winning mandates implementing policies on the economy (p_1) and gun control (p_2). We denote a policy by $p = (p_1, p_2)$ where $p_1 \in \{L, R\}$ and $p_2 \in \{A, P\}$: $p_1 = L$ and $p_1 = R$ corresponds to left type and right type policies, respectively and, $p_2 = P$ and $p_2 = A$ corresponds to pro-gun and anti-gun policies, respectively. There is also an incumbent (i) who top-ranks a policy pair p_i and implements it if the challenger loses, common knowledge to all. We assume that incumbent wins in case of tie.

The population of size 1 is partitioned into groups of voters based on policy preferences, $j = (L, P), (L, A), (R, P), (R, A)$. We denote the group of voters with the generic element, $j = (j_1, j_2)$ where group j constitutes fraction n_j of the electorate and j_1 and j_2 are the ideal policies of group-j voters on policy issues p_1 and p_2. For notational convenience, we write (j_1, j_2) as $j_1 j_2$. Voters in each group are further divided into two subgroups: those who care more about the economy, and those who care more about guns. Given implemented policy p, the utility of the group $j = j_1 j_2$ voters is:

$$W_j(p_1, p_2, \alpha) = \begin{cases} 1 + \alpha, & \text{if } p_1 = j_1, p_2 = j_2 \\ 1, & \text{if } p_1 = j_1, p_2 \neq j_2 \\ \alpha, & \text{if } p_1 \neq j_1, p_2 = j_2 \\ 0, & \text{if } p_1 \neq j_1, p_2 \neq j_2 \end{cases} \tag{1}$$

where $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$ represents importance of issue p_2 relative to issue p_1, with $\underline{\alpha} < 1 < \overline{\alpha}$. Voters in group j with $\alpha = \overline{\alpha}$ are the voters who care more about p_2 (or guns) and constitutes a proportion δ of the voters in group j.[8] All voters vote sincerely.

The challenger's ideal policy is given by p_c, which is common knowledge and α_c is the weight she assigns to guns (p_2): she has the same policy preferences as one of the voter groups. Election of the challenger requires a majority of the votes; the incumbent wins otherwise. The challenger only knows the distribution of the voters' preferences (n_j and δ), but cannot identify a given voter's preferences.

Special interest groups (IGs) Given the binary nature of our model, we assume that there are four IGs, two opposing IGs on each issue. The interest groups are denoted by $l = L, R, A, P$, where L and R are the IGs who only care about the economy, and strictly prefer policy L and R respectively. P and A are the IGs who only care about guns, and strictly prefer P and A respectively. The *like-minded* IGs are the ones whose ideal policies coincide with the challenger's ideal policy; the other IGs are *unlike-minded*.

[8] Heterogeneity within each group accounts for the fact that salience of an issue might be different for voters with the same policy preferences.

Each IG has private information on the identity of some voters. The IG can identify the group of voters who care more about the same issue as the IG and have the same policy preference on that issue. Such voters are called the members of that IG. For instance, the members of IG L are the voters in group LP and LA for whom $\alpha = \underline{\alpha}$. The policy preference of the members of any particular IG is known to all, but only their IG has their contact information and therefore, the members can only be contacted through the IG.

We assume that only one IG gets access to the challenger. We present two arguments to support the realism of this assumption. First, allowing only one IG to influence the challenger maintains accountability, as politicians, motivated by both office and ideology, may compromise on less important issues to appeal to a broader voter base. This helps candidates navigate voter preferences while maintaining core principles. Second, allowing two IGs to gain access (one on each issue) undermines MT, as the opposing IG (the one without access) can easily infer if the challenger has been compromised.

Nature then privately selects one IG, revealing the choice only to the chosen IG and the challenger.[9] The selected IG makes an offer prescribing its ideal policy and commits to endorsing the challenger if the offer is accepted. If accepted, the challenger commits to implementing a policy on that (lobbied) issue, free to choose any policy on the other.

We assume that the challenger implements her ideal policy on the uncommitted issue. Let p_l be the policy the challenger implements when IG l gets access and the challenger accept its offer (or when l endorses the challenger). The challenger can also *publicly* broadcast her commitment. The challenger's strategy specifies: (i) whether she accepts or rejects the offer she receives from each IG; and (ii) which policy pair to publicly commit to after rejecting an offer.

If l gains access and its offer is accepted, its members are informed of the challenger's endorsement and her commitment to policy p_l. Conversely, if l is denied access or if its offer is rejected, voters either observe the challenger's public commitment or no message, in which case it is one of the other IGs whose offer is accepted. IGs cannot conceal endorsement information, and voters share common knowledge of the access distribution assigned by Nature. The game described is played once.

Payoffs Given policy p, an IG earns 1 if its ideal policy is implemented and 0 otherwise, while a voter in group j earns $W_j(p, \alpha)$. The challenger belonging to group j earns $W_j(p, \alpha_c)$ if she wins the election and implements p, and $W_j(p_i, \alpha)$ if she loses where p_i is the policy implemented by the incumbent.

Timing In stage 0, Nature chooses IG l with probability $\pi_l \geq 0$ where $0 \leq \sum \pi_l \leq 1$. Nature's choice is privately revealed to the challenger and the chosen IG. In stage 1, the selected IG makes a private offer, p_l to the challenger. In stage 2, the challenger decides whether to accept or reject the offer. If she rejects then the challenger publicly announces her policy commitment. If she accepts, she promises to implement the offered policy conditional on winning and the IG then endorses the challenger and

[9] IG's years of experience and connection in the political system put them at the forefront of the line to access the political candidate. Since our focus is not on how a particular IG gains access but rather on the influence exerted, the most effective method is to assign access randomly. We can then analyse the influence exerted under each scenario of IG access.

informs its members of the endorsement. In stage 3, voters form their beliefs after receiving any information about the endorsement and vote either for or against the challenger. The winner is announced and policy is implemented.

Strategies and Equilibrium Since there is no lobbying cost involved, we assume that the IGs always makes an offer to the challenger when they get access. In that sense, IGs do not take any decision. The challenger decides whether to accept or reject the offered deal from the chosen IG. Let $\gamma = (\gamma_L, \gamma_R, \gamma_A, \gamma_P)$ denote the challenger's *acceptance strategy*, where $\gamma_l \in \{reject, accept\}$. The challenger publicly commits if she rejects. Let q denote the challenger's *public announcement strategy*

Voters' information (conditional on the history) is denoted by I_j^k where $I_j^k = 1$ if voters k in group j receive information from their representative IG that their IG endorsed the challenger and $I_j^k = 0$ if voters k in group j receive no information. Let $\beta_l = pr(l \mid I_j^k)$ denote the posterior belief of voter k belonging to group j that the IG l got access to the challenger. Voters use Bayes' rule to update their belief about which IG got access to the challenger. A voter k in group j calculates his expected payoff and votes for the challenger if and only if

$$\sum_l \beta_l (W_j(p_l; \alpha)) > W_j(p_i; \alpha) \tag{2}$$

We call the game described above the MT game. The equilibrium concept is Perfect Bayesian Equilibrium (PBE) in pure strategies. Loosely speaking, an equilibrium consists of strategies γ^* and q^*, and beliefs $\beta^*(\cdot)$ such that: (1) at every decision stage, each agent takes an action that maximises its expected payoff given its belief and the others' behaviour, and (2) beliefs are derived using Bayes' rule whenever possible.

3.1 Influence

Since all players care about which policy is implemented, we analyse how the introduction of MT influences the *policy outcome* of the election. Let $O = p$ be a generic outcome of the MT game. For every strategy profile σ of the MT game $O(\sigma) = (O_L(\sigma), O_R(\sigma), O_A(\sigma), O_P(\sigma))$ is the outcome vector of the game under σ where $O_l(\sigma)$ is the outcome conditional on IG l getting access in PBE σ. Throughout the discussion, we use this definition of *policy outcome* to measure the influence of lobbying, and focus on determining conditions under which this policy outcome is different in the presence of IGs.

In order to define influence, we study the benchmark case of the MT game in which $\pi_l = 0$ $(\forall l)$, i.e. in which MT is not feasible. Thus, the challenger cannot send group-specific messages. We refer to this special case of the MT game as the no-lobbying game; otherwise whenever we refer to MT game, it is with regard to the game where $\pi_l > 0$ for all l.

Direct influence

Let O^{NL} be an outcome of the no-lobbying game. Then, $O^{NL}(\sigma^{NL}) = \{O_l^{NL}(\sigma^{NL})\}$, where $O_l^{NL}(\sigma^{NL}) = O^{NL}(\sigma^{NL}) : \forall l$. We say that the MT is directly *influential* if $O_l(\sigma) \neq O^{NL}$ for at least one l for some PBE σ of the MT game where $\pi_l > 0$ $(\forall l)$, and that MT is otherwise not directly *influential*. We say that IG l has direct influence when $O_l(\sigma) \neq O^{NL}$ for IG l. The outcome is different when the implemented policy is different in the MT game where $\pi_l > 0$ $(\forall l)$ as compared to the no-lobbying game. We further examine how the presence of any particular IG affects the direct influence of other IGs. Such influence is called *indirect influence*.

Indirect influence

To measure indirect influence, we compare cases in which $\pi_l > 0$ for all l with cases in which $\pi_l = 0$ for some (exactly one) IG l. We call the latter case the modified game. We assume that in the modified game, no one gets access when Nature chooses l with probability 0. Formally, let $O_j^{-l}(\sigma^{-l})$ be the outcome of the MT game under σ^{-l} and $\pi_l = 0$ when $j \neq l$ gets access. Then, we say that l has indirect influence if $O_j^{-l}(\sigma^{-l}) \neq O_j(\sigma)$ for at least one $j \neq l$ for some PBE σ of the MT game where $\pi_l > 0$ (all l) and some PBE σ^{-l} of the modified game where $\pi_l = 0$ for some (exactly one) l. To find indirect influence of IG l on the direct influence of other IGs $j \neq l$, we consider the equilibrium of the MT game where direct influence from each IG is possible. Formally this requires that the challenger accepts offers from all IGs.

4 Influential micro-targeting

In this section, we identify conditions under which MT is influential. Throughout, the incumbent's ideal policy is fixed at $p_i = (R, A)$. We examine three cases of challengers with ideal policies that differ from those of the incumbent: (a) the *semi-alike* case: the challenger differs from the incumbent on only one policy issue i.e. $p_c = (R, P)$ (we focus on this semi-alike case wlog.); (b) The *polar* case: the challenger differs from the incumbent on both policy issues, i.e. $p_c = (L, P)$; and (c) The *alike* case: the challenger shares policy with the incumbent on both policy issues, i.e. $p_c = (R, A)$. To find direct influence, we assume that each IG has positive probability of getting access $(\pi_l > 0)$.

We investigate the influence of MT. We start by discussing direct and indirect influence in the three different cases: (4.1) alike, (4.2) semi-alike, and (4.3) polar. We then interpret and discuss the main results in Sect. 5. We seek to characterize the PBEs of the no-lobbying and the MT games. In the no-lobbying game, the challenger publicly commits to one of the four policy pairs. In the MT game, the challenger has to make two decisions: she first decides whether to accept or reject the offer she receives from each IG, and then decides which policy to publicly commit to if she rejects an offer. Note that the challenger does not have to commit to her ideal policy if she rejects an offer.

4.1 Alike case: $p_c = (R, A)$

When the challenger and incumbent share same interests, lobbying efforts (MT) by interest groups have no impact. Their aligned interests mean that MT does not alter policy outcomes. The challenger can get her ideal policy implemented, either by winning or strategically losing the election, in which case the incumbent implements (R, A). In cases of equal votes, the incumbent wins. In the MT game, the challenger rejects offers from unlike-minded interest groups, as accepting would not be rational since she could strategically lose the election for better outcomes. Consequently, the challenger only accepts offers if she anticipates losing, publicly committing to her ideal policy.

4.2 Semi-alike case: $p_c = (R, P)$

We now analyse the case in which the challenger and the incumbent are semi-alike: $p_c = (R, P)$ and $p_i = (R, A)$. Lemma 1 below identifies necessary conditions for MT to be influential in the semi-alike case.

Lemma 1 *If $p_c = (R, P)$ then MT is directly influential only if $n_{LP} + n_{RP} < 1/2$.*

The vote share the challenger gets if she publicly commits to her ideal policy is $n_{LP} + n_{RP}$ and the premise implies that the challenger must lose the election if she publicly committed to her ideal policy.[10] Consequently, the challenger may be motivated to accept offers from one or both unlike-minded IGs. If $n_{LP} + n_{RP} > 1/2$, the challenger can publicly commit to her ideal policy (R, P) and get the highest possible payoff, i.e. $1 + \alpha_c$. Accepting an offer from an unlike-minded IG means implementing a policy which is different from (R, P), earning a lower payoff.

Observation 1 *The like-minded IGs R and P have no direct influence.*

The first reason for this observation stems from the aversion of voters with policy preference (L, A), who are members of unlike-minded interest groups L and A. These voters would not support the challenger unless endorsed by their representative IG.[11] These voters, therefore, abstain from supporting the challenger if they neither see public commitment or an endorsement from their IG and they anticipate the challenger to accept offers from like-minded IGs in equilibrium. The second reason is that voters with policy preference (R, A) consistently support the incumbent, as her ideal policy yields them the highest payoff. They maintain this loyalty when they neither observe public commitment nor an endorsement from their IG, anticipating the challenger to accept offers from any other IG. When like-minded IGs endorse the challenger, voters in groups LA and RA still vote for the incumbent, limiting the challenger's maximum votes to $n_{LP} + n_{RP}$, which is less than majority from Lemma 1. Hence, like-minded IGs have no direct influence.

[10] Refer to Table 2 in the "Appendix" to see the proportion of votes the challenger gets for each of the four public policy commitment.

[11] In which case some might vote for the challenger and some vote for either candidate with equal probability depending on which IG they belong to.

Influence might depend on whether the challenger cares more about the economy or guns. These cases are discussed in the subsequent subsections.

Semi-alike case in which the challenger cares more about guns The challenger prioritizes the issue on which her policy preference differs from the incumbent's: $p_c = (R, P)$ and $\alpha_c > 1$.[12]

We first specify the equilibrium outcome of the no-lobbying game. The proof of no lobbying game is in the "Appendix". Given $\alpha_c > 1$, the challenger's policy preference order is: $(R, P) \succ (L, P) \succ (R, A) \succ (L, A)$. Note that her reservation utility is 1, which she can always get by publicly committing to the incumbent's ideal policy. Thus, any policy which gives her less than 1 will never be implemented. This is formally given below:

Outcome in no-lobbying game when $p_c = (R, P)$ and $\alpha_c > 1$. The outcome is:

$$O^{NL} = \begin{cases} (R, P) & \text{if } n_{LP} + n_{RP} > 1/2 \\ (L, P) & \text{if } n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2 > n_{LP} + n_{RP} \\ (R, A) & \text{otherwise} \end{cases}$$

The following lemmas identify necessary conditions for MT to be influential.

Lemma 2 *Let $p_c = (R, P)$ and $\alpha_c > 1$. MT is directly influential only if $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2$.*

Lemma 2 is a direct consequence of Lemma 1. The condition in the premise implies that the challenger's second best policy (L, P) must be unpopular among voters if she were to publicly commit to it. If not, then outcome of the no-lobbying game is the challenger wins and implements (L, P). In MT game, IG R and P have no direct influence from Observation 1. The equilibrium where the challenger accepts L's offer exists only if she would win after accepting $L's$ offer; otherwise, she could publicly commit to (L, P). Consequently, L has no direct influence. The challenger would not accept A's offer, as it would compel her to implement the incumbent's ideal policy, whereas she can achieve a better outcome by rejecting and announcing her second-best policy, and winning. Thus, A is not influential either. As a consequence, MT can only be influential if the challenger's second best policy is also unpopular among the voters.

Lemma 3 *Let $p_c = (R, P)$ and $\alpha_c > 1$. Unlike-minded IG L can have direct influence only if the challenger accepts the offer from like-minded IG P.*

[12] We account for differences in the challenger's salience on specific issues. For example, in 2024 US presidential political debates, Senator JD Vance (US elect vice-president) places significant emphasis on abortion rights, even though this issue might not be a top priority for most US citizens, who are more concerned with immigration, leadership, and other economic issues. This does not imply that politicians neglect issues of broader societal importance; there are instances where politicians passionately advocate for critical issues like climate control, which have widespread societal significance. In the current context, it could be that policy p_2 is more popular among voters ($\delta > 1/2$) but the challenger cares more about either p_2 ($\alpha_c > 1$) or p_1 ($\alpha_c < 1$). The analysis addresses both scenarios.

We know from Observation 1 that like-minded IGs do not have direct influence. Therefore, only unlike-minded IGs L and A can have direct influence. Interestingly, L can have direct influence only if the challenger accepts P's offer in equilibrium. While P itself does not exert direct influence, its presence and the potential acceptance of its offer can impact the direct influence of other unlike-minded IGs.

How voters vote when L endorses the challenger? All members of L vote for the challenger when L endorses the challenger: members in group LA get convinced and members in group LP vote for the challenger regardless.[13] We know from Observation 1 that all members of P and A vote for the challenger and the incumbent respectively, when they are uninformed.[14] Members of R in group RP vote for the challenger only if the challenger accepted P's offer in equilibrium.[15] This ensures that the challenger might not deviate from the economic issue, which they prioritise and also share an interest with the incumbent.

Proposition 1 below identifies the regions of the parameter space in which unlike minded IG L has direct influence. For each of the parameter configurations satisfying the conditions in the Proposition, equilibrium behaviour satisfies the necessary condition of Lemma 3 for unlike-minded IG L to have direct influence.

Proposition 1 *Let* $p_c = (R, P)$ *and* $\alpha_c > 1$. *Then,*
(P1.1) L has direct influence iff $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ *and* $n_{LP} + (1-\delta)n_{LA} + n_{RP} > 1/2$.
(P1.2) A does not have any direct influence.

To analyse L and A's direct influence, we first look at the equilibrium outcome in the no-lobbying game. Lemmas 1 and 2 must hold if MT is directly influential, in which case the outcome in the no-lobbying game is that the incumbent wins: $O^{NL} = (R, A)$.

Whenever MT is influential, lobbying by IGs L and A changes the outcome. This requires that, on gaining access, both L and A's offers are accepted, leading to a change in the implemented policy. As discussed above in Lemma 3, this also requires that the challenger accepts P's offer in equilibrium for L to have direct influence.

When L gets a policy commitment from the challenger,[16] it communicates this endorsement to its members, who all then vote for the challenger. The members of IG P, uncertain about the challenger's commitment, vote in favor if they anticipate the challenger to accept an offer from either L or R, whereas members of A vote for the incumbent if they receive no exclusive information. Some members of R must vote for the challenger, otherwise the challenger loses from Lemma 2. These members of R vote for the challenger if P endorsed the challenger; and vice-versa if L endorsed the challenger. Condition in P1.1 guarantees that those members of R have a large

[13] Note that the voters with ideal policy (L, A) least prefer the challenger's ideal policy and vote for the incumbent unless they a get policy commitment through their representative IG.

[14] Members of P with ideal policy (L, P) vote for the challenger because they least prefer the incumbent's ideal policy and those with ideal policy (R, P) also vote for challenger because they care more about guns and only the challenger implements their ideal policy on guns. Members of A with ideal policy (L, A) vote for the incumbent because they least prefer the challenger's ideal policy and those with ideal policy (R, A) also vote for the incumbent because her ideal policy is same as their ideal policy.

[15] The members of IG R in group RA vote for the incumbent because their ideal policy is completely aligned with the incumbent's.

[16] The challenger implements policy (L, P) if she accepts L's offer and wins.

enough incentive to vote for the challenger. This condition essentially means that P's probability of access is large enough compared to L's.[17] How much larger it need be depends on the value of $\underline{\alpha}$. The second part of P1.1 is the vote share when L endorses the challenger, given first part of the condition is satisfied. Condition P1.2 states that IG A does not have direct influence because, even if the challenger wins with IG A's endorsement, the implemented policy would be (R, A), which is the same as in the no-lobbying scenario.

To summarise, the equilibrium outcome in no-lobbying game is: $O^{NL} = (R, A)$. The equilibrium outcome of the MT game when L gets access is that the challenger wins, resulting in $O_L = (L, P)$ because condition P1.1 is satisfied. Let O^{MT} be the outcome vector in the MT game: $O^{MT} = (O_L, O_R, O_A, O_P)$. There is direct influence because $O^{NL} \neq O_l$ for $l = L$.

We now discuss indirect influence when the challenger in the semi-alike case cares more about guns.

Proposition 2 *Let $p_c = (R, P)$, $\alpha_c > 1$. Then, only like-minded IG P has indirect influence. IG P's presence affects the direct influence of IG L iff $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ and $n_{LP} + (1 - \delta)n_{LA} + n_{RP} > 1/2$.*

The presence of IG P affects the direct influence of unlike-minded IG L in that IG L does not have direct influence if $\pi_P = 0$. The purpose of MT is to promise a policy privately to some voters for their votes, leaving others uninformed. Uninformed voters might support the challenger based on their beliefs about access and influence of other IGs. It turns out that the crucial group of voters are in group RP who are members of IG R, whose vote depend on π_P.[18] These voters support the challenger if they believed that the challenger is likely to implement a policy closer to their ideal. Despite the potential risk of a worse policy if L endorses the challenger, their confidence to vote for the challenger is bolstered in the presence of IG P. However, if P is unable to secure access from the beginning, they are certain the challenger's implemented policy would be inferior to the incumbent's. The conditions are essentially the majority winning conditions for L to exert direct influence. Absence of these conditions results in the challenger's loss, leading to the absence of direct influence and, consequently, no indirect influence.

The challenger in the semi-alike case who cares more about the economy: the policy on which the challenger and the incumbent agree Specifically, $p_c = (R, P)$ and $\alpha_c < 1$.

[17] This asymmetry reflects the realistic scenario where certain IGs, due to their resources, experience, and established connections, are more likely to secure access. Voters, aware of these dynamics, infer the likelihood of different IGs gaining access. Yet only those directly informed by their IGs know the specific details. This understanding is shaped by voters' long-term observations of IG lobbying efforts, campaign contributions, grassroots activities, and other political engagements. This knowledge allows voters to make educated guesses about the influence exerted by different IGs, even if they do not have direct information about each instance of access.

[18] Voters in group LP always vote for the challenger as long as at least one of the following conditions holds: $\pi_L > 0$, $\pi_P > 0$, or $\pi_R > 0$. Voters in group RA always vote for the incumbent unless the challenger publicly commits to (R, A) or only $\pi_A > 0$. Voters in group LA who are members of A always vote for the incumbent.

When the challenger and the incumbent share the same policy preference on the issue that is more important to the challenger, there is no direct or indirect influence. If $\alpha_c < 1$, the challenger's second best policy is (R, A), which is completely aligned with that of incumbent. We know from Observation 1 that like-minded IGs do not exert any direct influence.

Direct influence from A does not alter the policy outcome. The challenger can always secure her second highest payoff by losing the election if her ideal policy is unpopular among the voters. If she wins by accepting L's offer, her payoff is α_c. Thus, she could profitably deviate to publicly commit to (R, P) or (R, A), which would result in losing the election by securing a payoff of 1. Consequently, unlike-minded IG L does not exert direct influence. It is irrational for the challenger to compromise on the issue she prioritises and agrees on with the incumbent.

4.3 Polar case: $p_c = (L, P)$

In the polar case, the challenger and incumbent hold opposing policies. The worst outcome for the challenger is if the incumbent wins, implementing policies she opposes on both issues. Hence, she is willing to compromise on any issue for a victory, enabling her to enact different policies. Consequently, influence is anticipated to be greater in the polar case than in the semi-alike one, implying less stringent conditions for influential equilibria. Lemma 4 below identifies a necessary condition for an influential MT.

Lemma 4 *Let $p_c = (L, P)$. MT is influential only if $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2$.*

Following the same argument as in the semi-alike case, the premise ensures that the challenger's ideal policy is unpopular among voters. The proportion of votes the challenger gets if she publicly commits to her ideal policy is $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$, which must be less than majority.

Observation 2 *The like-minded IGs have no direct influence.*

The argument parallels Observation 1. Firstly, if the challenger's ideal policy is widely favored, Lemma 4 demonstrates that no IGs hold direct influence. Secondly, when the challenger's ideal policy lacks popularity among voters (i.e., $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2$), the challenger can only secure a maximum of $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$ votes if like-minded IGs L or P endorsed her.[19] Observe that $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$ is the challenger's vote share if all the members of like-minded IGs vote for the challenger.[20] Therefore, when L or P endorses the challenger, the challenger must get votes from some members of unlike-minded IGs R and A to win, else there is no direct influence. When uninformed, members of unlike-minded IGs refrain from voting for the challenger due to two reasons. Firstly, some voters strongly support the

19 Note that if the challenger rejects any offer then the outcome is same as in the no-lobbying game. Thus, there is no change in the outcome when IG l gets access and the challenger rejects l's offer.

20 The members of L are voters in group LA and LP with $\alpha = \underline{\alpha}$. The members of P are voters in group RP and LP with $\alpha = \overline{\alpha}$. δ proportion of the voters have $\alpha = \overline{\alpha}$ and $1-\delta$ proportion of the voters have $\alpha = \underline{\alpha}$. If all the members of L and P vote for the challenger, the challenger gets a vote share of $(1-\delta)(n_{LA} + n_{LP}) + \delta(n_{RP} + n_{LP})$.

incumbent, sticking to their preferences even if their IG endorses the challenger with partial policy changes. Secondly, voters with strong preferences for the incumbent's ideal policy are reluctant to vote for the challenger when unlike-minded IGs endorse the challenger, as they anticipate the challenger would not implement their preferred policies without their representative IG's endorsement. This results in no support for the challenger from unlike-minded IG members even when their group endorses the challenger.

Challenger in the polar case who cares more about guns Specifically, $p_c = (L, P)$ and $\alpha_c > 1$: so the challenger's policy preference order is $(L, P) \succ (R, P) \succ (L, A) \succ (R, A)$. Below is the equilibrium outcome of the no-lobbying game.
Outcome in no-lobbying case when $p_c = (L, P)$ and $\alpha_c > 1$. The outcome is:

$$O^{NL} = \begin{cases} (L, P), & \text{if } n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2 \\ (R, P), & \text{if } n_{LP} + n_{RP} > 1/2 > n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} \\ (L, A), & \text{if } n_{LP} + n_{LA} > 1/2 \quad \text{and} \quad max\{n_{LP} + n_{RP}, n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}\} < 1/2 \\ (R, A), & \text{otherwise} \end{cases}$$

The proof is in the "Appendix". The following lemma identifies a necessary condition for MT to be influential in the polar case when the challenger cares more about guns.

Lemma 5 *Let $p_c = (L, P)$ and $\alpha_c > 1$. MT is influential only if $n_{LP} + n_{RP} < 1/2$.*

The premise establishes that the challenger's second best policy must be unpopular among the voters. Assuming the contrary, if the second-best policy is popular, the challenger would reject offers from L, P, and A, as accepting any would lead to suboptimal outcomes. She would accept R's offer only if she were to win, otherwise she has a profitable deviation to publicly committing to (R, P). In the absence of lobbying, the challenger commits to this policy and achieves her second-highest payoff, maintaining the same outcome in the MT game. Consequently, no direct influence is present when $n_{LP} + n_{RP} > 1/2$.

We consider two cases: 1. When the challenger's third best policy (L, A) is popular i.e. $n_{LP} + n_{LA} > 1/2$, and 2. When (L, A) is unpopular, i.e. $n_{LP} + n_{LA} < 1/2$: $n_{LP} + n_{LA}$ is the share of votes the challenger gets if she publicly commits to (L, A). Proposition 3 describes the equilibrium in which the challenger rejects offers from both like-minded IGs. The Supplementary Appendix details three additional influential equilibria as outlined in Propositions 7, 7a and 7b. These equilibria involve direct influence from both unlike-minded IGs, and although the analysis remains consistent, the conditions for influence vary across the different equilibria depending on the challenger's equilibrium strategy.

Proposition 3 *Let $p_c = (L, P)$, $n_{LP} + n_{LA} < 1/2$ and $\alpha_c > 1$. If the challenger rejects offers from both the like-minded IGs then R and A have direct influence if and only if each of the following conditions hold:*
(P3.1) $\underline{\alpha} < \frac{\pi_A}{\pi_R} < 1$ or $\overline{\alpha} > \frac{\pi_A}{\pi_R} > 1$ and,
(P3.2) $min\{n_{LP} + n_{RP} + (1-\delta)n_{LA}, n_{LP} + n_{LA} + \delta n_{RP}\} > 1/2$.

Lemmas 4 and 5, and condition $n_{LP} + n_{LA} < 1/2$ imply that the outcome in the no-lobbying game is that the incumbent wins: $O^{NL} = (R, A)$. Now we explain the voting patterns among various voter groups when they receive no public or private endorsement. The voters in group LP vote for the challenger unless they see a public commitment of (R, A) because their preferences are perfectly aligned with the challenger: they are anti-incumbent.[21] The opposite holds true for the voters in group RA.

The voters in group LA who are members of A vote for the challenger if their IG endorses the challenger, otherwise vote for the incumbent because their second best policy (R, A) is better than (R, P).[22] The voters in group LA who are members of L vote for the challenger if they believe that the challenger is more likely to implement their ideal policy ($\pi_A > \pi_R$) or the payoff they derive from the policy they share with the incumbent is low enough, i.e. $\underline{\alpha} < \frac{\pi_A}{\pi_R} < 1$.

The voters in group RP who are members of R vote for the challenger if their IG endorses the challenger; otherwise vote for the incumbent because their second best policy (R, A) is better than (L, A). The voters in group RP, who are members of P, vote for the challenger if they believe that the challenger is more likely to implement their ideal policy ($\pi_A < \pi_R$) or the payoff they derive from the policy they share with the challenger is high enough, i.e. $\overline{\alpha} > \frac{\pi_A}{\pi_R} > 1$.

When R gains access, voters in group LA who are members of L are pivotal for R's direct influence. Without their support, the challenger's maximum vote share is limited to $n_{LP} + n_{RP}$, which is less than a half by Lemma 5. Similarly, for A to wield direct influence, the essential voters are in group RP who are members of R. Without their vote, the challenger's maximum vote share is $n_{LP} + n_{LA}$, again insufficient for a majority based on the premise condition. Hence, condition P3.1, which is necessary for these group of voters to vote for the challenger. When condition P3.1 is satisfied, the challenger secures a higher vote share than possible with a public commitment, and direct influence occurs if this vote share constitutes a majority. Therefore, condition P3.2 must be satisfied: where $n_{LP} + n_{RP} + (1-\delta)n_{LA}$ is the vote share the challenger gets when R endorses the challenger, and $n_{LP} + n_{LA} + \delta n_{RP} > 1/2$ is her vote share when A endorses the challenger.

We now analyse the case where the challenger wins when she publicly announces policy (L, A), i.e. $n_{LA} + n_{LP} > 1/2$. In the no-lobbying game, the challenger announces (L, A) and wins the election: $O^{NL} = (L, A)$.

Proposition 4 *Let $p_c = (L, P)$, $n_{LP} + n_{LA} > 1/2$ and $\alpha_c > 1$. There is a unique influential equilibrium in which the challenger rejects the offers from both like-minded IGs and accepts the offers from both unlike-minded IGs. In such an equilibrium only unlike-minded IG R has direct influence iff $\frac{\pi_A}{\pi_R} > \underline{\alpha}$ or $\frac{\pi_A}{\pi_R} > 1$ and $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$.*

[21] The challenger only implements (R, A) when she publicly commits to it.

[22] Note that the challenger is committed to implement (L, A) [resp.(R, P)] if she accepts A's [resp.R's] offer and publicly commit if L or P gets access. Thus, if the challenger does not publicly commit, it must be that she would either implement (L, A) or (R, P).

We know from Observation 2 that like-minded IGs have no direct influence. The challenger will reject any offers from like-minded IGs because she loses and gets a payoff of 0. She could do better by deviating to publicly commit to (L, A) because she wins, which gives her a payoff of 1. The challenger needs to accept offers from a minimum of two IGs to enable any IGs to wield direct influence. Rejecting offers from more than two IGs allows voters to accurately deduce the challenger's policy stance in the absence of endorsements or public commitment. In this case, the challenger rejects the offers from both the like-minded IGs; so unlike-minded IGs can only have direct influence if the challenger accepts offers from both R and A.

An influential equilibrium exists if and only if the challenger wins after accepting A's and R's offers; otherwise she could profitably deviate to publicly commit to (L, A); so the challenger wins after accepting R's and A's offer in such an equilibrium. When the challenger accepts A's offer, she wins and commits to (L, A). Thus, A does not have direct influence because the challenger wins and implements (L, A) even without A's endorsement; so, only IG R can have direct influence. When the challenger accepts R's offer, she wins and implements (R, P). Thus, R has direct influence: $O_R = (R, P) \neq O^{NL}$ and the conditions are same as in proposition 3.

We now discuss indirect influence when the challenger in the polar case cares more about guns. Note that to find indirect influence, we only consider the case where the challenger accepts offers from all IGs. Now, if the challenger's third best policy is popular among the voters then the challenger would reject offers from the like-minded IGs and publicly commit to (L, A). Thus, to find indirect influence we must impose the condition that the challenger's third best policy is unpopular, i.e. $n_{LP} + n_{RP} < 1/2$.

Proposition 5 *Let $p_c = (L, P)$ and $\pi_A > \pi_R$. Then,*
(P5.1) Like minded P does not have indirect influence.
(P5.2) L can impact the direct influence exerted by unlike minded IGs A and R iff $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha} < \frac{\pi_A}{\pi_R}$.
(P5.3) R can impact the direct influence exerted by A iff $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha} < \frac{\pi_L+\pi_A}{\pi_L}$.
(P5.4) A can impact the direct influence exerted by R iff $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$ and $\underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$.

In the polar case, we observe more indirect influence as compared to the semi-alike scenario due to two sets of voters whose decisions rely on their prioritized issue and posterior beliefs shaped by IGs' access probabilities. These voter groups include members of L advocating for policy (L, A) and members of P supporting policy (R, P). Conversely, in the semi-alike case, such dependency is applicable to only one voter group. We explain why P lacks indirect influence. Assuming P cannot access the challenger ($\pi_P = 0$), this absence doesn't alter its own members' votes. Members of L supporting policy (L, A) still vote for the challenger since A has higher access probability than R ($\pi_A > \pi_R$), aligning with their economic priorities. Thus, P's presence does not influence their voting behaviour.

L's presence could impact the direct influence of R and A. The first part of the condition $\frac{\pi_A+\pi_L}{\pi_R+\pi_L} < \overline{\alpha}$ ensures that members of P with policy (R, P) vote for the challenger in the presence of all IGs.[23] Members of P with policy preference (R, P)

[23] This is so that the challenger wins when $\pi_l > 0$ for all l.

prioritize gun-related policies. They understand that voting for the challenger could lead to the implementation of their ideal policy under R's, the second-best under L's, and the worst under A's endorsement. Without L's presence, they anticipate either A or R endorsing the challenger, increasing the likelihood of their worst policy being implemented since A is more likely to get access. If $\overline{\alpha}$ fails to offset this risk, these voters favour the incumbent. Thus, in L's absence, they do not support the challenger if $\overline{\alpha} < \frac{\pi_A}{\pi_R}$, second part of the condition. If so, the challenger loses when endorsed by A or R, nullifying A or R's direct influence. Consequently, L indirectly influences the outcome when A or R would have directly influenced it, and voters oppose the challenger in L's absence.

The presence of R does not alter the voting behaviour of L's members with ideal policy (L, A), as they favour the challenger if $\pi_A > \pi_R$. However, R's presence may influence P's members with ideal policy (R, P), pivotal for A's direct influence. These voters' ideal policy is implemented only if R endorses the challenger, otherwise these voters face potential second-best (L, P) or worst policies (L, A) if they vote for challenger and third best policy (R, A) if vote for incumbent. Without R, they vote for the incumbent unless $\overline{\alpha}$ offsets risks from A endorsing the challenger, resulting in the worst policy. Consequently, R indirectly influences the outcome when A would have directly influenced it, and voters oppose the challenger in R's absence. Similarly, the presence of A does not alter the voting behaviour of P's members with ideal policy (R, P). However, A may influence L's members with ideal policy (L, A), crucial for R's direct influence. Without A's presence in endorsing the challenger, these voters vote for the incumbent if they prioritize gun-related policies i.e. $\underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$. This scenario gives A indirect influence, as without it, insufficient support for challenger leads to A losing direct influence.

Proposition 6 *Let $p_c = (L, P)$ and $\pi_A < \pi_R$. Then,*
(P6.1) Like minded L does not have indirect influence.
(P6.2) P can impact the direct influence exerted by unlike minded IGs A and R iff $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha} > \frac{\pi_A}{\pi_R}$.
(P6.3)R can impact the direct influence exerted by A iff $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha}$ and $\overline{\alpha} < \frac{\pi_A+\pi_L}{\pi_L}$.
(P6.4) A can impact the direct influence exerted by R iff $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha} > \frac{\pi_P}{\pi_P+\pi_R}$.

As discussed in previous proposition, two group of swing voters are members of L advocating for policy (L, A) and members of P supporting policy (R, P). We explain how the voting behaviour of these voters changes based on the access probabilities of a specific IG. Assuming L cannot access the challenger ($\pi_L = 0$), this absence does not alter its own members' votes. Members of P supporting policy (R, P) still vote for the challenger since R has higher access probability than A, aligning with their economic priorities. Thus, L's presence has no impact on swing voters' voting behaviour.

P's presence could impact the direct influence of R and A. The first part of the condition $\frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha}$ ensures that members of L with ideal policy (L, A) vote for the challenger in the presence of all IGs. Members of L with ideal policy (L, A) prioritize economy-related policies. They understand that voting for the challenger could lead to the implementation of ideal policy under A's, the second-best policy under L's, and the worst policy under R's endorsement. Without L's presence, they anticipate either A or R endorsed the challenger, increasing the likelihood of their worst policy being implemented since R is more likely to get access. If $\underline{\alpha}$ fails to offset this risk, these voters favour the incumbent. Thus, in P's absence, they vote against the challenger if $\underline{\alpha} > \frac{\pi_A}{\pi_R}$, second part of the condition. If so, the challenger loses when endorsed by A or R, nullifying A or R's direct influence. Consequently, P indirectly influences the outcome when A or R would have directly influenced it, and voters oppose the challenger in P's absence.

For the discussion of indirect influence of R and A, see the discussion in the last paragraph of Proposition 5.

Challenger in the polar case who cares more about the economy Specifically, $p_c = (L, P)$ and $\alpha_c < 1$. Here, we have analogous arguments as for the case where the challenger cares more about guns. We have the same claims to make where we observe direct influence from both of unlike-minded IGs and indirect influence. Thus, there is no difference in terms of finding influence in the polar case irrespective of whether the challenger cares more about the economy or guns.

As expected, we see that both unlike-minded IGs have direct influence when the challenger would definitely lose in the no-lobbying game. We also observe influence from one of the unlike-minded IGs when the challenger wins with her third favourite policy in the no-lobbying game. This is different from the semi-alike case where we observe influence only when the challenger definitely loses in the no-lobbying game. The reason why α_c does not change the result in the polar case is that the challenger and the incumbent are completely opposed to each other; so the challenger accepts any deal which gives her the opportunity to win. Irrespective of whether the challenger cares more about the economy or guns, the incumbent's ideal policy remains her least favourite policy, and therefore, this does not change the influence based on salience of an issue.

5 Discussion

We now summarize the results above and discuss the intuition.

Result 1 *Like-minded IGs have no direct influence.*

We identify two reasons underlying an IG's influence: (i) the dissemination of information regarding the challenger's private commitment to the IG among various voter groups, and (ii) the diversity in policy preferences among voters, both within and between groups. Consider a scenario where the challenger, a rightist advocate of gun rights, faces an incumbent who opposes such rights. In cases of direct influence, the IG persuades the challenger to align with its policy stance, particularly when their ideals conflict. Subsequently, the IG selectively shares this information with sympathetic members. Conversely, if the challenger and the IG share same policy ideals, direct influence becomes inconsequential. This suggests that like-minded IGs lack the capacity for direct influence. Alternatively, direct influence could manifest as the IG convincing the challenger to uphold her stance on their shared policy, thereby aiding in electoral success. However, this commitment may not secure sufficient votes for victory, particularly from leftist, anti-gun supporters who prioritize economic issues. The main argument lies in the inability of like-minded IGs to sway electoral outcomes beyond the challenger's inherent support base.

Partially, this outcome illustrates how influence is exerted within the policy-making process or during elections when interest groups aim to sway candidates in their favour. For instance, NRA's strategy concerning a candidate's stance on gun policy reveals a pattern: the NRA adopts an aggressive stance when faced with an anti-gun candidate, pressuring them to adjust their position under threat of losing votes. Conversely, if a candidate aligns with the NRA's position, the organization endorses them and mobilizes member votes. However, beyond its membership base, the NRA cannot significantly bolster a candidate's electoral prospects. This is because non-members largely consist of anti-gun voters who typically support the incumbent, and some pro-gun voters who prioritize alternative issues. Since these voters can be attracted through public commitments, the challenger doesn't rely on NRA's micro-targeting assistance.

In the given scenario, suppose the challenger, holding rightist and pro-gun views, faces defeat by openly supporting her stance. She secures backing from all pro-gun voters, whether rightist or leftist, due to alignment with their ideals. Even if endorsed by a pro-gun interest group, her voter base remains unchanged. However, efforts to court anti-gun voters prove futile, as both rightist and leftist constituents tend to support the incumbent. This is because they anticipate a potential alliance between the challenger and a like-minded interest group, thus denying the challenger any anti-gun votes.

Result 2 *If a pro-gun challenger, who prioritizes guns, faces an anti-gun incumbent, then the presence of a pro-gun IG becomes crucial for implementing pro-gun policies.*

An IG's influence depends on how the uninformed voters vote, which in turn depends on whether the challenger accepts offers from other IGs. That is how the presence of one IG and the challenger's private commitment to an IG affects the direct influence of other IGs. In Proposition 2, we showed that leftist, anti-gun voters consistently support the incumbent due to their opposition to the challenger's policy stance, whereas rightist, pro-gun voters who prioritize gun-related issues consistently support the challenger. They understand that any policy implemented by the challenger would either surpass or match the incumbent's preferred stance. However, those who prioritises economy are swing voters. They are members of R and their vote depends (when uninformed of challenger's commitment) on posterior access probabilities of L, A, and P. In P's absence they are sure that incumbent is better option for them, whereas in P's presence, there is a possibility that challenger serves them right.

Relating this result back to the NRA example underscores the significance of the NRA's presence for pro-gun voters not affiliated with the organization, particularly those prioritizing economic concerns. Without NRA involvement, the challenger risks losing support from pro-gun voters focused on the economy. Conversely, if the challenger lacks NRA's support, there is a likelihood of deviation on economic policy issue. However, with NRA endorsement, these voters may lean towards the challenger, hoping she maintains her pro-gun stance. Essentially, uninformed pro-gun voters, prioritizing the economy, tend to favor the incumbent in the absence of NRA's influence, leading to insufficient support for the challenger and victory for the incumbent, thereby implementing anti-gun policies unfavorable to the NRA. Thus, while the

NRA's endorsement may not attract additional voters, it can detract significant support from the challenger.

Result 3 *The indirect influence is more intense in the semi-alike case than in the polar case.*

Proposition 3 implies that in the polar case, the challenger does not have to accept offer from a like-minded IG for the unlike-minded IGs to have direct influence. This dynamic arises because in the polar case, like-minded IGs oppose the incumbent's ideals entirely, aligning with the challenger's objectives. Consequently, some members of like-minded IGs unconditionally support the challenger, sharing common interests with them.

In the semi-alike case, only one like-minded IG, i.e. pro-gun lobby group has members who disagrees with the incumbent's ideal policy on both issues. In fact, the other like-minded IG has members who share the incumbent's interests on both issues; in the polar case, none of the members of like-minded IGs share the incumbent's interests on both issues. Therefore, in the semi-alike case, the challenger makes sure to get their votes, and to do so she has to accept an offer from the other like-minded IG because this ensure those members that there is some chance that the challenger would stand by her ideal policy. Hence, the challenger must accept P's offer for there to be any meaning influence which benefits P (cf. Proposition 2).

Although, the IGs are passive players, the preferences of their members carry significant weight for the challenger to decide whether to accept or reject an offer, thereby influencing the potential impact on the outcome. Essentially, when the challenger is ideologically further away from the incumbent, she is not much concerned about losing the support of like-minded IG members if offers are refused. Some members of like-minded IGs have policy preference such that their ideal policy is implemented when one of the unlike-minded IGs endorses the challenger. For these voters, the presence of that unlike-minded IG is more important than the presence of like-minded IG. Therefore, the presence of like-minded IGs is not imperative for the challenger to secure the additional votes needed to win the election.

Result 4 *Even though direct influence is more pronounced in the polar-case, the direct influence leads to more polarisation between the competing candidates in the semi-alike case.*

Another interesting point is observed by comparing the implemented policies in Propositions 1, 3, and 4, which suggests that when candidates are relatively closer ideologically (i.e. they share same preference on one issue but the challenger cares more about the other issue), then MT leads to more polarisation between the candidates than when candidates are diametrically opposed to each other. Polarisation here measures how far the challenger's implemented policy is from the incumbent's ideal. This alignment can intensify partisan divide and incentivize candidates to adopt more extreme positions to cater to the demands of their affiliated lobbying groups. Additionally, when lobbying organizations exert significant financial or grassroots influence, they can further exacerbate polarization by amplifying ideological differences and promoting confrontational politics. At first glance, it may appear evident that when candidates are ideologically closer, the challenger's path to victory entails moving away from the incumbent's ideological position, whereas in the polar case, winning necessitates moving closer to the incumbent. However, as elaborated in Result 3, the intensity of indirect MT is more pronounced in the semi-similar case, serving as a significant factor contributing to polarization between the two candidates.

Our results supports the view that IG's ideological strategies i.e. strategy focused on supporting candidates who share particular policy views leads to more polarisation as compared to access oriented strategies in which IG prioritises obtaining favours from incumbents. This view is partially supported by an empirical study conducted by Phillips (2023) in which the author employs machine learning model to classify interest groups into access oriented and strategy focused groups. The results show that Legislatures in states where ideological interest groups play a larger role in campaign donations during the previous election tend to be more polarized. Additionally, individual lawmakers who heavily depend on contributions from ideological interest groups for their campaign funding tend to exhibit more extreme ideological positions. However, these patterns do not consistently apply to access-oriented interest groups. The strategy focused group in our model could be those like minded IGs who exert indirect influence, i.e. those in semi-alike case whereas in the polar case, the like minded IGs do not have indirect influence, however the unlike minded IGs have influence who focus on striking a deal with the challenger.

The findings of Garlick (2022) provide further validation for this assertion. Their empirical analysis, based on lobbying reports from the US Congress and three state legislatures, reveals that the correlation between lobbying and polarization is most pronounced when interest groups engage in negative lobbying efforts aimed at defeating proposed bills. In our framework, indirect influence can similarly be interpreted as negative lobbying. When an interest group is conspicuously absent from the lobbying process, it signals to its members the possibility of the challenger reconsidering their stance on a critical policy issue. We demonstrate that in a semi-alike scenario, this type of adverse inference is exacerbated because the challenger is compelled to accept offers from like-minded interest groups; failure to do so results in loss of support. However, this dynamic is not necessarily applicable in a polarized scenario. Essentially, negative lobbying exerts a greater impact in the semi-alike context compared to the polarized one.

6 Conclusion

Our results in Sect. 4, discussed in details in Sect. 5, are robust to positive rents from office. Let $x > 0$ be the rent from holding office, so the challenger has an additive utility function. Positive rents from office do not alter the analysis of influence in the polar case because the challenger is willing to deviate on both issues. Essentially, the challenger gains positive utility only by winning the election. In the semi-alike case, the only difference is that the challenger might be willing to deviate on the issue she values more if the rents are sufficiently high. Therefore, allowing for positive rents does not offer additional insights. Positive rents from office might motivate a candidate to publicly support a policy completely opposite to her ideals to avoid losing, which is unrealistic and another reason to assume away positive rents from office.

Our results may offer valuable insights into the real-world elections. First, MT can be a decisive factor in close elections by directly influencing key voter segments and potentially altering outcomes. In our model, a close election occurs when the ideological challenger is unpopular among voters. By reaching out to swing voters-those whose ideal policy does not perfectly align with either candidate-through an IG, the challenger can win by compromising on a less important issue. For example, in the 2016 U.S. Presidential Election, micro-targeting in swing states like Wisconsin, Michigan, and Pennsylvania played a crucial role in Trump's victory by shifting the preferences/votes of crucial voter segments. Second, MT can contribute to political polarization by reinforcing existing beliefs and influencing swing voters, as they are primarily exposed to information that to a large degree aligns with their views. In our model, polarization through MT manifests not only in the policies implemented due to lobbying but also in the polarization of voters themselves. For example, MT can persuade some anti-gun voters to support a pro-gun candidate by emphasizing other issues that are more important to them. Through targeted messaging, the challenger promises to implement the voters' ideal policies on these priority issues, leading to increased polarization between pro-gun and anti-gun voters.

Third, our results explain the mechanism through which like-minded IGs indirectly exert influence, particularly when a candidate is significantly invested in the policy advocated by the IG. If the candidate does not assure the like-minded IG that she will support its ideal policy on the relevant issue, she risks losing the IG's backing and, consequently, the votes of its supporters. This suggests why some policies are more likely to be implemented in the presence of MT by IGs. For example, the U.S. Congress is reluctant to introduce stricter regulation on gun control. There have been long obstructions to the Manchin-Toomey amendment to have stricter background checks for gun buyers in the wake of the Newtown massacre. On April 17 2013, it failed to get enough votes to pass in the U.S. Senate, even though the majority of Americans favoured these regulations according to contemporary opinion polls. On the same day, former U.S. President Obama urged voters to put more pressure on their representatives to pass gun control regulations:"Ultimately, you outnumber those who argued the other way. But they're better organized. They're better financed. They've been at it longer. And they make sure to stay focused on this one issue during election time. And that's the reason why you can have something that 90 percent of Americans support and you can't get it through the Senate or the House of Representatives".[24]

Our paper calls for more theoretical and empirical research on this topic to understand how influential MT is. We have measured influence by comparing the policy outcome in the presence of IGs to the outcome without IGs and made claims about whether MT is effective in changing the policy outcome. It would be interesting to study MT in the context of social welfare, i.e. to explore whether MT is detrimental or

[24] Before the defeat of gun-control amendments in April 2013, it was noted that "large majorities back expanding background checks to cover all purchases. (...) And yet, as Newtown disappeared further in the political rearview mirror, the same politics that had turned guns into a dormant issue on the national political stage for much of the 1990s and 2000s began to take hold. Senate Democrats up for re-election in Republican-leaning states in 2014—think Montana, North Carolina, Alaska, Arkansas and Louisiana—were loathe to vote on things like the assault weapons ban." ("Newtown didn't change the politics of guns," Washington Post, March 22, 2013)

welfare enhancing. Another avenue for future research is to explore the effectiveness of MT as compared to other lobbying strategies, such as campaign contributions. The NRA's financial contributions are nominal in the lobbying business; but the role that its contributions play is still unknown.

The model could be extended to cover cases where the incumbent can also be lobbied. In this extension, IGs have an active role to decide who to lobby. This might provide useful insights on why some interest groups are more likely to lobby a particular party or candidate: for instance why the NRA almost always endorses Republican candidates? In this setting it would be difficult to keep track of voters' beliefs about the policy of the candidate who is not lobbied. However, the analysis could be simplified by assuming that when candidates are not lobbied, they stand by their public commitment, as in the no-lobbying game. It seems that, on the one hand, we would expect to see more influence because IGs have more opportunities to push their agendas; but on the other hand, competition between the candidates might have negative effects on their influence. It would be interesting to see how these two mechanisms interact. Another interesting extension would be to allow for more than one IG to get access and endorse the candidate, say two opposing IGs get access. In this setting, competition between the IGs might lead to better informed voters. Although answers to these interesting questions would provide further insights on the use of MT in influencing policy-making process, the question remains of how to model them. One attraction of our model is that we do not have to address these questions in the analysis we are interested in.

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Declarations

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Appendix

Claim 1 *If the challenger rejects an offer from IG l then $O_l = O^{NL}$.*

Table 1 Voters' payoff

Voters	Payoff			
	(R, A)	(R, P)	(L, P)	(L, A)
RP with $\alpha = \overline{\alpha}$	1	$1 + \overline{\alpha}$	$\overline{\alpha}$	0
RP with $\alpha = \underline{\alpha}$	1	$1 + \underline{\alpha}$	$\underline{\alpha}$	0
LP with $\alpha = \overline{\alpha}$	0	$\overline{\alpha}$	$1 + \overline{\alpha}$	1
LP with $\alpha = \underline{\alpha}$	0	$\underline{\alpha}$	$1 + \underline{\alpha}$	1
LA with $\alpha = \overline{\alpha}$	$\overline{\alpha}$	0	1	$1 + \overline{\alpha}$
LA with $\alpha = \underline{\alpha}$	$\underline{\alpha}$	0	1	$1 + \underline{\alpha}$
RA with $\alpha = \overline{\alpha}$	$1 + \overline{\alpha}$	1	0	$\overline{\alpha}$
RA with $\alpha = \underline{\alpha}$	$1 + \underline{\alpha}$	1	0	$\underline{\alpha}$

This is straightforward. The challenger has to publicly commit if she rejects an offer. If she commits to policy p in the no-lobbying game, then she would commit to the same policy in the MT game after rejecting an offer. Then, the policy outcome is the same in both MT game when the challenger rejects l's offer and no-lobbying game.

Proof of no-lobbying game

Proof Table 1 shows the payoff voters in each group get for different implemented policies. For instance, in row 3 and column 2, voters with ideal policy (R, P) who care more about guns get a payoff of 1 if policy (R, A) is implemented.

Table 2 shows how voters vote for each of the four possible public policy commitment by the challenger. Column 1 has the the eight groups of voters with their proportion in the total population given in the brackets. Column 2 shows whether voters in a particular group vote for the challenger or the incumbent when the challenger announces policy (R, P), so on and so forth for policies (L, P), (L, A), and (R, A) in column 3, 4 and 5: where we write c if they vote for the challenger, i otherwise. The last row specifies the challenger's vote share for each public policy commitment. The outcome in no-lobbying game:

Outcome when $p_c = (R, P)$ **and** $\alpha_c > 1$.

The challenger's order of policy preference is

$$(R, P)[1 + \alpha_c] \succ (L, P)[\alpha_c] \succ (R, A)[1] \succ (L, A)[0]$$

where the payoff from the respective policy is given in the square brackets.

The challenger would commit to her ideal policy if her ideal policy is popular; the share of the vote she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + n_{RP} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} =, (R, P) \quad \text{if} \quad n_{LP} + n_{RP} > 1/2$$

The challenger commits to her second best policy if her ideal policy is unpopular but her second best policy is popular; the vote share she gets by committing to (R, P) is less than half and the vote share she gets by committing to (L, P) is more than half, i.e. $n_{LP} + \delta n_{RP} + (1 - \delta)n_{LA} > 1/2 > n_{LP} + n_{RP}$. The outcome then is that the

Table 2 Voters' vote in no-lobbying game

Voters' group (proportion of voters)	Public commitment			
	(R, P)	(L, P)	(L, A)	(R, A)
RP with $\alpha = \overline{\alpha}$ (δn_{RP})	c	c	i	1/2
RP with $\alpha = \underline{\alpha}$ $((1-\delta) n_{RP})$	c	i	i	1/2
LP with $\alpha = \overline{\alpha}$ (δn_{LP})	c	c	c	1/2
LP with $\alpha = \underline{\alpha}$ $((1-\delta) n_{LP})$	c	c	c	1/2
LA with $\alpha = \overline{\alpha}$ (δn_{LA})	i	i	c	1/2
LA with $\alpha = \underline{\alpha}$ $((1-\delta) n_{LA})$	i	c	c	1/2
RA with $\alpha = \overline{\alpha}$ (δn_{RA})	i	i	i	1/2
RA with $\alpha = \underline{\alpha}$ $((1-\delta) n_{RA})$	i	i	i	1/2
Total share of vote	$n_{LP} + n_{RP}$	$n_{LP} + \delta n_{RP} + (1-\delta) n_{LA}$	$n_{LP} + n_{LA}$	1/2

challenger wins and implements her second best policy. Formally,

$$O^{NL} = (L, P) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2 > n_{LP} + n_{RP}$$

If the conditions above do not hold then the challenger announces her third best policy, which is also incumbent's ideal policy. She loses because she gets half of the total votes. The challenger would never commit to her least preferred policy unless she were to lose. This happens because by implementing (L, A), she gets a payoff 0, but she can profitably deviate to announce (R, A), in which case her payoff is 1. In any case the outcome is incumbent wins and implements her ideal policy.[25]

$$O^{NL} = (R, A) \quad \text{otherwise}$$

Outcome when $p_c = (R, P)$ and $\alpha_c < 1$.

The challenger's order of policy preference is

$$(R, P)[1+\alpha_c] \succ (R, A)[1] \succ (L, P)[\alpha_c] \succ (L, A)[0]$$

where the payoff from each policy is given in the square brackets.

The challenger commits to her ideal policy if her ideal policy is popular; the share of vote she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + n_{RP} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} = (R, P) \quad \text{if} \quad n_{LP} + n_{RP} > 1/2$$

The challenger commits to her second best policy (R, A) if her ideal policy is unpopular. She loses because she gets half of the total votes. The challenger would never commit to policy (L, P) or (L, A) unless she were to lose. She can always deviate to publicly commit to (R, A) and get a higher payoff. The outcome is

$$O^{NL} = (R, A) \quad \text{otherwise}$$

Outcome when $p_c = (L, P)$ and $\alpha_c > 1$.

The challenger's order of policy preference is

$$(L, P)[1+\alpha_c] \succ (R, P)[\alpha_c] \succ (L, A)[1] \succ (R, A)[0]$$

The challenger commits to her ideal policy if her ideal policy is popular; the share of vote she gets by committing to (L, P) is more than a half, i.e. $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2$. The outcome then is that the challenger wins and implements her ideal policy. Formally,

$$O^{NL} = (L, P) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2$$

The challenger commits to her second best policy if her ideal policy is unpopular but her second best policy is popular; the vote share she gets by committing to (L, P)

[25] Remember that if both the candidates commit to the same policy then the incumbent wins because of the incumbency bias.

is less than a half and the vote share she gets by committing to (R, P) is more than a half, i.e. $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2 < n_{LP} + n_{RP}$. The outcome then is that the challenger wins and implements her second best policy. Formally,

$$O^{NL} = (R, P) \quad \text{if} \quad n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} < 1/2 < n_{LP} + n_{RP}$$

The challenger commits to her third best policy (L, A) if her first two best policies are unpopular but her third best policy is popular. The outcome then is that the challenger wins and implements (L, A). Formally,

$$O^{NL} = (L, A) \quad \text{if} \quad n_{LP} + n_{LA} > 1/2 \quad \text{and} \quad max\{n_{LP} + n_{RP}, n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}\} < 1/2$$

If none of the conditions above hold, i.e if the challenger cannot win with any of her best three policies then the incumbent wins and implements her ideal policy. Formally,

$$O^{NL} = (R, A) \quad \text{otherwise}$$

□

Proof of Lemma 1 The proof is by contradiction. Suppose that $n_{LP} + n_{RP} > 1/2$. Then, the outcome in no-lobbying game is that the challenger wins and policy (R, P) is implemented, i.e. $O^{NL} = (R, P)$. Let U_{max} denote the utility of the challenger when the challenger wins and implements her ideal policy. Thus, she earns $U_{max} = 1 + \alpha_c$ in the no-lobbying game.

In the MT game, if $n_{LP} + n_{RP} > 1/2$, then the challenger must reject offers from both unlike-minded IGs. No other strategy profile in which the challenger accepts offers from one or both unlike-minded IGs survives in the equilibrium. She commits to implement (L, P) if L endorses her and (R, A) if A endorses her. She could either win or lose the election after accepting offers from unlike-minded IGs. If she wins she implements a policy worse than her ideal or she could lose in which case the incumbent wins and implement a policy worse than the challenger's ideal. In any case, the challenger's payoff will be less than U_{max}; so, she could profitably deviate to publicly commit to her ideal policy, thereby earning U_{max}. The challenger accepts offers from like-minded IGs only if she were to win, otherwise she could profitably deviate to publicly announce her ideal policy. In any case, the challenger wins and implements her ideal policy; so $O_R = O_P = O_{NL}$.

We know from Claim 1 that if the challenger rejects an offer from l then $O_l = O^{NL}$. If $n_{LP} + n_{RP} > 1/2$, then the challenger rejects offers from both unlike-minded IG. Therefore, we have $O_L = O_A = O_R = O_P = O^{NL}$. □

Proof of Lemma 2 The proof is by contradiction. Suppose that $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP} > 1/2$. Then, by Lemma 1 the outcome in the no-lobbying game is that the challenger wins and implements her second best policy: $O^{NL} = (L, P)$.

In MT game, there exists only two equilibria if $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP} > 1/2$: one in which the challenger rejects all offers and publicly commits to (L, P); and one in which the challenger accepts the offer from IG L and rejects all other IGs' offers and publicly commit to (L, P). From Observation 1 and Lemma 1, we know that the challenger loses when like-minded IGs endorse her in which case policy (R, A) is implemented, earning 1. But, she could reject offers from like-minded IGs and publicly commit to (L, P), earning α_c. Since $\alpha_c > 1$, she rejects offers from like-minded IGs. If she accepts A's offer, she implements (R, A) if she wins, (R, A) is implemented otherwise. In any case, she earns 1. But, she could reject A's offer and publicly commit to (L, P), earning α_c. Thus, the challenger rejects offers from A, R and P and publicly commit to (L, P). The challenger accepts from L only if she were to win, otherwise she could profitably deviate to publicly committing to (L, P).

In both the equilibria, $O_l = (L, P) = O^{NL}$ for each l. Thus, $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP} < 1/2$ in any equilibrium where an IG has direct influence. □

Proof of Lemma 3 Assume by the way of contradiction that the challenger rejects P's offer. We show that L has no direct influence when L endorses the challenger. Now consider an equilibrium where the challenger accepts the offer from L. Note that by Lemmas 1 and 2, the challenger is indifferent between publicly announcing (R, P), (L, P) and (R, A) because she loses anyway. She strictly prefer to lose than to implement her least preferred policy (L, A). In case of indifference, we assume that she publicly commits to her ideal policy: so, $O^{NL} = (R, A)$.

Now we show what proportion of total votes the challenger gets when L endorses the challenger. Members of L see the endorsement from their IG and therefore know that the challenger would implement (L, P) if she wins. When L endorses the challenger, the members of IGs R, A and P neither receive any endorsement nor see a public announcement. Then,

Members of L vote for the challenger Members with ideal policy (L, P) vote for the challenger because $1 + \underline{\alpha} > 0$ and members with ideal policy (L, A) also vote for the challenger because $1 > \underline{\alpha}$.

Members of R vote for the incumbent Members of R know that either case (1) L or A endorsed the challenger if the challenger's strategy prescribes her to accept A's offer; or case (2) L endorsed the challenger if the challenger rejects A's offer. Payoff of the members with ideal policy (R, P) if they vote for the challenger is case (1) $\frac{\pi_L \underline{\alpha} + \pi_A 1}{\pi_A + \pi_L}$ or case (2) $\underline{\alpha}$. In either case, their payoff is less than 1 (payoff if they vote for the incumbent). Payoff of the members with ideal policy (R, A) if they vote for the challenger is case (1) $\frac{\pi_L 0 + \pi_A (1+\underline{\alpha})}{\pi_A + \pi_L}$ or case (2) 0. In either case, their payoff is less than $1 + \underline{\alpha}$ (payoff if they vote for the incumbent).

Members of A vote for the incumbent Members of A know that either case (1) L or R endorsed the challenger if the challenger's strategy prescribes her to accept R's offer; or case (2) L endorsed the challenger if the challenger rejects A's offer. Payoff of the members with ideal policy (L, A) if they vote for the challenger is case (1) $\frac{\pi_L 1 + \pi_R 0}{\pi_L + \pi_R}$ or case (2) 1. In either case, their payoff is less than $\overline{\alpha}$ (payoff if they vote for the incumbent). Payoff of the members with ideal policy (R, A) if they vote for the challenger is case (1) $\frac{\pi_L 0 + \pi_R 1}{\pi_L + \pi_R}$ or case (2) 0. In either case, their payoff is less than $1 + \overline{\alpha}$ (payoff if they vote for the incumbent).

Members of P vote for the challenger Members of P know that either case (1) L or R or A endorsed the challenger if the challenger's strategy prescribes her to accept both R's and A's offer; or case (2) L or R endorsed the challenger if the challenger's strategy prescribes her to accept R's offer and rejects A's offer; otherwise case (3)

L or A endorsed the challenger if the challenger's strategy prescribes her to accept A's offer and rejects R's offer. Payoff of the members with ideal policy (L, P) if they vote for the challenger is case (1) $\frac{\pi_L(1+\overline{\alpha})+\pi_A 0+\pi_R\overline{\alpha}}{\pi_A+\pi_L+\pi_R}$ or case (2) $\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}}{\pi_L+\pi_R}$ or case (3) $\frac{\pi_L(1+\overline{\alpha})+\pi_A 0}{\pi_A+\pi_L}$. In any case, their payoff is more than 0 (payoff if they vote for the incumbent). The payoff of the members with ideal policy (R, P) if they vote for challenger is case (1) $\frac{\pi_L\overline{\alpha}+\pi_A 1+\pi_R(1+\overline{\alpha})}{\pi_A+\pi_L+\pi_R}$ or case 2) $\frac{\pi_L\overline{\alpha}+\pi_R(1+\overline{\alpha})}{\pi_L+\pi_R}$ or case (3) $\frac{\pi_L\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L}$. In any case, their payoff is more than 1 (payoff if they vote for the incumbent).

Thus, only members of L and P vote for the challenger. Then, the vote share the challenger gets when L endorses the challenger is $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA}$, which is less than a half by Lemma 2. Thus, the challenger loses the election: so, $O_L =, (R, A) = O^{NL}$. Hence L has no direct influence. □

Proof of Proposition 1 (***Necessity***)

We first establish the necessity of condition P1.1. Assume by way of contradiction that $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. Now, direct influence from IG l is only possible if the challenger accepts offer from l. We know from Lemma 3 that in the MT game, the challenger must accept offer from P for L to have direct; so for unlike-minded IGs L and A to have direct influence, the challenger must accept offers from L, A and P. In such an equilibrium, the challenger either accepts offer from R or rejects. But, our premise holds irrespective of whether the challenger accepts or rejects R's offer.

When L endorses the challenger, members of L know that their IG endorsed the challenger so they know that the challenger would implement (L, P) if she wins; members of other IGs neither see an endorsement from their IG nor see a public commitment from the challenger. Then, the proportion of votes the challenger receives when L endorses her is calculated as follows:

(1) **Members of L vote for the challenger** Members in group LP vote for the challenger: the challenger implements (L, P) which earns them $1 + \underline{\alpha}$ and the incumbent implements (R, A) which earns them 0. Members in group LA vote for the challenger: the challenger implements (L, P) which earns them 1 and the incumbent implements (R, A) which earns them $\underline{\alpha}$

(2) **Members of P vote for the challenger**

(i) Members in group RP vote for the challenger because $\frac{\pi_L(\overline{\alpha})+\pi_R(1+\overline{\alpha})+\pi_A(1)}{\pi_L+\pi_A+\pi_R} > 1$, where the left hand side is the expected payoff from voting for the challenger and the right hand side is the payoff from voting for the incumbent.

(ii) Members in group LP vote for the challenger because $\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 0}{\pi_L+\pi_A+\pi_R} > 0$

(3) **Members of A vote for the incumbent**

(i) Members in group RA vote for the incumbent because $\frac{\pi_R+\pi_P}{\pi_L+\pi_P+\pi_R} < 1 + \overline{\alpha}$

(ii) Members in group LA vote for the incumbent because $\frac{\pi_L}{\pi_L+\pi_P+\pi_R} < \overline{\alpha}$

(4) **Members of R vote as follows:**

(i) Members in group RA vote for the incumbent because $\frac{\pi_A(1+\underline{\alpha})+\pi_P}{\pi_L+\pi_P+\pi_A} < 1 + \underline{\alpha}$

(ii) *Members in group RP vote for the challenger if and only if* $\frac{\underline{\alpha}(\pi_L+\pi_P)+\pi_A+\pi_P}{\pi_L+\pi_P+\pi_A} > 1 \Rightarrow \frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$

Thus, the challenger gets $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$ of the total votes if $\frac{\underline{\alpha}}{1-\underline{\alpha}} < \frac{\pi_L}{\pi_P}$. We know from Lemma 2 that $n_{LP}+(1-\delta)n_{LA}+\delta n_{RP} < 1/2$, which means challenger loses and L has no direct influence. Thus, direct influence from L requires $\frac{\underline{\alpha}}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$.

Table 3 The challenger's equilibrium strategy in Proposition 2

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(R, A)
P	Accepts	(R, P)

When $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$ holds, the challenger gets $n_{LP} + (1-\delta)n_{LA} + n_{RP}$ when L endorses her. The second part of the conditions ensures that the proportion of voters the challenger gets when L endorses her is greater than a half. The challenger otherwise loses, and there is no direct influence.

(***Sufficiency***)

For sufficiency, we show that the strategies described above are equilibrium strategies and that L has direct influence in such an equilibrium.

If conditions P1.1 is satisfied, then there exists an equilibrium in which the challenger accepts offers from L, A and P and wins when L endorses the challenger.

The challenger cannot profitably deviate to reject L's offer. Accepting L's offer gives the challenger a payoff of α_c. If the challenger rejects L's offer, the outcome is that the incumbent wins and implements (R, A), which gives the challenger a payoff of 1. Since $\alpha_c > 1$, the challenger cannot profitably deviate to rejecting L's offer.

The challenger has no profitable deviation to rejecting P's or A' offer. Note that P is like-minded and like-minded IGs have no direct influence. The outcome when P or A endorses the challenger is (R, A), giving her a payoff of 1. If she deviates to reject P's or A's offer, she loses the election and gets a payoff of 1. Thus, there is no profitable deviation from accepting P or A's offer.

It is easy to see that in such an equilibrium, L has direct influence. □

Proof of Proposition 2 Remember that to find indirect influence we only consider the equilibrium in which the challenger accepts offers from all IGs. For that we first look at the outcome of such an equilibrium.

Table 3 describes an equilibrium in which the challenger accepts all offers where column 3 has policies the challenger implements after accepting offer from each IG.

Given the challenger's strategy profile in Tables 3, 4 describes how voters vote in the MT game. Column 4 specifies whether voters vote for the challenger (c) or the incumbent (i) when their IG endorses the challenger, where the payoff from voting for the challenger is given in the brackets. The payoff from voting for the incumbent is given in column 5. Column 6 specifies whether voters vote for the challenger or the incumbent when they neither see public commitment nor do they see endorsement from their IG; (expected) payoff from voting for the challenger is given in brackets. When payoffs are the same, the voters vote for each candidate with equal probability, which is written as $1/2$. Given $\frac{\alpha}{1-\underline{\alpha}} > \frac{\pi_L}{\pi_P}$, members of R with ideal policy (R, P) vote for the challenger.

Lemmas 1 and 2 imply that the challenger would only accept offer from unlike-minded IGs if her ideal policy and second best policy are unpopular. Thus, this equilibrium would only exist if Lemmas 1 and 2 hold. Then, the outcome in MT game is:

Table 4 Voters' vote for strategy profile given in Table 3

IGs	α	Members	Vote (endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha}$	$LP\ (n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{(\pi_R+\pi_P)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_P+\pi_R})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$\mathbf{i}(\frac{(\pi_P+\pi_R)0+\pi_A\underline{\alpha}}{\pi_A+\pi_P+\pi_R})$
R	$\alpha = \underline{\alpha}$	$RP\ (n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{c}(\frac{\pi_L\underline{\alpha}+\pi_A 1+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_L+\pi_P})$
		$RA\ (n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\frac{\pi_L 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_L+\pi_P})$
A	$\alpha = \overline{\alpha}$	$LA\ (\delta n_{LA})$	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 1+(\pi_R+\pi_P)0}{\pi_L+\pi_P+\pi_R})$
		$RA\ (\delta n_{RA})$	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(\frac{\pi_L 0+(\pi_R+\pi_P)1}{\pi_L+\pi_P+\pi_R})$
P	$\alpha = \overline{\alpha}$	$LP\ (\delta n_{LP})$	$\mathbf{c}(\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_L 1+\pi_A 0+\pi_R\underline{\alpha}}{\pi_A+\pi_L+\pi_R})$
		$RP\ (\delta n_{RP})$	$\mathbf{c}(1+\overline{\alpha})$	1	$\mathbf{c}(\frac{\pi_L\overline{\alpha}+\pi_A 1+\pi_R(1+\underline{\alpha})}{\pi_A+\pi_L+\pi_R})$

- $$O_L = \begin{cases} (L, P) & \text{if } n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2 \\ O^{NL}, & \text{otherwise} \end{cases}$$
- $O_R = O^{NL}$
- $O_A = O^{NL}$
- $O_P = O^{NL}$

Any IG would have indirect influence if its absence would alter voting behavior. Since voters know which policy is implemented when their IG endorses the challenger, their vote does not change in column 4 when we change access probabilities. Therefore, we just need to check whether voting changes in column 6.

We first prove that L, R and A cannot have indirect influence.

Let $\pi_L = 0$. Then, voting does not change. Thus, L has no indirect influence.

- It does not affect the payoff (in column 6) of members of L.
- For members of R, setting $\pi_L = 0$
  - Members in group RP do not change their vote because $\frac{\pi_A 1 + \pi_P(1+\underline{\alpha})}{\pi_A + \pi_P} > 1$
  - Members in group RA do not change their vote because $\frac{\pi_A(1+\underline{\alpha}) + \pi_P 1}{\pi_A + \pi_P} < 1 + \underline{\alpha}$
- For members of A, setting $\pi_L = 0$
  - Members in group LA do not change their vote because $\frac{(\pi_R + \pi_P)0}{\pi_P + \pi_R} < \overline{\alpha}$
  - Members in group RA do not change their vote because $\frac{(\pi_R + \pi_P)1}{\pi_P + \pi_R} < 1 + \underline{\alpha}$
- For members of P, setting $\pi_L = 0$
  - Members in group LP do not change their vote because $\frac{\pi_A 0 + \pi_R \underline{\alpha}}{\pi_A + \pi_R} > 0$
  - Members in group RP do not change their vote because $\frac{\pi_A 1 + \pi_R(1+\underline{\alpha})}{\pi_A + \pi_R} > 1$

Let $\pi_R = 0$. Then, voting does not change. Thus, R has no indirect influence.

- It does not affect the payoff (in column 6) of members of R.
- For members of L, setting $\pi_R = 0$
  - Members in group LP do not change their because $\frac{\pi_P \underline{\alpha} + \pi_A 0}{\pi_A + \pi_P} > 0$
  - Members in group LA do not change their because $\frac{\pi_P 0 + \pi_A \underline{\alpha}}{\pi_A + \pi_P} < \underline{\alpha}$
- For members of A, setting $\pi_R = 0$
  - Members in group LA do not change their because $\frac{\pi_L 1 + (\pi_P)0}{\pi_L + \pi_P + \pi_R} < \overline{\alpha}$
  - Members in group RA do not change their because $\frac{\pi_L 0 + (\pi_P)1}{\pi_L + \pi_P} < 1 + \underline{\alpha}$
- For members of P, setting $\pi_R = 0$
  - Members in group LP do not change their because $\frac{\pi_L 1 + \pi_A 0}{\pi_A + \pi_L} > 0$
  - Members in group RP do not change their because $\frac{\pi_A 1 + \pi_R(1+\underline{\alpha})}{\pi_A + \pi_R} > 1$

Let $\pi_A = 0$. Then, voting does not change. Thus, A has no indirect influence.

- It does not affect the payoff (in column 6) of members of A.
- For members of L, setting $\pi_A = 0$
  - Members in group LP do not change their vote because $\frac{(\pi_R+\pi_P)\underline{\alpha}}{\pi_P+\pi_R} > 0$
  - Members in group LA, do not change their vote because $\frac{(\pi_P+\pi_R)0}{\pi_P+\pi_R} < \underline{\alpha}$
- For members of R, setting $\pi_A = 0$
  - Members in group RP do not change their vote because $\frac{\pi_L\underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_L+\pi_P} > 1$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+\pi_P 1}{\pi_L+\pi_P} < 1+\underline{\alpha}$
- For members of P, setting $\pi_A = 0$
  - Members in group LP do not change their vote because $\frac{\pi_L 1+\pi_R\underline{\alpha}}{\pi_L+\pi_R} > 0$
  - Members in group RP do not change their vote because $\frac{\pi_L\overline{\alpha}+\pi_R(1+\underline{\alpha})}{\pi_L+\pi_R} > 1$

We now prove that P could have indirect influence.

If $\pi_P = 0$, then the members of IG R with policy preference (R, P) change their vote from the challenger to the incumbent.

- It does not affect the payoff (in column 6) of members of P.
- For members of L, setting $\pi_P = 0$
  - Members in group LP do not change their vote because $\frac{(\pi_R)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_R} > 0$
  - Members in group LA do not change their vote because $\frac{(\pi_R)0+\pi_A\underline{\alpha}}{\pi_A+\pi_R} < \underline{\alpha}$
- For members of R, setting $\pi_P = 0$
  - Members in group RP, *vote for incumbent* because $\frac{\pi_L\underline{\alpha}+\pi_A 1}{\pi_A+\pi_L} < 1$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_L} < 1+\underline{\alpha}$
- For members of A, setting $\pi_P = 0$
  - Members in group LA do not change their vote because $\frac{\pi_L 1+(\pi_R)0}{\pi_L+\pi_R} < \overline{\alpha}$
  - Members in group RA do not change their vote because $\frac{\pi_L 0+(\pi_R)1}{\pi_L+\pi_R} < 1+\overline{\alpha}$

If $\pi_P = 0$ and L endorses the challenger, the challenger loses vote of the members of R who belong to group RP. The new vote share the challenger gets is $n_{LP}+\delta n_{RP}+(1-\delta)n_{LA}$. This is less than half by Lemma 2. The challenger loses and policy (R, A) is implemented: so, $O_L^P = O^{NL}$ where O_L^{-P} be the outcome when $\pi_P = 0$ and L endorses the challenger in the modified game. The outcome does not change if the challenger were to loses when $\pi_P > 0$; it changes only when the challenger wins when $\pi_p > 0$ and loses when $\pi_P = 0$. Hence, the conditions in the proposition must be satisfied. Then, $O_L = (L, P) \neq O_L^{-P} = (R, A)$ if $n_{LP} + n_{RP} + (1-\delta)n_{LA} > 1/2$. □

Proof of Lemma 4 The proof is by contradiction. Suppose $n_{LP}+\delta n_{RP}+(1-\delta)n_{LA} > 1/2$. Thus, the outcome in no-lobbying game is that the challenger wins and policy (L, P) is implemented, i.e. $O^{NL} = (L, P)$. Let U_{max} denote the utility of the chal-

Table 5 The challenger's equilibrium strategy in Proposition 3

IGs	Challenger's decision	Policy implemented
L	Rejects	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Rejects	(L, P)

lenger when she wins and implements her ideal policy. Thus, she $U_{max} = 1 + \alpha_c$ in the no-lobbying game.

In MT game, if $n_{LP} + \delta n_{RP} + (1-\delta)n_{LA} > 1/2$ then challenger must reject offer from both unlike-minded IGs, otherwise the challenger would have to promise to implement a policy different from her ideal but then she could deviate to reject and publicly commit to her ideal policy. Thus, unlike-minded IGs do not have direct influence. The challenger implements her ideal policy when like-minded IGs endorse the challenger. She would accept offers from like-minded IGs only if she were to win, otherwise she can deviate to publicly commit to her ideal policy. Thus, if like-minded IGs gets access, she either accepts their offers and win or she rejects and publicly commits to her ideal policy and wins. Thus, like-minded IGs do not have direct influence either. □

Proof of Lemma 5 $n_{LP} + n_{RP}$ is the proportion of votes the challenger gets if she publicly commit to policy (R, P), which is the challenger's second best policy. Lemma 4 implies that the challenger loses the election if she publicly commits to her ideal policy in the no-lobbying game. Now, assume by way of contradiction that $n_{LP} + n_{RP} > 1/2$. Then, the outcome in the no-lobbying game is that the challenger wins and policy (R, P) is implemented; $o^{NL} = (R, P)$. Thus, she earns α_c.

In the MT game, the challenger rejects the offer from unlike-minded IG A, otherwise she would have to promise to implement (L, A), which is her third best policy; but then she could do better by rejecting and publicly committing to (R, P). Thus, A has no direct influence. If she accepts R's offer, she would have to commit to implement (R, P). She would only accept R's offer if she were to win. If she accepts R's offer and loses then incumbent wins, earning 0. But, she could then deviate to publicly announce her second best policy, earning α_c. In any case, the outcome is the same. Thus, R has no direct influence. From Observation 2, we know that like-minded IGs never have direct influence. Hence, MT is not directly influential. □

Proof of Proposition 3 (***Necessity***)

We first establish the necessity of conditions P3.1 and P3.2. Given that $n_{LP} + n_{LA} < 1/2$ and the challenger rejects from both like-minded IGs, unlike minded IGs could only have direct influence if the challenger accepts their offers. Table 5 specifies the equilibrium decision of the challenger and policies she would implement for each of the offers she accepts. Note that $n_{LP} + n_{LA} < 1/2$; so by Lemmas 4 and 5, the challenger loses if she publicly commits to any policy. In case of indifference, the challenger commits to her ideal policy: so $O^{NL} = (R, A)$.

Table 6 specifies how voters vote. Column 4 specifies whether they vote for the challenger (c) or incumbent (i) when they see endorsement from their IG. Column 5 specifies their payoffs if they vote for the incumbent. Column 6 specifies whether they vote for the challenger (c) or incumbent (i) when they neither see public commitment nor endorsement. Note that the challenger only accepts offers from unlike-minded IGs R and A so, members of L and P do not see endorsement on the equilibrium path.[26]

In Table 6, we can see that: members of L with policy preference (L, A) vote for the challenger iff $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_R} > \underline{\alpha} \Rightarrow \frac{\pi_A}{\pi_R} > \underline{\alpha}$; members of P with policy preference (R, P) vote for the challenger iff $\frac{\pi_R(1+\overline{\alpha})+\pi_A 0)}{\pi_A+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A}{\pi_R}$.

Note that the voters in at least one of these groups vote for the challenger. If $\pi_A > \pi_R$, then the members of L vote for the challenger since $1 > \underline{\alpha}$. If $\pi_A < \pi_R$, then the members of P vote for the challenger since $1 < \overline{\alpha}$.

When R endorses the challenger, only members of R see endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP}+n_{RP}$, if members of L do not vote for the challenger. This vote share is less than half by Lemma 5. Thus, the members of L must vote for the challenger for R to have direct influence. This requires either $\frac{\pi_A}{\pi_R} > 1$ or $1 > \frac{\pi_A}{\pi_R} > \underline{\alpha}$.

When A endorses the challenger, only members of A receive an endorsement and members of other IGs neither see endorsement nor public commitment. Then, the maximum vote share the challenger gets is $n_{LP} + n_{LA}$, if members of P do not vote for the challenger. This vote share is less than half the given condition in the premise. Thus, the members of P must vote for the challenger for A to have direct influence. This requires either $\frac{\pi_A}{\pi_R} < 1$ or $1 < \frac{\pi_A}{\pi_R} < \overline{\alpha}$. These two conditions gives condition in P3.1.

When condition P3.1 is satisfied, the challenger gets a vote share of $n_{LP} + n_{RP} + (1-\delta)n_{LA}$ or $n_{LP} + n_{LA} + \delta n_{RP}$ when R or A endorses the challenger respectively. The challenger wins if these vote shares form majority. Otherwise, the challenger loses and the IGs do not have direct influence. Hence, condition P3.2 must be satisfied.

(***Sufficiency***)

For sufficiency, we prove that the challenger does not have profitable deviation from the prescribed strategy profile.

The challenger rejects offers from both like-minded IGs L and P and loses, earning 0. The challenger cannot profitably deviate to (accepting) L's offer. If she accepts L offer, she gets votes from members of L and P. This gives her a total vote share of $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 4. Thus, the challenger loses and earns 0. If she accepts P's offer, she gets votes from members of L and P. This gives her a total vote share of $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 4. Thus, the challenger loses and earns 0.

The challenger accepts offers from unlike-minded IGs R and A. If conditions P3.1 and P3.2 are satisfied, the challenger wins and implements (R, P) or (L, A) when R or A endorses the challenger respectively. Implementing (R, P) gives her a payoff of α_c and implementing (L, A) gives her a payoff of 1. The challenger cannot profitably deviate (to rejecting) R's or A's offer. If she rejects, she publicly commits to (L, P) and thus loses the election and gets a payoff of 0. □

[26] But they can only see endorsement off the equilibrium path if their IGs got access in which case they know that the challenger would implement her ideal policy.

Table 6 Voters' vote for strategy profile given in Table 7

IGs	α	Members	Vote (endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	0	$\mathbf{c}(\frac{\pi_R \underline{\alpha} + \pi_A 1}{\pi_A + \pi_R})$
		$LA(n_{LA}(1-\delta)$	$\mathbf{c}(1)$	$\underline{\alpha}$	$(\frac{\pi_R 0 + \pi_A (1+\underline{\alpha})}{\pi_A + \pi_R})$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$\mathbf{c}(1+\underline{\alpha})$	1	$\mathbf{i}\ (0)$
		$RA\ (n_{RA}(1-\delta)$	$\mathbf{i}(1)$	$1+\underline{\alpha}$	$\mathbf{i}(\underline{\alpha})$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$\mathbf{1/2}(\overline{\alpha})$	$\overline{\alpha}$	$\mathbf{i}(0)$
		$RA\ (\delta n_{RA})$	$\mathbf{1/2}(1+\overline{\alpha})$	$1+\overline{\alpha}$	$\mathbf{i}(1)$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$\mathbf{c}(1+\overline{\alpha})$	0	$\mathbf{c}(\frac{\pi_A 1 + \pi_R \underline{\alpha}}{\pi_A + \pi_R})$
		$RP\ (\delta n_{RP})$	$\mathbf{c}(\overline{\alpha})$	1	$(\frac{\pi_R (1+\overline{\alpha}) + \pi_A 0)}{\pi_A + \pi_R})$

Table 7 The challenger equilibrium strategy in Proposition 6

IGs	Challenger's decision	Policy implemented
L	Rejects	(L, A)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Rejects	(L, A)

Proof of Proposition 4 First, we show the uniqueness. Given $n_{LA} + n_{LP} > 1/2$, $O^{NL} = (L, A)$, so the challenger earns 1 in the no-lobbying game. We know from Observation 1 that like-minded IGs have no direct influence. If the challenger accepts like-minded IGs offer then she loses and gets a payoff of 0. But she could profitably deviate to reject their offers and publicly commit to (L, A). Thus, the challenger rejects offers from both like-minded IGs and publicly commit to (L, A). We also know that for any IG to have direct influence, the challenger must accept offers from at least two IGs. Since the challenger rejects offers from both like-minded IGs, the challenger must accept offers from both unlike-minded IGs for their direct influence.

(***Necessity***)

Next we show the necessary conditions for such an equilibrium to exist. Table 7 describes the equilibrium in which the challenger rejects offers from both like-minded IGs and accepts offers from both unlike-minded IGs. The challenger publicly commits to (L, A) in the no-lobbying game and when she rejects an offer in MT games. For such an equilibrium to exist, the challenger must win the election after accepting offers from unlike-minded IGs R and A, otherwise she could profitably deviate to publicly committing to (L, A): so, the challenger must get enough votes when R or A endorses the challenger.

When R endorses the challenger, members of R get to know that their IG endorsed the challenger and that the challenger would implement (R, P); members of other IG do not see endorsement or public commitment. In such an equilibrium, only R and A can endorse the challenger: so when R endorses the challenger, members of A know that R must have endorsed the challenger and members of L and P know that one of R or A must have endorsed the challenger. Then, the proportion of votes the challenger receives when R endorses the challenger is given below.

(1) **Members of R with ideal policy (R, P) vote for the challenger because $1+\overline{\alpha} > 1$, and members with ideal policy (R, A) vote for the incumbent because $1+\overline{\alpha} > 1$.**

(2) **Members of L vote as follows:**

(i) Members with ideal policy (L, P) vote for the challenger because $\frac{\pi_A 1+\pi_R \underline{\alpha}}{\pi_A+\pi_R} > 0$.

ii) *Members with ideal policy (L, A) vote for the challenger iff* $\frac{\pi_R 0+\pi_A(1+\underline{\alpha})}{\pi_A+\pi_R} > \underline{\alpha} \Rightarrow \frac{\pi_A}{\pi_R} > \underline{\alpha}$.

(3) **Members of A vote for the incumbent**

(i) Members with ideal policy (R, A) vote for the incumbent because $1 < \overline{\alpha}$.

(ii) Members with ideal policy (L, A) vote for the incumbent because $0 < \overline{\alpha}$.

(4) **Members of P vote as follows:**

(i) Members with ideal policy (R, A) vote for the challenger because $\frac{\pi_R\overline{\alpha}+\pi_A 1}{\pi_A+\pi_R} > 0$.

Table 8 The challenger equilibrium strategy in Propositions 5 and 6

IGs	Challenger's decision	Policy implemented
L	Accepts	(L, P)
R	Accepts	(R, P)
A	Accepts	(L, A)
P	Accepts	(L, P)

(ii) *Members with ideal policy* (R, P) *vote for the challenger if and only if* $\frac{\pi_R(1+\overline{\alpha})+\pi_A 0}{\pi_A+\pi_R} > 1 \Rightarrow \overline{\alpha} > \frac{\pi_A}{\pi_R}$

Thus, the maximum vote share the challenger gets is $n_{LP} + n_{RP}$ if the members of L with ideal policy (L, A) do not vote for the challenger. We know from Lemma 5 that $n_{LP} + n_{RP} < 1/2$, which means the challenger loses and we know that the challenger must not lose the election (when R endorses her) for this equilibrium to exist. Thus, direct influence from R requires $\frac{\pi_A}{\pi_R} > \underline{\alpha}$ or $\frac{\pi_A}{\pi_R} > 1$. If this condition is satisfied, the challenger gets a vote share of $n_{LP}+n_{RP}+(1-\delta)n_{LA}$ when R endorses the challenger. For R to have direct influence, the challenger must win, meaning that this vote share must form a majority. Hence, the conditions must be satisfied.

We know that the challenger must win the election when R or A endorses the challenger. Then, A has no direct influence because the challenger wins either way and implements the same policy. The outcome when R endorses the challenger is $O_R = (R, P)$. Thus, only R has direct influence.

(***Sufficiency***)

The challenger cannot profitably deviate to rejecting R's offer (to publicly committing to (L, A)). She wins when R endorses the challenger and gets a payoff of α_c. Rejecting R's offer and publicly committing to (L, A) would give her a payoff of 1. Since $\alpha_c > 1$, there is no profitable deviation. Similarly, there is no profitable deviation for the challenger to reject A's offer. When A endorses the challenger, she wins and implements (L, A), which earns her 1. If she deviates, she would still earn a payoff of 1.

The challenger has no profitable deviation to accept L's or P's offer. If she reject L's offer, she gets a payoff of 1 by publicly committing to (L, A). The maximum vote she can get if she accepts L or P's offer is $n_{LP} + (1-\delta)n_{LA} + \delta n_{RP}$, which is less than a half by Lemma 4, earning 0. □

Arguments and Tables we use to prove Propositions 5 and 6 Because we want to find indirect influence, we look at the equilibrium of the MT game in which the challenger accepts offers from all IGs. We know that such an equilibrium would only exist only if $n_{LP} + n_{LA} < 1/2$; otherwise the challenger can profitably deviate to reject like-minded IGs' offer and publicly commit to (L, A). Table 8 specifies the challenger equilibrium strategy and the policy she implements if she wins after an IG endorses her.

Given the challenger's equilibrium strategy, we specify in Table 9 the payoff voters get if they vote for the challenger after they see endorsement from their IG (column 4), payoff they get if they vote for the incumbent (column 5), and (expected) payoff by voting for the challenger if they see no endorsement and no public commitment (column 6).

Vote share when L endorses the challenger When L endorses the challenger, members of L see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of L vote for the challenger (column 4 > column 5).
- Members of R and A vote for the incumbent (column 5 > column 6).
- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy* (R, P) *vote for the challenger iff* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$

Vote share when P endorses the challenger When P endorses the challenger, members of P see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of P vote for the challenger (column 4 > column 5).
- Members of R and A vote for the incumbent (column 5 > column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy* (L, A) *vote for the challenger iff* $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$

Vote share when R endorses the challenger. When R endorses the challenger, members of R see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of R
  - with ideal policy (R, P) vote for the challenger (column 4 > column 5).
  - with ideal policy (R, A) vote for the incumbent (column 4 < column 5).
- Members of A vote for the incumbent (column 5 > column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy* (L, A) *vote for the challenger iff* $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$
- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 < column 6).
  - *With ideal policy* (R, P) *vote for the challenger iff* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$

Vote share when A endorses the challenger When A endorses the challenger, members of A see endorsement and members of all other IG see no endorsement and no public commitment. Voters then vote as follows:

- Members of A
  - with ideal policy (L, A) vote for the challenger (column 4 > column 5).
  - with ideal policy (R, A) vote for the incumbent (column 4 < column 5).

Table 9 Voters' vote for the strategy profile given in Table 10

IGs	α	Members	Vote (endorsement)	Incumbent	Vote (no endorsement)
L	$\alpha = \underline{\alpha} < 1$	$LP\ (n_{LP}(1-\delta))$	$1+\underline{\alpha}$	0	$\frac{\pi_A 1+\pi_R\underline{\alpha}+\pi_P(1+\underline{\alpha})}{\pi_A+\pi_R+\pi_P}$
		$LA(n_{LA}(1-\delta)$	1	$\underline{\alpha}$	$\frac{\pi_R 0+\pi_A(1+\underline{\alpha})+\pi_P 1}{\pi_A+\pi_R+\pi_P}$
R	$\alpha = \underline{\alpha} < 1$	$RP\ (n_{RP}(1-\delta))$	$1+\underline{\alpha}$	1	$\frac{(\pi_L+\pi_P)\underline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_P}$
		$RA\ (n_{RA}(1-\delta)$	1	$1+\underline{\alpha}$	$\frac{(\pi_L+\pi_P)+0+\pi_A\underline{\alpha}}{\pi_A+\pi_L+\pi_P}$
A	$\alpha = \overline{\alpha} > 1$	$LA\ (\delta n_{LA})$	$1+\overline{\alpha}$	$\overline{\alpha}$	$\frac{(\pi_L+\pi_P)1+\pi_R 0}{\pi_R+\pi_L+\pi_P}$
		$RA\ (\delta n_{RA})$	$\overline{\alpha}$	$1+\overline{\alpha}$	$\frac{(\pi_P+\pi_L)0+\pi_R 1}{\pi_R+\pi_L}$
P	$\alpha = \overline{\alpha} > 1$	$LP\ (\delta n_{LP})$	$1+\overline{\alpha}$	0	$\frac{\pi_L(1+\overline{\alpha})+\pi_R\overline{\alpha}+\pi_A 1}{\pi_A+\pi_L+\pi_R}$
		$RP\ (\delta n_{RP})$	$\overline{\alpha}$	1	$\frac{\pi_R(1+\overline{\alpha})+\pi_L\overline{\alpha}+\pi_A 0}{\pi_A+\pi_L+\pi_R}$

- Members of R vote for the incumbent (column 5 $>$ column 6).
- Member of L:
  - With ideal policy (L, P) vote for the challenger (column 5 $<$ column 6).
  - *With ideal policy* (L, A) *vote for the challenger iff* $\frac{\pi_A+\pi_P}{\pi_P+\pi_R} > \underline{\alpha}$
- Member of P:
  - With ideal policy (L, P) vote for the challenger (column 5 $<$ column 6).
  - *With ideal policy* (R, P) *vote for the challenger iff* $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$

Proof of Proposition 5 Condition $\pi_A > \pi_R \Rightarrow \frac{\pi_A+\pi_P}{\pi_R+\pi_P} > \underline{\alpha}$. We know from Observation 2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (R, A)$.

We first prove why P cannot have indirect influence (P5.1). Setting $\pi_P = 0$ does not change vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their vote: setting $\pi_P = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of L do not change their vote: setting $\pi_P = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. So setting $\pi_P = 0$ does not change any vote. Hence P has no indirect influence.

We now prove L's indirect influence (P5.2). The first part of the condition is imposed so that the challenger wins after unlike minded IG R or A endorsed the challenger, i.e. $\overline{\alpha} > \frac{\pi_A+\pi_L}{\pi_R+\pi_L}$ guarantees that the members of R with policy preference (R, P) vote for the challenger when all IGs can lobby. If this is not the case then the challenger loses irrespective of the presence of one of the IGs. L might have indirect if setting $\pi_L = 0$ changes votes when voters neither see endorsement nor a public commitment. Setting $\pi_L = 0$ does not change the vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, their payoff in column 5 is still greater than their expected payoff in column 6. Members of P with ideal policy (L, P) do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of* P *with ideal policy* (R, P) *might change their vote: setting* $\pi_L = 0$, *their payoff in column 5 is greater than their expected payoff in column 6 if* $\overline{\alpha} < \frac{\pi_A}{\pi_R}$, *in which case they vote for the incumbent*. Thus, 2nd part of the condition for the change in outcome of the election.

We now prove R's indirect influence (P5.3). The first part of the condition is imposed for the reason as in the previous paragraph. Next, we establish the second part of the condition. Setting $\pi_R = 0$ does not change vote of the members of R: their payoff in column 6 is independent of π_R. Members of IGs L and A do not change their: setting $\pi_R = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of L in column 6 is still greater than payoff in column 5. Members of P with ideal policy (L, P) do not change their vote: setting $\pi_R = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of* P *with ideal policy* (R, P) *might change their vote: setting* $\pi_R = 0$, *their payoff in column 5 is greater than their expected payoff in column 6, if* $\overline{\alpha} < \frac{\pi_A+\pi_L}{\pi_L}$, *in which case they vote for the incumbent*. So, when A endorses the challenger when $\pi_R = 0$, she loses votes of the members of R with ideal policy (R, P) and therefore loses the election because $n_{LP} + n_{LA} < 1/2$. If the challenger won when A endorsed her when $\pi_l > 0$ for all l and loses when $\pi_L = 0$ then the outcome O_A changes.

Next, we prove A's indirect influence. The first part of the condition is imposed for the reason as in the previous paragraph. Setting $\pi_A = 0$ does not change vote of the members of A: their payoff in column 6 is independent of π_A. Members of IGs L and P do not change their vote: setting $\pi_A = 0$, payoff of members of A in column 5 is still greater than their expected payoff in column 6; payoff of members of P in column 6 is still greater than payoff in column 5. Members of L with ideal policy (L, P) do not change their vote: setting $\pi_A = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of L with ideal policy* (L, A) *might change their vote: setting* $\pi_A = 0$*, their payoff in column 5 is greater than their expected payoff in column 6, if* $\underline{\alpha} > \frac{\pi_P}{\pi_P + \pi_R}$*, in which they vote for the incumbent*. So, when R endorses the challenger when $\pi_A = 0$, she loses votes of the members of L with ideal policy (L, A) and therefore loses the election because $n_{LP} + n_{RP} < 1/2$ by Lemma 5. If the challenger won when R endorsed her when $\pi_l > 0$ for all l and loses when $\pi_A = 0$ then the outcome O_R changes. □

Proof of Proposition 6 Condition $\pi_A < \pi_R \Rightarrow \overline{\alpha} > \frac{\pi_A + \pi_L}{\pi_R + \pi_L}$. We know from Observation 2 that the challenger loses when like-minded IGs endorse the challenger. Thus, the outcome when L or P endorses the challenger is $O_L = O_P = (R, A)$.

First we prove why L cannot have indirect influence (P6.1). Setting $\pi_L = 0$ does not change vote of the members of L: their payoff in column 6 is independent of π_L. Members of IGs R and A do not change their vote: setting $\pi_L = 0$, payoff of members of A and R in column 5 is still greater than their expected payoff in column 6. Members of P do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. So setting $\pi_L = 0$ does not change any vote. Hence L has no indirect influence.

We now prove P's indirect influence (P6.2). The first part of the condition is imposed so that the challenger wins after unlike minded IG R or A endorsed the challenger, i.e. $\frac{\pi_A + \pi_L}{\pi_R + \pi_L} > \underline{\alpha}$ guarantees that the members of R with policy preference (R, P) vote for the challenger when all IGs can lobby. If this is not the case then the challenger loses irrespective of the presence of one of the IGs. P might have indirect if setting $\pi_P = 0$ changes votes when voters neither see endorsement nor a public commitment. Setting $\pi_P = 0$ does not change the vote of the members of P: their payoff in column 6 is independent of π_P. Members of IGs R and A do not change their: setting $\pi_P = 0$, their payoff in column 5 is still greater than their expected payoff in column 6. Members of L with ideal policy (L, P) do not change their vote: setting $\pi_L = 0$, their payoff in column 6 is still greater than their expected payoff in column 5. However, *members of L with ideal policy* (L, A) *might change their vote: setting* $\pi_P = 0$*, their payoff in column 5 is greater than their expected payoff in column 6 if* $\underline{\alpha} > \frac{\pi_A}{\pi_R}$*, in which case they vote for the incumbent*. Thus, 2nd part of the condition for the change in outcome of the election.

To prove R and A's indirect influence, the first part of the condition is imposed for the same reason as in the previous paragraph. To prove the second part of the equation in P6.3 and P6.4, please refer to proofs of R and A's indirect influence in Proposition 5. □

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