Exchange rate and commodity prices

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Abstract

Chapter 1 discusses how fluctuations in crude oil prices deeply influence the global economy, particularly in oil-exporting countries.

Chapter 2 explores the economic dynamics of crude oil prices and their impact on the Real Effective Exchange Rate (REER) in oil-exporting countries. The analysis covers a diverse group of oil-exporting countries. We determine the REER based on the Behavioural Equilibrium Exchange Rate (BEER) model for these countries. Employing both time-series and panel methods, such as Johansen cointegration and the Autoregressive Distributed Lag (ARDL) bounds testing, the study investigates the real oil price on REER. While time-series results are mixed, Pooled Mean Group (PMG) methods reveal a significant positive relationship between real oil price, Net Foreign Assets (NFA), and six-sector value-added deflator (6SECT) with REER.

Chapter 3 examines whether exchange rates can reliably forecast international oil prices for oil-exporting countries. We focus on forecasting models for crude oil and its derivatives. We evaluate both in-sample and out-of-sample performance across global oil futures and major benchmarks (Brent, WTI, Dubai), using nominal exchange rates from Brazil, Canada, Colombia, Indonesia, Mexico, and Norway. Our findings reveal that the exchange rates of Brazil, Colombia, Mexico, and Norway possess substantial short-term forecasting power for crude oil and its derivative financial products, likely due to the significance of oil in their economies. In contrast, Canada and Indonesia have relatively small oil rents, which may explain why their exchange rates are less effective at predicting oil prices.

Chapter 4 further investigates whether crude oil price predictability exists within a small window. We find that most of these windows occur during economic downturns, suggesting that oil-exporting countries' exchange rates may gain predictive power in times of crisis.

Chapter 5 concludes this thesis by highlighting key findings, addressing limitations, and suggesting avenues for future research.

Declaration

I declare that, except where explicit reference is made to the contribution of others, that this dissertation is the result of my own work and has not been submitted for any other degree at the University of Essex or any other institution. Yanfeng Xu

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Chapter 1

Introduction

Over time, high global liquidity has allowed commodity prices to exert a significant influence on many economies. Numerous studies have focused on the predictability of international commodity prices. However, the uncertainty and rapid fluctuations in commodity prices have had adverse effects, particularly on emerging economies, where the impact tends to be more pronounced and enduring. In some regions, oil is referred to as the "king of commodities". Being the world's primary energy source, it is also one of the most volatile commodities traded en masse. While the Middle East has long been a key provider of oil, other regions also contribute to global crude oil exports. Certain countries in Southern Africa, along with developing oil exporters like Brazil, possess extensive oil reserves. However, technological constraints in the past prevented the full development and extensive exportation of these resources. With advancements over the last two decades, the oil exports from these countries have gradually increased. Table 1 below presents data on the crude oil export volumes of select countries every 20 years and their share of global exports.

The table indicates that certain emerging exporters, including Nigeria, Colombia, and Brazil, have experienced growth in oil exports. Conversely, nations like Indonesia have witnessed a decline in oil exports due to the depletion of their developed proven oil reserves. In contrast, Saudi Arabia's export volume remains substantial, consistently accounting for

Table 1: Some examples of oil exporting during 60 years

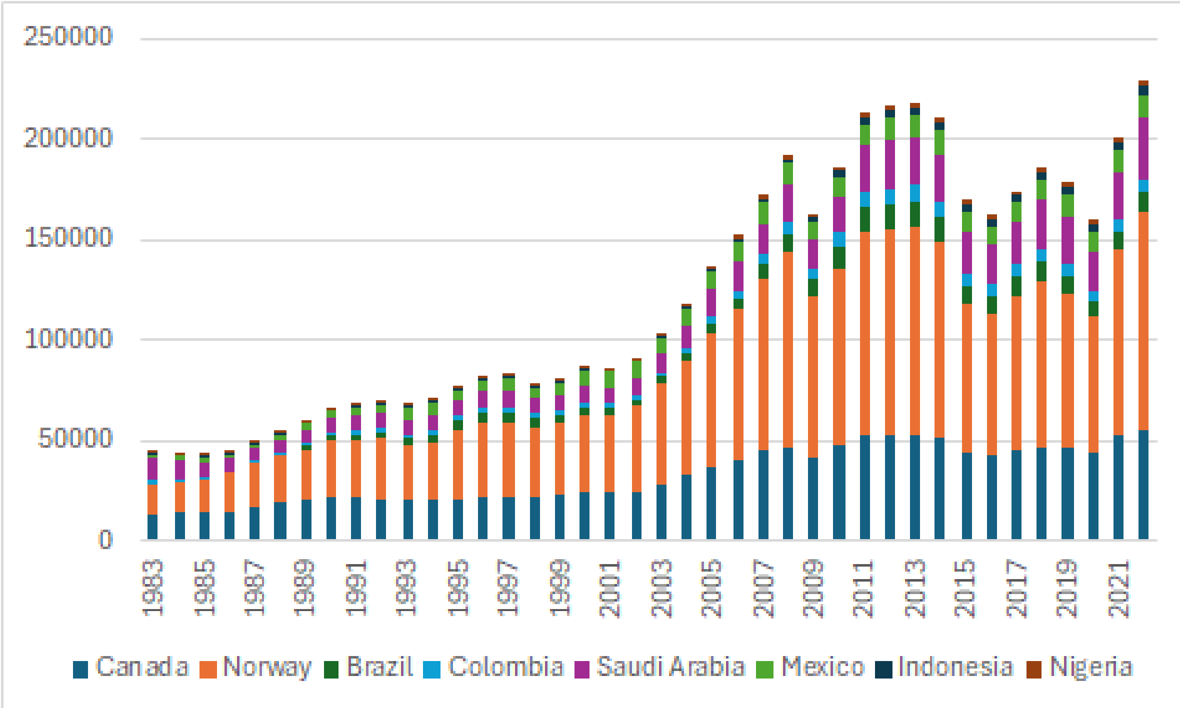
Crude oil export (1,000 B/D)	1982		2002		2022	
Country	Export	Percentage	Export	Percentage	Export	Percentage
Brazil	22.3	0.0868	233.0	0.633	1346.4	3.0968
Canada	230.186	0.896	1153.960	3.137	3350.2	7.705
Colombia	0.0	0	278.9	0.758	487.0	1.120
Indonesia	1108.03	4.313	614.9	1.6716	39.6	0.0910
Mexico	1666.8	6.489	1955.8	5.3166	1011.8	2.327
Norway	375.2	1.4607	2821.7	7.671	1558.2	3.5838
Nigeria	1002.8	3.904	1,798.2	48882	1388.3	3.193
Saudia Arabia	5639.4	21.955	5284.6	14.3657	7363.6	16.9367

Note:(B/D) means Barrels per day

approximately 15% – 20% of the global export market. Despite the high revenues generated from oil exports, not every oil-exporting nation has achieved sustainable economic growth. Figure 1 shows that, with the exceptions of Canada and Norway, per capita income and quality of life in many oil-exporting countries remain low. By contrast, East Asian economies such as Japan, South Korea, Taiwan, Singapore, and Hong Kong have attained relatively high standards of living despite their geographical limitations and scarcity of exportable natural resources—a phenomenon often described as the “natural resource curse.” This thesis aims to address several key questions: How do exchange rates and crude oil prices influence each other? What are the predictive capabilities of exchange rates on crude oil prices? How do geopolitical and economic uncertainties affect these predictions? These questions are explored through a multifaceted approach that considers historical data, economic theories, and modern econometric methods.

Chapter 2 explores the long-term relationship between crude oil prices and REER, addressing gaps in existing research that have largely overlooked the granular impact of the importance of oil exports for exporting countries on their real exchange rates. It illuminates the situation and policy approaches of nations heavily reliant on crude oil exports, and the importance and proportion of exports of a particular commodity may be significant factors in determining exchange rates. Many of the world’s oil-exporting countries, not limited to

Figure 1: GDP per capita for some oil exporting countries



the Middle East, experience inflation and have adopted fixed exchange rates (pegged to the US dollar), which has led to the phenomenon known as Dutch disease. In these countries, exchange rates are critical in underpinning economic stability and propelling growth. While all the mentioned countries export commodities, the situation for oil-exporting nations, particularly in the Middle East, differs from that of developed multi-commodity exporters like Australia and Canada, where the importance of oil exports to the economy varies significantly. Consequently, in contrast to previous studies, the updated model in Chapter 2 specification—substituting country-specific terms of trade with a real global oil price indicator and incorporating 6SECT as a proxy for internal price pressures—constitutes an improvement over existing BEER-type frameworks and offers a clearer and more consistent basis for analysing real exchange rate dynamics in oil-exporting economies. In the time-series analysis, evidence of cointegration was mixed: the Johansen test indicated cointegration in most countries except Gabon and the United States, while the Autoregressive Distributed Lag (ARDL) bounds test failed to confirm cointegration for Gabon and Nigeria. Given the limited data spans and small-sample concerns, we rely primarily on panel estimators. The Hausman test favours the PMG estimator over the MG, supporting the assumption of long-run coefficient homogeneity. PMG estimates yield robust and theoretically consistent results: oil price, 6SECT, and NFA all exhibit positive and statistically significant long-run effects on the REER, reflecting the roles of commodity income, sectoral inflation, and external wealth in determining exchange rate dynamics.

Numerous studies have attempted to determine the existence of a bidirectional relationship between the real exchange rate and the commodity price index. Therefore, Chapter 3 investigates whether the oil-exporting countries' exchange rate is a readily accessible and reliable predictor of international oil prices. Our analysis includes both in-sample and out-of-sample evaluations of global oil futures and three principal oil price benchmarks: Brent, WTI, and Dubai. We utilise a present value (PV) approach, applying exchange rates from Brazil, Canada, Colombia, Indonesia, Mexico, and Norway. The present value model posits that current crude oil prices should mirror the market's expectations of future prices. If oil-exporting countries' exchange rates successfully integrate anticipated trends in crude oil prices, then fluctuations in these currencies should correspond with shifts in crude oil prices and expectations of oil derivatives. Our findings reveal that the exchange rates of Brazil, Colombia, Mexico, and Norway have substantial short-term predictive power for crude oil and its derivative financial products. As highlighted in Chapter 2, the significance of crude oil to a local economy might heighten the currency's responsiveness to future crude oil price movements. For example, our research demonstrates that the exchange rates of Colombia and Brazil have strong predictive abilities in forecasting oil futures and crude oil prices. This also could explain why, in our forecasting method analysis, Canada and Indonesia do not forecast oil futures and prices effectively; the proportion of oil export revenues to their GDP is smaller compared to the other countries in our sample.

The research presented in the preceding chapters indicates that only the exchange rates of certain oil-exporting countries possess the predictive ability for international oil prices.

We attribute this to the inherent volatility and time-variability of oil prices, which might allow for short-range predictions but escape detection by standard forecasting regressions. Therefore, we think that this predictive capability typically might exist only within a short window. The fourth chapter of this study examines the predictability of crude oil prices, employing a range of predictive regression tests and Instrumental Variable (IV) estimation methods. We use three different exchange rates—Nominal Exchange Rate (NER), Nominal Effective Exchange Rate (NEER), and REER—for Brazil, Canada, Colombia, Indonesia, Mexico, and Norway to predict international oil prices. We observe limited evidence of predictability across the entire sample. However, a real-time monitoring program reveals that the exchange rates of oil-exporting countries demonstrate significant predictive power for oil prices amidst economic crises or downturns. And, the predictive capabilities of the three exchange rates—NER, NEER, and REER—are fundamentally similar. Appendices contain data information and other test results of Chapter 3 and Chapter 4.

Chapter 2

The Determination of Real Effective Exchange Rate for Crude Oil Exporting Countries

Abstract

The real exchange rate plays a central role in determining external competitiveness, especially for economies that are heavily reliant on resource exports. Among commodities, crude oil is uniquely influential for oil-exporting countries, yet much of the earlier literature did not isolate the effect of oil prices on the real effective exchange rate (REER). This chapter focuses specifically on how crude oil prices affect the long-run REER in 15 major oil-exporting countries from 1981 to 2017. To capture key macroeconomic fundamentals, the model includes three explanatory variables: global real oil prices, net foreign assets (NFA), and a six-sector value-added deflator (6SECT) that reflects Balassa–Samuelson effects.

Both time series and panel cointegration methods are employed. The Johansen and ARDL bounds tests yield mixed results at the individual country level, likely due to the limited time span. However, most countries show evidence of cointegration between REER and the selected fundamentals. At the panel level, both the Kao and Westerlund tests confirm a robust long-run relationship. To estimate the long-run coefficients, both the Mean Group (MG) and Pooled Mean Group (PMG) estimators are applied. The Hausman test supports the use of PMG, which shows that all three variables—oil prices, 6SECT, and NFA—have significant and positive effects on REER. These results align with economic theory and highlight the structural importance of oil, internal price dynamics, and external wealth in shaping the equilibrium real exchange rate of oil-exporting economies.

1 Introduction

Crude oil is still one of the most important natural resources and a politically important commodity globally. High fluctuations in oil prices also cause significant shifts in the wealth of nations and large current account surpluses in crude oil-exporting countries. Therefore, this chapter investigates the long-term relationship between the real exchange rate and crude oil. From the 1860s to the 1970s, the price of crude oil grew with the increase in demand. In recent years, the co-movements of crude oil prices and real exchange rates following the 2008 financial crisis have revived interest in their relationship. The relationship between crude oil and real exchange rate is widely discussed not only because of the huge current account surpluses for oil-exporting countries but also because of the 'Resource Curse' of crude oil. According to this curse, stronger natural endowments do not necessarily benefit national economic growth. Many studies try to determine the real exchange rate by the movement of oil prices. However, the relationship between the real exchange rate and the crude oil price has yet to be well-proven in many countries in previous studies. The relevant research is still full of challenges.

In the early 1970s, President Nixon suspended the convertibility of the dollar with the gold standard, and the Bretton Woods system collapsed. Many countries no longer peg their currencies to the value of the U.S. dollar. The era of floating exchange rates had begun. However, the floating exchange rate may not reflect the value of one country's currency in exchange for a foreign good or service. Hence, the real exchange rate is used to reflect the relative prices of domestically produced goods in comparison to international goods. It is not hard to imagine the presence of a relationship between the commodity price and the real exchange rate if a country has a large quantity of specific exports or imports of that commodity. As a result, the determination of the real exchange rate has sparked extensive research.

Among various commodities, crude oil could be recognised as the most unique one. Although the development of renewable energy has made considerable progress, fossil fuels are still one of the most important resources in the modern economy. On the demand side, Pershin et al. (2016) pointed out that the increase in oil demand from developing countries' economies might be why oil consumption keeps increasing. On the supply side, Basher et al. (2016) claimed that oil-exporting countries will have an appreciation in their exchange rate after an oil demand shock due to three structural shocks: global economic shock, oil supply and oil-market specific demand shock. This theory shows that because commodity prices are mainly determined by global supply and demand, an increase in commodity prices improves the terms of trade (TOT) and economic fundamentals of the commodity exporter. This, in turn, produces upward pressure on its currency, leading to an appreciation of the currency (Chen and Rogoff, 2003). This chapter specifically focuses on the determination of the equilibrium exchange rate in oil-exporting countries.

Why is understanding the determination of equilibrium real exchange rates so important? For countries' monetary policymakers, misalignment of the equilibrium real exchange rate and fixed exchange rate can have adverse effects on the local economy in the form of either it is overvaluation or undervaluation. In particular, overvaluation can be more harmful to a commodity-exporting country. Several studies, such as Frankel (2012) and Coudert et al. (2011), have pointed out that a country with rich natural resources or that has just discovered a new valuable natural resource will have massive inflows from exports and lead to a possible appreciation. If the spike in the country's currency negatively affects the competitiveness of its manufacturing and agricultural sectors, this could be called the 'Dutch Disease'. In general, developing countries with extensive mineral or energy resources might easily suffer from the Dutch Disease. For example, Goda and Torres (2013) demonstrated the negative effects of the 2004 to 2012 energy and mining boom on the real effective exchange rate (REER) in Colombia and the sectoral composition of its economy. Labour flowed into the tradeable sector from the non-tradeable sector. Hence, we chose countries with sizeable crude oil exports because those countries may be more vulnerable to crude oil price movements.

However, other factors besides the change in crude oil price might explain the real exchange rate movement. In practice, the complex interaction of unforeseen financial crises or other crises could influence economic performance. Therefore, it is also unavoidable to include those variables that influence the real exchange rate movement, although some variables might not be suitable for every country. In addition, the determination of equilibrium real exchange rates might be affected by many substantial factors, such as the time span, choice of sample countries, definition of the dependent variable (real exchange rate), the selected model and the econometrics method. Thus, based on the vast literature, this chapter investigates the following questions. What factors could affect the real exchange rates for oil-exporting countries? Does the importance of oil affect the REER? The model should include not only crude oil but also other explanatory variables. However, choosing the independent and dependent variables to investigate these questions is not straightforward. The discussion of other variables is presented in Section 4.

The sample countries examined in this chapter are 15 large oil-exporting countries: Brazil, Canada, Colombia, Ecuador, Gabon, Indonesia, Iran, Kuwait, Mexico, Nigeria, Norway, Oman, Saudi Arabia, Trinidad & Tobago, and the United States.. The total crude oil exports of these countries were over 80% of the world's oil exports during the sample period. We applied both time series and panel methods using the behavioural equilibrium exchange rate (BEER) method introduced by Clark and MacDonald (1999) to analyse the relationship between the fundamental variables and REER from the 1981s to the 2020s. The structure of this chapter is as follows. Section 2 explains the relevant concept of exchange rate selection. In Section 3, the key theoretical issues are stated. We then review the Empirical Literature in Section 4 and present our Methodology in Section 5. We describe the data in Section 6 and report the results in Section 7. Finally, the conclusions are presented in Section 8.

2 Concepts and definitions

This section explains different exchange rates (the spot exchange rate, real exchange rate and REER) and describes the most suitable choices for our case. The first priority was determining the applicable exchange rate for our model. Spot rates are the current exchange rates at which specific currencies can be bought or sold on currency exchange markets. In other words, the spot exchange rate is a current market price without considering historical inflation, making it inappropriate for analysing a long-term relationship. Conversely, the real exchange rate represents the nominal exchange rate adjusted by the relative price of domestic and foreign goods and services. Thus, it reflects the competitiveness of a country with respect to the rest of the world. The mechanism behind this is that higher inflation and the appreciation of the nominal exchange rate will increase the real exchange rate. This will lead to the home country's goods and services becoming less competitive than those of foreign countries. This will result in high unemployment and a foreign trade deficit. In fact, many studies prefer the real exchange rate rather than the nominal exchange rate or spot exchange rate, because the real exchange rate indicates the extent to which the goods and services in the domestic country can be exchanged for the goods and services in a foreign country. The definition of the real exchange rate between two countries is:

$$R \equiv \frac{EP^*}{P} \tag{2.1}$$

where P and P^* are the goods price levels of domestic and foreign countries, respectively; E is the domestic nominal exchange rate as per unit of foreign currency; and R is the domestic real exchange rate. Equation (2.1) can be rewritten in logarithm form as:

$$r \equiv e + p^* - p \tag{2.2}$$

Assume that all countries' goods may be divided into tradeable and non-tradeable goods, which are produced in both a domestic and a foreign country. The consumer price indexes of domestic (p) and foreign (p^*) countries are presented in logarithm form in Equations (2.3) and (2.4), respectively:

$$p = \alpha P^T + (1-\alpha)P^N \tag{2.3}$$

$$p^* = \alpha^* P^{T^*} + (1-\alpha^*)P^{N^*} \tag{2.4}$$

where P^T and P^{T^*} are the prices of tradeable goods in the domestic and foreign countries, respectively; P^N and P^{N^*} are the prices of non-tradeable goods in the domestic and foreign countries, respectively; and α and α^* correspond to the expenditure shares on the tradeable goods near the point of approximation for the domestic and foreign countries, respectively. Combining Equations (2.2), (2.3) and (2.4), we obtain Equation (2.5):

$$r = (e + p^{T^*} - P^T) + (1-\alpha)(P^T - P^N) - (1-\alpha^*)(P^{T^*} - P^{N^*}) \tag{2.5}$$

Based on Equation (2.5), we can divide the real exchange rate into two components: the real exchange rate of tradeable goods and the ratio of the domestic to the relative foreign prices of non-tradeable and tradeable goods. Thus, the weights of various sectors of the domestic and foreign economies could affect the movement of the real exchange rate between two countries. We can easily apply the real exchange rate as the dependent variable to investigate the relationship between two countries. In practice, countries trade with more than one country, so the REER is needed to account for this. A country's REER can be calculated by taking the average of the bilateral real exchange rate between itself and its trading partners by first weighting the bilateral real exchange rate using the trade allocation of each partner and then adjusting it for price deflators. The weights are determined by comparing the relative trade balance of a country's currency against another country in the basket. The equation of REER of a country i in the time t is:

$$REER_{it} = \prod_{j=1}^{n} P_{it}(\frac{E_{ijt}}{P^*_{jt}})^{w_{ij}} \quad (2.6)$$

where P_i measures the domestic price level in country i; P^*_j is the foreign price level in country j; and E_{ij} is the relevant nominal exchange rate, which is defined as the foreign currency per unit of domestic currency between countries i and j, and w_{ij} is the weight of country j in effective exchange rate index of country i[1].

Based on our Equation (2.6), an increase in REER will lead to a real appreciation of the local currency to its trading partners. In the long run, the movement of REER should reflect the fundamentals of the local country. However, monetary policy and other factors may move a currency temporarily away from the long-run equilibrium value. Thus, the challenge is to find the common determination of REER for many different countries. The possible fundamental variables and determination of REER are described in Section 4.

3 Theory

In this section, the key theoretical issues are described. The literature on the determinants of the equilibrium exchange rates and commodity prices is extensive. Many scholars have tried to establish the relationship between commodity prices and local currency (e.g., Chen and Rogoff, 2003; Cashin et al., 2004; Chen and Chen, 2007; Coudert et al., 2011; Frankel, 2011; Bodart et al., 2012; Basher et al., 2016; Pershin et al., 2016; Ayres et al., 2020). Although the literature has used many different methodologies, almost all of them start with Purchasing Power Parity (PPP). Hence, this section first discusses the literature review on PPP. Then, the Balassa-Samuelson Hypothesis (BSH) is explained to expand the PPP theory.

[1] The weights satisfy $\sum_{j=1}^{n} w_{ij} = 1$. For bilateral exchange rates ($n = 1$), this implies $w_{ij} \equiv 1$.

3.1 PPP theory

PPP was proposed by Swedish scholar Gustav Cassel in the early 19th century. The theory assumes that two identical goods in two countries with the same common currency should be equal in price because they are traded as the same goods. Gustav Cassel suggested using the consumer price index (CPI) for each country and using it to calculate the exchange rate that PPP would hold between countries. This theory has been the basis of most research into the real exchange rate. The simple theorem assumes the absence of many imperfections (e.g., transportation costs, different taxes and non-traded goods) to equate price levels and freely adjust nominal exchange rates to keep the real exchange rate constant. To account for this, PPP has two versions: absolute and relative. The absolute PPP is derived from the ratio of the index price between the home and foreign countries in their local currencies. The following formula presents the absolute PPP:

$$E = \frac{P}{P}* \tag{2.7}$$

where P is the price of a bundle of goods expressed in the domestic currency, P^* is the price of an identical bundle of goods in the foreign country expressed in terms of the foreign currency, and E is the exchange rate defined as domestic currency units per unit of foreign currency. Equation (2.7) may be expressed in logarithm form as:

$$e = p - p^* \tag{2.8}$$

If absolute PPP holds between the local country and the foreign country, the equilibrium real exchange rate is equal to 1. Thus, the identity of Equation (2.1) becomes:

$$R^* = \frac{EP^*}{P} = 1 \tag{2.9}$$

Even proponents of the absolute version of PPP acknowledge the theory's problems.

Firstly, the existence of transport costs and tariffs drives a wedge between the prices of an identical good in two trading countries' markets. Second, imperfect information exists between countries. Perhaps some price deviations are known to traders, but other deviations are not known. Perhaps only a small group of traders know about a price discrepancy, and that group is unable to achieve the scale of the trade needed to equalise the prices for that product. Third, the measurement of price levels differs from country to country. Inflation data from different countries are based on different commodity baskets. Therefore, exchange rate changes do not offset official measures of inflation differences (Pilbeam, 2013; MacDonald, 2007). Thus, a weaker form of PPP, known as relative PPP, may hold even in the presence of the above distortions, which remain constant over time. In the relative form of PPP, the dynamics of the nominal exchange rate follow the inflation differential between local and foreign economies and keep the real exchange rate at a stable level. The relative PPP is expressed as:

$$\triangle e_t = \triangle p_t - \triangle p_t^* \tag{2.10}$$

where $\triangle e_t$ is the percentage change in the log exchange rate, $\triangle p_t$ is the domestic inflation rate in logs, and $\triangle p_t^*$ is the foreign inflation rate in logs. The relative PPP equation can also be written as:

$$\triangle e_t = \theta + \ \alpha_1 \triangle p_t + \alpha_2 \triangle p_t^* + u_t \tag{2.11}$$

where θ is the constant; α_1 and α_2 are the coefficient of $\triangle p_t$ and $\triangle p_t^*$ respectively; and u_t is the error term. To satisfy the absolute PPP hypothesis, α_1 and α_2 should be 1 and -1, respectively. If e_t, p_t and p_t^* are integrated of order 1, and the residual is stationary in Equation (2.11), then the relative PPP could be confirmed as existing in the long run. Many early empirical studies could not find evidence of the existence of PPP. As stated before, the PPP hypothesis has many restrictive assumptions, and the PPP theory is the most straightforward determination of the real exchange rate. Balassa (1964) and Samuelson (1964) published BSH as an expansion of PPP. The BSH suggests that the increase in productivity in the tradeable goods sector compared to the non-tradeable goods sector is linked to the real appreciation of the domestic currency.

3.2 BSH

Early empirical studies of Purchasing Power Parity (PPP) often used unit root and cointegration tests but yielded mixed results. One explanation for the failure of PPP to hold in the long run is the presence of persistent productivity differentials across countries. The Balassa–Samuelson Hypothesis (BSH) provides a theoretical framework to explain how such differentials can lead to sustained deviations in the REER.

The BSH posits that countries with higher productivity growth—especially in the tradable sector—tend to experience faster increases in real wages. Since wages tend to equalise across sectors in the long run, higher productivity in the tradable sector pushes up wages and, in turn, prices in the non-tradable sector. This leads to a rise in the overall price level, contributing to REER appreciation in high-growth economies.

Let W_T and W_N denote wages in the tradable and non-tradable sectors, and Q_T and Q_N denote labour productivity in the respective sectors. Assuming perfect labour mobility between sectors, wages equalise across sectors in the long run: $W_T = W_N$. Prices are then given by:

$$P_T = \frac{W_T}{Q_T}, \quad P_N = \frac{W_N}{Q_N} \tag{2.12}$$

In high-productivity countries (indicated by asterisks), the same relationships hold:

$$P_T^* = \frac{W_T^*}{Q_T^*}, \quad P_N^* = \frac{W_N^*}{Q_N^*} \tag{2.13}$$

If $Q_T^* > Q_T$ while $Q_N^* \approx Q_N$, and wages equalise across sectors, then $P_N^* > P_N$, even if $P_T^* = P_T$. Thus, even under PPP for tradables:

$$E = \frac{P_T}{P_T^*}. \tag{2.14}$$

The overall REER can still appreciate due to higher non-tradable prices. Hence, the BSH explains why richer countries often have stronger real exchange rates despite PPP holding only in the tradable sector. This provides a structural explanation for persistent deviations from overall PPP.

4 Empirical literature

As mentioned above, the commodity prices with respect to the exporters' exchange rate have attracted the attention of many researchers examining many countries. The determination of REER has been broadly researched by Chen and Rogoff (2003), Cashin et al. (2004), Chen and Chen (2007), Coudert et al. (2011), Frankel (2011), Bodart et al. (2012), Basher et al. (2016), Pershin et al. (2016), and Ayres et al. (2020). However, determinants are still controversial. Existing research suggests that the real effective exchange rate (REER) is determined by a set of long-term fundamental factors, such as terms of trade, net foreign assets, relative prices, and productivity. We argue that specific commodity prices have additional importance. Oil prices play a pivotal role in explaining exchange rate dynamics in net oil-exporting countries. This section reviews the empirical literature on REER determination for oil-exporting countries in the context of the theoretical framework by examining different variables. First, we discuss the impact of oil price volatility on nominal exchange rates, highlighting the wealth transfer and capital flow channels. Secondly, we discuss the use of the sectoral price approach to explain the BSH effect. Finally, we analyse the role of NFA in determining the REER of oil-exporting countries.

4.1 Oil Prices and REER

In the short run, fluctuations in oil prices can lead to immediate adjustments in nominal exchange rates through wealth transfer and associated capital flow channels. An increase in oil prices produces a wealth transfer from oil-importing nations to oil-exporting nations. This concept, first formalised in the models of Krugman (1980) and Golub (1983), implies that a surge in oil export revenue rapidly improves the trade balance and net foreign asset position of oil exporters, while deteriorating those of importers. As a result, the currencies of net oil exporters tend to face upward pressure. Higher petrodollar revenues are converted into local currency for domestic use, boosting demand for the exporter's currency and causing a nominal appreciation (all else equal), whereas oil-importing countries may see their currencies weaken (Bénassy-Quéré et al., 2009). Empirical evidence supports this wealth-effect mechanism: commodity currency behaviour is often observed in practice, with oil-exporting countries' exchange rates appreciating in response to oil windfalls (Amano and Van Norden, 1998; Chen and Chen, 2007). For instance, Beckmann and Czudaj (2013) find that positive oil price shocks lead to an effective appreciation of oil exporters' currencies in the short term, reflecting the influx of export earnings.

Alongside the wealth channel, oil price movements trigger capital flow and portfolio reallocation effects that further influence nominal exchange rates. When oil exporters accumulate large surpluses from high oil prices, they often invest these funds internationally, a process sometimes termed "petrodollar recycling". The destination of these outflows can affect major currencies. If a significant portion of oil wealth is reinvested in assets denominated in a reserve currency (such as US dollars or euros), demand for those financial assets increases. This portfolio effect can lead to a short-run support (or even appreciation) of the reserve currency. At the same time, to the extent that oil-exporting countries retain or repatriate oil revenues, their domestic currencies appreciate due to both improved fundamentals and investors' revised expectations of future returns Bodenstein et al. (2011); Coudert et al. (2008). The net outcome depends on how wealth is allocated: for example, a spike in oil prices might initially weaken an oil-importer's currency like the U.S. dollar due to trade losses, but if oil exporters subsequently channel their windfall into U.S. assets (Treasury bonds, equities, etc.), the dollar can rebound via the portfolio channel (Bénassy-Quéré et al., 2009; Beckmann and Czudaj, 2013). In summary, oil price volatility translates quickly into nominal exchange rate volatility through these channels of wealth redistribution and capital flows. These short-run effects help explain why the currencies of oil-dependent economies (e.g. Russia, Norway, Gulf states) often move in tandem with oil markets on a high-frequency basis (Lizardo and Mollick, 2010). Policymakers in such countries are acutely aware of this linkage, sometimes responding with stabilisation funds or monetary interventions to buffer excessive currency movements stemming from oil price swings.

Over longer horizons, the influence of oil prices on exchange rates becomes deeply embedded in the REER through structural and persistent effects. For economies where oil exports constitute a significant share of GDP and exports, oil price trends fundamentally alter the equilibrium value of the currency. A sustained rise in oil prices effectively raises the permanent income and wealth of a net oil-exporting country, which in turn tends to appreciate its REER over time (Cashin et al., 2004; Koranchelian, 2005). This can be understood through the lens of a two-sector Balassa-Samuelson type model: the oil sector (tradable) boom drives up wages and demand for labour, and under labour mobility, wages rise economy-wide. Higher incomes spur greater demand for non-tradable goods and services, bidding up their prices (Asea and Mendoza, 1994). The result is an increase in the overall price level at home relative to abroad (especially in non-tradables), producing a persistent REER appreciation. In other words, a positive terms-of-trade shock due to oil improves the country's capacity to import and spend, causing an enduring shift in relative prices that strengthens the home currency's real value Amano and Van Norden (1998); Habib and Kalamova (2007). Classic theoretical work by Corden (1984) formalised how a resource boom leads to real appreciation and a reallocation of resources away from manufacturing and agriculture toward the booming sector and non-tradables. In oil-rich economies, the importance of oil exports means that oil price levels become a key structural driver of the REER, not just a cyclical one.

Empirically, numerous studies have documented a robust long-run relationship between oil prices and the REER in oil-exporting nations. Early evidence by Amano and Van Norden (1998) found a stable cointegrating relationship between real oil prices and the U.S. dollar's real exchange rate over the post-Bretton Woods era, suggesting that oil price trends explain persistent deviations in the dollar's value. Similarly, Chaudhuri and Daniel (1998) and Chen and Chen (2007) confirmed that for many advanced economies, especially those with commodity export exposure, real exchange rates and real oil prices move together in the long run (i.e., they are cointegrated). Focusing on oil exporters specifically, Koranchelian (2005) (for Algeria) and Zalduendo (2006) (for Venezuela) found that incorporating oil prices is essential to determining the equilibrium REER; higher oil prices are associated with an appreciation of the equilibrium REER, reflecting increased productivity and spending in those economies. Habib and Kalamova (2007) show that in countries like Russia, Saudi Arabia, and Norway, a large share of oil in exports makes their real exchange rates highly sensitive to oil price fluctuations, with sustained oil price increases yielding a stronger long-run REER. More recent panel studies and IMF assessments reinforce this point: the elasticity of the REER with respect to oil terms-of-trade is significantly positive for oil-exporting countries, indicating that structural reliance on oil elevates the currency's long-run real value Coudert et al. (2011). These persistent effects also highlight why some oil exporters, especially those with fixed or managed nominal pegs, often experience pro-cyclical inflation, since the nominal exchange rate cannot fully adjust to oil shocks, domestic prices rise, leading to a higher REER during oil booms Bodenstein et al. (2011). In sum, the long-run importance of oil exports manifests in a structural linkage between oil price levels and the country's REER, capturing phenomena such as prolonged Balassa-Samuelson effects, spending booms, and competitiveness losses in non-oil sectors when oil prices are high.

Overall, the short-run and long-run perspectives above together explain why oil prices serve as a powerful explanatory variable for the REER in net oil-exporting countries. In the short term, oil price volatility directly feeds into nominal exchange rate movements via wealth transfers and capital flow adjustments, causing immediate real exchange rate effects as prices and nominal rates respond. In the long term, the prominence of oil in the economy means that prolonged changes in oil prices alter the country's productive structure and consumption patterns, effectively anchoring the REER to oil-driven fundamentals. Thus, oil prices influence both the cyclical fluctuations and the underlying equilibrium path of exchange rates. Empirical models of equilibrium REER for oil exporters invariably include oil price or terms-of-trade measures as key fundamentals, and these models consistently find oil to be a statistically and economically significant driver of real exchange rate behaviour (Cashin et al., 2004; Korhonen and Mehrotra, 2009). In net oil-exporting countries, ignoring oil prices would leave a gaping hole in the explanation of REER dynamics – oil is the common factor that ties together the volatile short-run movements and the enduring long-run equilibrium of the exchange rate. Hence, oil prices emerge as a uniquely potent explanatory variable for the REER, encapsulating both transitory and persistent forces shaping external competitiveness in oil-dependent economies.

4.2 Six-sector relative price indices (6SECT) and REER

As mentioned before, the BSH extends the traditional PPP framework by incorporating the role of non-tradable goods. While PPP assumes that tradable goods prices equalise across countries, the BSH insight is that faster productivity growth in the tradable sector—relative to the non-tradable sector—raises wages across the economy, which in turn increases the relative price of non-tradeables and leads to REER appreciation. This mechanism has been incorporated into many models of equilibrium exchange rates. Early empirical support comes from De Gregorio et al. (1994), who use sectoral data to show that relative productivity growth between sectors is associated with REER appreciation. More recent studies, including Choudhri and Khan (2005) and Berka et al. (2018), confirm that relative sectoral productivity is a key driver of long-run REER differences, particularly across income groups.

To operationalise the BSH mechanism empirically, researchers often proxy productivity differentials using the relative price of non-tradeables to tradeables. Direct measurement of productivity can be data-intensive and difficult to compare internationally, whereas sectoral price indices are more readily available and informative. A common approach is to disaggregate value-added deflators by industry and construct composite indices that capture the price dynamics of tradable and non-tradable sectors. One such measure is the 6SECT, proposed by Lee and Tang (2007), which classifies economic activity into six broad categories—such as agriculture, manufacturing, services—and aggregates them into tradable and non-tradable components. The ratio of the price index for non-tradeables to that for tradeables serves as a proxy for internal price pressures stemming from productivity trends.

This type of sectoral price index has proven effective in tracking BSH dynamics. An increase in the index typically indicates rising non-tradable prices relative to tradeables, reflecting domestic demand pressures or wage growth driven by productivity gains. Empirical applications, such as Bénassy-Quéré et al. (2009) and Couharde et al. (2020), demonstrate that multi-sector price indices are useful in explaining long-run REER movements across countries and over time.

Nevertheless, the explanatory power of the BSH effect is more limited at short horizons or among economies with similar productivity growth. As Devereux (2014) points out, the pure BSH model struggles to account for REER fluctuations at business-cycle frequencies or in advanced economies without large productivity differentials. Likewise, Bahmani-Oskooee and Nasir (2004) find that the productivity bias hypothesis often fails in short-run time-series regressions, particularly when structural shifts are present. These limitations, however, do not invalidate the long-run relevance of the BSH mechanism. In fact, sectoral price proxies such as 6SECT continue to provide meaningful insights into long-run REER dynamics, especially in economies where structural changes are concentrated in specific sectors.

In oil-exporting countries, long-run REER movements are shaped not only by internal price structures—captured by measures like 6SECT—but also by external wealth dynamics arising from resource revenues. These countries often accumulate large stocks of net foreign assets (NFA) through current account surpluses or sovereign wealth funds. Such external positions influence the REER through intertemporal trade-offs and terms-of-trade effects. For example, Norway's sovereign wealth fund plays a stabilising role by smoothing income and mitigating inflationary pressures from oil windfalls. As shown in Lane and Milesi-Ferretti (2018) and Ricci et al. (2013), NFA is a key long-run determinant of REER in economies exposed to commodity price volatility.

From a modelling perspective, 6SECT and NFA capture distinct but complementary channels: the former reflects domestic cost structures shaped by productivity dynamics, while the latter reflects a country's external wealth and saving behaviour. Together, they provide a more complete picture of REER fundamentals, especially in resource-rich economies. This dual-channel view is consistent with the Behavioural Equilibrium Exchange Rate (BEER) framework, which models the REER as a function of macroeconomic fundamentals—including productivity, terms of trade, and net foreign assets. The BEER approach provides a widely used empirical framework for assessing equilibrium exchange rates based on these underlying economic drivers. The following 2 sections formally introduce the BEER framework applied in this study and outline how the 6SECT and NFA indicators are incorporated into the empirical analysis for oil-exporting countries.

4.3 Net Foreign Assets and REER

The BEER approach posits that a country's REER is driven in the long run by a set of economic fundamentals. Pioneering BEER studies (Clark and MacDonald, 1999) typically include variables such as relative productivity, terms of trade, and NFA in explaining equilibrium REER movements. The inclusion of NFA captures the stock of a country's cumulative external balances – essentially its creditor or debtor position vis-à-vis the rest of the world. In theory, a nation's NFA position influences the long-run REER through what is often termed the "transfer effect." Countries with large external liabilities must eventually run trade surpluses (or have higher export competitiveness) to service those liabilities, which requires a more depreciated real exchange rate, whereas countries with substantial external asset holdings can afford stronger (appreciated) currencies while still maintaining external balance (Coudert et al., 2008). This theoretical mechanism was articulated by Faruqee (1995) in a stock-flow model, who found for the United States and Japan that the long-run path of the real exchange rate is cointegrated with net foreign asset positions and other structural trade factors. In other words, permanent shifts in NFA induce proportional adjustments in the equilibrium REER, as a country's currency value must adjust to ensure its intertemporal budget constraint (external balance) is met in the long run.

More recent multi-country studies reinforce the role of NFA as a fundamental driver of REER. Using panel cointegration techniques for 48 industrial and emerging economies, Ricci et al. (2013) find that increases in NFA are associated with an appreciating CPI-based REER in the long run, even after controlling for other factors. In their estimates, a 10 % of GDP improvement in the NFA position leads to a statistically significant REER appreciation, reflecting the reduced need for future trade surpluses. Likewise, MacDonald (1999) BEER framework incorporates NFA as a key "stock" fundamental alongside terms of trade and productivity. They and others (Faruqee (1995); MacDonald and Nagayasu (2000)) show that equilibrium real exchange rate models better fit the data when they include the NFA term, capturing the long-run influence of a country's external wealth on its currency's value.

Most of the early literature focuses on broad cross-national panel data or large developed economies, while studies focusing on specific countries (especially developing countries and commodity-dependent economies) are relatively rare. Oil exporters, for instance, have unique institutional setups (sovereign wealth funds, pegged exchange rates, fiscal policy dynamics) that might modulate how NFA influences the REER. These economies often run large and persistent current account surpluses during oil booms, leading to rapid accumulation of foreign assets such as official foreign exchange reserves and sovereign wealth funds. In the 2000s, surging oil prices resulted in oil exporters as a group attaining the world's largest current account surpluses.

For example, Gulf states and Norway invested oil revenues in sovereign wealth funds abroad, effectively converting volatile oil income into a growing external asset stock. This strategy of accumulating NFA has important implications for the equilibrium REER. The wealth effect of rising NFA tends to put appreciation pressure on the REER over time – as external assets generate income and bolster national wealth, domestic absorption can be higher without incurring deficits, warranting a stronger currency in equilibrium. Therefore, omitting NFA from models of petroleum-based economies overlooks a key driver, as the accumulation of external assets is a defining feature of these countries.

Consistent with this, Coudert et al. (2011) find that cointegration equations for oil-exporters' REER have correctly signed and significant NFA coefficients, reflecting the strong contribution of external asset accumulation to long-run currency values. By incorporating NFA, one accounts for the influence of oil-funded sovereign wealth and reserve accumulation on exchange rates, which helps explain why oil exporters often exhibit higher equilibrium REERs than otherwise similar non-oil economies during prolonged boom periods.

4.4 Fundamental REER determination

As described before, the failure to establish the existence of PPP in the last century implies that the real exchange rate is non-stationary. Hence, many academics have tried to explain the long-run relationship in terms of other economic fundamentals. Unlike the studies by Chen and Chen (2007) and Bodart et al. (2012), many previous studies tried to determine the real exchange rate from many aspects. For oil-exporting countries, the upward demand for crude oil may create upward pressure on their price. Thus, Basher et al. (2016) used oil demand and supply as variables and applied the Markov-switching models to investigate the impact of the real exchange rate movement after an oil price shock. They found that a rise in crude oil prices would lead to real appreciation for the local currencies of the oil-exporting countries. Another famous model is the BEER model by Clark and MacDonald (1999). In their BEER model, the observed REER q_t is explained by a set of fundamental variables Z_{1t} and Z_{2t}, which affect the exchange rate in the long-term and medium-term, respectively. The

REER in the BEER model is shown as follows:

$$q_t = \theta_1 Z_{1t} + \theta_2 Z_{2t} + \delta T_t + \epsilon_t \tag{2.15}$$

where Z_{1t} is a vector of economic fundamentals expected to have persistent effects over the long run; Z_2 is a vector of economic fundamentals that affect the real exchange rate over the medium term, such as those coinciding with the business cycle; θ_1 and θ_2 are the coefficients; T is a transitory factor affecting the exchange rate in the short term; and ϵ is a random error term. The current equilibrium exchange rate does not contain the short-term transitory factors and the random disturbance term. Therefore, the current equilibrium exchange rate q'_t is derived from:

$$q'_t = \theta_1 Z_{1t} + \theta_2 Z_{2t} \tag{2.16}$$

Hence, the current misalignment, cm_t, is the difference between the actual real exchange rate and the equilibrium real exchange rate given by the current values of all the fundamental variables, as shown below:

$$cm_t = q_t - q'_t = q_t - \theta_1 Z_{1t} - \theta_2 Z_{2t} = \delta T_t + \epsilon_t \tag{2.17}$$

Clark and MacDonald (1999) also defined the total misalignment as the difference between the actual and real exchange rate in the long run. The equation of total misalignment is given by:

$$tm_t = q_t - \theta_1 \hat{Z_{1t}} - \theta_2 \hat{Z_{2t}} \tag{2.18}$$

Here, $\widehat{Z}_{1t}$ and $\widehat{Z}_{2t}$ represent the long-run or equilibrium values of the corresponding economic fundamentals, typically obtained through filtering techniques or long-run estimations. Equation (2.18) can be rewritten as:

$$tm_t = \delta T_t + \epsilon_t + [\theta_1 (Z_{1t} - \hat{Z_{1t}}) + \theta_2 (Z_{2t} - \hat{Z_{2t}})] \tag{2.19}$$

Equation (2.19) is the difference between the actual real exchange rate and the equilibrium real exchange rate by the long-run values of the economic fundamentals, $\hat{Z_{1t}}$ and $\hat{Z_{2t}}$. Equation (2.19) is divided into two components. The first part is the current misalignment, shown as Equation (2.17). The second component illustrates the effect of departures of the current values of the fundamentals from the long-run values. This equation explains that the total misalignment at any time can be decomposed into the effects of the transitory factors, the random error and the economic fundamentals by their sustainable values (long-run values).

Following Clark and MacDonald (1999), the BEER approach is built on the Uncovered Interest Parity (UIP):

$$E_t[\triangle e_{e+k}] = -(i_t - i_t^*) + \pi_t \tag{2.20}$$

where e_t is the foreign currency price of a unit of domestic currency, i_t denotes a nominal interest rate, $\pi_t = \lambda_t + c$ is the risk premium that has a time-varying composition, λ_t, $\triangle$ is the first differences operator, E_t is the conditional expectations operator and t+k defines the maturity horizons of the bonds.

By subtracting the expected inflation differential, $E_t(\triangle p_{t+k} - p_{t+k}^*)$, from both sides of Equation (2.20), we get the real relationship:

$$q_t = E_t[q_{t+k}] + (r_t - r_t^*) - \pi_t \tag{2.21}$$

where $r_t = i_t - E_t[\triangle p_{t+k}]$ is the ex ante real interest rate. Based on Equation (2.21), the current equilibrium exchange rate can be determined by three factors: the expectation of the real exchange rate in period t+k, the real interest rate differential with a maturity t+k, and the risk premium. Clark and MacDonald (1999) continue by assuming the time-varying component of the risk premium term as a function of the relative supply of domestic to foreign government debt:

$$\lambda_t = g(gdebt_t/gdebt_t^*) \tag{2.22}$$

Assuming $E_t[q_{t+k}]$ is determined solely by the long run economic fundamentals, Z_1, they denote the long run equilibrium exchange rate as:

$$\hat{q}_t = E_t[q_{t+k}] = E_t[\theta_1 Z_{1t}] = \theta_1 Z_{1t} \tag{2.23}$$

In different studies, a wide range of variables are assigned for fundamentals Z_{1t}, these could be the terms of trade (commodity price), the productivity differentials (BSH effect), NFA, Output Gaps, Openness and Government Spending. Clark and MacDonald (1999) focus on three variables:

$$q_t = f[tot_t, tnt_t, nfa_t] \tag{2.24}$$

where tot is the terms of trade, tnt is the BSH effect, i.e. relative price of non-traded to traded goods, and NFA is the net foreign assets.

Equation (2.20) to (2.24) imply the following general Equation:

$$q_t = f[r - r_t^*, tnt_t, tot_t, nfa_t, gdebt_t/gdebt_t^*] \tag{2.25}$$

Equation (2.25) represents a general expression linking the current real exchange rate to both the real interest rate differential and a set of fundamental factors that drive the long-run equilibrium exchange rate. As previously noted, this equation is derived from the UIP condition. However, a key concern is the validity of the UIP itself. This stands in contrast to studies like Meese and Rogofp (1988) and Froot (1990), which find no empirical support for a relationship between the real exchange rate and interest rate differentials. On the other hand, in developing countries, where economic structures often undergo significant changes, historical data may not accurately reflect the current state of the economy. Maeso-Fernandez et al. (2005) and Maeso-Fernandez et al. (2006) point out that the standard BEER framework is more suited to advanced economies, where past data can more reliably capture exchange rate dynamics. Therefore, we adopt a reduced-form approach but incorporate a different set of fundamental variables, as discussed in the following subsection.

4.5 Justification for Reduced-Form BEER Specification

The decision to exclude the real interest–rate differential and relative public debt from our long-run BEER specification rests on both practical and theoretical grounds. Data constraints are paramount in our diverse panel of commodity exporters. Many of the 15 countries (Brazil, Canada, Colombia, Ecuador, Gabon, Indonesia, Iran, Kuwait, Mexico, Nigeria, Norway, Oman, Saudi Arabia, Trinidad and Tobago, and the United States) lack long, consistent series for real interest rates or government debt on a comparable basis. For example, nominal policy rates and inflation measures (needed to construct real rates) are often missing or unreliable in earlier periods for smaller economies (e.g., Gabon, Oman, Trinidad and Tobago) and differ in frequency and definition. Likewise, public debt data (gross vs. net, general government vs. central government) are patchy across these countries. By constructing a balanced panel (i.e., requiring each country-year to have all regressors present), we have to omit any variable that would force us to drop entire countries or periods.

The theoretical BEER literature further justifies omitting these variables. Clark and MacDonald (1999) described the Behavioural Equilibrium Exchange Rate as driven by a set of long-run real fundamentals (e.g., terms of trade, net foreign assets, relative prices, productivity) together with UIP determining short-run fluctuations. In this framework, the real interest rate differential enters only as a short-term (cyclical) factor, not as a stationary equilibrium determinant. In other words, UIP implies that interest gaps cause only transitory exchange rate movements around the BEER, so they need not appear in the long-run cointegration equation. Similarly, while Clark and MacDonald (1999) mentioned government debt as a possible fundamental, there is no strong theoretical mechanism linking relative debt levels to the steady-state real exchange rate, especially once net foreign wealth is controlled. In commodity exporters, the exchange rate is much more sensitive to real demand (terms of trade) and supply (productivity) shocks than to fiscal debt per se. Thus, most BEER applications for resource economies focus on commodity prices, income or spending measures, and net asset positions, treating interest and debt terms as secondary or omitted, as in many IMF or ECB BEER studies (e.g., Coutinho et al. (2021)). Our specification follows this guidance by excluding variables that are not expected to shift the long-run REER.

Empirically, prior studies find the interest-rate fundamental to be weak or unstable in long-run REER models. For instance, studies of major commodity-linked currencies often show no robust cointegration of REER with interest differentials alone. One early study for Australia (a commodity-exporter) finds "no stable long-run relationship" between the real exchange rate and either short or long real interest differentials; in that work, terms of trade alone carried the signal of equilibrium, while interest gaps proved insignificant or spurious (Gruen and Wilkinson, 1994). More broadly, panel cointegration analyses (e.g., MacDonald and Nagayasu (2000)) suggest that an interest differential may help short-term predictions but is not a reliable long-horizon driver across countries. Likewise, fiscal or debt variables seldom appear as significant long-run fundamentals once net asset or productivity effects are included. Few BEER studies impose debt ratios, and where they do, the results vary. In practice, exchange rate models for oil exporters (Mexico, Nigeria, etc.) typically emphasise oil prices and spending or saving, not debt-to-GDP. The instability of interest and debt coefficients in long-horizon regressions means they add noise rather than explanatory power. To avoid weak instruments or omitted-variable bias, we therefore follow the BEER tradition of omitting these fundamentals that do not show a consistent influence on the REER's permanent component.

Finally, we ensure the panel remains balanced and economically interpretable. In our 15-country panel, insisting on full data coverage (as in other large-sample BEER implementations) means excluding any variable with gaps. Dropping the interest differential and debt ratios thus allows us to exploit the entire panel and time span without losing observations. In sum, the exclusion is driven by data availability concerns, BEER-theoretic guidance, and empirical precedent.

5 Methodology

If all variables in an ordinary least squares (OLS) regression are stationary, conventional statistical measures, such as the t statistic, are the standard approach. In most cases, if two variables are I(1) with a linear combination, then the combination will also be I(1). If these two variables are cointegrated, the linear combination of I(1) will be I(0). In general, most financial variables contain a unit root and are thus I(1) variables. The sample variables in our case, REER, NFA, OIL and 6SECT, are all usually non-stationary variables. This section describes all methods and mechanisms involved in this chapter. The augmented Dickey-Fuller (ADF) and Phillips–Perron (PP) unit root tests were applied to determine the properties of all variables. Then, Johansen (1988, 1991, 1995) maximum likelihood approach is introduced as the primary method in this chapter to discover the presence of cointegration in our model. Finally, Pesaran et al. (2001) autoregressive distributed lag (ARDL) approach was applied to calculate the long-run relationship. Given the short time dimension of country-level series and the inconclusive patterns across time series regressions, we focus our main analysis on panel cointegration techniques, which allow for more robust inference across heterogeneous economies. Section 5.1 describes different panel methods.

5.1 Panel methods

Cross-sectional dependence

Panel data often exhibit cross-sectional dependence due to shared shocks or institutional linkages. Despite the heterogeneous economic structures of crude oil exporters, unobserved common factors—such as environmental agreements or global energy transitions—may induce such dependence. To address this, we employ estimators robust to cross-sectional dependence. The Common Correlated Effects estimator developed by Pesaran (2006) addresses this issue by augmenting the regression with cross-sectional averages of both the dependent and independent variables, thereby accounting for unobserved common factors. This approach improves upon standard Mean Group (MG) and Pooled Mean Group (PMG) estimators (Pesaran and Smith, 1995; Pesaran et al., 1999), which assume cross-sectional independence.

Panel Unit Root Tests

Panel unit root tests extend single-series unit root tests to panel data by pooling information across cross-sectional units. Let $y_{i,t}$ denote the series for unit $i = 1, \dots, N$ over $t = 1, \dots, T$. A common starting point is the panel ADF regression:

$$\Delta y_{i,t} = \phi_i \, y_{i,t-1} + z_{i,t}' \gamma_i + \sum_{j=1}^{p_i} \theta_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t},$$

where $z_{i,t}$ captures deterministic components (e.g., intercepts or trends), and $\varepsilon_{i,t}$ is the idiosyncratic error term. We compare three widely used first-generation panel unit root tests: Levin–Lin–Chu (LLC) (Levin et al., 2002), Im–Pesaran–Shin (IPS) (Im et al., 2003b), and Hadri LM (Hadri, 2000).

The LLC test (Levin et al., 2002) assumes a homogeneous autoregressive coefficient ϕ across all panels. The model is:

$$\Delta y_{i,t} = \phi \, y_{i,t-1} + z_{i,t}' \gamma_i + \sum_{j=1}^{p_i} \theta_{i,j} \Delta y_{i,t-j} + u_{i,t}, \tag{2.26}$$

where ϕ is common across i, and $z_{i,t}$ typically includes individual effects and trends. The null hypothesis is $H_0 : \phi = 0$ (unit root in all panels) versus $H_1 : \phi < 0$ (stationarity). The test is most suitable when the time dimension T is sufficiently large and cross-sectional dependence is negligible or mitigated via demeaning or cross-sectional averages. However, the homogeneity assumption can be restrictive and may lead to biased inference when dynamics vary across units.

The Im–Pesaran–Shin (IPS) test (Im et al., 2003b) relaxes the LLC assumption by allowing heterogeneous autoregressive coefficients. Each panel follows its own ADF regression:

$$\Delta y_{i,t} = \phi_i \, y_{i,t-1} + z'_{i,t}\gamma_i + \sum_{j=1}^{p} \psi_{i,j} \Delta y_{i,t-j} + \varepsilon_{i,t}, \tag{2.27}$$

where ϕ_i may differ across i. The null hypothesis remains $H_0 : \phi_i = 0$ for all i (unit root), while the alternative is H_1: $\phi_i < 0$ for at least some i. The IPS statistic is computed as the standardised average of individual ADF t-statistics:

$$W_{t\text{-bar}} = \frac{\sqrt{N}\left(\frac{1}{N}\sum_{i=1}^{N} t_i - \bar{\mu}_T\right)}{\bar{\sigma}_T},$$

where t_i is the ADF t-statistic for unit i, and $\bar{\mu}_T$, $\bar{\sigma}_T$ are mean and variance under the null. The test accommodates heterogeneity in dynamics, offering greater flexibility than LLC. However, it assumes cross-sectional independence and requires large N and T for reliable inference.

Hadri's LM test (Hadri, 2000) reverses the null hypothesis, testing for stationarity rather than unit roots. The null hypothesis is H_0: all panels are (trend-)stationary, versus H_1: some panels contain unit roots. It is based on the KPSS framework, where the data-generating process is modelled as:

$$y_{i,t} = r_{i,t} + \beta_i t + \varepsilon_{i,t},$$
$$r_{i,t} = r_{i,t-1} + u_{i,t},$$

and the presence of a unit root corresponds to $\sigma_u^2 > 0$. The LM statistic is:

$$LM = \frac{1}{NT^2\hat{\sigma}_\varepsilon^2} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(\sum_{s=1}^{t} \hat{e}_{i,s} \right)^2,$$

where $\hat{e}_{i,t}$ are residuals from the stationary regression and $\hat{\sigma}_\varepsilon^2$ is an estimate of the long-run variance. The test is useful for confirming the absence of unit roots, though it may suffer from size distortions under cross-sectional dependence.

Panel cointegration tests

After establishing the presence of unit roots, we proceed to test for cointegration relationships among the non-stationary variables using two complementary approaches: the residual-based Kao test (Kao, 1999) and the error-correction-based Westerlund test (Westerlund, 2007).

The Kao test extends the Engle–Granger methodology to the panel data setting by testing whether residuals from a pooled cointegrating regression are stationary. Specifically, it estimates the long-run relationship as

$$y_{it} = \gamma_i + \beta' x_{it} + e_{it},$$

where γ_i captures individual fixed effects, and β is assumed to be homogeneous across all cross-sectional units. No deterministic trends are included. After estimating the residuals $\hat{e}_{it}$, an ADF regression is conducted:

$$\hat{e}_{it} = \rho\,\hat{e}_{i,t-1} + \sum_{j=1}^{p} \rho_j\,\Delta\hat{e}_{i,t-j} + \nu_{it}, \tag{2.28}$$

testing the null hypothesis $H_0 : \rho = 1$ (no cointegration) against the alternative $H_1 : \rho < 1$ (cointegration). The Kao test assumes strict exogeneity of regressors, no cross-sectional dependence, and identical long-run relationships across units.

While the Kao test is simple and analytically tractable, it imposes restrictive assumptions that limit its applicability in heterogeneous panel settings. In particular, the homogeneity of the cointegrating vector across units, the exclusion of deterministic time trends, and the assumption of cross-sectional independence can all reduce the test's power or lead to biased inference when violated. These limitations are especially relevant in macroeconomic panels, where countries may follow structurally different long-run adjustment paths.

To address these concerns, we also implement the Westerlund test, which is based on an error-correction model framework. Unlike residual-based tests, the Westerlund approach directly tests for the existence of a long-run relationship by evaluating whether the error-correction term is significant in a conditional dynamic specification. The ECM for each cross-sectional unit is specified as

$$\Delta y_{it} = \alpha_i + \phi_i(y_{i,t-1} - \beta_i' x_{i,t-1}) + \sum_{j=1}^{p} \gamma_{ij} \Delta y_{i,t-j} + \sum_{j=1}^{q} \delta_{ij} \Delta x_{i,t-j} + \varepsilon_{it},$$

where cointegration is indicated by a significantly negative adjustment coefficient ϕ_i. The Westerlund test evaluates the null hypothesis of no cointegration (i.e., $\phi_i = 0$ for all i) against the alternative that at least some panels exhibit a restoring force to equilibrium (i.e., $\phi_i < 0$ for some i).

This approach accommodates heterogeneity in both the short-run dynamics and the long-run relationships, and can incorporate unit-specific intercepts and deterministic trends. Furthermore, Westerlund's framework permits weak cross-sectional dependence under certain conditions and is robust to structural breaks in the cointegrating relationship. Simulation studies have shown that the test has favourable size and power properties, especially when T is reasonably large.

In summary, the Kao test provides a convenient benchmark under strong homogeneity assumptions, while the Westerlund test offers a more flexible and robust alternative by leveraging the dynamics of the error-correction process. The combination of both methods enables a more comprehensive evaluation of long-run relationships in heterogeneous panel settings.

The Error correction PMG

Pesaran et al. (1999) propose the PMG and MG estimators for dynamic heterogeneous panels, based on the ARDL framework. The general panel ARDL(p, q) model is specified as:

$$y_{it} = \sum_{j=1}^{p} \lambda_{ij} y_{i,t-j} + \sum_{j=0}^{q} \delta_{ij}' x_{i,t-j} + \mu_i + \epsilon_{it}, \tag{2.29}$$

where y_{it} is the dependent variable, x_{it} is a $k \times 1$ vector of explanatory variables, μ_i denotes group-specific fixed effects, and ϵ_{it} is the error term. If the variables are integrated of order one and cointegrated, the error term is stationary. The model can then be rewritten in an error-correction form as:

$$\Delta y_{it} = \theta_i (y_{i,t-1} - \beta_i x_{i,t-1}) + \sum_{j=1}^{p-1} \lambda_{ij}^{*} \Delta y_{i,t-j} + \sum_{j=0}^{q-1} \delta_{ij}^{*} \Delta x_{i,t-j} + \mu_i + \epsilon_{it}, \tag{2.30}$$

where:

- Adjustment coefficient: $\theta_i = -\left(1 - \sum_{j=1}^{p} \lambda_{ij}\right)$
- Long-run coefficient: $\beta_i = \sum_{j=0}^{q} \delta_{ij} / \left(1 - \lambda_{ik}\right)$
- Short-run dynamic coefficients:

$$\lambda_{ij}^{*} = -\sum_{m=j+1}^{p} \lambda_{im}$$
$$\delta_{ij}^{*} = -\sum_{m=j+1}^{q} \delta_{im}$$

The PMG estimator imposes homogeneity on the long-run parameters, assuming $\beta_i = \beta$ across all units, while allowing the short-run coefficients, adjustment speeds, and error variances to vary. In contrast, the MG estimator estimates the ARDL model separately for each cross-sectional unit and averages the coefficients. The existence of a long-run relationship requires $\theta_i \neq 0$, with $\theta_i < 0$ indicating adjustment toward equilibrium. The significance of θ_i is typically assessed using standard critical values.

To determine whether the PMG or MG estimator is more appropriate, we apply the Hausman specification test. The test evaluates the null hypothesis that the long-run coefficients are homogeneous across groups, implying that the PMG estimator is both consistent and efficient. Under this null, MG remains consistent but is less efficient. Rejection of the null hypothesis indicates that the homogeneity restriction does not hold, rendering PMG inconsistent and favouring the use of the MG estimator instead. Thus, failure to reject supports the PMG specification, while rejection suggests preference for the more flexible MG approach.

6 Data

We construct a balanced panel of major oil-exporting countries covering the period 1981-2017. The sample comprises 15 economies: Brazil, Canada, Colombia, Ecuador, Gabon, Indonesia, Iran, Kuwait, Mexico, Nigeria, Norway, Oman, Saudi Arabia, Trinidad & Tobago, and the United States. Each country is observed annually from 1981 through 2017, resulting in 37 observations per country and a total of 555 observations. These countries were selected due to their substantial roles as net exporters of crude oil. Collectively, the oil exports of our selected countries account for approximately 80% of global oil exports. Given their significant share of global oil trade, oil-market shocks are particularly relevant for understanding the dynamics of their real exchange rates.

Although the United States was not consistently a net oil exporter throughout the sample period, it has long been one of the world's largest oil producers. Over the past decades, the U.S., along with Saudi Arabia and Russia, has accounted for a substantial share of global crude oil output. These three countries together have consistently produced over 40% of the world's oil supply. Given this dominant production role, global oil price fluctuations have considerable macroeconomic implications for the U.S. economy, particularly through their effects on terms of trade and the REER.

Empirical studies support the notion that oil prices are an important determinant of the U.S. REER. For instance, Amano and Van Norden (1998) show that oil price shocks are cointegrated with the U.S. REER and Granger-cause its long-run movements. Similarly, Beckmann et al. (2017) find that higher real oil prices are generally associated with appreciation of the U.S. dollar in real effective terms. Other studies, such as Basher et al. (2012) and Bai and Koong (2018), document that oil price increases tend to weaken the nominal dollar—implying an appreciation of the REER. These findings justify the inclusion of the U.S. in our panel of oil-exporting economies, as oil price dynamics are empirically relevant for understanding U.S. real exchange rate behaviour despite its mixed net export status during the sample period.

Based on the previous discussion, we apply OIL, NFA, and 6SECT as fundamental variables to analyse the long-run relationship with the REER. Accordingly, our baseline model is specified as:

$$REER = f(\mathrm{OIL}_t, \mathrm{NFA}_t, 6\mathrm{SECT}_t) \tag{2.31}$$

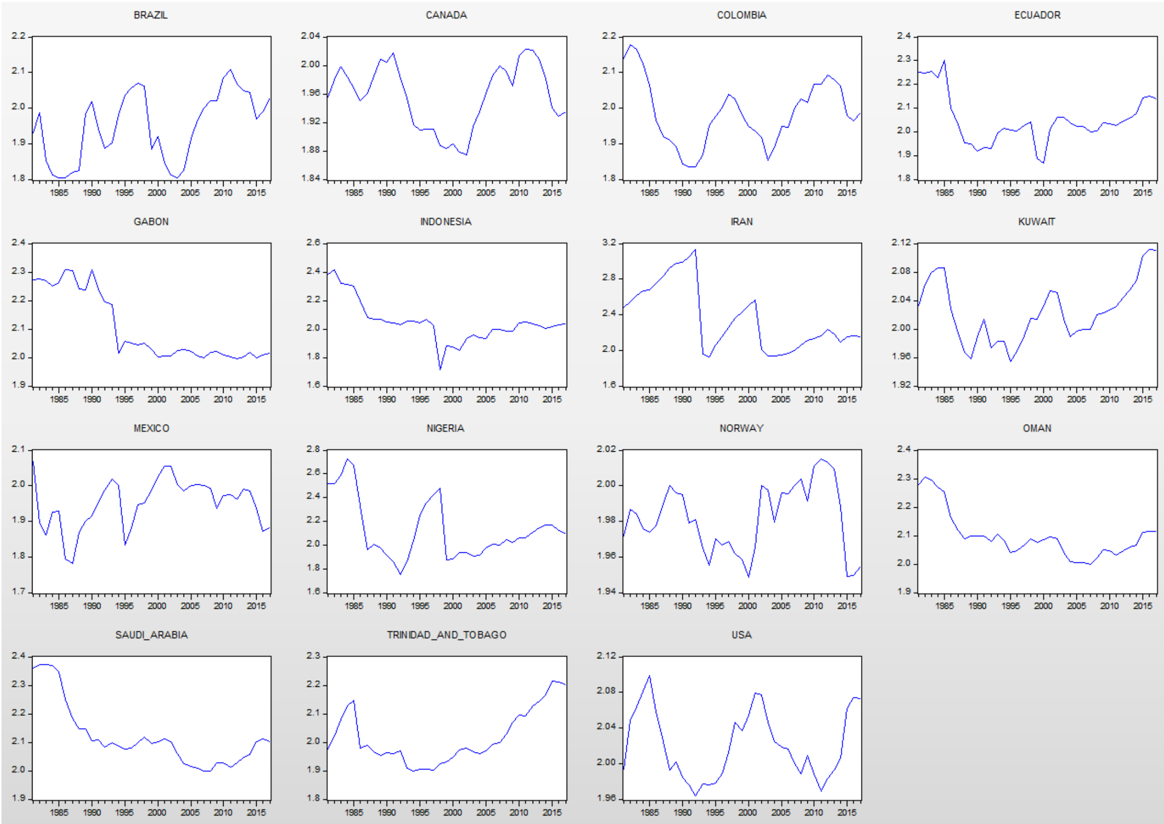
The data sources are as follows: the REER from Darvas (2021), the Brent crude oil price from the Federal Reserve Economic Data (FRED) database, NFA from the International Monetary Fund (IMF) databases (Lane and Milesi-Ferretti, 2018), and the six-sector value-added deflator (6SECT) from the CEPII RPROD dataset (Couharde et al., 2020).

- **Real Effective Exchange Rate (REER):** The dependent variable, REER, is a trade-weighted multilateral real exchange rate index with a base year of 2010 = 100. It measures real exchange rate movements accounting for relative price differences across countries. Figure 2 shows the REERs from 1981 to 2018 as a log scale.

- **Oil Price (Brent):** We use the global benchmark Brent crude oil price (USD per barrel), obtained from the IMF via the FRED database. To account for the effects of inflation, we deflate the nominal Brent price using the U.S. Consumer Price Index (CPI), thereby constructing a real oil price series in constant U.S. dollars. The log of this deflated Brent price is incorporated as an external fundamental affecting all exporters uniformly.

- **Net Foreign Assets (NFA):** Each country's net foreign asset position is expressed as a percentage of GDP, sourced from the IMF International Financial Statistics. Positive

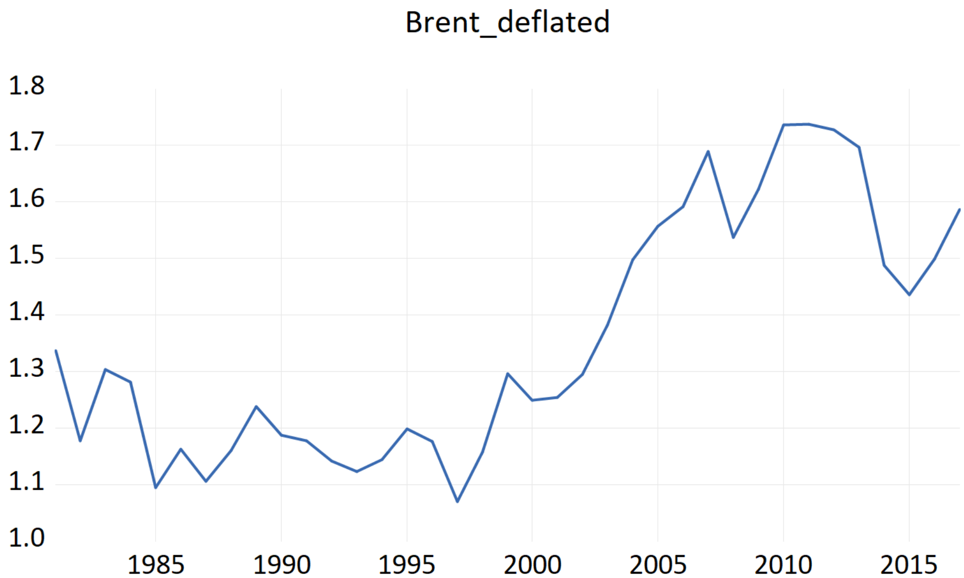
Figure 2: The REERs from 1981 to 2017 in log scale



NFA values indicate net creditor positions. This variable measures external sustainability and wealth.

- **Six-Sector Value-Added Deflator (6SECT):** The 6SECT variable captures the Balassa-Samuelson effect via the relative price changes between non-tradable and tradable sectors. Constructed from the CEPII RPROD database (Couharde et al., 2020), the index aggregates six sector-specific value-added deflators (agriculture, industry, construction, trade, transport, and other services), with a base year of 2010 = 100. A higher value implies relatively higher prices in non-tradable sectors, theoretically leading to real exchange rate appreciation.

Figure 3: The log real Brent oil price from 1981 to 2017

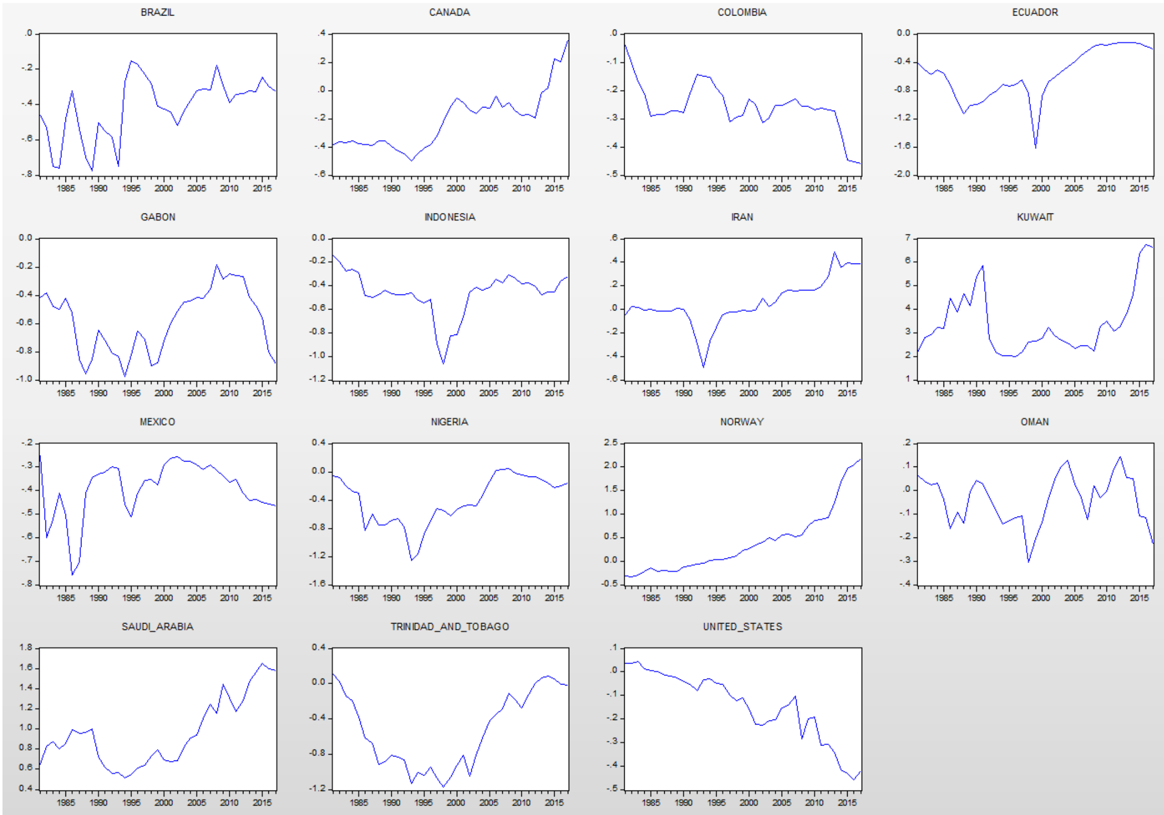


6.1 Unit-root tests

To examine the order of integration of each variable, we apply the ADF unit root test individually to each country-level series, including the REER, NFA, 6SECT, and real crude oil prices. The ADF test evaluates the null hypothesis of a unit root against the alternative of stationarity, with lag length selected using the Bayesian information criterion (BIC). The null hypothesis is that the series contains a unit root, whereas the alternative hypothesis is that this series is stationary.

As shown in Table 2, the majority of the variables fail to reject the null hypothesis at levels under both specifications (with drift and with drift plus trend), indicating the presence of unit roots. Only a few isolated cases—such as NFA in Brazil or 6SECT in Kuwait—show weak evidence against non-stationarity at conventional significance levels. However, when the variables are first differenced, the null hypothesis is strongly rejected at the 1 % level across nearly all countries and specifications. This pattern is consistent across REER, NFA, 6SECT, and real oil prices, supporting the inference that these series are integrated of order one, I(1). These findings align with empirical expectations for macroeconomic and trade-related time series, which are typically non-stationary in levels but become stationary after

Figure 4: The Net Foreign Assets (NFA) from 1981 to 2017 in log scale



differencing.

6.2 Panel unit root tests

Given the relatively short time dimension of our panel (37 years per country), we supplement the time-series unit root analysis with panel-based tests, which provide increased statistical power by pooling information across cross-sectional units. We employ three widely used first-generation panel unit root tests: LLC(Levin et al., 2002), IPS (Im et al., 2003a), and Hadri's LM test (Hadri, 2000). The LLC and IPS tests share the null hypothesis that all panel members contain a unit root, but differ in their treatment of heterogeneity: LLC imposes a common autoregressive parameter, while IPS allows for individual unit root processes. By contrast, Hadri's test assumes the null hypothesis of stationarity.

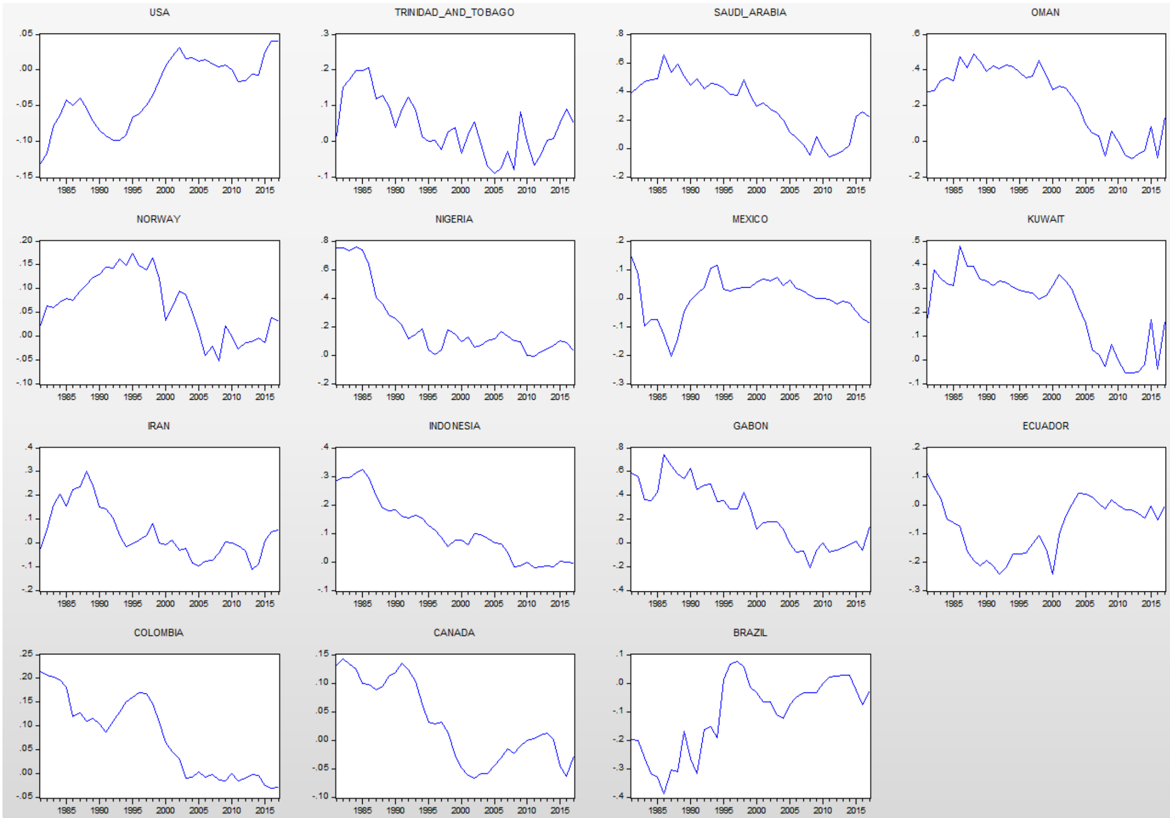
The results, presented in Table 3, confirm the time-series findings. Tables 3 are divided

Table 2: ADF Unit Root Tests (Time Series)

Country	$Z(t)$ with drift			$Z(t)$ with drift and trend		
	Real Effective Exchange rate ($reer$)	Net foreign Assets (nfa)	Six-Sector with Deflator ($6SECT$)	Real Effective Exchange rate ($reer$)	Net foreign Assets (nfa)	Six-Sector with Deflator ($6SECT$)
Brazil	0.171	-2.561	-1.420	-2.356	-3.435**	-2.202
Ecuador	-0.426	-1.579	-1.257	-1.907	-2.764	-2.854
Gabon	-1.188	-1.419	-1.655	-1.648	-1.375	-2.338
Indonesia	-0.982	-2.115	-2.514	-2.062	-1.997	-1.453
Kuwait	0.577	-1.008	-0.954	-1.190	-1.162	-3.627**
Nigeria	-0.715	-1.498	-3.012**	-2.042	-2.376	-1.526
Saudi Arabia	-1.723	-0.326	-0.936	-0.793	-1.261	-2.051
Trinidad & Tobago	0.985	-1.133	-1.862	-1.001	-2.411	-3.030
USA	0.553	-0.444	-2.518	-1.534	-2.885	-1.815
Canada	-0.189	1.049	-1.740	-1.350	-0.826	-1.126
Norway	-0.226	3.621	-0.974	-1.809	0.867	-2.419
Iran	-0.512	-0.824	-1.304	-2.629	-1.942	-2.473
Mexico	-0.624	-2.885**	-2.350	-3.323**	-3.126	-2.335
Oman	-1.160	-2.259	-0.977	-1.010	-2.202	-2.604
Colombia	-0.703	-1.727	-2.349	-1.894	-2.088	-1.575
Brent oil	$Z(t)$ with drift		-0.9971	$Z(t)$ with drift and trend		-2.62
First Difference	$D.reer$	$D.nfa$	$D.6SECT$	$D.reer$	$D.nfa$	$D.6SECT$
Brazil	-5.186***	-5.562***	-6.113***	-5.066***	-5.473***	-5.967***
Ecuador	-5.277***	-6.389***	-5.032***	-5.478***	-6.377***	-5.167***
Gabon	-6.671***	-4.289***	-6.158***	-6.886***	-4.229***	-6.102***
Indonesia	-6.829***	-4.879***	-3.957***	-7.440***	-4.984***	-4.456***
Kuwait	-4.261***	-5.536***	-8.828***	-4.465***	-5.610***	-8.500***
Nigeria	-4.612***	-5.372***	-4.330***	-4.563***	-5.468***	-4.888***
Saudi Arabia	-3.454***	-5.603***	-6.563***	-4.291***	-5.774***	-6.419***
Trinidad & Tobago	-5.212***	-4.616***	-7.244***	-5.457***	-5.340***	-7.084***
USA	-4.556***	-7.634***	-3.546***	-4.530***	-7.631***	-3.649**
Canada	-3.794***	-5.177***	-3.113**	-3.669**	-5.647***	-3.201**
Norway	-5.200***	-3.367**	-6.345***	-5.070***	-4.080***	-6.178***
Iran	-5.752***	-5.065***	-4.769***	-5.592***	-5.123***	-4.603***
Mexico	-6.085***	-6.472***	-4.615***	-5.892***	-6.433***	-4.490***
Oman	-3.847***	-5.556***	-8.287***	-4.680***	-5.479***	-8.195***
Colombia	-3.631***	-4.122***	-4.041***	-3.789**	-4.061***	-4.306***
D.Brent oil	$Z(t)$ with drift		-6.275***	$Z(t)$ with drift and trend		-6.180***

Note: Drift is included in the test shown in the upper-left part. The trend is included in the test shown in the upper-right part. $*, **$ and $***$ denote significance at the 10, 5 and 1 percent levels. The finite sample critical values, when including a mean, are -2.612, -2.946, and -3.632 at the 10, 5, and 1 percent levels and the critical values are -3.211, -3.552, and -4.270 for including a trend.

Figure 5: The Six-Sector Value-Added Deflator (6SECT) from 1981 to 2017 in log scale



into two parts; the upper part is on the level, and the lower part is after the first differencing. At levels, both LLC and IPS fail to reject the null for most variables, indicating non-stationarity, while Hadri strongly rejects the null of stationarity at the 1% level. These results provide consistent evidence that REER, NFA, 6SECT, and real oil prices are non-stationary in levels. After first differencing, however, both LLC and IPS reject the unit root null with high significance, and Hadri no longer rejects stationarity. This convergence across test types and differencing supports the conclusion that the variables are I(1) in panel as well. Taken together, these panel unit root tests validate the use of dynamic panel models and cointegration techniques in subsequent empirical sections, under the assumption that each series becomes stationary after differencing.

Table 3: Panel Unit Root Tests For Oil-Exporting Countries

	LLC	ips	Hadri
REER	-4.48(0.0000)***	-1.5921(0.0557)*	33.9430(0.0000)***
$SECT$	-1.4983(0.0670)*	-0.2406(0.4049)	62.9881(0.0000)***
NFA	2.4980(0.9938)	1.0819(0.8603)	26.7001(0.0000)***
OIL	0.4906(0.6881)	2.2212(0.9868)	63.4059(0.0000)***
D.REER	-9.4207(0.0007)***	-11.0894(0.1009)	-1.7226(0.9575)
$D.SECT$	-9.6449(0.0000)***	-12.0606(0.0000)***	-0.7228(0.7651)
$D.NFA$	-10.7684(0.0000)***	-11.9353(0.0000)***	0.7184(0.2362)
$D.OIL$	-11.3830(0.0000)***	-14.0020(0.000)***	-0.8866(0.8123)

Note: $*, **\ and\ ***$ denote significance at the 10, 5 and 1 percent levels, respectively. Remarks that the Hardi test has an adverse null hypothesis from the other tests.

7 Results

All empirical results are presented in this section. Since the non-stationarity for our explanatory variables and dependent variables was demonstrated in the previous sections, the next step was the cointegration tests. Based on the maximum likelihood approach by Johansen (1988, 1995), and Pesaran et al. (2001), we applied the Johansen and ARDL bounds cointegration tests to check the existence of cointegration among the variables for each country. Time series results are presented first, followed by panel results.

7.1 Cointegration tests

The results in the last section suggested that our explanatory variables and REER are I(1) for most countries. Engle and Granger (1987) proposed a cointegration test based on the hypothesis that although multiple series may not be stationary, there could be a linear combination, which is stationary. We applied the Johansen and ARDL bound tests. Johansen (1991) introduced a cointegration test by applying a full information maximum-likelihood estimator. The advantage of the Johansen test is that it allows for more than one explanatory variable. The interpretation of the Johansen cointegration test is different from the ordinary test. The null hypothesis of the Johansen cointegration test in rank 1 is no cointegration, and the alternative hypothesis of rank 1 is that there is cointegration between the tested series. The null hypothesis of rank 2 has one cointegration from the tested series and so on. The number of ranks is based on the number of variables; the maximum number of ranks for this study was four. As described before, the result of the Johansen test is sensitive to lag selection in limited data. Given this test's sensitivity to lag-length specification in finite samples and uncertainties regarding the integration order of Brent oil price (a globally common regressor), we augment it with the ARDL bounds test.

The ARDL bounds test introduced by Pesaran et al. (2001) is used to determine the existence of a long-run relationship based on the error correction representation. This test can draw conclusive inferences without knowing the integrated order of series because Pesaran et al. (2001) tabulated asymptotic critical values that span a band from all regressors being purely I(0) to all regressors being purely I(1) but must not be I(2). Compared with the Johansen test, the ARDL bounds test allows different variables to carry different lag lengths in the model. The null hypothesis of the ARDL bounds method is that no long-run relationships exist. The critical values of upper and lower bounds are calculated by Pesaran et al. (2001). The lower bound is based on the assumption of purely I(0), and the upper bound is based on purely I(1). If the statistics of the Wald test are higher than the I(1) bound, we can conclude that cointegration exists. In contrast, if the statistics of the Wald test are lower than the I(0) bound, the null hypothesis is accepted, which means there is no cointegration. If the statistics of the Wald test lie between I(0) and I(1), the cointegration results are inconclusive. The country-by-country cointegration results based on the Johansen and ARDL bounds tests are shown in Tables 4 and 5.

In Table 4, if the country's trace statistic number is higher than the 1%,5% or 10% critical value, we can reject the null hypothesis. Johansen trace test results (Table 4) reveal significant cointegration at rank 1 for most petroleum exporters. At the 5% significance level, 12 countries reject the null of no cointegration (rank 0). Specifically, Ecuador, Saudi Arabia,

Table 4: Johansen cointegration test for each country (Trace)

rank	Brazil	Colombia	Ecuador	Gabon	US
1	49.95*	51.42*	60.41*	52.45*	51.55*
2	19.17	20.37	23.43	23.30	22.88
3	8.13	7.05	11.88	10.37	6.56
4	1.97	0.19	1.88	0.33	0.39
rank	Indonesia	Kuwait	Nigeria	Saudi Arabia	Trindad & Tobago
1	50.25*	40.54	47.85*	91.86*	49.18*
2	20.67	10.46	22.87	29.43	19.47
3	7.39	3.58	9.66	12.34	3.25
4	1.33	0.29	1.64	1.18	0.15
rank	Oman	Norway	Mexico	Iran	Canada
1	70.77*	47.66*	81.37*	52.65*	44.34
2	21.18	27.45	29.48	24.51	19.28
3	8.05	11.84	10.34	9.66	6.91
4	2.66	2.62	1.31	1.48	0.93

Note: $*$ denotes significance at the 5 percent level (47.21 at rank 1, 29.68 at rank 2, 15.41 at rank 3, 3.76 at rank 4), respectively. Lags selected by BIC.

Mexico, Oman, Indonesia, Nigeria, Trinidad and Tobago, Norway, Iran, Colombia, Brazil, and the United States all show significant evidence of at least one cointegrating vector. By contrast, Gabon, Kuwait, and Canada do not reject the null hypothesis at rank 1, suggesting the absence of a cointegrating relationship in those economies. Lag lengths were selected using the Bayesian Information Criterion (BIC), although the Johansen test can be sensitive to lag choice in small samples. Thus, the Johansen cointegration test might not be accurate enough in this case. Therefore, the empirical analysis also employs the ARDL approach to investigate the long-run relationship between the REER, 6SECT, real oil price, and NFA. The model is estimated with REER as the dependent variable and includes one lag for each variable based on the Akaike Information Criterion. The corresponding unrestricted error correction form of the ARDL is specified as:

$$\Delta\text{REER}_t = c + \alpha t + \phi\,\text{REER}_{t-1} + \theta_1\,\text{SECT}_{t-1} + \theta_2\,\text{OIL}_{t-1} + \theta_3\,\text{NFA}_{t-1} + \sum_{j=1}^{p}\lambda_j\Delta\text{REER}_{t-j} + \sum_{j=0}^{q_1}\varpi_{1j}\Delta\text{SECT}_{t-j} + \sum_{j=0}^{q_2}\varpi_{2j}\Delta\text{OIL}_{t-j} + \sum_{j=0}^{q_3}\varpi_{3j}\Delta\text{NFA}_{t-j} + \varepsilon_t \tag{2.32}$$

Where, REER_t is the dependent variable; SECT_t, OIL_t, and NFA_t are explanatory variables; ε_t is the residual; c and t denote the drift and deterministic trend, respectively; ϕ and θ_i are long-run coefficients; λ_j and ϖ_{ij} are short-run coefficients; and p, q_1, q_2, and q_3 represent the lag orders of each variable. A long-run relationship is inferred if ϕ (the coefficient on REER_{t-1}) is negative and statistically significant.

As before, we use the Bayesian Information Criterion (BIC) to select the optimal lag structure. Once the model is estimated and the error correction structure is correctly specified, the next step is to test for the existence of a long-run relationship among the variables. This is carried out by conducting a Wald-type F-test on the joint significance of the lagged level terms in the model. The null hypothesis states that no long-run relationship exists (i.e., $\phi = \theta_i = 0$), while the alternative hypothesis suggests the presence of cointegration (i.e., at least one of ϕ or θ_i is nonzero).

In the ARDL framework, the F-statistics used to test for cointegration do not follow a standard distribution. Instead, they are evaluated against critical value bounds developed by Pesaran et al. (2001), which account for uncertainty about the integration order of the regressors. Two sets of critical values are provided: one assuming that all variables are integrated of order zero (I(0)), and the other assuming all are integrated of order one (I(1)). If the computed F-statistic exceeds the upper bound, the null hypothesis of no cointegration is rejected, indicating evidence of a long-run relationship. If it falls below the lower bound, the null cannot be rejected. When the F-statistic lies between the two bounds, the test result is inconclusive.

To complement these results and to allow for variables with potentially mixed orders

Table 5: ARDL Bounds test(Time series)

Countries	Brazil	Canada	Ecuador	Gabon	Trinidad and Tobago
F-stat	5.245**	5.026**	8.370***	7.153***	6.054***
Countries	Kuwait	Indonesia	Nigeria	Saudi Arabia	US
F-stat	4.674**	5.211**	3.282	16.267***	4.847**
Countries	Colombia	Iran	Mexico	Norway	Oman
F-stat	5.357**	5.034**	14.59***	4.704**	4.119*

Note: The lag selection is based on the BIC. $*, **and***$ denote significance at the 10, 5 and 1 percent levels. The upper bound values are 3.77, 4.35, and 5.61 at the 10, 5, and 1 percent levels, respectively.

of integration, we employ the ARDL bounds testing procedure proposed by Pesaran et al. (2001). Table 5 reports country-level F-statistics and their significance levels. The majority of countries reject the null hypothesis of no long-run relationship at the 5% level or better. Notably, Ecuador, Saudi Arabia, Mexico, Gabon, Trinidad and Tobago, and the United States report strong evidence of cointegration at the 1% level. Brazil, Canada, Indonesia, Colombia, Iran, Norway, and Oman also show significant results at the 5% or 10% levels. Only Nigeria failed to reject the null hypothesis under the ARDL framework. Based on the Johansen and ARDL bounds test results, we can conclude that most oil-exporting countries had a long-run relationship between REER and our independent variables: oil, NFA and 6SECT.

7.2 Panel results

The panel analysis was applied to overcome the drawbacks of the limited length of the sample data. The whole process was similar to the time series part. First, we applied the cointegration test to investigate the cointegration relationship between the fundamental variables and the REER. If the cointegration tests passed, then we analysed the long-run coefficient between our factors and REER by the PMG or MG estimator.

Panel cointegration tests and estimated long-run relationships

After confirming the order of integration for our data in Section 6, Kao and Westerlund's panel cointegration approaches were used to test the presence of long-run equilibrium relationships among our variables. Overall, the results of the Kao and Westerlund tests provide evidence of the determination of REER in oil-exporting countries. The result of the cointegration test is shown in Table 6.

Table 6: Kao and Westerlund Panel Cointegration Tests for Oil-Exporting Countries with The Real Effective Exchange Rate (REER) as the Dependent Variable

	Kao Panel Cointegration Tests
Modified DF_t	-5.1604(0.0000)***
DF_t	-3.8884(0.0010)***
ADF_t	-4.4757(0.0000)***
Westerlund test	-1.9686(0.0245)**

Note: The upper part of this table is the result of the Kao test with the cross-section and without the cross-sectional mean. The lower part is the result of the Westerlund test. Lag of 1 selected by BIC. $*, **$ and $***$ denote significance at the 10, 5 and 1 percent levels, respectively.

The null hypothesis of Kao is the non-stationarity of residuals, and its alternative hypothesis is the stationarity of residuals. The stationarity of residuals indicates a cointegrating relationship among the variables. The hypothesis of the Westerlund test is the same as the Kao test. According to Table 6, the Kao test rejected the null hypothesis at the 1% significance level regardless of whether it contained cross-sectional means or not. In addition, the Westerlund test result also denoted the presence of cointegration between variables since it was rejected at the 5% significance level. To conclude, all the cointegration tests' results indicate the existence of cointegration between REER, OIL, NFA and 6SECT.

Based on the strong evidence from the previous tests, the next step was to estimate the long-run coefficients and investigate the relationships between variables for our countries. Pedroni (1999) introduced the PMG and MG to the ARDL model. PMG and MG have different features. PMG allows short-run and long-run coefficients with the intercepts. The PMG estimator is built from an ARDL model in order (p, q), where p and q are the autoregressive orders of the dependent and independent variables, respectively. In contrast, MG estimates each country separately initially and then averages the countries' specific coefficients at the end. Thus, the MG estimator might be more suitable for extensive data than small-sized data (Samargandi et al., 2015). Therefore, the MG estimator may not be appropriate for our data since the MG estimator is sensitive to a low number of cross-sectional data.

The panel model is estimated in an error correction form, normalised on the REER. The estimated coefficients reported in Table 7 correspond to the long-run equilibrium relationship of the form:

$$\begin{aligned}\Delta \mathrm{REER}_{it} = {} & \phi_i \left(\mathrm{REER}_{it-1} - \beta_1 \cdot \mathrm{Oil}_{it-1} - \beta_2 \cdot 6\mathrm{SECT}_{it-1} - \beta_3 \cdot \mathrm{NFA}_{it-1}\right) \\ & + \sum \gamma_j \Delta X_{it-j} + \epsilon_{it}\end{aligned} \tag{2.33}$$

where ϕ_i is the error correction term for country i, capturing the speed of adjustment toward the long-run equilibrium. The signs of the reported long-run coefficients (β_j) should be interpreted as the impact of each variable on the REER, with REER normalised to have a coefficient of one. A positive coefficient implies that an increase in the explanatory variable leads to an increase in the REER (i.e., an appreciation), while a negative coefficient implies a depreciation.

Given that the panel consists exclusively of net oil-exporting countries, it is reasonable to assume a similar structural relationship between REER and its key drivers—real oil prices, NFA, and 6SECT. These economies share comparable macroeconomic exposure to real oil price fluctuations, which systematically affect their trade balances, national income, and exchange rate dynamics. The assumption of a common long-run relationship across countries is consistent with the BEER literature (e.g., Cashin et al. (2004), Ricci et al. (2013)). To test the validity of this assumption, a Hausman test is conducted to compare the PMG and MG estimators. We applied the Hausman test to check our data's homogeneity or heterogeneity. Since the P-value of the Hausman test was 0.1394, which is greater than 0.05, we could not reject the null hypothesis of homogeneity, suggesting that the PMG estimator is statistically more efficient and appropriate in our setting. Therefore, we focus on interpreting the long-run results from the PMG estimator.

The PMG estimates in Table 7 indicate that real oil price, the 6SECT, and NFA are all significant and positively associated with the REER in the long run. The reported long-run coefficients represent adjusted estimates, where positive values unambiguously indicate that an increase in the explanatory variable leads to a rise in REER. In particular, when our new explanatory variable, real oil price, increases by 10%, it results in a 1.22% increase in REER, which is consistent with our predictions. A positive long-run coefficient implies that increases in these variables tend to generate a real appreciation of the domestic currency. For NFA and 6SECT, a 10% increase in either of these variables resulted in 1.85% and 4.56% increases, respectively, in REER.

First, the coefficient on real oil price is positive and highly significant, supporting the notion that higher real oil prices strengthen the real exchange rate of oil-exporting countries. This is consistent with the theoretical mechanism whereby oil windfalls improve the terms of trade, boost export revenues, and increase domestic absorption, leading to REER appreciation. This "petrodollar" effect is well documented in the literature. For instance, Coudert et al. (2011) report oil–REER elasticities between 0.2 and 0.3 for major exporters, while Amano and Van Norden (1998) and Cashin et al. (2004) also find long-run cointegration between real oil prices and the REER in similar contexts.

Second, the coefficient on 6SECT is also positive and significant. Since this variable captures the relative price dynamics between non-tradeables and tradeables, it proxies for the Balassa-Samuelson effect. According to this theory, productivity growth in the tradable sector raises wages in both sectors (under factor mobility), which in turn drives up prices in the non-tradable sector, leading to real appreciation. Our finding supports this: countries

Table 7: Results of Long-Run Relationship

Long-run relationship	PMG		MG	
Explanatory Variables	Coef	P-Value	Coef	P-Value
Six-Sector with Deflator ($6SECT$)	0.4555	0.000	0.6903	0.181
real Oil price (oil)	0.1217	0.001	0.008	0.915
Net Foreign Assets (NFA)	0.1848	0.000	0.1032	0.035
Adjustment terms (ec)	-0.1741	0.000	2.0383	0.000
Hausman Test			chi^2	0.1394

experiencing faster growth in domestic sectoral prices (relative to tradeables abroad) tend to exhibit a stronger REER. Similar interpretations are found in De Gregorio et al. (1994), Berka et al. (2018), and Bénassy-Quéré et al. (2009), who use sectoral deflators to model Balassa–Samuelson-type dynamics.

Third, the long-run NFA coefficient is positive and statistically significant. This suggests that countries with higher net foreign asset positions tend to have more appreciated real exchange rates. The economic rationale is that external wealth reduces the need for future trade surpluses, allowing the REER to appreciate while maintaining external balance. This result is consistent with the intertemporal budget constraint framework of Faruqee (1995), and panel evidence in Ricci et al. (2013) and Lane and Milesi-Ferretti (2018), which all report a robust relationship between NFA and REER.

The adjustment coefficient (error-correction term) in the PMG model is negative and statistically significant, indicating that deviations from the long-run equilibrium are corrected over time. Its magnitude, approximately 17%, implies that about 17% of any disequilibrium between the actual and long-run REER is corrected within a year. This speed of adjustment is broadly comparable to the findings in Dauvin (2014), who report adjustment speeds around 16% for similar samples of oil-exporting countries. Ca'Zorzi et al. (2020) also found a similar adjustment term of 27%.

In sum, our results indicate that the REER in oil-exporting countries is driven by three main fundamentals: Real oil price, BSH effects (as measured by 6SECT), and NFA. These findings align well with both theoretical expectations and prior empirical literature. They highlight the importance of commodity cycles, domestic price structures, and external wealth in shaping the equilibrium path of the real exchange rate. The BEER model we developed is specifically adapted to the context of oil-exporting economies and demonstrates empirical feasibility in capturing the key long-run determinants of REER in these countries.

8 Conclusion

We followed the BEER approach developed by Clark and MacDonald (1999) to investigate the long-run determinants of the REER for 15 oil-exporting countries: Brazil, Canada, Colombia, Ecuador, Gabon, Indonesia, Iran, Kuwait, Mexico, Nigeria, Norway, Oman, Saudi Arabia, Trinidad & Tobago, and Venezuela. We estimated the relationship between REER and key macroeconomic fundamentals—namely, the real oil price, net foreign assets, and a six-sector value-added price deflator —using both time series cointegration techniques (Johansen and ARDL bounds tests) and panel estimators (MG and PMG).

In the time series framework, evidence of a stable cointegrating vector was mixed: the Johansen test showed cointegration in most countries except Gabon and the United States, while the ARDL bounds test confirmed cointegration in all but Gabon and Nigeria. However, due to the relatively short span of data available for many of these countries and concerns over small-sample reliability in time series methods, we primarily focus on the panel mean group estimators. In particular, the Hausman test supports the use of the PMG estimator over MG, suggesting the assumption of homogeneity in long-run coefficients across countries is not rejected.

The PMG results offer robust and economically intuitive findings. All three explanatory variables—real oil price, 6SECT, and NFA—enter the long-run REER equation with positive and statistically significant coefficients. These results are in line with theoretical expectations:

higher oil prices boost export revenues and demand for domestic non-tradables, leading to real appreciation; increases in 6SECT are associated with sector-specific domestic inflation consistent with BSH effects; and higher net foreign assets reflect improved external wealth, allowing for a stronger equilibrium REER. These effects are consistent with prior empirical findings on commodity-exporting countries (e.g., Cashin et al., 2004; Dauvin, 2014; Lane and Milesi-Ferretti, 2018).

Our findings also offer clarification and refinement over previous literature. For instance, Bodart et al. (2012) argue that commodity price effects on REER are only visible when the leading commodity constitutes a sufficiently large share of exports (e.g., more than 20%). By focusing specifically on oil-exporting countries and directly incorporating the international oil price, our framework avoids such ad hoc thresholds and shows a consistently significant long-run oil price effect across the panel. Similarly, our results help explain the mixed findings in Coudert et al. (2011), who report weaker oil price effects possibly due to the broader sample and commodity aggregation. By isolating crude oil exporters and updating the fundamentals, we demonstrate that oil price plays a central and statistically robust role in determining the REER.

In conclusion, we find strong evidence that the REER of oil-exporting countries is driven by global oil price movements, domestic sectoral inflation, and external wealth accumulation. These insights reinforce the importance of commodity cycles, structural inflation pressures, and international financial positions in shaping long-run exchange rate dynamics. Our updated model specification, which replaces the country-level terms of trade with a global oil price indicator and introduces 6SECT as a proxy for internal price pressures, improves upon prior BEER-type models and provides a clearer, more consistent framework for understanding real exchange rate behaviour in oil-exporting economies.

Chapter 3

Forecasting crude oil and oil derivatives price with the crude oil exporters' exchange rates

Abstract

In this paper, we perform in-sample and out-of-sample analyses of global oil futures and three major oil price benchmarks (Brent, WTI, and Dubai) using the exchange rates of Brazil, Canada, Colombia, Indonesia, Mexico, and Norway. This paper reveals that the exchange rates of certain smaller oil-exporting countries, such as Brazil, Colombia, and Mexico, have surprisingly good forecasting power for global oil futures prices and oil prices, both in-sample and out-of-sample. Our analyses are based on the Present value (PV) approach. This approach is largely based on the forward-looking ability of the exchange rates of oil-exporting countries. According to the present value model, the price of crude oil today should reflect the market's expectations of its future price. If oil-exporting countries effectively incorporate information of potential expectations for crude oil prices into their exchange rate, the currencies of oil exporters should reflect any relevant changes in expectations for crude oil prices and oil derivatives. Among our samples, the exchange rates of Brazil, Colombia, and Mexico all demonstrate strong short-horizon forecasting capabilities for spot and future prices of crude oil. We found that the Norwegian exchange rate has strong predictive power for oil spot prices, but not for oil futures prices. We posit that the more significant the role of crude oil in a local economy, the more sensitive the local currency might be to capturing future movements of crude oil prices. This could be why Canada and Indonesia cannot forecast oil futures and oil prices in our case, due to these countries' lower ratio of income from oil exports to GDP compared to our other sample countries. This ratio in Indonesia is observed to be decreasing year by year. We also tried long-horizon forecasting for oil and oil derivatives, but based on the results, it's evident that our long-horizon forecasts may not be reliable.

1 Introduction

In recent years, commodities have emerged as a popular asset for many financial institutions due to the financialization of commodity markets. In the world of commodities, crude oil is a highly sought-after product, and its price holds great significance for the global economy. A number of factors, including macroeconomic conditions, political stability, supply and demand, and currency exchange rates, can impact the price of crude oil. Because crude oil is priced in US dollars when traded even macroeconomic determinants such as U.S. interest rates, U.S. inflation, or global economic growth, these factors can affect oil prices (Alquist and Kilian, 2010). Since commodity prices can provide timely information about economic conditions, an increasing number of scholars have begun to study the use of financial variables to predict commodity prices.

There are many forecasting methods based on different theories in the literature, such as using the GC and PV approaches. The different macroeconomic and financial variables used to forecast commodity prices also vary based on the different types of commodity prices. For example, Chen et al. (2010b), Chen et al. (2010a) and Groen and Pesenti (2011)'s findings support the idea that commodity currencies have some forecasting power for commodity prices. Beckmann et al. (2020) think the interest rate could be a possible variable to forecast commodity prices. Alquist et al. (2013) find that futures prices of commodities could also help forecast spot prices. It is reasonable to posit that if previous studies have successfully employed various data sets and methodologies to predict commodity prices, analogous methods could be applied to the forecasting of oil prices. Among these, the exchange rate has been found to play a significant role in determining the price of crude oil, particularly for crude oil exporting countries, since Campbell and Shiller (1987) and Chen et al. (2010b) demonstrated that although commodity prices have an impact on the exchange rate of the commodity-exporting country, the latter has forecasting ability for the former. In this chapter, we mainly use the exchange rate to forecast oil prices based on the present value approach introduced by Campbell and Shiller (1987) and Chen et al. (2010b). This approach is largely based on the forward-looking ability of the exchange rates of oil-exporting countries.

There is extensive literature on forecasting commodity prices using the present value method with the exchange rates of commodity-exporting countries, such as Chen et al. (2010b) and Chen et al. (2010a). In addition, these recent studies are primarily focused on forecasting metal prices, such as those found in the works of Ciner (2017), Brown and Hardy (2019), and Pincheira and Hardy (2021a). However, the core underlying reason may indeed be the same across different commodities. Specifically, the significance of exporting a single bulk commodity for certain commodity-exporting countries might make it easier to observe the forecastability of those countries' commodity currencies on the price of that particular commodity.

Based on the price of crude oil, for most oil-exporting countries, the annual revenue from oil exports accounts for a relatively large proportion of the GDP for that year. But that doesn't mean all oil exporters can help forecast oil prices, because forecasting crude oil prices is notoriously challenging for several reasons: Complex Market Dynamics, Geopolitical Factors, Economic Conditions for local countries, OPEC Policies, Currency Fluctuations, etc... The most important thing is the past price of oil itself. Crude oil prices are known for their high volatility. Prices can swing dramatically in a short period due to various underlying factors. This high volatility makes it difficult to establish clear patterns that can be relied upon for forecasting (Hamilton, 2009; Kilian, 2009; Alquist et al., 2013). In this chapter, we discuss the choice of exporting country and explore what types of oil-exporting countries' exchange rates can help forecast oil prices. Then, we first prove that the exchange rates of oil-exporting countries can indeed predict the price of oil derivatives and oil by using the in-sample and out-of-sample Granger causality (GC) tests. We then apply the rolling window method to forecast crude oil and calculate the Mean Squared Forecasting Error (MSFE) to evaluate the performance of our model relative to a benchmark. Through this study, we aim to contribute to the ongoing discussion by exploring various samples and methods and providing a fresh perspective on forecasting oil prices. The findings and conclusions drawn could prove instrumental for investors, policymakers, and governments worldwide in their decision-making processes.

The sample countries examined in this chapter are 6 small open crude oil-exporting countries: Brazil, Canada, Colombia, Indonesia, Mexico and Norway. We applied the PV approach introduced by Chen et al. (2010a) to analyse the relationship between the nominal exchange rate (NER) and crude oil price from the 1993s to the 2022s. The structure of this chapter is as follows. In Section 2, we review the Empirical Literature and then present our Methodology in Section 3. We describe the data in Section 4 and report the results in Section 5. Finally, the conclusions are presented in Section 6.

2 Literature review

The considerable surge and volatility in commodity prices since the early 2000s have sparked significant debates on the causes and appropriate policy responses at both national and international levels. These debates are of critical importance given that fluctuations in crude oil prices can profoundly impact stock markets, commodity markets, and currency exchange rates. By accurately forecasting these oil price changes, investors can make more informed decisions about asset allocation, risk management, and portfolio optimisation. Similarly, governments, especially those heavily reliant on oil exports or imports, can use these forecasts to better plan their budgets. Accurate crude oil price forecasts are instrumental in shaping decisions around fiscal policy, energy subsidies, and infrastructure investments. Therefore, a substantial body of literature has emerged, exploring the factors influencing crude oil price dynamics and fluctuations.

Numerous studies have sought to establish a robust relationship in determining crude oil prices. However, it is essential to understand whether crude oil prices can be predicted and, if so, the reasons behind such predictability. Alquist et al. (2013) provided an explanation for the predictability of nominal and real oil prices. The predictability of nominal and real oil prices may differ due to inflation. For instance, if the U.S. Consumer Price Index is predictable, then changes in the real price of oil might also be predictable, as they are influenced by changes in inflation. Alternatively, if the real price of oil is unpredictable but inflation is predictable, one might be able to forecast the nominal price of oil based on the predictable inflation rate. This is because the nominal price of oil is affected by both the real price of oil and inflation. Consequently, various factors - the real price of oil, the nominal price of oil, and a country's local CPI - may be predictable to different extents. In the study by Gillman and Nakov (2009), it is stated that U.S. inflation possesses Granger Causality towards oil prices. Similarly, Funk (2018) utilised an oil inventories model to forecast Brent oil prices, achieving results that were more accurate than the no-change benchmark for prediction horizons up to 24 months. Furthermore, Pincheira-Brown et al. (2022) ascertained that the Chilean exchange rate could predict the returns of not only oil prices but also three additional oil-related products: gasoline, propane, and heating oil. Based on these papers, we may conclude that crude oil might be predictable.

Early research, such as Adams and Marquez (1984), developed a fundamental cartel model for the Organisation of the Petroleum Exporting Countries (OPEC) to elucidate the process of optimal oil price determination. utilising data from 1960 to 1979, they posited that the OPEC cartel establishes oil prices with the objective of maximising the joint revenue of its member nations. However, several factors currently limit OPEC's influence on global oil markets.

Verleger (1987) argued that during the 1960s and 1970s, major oil companies exercised significant control over the flow of oil to the market through their integrated networks, resulting in a relatively stable oil supply and demand, and minimal price fluctuations. Nevertheless, since 1973 and particularly after 1979, the oil market has experienced frequent and unpredictable shifts in demand and supply, leading to increased price volatility. Furthermore, Stevens (1995) and Frankel (2012) highlighted the changes in OPEC's internal dynamics, such as increased collusion following the Arab Oil Embargo of 1973 and the breakdown in the 1980s when members ceased adhering to their agreed quotas. Simultaneously, the emergence of new oil producers, such as Brazil, outside of OPEC has contributed to heightened price volatility.

Technological advancements are also challenging the hegemony of traditional oil producers and OPEC. Ansari (2017) noted that the shale revolution, characterised by the rapid expansion of unconventional oil and gas production in the United States due to advances in hydraulic fracturing and horizontal drilling technologies, has weakened OPEC's capacity to regulate oil prices. In addition to the aforementioned factors, several other elements limit OPEC's influence: (1) Non-OPEC oil producers, such as the United States, Russia, and Canada, are not subject to OPEC quotas, and their production levels can significantly impact global oil prices and supply; (2) Geopolitical factors, including tensions between OPEC member countries or other global political events, can hinder the organisation's ability to maintain a united stance on production decisions; and (3) Market forces, where OPEC's control over oil prices is circumscribed by the interplay of supply and demand in global oil markets. When demand is elevated, OPEC's influence is more pronounced; conversely, when demand is low, OPEC's influence wanes. Consequently, OPEC experiences a reduction in its collective monopoly power even when acting in unison. As such, OPEC does indeed face limitations regarding crude oil exports. Thus, when forecasting oil prices, we should not solely focus on OPEC members. We will discuss the crude oil exporter in Section 2.3.

Given the limitations of OPEC's influence on crude oil prices and the various factors affecting the global oil market, many scholars consider alternative approaches for forecasting oil prices. Among them, the exchange rate of oil-exporting countries is regarded as relatively valuable data and widely used to predict the price of oil in many papers because the exchange rate of a commodity-exporting nation is a forward-looking variable as it encapsulates market participants' expectations concerning the future commodity price movements, risk premiums, interest rate differentials, and the overall informational efficiency of financial markets. As a result, commodity exporters' exchange rates exhibit the potential to forecast commodity prices. For example, Yousefi and Wirjanto (2004) explored the role of exchange rates in crude oil price formation. They found that exchange rate fluctuations have a significant impact on oil prices and suggest that incorporating exchange rates in oil price forecasting models can improve their accuracy. In the multitude of studies examining the effect of exchange rates on commodity prices, one of the most influential works is by Chen et al. (2010b). This paper is recognised for pioneering the PV approach to forecasting commodity prices by commodity currencies. We will mainly describe the concept behind the PV approach in the next part.

2.1 Present-value (PV) approach

Many studies explored this forecasting relationship between the exchange rates of some commodity exporter countries and the spot price of commodities based on the PV approach. The most influential articles might be Chen et al. (2010b)'s paper. They demonstrated that the floating exchange rates of certain small commodity-exporting countries (such as Australia, Canada, New Zealand, South Africa, and Chile) relative to the US dollar could exhibit impressive predictive strength for the worldwide prices of their exported commodities in some instances. In addition, Chen et al. (2010a) found more evidence in agricultural commodities. Some papers also have followed with additional supportive evidence. Despite this evidence, the empirical implications of the present-value model for exchange rate determination applied to commodity currencies are not exempt from controversy. For example, Chan et al. (2011) find no evidence of predictability from commodity-currencies to future contracts of commodities. Lof and Nyberg (2017) found that Non-causal autoregressions fit commodity prices better than causal models. These failures may arise from the stringent assumptions of the PV model. Given that real-world factors are diverse and complex, this model may not be capable of accounting for all of them. The PV approach might not be suitable for all countries. The study of Alquist et al. (2013) concludes that neither short-term interest rates nor trade-weighted exchange rates have significant predictive power for the nominal price of oil in terms of point forecasts.

Engel and West (2005a) and Chen et al. (2010b) suggested that a key implication of the present-value approach is that exchange rates may Granger-cause their own fundamentals. The underlying premise of their research is that exchange rates, as fundamentally forward-looking variables, likely contain information about future commodity price movements and do not directly depend on the variables explaining commodity prices. This implies that historical values of the exchange rate can contribute to the prediction of future values of its fundamentals. Although this notion may seem counter-intuitive, it is typically assumed that changes in fundamentals drive changes in the exchange rate, rather than the inverse. However, this bidirectional relationship is plausible. Particularly for commodity-exporting countries, the exchange rate may harbour information about respective commodity prices, information that is not yet factored into historical commodity prices or local commodity prices (Chen and Rogoff, 2003). For example, Australia is the largest iron ore exporter. If the domestic market players shape their expectations with information about future developments in the iron market, when market participants foresee future commodity price shocks, this expectation will be priced into the current exchange rate through its anticipated impact on future export income and exchange rate values. If the market effectively incorporates the information, Australian dollars will reflect any relevant change in expectations about the future iron price. Hence, commodity exporters should contain extra predictive information about respective commodity prices that cannot be captured by a random walk or AR(1).

Consider the simplest present value model for the log of the exchange rate s_t. It can be expressed in terms of its underlying fundamentals f_t and expectations about its future value as:

$$s_t = (1-\beta)f_t + \beta E_t(s_{t+1}) = (1-\beta)E_t \sum_{j=0}^{\infty} \beta^j f_{t+j} \tag{3.1}$$

where $\beta \in (0,1)$ is a discount factor. E_t is the expectation operator given information at time t. For commodity exporters, fundamentals include terms-of-trade shocks driven by commodity prices (Chen and Rogoff, 2003). Equation (3.1) leads to a pricing relation between the current log exchange rate and its expected future fundamentals [1]. Therefore, this relationship should also be applied between commodity price (as one of the fundamentals) and the exchange rate. Hence, the exchange rate should help forecast the fundamentals, including crude oil prices and its financial derivatives. If the exchange rate shows sensitivity or anticipates projections concerning future economic fundamentals, we could propose that the exchange rate might serve as an effective tool in forecasting these fundamentals. In the research conducted by Chen et al. (2010b), a connection is established between the nominal exchange rate, denoted as s_t, its corresponding fundamentals f_t, and its anticipated future value $E_t s_{t+1}$. This methodology forms a present-value relationship, aligning the nominal exchange rate with the discounted sum of its future expected fundamentals for commodity exporters, as shown in the subsequent equation:

$$s_{t+1} = \gamma \sum_{j=1}^{\infty} \psi^j E_t(f_{t+j}|I_t) \tag{3.2}$$

where ψ and γ are parameters determined by the specific structural model, and E_t is still the expectation given the information I_t. In Chen et al. (2010b) research, the commodity price cp_t is also one of the fundamentals. This equation shows why exchange rates can forecast exogenous world commodity prices because exchange rates directly embody information about future commodity prices. However, it is not only commodity price that drives exchange rate fluctuations shown as follows:

$$\triangle s_{t+1} = \gamma \sum_{j=1}^{\infty} \psi^j \triangle cp_{t+j} + z_{t+1} \tag{3.3}$$

where $\triangle s_{t+1}$ and $\triangle cp_{t+j}$ represent changes in the exchange rate and commodity prices, respectively. We assume that z_t could represent the other exchange rate determinants that are independent of the commodity prices. If the commodity prices, which we observe as the fundamentals, are not the primary determinants of the exchange rate, and instead, unobserved

[1] A more concrete example from Engel and West (2005b) can be found in the Appendix 2

factors are driving exchange rates (shown as variable z_{t+1} in Equation (3.3)), then the exchange rate shouldn't help in forecasting commodity prices. Conversely, if these unobserved fundamentals aren't the main drivers of exchange rates, then fluctuations in exchange rates might aid in forecasting both fundamentals and commodity prices (Engel et al., 2007). This causal relationship should be particularly evident for commodity currencies, as the considerable wealth generated by exporting commodities often accounts for a large part of their GDP (especially for oil-exporting countries). As in Chen et al. (2010b), our results follow directly from the fact that exchange rates are strongly forward-looking and do not directly depend on the variables that explain commodity prices. The dependency arises only through the net present value relationship. In particular, as in Campbell and Shiller (1987), when a variable s_t is the present value of a variable cp_t, either s_t Granger-causes cp_t relative to the bivariate information set consisting of lags of s_t and cp_t, or s_t is an exact distributed lag of current and past values of cp_t. In general, Equation (3.3) implies that exchange rates Granger-cause an infinite series of future commodity prices, and the exact expression in Equation (3.4) follows under special assumptions. For example, from Equation (3.3), assuming that $E_t z_{t+1} = 0$ and that the most significant forecastable information about commodity price changes is concentrated at horizon $t+1$ (i.e., $E_t \triangle cp_{t+j} = 0$ for $j \geq 2$). This relationship suggests that exchange rate movements contain information about expected commodity price changes. Motivated by this insight and following the empirical approach in Chen et al. (2010b), we specify the forecasting equation:

$$E_t \triangle cp_{t+1} = \beta_0 + \beta_1 \triangle s_t + \beta_2 \triangle cp_t \tag{3.4}$$

Therefore, if the exchange rate reacts to news about future commodity prices, it could also potentially be useful in forecasting crude oil. This extension is theoretically justified because oil represents a significant component of the commodity export basket for oil exporters. For example, Ferrarini and Grote (2015) find that the Norwegian krone contains significant predictive information for Brent crude futures. If the exchange rate of Norway can predict

Brent crude futures, it is reasonable to expect that the exchange rates of other oil-exporting countries might also contain predictive power for crude oil prices more generally. So, we replace the commodity price cp_{t+1} in the earlier equations with oil_{t+1}, which represents the inflation of crude oil futures prices or three crude oil prices:

$$E_t \triangle oil_{t+1} = \beta_0 + \beta_1 \triangle s_t + \beta_2 \triangle oil_t \tag{3.5}$$

There are also other mechanisms that define the relationship between the exchange rate of commodity-exporting countries and commodity prices. For instance, the denomination channel, which is tied to expectation effects, plays a role. If oil futures are seen as a reliable hedge against expected dollar depreciation, this could induce volatility in future markets, which could then influence the spot market for oil and affect the exchange rates of commodity-exporting countries. Moreover, higher commodity prices, or the expectation of such price increases, can attract investment into commodity-producing firms or countries (Chen et al., 2010b; Fratzscher et al., 2014). These channels interconnect both nominal exchange rates and oil prices in commodity-exporting countries. Thus, as mentioned before the exchange rate of oil-exporting countries is intricately linked to the oil exchange rate, and should provide some information on crude oil price movements. Previous researches have varied standards in the choice of oil price, forecasting methods, and the selection of oil-exporting countries, and may lead to different results. These three aspects will be delved into in the subsequent sections of this chapter.

2.2 The forecasting relationship between commodity currencies and commodity price

As previously described, not every country's exchange rates necessarily contribute to the forecasting accuracy of commodity prices. Many studies stated that small commodity-exporting nations exhibit impressive predictive power over global commodity prices. For example, using quarterly data, Chen et al. (2010b) demonstrated that the exchange rates of numerous small commodity-exporting countries robustly predict global commodity prices. They underscore the possibility that this predictive power inherent in commodity currencies could stem from the forward-looking nature of commodity-exporting countries' currencies, which anticipate fluctuations in corresponding commodity prices. Nonetheless, our research trajectory diverges from Chen et al. (2010b) in several aspects. They concentrate on global and country-specific commodity price indices. Consequently, when it comes to a specific commodity price, their predictive relationship might not hold as firmly when considering individual commodity prices.

Recently, some scholars have begun to utilise the present value method to explore the forecasting relationship between commodity-exporting countries with individual commodity prices. For example, Ciner (2017) provided evidence of a predictive relationship between the South African Rand and the price of white metals. Similarly, Brown and Hardy (2019) showed strong results when predicting base metal returns with either the Chilean exchange rate or survey-based expectations of the Chilean currency. Pincheira and Hardy (2021a) found that the exchange rates of some commodity exporters can predict the price of spot and future contracts of aluminium.

Most of the above studies use the exchange rates of metal-exporting countries to predict the price of a particular metal. Moreover, the economy of the sample countries in the above article has a strong dependence on exporting metals. According to the intuition of our research, this predictive relationship should also be used to predict oil prices. Numerous studies suggest that commodity currencies could serve as suitable variables to forecast crude oil prices. For instance, Alquist et al. (2013) utilised the currencies of Australia, Canada, New Zealand, and South Africa for forecasting crude oil prices. Another example, Belasen and Demirer (2019) also found that the Brazilian real has significant implications for Brent oil. After reviewing the existing literature, it becomes apparent that the present value method suggests a potential predictive relationship between the exchange rates of numerous commodity-exporting countries and their respective commodities. Intriguingly, research by Pincheira-Brown et al. (2022) indicates that even in a country like Chile, which does not export many fossil fuels, an enhancement in the precision of global crude oil price forecasts is possible. They posit that this phenomenon can be attributed to the connection between Chile's metal commodity exports and fuel prices, thereby enabling Chile's exchange rate to bolster the accuracy of crude oil forecasts. However, this paper contends that not every country's exchange rate can contribute to improving the accuracy of crude oil price forecasts. As Pincheira and Hardy (2021b) pointed out, the forecast relationship between commodity currencies and commodity prices should not be assumed as a given. The importance of the export commodity to the country may also be an important factor. Therefore, this study primarily employs the exchange rates of oil-exporting countries to forecast oil prices, involving countries that may have either a high or low dependency on exporting crude oil.

2.3 Oil exporters and oil price selection

While forecasting crude oil prices is a complex task involving numerous variables, currencies of oil-exporting countries can provide useful predictive signals. However, there are many factors that can affect the forecast of oil prices.

Oil price selection

There are three widely used criteria for oil prices: West Texas Intermediate (WTI), Brent, and Dubai oil prices. The choice between these three criteria in forecasting oil prices is not absolute and often depends on their geographical significance and the type of oil they represent. For instance, Baumeister et al. (2014) pointed out that the WTI price was subject to U.S. government regulation until the early 1980s, making it less suitable for a VAR analysis of the global market for crude oil. In general, the choice of WTI, Brent, and Dubai oil prices in forecasting oil prices is often based on their geographical significance and the type of oil they represent. WTI is a grade of crude oil produced in Texas and is widely used as a benchmark for oil prices in the United States. Brent oil is produced in the North Sea and serves as a benchmark for oil prices in Europe, Africa, and the Middle East. Dubai oil is a light sweet crude oil produced in the Middle East and is used as a benchmark for oil prices in Asia.

Each of these benchmark oil prices reflects the specific characteristics of the oil produced in its respective regions and therefore provides a unique insight into global oil markets, but the problem of the large volatility of individual prices may be more serious. Baumeister and Kilian (2016) pointed out that price expectations play a key role in a wide range of forward-looking economic models. Futures markets are one of the main sources of information about price expectations. Beckmann et al. (2020) suggested that, as an alternative to forecasting current or spot crude oil prices, one could focus on the dynamics of futures prices, given that they also encapsulate market expectations. If international crude oil prices can be forecasted using oil-exporting countries' exchange rates or domestic oil market expectations, then future contracts should also be predictable through the exchange rates of crude oil-exporting countries. There are three main advantages by using crude oil futures prices. Firstly, this is particularly true for commodity currencies that are closely tied to the price of a specific export, like oil. Thus, changes in these currencies can provide early signals about anticipated shifts in crude oil futures. Secondly, currency values reflect a wide array of economic data and expectations, including inflation rates, interest rates, political stability, and overall economic performance. As such, they may capture information relevant to the expectation of investors in oil futures that isn't directly reflected in the spot price. Last but not least, spot prices can be highly volatile and influenced by short-term fluctuations in supply and demand, which might not reflect longer-term trends. Currency values, while also volatile, are influenced by a broader set of factors and might offer a less noisy signal for forecasting purposes.

However, this does not imply that using futures prices is necessarily superior to using oil prices. Futures contracts, particularly those with longer maturities, can sometimes be less liquid than the spot market. This lack of liquidity can result in wider bid-ask spreads and larger price swings in response to trading activities, which may impact the precision of a forecasting model based on exchange rates. Exchange rates mirror the current and short-term expectations about an economy, while futures prices embody predictions about conditions at a specified future date. If we utilise exchange rate forecasts from the previous period, the results may not be accurate. This discrepancy could lead to a mismatch in the data's time horizons, making it more difficult to establish robust and dependable relationships for forecasting purposes. Thus, we'll use both prices in this chapter.

Pegging exchange rate regime

The floating and pegging exchange rate regimes are two fundamental systems that govern how a currency's value is determined. For many crude oil exporters, it is common to peg their currencies to the U.S. dollar. Since oil revenues are in U.S. dollars, a peg to the dollar helps stabilise those revenues in terms of the local currency. This practice can make budget planning more predictable and manageable, particularly in countries where oil exports constitute a significant portion of government revenue, such as Saudi Arabia, the United Arab Emirates (UAE), Oman, Venezuela, Qatar, etc... However, a pegged exchange rate can also be problematic. For instance, if oil prices fall, a fixed exchange rate does not allow for domestic depreciation to absorb negative demand shocks. It can also be costly due to the need for constant interventions to maintain the peg, resulting in a loss of monetary policy independence (Husain et al., 2015).

The drawback to using pegging exchange rate countries for forecasting crude oil prices is that maintaining a pegged exchange rate can lead to a situation where the fixed rate does not align with underlying economic realities. If the pegged rate is not in line with what the exchange rate would be in a freely floating system, it may not reflect the true economic relationship between the country's currency and the global oil market. Pegged exchange rates also make it difficult to identify specific shocks affecting the oil market, as the exchange rate does not fluctuate in response to these shocks.

Applying a pegged exchange rate regime will increase dependence on foreign currency reserves since maintaining a peg requires substantial foreign currency reserves. Sudden changes in the oil market could lead to pressure on these reserves, leading to potential instability that further clouds the forecasting picture. As Beckmann et al. (2020) mentioned, the case of Nigeria, where an unsuccessful attempt to defend a pegged exchange rate resulted in substantial exchange rate fluctuations, serves as an example of the potential pitfalls of this approach.

Lastly, but perhaps most importantly, are the issues of lack of responsiveness to market signals and limited information content. In a pegged exchange rate regime, the currency's value is fixed relative to another currency, often the U.S. dollar. This means that changes in the global oil market may not be reflected in the exchange rate, as the rate is maintained at a predetermined level. The pegged exchange rate might not provide useful information for forecasting oil prices, as it's not reacting to changes in the oil market itself. Floating exchange rates, on the other hand, respond to market dynamics and may offer a more accurate reflection of the factors influencing oil prices, making them potentially more useful in forecasting. Thus, in our sample countries, we may not involve the pegged exchange rate countries, although those countries might export significant amounts of crude oil compared to other nations[2].

Crude oil exporter selection

The choice of crude oil-exporting countries is a crucial aspect of this study, as not all oil-exporting countries are equally suitable for analysing the forecasting relationship between exchange rates and crude oil prices. Several practical and theoretical considerations have guided the selection of the six countries included in this chapter: Brazil, Canada, Colombia, Indonesia, Mexico, and Norway.

First, this study focuses on small to medium-sized open economies rather than large, systemically important oil producers. The reason is twofold. On the one hand, these na-

[2]The five countries with pegged exchange rates that we mentioned Saudi Arabia, UAE, Oman, Venezuela, and Qatar, account for approximately 50% of total crude oil exports

tions exemplify small to medium-sized open economies where external shocks—particularly oil price fluctuations—transmit rapidly to domestic variables like exchange rates due to high trade openness. Their limited market size prevents individual actions from influencing global oil prices, ensuring exchange rate movements primarily reflect exogenous shocks rather than endogenous policy interventions. This characteristic is essential for isolating the predictive relationship between exchange rates and oil prices (Chen et al., 2010b; Kohlscheen et al., 2017). On the other hand, countries with relatively open capital accounts and fewer capital controls provide exchange rates that better reflect market expectations. For example, countries like Iran or Russia, despite being major oil exporters, are unsuitable for this type of analysis due to extensive capital controls, limited financial market openness, and geopolitical tensions that distort exchange rate dynamics.

The second point is the Policy Transparency & Geopolitical Stability. Countries with relatively high institutional quality, transparent policymaking, and stable political environments are preferred. Countries experiencing institutional opacity or active conflicts introduce more structural breaks that contaminate forecasting models through unpredictable interventions (e.g., capital controls, sanctions, supply disruptions). For instance, countries such as Venezuela or Libya, despite their significant oil export volumes, have experienced periods of extreme political and economic turmoil, rendering their exchange rate movements unreliable indicators of market expectations regarding oil prices. In contrast, the selected economies have avoided major conflicts during the sample period (1993–2022), minimising noise in the exchange rate–oil price transmission channel (Hamilton, 2009; Ansari, 2017).

Third, this study deliberately excludes OPEC members with pegged exchange rate regimes, as discussed in Section 2.3. Pegged exchange rates limit exchange rate fluctuations, suppress market-driven price signals, and thus diminish the ability of exchange rates to contain forward-looking information relevant for forecasting oil prices. Countries such as Saudi Arabia, the United Arab Emirates, and Qatar fall into this category. Besides, OPEC members face production quotas that distort the market-driven link between exchange rates and oil prices. Brazil, Canada, Colombia, Mexico, and Norway are non-OPEC producers whose supply decisions respond competitively to price signals. Indonesia, though historically OPEC-affiliated, suspended active membership during critical sample years (2009–2015) and operates as a price-taker, aligning with non-cartel dynamics (Frankel, 2012; Baumeister and Kilian, 2016). In addition, as pegged currencies (e.g., Saudi Riyal, UAE Dirham) artificially suppress exchange rate volatility, they fail to embed market expectations about future oil prices. All six countries float their currencies, allowing nominal exchange rates to freely reflect forward-looking fundamentals—a prerequisite for the present-value approach (Husain et al., 2015; Beckmann et al., 2020).

The fourth point is the data availability and quality. Consistent, long-term, high-frequency data on exchange rates, crude oil prices, and other relevant macroeconomic variables is essential for implementing the forecasting models in this chapter. To ensure the comparability of panel estimations, countries with poor data coverage or inconsistent economic statistics are excluded.

Finally, the selected countries—Brazil, Canada, Colombia, Indonesia, Mexico, and Norway—represent a diverse set of oil exporters with varying degrees of reliance on oil production and exports. While some of these countries, such as Colombia and Brazil, have economies that are highly sensitive to oil price fluctuations due to the significant contribution of oil exports to GDP, others like Canada and Norway, although major oil exporters, have more diversified economies. This heterogeneity allows for a more nuanced investigation into whether the strength of the exchange rate–oil price forecasting relationship depends on a country's reliance on oil exports.

3 Methodology

3.1 Benchmark model

Rossi (2013) pointed out that the forecasting performance heavily depends on the choice of benchmark. Many papers want to discover the best benchmark. The high autocorrelation of commodity prices, in general, has been widely recognised (Deaton and Laroque, 1996; Brown and Hardy, 2019; Pincheira and Hardy, 2021a). As Alquist and Kilian (2010) said, the best forecast of the future spot price of crude oil is simply the current spot price. The Autoregressive Model (AR(1)) and Random walk (RW) are the majority benchmarks in many forecasting studies, such as exchange rate and commodity price. Nowadays, some recent studies also employ an alternative benchmark, the random walk with drift (RWWD) (Liu et al., 2020). Engel and West (2005b), Baumeister and Kilian (2012) and Alquist et al. (2013) stated that AR(1) might be more accurate than the RW for horizons up to 1 year. Based on a wide literature review, simple autoregressions are typically difficult benchmarks to beat when forecasting commodity futures prices. Therefore, in this chapter, we will apply AR(1) as the benchmark model just like Brown and Hardy (2019) and Pincheira and Hardy (2021a). The AR(1) is shown as follows:

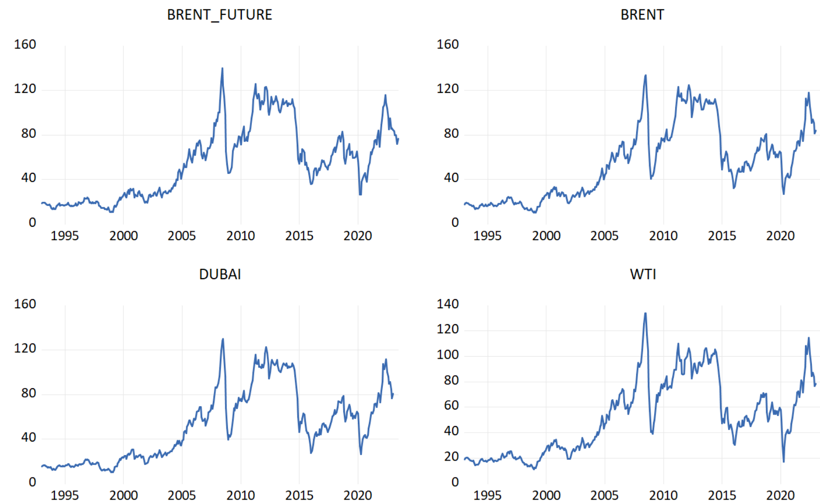
$$E_t \triangle oil_{t+1} = \beta_0 + \beta_1 \triangle oil_t \tag{3.6}$$

where β_1 is the coefficient for crude oil prices or future prices, and β_0 is a constant.

3.2 Granger Causality Test

We used Granger causality tests to explore predictive relationships between variables. This method assesses whether past values of one variable (X) significantly improve forecasts of another variable (Y). If including X's history yields better predictions of Y than using only Y's own history, we conclude "X Granger-causes Y". Statistical significance was determined through standard hypothesis testing. It should be emphasised that the validity of the PV approach relies on the presence of Granger Causality.

Figure 6: Three oil prices and Brent oil future price during the sample period



Rossi (2005) Robust Granger-Causality Tests

However, not accounting for potential parameter instabilities is the limitation of traditional Granger causality tests. In many real-world situations, particularly in economics and finance, this may not be a valid assumption. The relationship between variables can change over time due to various factors like changes in policy, technology, market conditions, and so on. This is referred to as parameter instability. While we will employ the ADF unit root test to evaluate the stationarity of the time series before forecasting, it's important to note that even for a stationary time series, the absence of structural breaks cannot be fully guaranteed.

Over the past few decades, crude oil prices have experienced significant fluctuations. These variations reflect many factors, including shifts in supply and demand dynamics, geopolitical events, technological advancements, and market speculations among others. Three oil and Brent oil futures prices on the level are shown in Figure 6, the first differenced prices in Figure 7. According to both figures, Brent crude oil futures have exhibited significant fluctuations. For example, there were two large spikes around 2008 and 2020, corresponding to the financial crisis and the COVID-19 pandemic. Chen et al. (2010b) also mentioned the important effects of a structural break in forecasting commodity price, so they use Rossi (2005)'s Robust $Exp - W$ Test, which tests for the null hypotheses of no time variation and no Granger causality.

Let the model be $y_t = x'_{t-1}\beta_t + \epsilon_t,\ t = 1, ...T$, where x_{t-1} is a $p \times 1$ vector of explanatory variables. Traditional GC regressions assume that the parameter $\beta_t = \beta$; that is, β is constant. Among the various forms of instability considered, the $Exp-W$ test focuses on the case in which β_t may break or shift at some unknown point in time. This test assumes that the shift occurs at a particular point in time, τ: $\beta_t = \beta_1 \times 1(t \leq \tau) + \beta_2 \times 1(t > \tau)$. Let $\hat{\beta}_{1\tau}$ and $\hat{\beta}_{2\tau}$ denote the OLS estimators before and after the time of the shift, and shown as follows:

$$\begin{aligned} \hat{\beta}_{1\tau} &= (\frac{1}{\tau}\sum_{t=1}^{\tau-1} x_{t-1}x'_{t-1})^{-1}(\frac{1}{\tau}\sum_{t=1}^{\tau-1}(x_{t-1}y_t)) \\ \hat{\beta}_{2\tau} &= (\frac{1}{T-\tau}\sum_{t=\tau}^{T-1} x_{t-1}x'_{t-1})^{-1}(\frac{1}{T-\tau}\sum_{t=\tau}^{T-1}(x_{t-1}y_t)) \end{aligned} \tag{3.7}$$

The robust test builds on two components: $(\tau/T)\hat{\beta}_{1\tau} + (1-\tau/T)\hat{\beta}_{2\tau}$ and $\hat{\beta}_{1\tau} - \hat{\beta}_{2\tau}$. The first component is the full-sample estimate of the parameter, $(\tau/T)\hat{\beta}_{1\tau} + (1-\tau/T)\hat{\beta}_{2\tau} = \hat{\beta}$, where $\hat{\beta}$ is the full sample OLS estimator. A test of whether the first component is zero can detect situations where the parameter is constant but differs from zero. However, if the regressor Granger-causes the dependent variable in such a way that the parameter changes but the average of the estimates equals zero, then the first component will not be able to detect such situations. The second component is introduced to perform that task. It is the difference of the parameters estimated in the two subsamples; a test on whether this component is zero is able to detect situations in which the parameter changes at time τ[3]. The test statistic is the following:

[3]The $Exp-W$ test statistic integrates information over all possible break dates τ within a predefined search interval $[\tau_{min}, \tau_{max}]$, following the approach in Chen et al. (2010b). We set $[\tau_{min}, \tau_{max}] = [15\%, 85\%]$ trimming range follows Andrews (1993) to ensures sufficient observations for reliable subsample estimation

$$
\begin{aligned}
\text{Exp - W Test statistic} &= \frac{1}{T}\sum_{\tau=0.15T}^{0.85T}\frac{1}{0.7}\exp\left(\frac{1}{2}\right)\times \\
&\left((\hat{\beta_{1\tau}}-\hat{\beta}_{2\tau})'\left(\frac{\tau}{T}\hat{\beta}_{1\tau}+(1-\frac{\tau}{T})\hat{\beta}_{2\tau}\right)\right)\hat{V}^{-1} \\
&\times\begin{pmatrix}(\hat{\beta}_{1\tau}-\hat{\beta}_{2\tau}) \\ \\ (\frac{\tau}{T}\hat{\beta}_{1\tau}+(1-\frac{\tau}{T})\hat{\beta}_{2\tau})\end{pmatrix}
\end{aligned}
\tag{3.8}
$$

where $\hat{V}$ is a consistent estimate of the covariance of $\hat{\beta}$ equals to $\begin{pmatrix}\frac{\tau}{T}A & 0 \\ \\ 0 & \frac{T-\tau}{T}B\end{pmatrix}$, with $A = S'_{xx}\hat{S}_1^{-1}S_{xx}$ and $B = S'_{xx}\hat{S}_2^{-1}S_{xx}$. S_{xx} symbolises the sample covariance of the lagged independent variables throughout the time series: $S_{xx} = \frac{1}{T-1}\sum_{t=1}^{T-1}x_{t-1}x'_{t-1}$.

$$
\hat{S}_1 = \left(\frac{1}{\tau}\sum_{t=2}^{\tau}x_{t-1}\hat{\epsilon}_t\hat{\epsilon}_t x'_{t-1}\right) + \sum_{j=2}^{\tau-1}\left(1-\left|\frac{j}{\tau^{1/3}}\right|\right)\times\left(\frac{1}{\tau}\sum_{t=j+1}^{\tau}x_{t-1}\hat{\epsilon}_t\hat{\epsilon}_{t-j}x'_{t-j-1}\right) \tag{3.9}
$$

$$
\begin{aligned}
\hat{S}_2 = &\left(\frac{1}{T-\tau}\sum_{t=\tau+1}^{T-\tau}x_{t-1}\hat{\epsilon}_t\hat{\epsilon}_t x'_{t-1}\right) \\
&+\sum_{j=\tau+1}^{T-\tau}\left(1-\left|\frac{j}{(T-\tau)^{\frac{1}{3}}}\right|\right)\times\left(\frac{1}{T-\tau}\sum_{t=j+1}^{T-\tau}x_{t-1}\hat{\epsilon}_t\hat{\epsilon}_{t-j}x'_{t-j-1}\right)
\end{aligned}
\tag{3.10}
$$

In other words, with respect to the null hypothesis of no Granger causality at any point in time, as follows $H_0 : \beta_t = \beta = 0$. The Rossi (2005)'s robust test has a distribution whose critical values are tabulated in Rossi (2005)'s paper Table B1.

By considering time-varying parameters, the test becomes more flexible and can potentially capture these changes, providing more accurate and realistic forecasts. This is particularly important in areas such as commodities forecasting where prices can be heavily influenced by a range of unpredictable factors, from geopolitical events to sudden shifts in supply and demand. This test has a distribution whose critical values are tabulated in Rossi (2005)'s paper.

3.3 Forecasting performance evaluation

MSFE

The Mean Squared Forecasting Error (MSFE) is a key metric used in forecast evaluation. The MSFE is calculated as the average of the squared differences between the predicted and the actual observed values. Mathematically, if we have a series of actual values denoted by $y_1, y_2, ..., y_T$ and corresponding forecasted values denoted by $\hat{y}_1, \hat{y}_2, ..., \hat{y}_T$, the MSFE can be calculated as follows:

$$MSFE = \frac{1}{T}\sum_{T=1}^{T}(y_t - \hat{y}_t)^2 \tag{3.11}$$

where T is the total number of observations. The main purpose of the MSFE is to quantify the difference between forecasted and observed values, providing a single measure of the overall accuracy of the forecasting model. A smaller MSFE indicates a better fit of the model to the data, while a larger MSFE suggests a poor fit. While the MSFE is useful, it should not be the only metric to rely on when evaluating forecast accuracy, especially because it gives more weight to large errors due to squaring. Therefore, we will also apply the other two tests in forecasting evaluation.

DM test

We employ the Diebold-Mariano (DM) test to statistically compare the forecast accuracy of competing models. This standard procedure evaluates whether two models exhibit significantly different predictive performance (using MSFE as the accuracy metric). The test formally assesses the null hypothesis that the models have equal forecasting ability against the alternative that one model outperforms the other. Where the DM test rejects the null, we conclude that one model provides significantly more accurate forecasts than its competitor. All comparisons are made against the AR(1) benchmark model.

ENCNEW test

Many studies, such as those by Chen et al. (2010b) and Pincheira Brown and Hardy (2019), have utilised the ENCNEW test to compare the forecasting performance of nested forecasting equations. Researchers favour the ENCNEW test for its potential to mitigate finite sample bias. Introduced by Clark and McCracken (2001), the ENCNEW test specifically addresses challenges associated with nested models. It is a modified version of the ENC-T, initially proposed by Harvey et al. (1998), who introduce an encompassing test that employs a t-statistic for the covariance between $\hat{u}_{1,t+1}$ and $\hat{u}_{1,t+1} - \hat{u}_{2,t+1}$. Define $c_{t+1} = \hat{u}_{1,t+1}(\hat{u}_{1,t+1} - \hat{u}_{2,t+1}) = \hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1}$ and $\bar{c} = P^{-1}\sum_t c_{t+1}$. Their encompassing test, termed ENC-T, is expressed as follows:

$$\text{ENC-T} = (P-1)^{1/2}\frac{\bar{c}}{\sqrt{P^{-1}\sum_t (c_{t+1}-\bar{c})^2}} \tag{3.12}$$

$$= (P-1)^{1/2}\frac{P^{-1}\sum_t(\hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1})}{\sqrt{P^{-1}\sum_t\left(\hat{u}^2_{1,t+1} - \hat{u}_{1,t+1}\hat{u}_{2,t+1}\right)^2 - \bar{c}^2}} \tag{3.13}$$

The factor in front is $(P-1)^{1/2}$ instead of $P^{1/2}$ because the test is computed using standard regression techniques (regressing c_{t+1} on a constant). If model 1's forecast encompasses model 2, the covariance between $u_{1,t}$ and $u_{1,t} - u_{2,t}$ will be zero or negative. Conversely, if model 2 adds information, the covariance should be positive. Thus, the ENC-T test, along with the other encompassing tests discussed later, is one-sided. Clark and McCracken (2001) demonstrated that simpler models often have an inherent advantage over more complex ones. This advantage arises because simpler models impose certain parameters instead of estimating them from the data, reducing variability from parameter estimation. Consequently, forecasts from simpler models may be more stable, though not necessarily more accurate. In contrast, larger models, which estimate more parameters, introduce additional information but also potential estimation errors.

Furthermore, Clark and McCracken (2001) noted that despite theoretical equivalences, the larger model's sample mean squared forecast error often exceeds that of the smaller model. Without proper adjustments, researchers might inadvertently favour the smaller model, leading to a potential bias where the larger model is frequently and erroneously set aside. This highlights the pitfalls of the standard DM test in certain scenarios. Specifically, the standard DM test statistic might not adhere to a standard normal distribution when: (1) The loss differential series lacks covariance stationarity, which can emerge when the compared models are misspecified or when the optimal forecast evolves over time. (2) The models under comparison are nested. In such contexts, the errors from the smaller (nested) model are essentially linear combinations of the errors from the larger (unrestricted) model. This relationship implies that the loss differential series becomes contingent on the models' estimated parameters, leading to potential non-standard distributions even in large samples. To remedy these complications, Clark and McCracken (2001) and Clark and West (2006) proposed the ENCNEW test in recursive and rolling window forecasts. It serves as a modification of the DM test statistic, taking into account parameter dependence and ensuring the correct asymptotic size. Suppose we are doing the one-step-ahead forecasting. The total sample $T+1$ is divided into in-sample and out-of-sample portions: the in-sample observations span 1 to R and P denotes the number of 1-step ahead predictions. Then the total number of observations in the sample is $R+P=T+1$. The first forecast is for observation $R+1$, the final for $T+1$. Consider a simple model to forecast the y_{t+1} by using two linear models in the form $x'_{i,t+1}\beta^*_{i,t}, i=1,2$. Then, we can denote the forecasting error for Model 1 and Model 2 as $\hat{u}_{1,t+1}=y_{t+1}-x'_{1,t+1}\hat{\beta^*_{1,t}}$ and $\hat{u}_{2,t+1}=y_{t+1}-x'_{2,t+1}\hat{\beta^*_{2,t}}$, where $\hat{\beta^*_{i,t}}, i=1,2$ is the parameter estimate. In the recursive scheme, the sample size used to estimate β grows as one makes predictions for successive observations. One first estimates β_1 and β_2 with data from 1 to R and uses the estimate to predict; one next estimates β'_1 and β'_2 with data from 1 to $R+1$, with the new estimate used to predict, and so on. In the rolling scheme, the sequence of regression estimates is always generated from a fixed sample of size R. The first estimates of β_1 and β_2 are obtained with a sample running from 1 to R, the next with a sample running from 2 to $R+1$, ..., and so on. The ENC-NEW tests are formed using these two sequences of P forecast errors. Clark and McCracken (2001) proposed the ENC-NEW test by using an estimate of the variance of $\bar{c}$ [4], it captures the difference in squared forecast errors of the smaller model and the product of forecast errors of both models. By leveraging the variance of the forecast errors of the larger model as the scaling factor, ENCNEW aims to offer more consistent evaluations, especially pertinent when dealing with smaller sample sizes. It essentially provides a measure of how differently the two models forecast. The formula of ENCNEW is provided as follows:

$$ENCNEW = P\frac{\bar{c}}{P^{-1}\sum_t \hat{u}_{2,t+1}^2} \tag{3.14}$$

If the value of $ENCNEW$ is significantly less than zero, it provides evidence against the null hypothesis that the benchmark model cannot be improved upon, suggesting the new model provides better forecasts. Conversely, if the ENCNEW is not significantly less than zero, there is no evidence to suggest that the new model's forecasts are superior to those of the benchmark model. ENCNEW does not follow the normal distribution, the critical values are provided in Clark and McCracken (2001). In our out-of-sample evaluation, we test H_0 using the ENCNEW test proposed by Clark and McCracken (2001). In this chapter, we employ the rolling window method.

The primary application behind ENC-NEW is its suitability for one-step-ahead forecasting. However, the model in this chapter will also deal with multi-step-ahead forecasting. Therefore, the GW test is employed, as it serves as a potential long-horizon test method. The GW test (Giacomini and White, 2006) and the DM test both compare the forecasting performance of two competing forecasts. The GW test builds on the result of the DM test.

[4] Because the population prediction errors from models 1 and 2 are exactly the same under the null hypothesis (making c_{t+1}, in the population, identically 0), the sample variances in the denominators of the ENC-T statistics are 0. This characteristic of the ENC-T statistics can negatively impact the small-sample properties of the tests. Therefore, Clark and McCracken (2001) propose a modified version of the ENC-T statistics in which $\bar{c}$ (the covariance between $\hat{u}_{1,t+1}$ and $\hat{u}_{1,t+1} - \hat{u}_{2,t+1}$) is scaled by the variance of one of the forecast errors rather than using an estimate of the variance of $\bar{c}$.

This difference shows up in the calculation of the loss differential d_t. The loss function of GW for each period t:

$$d_t = L_t^1 - L_t^2 \tag{3.15}$$

where L_t^1 and L_t^2 are the loss functions for models 1 and 2, respectively. Commonly used loss functions include the squared error loss $(e_t^i)^2$ or the absolute error loss $|e_t^i|$.

Next, calculate the average of the loss differentials and the variance of the loss differentials:

$$\begin{aligned} \overline{d} &= \frac{1}{T}\sum_{t=1}^{T} d_t \\ s^2 &= \frac{1}{T-1}\sum_{t=1}^{T}(d_t - \overline{d})^2 \end{aligned} \tag{3.16}$$

where T is the total number of observations.

In the GW test, the long-run variance of the loss differentials is estimated using the Newey-West estimator. In contrast, the DM test typically uses a simpler approach to estimate the variance, such as the sample variance or an autoregressive estimator. Now, we can estimate the autocovariance of the loss differentials at each lag h and compute the long-run variance of the loss differentials using the Newey-West estimator:

$$\begin{aligned} \gamma(h) &= \frac{1}{T}\sum_{t=h+1}^{T}(d_t - \overline{d})(d_{t-h} - \overline{d}) \\ V_T &= \gamma(0) + 2\sum_{h=1}^{H}\left(1 - \frac{h}{H+1}\right)\gamma(h) \end{aligned} \tag{3.17}$$

Finally, we can calculate the test statistic as follows:

$$GW_{stat} = \frac{\overline{d}\sqrt{T}}{\sqrt{V_T}} \tag{3.18}$$

The test statistic for the GW test follows a chi-square distribution asymptotically, while the DM test statistic follows a standard normal distribution only if errors are normally distributed for finite samples. In general, the DM test statistic follows a Student's t-distribution with

$(T - 1)$ degrees of freedom (Harvey et al., 1997). These differences make the GW test more suitable for dealing with time-varying parameters and structural changes in the data-generating process. In contrast, the DM test is more appropriate for stationary data and constant model parameters. If the test statistic GW_{stat} is greater than the critical value from the chi-square distribution, we can reject the null hypothesis, indicating that the two models have different conditional forecasting abilities.

3.4 Rolling window

While structural breaks pose significant challenges for crude oil price forecasting (Perron, 2006), our methodology prioritises the rolling window approach over alternative break-adaptation techniques for three reasons. First, specialised methods like time-varying parameter models (Koop and Korobilis, 2015) or Markov-switching frameworks (Hamilton, 1989)—while theoretically appealing—require substantially longer data series than available in our post-1990s deregulated market period to reliably estimate break dynamics. Second, endogenous breakpoint detection methods (Bai and Perron, 2003) would fragment our limited sample into economically insignificant subperiods given the frequency of oil market disruptions (Kilian, 2009). Crucially, the rolling window provides an empirically validated compromise: it implicitly accommodates gradual regime evolution through continuous re-estimation while avoiding strong assumptions about break timing or functional form that underlie many dedicated break models.

Our rolling specification (fixed at one-fourth sample size) optimises this balance by sufficiently excluding obsolete structural regimes (e.g., pre-2000s price-control periods) while retaining adequate observations for estimation. This aligns with Chen et al. (2010b)'s commodity forecasting framework but adjusts for oil's higher volatility through tighter recency-weighting. As noted in Section 3, we complement this with Rossi (2005)'s robust Granger causality to explicitly test breakpoints—a targeted application where dedicated break methods offer unique advantages over rolling estimation.

4 Data

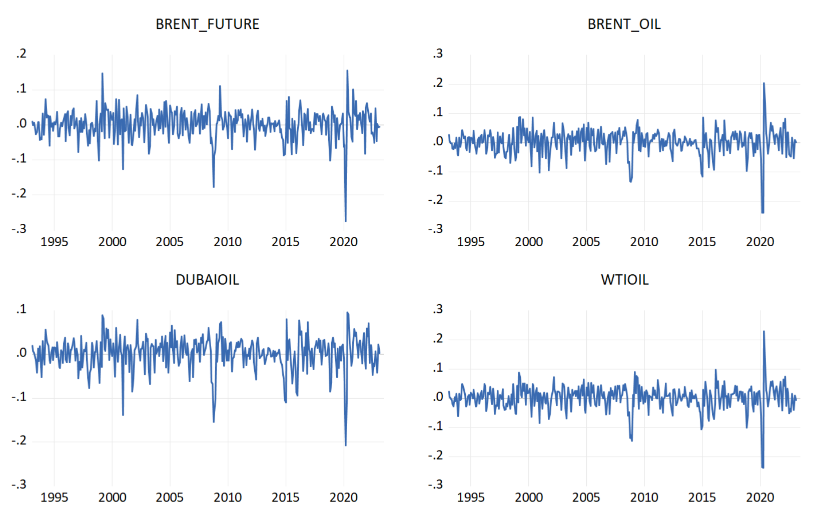
4.1 Data description

In this section, we explain our choice of price and NER series as well as the time span of the analysis. We apply monthly data on the sample countries' exchange rates relative to the US dollar. We consider three different crude oil prices—Dubai, WTI, and Brent. Additionally, crude oil futures prices are also included in the analysis at the same frequency. Brent crude oil future price and three crude oil prices are all monthly covered from 1993:01 to 2023:02, which gives a total of 362 monthly observations. While the period for oil prices is relatively easy to collect, the same period may not be readily available for the exchange rates of oil-exporting countries for a number of reasons. Hence, the availability and time span of exchange rate data for oil-exporting countries might differ due to various country-specific and external factors. The three most important reasons are exchange rate regimes, geopolitical considerations, and data collection infrastructure. As mentioned before, many oil-exporting countries peg their currencies to a stable currency like the U.S. dollar to mitigate volatility, such pegged exchange rates can be meaninglessly forecasting crude oil prices. In addition, political instability, sanctions, or regional tensions may contain a lot of unnecessary information, which can influence forecasting. Data collection infrastructure is a practical reason. Some oil exporters still developing economically might not have the infrastructure to collect and disseminate high-frequency, reliable exchange rate data compared to more developed oil exporters. Based on the above reasons, there are 6 selected oil exporters: Brazil, Canada, Colombia, Indonesia, Mexico and Norway. The data of Canada, Colombia, Indonesia and Norway are monthly and cover the period from 1993:01 to 2023:02. For Mexico, it is also monthly from 1994:01 to 2023:02. Brazil is also monthly covered from 1995:01 to 2023:02. We apply the largest period for each in single country forecasting. The training period and forecasting period are shown in Appendix 1. We examined three data series: (1) the NER of our sample data taken from the Federal Reserve Bank of St.Louis (Fred) and the Bank for International. (2) the Brent Crude oil future price is real-time derived on the Intercontinental Exchange (ICE). (3) the Brent, WTI and Dubai spot prices taken from the Fred. Note that all data are available in real time and are never revised. And then, we can define the inflation of crude oil futures prices and NER. Since we are using the inflation of crude oil prices and the spot prices, the total observations will reduce one time period to 361. The framework of calculating inflation is the log difference as described in Equation (3.19):

$$\begin{aligned} \triangle log(oil_t^i) &\equiv log(oil_t^i) - log(oil_{t-1}^i) \\ \triangle log(EX_t^i) &\equiv log(EX_t^i) - log(EX_{t-1}^i) \end{aligned} \tag{3.19}$$

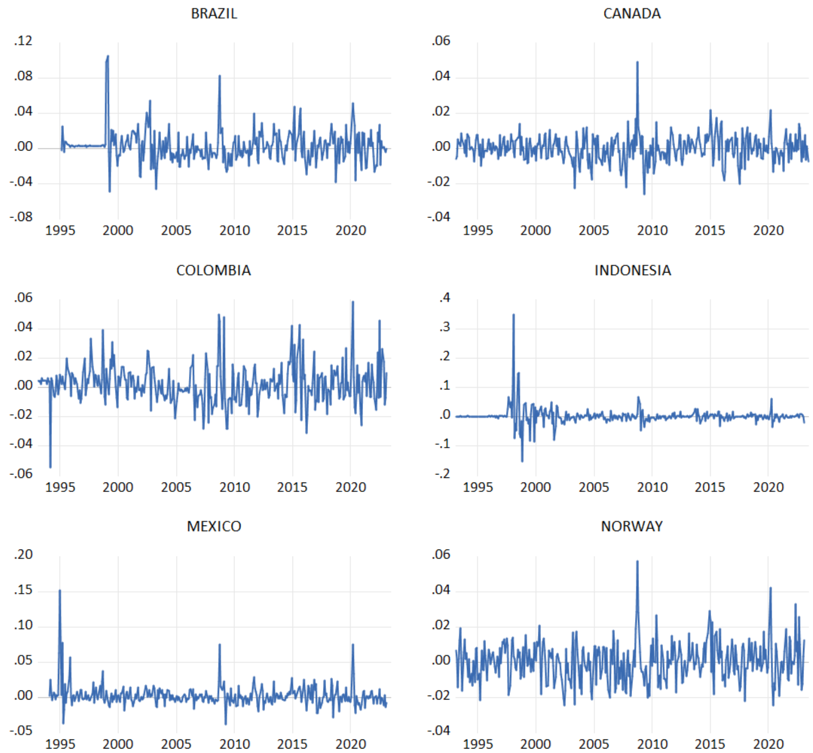
where EX_t^i and oil_t^i are the NER at time t for country i and the Brent oil future or Brent oil price at time t. In our context, EX_t could refer to the NER of the Brazilian Real, the Indonesian rupiah, the Colombian Peso, the Mexican peso and the Norwegian Krone. The following Figures 7 and 8 below show the log difference of $oilprice_t$, and EX_t during our sample period.

Figure 7: $\triangle log(oil_t)$ during sample period



In order to incorporate as much information as possible in our oil price predictions, we aim to use the longest available period as possible as we can for one-step-ahead forecasts, both in-sample and out-of-sample. However, due to the unique national conditions of developing

Figure 8: $\triangle log(EX_t)$ during sample period



countries, we need to pre-process our data. This is because the Mexican financial crisis of 1994 and the Asian financial crisis of 1997 quickly spread to other developing regions. These financial crises led to significant exchange rate movements for many developing countries. These movements may not help forecast crude oil prices.

4.2 Financial crisis

As evidenced by Figure (8), the exchange rates of Brazil, Indonesia, and Mexico in our sample may have been significantly influenced between 1997-1999 and 1994-1996. Given the significant financial turbulence and volatility experienced in those periods, using the exchange rate of this period to forecast global crude oil prices may introduce distortions. The specific economic and political circumstances during that time caused large fluctuations in the exchange rate that were not necessarily tied to changes in global crude oil prices. Using data from this period might therefore introduce noise and make it more challenging to model and forecast oil prices accurately. In this section, we explored the impacts of these financial crises on the exchange rates. Additionally, we assessed whether these impacts could have introduced unnecessary disturbances to our utilisation of exchange rates for oil price forecasting country by country.

Although Brazil's financial crisis didn't stem solely from the Asian and Russian financial crises that began in 1997, it was significantly influenced by Brazil's economic vulnerabilities. At the time of the Asian financial crisis, Brazil was running sizeable fiscal deficits, and the majority of its government debt, which was financed short-term, amounted to 40 percent of GDP. The country also had a burgeoning current account deficit. These vulnerabilities rendered the Brazilian economy highly susceptible to changes in investor sentiment. When financial crises engulfed Asia in 1997 and Russia in 1998, investors divesting from those countries also began to withdraw from Brazil. This created a situation that Brazil couldn't manage, which resulted in the forced floating of the Brazilian real and a financial crisis. Therefore, while the 1999 Brazilian financial crisis wasn't directly caused by the Asian financial crisis, the global shift in investor sentiment and capital flows that ensued from the Asian and Russian crises played a pivotal role in exacerbating Brazil's pre-existing economic vulnerabilities, eventually leading to the 1999 crisis (Fraga, 2000; Palma, 2006). The financial crisis resulted in significant volatility in its exchange rate. The factors influencing this volatility were numerous and complex, including fiscal deficits, investor sentiment, short-term debt servicing issues, and changes in monetary policy, among others such as panic selling, government intervention, and speculation. These factors were generally specific to the time and place of the crisis and were unlikely to provide useful information for predicting global oil prices. These factors would have caused fluctuations in the exchange rate that were not necessarily related to changes in the global crude oil price. Therefore, these fluctuations might not have been helpful for forecasting. Consequently, for Brazil, we excluded the data from before 2000:01 to avoid the distortions caused by the crisis.

This logic similarly applies to the impact of the Mexican financial crisis as called 'Tequila' on exchange rates. The Mexican peso crisis was a currency crisis sparked by the Mexican government's sudden devaluation of the peso against the U.S. dollar in December 1994, which became one of the first international financial crises ignited by capital flight. With dwindling foreign reserves and unable to maintain the fixed exchange rate peg, the Mexican government was forced to devalue the peso in December 1994. This decision was meant to make Mexican exports cheaper, reduce the current account deficit, and restore investor confidence. However, it ended up sparking a financial crisis because the devaluation was much larger than expected, and the government mishandled the situation, leading to even more capital flight (Lustig et al., 1995). This financial crisis and its aftermath extended into 1996. Therefore, to improve the accuracy and reliability of oil price forecasting using exchange rates, it would be beneficial to exclude the period from 1994 to 1996 for Mexico from the dataset. This would help ensure that the exchange rate data used for forecasting is more reflective of changes in global crude oil prices and not unduly influenced by country-specific financial crises.

In fact, at the close of the 20th century, numerous developing countries, including Indonesia, grappled with economic crises spurred by similar factors. The economic upheaval in Indonesia during this period was closely tied to specific circumstances, such as a fragile financial sector and excessive dependence on short-term foreign debt. Sharma (2001) in their study highlighted the significant role of crony capitalism in Indonesia's crisis. Crony capitalism describes an economic system where specific business people and government officials maintain close ties, leading to favouritism in the distribution of legal permits, government grants, special tax breaks, and other forms of state intervention. This favouritism undermined economic competitiveness and fostered economic instability, eventually precipitating the financial crisis and causing substantial volatility in Indonesia's currency exchange rate. However, while these factors significantly influenced Indonesia's exchange rate at the time, they may not have a direct bearing on global crude oil prices. By excluding the data for Indonesia's exchange rate before January 1999 from our dataset, we aim to sidestep these distortions and enhance our model's ability to discern and learn from the underlying relationship between exchange rates and crude oil prices. Finally, after pre-processing the data for the three countries, the starting points for the time series data are as follows: Brazil from January 2000 with 278 observations, Indonesia from January 1999 with 290 observations, and Mexico from January 1996 with 326 observations, and all ending in February 2023. From Figure 8, for Colombia, Canada, and Norway, aside from the pandemics in 2009 and 2019-2021, there are no particularly abnormal periods of fluctuation, so we do not make additional deletions, still as 361 observations[5].

4.3 Stationarity

After pre-processing the data, we conduct a unit root test for all data series to ensure their stationarity before initiating the forecasting process. We employ the Augmented Dickey-Fuller (ADF) test for this purpose, and the results are presented in Table 8:

Table 8: ADF test of $\triangle log(oil_t)$ and $\triangle log(EX_t)$

ADF test	Brent Future	Colombia EX	Indonesia EX	Canada EX	Brent oil
T-statistic	-15.70(0.00) ***	-17.38(0.00) ***	-17.17(0.00) ***	-14.39(0.00)***	14.99(0.00)***
ADF test	Norway EX	Brazil EX	Mexico EX	WTI oil	Dubai oil
T-statistic	-11.97(0.00) ***	-11.46(0.00)***	-13.00(0.00)***	-14.55(0.00)***	-13.60(0.00) ***

Note: In our specific application, we utilise the ADF test with an intercept. The P-value of the ADF test is in parentheses. The one-sided ADF test critical value for $1\%, 5\%$ and 10% is -3.45, -2.86 and -2.57. $*, **$ and $***$ denote significance at the 10, 5 and 1 percent levels.

The table above presents the results of the ADF test applied to the log differences of three oil spot prices, Brent futures prices, and various NERs, including Brazil, Canada, Colombia, Indonesia, Mexico, and Norway. The T-statistics reported for each series reflect the T-statistics are negative and highly significant, as indicated by the p-values in parentheses (all equal to 0.00). This strong statistical significance suggests that we can confidently reject

[5]More information can be found in Appendix 1

the null hypothesis of a unit root for all series. In other words, all the series are likely to be stationary (see Table 26).

5 Results

5.1 In-sample analysis

We first focus on the future price. Initially, we explore the in-sample Granger-Causality tests on the future price for our univariate outcomes, as detailed in Tables 9 and 10. For Brazil, Indonesia, and Mexico, our starting points are established as 2000:02, 1999:02, and 1996:02 respectively, extending up to 2023:02. Conversely, for Colombia, Canada, and Norway, the period considered spans from 1993:02 to 2023:02. The test statistics rely on Newey and West (1987)'s method, utilising a bandwidth of $T^{\frac{1}{3}}$. For Columbia, we find a p-value of 0. Consequently, we can reject the null hypothesis for Colombia, inferring that shifts in the exchange rate indeed exert a Granger-causal impact on the fluctuations in Brent's future price. The p-value of Brazil also could reject the null hypothesis at the 10% significance level, but the evidence is weak. Yet, for other countries, we cannot reject the null hypothesis. Therefore, we uncover scant evidence of oil exporters Granger-causing the Brent oil future price in the traditional in-sample GC test. As previously mentioned, the traditional GC test is unable to account for possible parameter instabilities. Chen et al. (2010b) and Pincheira Brown and Hardy (2019) discovered that the movements of many commodity prices exhibit some instability. Thus, we also employ the robust instability GC test introduced by Rossi (2005), and its results are presented in Table 10.

Colombia and Brazil display p-values of 0.0138 and 0.0129. Mexico has p-values of less than 0.001. All these values denote statistical significance at 5% and 1% levels, suggesting that we can reject the null hypothesis for these countries, which implies that alterations in the exchange rate do Granger-cause variations in the crude oil future price. Nevertheless, Canada, Indonesia and Norway continue to exhibit p-values that surpass the conventional sig-

Table 9: univariate Granger-Causality Tests

P-value of $H_0 : \beta_0 = \beta_1 = 0$ in $\triangle oil_{t+1} = \beta_0 + \beta_1 \triangle EX_t + \beta_2 \triangle oil_t$						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
Brent	0.0731*	0.000***	0.5787	0.2730	0.3616	0.1830

Note: The table reports p-values for testing the null of no Granger causality that are robust to parameter instabilities. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively.

Table 10: Rossi (2005) Granger-Causality Tests Robust to Instabilities

P-value of $H_0 : \beta_0 = \beta_1 = 0$ in $\triangle oil_{t+1} = \beta_0 + \beta_1 \triangle EX_t + \gamma_2 \triangle oil_t$						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
Brent	0.0129**	0.0138**	0.6148	0.000***	0.7457	0.3735

Note: The table reports p-values for testing the null of no Granger causality that are robust to parameter instabilities. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. If the test statistic is less than or equal to the smallest critical value in the table, this implies that the test statistic is not large enough to reject the null hypothesis at any of the significance levels represented in the table. Therefore, the p-value is set to 1 for Indonesia.

nificance thresholds, indicating that we cannot reject the null hypothesis for these countries. The results infer that for Brazil, Colombia and Mexico, fluctuations in crude oil exporters' exchange rate do Granger-cause changes in crude oil future price, although this might not hold true for Canada, Indonesia and Norway. We also estimated R^2 for the in-sample regressions in Appendix 6.

5.2 Out-of-sample forecasting

Even though we lack evidence that every country's NER could assist in forecasting the crude oil future price in the in-sample test, as Chen et al. (2010b) articulated, the performance of in-sample forecasting does not necessarily translate into out-of-sample forecasting ability. Recall that our rolling window size for all one-step forecasts is one-fourth of the data for each country, resulting in 70 data points for Brazil, 91 for Colombia, 73 for Indonesia, 82 for Mexico, 91 for Norway, and 91 for Canada. The first prediction results for each country correspond to the following dates: Brazil (2000:02), Colombia (1993:02), Indonesia (1999:02), Mexico (1996:02), Norway (1993:02), and Canada (1993:02). Table 11 presents the results of out-of-sample forecast capabilities, based on Equation (3.5). We opted for the rolling forecast procedure due to the high volatility of crude oil futures prices. Rolling window forecasts may lead to more reliable forecasting over time, as they continuously adjust to the most recent data, ensuring outdated observations don't unduly influence forecasts. This approach is particularly beneficial in unstable markets such as crude oil futures, where price changes can be abrupt and considerable. We compared the out-of-sample forecasting results between the AR(1) benchmark and our model by calculating the Mean Squared Forecast Error (MSFE) for each. Specifically, we documented the difference between the MSFE of our forecast and the MSFE of the benchmark. In our context, a negative value signifies that the model outperforms the benchmark. Note that we are applying the ENCNEW test

Table 11: Tests for Out-of-Sample Forecasting Ability

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
Brent future	-0.0275	-0.2385	2.3417	-0.1087	0.7072	1.9598
ENCNEW test	***	***	NR	***	NR	NR
start date	(2000:02)	(1993:02)	(1999:02)	(1996:02)	(1993:02)	(1993:02)
Training size	70	91	73	82	91	91

Note: For the MSFE part, we are reporting the loss difference between the two models. A negative value signifies that the model outperforms the benchmark. The ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

for our nested model instead of the traditional DM test, but the null hypothesis is akin to the DM test. A rejection means that one model will statistically outperform another significantly. The results of the ENCNEW test are indicated by asterisks. $*$,$**$, and $***$ denote statistical significance at the 1%, 5% and 10% levels, respectively. According to Table 11, oil exporters in Latin America, specifically Brazil and Mexico, exhibit MSFE differences of -0.0275 and -0.1087, respectively. The ENCNEW test also shows significance at the 1% level for these countries. These negative values indicate that the current model provides more accurate out-of-sample forecasts. For Colombia, the relatively large negative MSFE difference (-0.2385) further suggests that the model significantly enhances forecasting accuracy. The 1% significance level also suggests that the exchange rate model provides greater accuracy. Although the traditional in-sample GC test for Mexico does not establish a GC relationship between the Mexican peso and the Brent crude oil future price, the Robust Granger-Causality Test demonstrates that the exchange rate of Mexico could assist in forecasting the Brent crude oil future price. Therefore, we may admit that fluctuations in the Mexican exchange rate do contribute to the forecasting of crude oil futures prices. Conversely, for Canada, Indonesia and Norway, the positive MSFE difference suggests that the current model gives less accuracy for out-of-sample forecasts. There is no evidence to support that the currencies of Canada, Indonesia and Norway could help forecast crude oil futures prices under our sample period.

5.3 Robustness check

We conduct robustness checks using NER to forecast Brent crude oil prices, reasoning that if an oil producer's currency can forecast crude oil derivatives, it should also be able to forecast crude oil itself. As in our previous approach, we will first utilise the NER of the oil producer and apply the same test as in the last section. The only difference is that we will replace the oil futures price with the price of Brent crude oil. Furthermore, we will also consider longer-horizon forecasting ability.

Forecasting Crude Oil Prices

In our robustness test, we examine the forecasting ability of the exchange rates of Brazil, Canada, Colombia, Indonesia, Mexico, and Norway on crude oil prices, using the same period as the exchange rate in the previous test. As Baumeister and Kilian (2016) mentioned, the futures price may deviate from the market expectation of the price of the underlying asset. Therefore, sometimes direct forecasting of oil prices itself may lead to better results. The NER of oil exporting countries forecasting ability in crude oil price is better than oil financial derivatives according to our new results. These results are presented in Table 12. For the in-sample GC Tests, we observe statistically significant p-values of less than 0.001 for Colombia and Mexico. This allows us to reject the null hypothesis that exchange rates do not provide any statistically significant information for predicting future crude oil prices. This implies that the Colombian and Mexican exchange rates Granger-cause crude oil prices. Brazil and Norway also exhibit statistical significance at the 5% level with p-values of 0.0101 and 0.0124. And Canada also displays statistical significance at the 10% level with 0.0766. Although this is weak evidence, it might also suggest that the price of oil is more forecastable than the futures price of oil by the exchange rates of oil-exporting countries. However, for Indonesia, we cannot reject the null hypothesis. Under the Rossi GC test, Brazil, Colombia and Mexico once again demonstrate strong statistical significance at the 1% level. In addition, this test reinforces the significance for Norway at the 1% level, which was not indicated in the traditional GC test. It is important to note that the Rossi GC test does not always yield consistent results. For Canada, the Rossi GC test does not show statistical significance, which differs from the traditional GC test results. This inconsistency may suggest that past exchange rates for Canada do not consistently help forecast future Brent oil prices. And, we cannot reject the null hypothesis for Indonesia.

The results of the out-of-sample test are consistent with the results of the previous traditional GC and robust GC tests. Brazil, Colombia, Mexico, and Norway significantly outperform the benchmark model, with MSFE differences of -0.5403, -1.0545, -0.7997, and -0.6001,

Table 12: Tests for Brent Crude oil case

In-sample GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.0101**	0.000***	0.3395	0.000***	0.0124**	0.0766*
Rossi GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.0000***	0.000***	0.4843	0.027**	0.000***	0.6243
MSFE Differences between Model: $\triangle oil_{t+1}^i = \beta_0 + \beta_1 \triangle EX_t^i + \beta_2 \triangle oil_t^i$ and AR(1)						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
MSFE	-0.5403	-1.0545	2.8952	-0.7997	-0.6001	0.7201
ENCNEW test	***	***	NR	***	***	NR
Start date	(2000:02)	(1993:02)	(1999:02)	(1996:02)	(1993:02)	(1993:02)
Window size	70	91	73	82	91	91

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

respectively. And the ENCNEW test rejects the null hypothesis at the 1% significance level for these countries. This robustness test provides compelling evidence that the exchange rates of Brazil, Colombia, Mexico, and Norway can significantly forecast crude oil prices, both in- and out-of-sample. The Canadian and Indonesian exchange rates do not enhance crude oil price forecasts within our sample period.

Brent, WTI, and Dubai crude oil are produced in different parts of the world and have different chemical properties, production costs, and supply-demand dynamics. They also often trade at different prices. This means the exchange rate dynamics of oil-exporting countries could react differently to price changes in these different types of crude oil. Therefore, examining the forecasting ability for WTI and Dubai, as well as Brent, allows for a more comprehensive understanding of the relationship between crude oil prices and exchange rates across different regions and types of oil. For this reason, we briefly summarise similar analyses conducted using WTI and Dubai crude oil benchmarks. Results align closely with those obtained for Brent oil. Specifically, Brazil, Colombia, Mexico, and Norway consistently exhibit significant forecasting power both in-sample and out-of-sample, indicated by negative MSFE values and ENCNEW test rejections at 1% to 5% significance levels in most cases. However, Canada and Indonesia remain unable to consistently forecast these benchmarks, despite minor variations reflecting their trade relationships. The results for WTI and Dubai oil are shown in Table (28) and Table (29) in Appendix 4.

Out-of-sample forecasting with different rolling window size

As previously mentioned, the window size utilised for creating out-of-sample rolling forecasts can significantly impact model performance. Varying window sizes may either enhance or diminish the model's predictive accuracy. Unlike the previous section where a fixed quarter sample size window was used[6], we have employed four different window sizes, ranging from 72 to 108, to compare the MSFE differences across these multiple window sizes for crude oil future price and Brent crude oil price cases. The multi-rolling window forecasting results for the crude oil future case and crude oil case are presented in Table 13 and Table 14, respectively.

In Brent Crude oil future cases in Table 13, Brazil, Colombia and Mexico demonstrate noticeable forecasting capability. Mexico consistently shows negative MSFE values of -0.5122, -0.9832, -0.8038, and -0.5436 for window sizes of 72, 84, 96, and 108, respectively. The ENCNEW test indicates improved forecasting abilities at the 1% significance level for all window sizes compared to the benchmark. Brazil and Colombia continue to show negative MSFE values for most window sizes. For Colombia, the ENCNEW test yields significant results at the 1% level for all window sizes except 72. Similarly, Brazil's exchange rate

[6] In the previous section, our rolling window size for all one-step forecasts was one-fourth of the data for each country, resulting in 70 data points for Brazil, 91 for Colombia, 73 for Indonesia, 82 for Mexico, 91 for Norway, and 91 for Canada. Including these previous window sizes in our comparison allows us to better evaluate the model's performance across different contexts.

Table 13: Multi-rolling window forecasting for Brent Crude oil future case

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.0096	0.0211	-0.3570	0.5782
	ENCNEW	***	**	***	NR
Canada	MSFE	2.2278	1.4992	0.4507	0.3650
	ENCNEW	NR	NR	NR	NR
Colombia	MSFE	0.1824	-0.2394	-0.3572	-0.5214
	ENCNEW	NR	***	***	***
Indonesia	MSFE	2.5025	2.3757	2.1218	1.7163
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	-0.5122	-0.9832	-0.8038	-0.5436
	ENCNEW	***	***	***	***
Norway	MSFE	1.2052	0.6820	-0.0318	0.3987
	ENCNEW	NR	NR	**	NR

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

demonstrates significant forecasting ability at the 1% level for window sizes of 72 and 96 months. Notably, for Colombia, statistical significance and forecasting ability increase with larger window sizes. However, this feature was not found in other countries. Norway only shows a negative MSFE for the window size of 96. Conversely, Canada and Indonesia fail to display any forecasting ability across all the evaluated window sizes.

In the case of Brent crude oil in Table 14, the NER of oil-exporting countries continues to display better forecasting ability for crude oil prices than for oil financial derivatives across all window sizes, regardless of whether it's Brent, WTI, or Dubai. In the Brent oil case, Colombia, Mexico, and Norway consistently demonstrated better forecasting ability at the 1% level for all window sizes. These countries also show negative MSFE differences, implying

Table 14: Multi-rolling window forecasting for Brent Crude oil case

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.6854	-0.5040	-0.6848	0.6823
	ENCNEW	***	***	***	NR
Canada	MSFE	1.3524	0.8351	0.1419	0.1442
	ENCNEW	NR	NR	NR	NR
Colombia	MSFE	-0.9697	-1.2020	-1.0913	-1.0966
	ENCNEW	***	***	***	***
Indonesia	MSFE	2.8895	2.6253	2.5040	2.5204
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	-0.5064	-0.9366	-0.7910	-0.5388
	ENCNEW	***	***	***	***
Norway	MSFE	-0.4886	-0.4859	-0.7213	-0.6514
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

that their predictions are more accurate than the benchmark. For example, Brazil's MSFE differences for Brent oil are -0.6854, -0.5040, and -0.6848 for window sizes of 72, 84, and 96, respectively. The MSFE differences for futures with the same window sizes are -0.0096, 0.0211, and -0.3570. Similarly, for Colombia, the MSFE differences for Brent oil are -0.9697, -1.2020, -1.0913, and -1.0966 for window sizes of 72, 84, 96, and 108, respectively. The MSFE differences for futures with the same window sizes are 0.1824, -0.2394, -0.3572, and -0.5214. Additionally, for Noway, the MSFE values for the NER in the futures case are all positive, whereas, in the Brent oil case, the values are -0.4886, -0.4859, -0.7213, and -0.6514 for the four window sizes. Just as with the case of the future, Canada and Indonesia do not show any predictive power across all window sizes. Canada and Indonesia's exchange rates do not show predictive power in any scenario, suggesting that their currency movements may not be closely tied to oil price dynamics. Results of WTI and Dubai consistently affirm previous findings: the exchange rates of Brazil, Colombia, Mexico, and Norway systematically improve crude oil price forecasts for most windows, indicated by consistently negative MSFE differences and robust ENCNEW test rejections. Conversely, Canada and Indonesia exhibit persistent lack of predictive ability, with positive MSFE differences and no significant ENCNEW rejections across varying windows. The results for WTI and Dubai oil are shown in Table (30) and Table (31) in Appendix 5.

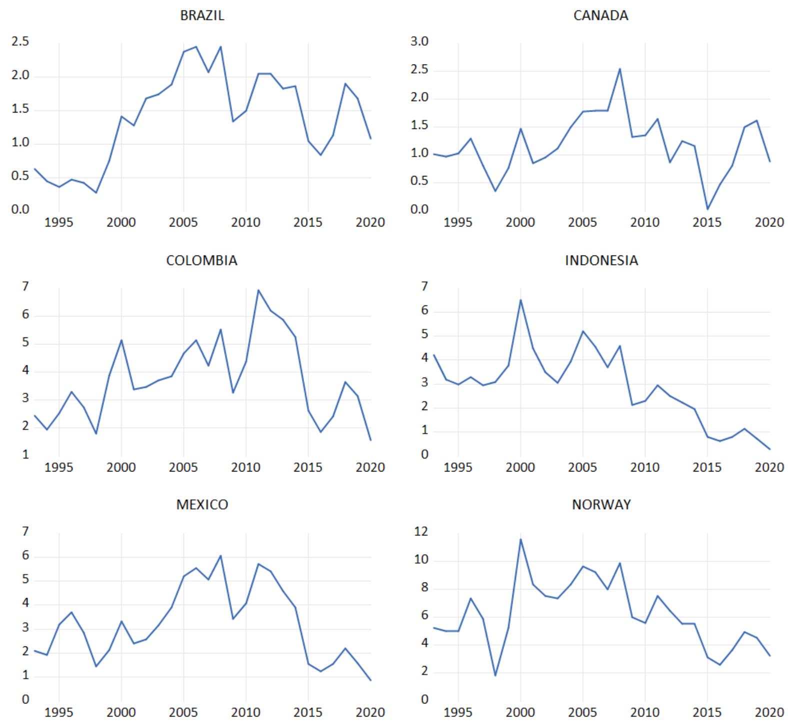
Based on the results, the forecasting power of exchange rates in oil-exporting countries varies significantly depending on the country, the window size applied, and the type of oil (Brent, WTI, or Dubai). All rolling window sizes in this section are based on one-fourth of the country-specific data. Consequently, the evaluation of prediction performance between countries may not be entirely fair, as the size of the rolling window significantly impacts the prediction results. In Appendix 7, we present results using the same sample size, which yields nearly identical outcomes. We found that Brazil, Colombia, Mexico, and Norway consistently demonstrated strong forecasting ability in most scenarios, exhibiting negative MSFE values and the ENCNEW test rejected at the 1% level across most window sizes, indicating robust predictive power. Among them, Norway's exchange rate, despite its strong forecasting ability in oil cases, did not consistently demonstrate this ability across all window sizes in the futures case. Conversely, the exchange rates of countries like Canada and Indonesia failed to demonstrate forecast power in all scenarios, indicating that their currency movements might not be as closely tied to oil price dynamics. The impact of window size on the model's forecasting ability was also evident. Larger window sizes typically enhanced the statistical significance and forecasting ability of countries like Colombia, while others, like Brazil, only showed significant forecasting abilities at certain window sizes. The relationship between window size and forecasting power was not uniform across countries, reinforcing the complexity of the relationship between exchange rates and oil prices. This observation aligns with established literature emphasising the connection between commodity dependence, terms-of-trade shocks, and currency valuation (Kohlscheen et al., 2017; Chen et al., 2010a). Countries heavily dependent on oil revenues typically experience a stronger linkage between exchange rates and oil prices, enhancing forecasting power. Our empirical results thus strongly support this theoretical expectation.

As previously mentioned, the significance of oil to a country may impact the relationship between its currency and oil, thereby reducing the predictive power of oil (Kohlscheen et al., 2017). This could explain why Canada and Indonesia cannot predict oil and its derivatives, a potential explanation for which may lie in oil rents. This finding is consistent with the theoretical framework of commodity currencies (Chen and Rogoff, 2003; Cashin et al., 2004), which suggests that the currencies of economies heavily dependent on commodity exports exhibit stronger co-movements with commodity prices due to terms-of-trade shocks. In our sample, countries with high observed oil rents [7](see Figure 9) demonstrate greater economic exposure to oil price dynamics.

Figure (9) presents oil rents in our sample countries. The figure illustrates that all countries experienced a sharp decline in oil rents due to significant oil price reductions in 2008 and 2015. Except for these two substantial price reductions, the oil rents for each country demonstrate varying trends. Brazil, for instance, had relatively low oil rents before 2000, but this has been increasing over the past two decades. This increase corresponds with Brazil's growing oil production and the discovery of more offshore oil fields. For Colombia and Norway, their oil rents have consistently stayed above 1.5%. In Mexico, even though recent years have seen a considerable decline in oil rents, it maintained a level above 2.5% for most of our sample period. Lastly, Canada and Indonesia present different cases. Canada, being an exporter of a diverse range of products, has never considered petroleum products as core to its export portfolio. In contrast, Indonesia, a former member of OPEC and once

[7]Oil rent refers to the difference between the value of crude oil production at world prices and total production costs. Often expressed as a percentage of a country's GDP, it represents the economic advantage a country receives from oil extraction.

Figure 9: Oil rent for sample countries



a significant net exporter of oil, has been grappling with becoming a net oil importer in recent years. This shift is due to a combination of factors: decreasing oil production, rising domestic energy demands triggered by the country's growing economy and population, and the effects of ageing oil fields and limited investments in exploration and production. This complex interplay of factors has led to a significant decrease in Indonesia's oil output and, consequently, its oil rents. This exposure creates a feedback mechanism, where exchange rate fluctuations partially encode information about future oil market movements (Ferraro et al., 2015). By contrast, the weaker forecasting performance observed for Canada and Indonesia can be explained by their structural characteristics. Canada's diversified export base reduces the economy's overall dependence on oil, weakening the link between oil prices and exchange rates. Similarly, Indonesia's transition to a net oil-importing country in recent decades (Lucky and Su, 2021) diminishes the transmission channel from oil price fluctuations to the domestic currency. Although formally testing the threshold effect of oil dependence is beyond the scope of this study, our results align with existing theoretical expectations and suggest that oil dependence is a key moderating factor influencing the extent to which exchange rates contain predictive information for oil price movements.

Longer-horizon forecasting ability

As mentioned before, the rolling window forecast method used in this chapter may yield different results for different window choices. Based on the results of the previous section, in this section, we will exclude Canada and Indonesia because of less forecasting evidence in the previous sections, and conduct a rolling window long-horizon test to evaluate the forecasting capabilities of the exchange rate of Brazil, Colombia, Mexico, and Norway for forecasting oil and its derivatives. Generally speaking, predictions for short horizons tend to

Table 15: Long-Horizon Forecasting Regressions for Brent Oil Future Prices

GW and DM's test statistics between Model: $\triangle oil^i_{t+h} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and Benchmark						
Steps ahead	:	h=4	h=6	h=12	h=18	h=24
Brazil	MSFE	1.2888	1.3567	0.3748	-0.4234	1.1688
	GW_{stat}	2.45	2.94	2.35	2.48	3.92
Colombia	MSFE	1.7966	1.1891	1.3146	1.7868	0.6303
	GW_{stat}	4.59	2.13	1.94	4.93	2.13
Mexico	MSFE	0.1514	0.3057	0.6018	1.8271	1.4361
	GW_{stat}	0.08	3.49	1.76	4.21	3.23
Norway	MSFE	-0.4190	-0.1388	-1.1786	1.4731	0.8395
	GW_{stat}	-0.71	-0.62	-1.39	2.88	2.21

Note: we report the GW and DM's test statistics. The GW rejections are indicated by asterisks, with $***$, $**$ and $*$ denoting rejection at the 1%, 5%, and 10% significance levels, respectively. The corresponding critical values are 6.6349, 3.8415, and 2.7055.

be more accurate due to the accumulation of model uncertainty. In multi-step forecasts, each subsequent step relies on the previous forecasts, and errors from earlier steps can compound in later steps, leading to increased uncertainty. Lack of information also contributes to this issue; as forecasts extend further into the future, there is less available information about potential influencing factors. This scarcity of information can lead to more assumptions and, consequently, greater uncertainty. However, it is clear that for some applications, longer horizons are of interest. The structure of our forecasting model has not changed compared to the previous part, as shown in Equation 3.20. The length of the sample is also the same as Section 5.2, just changing the length of the forecasting. The size of the rolling window is set to 96, since in the previous section it appeared that most countries demonstrated relatively better forecast power with this window size in the short horizon. We forecast oil and its derivatives with 4, 6, 12, 18, and 24 steps ahead, as shown in the following figures. The first predicted values are May 2001, July 2001, January 2002, July 2002, and January 2003 for the different steps ahead. Note that we are applying the GW test for our nested model instead of the traditional DM test, but the null hypothesis is akin to the DM test. The results show that our long-term forecasts for the exchange rates of most countries are not as accurate as the one-step forecasts.

$$\begin{aligned} Benchmark: \triangle oil_{t+h} &= \beta_0 + \beta_1 \triangle oil_t + \epsilon_t \\ Model\ with\ NER: \triangle oil_{t+h} &= \beta_0 + \beta_1 \triangle oil_t + \beta_2 \triangle NER_t + \epsilon_t \end{aligned} \tag{3.20}$$

where β_0 is the constant and ϵ is the error term, t is the total sample period and h is the forecasting horizon.

According to Tables 15 to 18, Brazil and Mexico consistently show positive MSFE differences across various forecast horizons for Brent oil future prices, as well as for all other types of crude oil prices. An exception is Brazil, which shows a negative MSFE at the 18-step horizon for Brent futures, but the GW_{stat} result is not significant. Similarly, the long-term forecast for Mexico's exchange rate is also less accurate than the one-step forecast. Only in the six-step forecast for Dubai oil does it show a negative MSFE, but the GW_{stat} result is not significant. This indicates that the country's forecasting ability may not outperform the benchmark, reflecting challenges in its predictive model in long-horizon forecasting. Colom-

Table 16: Long-Horizon Forecasting Regressions for Brent oil prices

GW and MSFE Differences test statistics between Model: $\triangle oil^i_{t+h} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and Benchmark						
Steps ahead	:	h=4	h=6	h=12	h=18	h=24
Brazil	MSFE	2.2807	1.3898	1.1528	0.9504	0.6696
	GW_{stat}	5.68	4.68	4.92	2.45	2.11
Colombia	MSFE	2.9128	2.3421	2.6346	1.2861	0.0964
	GW_{stat}	11.15	3.42	6.02	1.17	1.20
Mexico	MSFE	2.2807	1.3898	1.1528	0.9054	0.5809
	GW_{stat}	2.02	1.42	1.33	4.18	1.94
Norway	MSFE	1.0149	-0.6366	0.6694	0.6017	0.6177
	GW_{stat}	1.54	-0.76	1.59	0.25	0.41

Note: This table reports the GW and DM's test statistics. The GW rejections are indicated by asterisks, with $***$, $**$ and $*$ denoting rejection at the 1%, 5%, and 10% significance levels, respectively. The corresponding critical values are 6.6349, 3.8415, and 2.7055.

bia behaves quite differently from short-term forecasts, consistently showing positive MSFE divergence across different forecast horizons for Brent futures prices as well as for all other types of crude oil prices. Finally, the results using the Norwegian exchange rate for the three oil spot prices and oil futures are mixed. The best results are observed for the six-step forecast, which shows negative MSFE for all types of oil. However, only WTI and Dubai have GW_{stat} values that reject the null hypothesis at the 5% and 10% significance levels. Negative MSFE is also observed for both Dubai oil and Brent future prices in the 12-step forecast, but the GW_{stat} results are not significant.

In conclusion, as we expected, the long-horizon forecasting of oil and its derivatives for Brazil, Colombia, Mexico, and Norway reveals complex and varied results. These findings emphasise the intricate nature of long-horizon forecasting, particularly when it comes to oil prices. The possible reason for this result is the selection of the window size. Although the 96-step rolling window may yield relatively significant results in short-horizon forecasting, it

Table 17: Long-Horizon Forecasting Regressions for WTI oil prices

GW and DM's test statistics between Model: $\triangle oil^i_{t+h} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and Benchmark						
Steps ahead	:	h=4	h=6	h=12	h=18	h=24
Brazil	MSFE	0.6784	1.7102	2.0185	0.6483	0.6152
	GW_{stat}	1.33	4.72	6.36	2.04	1.70
Colombia	MSFE	2.8131	2.9178	2.4270	0.8055	0.6081
	GW_{stat}	9.80	4.92	9.36	0.89	2.23
Mexico	MSFE	0.7188	0.4040	1.1717	2.4985	0.9682
	GW_{stat}	2.24	1.26	3.24	9.27	2.47
Norway	MSFE	1.1445	-0.8862	0.8193	0.1494	0.7764
	GW_{stat}	2.26	-4.05**	0.84	0.06	1.52

Note: This table reports the GW and DM's test statistics. The GW rejections are indicated by asterisks, with $***$, $**$ and $*$ denoting rejection at the 1%, 5%, and 10% significance levels, respectively. The corresponding critical values are 6.6349, 3.8415, and 2.7055.

is still unclear whether this approach is also applicable to long-term forecasting. Our study demonstrates the importance of careful window selection and the inherent challenges of long-horizon forecasting, particularly in complex commodities like oil. Long-horizon forecasting of crude oil prices may require more complex models that capture various lagged values, seasonal patterns, nonlinear relationships, etc. Handling this complexity is not trivial.

6 Conclusion

The literature on using PV models introduced by Chen et al. (2010b) to forecast commodity prices seems to have expanded significantly. This may be due to more countries now recording their economic data in a standardised manner, coupled with the present economic system being more robust and comprehensive than before. In this paper, we aim to understand the role of oil exporters' NER in forecasting Brent oil future prices and three different types

Table 18: Long-Horizon Forecasting Regressions for Dubai oil prices

GW and DM's test statistics between Model: $\triangle oil^i_{t+h} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and Benchmark						
Steps ahead	:	h=4	h=6	h=12	h=18	h=24
Brazil	MSFE	1.6450	1.0258	0.6035	0.7296	0.8406
	GW_{stat}	3.02	1.42	3.62	5.43	2.49
Colombia	MSFE	1.8045	1.8266	3.0470	2.0008	0.4653
	GW_{stat}	6.12	2.79	6.49	3.19	1.90
Mexico	MSFE	0.5104	-0.3452	1.2904	1.5947	0.7038
	GW_{stat}	2.01	-2.42	8.16	2.11	2.01
Norway	MSFE	0.7410	-1.1056	-0.1155	0.4962	1.3881
	GW_{stat}	0.80	-3.83*	-0.70	0.17	1.61

Note: This table reports the GW and DM's test statistics. The GW rejections are indicated by asterisks, with $***$, $**$ and $*$ denoting rejection at the 1%, 5%, and 10% significance levels, respectively. The corresponding critical values are 6.6349, 3.8415, and 2.7055.

of oil prices known as Brent, WTI and Dubai. We applied both the traditional GC and Rossi (2005) instability GC test. In traditional GC, our in-sample results highlight that only for Colombia, changes in the exchange rate significantly affect oil futures prices, and Brazil is only shown at the 10% significance level. While for other countries, the evidence is not strong enough to reject the null hypothesis. However, it is critical to note that traditional in-sample GC tests fail to consider parameter instabilities, a situation observed in commodity price movements. When considering potential instabilities, our robust instability GC test reveals more significant results for Brazil and Mexico. For these countries, shifts in crude oil exporters' exchange rates demonstrate a Granger-causal relationship with crude oil future prices. Nevertheless, the null hypothesis couldn't be rejected for Canada, Indonesia, and Norway, suggesting this relationship may not hold for these countries.

Despite the limited in-sample evidence, the out-of-sample forecast results based on a rolling window forecast procedure provide interesting insights. To mitigate the influence on our forecasting of the high volatility of oil prices, we primarily employ the rolling window method in this chapter. Our methods led us to identify that the oil exports of Brazil, Colombia, and Mexico significantly outperform the AR(1) benchmark, with their MSFE showing negative values and most ENCNEW tests rejecting the null hypothesis at the 1% significance level. Among these, the results for Colombia are more pronounced. For Norway, despite lacking strong evidence from in-sample GC tests, the current model offers a weakly improved out-of-sample forecast accuracy. In the case of Canada, Indonesia and Norway, the absence of significant GC test results and a lack of evidence to support their currencies' role in forecasting crude oil futures necessitate further exploration. And then, we examine the forecasting ability of the oil exporter's NER on crude oil prices, using the same period. Then, according to our new results, we found that the NER of oil-exporting countries' forecasting ability in crude oil price is better than oil financial derivatives. The NER of Brazil, Colombia, and Mexico successfully forecasted crude oil prices for all three oil standards, both in-sample and out-of-sample. Notably, the NER of Norway also outperforms than the benchmark in most cases. The results of the ENCNEW statistics show that the forecasting ability of each country's exchange rate is significantly more favourable for the three oil spot prices than for the futures cases. This might be attributed to a variety of influences on these contracts, extending beyond simple oil price considerations and the anticipations of seasoned investors within each country. Factors such as market speculation, risk assessments, and time until contract expiration may all play a role in this divergence. Finally, we conducted long-horizon forecasts for all sample countries, but just like Beckmann et al. (2020) mentioned, the results indicate that the long-horizon forecasts based on the exchange rates of oil-exporting countries alone do not show a significant improvement over the benchmark.

Given an economy like Colombia, it is only natural that a strong link exists between international oil futures and the Colombian currency. This is due to oil and mining exports accounting for 55% of total exports, and approximately 30 % of the country's total oil and gas production is undertaken by foreign companies. Similarly, in Brazil, the oil and gas market has, for many years, constituted the majority of investments, accounting for about 10% of the country's GDP. Net Oil revenues account for 6.06% of Norway's total GDP in 2021. and Mexico's net oil revenues also accounted for a share of 2.06 % of the country's GDP, and only 0.76% in 2021 for Indonesia. Also, for Canada, as a developed country with diverse exports, crude oil has never been the most important export. Perhaps the importance of oil to a country is the key reason why a country's currency can predict oil futures, since countries with economies heavily dependent on oil may have currencies that respond more sensitively to fluctuations in oil prices, thus providing predictive power for oil futures in our sample countries.

In conclusion, our research demonstrates that the currencies of Colombia and Brazil, as oil exporters, can help predict oil futures prices and crude oil prices, while the exchange rates of Canada and Indonesia did not. The intuition behind this result stems from the forward-looking nature of the present value model. Market participants in Brazil, Colombia, and Mexico adjust their expectations based on information about future developments in the oil market(Chen et al., 2010b). If they effectively incorporate this information, the currencies of Brazil, Colombia, and Mexico should reflect any relevant changes in expectations for crude oil prices and their derivatives. Therefore, these currencies should contain predictive information that cannot be captured by standard univariate models. While some success was noted, the general inconsistency across different horizons and oil types emphasizes the need for continued refinement and exploration of forecasting techniques. It is worth mentioning that both Brazil, Colombia, and Mexico are all Latin American countries and a recent paper by Pincheira-Brown et al. (2022) found that Chile, also as a Latin American non-oil exporting country, can also predict three different oil products. This suggests that countries in South America may contain predictive information not found in other countries. This study provides valuable insights into the complex dynamics between crude oil prices and exchange rates, demonstrating a compelling need for more nuanced and region-specific analysis in this field.

Chapter 4

Predictability Between Crude Oil Price Return and Exporters' Exchange Rates During Structural Breaks and Economic Downturns

Abstract

Acknowledging the pivotal role of crude oil in the global economy and its impact on macroeconomic activities, our study delves into the complexities of predicting international crude oil price fluctuations. We utilise various predictive regression tests, including the Instrumental Variable (IV) estimation test, to assess the predictability of crude oil price return, finding limited evidence of such predictability for the full sample. This observation leads us to consider the oil market's similarity to some stock markets, which are unpredictable over most periods but may exhibit brief regimes of local predictability. This phenomenon could arise if a variable begins to demonstrate predictive power for crude oil returns, creating a short window of predictability before investors adapt to the new relationship between that variable and returns, causing the predictive power to dissipate. Therefore, through the use of a (pseudo) real-time monitoring procedure, we discover that the exchange rate of an oil-exporting country exhibits significant predictive ability for oil prices during periods of domestic economic crises or downturns. This paper further explores how different types of financial crises influence forecasting capabilities, providing insights into the dynamic relationship between economic indicators and oil price predictability.

1 Introduction

As commodity financialization advances, the appeal of investing in commodities has grown significantly (Gorton and Rouwenhorst, 2006; Gorton et al., 2013). Crude oil plays a central role in the global economy, not only as a primary energy source but also as a key determinant of macroeconomic stability. In particular, oil-exporting countries experience close linkages between energy prices, fiscal revenues, and exchange rate dynamics. Accurate predictions of crude oil price return are crucial for economic policymakers, investors, and businesses worldwide. Especially for oil-exporting countries, changes in energy prices are an important determinant of economic growth, affecting multiple segments of the economy. Despite the critical need for reliable predictions, the endeavour to forecast crude oil price return has been a persistent challenge, attributed to the market's inherent complexity and the myriad factors influencing price movements (Hamilton, 2009; Alquist et al., 2013).

Many existing studies highlight exchange rates and monetary policy as important factors in forecasting global oil prices. The exchange rates of oil-exporting countries are forward-looking, incorporating expectations about monetary policy changes and production decisions, so it may be possible to use an oil exporter's currency returns to predict trends in international oil prices (Chen and Chen, 2007). Some scholars have indeed identified a transmission of fluctuations between crude oil markets and exchange rates—especially in emerging markets—underscoring the importance of accounting for exchange rate movements in oil price return prediction (Chen et al., 2014; Rosa, 2014; Gospodinov and Jamali, 2015). For example, Chen and Chen (2007) find that "commodity currency" exchange rates often move in tandem with world commodity prices, implying that these currencies reflect information about future commodity market conditions. Recent work has also examined this relationship during turbulent periods. Bigerna (2023) argue that in times of heightened uncertainty—particularly in emerging economies—exchange rate policy can help stabilize inflation and even dampen the covariance between exchange rates and oil prices, highlighting a nuanced interaction between currency values and oil market dynamics.

As discussed in Chapter 3, initial evidence indicated that the exchange rates of certain oil-exporting nations can indeed serve as predictive indicators for future oil price movements. This finding suggests that currency markets in these petro-economies embed forward-looking information about oil market fundamentals. In particular, the previous chapter's analysis showed that both an emerging-market currency (such as the Brazilian real) and a developed oil-producer's currency (such as the Norwegian krone) exhibited measurable power to forecast subsequent changes in international crude prices (Chen and Chen, 2007). Recent research using causality methods lends support to this "oil currency" phenomenon: large swings in the exchange rates of major oil exporters have been shown to precipitate significant oil price variations. However, the predictive ability observed in Chapter 3 was not uniform across all oil exporters, which raises further questions about its drivers and scope. For instance, how widespread is this predictability across different countries? Second, is this predictive ability affected by periods of global volatility or heightened macroeconomic uncertainty? Third, is the forecasting power short-lived or persistent across different time horizons? Since energy prices are a central component of the global macroeconomic outlook, understanding when and how exchange rates serve as forward-looking signals is of practical importance to both policymakers and investors.

In this chapter, we focus on the impact of exchange rates in oil-exporting countries on crude oil price returns and address the following questions: (i) Do the exchange rates of most oil-exporting countries possess predictive ability for international oil prices—whether the exporter is an emerging economy (such as Brazil) or an established one (such as Norway)? (ii) Does global volatility or uncertainty influence this predictive ability, either by strengthening it or undermining it? (iii) Is this predictive ability a long-term phenomenon or only evident in the short run, and does it manifest only during specific periods? In addition, our analysis explores whether different types of exchange rate regimes lead to variations in predictive power regarding crude oil price returns. Given that energy prices are a key determinant of the global economic outlook, an investigation of this nature is crucial. The results shed light on whether and when exchange rates can serve as advance signals of oil market shifts, informing monetary policymakers and investors about appropriate strategies to bolster global economic stability.

The predictive relationship between exchange rates and oil prices, which constitutes the core focus of this study, operates within an environment characterised by complex oil price dynamics. Crude oil markets exhibit pronounced volatility that significantly complicates forecasting efforts, a well-documented phenomenon arising from economic and political events, supply-demand imbalances, and shifting market sentiment (Hamilton, 2009; Kilian, 2009; Alquist et al., 2013). This volatility manifests in two distinct forms with critical implications for empirical modelling: structural breaks and persistent stochastic volatility. First, structural breaks—typically induced by geopolitical crises, major supply disruptions, or OPEC policy shifts—generate abrupt level changes. These breaks introduce instability in parameter estimates and render conventional linear models unreliable if breaks are not accounted for (Andrews, 1993; Rossi, 2005). Second, oil prices also exhibit persistent stochastic volatility, where high-volatility periods (e.g., during financial crises or pandemics) tend to cluster. The co-existence of these phenomena induces non-stationarity in both conditional mean and variance processes, violating foundational assumptions of standard predictive regressions (Hamilton, 1994; Kim et al., 1998). Consequently, the time series of oil prices and exchange rates are non-stationary. This non-stationarity poses challenges for analysis and forecasting because many standard statistical and econometric models assume that the underlying time series are stationary. In this chapter, the IVX predictor used in this chapter is known to be robust to persistent regressors and long-memory behaviour in the predictor variables (Kostakis et al., 2015). Furthermore, the use of real-time monitoring techniques (e.g., Harvey et al., 2021) allows us to assess whether predictable regimes persist during periods of unexpected events. These methods do not necessarily require the independent variable to be stationary.

Numerous oil-exporting nations commonly anchor their currencies to the U.S. dollar, a strategy that renders fiscal planning more stable and controllable. This approach is particularly pertinent in nations where oil exports are a major contributor to state income, such as Saudi Arabia, the UAE, Oman, Venezuela, Qatar, and others. Under a fixed exchange rate system, a country's currency is set at a certain value against another currency, typically the U.S. dollar. Consequently, fluctuations in the international oil market may not influence the exchange rate due to its constant, pre-established rate. Therefore, the static exchange rate under this regime may not be a reliable indicator for predicting oil price movements, given its insensitivity to the oil market dynamics. As a result, we may exclude nations with pegged exchange rates in our analysis of sample countries, despite these countries potentially exporting substantial volumes of crude oil relative to others.

The sample countries examined in this chapter are 6 crude oil-exporting countries: Brazil, Canada, Colombia, Indonesia, Mexico and Norway. The structure of this chapter is as follows. In Section 2, we review the Empirical Literature and then present our Methodology in Section 3. We describe the data in Section 4 and report the results in Section 5 and Section 6. Finally, the conclusions are presented in Section 7.

2 Literature review

Numerous studies in applied economics and finance have delved into the predictability of asset returns. Crude oil, with its fluctuating prices and central role in the global economy, is frequently highlighted as a key asset in these predictive endeavours. Due to its significant impact on macroeconomics, there is a broad consensus that fluctuations in oil prices influence real economic activities (Jo, 2014). Hence, the volatility of oil prices remains a key issue for policymakers, market investors, and workers in the oil industry, among others. It garners significant attention from researchers and economists alike. However, predicting oil prices remains a formidable challenge (Verleger, 1987; Alquist et al., 2013). In this chapter, we review key literature on oil return predictability, introducing new strands of research on persistent regressors and structural breaks while building on concepts from earlier chapters.

2.1 Exchange Rates as Predictors of Commodity Prices

Hamilton (2009) pointed out this complexity, emphasising the myriad economic, geopolitical, and natural factors that contribute to oil price volatility. Recent articles on oil forecasting relationships predominantly rely on highly financialized crude oil futures. Building on ideas introduced in the previous chapter, one potential predictor for commodity prices (like oil) is the exchange rate of commodity-exporting countries. Exchange rate data are widely available and relevant across a broad spectrum of countries, unlike certain financial variables (e.g. dividend yields or term spreads) commonly used for predicting stock returns in developed markets (Welch and Goyal, 2008). Studies by Chen and Chen (2007) and Chen et al. (2010a) suggest that the exchange rates of certain commodity-exporting nations (often termed commodity currencies) contain information about future commodity price movements. These exchange rates are forward-looking, adjusting to news about global demand and supply conditions. Thus, an oil-exporting country's NER might signal upcoming trends in international oil prices.

However, this predictive relationship may not be uniform across all oil-exporting countries. Some currencies exhibit a strong co-movement with oil prices, while others do not. For example, Beine and Senga (2015) find that while the Russian ruble appreciates with rising oil prices in the long run, the Norwegian krone and Saudi Arabian riyal show virtually no such oil price impact. These differences have been attributed to factors like policy interventions (e.g. sterilisation of oil revenues via sovereign wealth funds) and institutional characteristics, rather than just exchange rate regime choices. This implies that the predictive power of exchange rates for oil prices can vary significantly by country. Even for currencies that do not consistently track oil fundamentals, there may be specific periods or regimes where the relationship strengthens. Empirical evidence indeed suggests that exchange rate–oil price linkages are time-dependent and often crisis-sensitive. For instance, Bigerna (2023) document that the causal influence of oil prices on currencies tends to intensify during periods of economic and financial uncertainty. In other words, while even some oil exporter's exchange rate might not predict oil price movements in normal times, it could still exhibit episodic predictability—short-lived "windows" of predictive power during major oil shocks or global crises.

2.2 Structural Breaks and Time-Varying Predictability

Early studies that employed ordinary least squares (OLS) regression to assess predictability with highly persistent financial variables often resulted in misleading inferences. Specifically, these studies tended to over or under-reject the null hypothesis when using non-stationary data, leading to tests that exhibited severe size distortions. As Campbell and Yogo (2006) observed, the log dividend-price ratio is often used to predict returns. While the log dividend-price ratio can theoretically serve as a reliable predictor, the methods used to test for predictability are crucial. It is essential that these methods account for non-stationary predictors, as the practical application of the log dividend-price ratio is complicated by its high persistence. This statistical inference relies on first-order asymptotic distribution theory, where the autoregressive roots of the predictors are modelled as local to unity, i.e., $\rho = 1 - c/T$ with c as a constant. The test statistic only follows a standard normal distribution when the predictor is weakly stationary. However, both simulation and analytical studies have demonstrated that when the predictor is non-stationary, the limit distribution is entirely different from the weak predictor case. Large-sample theory inadequately approximates the actual finite-sample distribution of the test statistic when predictors exhibit persistence and their innovations are strongly correlated with returns. For instance, Elliott and Stock (1994) provides Monte Carlo evidence that, for reasonable parameter values and sample sizes similar to those in standard predictive regressions involving the dividend-price ratio, the size of a nominal 5% test is about 20 %, indicating a substantial size distortion. Stambaugh (1999) also found that high persistence in predictors leads to biased coefficients in predictive regressions when the innovations driving these predictors are correlated with returns, a common characteristic for many widely-used macroeconomic and financial predictors such as exchange rates and commodity prices. Similar studies by Mankiw and Shapiro (1986), Nelson and Kim (1993), Lanne (2002), Torous et al. (2004), and others support these findings. Therefore, numerous studies emphasise employing valid tests to establish predictability, even in cases where predictors exhibit high persistence or contain unit roots.

As Breitung and Demetrescu (2015) and Harvey et al. (2021) pointed out, the validity of inferences in predictive regressions critically depends on the degree of persistence of the predictor variables. Several tests for predictability have been formulated, which are intended to remain valid in situations where the predictor exhibits strong persistence and is endogenous (Campbell and Yogo, 2006; Welch and Goyal, 2008; Jansson and Moreira, 2006). Among the various tests developed, the Campbell and Yogo (2006) test is often considered a good standard in literature for evaluating stock return predictability using highly persistent regressors. These tests are crafted to maintain a controlled size, ensuring that the probability of falsely detecting predictability (Type I error) remains within acceptable bounds, even when predictors are highly persistent, or when there is a high level of correlation between the innovations. However, these tests have a limitation: they don't work well if the predictor is only weakly persistent (or stationary). Nevertheless, it should be highlighted that recent studies by Astill et al. (2023) indicate that the size of the predictor's initial condition significantly influences the results of the Bonferroni Q test. They found that the outcomes are highly dependent on the correlation between the innovations. Specifically, when this correlation is less than zero, and the initial condition is large, the tests are undersized in the right tail and oversized in the left tail. This balance between avoiding Type I errors and detecting true predictability is also reflected in alternative tests that employ instrumental variable (IV) estimation, such as those proposed by Phillips and Magdalinos (2009), Kostakis et al. (2015), and Breitung and Demetrescu (2015). These methodologies include stochastic instruments derived from the predictors, accommodating the inclusion of regressors whose persistence is unknown. Thus, these IV-based tests maintain their validity regardless of the persistence level in the predictors, providing a controlled approach to predictive regression analysis. While these IV-based tests remain valid irrespective of the predictor's level of persistence, they may not be as powerful as CY's tests when the predictor is strongly persistent. In this chapter, we employ both two different predictive regressions to test the predictability of NERs for oil-exporting countries relative to crude oil prices, specifically the CY and IV tests, as detailed in Campbell and Yogo (2006), Kostakis et al. (2015) and Breitung and Demetrescu (2015).

The vast majority of early empirical studies of predictability are based on the assumption of a constant parameter predictive regression model (Harvey et al., 2021). The constant parameter assumption is made for simplicity and is based on the idea that underlying economic relationships are stable over time. However, in reality, commodity prices fluctuate due to unexpected changes in supply and demand. These demand/supply relationships can and often do change due to various reasons like changes in market structure, regulatory shifts, and macroeconomic shocks (Fernandez et al., 2023). The oil market particularly demonstrates that uncertainties are indeed key factors leading to the time-varying nature of economies heavily reliant on oil exports (Zhang et al., 2015; Su et al., 2020). As a matter of historical fact, oil, being a strategic asset, has seen numerous oil crises in the past because of geopolitical uncertainties such as the Persian Gulf War (Lee et al., 2023). Recently, there has been growing interest in models that allow for time-varying coefficients to capture potential changes in relationships over time. There are several reasons to suspect that if crude oil returns are predictable, then it is likely to be a time-varying phenomenon. If the true relationship between exchange rate and crude oil returns changes over time (e.g., due to significant geopolitical events), then a model with constant parameters might not capture this and could potentially lead to misleading inferences about predictability, since traditional methods of analysis may not adequately capture the time-varying nature of this relationship.

2.3 Predictability under Crisis and Precautionary Demand Shocks

As we mentioned before, crude oil prices are highly subject to variation over time due to a variety of factors (Zhang et al., 2015). A solid explanation for abrupt changes in the real price of oil is the oil-specific demand shock. This concept captures changes in anticipatory demand stemming from heightened concerns about future oil supplies, as outlined by Kilian (2009). This type of oil demand is often referred to as "precautionary demand" [1]. One primary way unexpected events can influence the actual oil price is by affecting this precautionary demand. Events such as political upheavals, conflicts that could disrupt the oil supply chain (particularly in key oil-producing regions) such as the Persian Gulf War, global financial crises, and other oil industry-specific factors all play a role. In fact, as Frankel (2006) noted, many victims of the emerging market crises of the 1990s were countries with economies heavily dependent on oil. These countries, including Mexico, Indonesia, and Russia, faced significant challenges, especially due to low oil prices.

As Kilian (2009) pointed out, while no two oil price shocks are exactly the same historically, there are some regularities. Baffes et al. (2015) has found critical similarities and differences among the four oil price collapses, which include the 1985–86 oil price crash, the 2014-2016 downturn, and the 1990–1991 and 2008–2009 crashes in the history of crude oil trading. They categorised the reasons for the collapse of oil prices into two broad categories. The first category pertains to factors unique to the oil industry, while the second category relates to global crisis. Among those crises, the 1985–86 oil price crash bears similarities to the one from 2014-2016. Both were influenced by factors unique to the oil industry and followed periods of a rapid expansion in unconventional supplies and a shift in policy by OPEC after a period of high prices. The high production is driven by non-OPEC countries — with the earlier surge resulting from Alaska, the North Sea, and Mexico, and the latter due to

[1] Precautionary demand can lead to both upward and downward movements in crude oil prices. When market participants are concerned about potential disruptions to future oil supplies, precautionary demand increases. This can be due to a variety of reasons, such as geopolitical tensions, natural disasters affecting oil production, or significant changes in policies of oil-producing nations. An increase in precautionary demand typically pushes oil prices up because buyers are willing to pay more to secure supply amid uncertainty. Conversely, if the concerns about future supply disruptions diminish, the precautionary demand will decrease. This might happen when geopolitical situations stabilise when there's an announcement of new oil reserves being discovered, or when technological advancements make it easier to extract and distribute oil. A decrease in precautionary demand can lead to a fall in oil prices as the urgency to secure oil diminishes and market fears subside. Therefore, the concept of precautionary demand accounts for both the fear of future supply shortages, which can drive prices up and the alleviation of those fears, which can cause prices to fall.

U.S. shale oil, Canadian oil sands, and biofuels. While oil supplies were projected to increase rapidly, oil demand forecasts were revised downwards due to consistently disappointing global growth. Therefore, supply, far more so than demand, largely accounted for the fluctuations in oil prices during the periods of 1985–1986 and 2014-2016. In this instance, OPEC transitioned its strategy from maintaining prices through output regulation to prioritising the preservation of its market share. Another situation, the oil price downturns of 1990-1991, 2008-2009, and the Covid period were rooted in broader concerns such as a significant slump in demand post-crisis, global uncertainties, and liquidity shortages. Specifically, during the 2008-2009 and Covid periods, a severe global recession prompted a decline in nearly all commodity prices, which rebounded quickly once the downturn began to stabilise. Since both situations illustrate the impact of precautionary demand on oil prices, various causes of oil price fluctuations may yield divergent outcomes in oil price prediction.

These events can disrupt supply chains, alter production levels, impact global supply and demand dynamics, and introduce uncertainty into the markets, all of which can lead to fluctuations in oil prices. The situation presented an unforeseen, unparalleled jolt to the global economy, with no previous data hinting at its potential economic effects. Hence, during the crisis outbreak, conventional prediction models quickly became obsolete, with their efficacy declining sharply. Multiple issues contributed to their downfall. However, for effective policy formulation and assessment during these challenging times, leaders needed immediate forecasts and real-time insights into the economic state (Ferrara and Sheng, 2022; Barbaglia et al., 2023). However, achieving accurate predictions was particularly difficult during such global upheaval. Discovering the novel methods and knowledge gained during this period could assist policymakers in better real-time economic analysis, providing a more current foundation for forecasting and scenario evaluations.

Bai and Perron (1998) and Bai and Perron (2003)'s studies provide evidence suggesting that parameter instability can cause predictions to be significantly biased. They emphasize that when structural breaks or parameter instabilities exist in the data and are not accounted for in the model, the model's predictions could deviate considerably from the actual outcomes. Many other studies also suggest that parameter instability is a characteristic of return prediction models (Goyal and Welch, 2003; Paye and Timmermann, 2006; Harvey et al., 2021). While the Bai–Perron tests introduced methods to compute multiple breakpoints, they are not suitable for use with highly persistent, endogenous predictors. Harvey et al. (2021) develop new statistical monitoring techniques for change in real-time to avoid spurious detection problems. Their identification methods revolve around the sequential use of straightforward heteroskedasticity-robust regression t-statistics, which measure the significance of the predictor variable over a fixed-length subsample, m. Their test determines whether the predictive behaviour observed in one subsample is statistically distinct from the predictive behaviour in all other subsamples, thereby identifying changes in the predictive relationship across the dataset. This is because a rejection will occur where the estimated slope coefficient on the predictor differs significantly between the subsample over which the one-shot test is based and the subsamples used in the critical value generation. This test will exhibit significant power when predictability exists within the latest m observations, but not within the training period. Harvey et al. (2021) introduced two detection methods: The first method, referred to as the MAX procedure, compares the series of statistics from the monitoring data with the most extreme value from the training data. A predictability regime is indicated when the monitoring data's predictability statistic surpasses the extreme value from the training set. The second procedure is denoted SEQ: In this method, a predictability regime is recognised when the number of consecutive rejections by one-shot tests in the monitoring data exceeds the longest series of such rejections in the training data. The critical value for these tests is determined by subsampling from the training data. More details for the two methods will shown in the Methodology section.

3 Methodology

3.1 KMS's test

We focus our analysis on a predictive regression model for oil price returns, denoted by y_t, using the log exchange rates as the lagged potential predictor, x_{t-1}, and its coefficient, β. The model is specified as follows:

$$\begin{aligned} y_t &= \alpha + \beta x_{t-1} + u_t \quad & t = 1, ..., T \\ x_t &= \mu_x + \xi_t \quad & t = 0, ..., T \\ \xi_t &= \phi \xi_{t-1} + v_t \quad & t = 1, ..., T \end{aligned} \tag{4.1}$$

We denote by $\delta = \sigma_{uv}/\sigma_u\sigma_v$ the contemporaneous correlation coefficient between u_t and v_t. The covariance vector $\epsilon_t = (u_t, v_t)'$ is distributed as $N(0, \Sigma)$, where

$$\Sigma = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}$$

The underlying principle of the IVX procedure, as detailed by Phillips and Magdalinos (2009), is to instrument the regressor x_{t-1} with a variable that exhibits controlled persistence, defined as follows:

$$z_0 = 0 \quad and \quad z_t = \sum_{j=0}^{t-1} \rho^j \triangle x_{t-j}, t = 1, ..., T, \tag{4.2}$$

where $\rho := 1 - \alpha T^{-\eta}$ with $\alpha > 0$ and $0 < \eta < 1$. We set $\alpha = 1$ and $\eta = 0.95$, the IVX t-statistic is given by:

$$t_{zx} := \frac{\hat{\beta}_{zx}}{s.e.(\hat{\beta}_{zx})} \tag{4.3}$$

where $\hat{\beta}_{zx}$ is the IVX estimator of β.

$$\hat{\beta}_{zx} = \frac{\sum_{t=1}^{T} z_{t-1}(y_t - \bar{y})}{\sum_{t=1}^{T} z_{t-1}(x_{t-1} - \bar{x}_{-1})} \tag{4.4}$$

$$s.e.(\hat{\beta}_{zx}) = \frac{\sqrt{\hat{\sigma}_u \sum_{t=1}^{T} z_{t-1}^2 - \Xi}}{\sum_{t=1}^{T} z_{t-1}(x_{t-1} - \bar{x}_{-1})} \tag{4.5}$$

with $\hat{\sigma}_u^2 = \sum_{t=1}^{T} \hat{u}_t^2$, $\bar{y} = T^{-1} \sum_{t=1}^{T} y_t$, $\bar{x}_{-1} = T^{-1} \sum_{t=1}^{T} x_{t-1}$, and $\Xi := T\bar{z}_{-1}^2(\hat{\sigma_u}^2 - \hat{\sigma}_{uv}^2 \hat{\sigma}_v^{-2})$ with $\bar{z}_{-1} := T^{-1} \sum_{t=1}^{T} z_{t-1}$ and where $\hat{\sigma}_v^2$ and $\hat{\sigma}_{uv}$ are estimates of the long run variance of v_t and the long run covariance between u_t and v_t, respectively. In the regression model specified by Equation (4.4), the null hypothesis, denoted as H_0, posits that the coefficient β is equal to zero. KMS show that the regression t-statistic from this IVX estimation is asymptotically normally distributed, allowing for comparison to Gaussian critical values. If H_0 is rejected, it indicates a statistically significant predictive relationship between the lagged exchange rate, x_{t-1}, and the crude oil returns, y_t. Conversely, if H_0 is not rejected, it signifies that the lagged exchange rate x_{t-1} fails to provide any actionable information for predicting crude oil returns, indicating an absence of predictability within the context of the model.

3.2 CY's test

The difference between the approach of Campbell and Yogo (2006) and that of Kostakis et al. (2015) lies not merely in how they address endogeneity and weak instruments, but in the fundamental methodology employed. Campbell and Yogo (2006) developed a method to calculate a confidence interval for β by employing a t-statistic that is derived from a predictive regression, which is enhanced by incorporating the covariate $(x_t - \rho x_{t-1})$, which does not hinge on the use of instrumental variables. This technique provides a robust framework for constructing confidence intervals for the predictive regression coefficient, ensuring the test's validity even when faced with persistent predictors and the challenges of endogeneity. Their approach diverges from typical IV-based methods, offering an alternative strategy. The Q-statistic outlined in this chapter is tailored for an AR(1) predictor. However, in the actual application, we used the AR(p) version as detailed by Campbell and Yogo (2006) in their Appendix A. They assume that the predictor x_t is strongly persistent, and the innovations are independently and identically distributed (i.i.d.) as multivariate normal with an estimated covariance matrix.

As mentioned in previous subsection, σ_u^2 represents the variance of the regression errors u_t, σ_v^2 represents the variance of the innovations v_t, and σ_{uv} represents the covariance between u_t and v_t. Campbell and Yogo (2006) propose testing for predictability based on Bonferroni procedures that make use of confidence intervals for the unknown autoregressive parameter $\rho = 1 - c/T$, with these confidence intervals constructed by inverting unit root tests. The null hypothesis of CY is that there is no hypothesis of no predictability, $H_0 : \beta = 0$, against the alternative $H_1 := \beta \neq 0$. CY derive the optimal test for H_0 against H_1:

$$Q(\beta_0, \rho) := \frac{\sum_{t=1}^{T} x_{t-1}^{\mu} \left[y_t - \beta_0 x_{t-1} - \beta_{uv}(x_t - \rho_{x_{t-1}}) \right]}{\sqrt{\sigma_u^2 (1 - \delta^2) \sum_{t=1}^{T} (x_{t-1}^{\mu})^2}} \tag{4.6}$$

where $\beta_{uv} = \sigma_{uv}/\sigma_v^2 . Q(\beta_0, \rho)$ is normally distributed but only under the null and when the true value of rho is known. A $100(1-\alpha)\%$ confidence interval for β is given by $[\underline{\beta}(\rho, \alpha), \bar{\beta}(\rho, \alpha)]$ where

$$\beta(\rho) := \frac{\sum_{t=1}^{T} x_{t-1}^{\mu} \left[y_t - \beta_{uv}(x_t - \rho_{x_{t-1}}) \right]}{\sqrt{\sigma_u^2 (1 - \delta^2) \sum_{t=1}^{T} (x_{t-1}^{\mu})^2}} \tag{4.7}$$

$$\underline{\beta}(\rho, \alpha) = \beta(\rho) - z_{\alpha/2} \sigma_u \sqrt{\frac{1 - \delta^2}{\sum_{t=1}^{T} (x_{t-1}^{\mu})^2}}$$

$$\bar{\beta}(\rho, \alpha) = \beta(\rho) + z_{\alpha/2} \sigma_u \sqrt{\frac{1 - \delta^2}{\sum_{t=1}^{T} (x_{t-1}^{\mu})^2}}$$

with $z_{\alpha/2}$ denoting the $\alpha/2$ quantile of the standard normal distribution.

3.3 IV combining test

Breitung and Demetrescu (2015) investigated the feasibility of instrumental variable tests in predictive regression models, also considering issues with endogeneity and unknown regressor persistence. The IV_{comb} test is also based on the procedure outlined by Phillips and Magdalinos (2009). They described several instruments with either less persistence than the predictor variable (type-I instruments) or those that are completely independent of the error term (type-II instruments), to enhance the estimation process. Type-I instruments are divided into four different types. Each instrument demonstrates inherently lower persistence compared to a nearly integrated process and is devoid of stochastic trends. This attribute holds whether x_{t-1} is nearly integrated or stationary and is driven by the same innovations. Type-I instruments are shown as below:

1. A short memory instrument where $z_{t-1} = (1 - \overline{\alpha}L)_{+}^{-1}\Delta x_{t-1} = \Delta x_{t-1} + \overline{\alpha}\Delta x_{t-2} + \cdots + \overline{\alpha}^{t-2}$. Here, L is the lag operator and $|\overline{\alpha}| < 1$, ensuring the instrument has less persistence than the predictor.

2. A mildly integrated instrument defined by $z_{t-1} = (1 - \alpha_T L)_{+}^{-1}\Delta x_{t-1}$, with $\alpha_t = 1 - aT^{-\eta}$ where $a > 0$, and $0 < \eta < 1$.

3. A fractionally integrated instrument expressed as $z_{t-1} = (1 - L)^{d^*} x_{t-1}\mathbb{I}(t > 0)$for $d^* \in (0, 1/2)$;.

4. A long differences instrument takes the form $z_{t-1} = x_{t-1} - x_{t-k_T}$ for k_T set to $\min(kT^u, t-1)$, where $0 < v < 1$ and K is a positive constant, which effectively captures long-run movements in the predictor.

The IVX approach we mentioned before is an example of using a mildly integrated instrument for regression analysis. Breitung and Demetrescu (2015) argue that instruments such as trends or trigonometric functions can be effective even when they do not convey specific information about the regressor. Consequently, three Type-II instruments, which are completely independent of the error term, are exemplified as follows:

1. A generated random walk instrument shown as:

$$z_{t-1} = (1-L)_{+}^{-1} w_{t-1}, \quad \text{where} \quad w_{t-1} \sim IID(0, \sigma_w^2)$$

   with w_t independent of u_t and v_t.

2. Deterministic functions of time, such as:

$$z_{t-1} = (t-1) \quad \text{or} \quad z_{t-1} = \sin\left(\frac{\pi(t-1)}{2T}\right)$$

   which are exogenous with respect to u_t by construction.

3. Cauchy instruments:

$$z_t = \text{sign}(x_t)$$

   which, due to their construction, are also exogenous with respect to u_t.

As we mentioned before, each instrument is exogenous to u_t. However, they do not utilise specific information about x_t, except in scenarios where x_t is near-integrated, in which case there may be a correlation with x_t. Breitung and Demetrescu (2015) simulations suggest type-II instruments outperform type-I when the predictor is highly persistent, although type-I instruments may be nearly uncorrelated with stationary predictors. They advocate for a combined test using both instrument types to exploit the strengths of each. The technical formula provided represents a test statistic used to evaluate the efficacy of these instruments, accounting for heteroskedasticity by using Eicker-White heteroskedasticity consistent standard errors (Eicker, 1967; White, 1980)[2]. The IV combining test's t-statistic by using Eicker-White heteroskedasticity-robust standard errors is shown as the following Equation (4.8):

[2] The t-statistic by using Eicker-White heteroskedasticity consistent standard errors is given by $t_i = \left(\sum_{t=2}^{T} \hat{z}_{t-1}^2 \hat{u}_t^2\right)^{-1/2} \sum_{t=2}^{T} \hat{z}_{t-1} y_t$, where $\hat{u}_t = y_t - \hat{\beta} x_{t-1}$ and $\hat{\beta}$ is the OLS estimator of β in Equation (4.1).

$$t_{IV_{comb}} = \frac{\left(\sum_{t=2}^{T} x_{t-1}z_{t-1}'\right)\left(\sum_{t=2}^{T} z_{t-1}z_{t-1}'\right)^{-1}\left(\sum_{t=2}^{T} z_{t-1}y_t\right)}{\sqrt{\left(\sum_{t=2}^{T} x_{t-1}z_{t-1}'\right)\left(\sum_{t=2}^{T} z_{t-1}z_{t-1}'\right)^{-1}\left(\sum_{t=2}^{T} z_{t-1}z_{t-1}'\hat{u}_t^2\right)\left(\sum_{t=2}^{T} z_{t-1}z_{t-1}'\right)^{-1}\left(\sum_{t=2}^{T} x_{t-1}z_{t-1}\right)}} \tag{4.8}$$

where z_t is vector of instruments and $\hat{u}_t$ representing the residuals from estimating Equation (4.1). This test is appropriately conducted as a two-tailed test exclusively. The statistic is characterised by a standard normal asymptotic null distribution, which guarantees the test's validity regardless of the predictor's persistence or the presence of heteroskedasticity in the error terms.

3.4 Real-time monitoring procedures

The inherently volatile nature of crude oil markets, influenced by complex geopolitical, economic, and environmental factors, renders the task of accurately predicting crude oil prices particularly challenging (Hamilton, 2009; Kilian, 2009). Consequently, long-term forecasts may not yield highly accurate results, especially since oil prices can be time-varying. Another related hypothesis posits that the discovery of a new variable's predictability leads to its arbitrage, ultimately nullifying its predictive power. Hence, predictability exists prior to its discovery, typically within a brief short window. Given this ever-changing landscape, it is not practical to assume a same predictive DGP across the entire sample period. Instead, identifying distinct intervals of predictability within the market fluctuations is more prudent. Therefore, we also utilised real-time monitoring procedures for the emergence of end-of-sample predictive regimes (Harvey et al., 2021). The model responsible for generating data for the dependent variable y_t and its predictor x_t is presented as:

$$y_t = \alpha + \sum_{j=1}^{n} \beta_j d_t(e_j, m_j) x_{t-1} + u_{y,t}, \quad t = 1, ..., T. \tag{4.9}$$

where n is the number of predictive regimes and their duration, $m_j, j = 1..., n$, the predictor x_t is articulated as:

$$\begin{aligned} x_t &= \mu_x + \xi_{x,t}, \quad t = 0, ..., T \\ \xi_{x,t} &= \phi \xi_{x,t-1} + v_{x,t}, \quad t = 1, ..., T. \end{aligned} \tag{4.10}$$

with the starting condition $\xi_{x,0} = 0$. It should be noted that the methodology remains valid for an AR(p) predictor for $\xi_{x,t}$, as discussed in Harvey et al. (2021). The function $d_t(e_j, m_j)$ stands as a dummy variable that equals 1 for a consecutive string of m_j >0 dates ending at $t = e_j$. The perturbation vector u_t is bidimensional with components $[u_{y,t}, u_{x,t}]'$, and it is posited to be stationary. Recall that δ_{xy} denotes the correlation between $u_{y,t}$ and $v_{x,t}$ and is restricted such that $|\delta_{xy}|$ <1. Notably, the assumptions of Harvey et al. (2021) allow for potential conditional heteroscedasticity, evident in models like GARCH or stationary autoregressive stochastic volatility, for both disturbances. Acknowledge that it is challenging to match assumptions across all these tests. In the context of Equation (4.9), a predictive regime for y_t via x_{t-1} is identified, spanning m_j data points from $t = e_j - m_j + 1$ through to $t = e_j$. It is conceivable that the predictor variable may not be capable of predicting y_t, or that there could exist multiple, relatively small regions capable of predicting y_t throughout the entire sample period. Based on the DGP Equation (4.9), $n \geq 0$. This means the model can accommodate any number of predictive regimes, including the possibility of having none. The predictive regime detection procedure will take into account the variables e_j and m_j, which define the start and end dates of the predictive regimes, as well as the number n[3]. There are two pivotal time points: the end of the monitoring, indicated by $t = E$, and the theoretical endpoint of the DGP for y_t, symbolised by T, where it is understood that E does not exceed T.

[3]The Real-time monitoring procedure is applicable across a spectrum of scenarios, encompassing weakly dependent ($|\rho| < 1$), strongly persistent ($\rho = 1 - \frac{c}{T}$ with $c \geq 0$), and moderately persistent ($\rho = 1 - cT^{-\theta}$ with $c > 0$ and $\theta \in (0, 1)$) time series data.

Harvey et al. (2021) propose using subsample regression t-statistics to detect the presence of a predictive regime in y_t. The OLS regression in Harvey et al. (2021) is shown as follows:

$$y_t = \alpha + bx_{t-1} + u_t, \quad t = e - m + 1, ..., e. \tag{4.11}$$

This subsample initiates m periods prior to the endpoint e and continues through to e. For our analysis, we have selected the values {15, 30, 45, 60, 90} for m, with each subsample ranging from $t = e - m + 1$ to $t = e$. The regression t-statistic for the significance of x_{t-1} over a window of m observations ending at time e is shown as:

$$\tau_{e,m} := \hat{b}[\hat{V}(\hat{b})]^{\frac{1}{2}} \tag{4.12}$$

where

$$\hat{b} := \frac{\sum_{t=e-m+1}^{e}(x_{t-1} - \overline{x}_{-1})(y_t - \overline{y})}{\sum_{t=e-m+1}^{e}(x_{t-1} - \overline{x}_{-1})^2}, \quad \hat{V}(\hat{b}) := \frac{\sum_{t=e-m+1}^{e}(x_{t-1} - \overline{x}_{-1})^2 \hat{u}_t^{2}}{\left\{\sum_{t=e-m+1}^{e}(x_{t-1} - \overline{x}_{-1})^2\right\}^2} \tag{4.13}$$

$$\begin{aligned} \hat{u} &:= (y_t - \overline{y}) - \hat{b}(x_{t-1} - \overline{x}_{-1})^2 \\ \overline{y} &:= m^{-1} \sum_{t=e-m+1}^{e} y_t, \ \overline{x}_{-1} := m^{-1} \sum_{t=e-m+1}^{e} x_{t-1}. \end{aligned} \tag{4.14}$$

Because this method aims to detect short-term predictive mechanisms, it may not be suitable to employ asymptotic (in terms of sample size T) distribution theory to approximate the test's critical value. As Harvey et al. (2021) point out, if m is considered a function of T, the limiting distribution of $\tau_{e,m}$ will be influenced by nuisance parameters in the DGP. Thus, instead, we need to use a subsampling approach that accounts for the small window size. For the specified subsamples $t = e - m + 1, \ldots, e$, the detection of predictive states between y_t and x_{t-1} can be based on $\tau_{e,m}$, as demonstrated in Equations (4.12) to (4.14). To illustrate, assume that we have data available for $t = 1, \ldots, T^*$ with $T^* + m \leq T$ where T^* denotes the end of the training period. The existence of a predictive mechanism for the last m sample observations would be determined using the statistic $\tau_{T^*+m,m}$.

To obtain the critical values, Harvey et al. (2021) employs the training period $t = 1, \ldots, T^*$ to compute statistics $\tau_{e,m}$, analogous to test statistics of interest for $e = m+1, \ldots, T^*$. The $1-\pi$ quantile of these statistics provides the estimated significance level-π critical value for tests of predictability at the end of the sample. These tests, asymptotically robust to the sample size T, are robust to nuisance parameters in the DGP since the training period's statistics share the same functional dependency as $\tau_{T^*+m,m}$. Such a test possesses notable power when there is predictability in the last m observations, absent during the training period. Consequently, for every conceivable end date of the subsample, $e = T^* + m, \ldots, E$, where E signifies the monitoring period's end, $\tau_{e,m}$ as depicted in Equation (4.12) can discern predictive regimes within the monitoring time frame. The predictive regime detection procedures, MAX and SEQ, which we introduce subsequently, hinge on contrasting this sequence of statistics computed using data in the monitoring period with statistics computed using data in the training period. These procedures are methodically crafted to ensure that the theoretical (large-sample) False Positive Rate (FPR) representing the likelihood of erroneously signalling at least one predictive regime during the monitoring phase is known, with Harvey et al. (2021) showing it is a function of the relative lengths of the training and monitoring periods

MAX and SEQ procedure

This section presents Harvey et al. (2021)'s methodologies for detecting short regimes of predictability. These methodologies are initially applied to scenarios characterised by upper tail outcomes, where the coefficient β_j is positive. Nevertheless, these methods are versatile and can be easily modified for scenarios that require lower tail or both tails consideration. We will discuss two approaches MAX and SEQ. These criteria are based on the characteristics of the sequence of $\tau_{e,m}$ statistics acquired during the training phase. In outlining the subsequent procedures, the duration designated as the training period spans as $t = 1..., T^*$.

with the underlying assumption being the absence of any predictive regime during this phase; specifically, the end of the training period T^* precedes the beginning of any potential predictive regime. The initial approach is predicated on the maximum value attained by the $\tau_{e,m}$ statistics in the training phase, while the subsequent approach considers the most extended sequence in which the $\tau_{e,m}$ statistics surpass a predetermined critical threshold. Further assumptions include setting T^* and E according to $T^* := \lfloor \lambda_1 T \rfloor$, and $E := \lfloor \lambda_2 T \rfloor$, with the notation $\lfloor \ldots \rfloor$ representing the floor function, which returns the greatest integer less than or equal to a given number, and λ_1 and λ_2 satisfying $0 < \lambda_1 < \lambda_2 \leq 1$.

As we mentioned before, MAX operates by assessing the maximum of a sequence of test statistics, $\tau_{e,m}$, across designated training and monitoring periods. The procedure flags a predictive regime if the peak statistic in the monitoring period surpasses the highest statistic observed in the training period.This approach is highly sensitive to isolated strong signals (e.g., sudden large β_j shifts) and typically detects regimes earlier than SEQ when predictability emerges late in the monitoring period (Harvey et al., 2021).

The procedure's validity is gauged by its FPR. The FPR at monitoring time t is defined theoretically as the probability of observing at least one false rejection of the null hypothesis up to time t when no predictive regime exists. This corresponds to the limiting probability $T \to \infty$ that the procedure spuriously detects a regime shift during the monitoring period. Formally, this rate is expressed through the asymptotic ratio of the number of exceedance statistics in the monitoring period to the total statistics from both calibration and monitoring periods, as illustrated in the following equations. The FPR (α) of the procedure for a given training sample length, T^*, window width, m, and monitoring horizon E is given by:

$$\lim_{T \to \infty} P\left(\max_{e \in [T^*+m, E]} \tau_{e,m} > \max_{e \in [m+1, T^*]} \tau_{e,m} \right) = \alpha^*, \tag{4.15}$$

where α^* is derived from the ratio of the period lengths, encapsulated in Equation (4.16):

$$\alpha := \frac{E - T^* - m + 1}{E - 2m + 1}. \tag{4.16}$$

The maximal monitoring horizon E for a stipulated FPR level α is shown as Equation (4.17):

$$E = \frac{T^* + m - 1 - \alpha(2m - 1)}{1 - \alpha}. \tag{4.17}$$

Differing from the MAX procedure, SEQ is predicated on identifying the lengthiest contiguous sequence of exceedances above a practitioner-defined threshold within the statistics $\tau_{e,m}$ during the monitoring period, contrasted with the training period measure. Then, cv_π is defined such that $cv_\pi := \tau_{((1-\pi)(T^*-m))}$ where $\tau_{(j)}, j = 1, \ldots, T^* - m$ are the ascending order statistics of $\tau_{e,m}, e = m + 1, \ldots, T^*$ (i.e., $\tau_{(j+1)} > \tau_{(j)}$ for $j = 1, \ldots, T^* - m - 1$). Based on cv_π, $R_{\pi,e}$ is represented by the following measure:

$$R_{\pi,e} = 1(\tau_{e,m} > cv_\pi), \tag{4.18}$$

where $1(\cdot)$ denotes the indicator function,and the measure over $e = L$ to $e = U$ with $U \geq L$ is given by:

$$R_\pi(L, U) = (U - L + 1) \prod_{e=L}^{U} R_{\pi,e}. \tag{4.19}$$

It leverages an empirical critical value cv_π [4], which is derived from subsampling within the training period using the order statistics of $\tau_{e,m}$. And, the SEQ procedure does not depend on the consistency of cv_π. The primary statistic for the SEQ method is $R_\pi(L, U)$, representing the maximum count of contiguous $\tau_{e,m}$ values exceeding cv_π within the training period. A predictive regime during the monitoring period is signalled if the maximum sequence length of exceedances in the monitoring period outstrips that of the training period's maximum. Therefore, SEQ may excel in identifying persistent regimes (e.g., longer m_j) and demonstrates superior power when predictability begins before monitoring starts or involves gradual parameter changes. If there is an absence of predictability in the training or monitoring periods, the limiting probability that the SEQ procedure will erroneously signal a predictive regime is capped by α^*, as established in Equation (4.20).

[4] π could be any conventional significance level, such as 0.10 or 0.05.

$$\lim_{T\to\infty} \Pr\left(\max_{L,U\in[T^*+m,E]} R_\pi(L,U) > \max_{L,U\in[m+1,T^*]} R_\pi(L,U)\right) \le \alpha^*, \tag{4.20}$$

where α^* is defined in Equation (4.15) to (4.16). Harvey et al. (2021) have assumed that no predictive regimes exist within the training period up to this point. However, they anticipate that the presence of such regimes could reduce the effectiveness of these procedures in detecting new predictive regimes in the monitoring period. In addition, the discussion has so far implied that the training period is chosen to start from the earliest available time in the dataset up to just before the start of the monitoring period. This maximises the length of the training period, ensuring that the FPR is as small as possible for a given monitoring horizon E. Harvey et al. (2021) suggest that in cases with a very long history of data, it may be wise to use only recent data to avoid including historical predictive regimes in the training period. This test will exhibit significant power when predictability exists within the latest m observations, but not within the training period.

The selection between MAX and SEQ monitoring procedures fundamentally depends on the anticipated characteristics of predictability regimes. MAX is preferable when expecting abrupt, strong predictability regimes emerging during monitoring (e.g., post-structural breaks). It requires no significance level (π) choice. SEQ is better suited for detecting persistent regimes starting before monitoring or evolving gradually. For analysing oil-exporting currencies' predictive power over crude oil prices, MAX may emerge as the theoretically preferred approach due to petroleum markets exhibiting pronounced short-term shock-driven behaviour (Hamilton, 2009), where predictability typically materializes abruptly through geopolitical crises, supply disruptions, or OPEC+ policy shifts. Although the advantages of both methods are well understood, it remains unclear which is more appropriate for testing the predictive ability of exchange rates from oil-exporting countries on international oil prices. While the MAX procedure may be more suitable for the characteristics of our sample, it remains uncertain in practice which method is most appropriate. Therefore, both the MAX and SEQ procedures are employed in this chapter.

4 Data

The selection of crude oil-exporting countries follows the same criteria outlined in Subsection 2.3 of Chapter 3, guided by specific theoretical and practical considerations to ensure a robust analysis of the relationship between exchange rates and oil price forecasting. First, these economies represent small-to-medium open systems where external oil shocks rapidly transmit to exchange rates due to trade openness, without significantly influencing global prices (Chen et al., 2010b; Kohlscheen et al., 2017). Second, their institutional transparency, political stability, and absence of major conflicts during 1993–2022 minimise structural breaks in price transmission channels (Hamilton, 2009; Ansari, 2017). Third, non-OPEC status (or

Figure 10: log NEER during sample period

Indonesia's suspension during 2009–2015) and freely floating exchange rates ensure market-driven dynamics, unlike OPEC producers with pegged regimes or quota distortions (Frankel, 2012; Baumeister and Kilian, 2016; Husain et al., 2015; Beckmann et al., 2020). Fourth, consistent long-term data availability supports reliable modelling. Finally, their heterogeneous oil reliance—from highly sensitive (Colombia, Brazil) to diversified (Canada, Norway)—enables a nuanced examination of how export dependence shapes forecasting linkages.

To obtain more comprehensive results, we utilise the Nominal Effective Exchange Rate (NEER), Real Effective Exchange Rate (REER), and Nominal Exchange Rate (NER) of crude oil-exporting countries to predict Brent crude oil returns. Specifically, NEER is defined as the geometrically weighted average of the bilateral nominal exchange rates between the home country and its trading partners. [5] The rationale for including NEER, as opposed to using only the nominal exchange rate, is that short-run fluctuations in the real effective exchange rate (REER) are primarily dominated by movements in nominal exchange rates, while inflation rates tend to be sticky and contribute relatively little to short-run changes in the real exchange rate. Consequently, NEER may capture a more comprehensive picture of currency movements against multiple trading partners, providing greater robustness and clarity for analyzing exchange rate dynamics than a single bilateral nominal exchange rate would. Our dataset encompasses monthly observations from January 1995 to July 2023, totalling 343 observations. The sources of our data are detailed as (1) The NER is sourced from the Federal Reserve Bank of St. Louis (FRED) [6]. (2) The Brent crude oil price is also obtained from FRED. (3) The NEER and REER data are cited from Darvas (2021). Figures 10 to 13 below showcase the three logarithmic exchange rates, along with the Brent oil return, during our sample period.

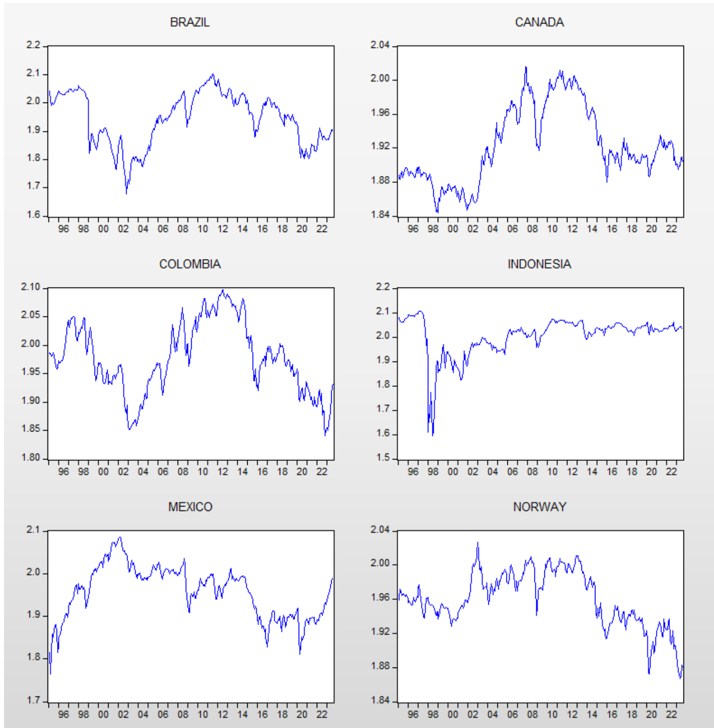
[5] NEER is calculated as:

$$NEER_{i,t,E} = \prod_{j=1}^{N} s_{i,j,t}^{w_{i,j}} \tag{4.21}$$

where: $NEER_{i,t,E}$ denotes the nominal effective exchange rate for country i at time t; $s_{i,j,t}$ represents the nominal bilateral exchange rate between country i and trading partner j, measured as the foreign currency price of one unit of domestic currency (an increase indicates appreciation of the home currency); $w_{i,j}$ indicates the weight of trading partner j, with the weights summing to one ($\sum_{j=1}^{N} w_{i,j} = 1$).

[6] The original NER from FRED is expressed as the domestic currency price of one unit of foreign currency; we have converted this to represent the foreign currency price of one unit of domestic currency for consistency. Since most oil-exporting countries exhibit relatively small nominal values, a majority of these nations tend to have negative values after taking logarithm.

Figure 11: log REER during sample period



5 Preliminary results

5.1 Unit root test

We conduct a unit root test for all predictor variables to check their persistence in levels before initiating the predictive process. We employ the Augmented Dickey-Fuller (ADF) test for this purpose. The presence of a unit root suggests non-stationarity in the series. The results of the ADF test are presented in Table 19.

The ADF test statistics for the log exchange rates (NER, NEER, REER) for countries such as Brazil, Canada, Colombia, Indonesia, Mexico, and Norway are presented in Table 19. As we expected, most of the ADF tests cannot reject the null hypothesis of a unit root, indicating that the majority of the exchange rates are not stationary. Significant p-values, denoted by asterisks, indicate a rejection of the null hypothesis of a unit root, which implies stationarity within the series. Notably, all three Indonesian exchange rates demonstrate significant statistics, with NER and NEER only rejecting the null at the 10% significance

Figure 12: log NER during sample period

level, and REER at the 5% level. Additionally, the Mexican REER is rejected at the 5% significance level. However, several series, like the Brazilian NER, display p-values above 0.10, providing insufficient evidence to reject the null hypothesis of a unit root at the conventional significance levels. It is observable that the exchange rates of most oil-exporting countries exhibit relatively high persistence.

After the ADF test, we begin our study with a preliminary analysis, using well-established methods to assess predictability, as shown in the following Table 20. This table lists the estimated slope parameters ($\hat{\beta}$), along with standard and adjusted R^2 values, for traditional bivariate regression models. These models are applied using OLS for parameter estimation, alongside the IV_{comb} test by Breitung and Demetrescu (2015) and the IVX test introduced by Kostakis et al. (2015). Additionally, we applied the CY test in Table 21, finding its results to be largely consistent with those from the KMS and IV_{comb} analysis.

Figure 13: oil return during sample period

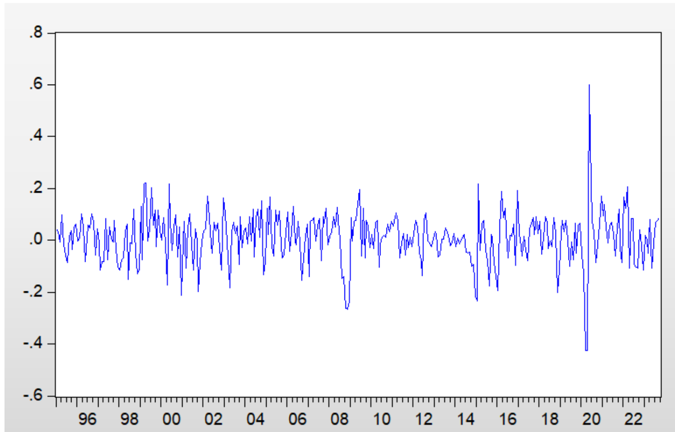


Table 19: ADF test of $log(EX_t)$

ADF test	Brazil ner	Canada ner	Colombia ner	Indonesia ner	Mexico ner	Norway ner
T-statistic	-1.47(0.55)	-1.64(0.46)	-2.06(0.26)	-2.73(0.07)*	-1.64(0.46)	-1.49(0.54)
ADF test	Brazil neer	Canada neer	Colombia neer	Indonesia neer	Mexico neer	Norway neer
T-statistic	-2.06(0.26)	-1.82(0.37)	-2.08(0.25)	-2.78(0.06)*	-1.65(0.45)	-1.09(0.72)
ADF test	Brazil reer	Canada reer	Colombia reer	Indonesia reer	Mexico reer	Norway reer
T-statistic	-2.43(0.13)	-1.75(0.41)	-2.22(0.20)	-3.43(0.01)**	-3.25(0.02)**	-1.45(0.55)

Note: In our specific application, we utilise the ADF test with an intercept. The P-value of the ADF test is in parentheses. The one-sided ADF test critical value for $1\%, 5\%$ and 10% is -3.45, -2.86 and -2.57.

5.2 Full sample predictability tests

In Chapter 3, we employed in-sample and out-of-sample forecasting approaches to examine whether exchange rates could predict crude oil futures prices. Those results indicated that only a few countries (notably Brazil, Colombia, and Mexico) exhibited significant predictive power, whereas most other oil-exporting countries' exchange rates did not show a statistically meaningful ability to forecast oil prices. This lack of strong predictability using straightforward forecasting methods suggests that a simple full-sample predictive regression may also yield limited evidence of predictability for many cases. Consequently, our focus shifts in this chapter from the pure forecasting methods of Chapter 3 to a predictive regression framework to further investigate the exchange rate–oil price nexus.

NEER, REER and NER in Table 20 report the KMS and IV_{comb} results for the full sample

January 1995–July 2023 (T=343), by comparing the test statistic between KMS and IV_{comb} with our results for three different exchange rates. In Table 21, we report the 90% Bonferroni

Table 20: Preliminary results for the full sample for Brent

NEER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.050475	0.032605	0.46917	0.17643	-1.632
Canada	-0.05964	0.87155	0.10445	-0.1893	-0.310
Colombia	-0.033669	0.7376	0.15749	-0.13616	-1.114
Indonesia	-0.0022856	0.66605	0.0020215	-0.29209	-0.310
Mexico	-0.025357	0.19242	0.11702	-0.17676	-0.877
Norway	-0.11046	0.94184	0.18253	-0.11106	-0.548
REER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.11953	-0.17754	1.2084	0.91781	-1.934*
Canada	-0.075936	1.3345	0.11419	-0.17959	-0.339
Colombia	-0.099718	1.5591	0.38039	0.087394	-1.018
Indonesia	0.0055488	1.434	0.0019207	-0.29219	0.145
Mexico	-0.092431	-0.090265	0.32223	0.02906	-0.666
Norway	-0.1022	1.0867	0.1035	-0.19031	-0.403
NER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.233	0.21982	0.23609	-0.0573	-1.1456
Canada	-0.0556	1.56	0.110	-0.1837	-0.332
Colombia	-0.0205	0.78623	0.114	-0.17962	-0.875
Indonesia	0.003	1.101	0.003	-0.2911	0.001
Mexico	-0.022033	0.25878	0.097776	-0.19605	-0.757
Norway	-0.026962	1.5333	0.043861	-0.25013	-0.192

Note: The 10% critical value used for IV_{comb} and KMS is ±1.645.

confidence intervals for β using the Q test in the last column. As expected, we found no evidence of significant predictive ability of most NER, REER, and NEER for crude oil price returns in the full sample period for all three tests. Interestingly, we find weak predictability evidence for Brazil's REER that the null of no predictability can be rejected at the 10% level when the lagged Brazil's REER is used as a predictor by the KMS test. Clearer results are evident in the Bonferroni Q test as applied by CY.

The CY test does agree with KMS for Brazil REER, the upper bound is below 0 from the CY test. Specifically, the Bonferroni confidence interval for β based on the Q-test is entirely below zero, signalling a rejection of the null hypothesis of no predictability since zero is not included within the confidence interval. However, the evidence for this predictability is still relatively weak because the Bonferroni Q confidence intervals are very close to zero, which is considered insignificant according to our test. Overall, we find little evidence of predictability in the exchange rates of the examined countries.

The lack of predictability may be due to the time-varying nature of the data, which could limit predictability to brief short windows. This suggests that accurately predicting the data's behaviour over extended periods is difficult, yet more reliable forecasts may still be possible over a shorter duration. Overall, based on our preliminary results, the three different exchange rates for oil-exporting countries yield the almost same results. Consequently, we have adopted real-time monitoring procedures to identify the emergence of short-lived predictive regimes. As previously mentioned, these procedures involve the sequential application of standard heteroskedasticity-robust regression t-statistics for predictability, over relatively short periods, as proposed by Harvey et al. (2021). Thus, these techniques can also help in identifying historical periods of temporary predictability.

As we mentioned before in Section 3, the real-time monitoring procedures hypothesised that there was no predictability over the training periods. To determine the relationship between this hypothesis and our data set, we tested the training sample for predictability in the same way that we previously tested the full sample for predictability, as shown in Tables 20 and 21. We have chosen specific training periods: January 1995 to September 2012 (for $m = 15$), January 1995 to June 2011 (for $m = 30$), January 1995 to March 2010 (for $m = 45$), January 1995 to December 2008 (for $m = 60$), and January 1995 to June 2006 (for $m = 90$). These intervals correspond to $T^* = 228 - m$, where T^* represents the training period length, and the observation $t = 228$[7]. This setup ensures that the monitoring process commences at $t = 228$ (regardless of the window length), allowing us to consistently evaluate

[7] It is important to clarify that all monitoring procedures begin in December 2013. Furthermore, these tests apply to the full sample within the training period to ascertain the absence of predictability, adhering to the test assumptions for the monitoring phase that follows.

Table 21: CY test results for the full sample for Brent

NEER	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.0536	-1.32	0.5128	[-0.107, 0.029]
Canada	-0.0573	-0.57	0.0956	[-0.184, 0.169]
Colombia	-0.0381	-0.82	0.199	[-0.095, 0.061]
Indonesia	-0.004103	-0.15	0.001	[-0.041, 0.053]
Mexico	-0.027823	-0.68	0.136	[-0.075, 0.069]
Norway	-0.11947	-0.86	0.216	[-0.428, 0.106]
REER	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.1215	-2.07	1.24	[-0.211, -0.011]*
Canada	-0.0760199	-0.62	0.11410	[-0.277, 0.171]
Colombia	-0.102162	-1.17	0.3996	[-0.252, 0.048]
Indonesia	0.0057689	0.08	0.00207	[-0.102, 0.129]
Mexico	-0.09271	-1.03	0.31415	[-0.187, 0.120]
Norway	-0.11489	-0.67	0.13296	[-0.509, 0.147]
NER(local)	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.0254	-0.97	0.275	[-0.059, 0.031]
Canada	-0.0558	-0.61	0.111	[-0.214, 0.124]
Colombia	-0.0236	-0.72	0.15	[-0.060, 0.052]
Indonesia	0.0016549	0.07	0.0013	[-0.032, 0.052]
Mexico	-0.0244723	-0.63	0.1166	[-0.066, 0.070]
Norway	-0.0315033	-0.45	0.060	[-0.155, 0.100]

*Note:The 90% CI columns report the 90% Bonferroni confidence intervals for $\hat{\beta}$ using the Q-test. Confidence intervals that reject the null are shown with a * mark.*

predictability starting from the same point in time for all procedures. Table 44 presents the IV_{comb} test results for each training period selected for our sample countries, while Table 45 shows the KMS and CY test results. Similar to the full sample period, we found no evidence of significant predictive ability for most NER, REER, and NEER with respect to crude oil price returns, except for Colombia and Indonesia in the REER case. In Table 44, there is statistically significant evidence of predictability at conventional significance levels from IV_{comb} for $m = 15$ and $m = 30$ for Colombia. And the null hypothesis also can be rejected for $m = 15$, $m = 30$, $m = 45$, and $m = 60$ for Indonesia. However, the KMS test and the CY Q test in Table 45 provide different results. Both the KMS test and the CY Q test for Colombia and Indonesia fail to reject the null hypothesis for any value of m. This indicates that the REERs for Colombia and Indonesia may not be predictive during any training period. We find that predictability is concentrated in the data for Colombia, likely from December 2013 to the end of the training period, while for Indonesia, it exists from September 2007 to the end of the training period based on IV_{comb} results[8]. Therefore, for data where no predictability was detected, we continue to use the initial training period for these predictors in the surveillance application in the next section. Because we continuously detect predictability in adjacent m subsamples, we use an adjusted training period for Indonesia and Colombia in the following tests to ensure accuracy.

6 Monitoring results

Based on the results of our three different predictive regressions from the previous section, we can now employ real-time monitoring procedures. We began monitoring in December 2013, corresponding to $T^* + m = 228$ for Brazil, Canada, Mexico and Norway. This point is approximately midway through our full sample period, strategically chosen to not only avoid the impact of the 2008 financial crisis but also to precede the early 2015 drop in commodity prices. Additionally, occurring amidst the COVID-19 pandemic, this period was also characterised by unprecedented oil price volatility. We present results assuming that monitoring continues through to the last data observation on 07/23. The training periods are as follows: January 1995 to September 2012 (for $m = 15$), January 1995 to June 2011 (for $m = 30$), January 1995 to March 2010 (for $m = 45$), January 1995 to December 2008 (for $m = 60$), and January 1995 to June 2006 (for $m = 90$). For the SEQ procedure, we adopted relatively stringent critical values of $\pi = 0.05$. Recall that for Colombia, we reset to

[8] In REER case, for Colombia, we reset to $T^* = 198 - m$, with the adjusted IV_{comb} test results being 1.5386, 1.4519, -0.019867, 1.5322, and -0.094309 for $m = 15, 30, 45, 60, 90$ respectively. For Indonesia, we reset to $T^* = 153 - m$, with the adjusted IV_{comb} test results being 1.5322, 1.3128, 1.549, 1.148, and 0.84406 for $m = 15, 30, 45, 60, 90$ respectively. It can be seen that no predictability is detected in each training cycle after adjustment.

begin monitoring in June 2011 as $T^* + m = 198$. For Indonesia, we reset as $T^* + m = 153$[9], so we begin monitoring in May 2007. All other conditions remain the same.

Table 22 displays the number of predictive regimes identified by the MAX and SEQ methods (with $\pi = 0.05$) for different lengths of the training period (m=15,30,45,60,90) and across three sample exchange rates. A key observation is the difference between the

Table 22: Number of predictive regimes detected by SEQ with $\pi = 0.05$ and MAX Brent crude oil price

	SEQ					MAX				
neer	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	1	1	1	1	1	1	3	3
Canada	0	1	0	1	1	0	1	3	2	1
Colombia	0	1	0	0	0	0	3	0	0	0
Indonesia	1	1	3	1	2	1	0	0	3	3
Mexico	1	2	1	1	1	3	2	1	1	2
Norway	0	0	0	0	3	1	2	3	3	3
reer	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	0	2	2	1	1	1	3	2
Canada	0	1	0	2	3	0	0	3	3	3
Colombia	0	1	0	0	3	1	3	0	0	3
Indonesia	1	0	3	1	(N/A)	3	0	3	2	(N/A)
Mexico	1	0	0	0	0	1	0	1	1	0
Norway	1	0	0	1	2	2	2	3	3	2
ner	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	0	1	1	2	0	1	3	3
Canada	1	1	0	1	1	0	1	2	2	1
Colombia	0	1	0	0	0	2	2	0	1	0
Indonesia	1	1	0	2	2	0	0	0	0	2
Mexico	1	1	2	1	1	3	3	0	3	2
Norway	0	1	0	0	0	2	3	2	3	2

Note: For only the REER case, since we detected possible predictability for Indonesia during training in Section 5, we reset to $T^* + m = 153$. This makes it impossible to calculate results for Indonesia when $m = 90$, as m is greater than the training period. Therefore, we use (N/A) to indicate it.

[9]For Colombia, the training periods are as follows: January 1995 to June 2010 (for $m = 15$), January 1995 to January 2009 (for $m = 30$), January 1995 to June 2007 (for $m = 45$), January 1995 to December 2006 (for $m = 60$), and January 1995 to September 2004 (for $m = 90$). For Indonesia, the training periods are as follows: January 1995 to July 2006 (for $m = 15$), January 1995 to June 2005 (for $m = 30$), January 1995 to September 2004 (for $m = 45$), and January 1995 to June 2003 (for $m = 60$). It is impossible to calculate results for Indonesia when $m = 90$, as m is greater than the training period.

two methods in Table 22: The MAX procedure can typically identify predictive regimes at various subsample (m) sizes, whereas the results of the SEQ procedure are more sensitive to the size of m. but in general, the number of regimes is quite variable for MAX and SEQ. SEQ shows variability in the detected predictive regimes, influenced by the training period length. This should be caused by the different mechanisms of the two methods. We present the first month where a predictive regime is detected by MAX and SEQ for all three exchange rates in Figures 23 to 36. The following three subsections will introduce the predictive period results for each of our three exchange rates. While Table 23 to Table 36 only displays the first identified predictability regime for all countries. As mentioned in Section 3, real-time monitoring procedures from Harvey et al. (2021) allow for the detection of multiple predictability regimes, as shown in Table 22. To further illustrate the location of these regimes, we also provide a line chart visualisation of the predictability for each country.

6.1 NEER results

For the NEER analysis, Table 22 shows that MAX identified one or more predictive regimes in all countries. Brazil, Mexico, and Norway detected predictive regimes for oil returns at every m size. Canada detected no predictive regimes only when $m = 15$. Colombia detected predictive regimes only when $m = 30$. Indonesia detects predictive regimes at $m = 15, 60, 90$. SEQ provided similar results, confirming predictive regimes across all countries but did not detect any for Norway until $m = 90$, where it found 3 regimes. And Indonesia, in particular, found predictive regimes in every training period. Clearly, for NEER, the larger the m value, the more evident the predictive ability. For example, when $m = 15$, SEQ and MAX detected 2 and 6 prediction regimes, respectively, while when $m = 90$, SEQ and MAX detected 8 and 12 prediction regimes, respectively. In the paper by Harvey et al. (2021), it was also observed that a larger value of m tends to be associated with a higher number of predictive regimes. It highlights that the subsample size (m) plays a pivotal role in determining the sensitivity, specificity, and robustness of predictive regime detection, as evidenced in the previous Table.

Table 23: First month where a predictive regime is detected by MAX and SEQ with $\pi = 0.05$ for NEER case

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	04/14	0.041667	03/15	0.1	03/15	0.2
Canada	NaN	NaN	02/20	0.29707	NaN	NaN	03/16	0.18182	05/16	0.35135
Colombia	NaN	NaN	05/23	0.39568	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	02/15	0.052632	05/16	0.13402	04/14	0.041667	01/15	0.084746	01/15	0.172
Mexico	09/18	0.21429	02/19	0.25991	05/20	0.34906	04/16	0.18797	12/16	0.407
Norway	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	05/15	0.23
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	04/20	0.28	04/20	0.31429	04/20	0.35814	09/14	0.084746	12/14	0.21311
Canada	NaN	NaN	12/14	0.071823	12/14	0.086093	12/14	0.10744	01/16	0.35135
Colombia	NaN	NaN	01/15	0.076923	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	09/14	0.048077	NaN	NaN	NaN	NaN	10/14	0.092437	10/14	0.18644
Mexico	01/15	0.066038	07/19	0.28814	10/19	0.33971	01/16	0.19403	07/16	0.4
Norway	01/15	0.066038	12/14	0.071823	12/14	0.086093	01/15	0.11475	12/14	0.21311

Tables 23 delineate the first month when the SEQ and MAX identify a predictive regime for the NEER case. From the tables, our procedures pinpoint predictability regimes that not only coincide in timing but, in some cases, are identical to the month. Many of these regimes converge around the period of the global commodity price crash from late 2014 to early 2016. This was a time when the world experienced one of the most severe oil price declines in contemporary history. Oil prices plummeted by 70%, marking this downturn as one of the three steepest declines post-World War II and the lengthiest since the supply-driven collapse of 1986.

Starting with the SEQ procedure, Norway's predictive regime was found to start in January 2015 for $m = 90$. Similarly, SEQ identified its first predictive regime for Mexico in April 2016 for $m = 60$, although this example may have been at the end of the 2015 commodity crisis. Mexico also found predictive regimes in May 2020, coinciding with COVID-19. Of course, not all predictive regimes occur during periods of economic turmoil. For example, for $m = 15$ and $m = 30$, Mexico found predictive regimes in September 2018 and February 2019. However, it is undeniable that most predictive regimes appear during periods of economic downturn. Perhaps more robust evidence is provided by Indonesia, which found predictive regimes in February 2015, May 2016, April 2014, January 2015, and January 2015 for $m = 15$, $m = 30$, $m = 45$, $m = 60$, and $m = 90$, respectively. Similar evidence can be found for Brazil with $m = 45$, $m = 60$, and $m = 90$, and for Canada with $m = 30$, $m = 60$, and $m = 90$.

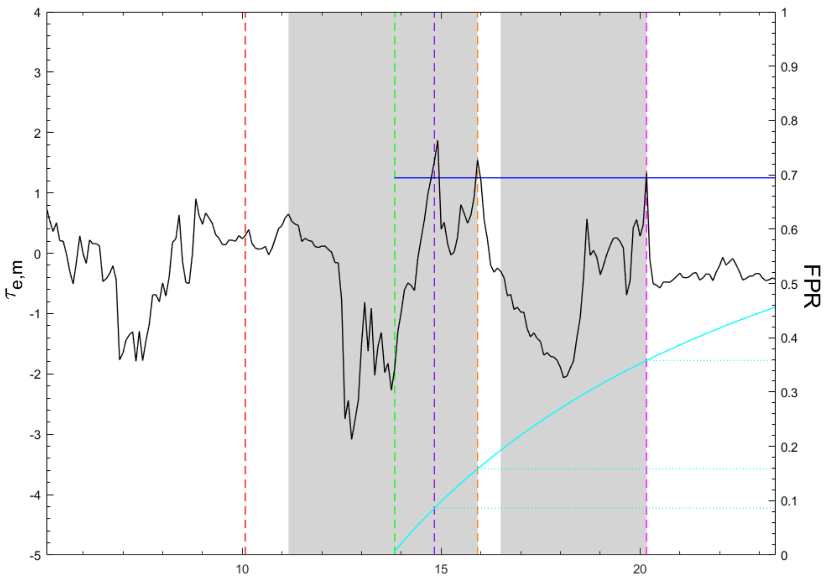
Furthermore, the MAX procedures for Brazil, Canada, Mexico, and Norway also provide results similar to SEQ in Table 23. For instance, for Canada, MAX identified predictive regimes at the same time points in December 2014 for $m = 30$, $m = 45$, and $m = 60$. A predictive regime was also found in January 2016 for $m = 90$. Norway finds the first predictive regime from early 2014 to 2015 for all different m sizes. Brazil's first detected predictive regime is during the COVID-19 period for $m = 15$, $m = 30$, and $m = 45$, and around the 2015 commodity crisis for $m = 60$ and $m = 90$. Indonesia found similar predictive regimes during the 2014-2015 financial crisis for $m = 15$, $m = 60$, and $m = 90$. These results align with the onset of the global financial crisis and the commodity price crash from previous examples, indicating the predictive ability of many of our exporters' exchange rates during economic downturns.

We also provide a line chart visualization of the predictability for each country, as shown in Figures 23 to 52 in Appendix 10. Due to the large number of figures, we present Figures 14 and 15 as examples. These figures illustrate the location of the regimes for NEER using two real-time monitoring procedures. The figures also display key points including the end of the training period T^*[10], the date monitoring begins $T^* + m$, the maximum value of $\tau_{e,m}$ during the training period ($\max_{e \in [m+1, T^*]} \tau_{e,m}$), the date of the first significant rejection for the i-th predictive regime j_i (the date when the i-th predictive regime is identified), and the FPR, calculated based on Equation 4.17, as a function of E. The shaded areas in Figure 14 and 15 are referred to as the weak set of dates. This represents the start of a liberal or

[10]For clarity, these graphs begin the horizontal axis 5 years before the end of each training period, rather than displaying $\tau_{e,m}$ over the entire training period.

Figure 14: The location of the predictive regimes for Canada's NEER, MAX procedure, m=45

(a) Canada



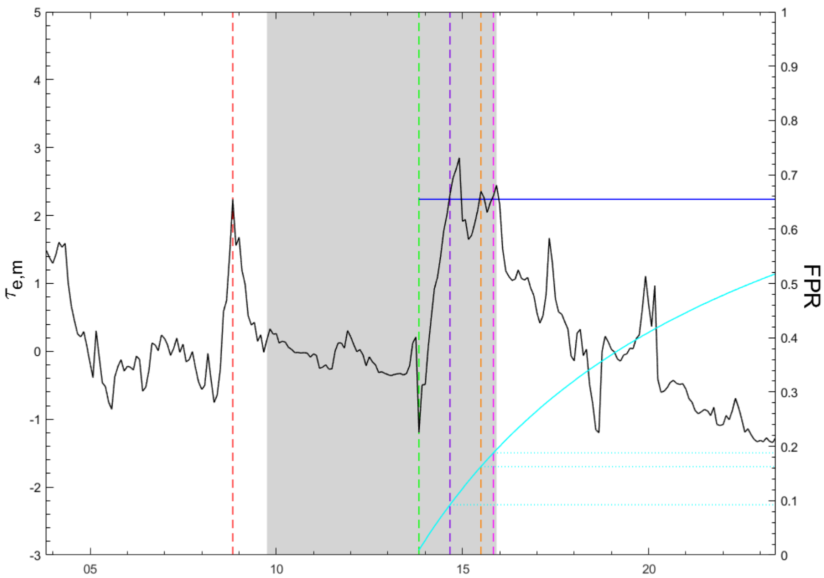
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

weak predictive regime before each found significant rejection. This set is termed "weak" because it offers a broader, more inclusive estimate of the period during which the predictive regime could have been active. The actual predictive regime might start later and end earlier than indicated by this set of dates. Typically, weak sets of dates often conclude shortly after the predictive regime is initially rejected. Consequently, the weak set can overlap with consecutive regimes, meaning some dates might belong to multiple predictive regimes.

As shown in Figure 14, for Canada's NEER, the MAX procedure with $m = 45$ detected two consecutive predictive regimes at the end of December 2014 and 2015. Due to the large subsample used, the weak set of dates appeared earlier; however, the figure indicates that the sharp increase in $\tau_{e,m}$ began in 2014. The two consecutive significant rejections cover the beginning and end of the commodity crisis. Therefore, based on this figure, we infer that during the commodity crisis, Canada's NEER had the ability to predict Brent oil price return.

Figure 15: The location of the predictive regimes for Indonesia's NEER, MAX procedure, m=60

(a) Indonesia



Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, $\cdots$ (T^*),— is the T^* +m, $\cdots$ shows the first rejection, $\cdots$ shows the second rejection, $\cdots$ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

At the same time, the predictive regime was also detected at the onset of the COVID-19 pandemic in early 2020. Figure 15 shows another MAX program example for Indonesia with $m = 60$. Compared with Figure 14, the MAX program detects three predictive mechanisms simultaneously: the first at the end of 2014, and the second and third at the end of 2015. This different subsample size and different oil-exporting countries provide such "coincidental" evidence, further proving the predictability of oil exporters' NEER to Brent crude oil return during the commodity price crisis period. More NEER results by MAX and SEQ are shown in Appendix 10.

A similar situation is observed in the REER analysis shown in Table 22: all countries exhibited one or more predictive regimes for various m values. Because we detected possible predictability for Indonesia during training in Section 5, we reset to $T^* + m = 153$. This makes it impossible to calculate results for Indonesia when $m = 90$, as m is greater than the training period. Therefore, we use (N/A) to represent the case of Indonesia in Table 22 and thereafter for $m = 90$. Similar to NEER, the predictive ability increases as the value of m increases in the REER case overall. For instance, SEQ identifies 10 predictive regimes for REER when $m = 90$, but only 3 when $m = 15$. Similarly, MAX found 8 predictive regimes for REER when $m = 15$ but 12 predictive regimes at $m = 60$. These findings underscore the nuanced differences in detected predictive regimes, highlighting how the choice of m influences the detection of short-lived predictability episodes and their real-time applicability.

To avoid redundancy, a detailed exposition of the results for NER is omitted in the main text, as the findings largely parallel those of the NEER case. Specifically, the monitoring procedures applied to NER series similarly identified predictive regimes clustered around major oil price dislocations—most notably the 2014–2016 commodity price collapse and the early 2020 period associated with the COVID-19 pandemic. These results are consistent across a range of training window lengths. For completeness, the full set of NER-based monitoring results, including summary tables and graphical illustrations, is provided in Appendix 8.

6.2 REER results

Table 24 shows the first month where a predictive regime is detected by MAX and SEQ for the REER case. Also starting from the SEQ procedure, the results of the found predictable regimes are generally similar to those of NEER, with most predictable regimes appearing during periods of the 2015 commodity crisis or the COVID-19 pandemic. The results for Brazil, Indonesia, and Norway are particularly clear, as these countries first identified predictable regimes during the 2014-2016 commodity crisis whenever a prediction regime was detected. Notably, even though we adjusted the monitoring start point for Indonesia, the detected prediction regimes for larger m values remained relatively unchanged, centring around 2014-2015. Except, Colombia identified a new prediction regime around 2012. The results of MAX are quite similar to those of SEQ, as we also find predictable regimes during financial crises and commodity price declines. When $m = 45$, $m = 60$, and $m = 90$, Brazil, Canada,

Table 24: First month where a predictive regime is detected by MAX and SEQ with $\pi = 0.05$ for REER case

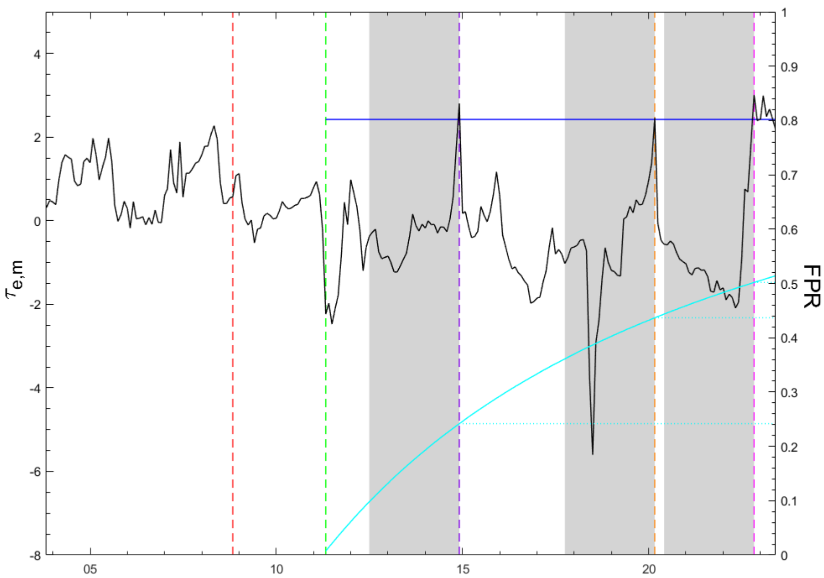
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	NaN	NaN	05/15	0.115	07/15	0.25
Canada	NaN	NaN	03/20	0.3	NaN	NaN	05/15	0.115	05/15	0.226
Colombia	NaN	NaN	03/23	0.51	NaN	NaN	NaN	NaN	03/12	0.35714
Indonesia	09/14	0.40865	NaN	NaN	12/14	0.58278	11/14	0.725	NaN	NaN
Mexico	10/18	0.21739	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Norway	03/15	0.057143	NaN	NaN	NaN	NaN	04/16	0.18797	01/15	0.172
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	04/20	0.28	04/20	0.31429	04/20	0.35814	12/14	0.10744	11/15	0.33333
Canada	NaN	NaN	NaN	NaN	12/14	0.086093	11/14	0.1	12/14	0.21311
Colombia	04/20	0.39	01/15	0.242	NaN	NaN	NaN	NaN	02/12	0.33333
Indonesia	07/14	0.40291	NaN	NaN	03/16	0.62048	10/14	0.72269	NaN	NaN
Mexico	05/18	0.21429	NaN	NaN	NaN	NaN	01/16	0.19403	NaN	NaN
Norway	01/15	0.066038	12/14	0.071823	12/14	0.086093	12/14	0.10744	09/14	0.17241

Indonesia, and Norway all found predictability windows from 2014 to 2016 at one or more different m sizes. However, in the results for relatively small subsample m, such as for Brazil and Colombia, predictability windows during the COVID period were found. Specifically, in MAX, Brazil detected predictability in April 2020 at $m = 15$, $m = 30$, and $m = 45$, while Colombia detected predictability at the same time at $m = 15$, with no predictability results in SEQ.

As shown in Figure 16, the location of the predictive regimes for Colombia's REER by the MAX procedure when $m = 30$ is very interesting. Predictable regimes are found in 2015, 2020, and 2023, corresponding to the 2015 commodity crisis, Covid-19, and the 2021 inflation surge. In this figure, it is obvious that there was a huge increase in $\tau_{e,m}$ when the 2015 commodity crisis began in mid-2014. Similarly, $\tau_{e,m}$ also suddenly increased in late 2019. Finally, we see $\tau_{e,m}$ rise in 2022 until mid-2023, which is when the worldwide inflation surge and the Russo-Ukrainian war began. In contrast, when the world is relatively peaceful

Figure 16: The location of the predictive regimes for Colombia' REER, MAX procedure, m=30

(a) Colombia



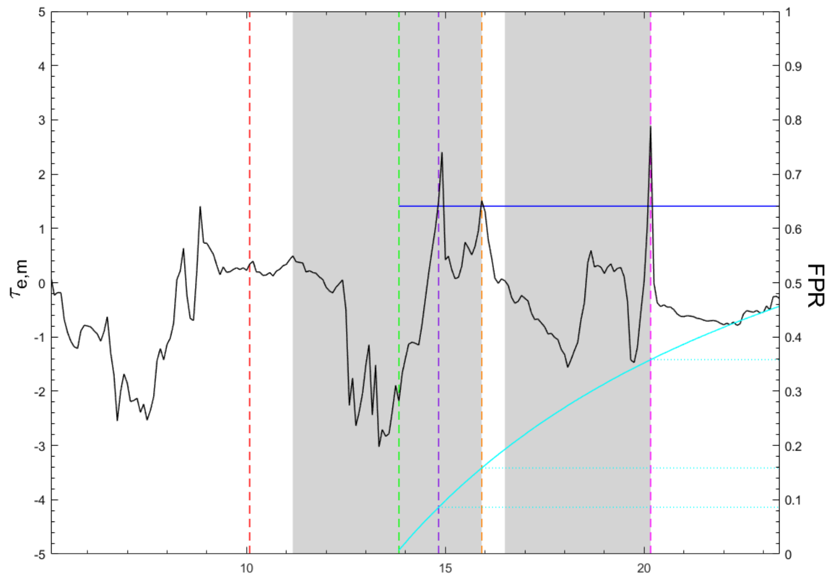
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

and there is no major economic crisis, such as from mid-2012 to 2018, $\tau_{e,m}$ remains at a relatively low level. Similar results are also found in Figure 17, where predictive mechanisms are detected at the end of 2014, the end of 2015, and the beginning of 2020. This result is similar to Figure 16, except that no predictable regime is found in 2023. The results of REER are similar to those of NEER in the previous subsection on the predictability of oil exporters' REER to Brent crude oil return during the commodity price crisis period. However, external crises seem to be more evident in the fluctuations of REER $\tau_{e,m}$.

As also detailed in Table 23 to 24, most countries have identified almost the same predictability regimes for the exchange rates of oil-exporting countries to Brent oil prices during significant downturns in either oil prices or the economy. This insight aligns with the findings of Alam et al. (2019), who found that increased economic and financial uncertainties accentuate the impact of currency fluctuations on crude oil prices. The underlying mechanism is

Figure 17: The location of the predictive regimes for Norway' REER, MAX procedure, m=45

(a) Norway



Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

rooted in the anticipatory nature of exchange rates coupled with the increased financialization of the oil market observed since the mid-2000s. Hence, the notable volatility in the oil market during the commodity price crash appears to have been mirrored in the exchange rates of oil-dependent nations, leading to temporary predictability.

7 Conclusion

The literature is replete with the nexus between aggregate crude oil market indices and the financial index or stock market, albeit with mixed findings. We differ from the existing literature by focusing on crude oil exporters' exchange rates. We use the NER of six major oil exporting countries Brazil, Canada, Colombia, Indonesia, Mexico, and Norway from January 1995 to July 2023, as well as the Brent oil benchmark and Brent oil futures prices over a similar period. We also compared the predictive capabilities between different exchange rates to crude oil. Although there are subtle differences, both the full sample test and the real-time procedures REER, NEER and NER give similar results. Oil prices are known for their unpredictability over longer periods, influenced by economic and political events, supply and demand imbalances, market sentiment, and regulatory changes and environmental policies. Additionally, the relationship between exchange rates and oil prices may vary over time due to their inherent instability but may exhibit short-lived predictability. We initially employed the widely evaluated IVX test as described by Kostakis et al. (2015) to assess. To account for potential time variability in the variables, we also applied the test developed by Campbell and Yogo (2006). Finally, for testing forecasting capabilities over short periods, we utilised the real-time monitoring program outlined by Harvey et al. (2021). We compared the predictability regime with the time of the international financial crisis and found that almost all prediction regimes had a financial crisis or commodity crisis.

Based on the results, as anticipated, the full sample analyses using the IVX and CY tests revealed no predictive capability. However, the predictive models based on MAX and SEQ offered intriguing insights. Specifically, the findings from both MAX and SEQ indicated that the exchange rates of oil-exporting countries for the entire sample from 2014 to early 2016, and from 2019 to 2021, demonstrated a predictive regime to Brent international oil prices and Brent oil futures. These periods correspond to the global commodity crisis and the COVID-19 pandemic, respectively. During these times, the oil-exporting countries experienced economic downturns. As Tiwari et al. (2024) noted, under normal circumstances, the monetary policy of no single country can significantly influence global oil prices. However, during extreme events, both crude oil markets and debt instruments may be substantially impacted by changes in monetary policy. This is because the exchange rates of oil-exporting countries are forward-looking, incorporating factors such as changes in monetary policy and the production decisions of these countries (Chen and Chen, 2007). Consequently, in the short term, it is possible to predict international oil prices based on the exchange rates of oil-exporting countries during extreme events. Overall, by focusing on the exchange rates of oil-exporting countries during economic downturns or extreme events, the study provides valuable insights into the factors influencing oil price fluctuations. This relationship suggests that exchange rates can serve as a barometer for future oil price movements, particularly during economic stress. Furthermore, the real-time monitoring procedure employed in this research highlights the potential for identifying short windows of predictability amidst the general unpredictability of the oil market. This approach reflects the adaptive nature of financial markets, where investors quickly respond to new information, thereby diminishing the predictive power of previously significant variables. This research not only enhances our understanding of the oil market's complexity but also offers practical implications for economic policy formulation and investment strategy development.

Chapter 5

Conclusion

As introduced in Chapter 1, this thesis provides a comprehensive examination of the intricate relationship between exchange rates and crude oil prices, with a particular focus on oil-exporting countries. Our research provides new insights into the multifaceted dynamics that underpin these relationships, offering both theoretical contributions and practical implications. Regarding the impact on REER, we find that the significance of oil exports in these countries is a potentially important explanatory variable, and for the crude oil price, for the six major oil-exporting countries we studied, we find that the most oil-exporting countries' NER have the ability to forecast oil prices in the short term, which we believe may be due to the importance of oil exports to their economies. While oil prices are notoriously unpredictable over long periods due to a multitude of factors such as economic and political events, supply-demand imbalances, market sentiment, and regulatory changes, the relationship between exchange rates and oil prices may exhibit short-lived predictability, especially during extreme economic events.

Chapter 2 employs the BEER approach (Clark and MacDonald, 1999) to analyse the long-run determinants of the REER for a panel of 15 oil-exporting countries. This study contributes to the literature by explicitly accounting for the structural role of real oil price in shaping exchange rate dynamics. Given the heterogeneity in domestic economic conditions across countries, constructing a standardized country-specific trade deflator as a price index presents significant challenges. To address this, we propose the use of the Six-Sector with Deflator as a proxy for sectoral productivity differentials, alongside the real Oil price and Net Foreign Assets, as key determinants of the equilibrium REER.

Our empirical analysis combines time-series cointegration techniques (Johansen and ARDL bounds tests) with panel estimators (MG and PMG). The time-series results yield mixed evidence: while the Johansen test confirms cointegration in all countries except Brazil, Gabon, and the United States, the ARDL bounds test supports cointegration in all cases except Gabon and Nigeria. As noted in Chapter 2, the reliability of these time-series estimates may be constrained by the limited span of annual data, particularly for countries with shorter historical records of REER, NFA, and other variables. To mitigate small-sample biases, we employ the panel estimators proposed by Pesaran et al. (1999). Although the MG estimates exhibit weaker significance, the Hausman test validates the PMG specification, which reveals statistically significant long-run relationships between REER and its fundamentals. The PMG estimates indicate that a 10% increase in the real Oil price raises the REER by 1.22%, while analogous increases in 6SECT and NFA lead to REER appreciations of 4.56% and 1.85%, respectively. These findings align with theoretical expectations and prior empirical work on commodity-exporting economies (e.g., Cashin et al., 2004; Dauvin, 2014; Lane and Milesi-Ferretti, 2018). Our results resolve ambiguities in the literature, such as those highlighted by Coudert et al. (2011), by demonstrating the distinct role of oil prices in driving REER dynamics for specialised oil exporters. The significant coefficients for 6SECT and NFA further underscore the importance of domestic structural factors (e.g., Balassa-Samuelson effects) and external imbalances in determining equilibrium exchange rates. This refined BEER framework offers a robust empirical foundation for analysing REER behaviour in oil-dependent economies.

In Chapter 3, I examined whether the exchange rates of oil-exporting countries can reliably predict international oil prices. I employed the PV model from (Chen et al., 2010b), which is widely used for predicting commodity prices. The underlying premise of the PV method is that exchange rates, as fundamentally forward-looking variables, likely contain information about future commodity price movements and do not directly depend on the variables that explain commodity prices. However, predicting crude oil prices is a complex task involving many variables. Numerous factors can influence oil prices, such as geopolitical risks, OPEC influence, technological changes, and more (Hamilton, 2009; Alquist et al., 2013; Frankel, 2012). In this chapter, we provide a detailed explanation of the selection of oil-exporting countries, the classification of various oil standards, the choice of exchange rates, and the relevant information for each oil-exporting country.

Therefore, we conduct in-sample and out-of-sample estimates of global oil futures and three major oil price benchmarks (Brent, WTI, and Dubai) using NER pairs for Brazil, Canada, Colombia, Indonesia, Mexico, and Norway. We employed both the traditional GC test and the instability GC test proposed by Rossi (2005). In the traditional GC test, our in-sample analysis showed that changes in the exchange rate significantly affected oil futures prices only for Colombia, with Brazil showing significance at the 10% level. For the other countries, the evidence was not strong enough to reject the null hypothesis. However, it is important to recognise that traditional in-sample GC tests do not account for parameter instabilities, which are common in commodity price movements. When these potential instabilities are considered, our robust instability GC test reveals more significant results for Brazil and Mexico, indicating that shifts in the exchange rates of these crude oil exporters have a Granger-causal relationship with crude oil futures prices. On the other hand, the null hypothesis could not be rejected for Canada, Indonesia, and Norway, suggesting that this relationship may not be applicable to these countries. Next, we conducted out-of-sample predictions using both the rolling window and recursive methods. The findings indicate that the NER of Brazil, Colombia, and Mexico effectively forecasted crude oil prices across all three oil benchmarks, both in-sample and out-of-sample. Notably, Norway's NER also outperformed the benchmark in most instances. The ENCNEW statistics reveal that the exchange rates of these countries have significantly stronger forecasting abilities for the three oil spot prices compared to the futures cases. As we expected, the greater the role of crude oil in a country's economy, the more sensitive its currency is likely to be to fluctuations in oil prices, which in turn enhances its ability to forecast oil prices.

It is widely recognised that oil prices are notoriously difficult to predict over extended periods, as they are influenced by a complex interplay of economic and political events, supply and demand imbalances, market sentiment, regulatory changes, and environmental policies (Verleger, 1987; Alquist et al., 2013). Consequently, the relationship between exchange rates and oil prices is likely to fluctuate over time due to their inherent instability. This significant volatility complicates the identification of reliable patterns for accurate forecasting (Hamilton, 2009; Kilian, 2009; Alquist et al., 2013). This variability in oil prices may be precisely why the forecasting results for Indonesia and Canada in Chapter 3 are suboptimal. Therefore, I speculate that like many other countries discussed in Chapter 3, Canada and Indonesia should also possess the ability to predict oil prices. However, this predictability may be limited to a specific, short period. Hence, in Chapter 4, we diverge from previous forecast methods by first conducting a full-sample predictability test using the IVX test introduced by Kostakis et al. (2015). To address potential temporal variations in the variables, we also apply the CY test developed by Campbell and Yogo (2006). Finally, to evaluate short-term forecasting capabilities, we utilise the real-time monitoring programs MAX and SEQ as outlined by Harvey et al. (2021). As expected, the full-sample analysis using the IVX and CY tests did not reveal predictive power. However, the forecasting models based on MAX and SEQ provided valuable insights. The results from these models show that the exchange rates of oil-exporting countries exhibited predictive mechanisms for Brent international oil prices and Brent crude oil futures during two significant periods: 2014 to early 2016 and 2019 to 2021. Even many predictive regimes start in the same month. These periods coincide with the global commodity crisis and the COVID-19 pandemic, both of which were marked by economic recessions in oil-exporting nations.

My research provides valuable insights for policymakers and commodity traders by deepening our understanding of the relationship between the oil market and the exchange rates of oil-exporting countries. If the exchange rates of countries heavily dependent on oil exports can be used as reliable predictors, our findings could enhance policymakers' ability to monitor and achieve financial stability, while also providing useful guidance for investors. For instance, when forecasting the price of a specific commodity, investors might consider the exchange rates of the countries most reliant on exporting that commodity.

While this thesis presents various independent and dependent variables and a complete picture, each chapter offers avenues for future research. For example, the study in Chapter 2 is somewhat constrained by the availability of data from sample countries, and the significance of oil may differ across nations, potentially explaining the weaker time series results. Additionally, Chapters 3 and 4 highlight the importance of oil reserves and technological advancements. For example, Brazil, a new oil-producing nation with abundant offshore reserves, could see increased production as technology advances. In contrast, Indonesia's oil reserves have dwindled to one-third of their peak levels, which might impact the predictability of oil prices. As resources diminish, oil becomes less crucial to the country's economy, attracting fewer foreign investors and leading to exchange rates that no longer reflect oil-related information. Finally, we intentionally limited the selection of oil-exporting countries in the Persian Gulf, as most are OPEC members with currencies pegged to the US dollar. This pegging adds complexity to the analysis of their exchange rate fluctuations compared to the countries in our sample. Consequently, these nations were not included in this thesis. Future research could further explore this area, especially considering that these countries have significantly higher oil exports and production than those in our sample.

Chapter 6

Appendices

1 Data Source and Collection for Chapter 3

For the three oil prices (Brent, WTI and Dubai), their values serve as benchmark prices representative of the global market. These prices are monthly average values from the global market during 1993:01-2023:02, expressed in nominal U.S. dollars per barrel. So, there are 362 observations before the log difference and 361 observations after the difference. Monthly exchange rates are averages of the daily data available for each country. We endeavoured to collect data for oil-exporting countries that aligned with our oil price data's time span, during monthly data from January 1993 to February 2023. However, complete data sets were not available for all countries. Specifically, Brazil's data range is from 1995 to 2023, giving 338 observations, while Mexico's data covers 1994 to 2023, giving 350 observations. However, not all of this data is suitable for forecasting analysis. We think that the exchange rate fluctuations stemming from past economic crises in certain countries might not be indicative of price fluctuations in oil and its by-products. This is primarily due to their limited correlation with import and export oil prices. Consequently, we have adjusted certain time spans to reflect this assessment. A comprehensive discussion of these adjustments can be found in Section 4.2. To summarise, the final starting points we established for Brazil, Indonesia, and Mexico are 2000:01, 1999:01, and 1996:01, respectively. After applying log differences, the data yielded 277, 289, and 325 observations for each country. Brent oil futures prices are the real-time data collected from ICE, and the Monthly data are also averages of the daily data available for the same period as crude oil prices (as 361 observations after log difference). In Section 5.2, we use rolling window forecasting by one-quarter of the total sample size, the information for the rolling window size is shown in Table 26. Oil rents represent the difference between the value of crude oil production at regional prices and the total costs of production. Natural resource rents are estimated as the difference between a commodity's price and the average cost of producing it. This is achieved by determining the price of specific commodity units and then subtracting the average unit costs of extraction or harvesting. These unit rents are subsequently multiplied by the quantities extracted or harvested by countries, yielding the rents for each commodity as a percentage of GDP. While we do not directly use these data in our forecasting models, we rely on them to verify our results. The data consists of annual figures spanning from 1993 to 2022.

Table 25: Data sources and collection

Variables name	period	sources	website
Brazil NER	Monthly:1995:01-2023:02	Fred	https://fred.stlouisfed.org/series/EXBZUS
Canada NER	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/EXCAUS
Colombia NER	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/COLCCUSMA02STM
Indonesia NER	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/CCUSSP02IDM650N
Mexico NER	Monthly:1994:01-2023:02	Fred	https://fred.stlouisfed.org/series/DEXMXUS
Norway NER	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/EXNOUS
Brent oil	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/POILBREUSDM
WTI crude oil	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/POILWTIUSDM
Dubai crude oil	Monthly:1993:01-2023:02	Fred	https://fred.stlouisfed.org/series/POILDUBUSDM
Brent futures	Monthly:1993:01-2023:02	ICE	https://uk.investing.com/commodities/brent-oil
Oil rent	Annual: 1993-2022	World Bank	https://data.worldbank.org/indicator/NY.GDP.PETR.RT.ZS

Table 26: One quarter for each country for forecasting after differencing

	Brazil	Canada	Colombia	Indonesia	Mexico	Norway
Total observation number	277	361	361	289	325	361
$\frac{1}{4}$ Rolling window size	70	91	91	73	82	91
start date	2000:02	1993:02	1993:02	1999:02	1996:02	1993:02
end date	2023:02	2023:02	2023:02	2023:02	2023:02	2023:02

Note: The rolling window size is calculated by rounding up.

2 An example of the determination of exchange rate

Present-value models have found widespread application in forecasting fundamentals, as illustrated in prior studies such as Campbell and Shiller (1987), Engel and West (2005b), and Chen et al. (2010b). The present value or asset-pricing approach entails a present-value relationship between a variable and the discounted sum of its expected future fundamentals. Campbell and Shiller (1987) applied this model to stock prices, demonstrating one such example. This present-value model has also been utilised for exchange rates, as suggested by Engel and West (2005b), who postulates that exchange rates can indeed encapsulate projections about future fundamentals, a conclusion that aligns with the principles of present-value models. The equation representing the present-value approach to exchange rates can be expressed as follows, as proposed in Engel and West (2005b)'s study:

$$s_t = (1-\beta)(f_{1t} + z_{1t}) + \beta(f_{2t} + z_{2t}) + \beta E_t s_{t+1} \tag{1}$$

In the equation, β represents a discount factor, s_t is the logarithm of the exchange rate, f_{it} and z_{it} (where i=1,2) are observable and unobservable fundamentals that ultimately drive the exchange rate, respectively, and E_t is a mathematical expectation, conditional on the full public information set I_t. The observable variables could be standard macroeconomic variables, such as money supply and demand. Let's assume the following relationship for the local country's money market:

$$m_t = p_t + \gamma y_t - \alpha i_t + \upsilon_{mt} \tag{2}$$

where m_t represents the log of the nominal money supply, p_t is the log of the price level, y_t is the log of real income or output, i_t is the nominal interest rate, γ is the income elasticity of money demand, α is the interest elasticity of money demand and υ_{m_t} represents a shock to the money market equilibrium equation, potentially reflecting shifts in money demand not accounted for by changes in income or interest rates. The nominal exchange rate equals its purchasing power parity value plus the real exchange rate, and the interest parity relationship is:

$$\begin{aligned} s_t &= p_t - p_t^* + q_t \\ E_t s_{t+1} - s_t &= i_t - i_t^* + \rho_t \end{aligned} \tag{3}$$

where i_t^*, p_t^* are the foreign nominal interest rate and price level, ρ_t is the deviation from rational expectations uncovered interest parity. Therefore, combining the money market equilibrium, uncovered interest parity and the purchasing power parity conditions as follows:

$$s_t = \frac{1}{1+\alpha}(m_t - m_t^* - \gamma(y_t - y_t^*) + q_t - (\upsilon_{mt} - \upsilon_{mt}^*) - \alpha\rho_t) + \frac{\alpha}{1+\alpha} E_t s_{t+1} \tag{4}$$

Where $\beta = \frac{\alpha}{1+\alpha}$, α represents the interest elasticity of money demand and υ_{mt}^* represents a shock but in the foreign market. Solving the model for the exchange rate (s_t) with respect to current and expected future fundamentals results in the exchange rate being dependent on expected future commodity prices. These expected future commodity prices are embodied in q_t, as the real exchange rate essentially reflects the relative price of a basket of goods between two countries. This can be influenced by commodity prices, particularly in countries where commodities comprise a significant portion of their exports.

3 An Overview of Recent Year Price Movements of Three Major Oil Prices and Brent Oil Future Prices

The movements of the three oil prices (Brent, WTI and Dubai) and Brent oil future prices have historically served as a reliable gauge for energy market dynamics. Over the past decade, crude oil prices have exhibited remarkable volatility. The inherent inelasticity of oil prices in response to short-term demand and supply shifts means that fluctuations are somewhat erratic. Numerous commercial developments since the 2009 financial crisis have amplified this volatility (Basher et al., 2016). For instance, the economic rise of BRICS nations [1], particularly China and India (Pershin et al., 2016), and the emergence of hydraulic fracturing and horizontal drilling in the U.S., have left notable imprints on price movements, evident in the spikes displayed in Figure (6). Simultaneously, fluctuations in the three major oil prices have converged significantly.

Geopolitical events, such as the COVID-19 pandemic outbreak and the Russia-Ukraine conflict, have also played crucial roles in determining oil prices. An energy supply shortage that began impacting Europe in late 2021, coupled with concerns regarding oil supply bottlenecks in the aftermath of the Russia-Ukraine war, led to sharp increases in crude oil prices across the board. To illustrate, the average annual price of Brent crude oil reached 100.93 U.S. dollars per barrel in 2022, a figure nearly sixfold its value in 1993.

WTI is another pivotal oil benchmark, representing oil derived from U.S. wells. Its futures are primarily traded on the New York Mercantile Exchange. Typically, WTI prices are either slightly above or below Brent, but this differential can change based on elements like U.S. oil production rates and inventory numbers. Factors like newly established U.S. crude transport infrastructures that commenced operations in early 2013, combined with record-breaking crude production rates by U.S. refineries, contributed to the elevation of WTI prices. Meanwhile, the accelerated domestic crude transportation in the U.S. introduced a downward

[1] BRICS is an acronym that refers to the developing countries of Brazil, Russia, India, China, and South Africa. It was updated from the original BRIC in 2010, when South Africa was invited to join the group.

force on Brent prices as domestically-produced crude began substituting some of the Brent imports. Between 2014 and 2016, UK Brent prices experienced a sharp decline, a trend that was observed across other crude oil types as well. On the other hand, Dubai crude oil, apart from the years 2009 and 2010, enjoyed a steady price climb per barrel over the decade leading up to 2015. However, the subsequent year saw prices halving. The average price per barrel was approximately 24 U.S. dollars in 2002, surging to more than 109 U.S. dollars by 2012, only to fall to 51.20 U.S. dollars per barrel by 2015.

It is evident from Figure (6) that the monthly oil price is not stationary. Taking the logarithm of a series and then differencing it offers specific advantages in this context. By doing so, we can capture the proportional or percentage change, as opposed to the absolute change. For many economic and financial variables, such as oil prices and exchange rates, these percentage changes are often more pertinent than the absolute ones. This transformation—taking the logarithm followed by differencing—not only makes the series more interpretable but also helps in achieving stationarity for the oil price.

Table 27: The coefficient of determination for in- and out-of-sample forecasting: one quarter case

	Brazil	Canada	Colombia	Indonesia	Mexico	Norway
Brent oil future case						
in-sample R^2	0.0432	0.0270	0.0477	0.0256	0.0384	0.0251
outR^2_{oos}	0.1551	0.0682	0.1053	0.0832	0.1742	0.0824
Brent crude oil case						
in-sample R^2	0.0841	0.0689	0.1165	0.0614	0.0832	0.0849
outR^2_{oos}	0.1641	0.1140	0.1704	0.1311	0.1395	0.1308
WTI crude oil case						
in-sample R^2	0.0873	0.0806	0.1082	0.0740	0.0810	0.1025
outR^2_{oos}	0.1873	0.1329	0.1790	0.1593	0.1561	0.1612
Dubai crude oil case						
in-sample R^2	0.1380	0.1107	0.1501	0.1148	0.1221	0.1269
outR^2_{oos}	0.2387	0.1509	0.2259	0.1541	0.1917	0.1631

4 WTI and Dubai Out-of-sample forecasting results

Table 28: Tests for WTI Crude oil case

In-sample GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.0560*	0.000***	0.5541	0.0329**	0.0253**	0.0675*
Rossi GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.0319**	0.0257***	0.5357	0.2380	0.0299**	0.2607
MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
MSFE	-0.3313	-0.1605	2.2129	-0.0588	-0.6163	0.5104
ENCNEW test	***	***	NR	**	***	NR
Start date	(2000:02)	(1993:02)	(1999:02)	(1996:02)	(1993:02)	(1993:02)
Window size	70	91	73	82	91	91

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

The first is the forecast of WTI in Table (28). It can be seen that our test has some subtle differences in the price of Brent oil. In the In-sample GC Test, Brazil, Colombia, Mexico, and Norway have significant results at the 10%, 1%, 5%, and 5%levels, respectively. Although the value for Canada rejects the null hypothesis of the GC test in the standard GC test, the evidence is still weak, as it is only significant at the 10% level. In the Rossi GC Test, the results for Brazil, Colombia, and Norway are significant at the 5%, 1%, and 5%levels respectively. While Mexico can not reject it at any significant level at this time. In the MSFE differences part, the same countries as in the Brent case—Brazil, Colombia, Mexico, and Norway—show negative values. Among them, the MSFE values for Brazil, Mexico, and Norway are -0.3313, -0.0588, and -0.6163, respectively. However, the prediction result for Colombia is slightly weaker, with an MSFE of -0.1605. According to the ENCNEW test results, Brazil, Colombia, and Norway all reject the null hypothesis at the 1% significance level, while Mexico does so at the 5% level. Meanwhile, Canada and Indonesia still show positive values. This indicates that for Brazil, Colombia, Mexico, and Norway, the oil exporter currency forecasts better than the benchmark. Despite the US being the main destination for Canada's oil exports, it appears that Canada's exchange rate is not forecasting WTI oil prices.

Finally, looking at the Dubai oil in the In-sample GC Test in Table (29), we see similarities to the previous two oil standards. In-sample GC TEST, Brazil, Colombia, and Norway continue to present significant results at the 5%, 1%, and 1% levels, respectively. This time, however, Mexico only rejects at the 10% significance level. Intriguingly, Canada's exchange rate is rejected at the 5% significance level in the GC test. Indonesia continues to demonstrate no significant improvement in forecasting Dubai oil. The outcomes of the Rossi GC Test closely resemble those of the traditional GC. Brazil and Colombia both exhibit significant results at the 1% level, whereas Mexico and Norway demonstrate significance at the 10% and 5% levels, respectively. Finally, based on the MSFE Differences between Models section of the table, Brazil, Colombia, Mexico, and Norway once again show negative values of -0.4434, -0.9848, -0.7286, and -1.1000, respectively, while Canada and Indonesia present positive values. The forecast results for Brazil, Colombia, Mexico, and Norway outperform the benchmark. The predictive ability of the exchange rates for Colombia and Norway is significantly higher for Brent and Dubai compared to WTI. In contrast, for Canada and Indonesia, the forecasts are worse than the benchmark in almost all cases.

Table 29: Tests for Dubai Crude oil case

In-sample GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.0148**	0.000***	0.3134	0.0734*	0.000***	0.0343**
Rossi GC TEST						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
	0.000***	0.000***	0.8546	0.0738*	0.0166**	0.2485
MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)						
Countries:	Brazil	Colombia	Indonesia	Mexico	Norway	Canada
MSFE	-0.4434	-0.9848	2.2617	-0.7286	-1.1000	1.2067
ENCNEW test	***	***	NR	***	***	NR
Start date	(2000:02)	(1993:02)	(1999:02)	(1996:02)	(1993:02)	(1993:02)
Window size	70	91	73	82	91	91

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

5 WTI and Dubai Out-of-sample forecasting with different rolling window size

For the ENCNEW test of the WTI Crude oil case in Table 30, Brazil, Colombia, Mexico and Norway present significant forecasting ability at the 5% level or better results with window sizes of 72 and 84 months. Brazil's exchange rate presents significant forecasting ability at the 1% level for window sizes of 72 months. Norway and Colombia demonstrate noticeable forecasting capability since the exchange rate rejects the null hypothesis for all window sizes at the 1% level for all window sizes in the WTI case. However, the forecasting ability of the exchange rates from Canada and Indonesia proved incapable of surpassing the benchmark across all window sizes.

Table 30: Multi-rolling window forecasting for WTI Crude oil case

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.4411	-0.0243	-0.2335	1.4749
	ENCNEW	***	**	**	NR
Canada	MSFE	0.8254	0.5569	0.1683	0.4319
	ENCNEW	NR	NR	NR	NR
Colombia	MSFE	0.0709	-0.2478	-0.2457	-0.2480
	ENCNEW	***	***	***	***
Indonesia	MSFE	2.3100	1.7365	1.7080	1.8106
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	0.0336	-0.2201	-0.0735	0.2706
	ENCNEW	**	***	**	NR
Norway	MSFE	-0.5551	-0.4978	-0.7458	-0.6703
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

Regarding the Dubai crude oil case in Table 31, Brazil, Colombia, Mexico, and Norway continue to demonstrate strong forecasting capabilities across different window sizes. Specifically, the exchange rates of Colombia, Mexico, and Norway consistently show significant forecasting ability at the 1% level for all window sizes. Compared to WTI, the exchange rates of almost all oil exporters show better forecasting ability for Dubai's results. Interestingly, Canada, despite having negative MSFE values for window sizes of 96 and 108 months, and the ENCNEW test rejecting at the 10% and 5% significance levels, shows improved performance for Dubai forecasts. Compared to Brent, Colombia and Mexico exhibit similar forecasting abilities, while other countries show mixed results with their own strengths and weaknesses. However, Indonesia's exchange rate consistently fails to surpass all benchmarks

Table 31: Multi-rolling window forecasting for Dubai Crude oil case

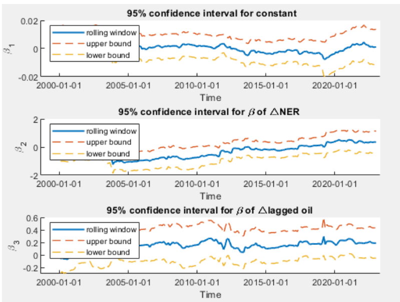
MSFE Differences between Model: $\triangle oil_{t+1}^{i} = \beta_0 + \beta_1 \triangle EX_t^i + \beta_2 \triangle oil_t^i$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.5001	-0.5080	-0.6861	0.6390
	ENCNEW	***	***	***	NR
Canada	MSFE	2.2070	1.3750	-0.03616	-0.4273
	ENCNEW	NR	NR	*	**
Colombia	MSFE	-1.0874	-1.2834	-1.0357	-1.1184
	ENCNEW	***	***	***	***
Indonesia	MSFE	1.9879	2.1213	1.3471	1.6012
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	-0.3622	-0.9267	-0.4766	-0.2881
	ENCNEW	***	***	***	***
Norway	MSFE	-0.9819	-0.7998	-1.4208	-1.3485
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

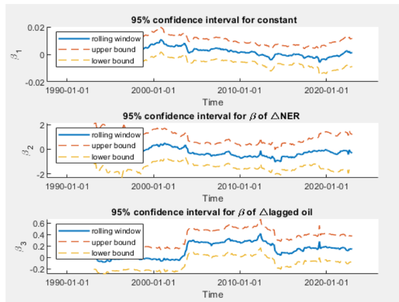
across all window sizes.

6 The coefficient of determination

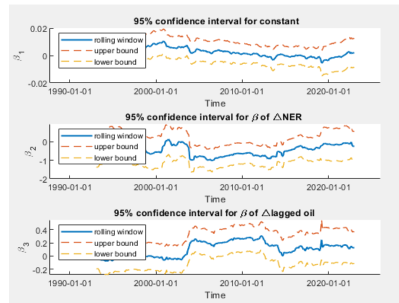
Upon evaluating the coefficient of determination, both in-sample and out-of-sample, for various oil benchmarks against the exchange rates of our sample countries, the result of R^2 can help discern the strength. Compared to the results from Chen et al. (2010b), which used the exchange rates of commodity currency countries to forecast the price of the world commodity index, the R2 of our study, forecasting oil prices using the exchange rates of oil-exporting countries, is notably higher. This aligns with our prior belief that for forecasting a single commodity, using the exchange rate of a country that exports a substantial quantity of that specific commodity or is highly dependent on its export, may be more effective than solely relying on commodity currencies.



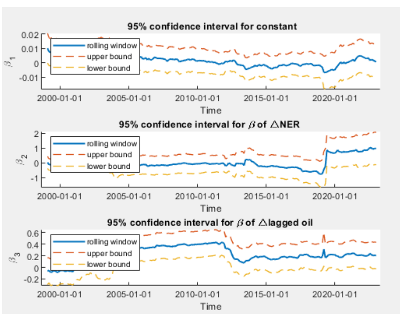
(a) Brazil



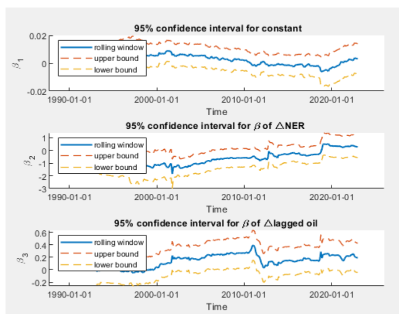
(b) Canada



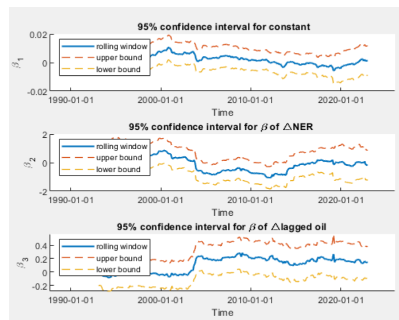
(c) Colombia



(d) Indonesia



(e) Mexico

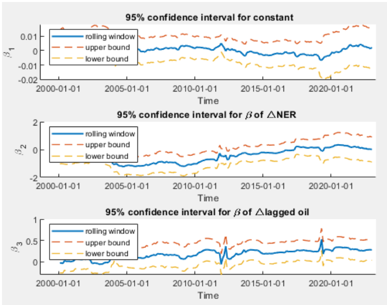


(f) Norway

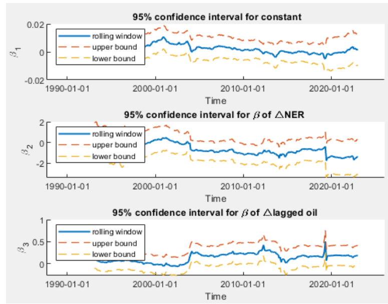
Figure 18: Upper and lower bounds for the coefficients in Brent oil future case: one-step-ahead out-of-sample forecasting model

The results of in-sample R^2 and out-of-sample R^2_{oos} are shown as Figure (27). For Brent crude oil, Colombia stands out with the highest in-sample R^2 value of 0.1165, indicating a strong linear relationship between its exchange rate and Brent crude oil prices. This strength is also mirrored in the out-of-sample R^2_{oos} with a value of 0.1704. It's intriguing to note that when it comes to forecasting ability, Mexico's out-of-sample R^2_{oos} for the Brent oil future case tops the list at 0.1742, despite its in-sample R^2 not being the highest. This could point to Mexico's exchange rate possessing predictive power, especially in a forecasting context for Brent oil futures.

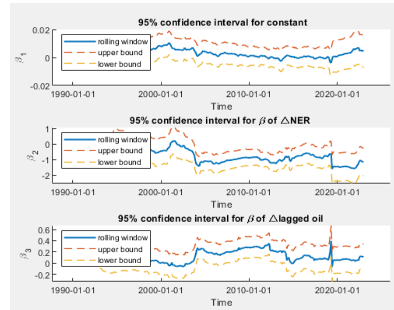
When we shift our attention to WTI crude oil, Brazil demonstrates a robust in-sample R^2 of 0.0873, but the out-of-sample R^2_{oos} peak for this oil type is held by Brazil at 0.1873. This



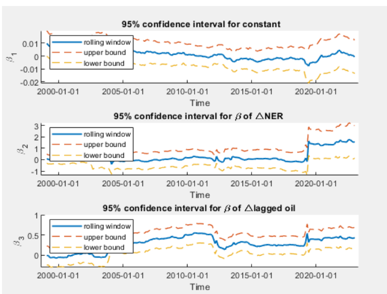
(a) Brazil



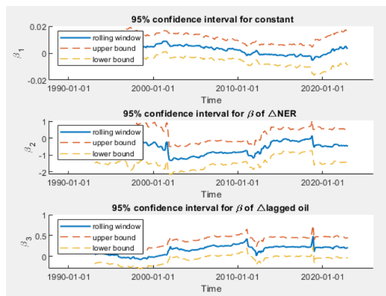
(b) Canada



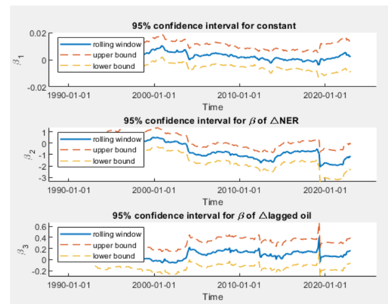
(c) Colombia



(d) Indonesia



(e) Mexico



(f) Norway

Figure 19: Upper and lower bounds for the coefficients in Brent oil case: one-step-ahead out-of-sample forecasting model

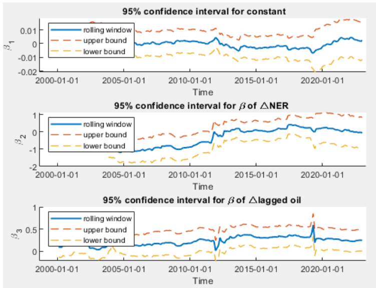
suggests Brazil's exchange rate is a reliable predictor for WTI crude oil prices, both within the sample and in a forecasting capacity.

Dubai crude oil sees Brazil taking the lead in the in-sample R^2 with a value of 0.1380. Yet, in the out-of-sample forecasting, Colombia shines again with an R^2_{oos} of 0.2259, underscoring the strength and consistency of the Colombian exchange rate in forecasting oil prices across different benchmarks.

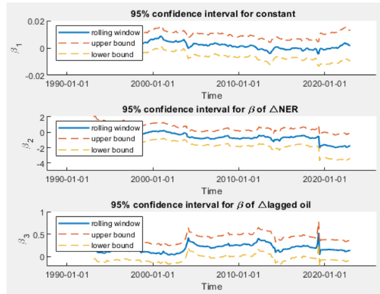
In general, while the in-sample R^2 provides insights into the linear relationship within the dataset, the out-of-sample R^2_{oos} offers a clearer picture of the forecasting ability of these exchange rates in real-world scenarios. It's notable that certain countries (such as Brazil and Colombia) consistently show a strong forecasting ability across different oil types, solidifying the assertion that their exchange rates might significantly help with global oil prices[2].

We also calculated the estimated coefficients with SE bounds for our one-step-ahead out-

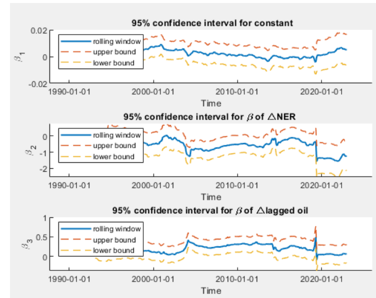
[2]Compared to the study by Chen et al. (2010b), our in-sample and out-of-sample R^2 values are almost double theirs.



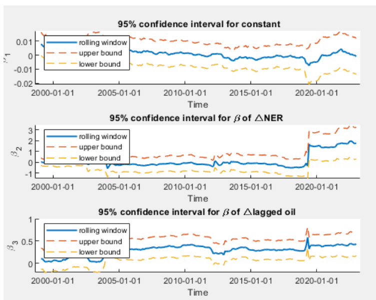
(a) Brazil



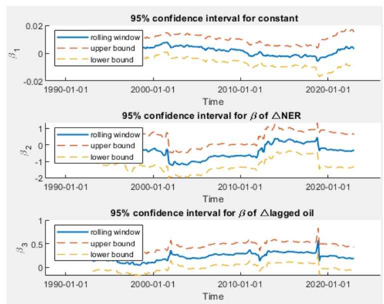
(b) Canada



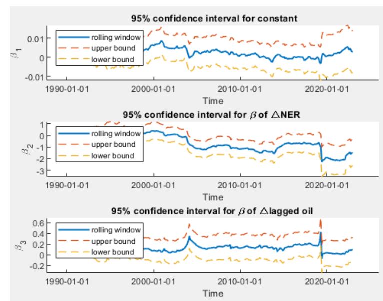
(c) Colombia



(d) Indonesia



(e) Mexico



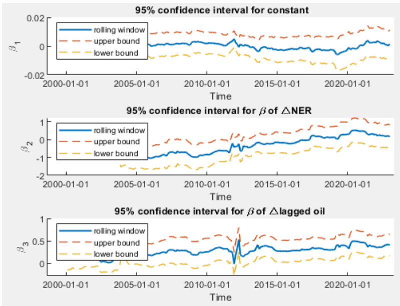
(f) Norway

Figure 20: Upper and lower bounds for the coefficients in WTI oil case: one-step-ahead out-of-sample forecasting model

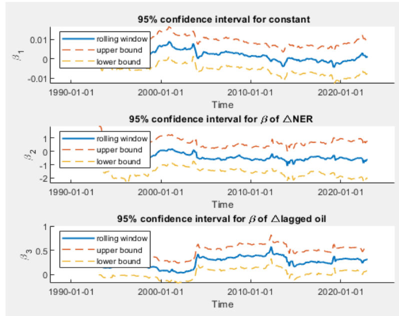
of-sample forecasting model, presented in tables 11, 12, 28, and 29. These are shown in figures 18, 19, 20, and 21, respectively. In general, the coefficients vary widely from one window to the next for all crude oil prices. This suggests that the relationships captured by the model may be changing over time, particularly around 2008 and 2019. This further underscores the benefits of using a rolling window and ENCNEW, given the inherent volatility of oil prices.

7 Multi-rolling window forecasting with the same sample size

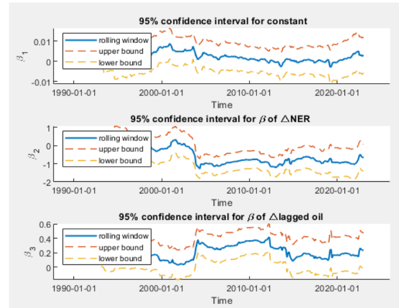
In Section 5.3, we used inconsistent sample numbers due to variations situations of national situations. This inconsistency in the sample length makes it challenging to determine whether the NER for most oil-exporting countries can be used to forecast crude oil prices over the same period. In this appendix, we use the same time to predict oil futures and three oil prices.



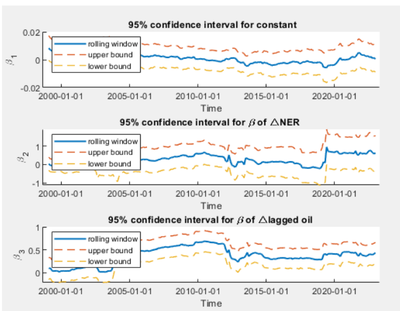
(a) Brazil



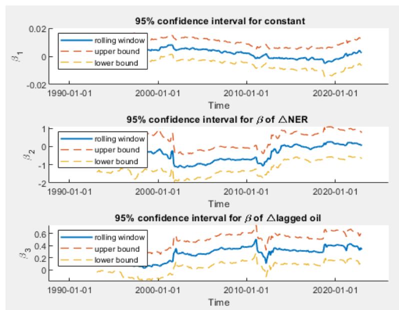
(b) Canada



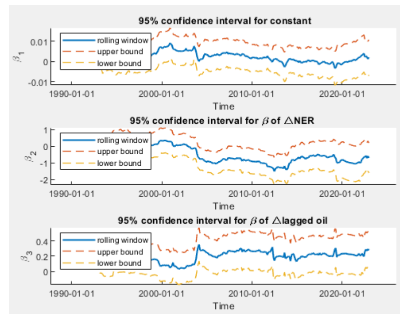
(c) Colombia



(d) Indonesia



(e) Mexico



(f) Norway

Figure 21: Upper and lower bounds for the coefficients in Dubai oil case: one-step-ahead out-of-sample forecasting model

For sample selection, to avoid the effects of early financial crises in many countries, we've set the time span for every country as the same size as from February 2000 to February 2023. Our same period results are shown in the following tables: 32, 33, 34, and 35. Our results are similar to those in Section 5.3. Generally, the exchange rate of oil-exporting countries still more readily forecasts the three oil prices than the Brent futures price. Regarding future prices, both Colombia and Brazil consistently show negative results in most window sizes. Meanwhile, Mexico and Norway only produce results that are negative and the ENCNEW test rejects the null hypothesis at the 1% significance level under the 96-rolling window. Canada and Indonesia, on the other hand, did not show any forecasting prowess for any of the window sizes in this case.

For the Brent oil case, Brazil, Colombia, Mexico, and Norway exhibited strong forecasting capabilities across most window sizes with relatively larger negative MSFE values than others, suggesting a consistently robust forecasting ability. Canada showed limited forecasting power

Table 32: Multi-rolling window forecasting for Brent Crude oil future case

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.0096	0.0211	-0.3570	0.5782
	ENCNEW	***	**	***	NR
Canada	MSFE	1.3902	0.7863	0.2767	0.3564
	ENCNEW	NR	NR	NR	NR
Colombia	MSFE	-0.1115	-0.4281	-0.3492	0.3721
	ENCNEW	***	***	***	*
Indonesia	MSFE	2.5051	2.3726	2.1416	1.7759
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	0.7793	0.0367	-0.2355	0.4065
	ENCNEW	NR	NR	***	**
Norway	MSFE	0.7376	0.3608	-0.3349***	0.5490
	ENCNEW	NR	NR	***	NR

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

only at the 96-month window. Indonesia, similar to the future case, did not demonstrate any significant forecasting ability. For the WTI case, Norway, again stood out with negative MSFE values across all window sizes, reinforcing its stronger forecast abilities than others. Brazil and Mexico exhibited a consistent forecasting capability for window sizes of 72, 84, and 96 months. This time Colombia only showed forecasting capabilities with the 84 and 96 months windows. Canada and Indonesia showed the same results as the previous case. For the Dubai case, the results are shown almost the same as in the WTI case. Judging from the results, there is not a significant difference compared to the 1/4 rolling window results. However, adjusting the window size does improve prediction results for Canada and Norway. This improvement may occur because the log difference exchange rate exhibits

Table 33: Multi-rolling window forecasting for Brent Crude oil case

MSFE Differences between Model: $\triangle oil_{t+1}^{i} = \beta_0 + \beta_1 \triangle EX_t^i + \beta_2 \triangle oil_t^i$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.6854	-0.5040	-0.6846	0.6823
	ENCNEW	***	***	NR	*
Canada	MSFE	0.9563	0.4526	-0.1622	0.5064
	ENCNEW	NR	NR	*	NR
Colombia	MSFE	-1.10175	-1.2461	-1.1013	-0.4471
	ENCNEW	***	***	***	***
Indonesia	MSFE	2.8775	2.6343	2.4921	2.4012
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	-0.4452	-0.9808	-0.8306	0.5437
	ENCNEW	***	***	***	*
Norway	MSFE	-0.6978	-0.5550	-0.7623	-0.2957
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

better predictive ability only within certain time windows, and selecting the appropriate windows can better capture these periods of enhanced predictive power.

8 NER results

In the NER results in Table 22, similar patterns are observed from NEER and REER. This pattern holds in the analysis of NEER, REER, and NER results, highlighting a general increase in predictive ability with longer training periods. Nevertheless, the NER results exhibit the least variation in the number of predictive windows among the three exchange rates. For instance, using MAX, 9 predictive regimes are found when $m = 15$, increasing

Table 34: Multi-rolling window forecasting for WTI Crude oil case

MSFE Differences between Model: $\triangle oil_{t+1}^{i} = \beta_0 + \beta_1 \triangle EX_t^i + \beta_2 \triangle oil_t^i$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.4411	-0.0243	-0.2335	1.4749
	ENCNEW	***	**	**	NR
Canada	MSFE	0.5942	0.2877	-0.0195	0.8185
	ENCNEW	NR	NR	*	NR
Colombia	MSFE	0.0787	-0.2942	-0.2496	0.6263
	ENCNEW	***	***	***	NR
Indonesia	MSFE	2.3074	1.7354	1.7078	1.7041
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	0.0858	-0.2371	-0.0994	1.6642
	ENCNEW	**	**	**	NR
Norway	MSFE	-0.7492	-0.5974	-0.8012	-0.3977
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

to only 10 when $m = 90$. Similarly, with SEQ, only 3 predictive regimes are found when $m = 15$, but this increases to 5 when $m = 90$.

Table 36 shows the first month where MAX and SEQ detect a predictive regime for the NER case. First, the result of the SEQ procedure for NER differs from the previous NEER and REER in that the predicted windows detected by NER do not entirely coincide with the economic downturn periods mentioned earlier. For instance, the NER of Indonesia and Norway did not find predictable regimes around 2015 and 2020 when $m = 15$ and $m = 30$. Additionally, Canada and Mexico detected predictable regimes in November 2018 and March 2019. However, most of the other detected predictable windows are still around 2015 and 2020 across the different sizes of m. Interestingly, the results of the MAX procedure differ

Table 35: Multi-rolling window forecasting for Dubai Crude oil case

MSFE Differences between Model: $\triangle oil^i_{t+1} = \beta_0 + \beta_1 \triangle EX^i_t + \beta_2 \triangle oil^i_t$ and AR(1)					
Window Number	:	72	84	96	108
Brazil	MSFE	-0.4993	-0.1832	-0.4063	1.4550
	ENCNEW	***	**	***	NR
Canada	MSFE	0.6798	0.3232	-0.1664	0.8054
	ENCNEW	NR	NR	*	NR
Colombia	MSFE	0.0383	-0.2902	-0.1594	0.5587
	ENCNEW	***	***	***	*
Indonesia	MSFE	2.2918	1.7689	1.7303	1.6995
	ENCNEW	NR	NR	NR	NR
Mexico	MSFE	0.0937	-0.2713	-0.1160	1.6736
	ENCNEW	**	**	**	NR
Norway	MSFE	-0.7112	-0.5463	-0.8206	-0.3914
	ENCNEW	***	***	***	***

Note: For the MSFE part, we are reporting the loss difference between the two models, the ENCNEW results are reported by the number of Asterisk. $***$, $**$ and $*$ mark means rejection at the 1%, 5% and 10% significance levels, respectively. The critical values of the ENCNEW test are provided by Clark and McCracken (2001). NR means no rejection.

from those of SEQ. Every predictive regime found by MAX is around 2014-2016, except for one instance with Brazil, when $m = 45$, a predictive regime for Brazil was found in April 2020.

Since the previous two subsections mainly describe relatively large subsample examples, Figure 22 shows that there are two predictive regimes for Brazil's NER by the MAX procedure when $m = 15$. The first regime is in December 2014 and the second one is in early 2020. It can be seen that the results for NER are not particularly different from NEER and REER. The results from Figures 15 to 22 show that the three oil-exporting countries' exchange rates have clear convergence in the prediction mechanisms around major economic events. Different countries also find predictive mechanisms at roughly the same time, especially in 2015 and

Table 36: First month where a predictive regime is detected by MAX and SEQ with $\pi = 0.05$ for NER case

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	NaN	NaN	04/15	0.10744	03/15	0.2
Canada	11/18	0.22047	02/20	0.29707	NaN	NaN	06/16	0.2	06/16	0.36
Colombia	NaN	NaN	06/23	0.39568	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	04/22	0.32881	06/23	0.39785	NaN	NaN	04/15	0.10744	04/15	0.213
Mexico	03/19	0.23256	01/20	0.29412	06/16	0.16364	06/16	0.2	12/16	0.407
Norway	NaN	NaN	10/23	0.40636	NaN	NaN	NaN	NaN	NaN	NaN
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	01/15	0.066038	NaN	NaN	04/20	0.35814	09/14	0.084746	12/14	0.21311
Canada	NaN	NaN	12/14	0.071823	12/14	0.086093	12/14	0.10744	01/16	0.35135
Colombia	01/15	0.066038	01/15	0.076923	NaN	NaN	01/15	0.11475	NaN	NaN
Indonesia	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	12/14	0.21311
Mexico	12/14	0.061611	01/15	0.076923	10/19	0.33971	01/15	0.11475	07/16	0.4
Norway	11/14	0.057143	01/15	0.076923	01/15	0.092105	01/15	0.11475	01/15	0.22581

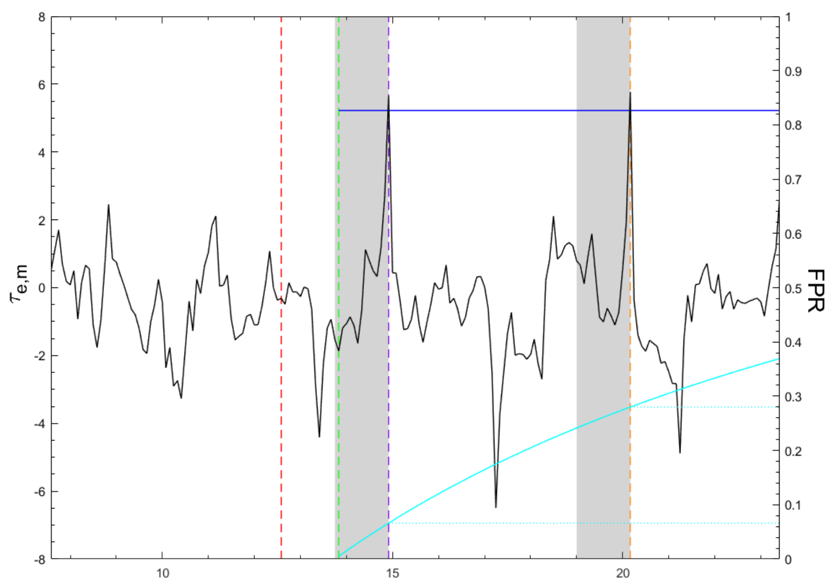
2020. These years are periods of significant economic fluctuations. As mentioned earlier, during periods of large fluctuations in oil prices, especially during economic downturns, the exchange rates of oil exporters can temporarily predict crude oil returns.

9 Appendices I: Brent's oil futures price

To conserve space, we have consolidated the results for Brent's oil futures price here, employing the same approach as that used for Brent. We begin our analysis with a preliminary examination aimed at testing the predictability of the full sample, setting the stage for the subsequent application of MAX and SEQ procedures. To detect predictability, we utilise the IVX method, as illustrated in the table below (see Table 37). This table presents estimated slope parameters $\hat{\beta}$, along with the standard and adjusted R^2 values, for orthodox bivariate regression models. These models were applied using OLS for parameter estimation and the IV test, as introduced by Kostakis et al. (2015). Except for Brazil in the NEER and REER

Figure 22: The location of the predictive regimes for Brazil' NER, MAX procedure, m=15

(a) Brazil



Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

case, which rejected the KMS test's null hypothesis, all other KMS tests failed to reject the null hypothesis, indicating a lack of predictability for Brent's oil futures price, similar to Brent's.

In Table 38, we present the Bonferroni 90% confidence intervals for the coefficient β derived via the Q test. Initial examination of the aggregated sample in NEER reveals that the Bonferroni Q-test upholds the null hypothesis, indicating an absence of predictability across all included nations. This absence of predictive power is also echoed in the subsamples pertaining to REER and NER, where the Bonferroni Q-test consistently implies no predictability across these nations. Notably, in parallel with the results from the KMS test and IV_{comb} test, the Bonferroni Q-test discerns predictability within Brazil's REER, delineating a strictly positive confidence interval for β at the 90% significance level. Overall, using various methods yields consistent results for Brent crude oil and Brent crude oil futures prices. The

Table 37: Preliminary results for the full sample for Brent's oil futures price

NEER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.049205	-0.16048	0.47061	0.17788	-1.653*
Canada	-0.069939	0.48477	0.1516	-0.14206	-0.401
Colombia	-0.036828	0.5625	0.1989	-0.094634	-1.203
Indonesia	-0.0026962	0.48329	0.0029692	-0.29114	-0.393
Mexico	-0.021528	0.2271	0.08903	-0.20483	-0.826
Norway	-0.14308	0.39216	0.32324	0.030071	-0.786
REER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.12078	-0.53045	1.3023	1.012	-2.015*
Canada	-0.092592	0.79459	0.17921	-0.11439	-0.476
Colombia	-0.11392	1.2186	0.52401	0.23144	-1.210
Indonesia	-0.0059131	0.69591	0.0023022	-0.29181	-0.024
Mexico	-0.076747	-0.12062	0.23448	-0.058948	-0.524
Norway	-0.15751	0.54193	0.25952	-0.033836	-0.747
NER(local)	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	adjusted $R^2(\%)$	KMS_{test}
Brazil	-0.02314	0.044402	0.24552	-0.047875	-1.174
Canada	-0.067591	1.0018	0.17167	-0.12194	-0.466
Colombia	-0.022956	0.57332	0.15162	-0.14206	-0.959
Indonesia	-0.00090133	0.73854	0.00040045	-0.29372	-0.243
Mexico	-0.020026	0.26217	0.085256	-0.20861	-0.732
Norway	-0.043154	0.90937	0.1186	-0.17517	-0.442

Note: The 10%critical value used for IV_{comb} and KMS is ± 1.645.

outcomes of the preliminary analysis of the data over the training periods are given in Table (46) in Appendices 10. This time, we observed that none of the sample countries exhibited predictability during the training period. Therefore, we do not modify the monitoring period in this case. Then we proceed to employ a real-time monitoring program to track the development of an end-of-sample prediction mechanism.

To evaluate the alignment of the monitoring application with our datasets, we employ the same ways used to obtain the results in Table (22) for the training periods used in the subsequent monitoring application as the selection of training periods includes January 1995 to September 2012 (for $m = 15$), January 1995 to June 2011 (for $m = 30$), January 1995 to March 2010 (for $m = 45$), January 1995 to December 2008 (for $m = 60$), and January

Table 38: CY test results for the full sample for Brent future price

NEER	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.05197	-1.31	0.5084	[-0.104, 0.029]
Canada	-0.0709642	-0.73	0.1549	[-0.200, 0.144]
Colombia	-0.03717	-0.82	0.1988	[-0.090, 0.063]
Indonesia	-0.0025193	-0.09	0.0025	[-0.040, 0.050]
Mexico	-0.023426	-0.59	0.102	[-0.075, 0.063]
Norway	-0.1426012	-1.04	0.3208	[-0.452, 0.069]
REER	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.12099	-2.12	1.3025	[-0.208, -0.012]*
Canada	-0.09312	-0.78	0.1809	[-0.290, 0.141]
Colombia	-0.1139	-1.34	0.5238	[-0.267, 0.032]
Indonesia	-0.0057	-0.09	0.0021	[-0.111, 0.113]
Mexico	-0.07672	-0.88	0.2271	[-0.181, 0.115]
Norway	-0.1577	-0.94	0.260155	[-0.556, 0.087]
NER(local)	$\hat{\beta}$	t-stat	$R^2(\%)$	90% CI
Brazil	-0.0254	-0.97	0.275	[-0.059, 0.031]
Canada	-0.0558	-0.61	0.111	[-0.214, 0.124]
Colombia	-0.0236	-0.72	0.150	[-0.060, 0.052]
Indonesia	0.00165	0.07	0.0013	[-0.032, 0.052]
Mexico	-0.02447	-0.63	0.1166	[-0.066, 0.070]
Norway	-0.0315033	-0.45	0.06039	[-0.155, 0.100]

Note: The 90% CI columns report the 90% Bonferroni confidence intervals for $\hat{\beta}$ using the Q-test. Confidence intervals that reject the null are shown with a * mark.

1995 to June 2006 (for $m = 90$). These periods correspond to $T^* = 228 - m$, where T^* is the training period, and observation $t = 228$ marks the commencement of monitoring in the application below, dated 2013:12.

The outcome of Brent oil future prices appears to be identical to that of Brent commodity prices. Table 39 presents the number of predictive regimes detected by the MAX and SEQ methods (with a significance level of $\pi = 0.05$ for SEQ), for Brent futures prices. The analysis encompasses three exchange rates—NEER, REER, and NER—across varying sizes of the moving window (m) set at 15, 30, 45, 60, and 90. In the NEER case, the SEQ method shows variability in the number of predictive regimes detected across different countries and

Table 39: Number of predictive regimes detected by SEQ with $\pi = 0.05$ and MAX for Brent futures price

	SEQ					MAX				
neer	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	0	2	1	0	0	1	3	3
Canada	0	0	0	2	0	0	1	1	2	0
Colombia	0	1	0	0	0	0	0	0	0	0
Indonesia	1	0	3	3	2	0	0	2	2	2
Mexico	1	0	1	0	1	2	1	1	1	1
Norway	0	0	2	1	2	1	2	3	2	3
reer	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	1	0	2	0	0	1	3	2
Canada	0	0	0	1	1	0	1	2	2	1
Colombia	0	1	0	0	0	0	1	0	0	0
Indonesia	1	1	3	1	1	0	0	3	3	2
Mexico	1	0	0	0	0	3	0	0	0	0
Norway	2	0	0	1	2	0	2	3	3	3
ner	m=15	m=30	m=45	m=60	m=90	m=15	m=30	m=45	m=60	m=90
Brazil	0	0	1	2	2	0	0	1	3	3
Canada	0	2	0	2	0	0	2	1	2	0
Colombia	1	1	0	0	0	2	0	0	0	0
Indonesia	0	1	0	2	2	0	0	0	1	3
Mexico	1	1	1	1	0	3	0	1	1	1
Norway	2	1	0	0	0	2	2	1	1	0

m values. This indicates a sensitivity to the training period's length. For instance, Brazil exhibits an increasing trend in detected predictive regimes as m increases, peaking at $m = 60$. On the other hand, the MAX method also demonstrates a relatively stable or increasing trend in detected regimes with the growth of m. Notably, Colombia emerges as an exception, with MAX failing to identify any predictive regimes across all m values. The REER results reflect a similar pattern, with SEQ revealing fluctuations in the number of detected predictive regimes based on the training period length. Countries like Brazil and Norway display an increase in the number of regimes with larger m values. Meanwhile, the MAX method's outcomes align with those observed in the NEER analysis, showcasing a degree of predictability that, while

Table 40: First month where a predictive regime is detected by SEQ and MAX with $\pi = 0.05$ for neer case

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	NaN	NaN	02/15	0.12195	12/14	0.21311
Canada	NaN	NaN	NaN	NaN	NaN	NaN	01/15	0.11475	NaN	NaN
Colombia	NaN	NaN	04/23	0.40214	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	04/19	0.24715	NaN	NaN	01/15	0.092105	10/14	0.092437	09/14	0.17241
Mexico	10/18	0.22957	NaN	NaN	12/19	0.34597	NaN	NaN	08/16	0.40741
Norway	NaN	NaN	NaN	NaN	01/15	0.092105	03/16	0.20588	01/15	0.22581
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	NaN	NaN	NaN	NaN	03/20	0.35514	11/14	0.1	12/14	0.21311
Canada	NaN	NaN	12/14	0.071823	12/14	0.086093	12/14	0.10744	NaN	NaN
Colombia	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	NaN	NaN	NaN	NaN	02/16	0.16364	10/14	0.092437	10/14	0.18644
Mexico	05/18	0.21429	07/18	0.25	10/19	0.33971	01/16	0.19403	07/16	0.4
Norway	12/14	0.061611	11/14	0.066667	11/14	0.08	12/14	0.10744	12/14	0.21311

generally stable, varies with the increase m value. For NER, the trends observed are akin to those in NEER and REER analyses. For SEQ, what is different from before is that the most regimes are found when $m = 60$. This also reflects how SEQ's detection of predictive regimes varies across different m values and countries. On the other hand, MAX also exhibits a similar pattern in regime detection across m values. The predictable regime usually increases with m, peaking at $m = 60$ and then declining, with certain exceptions such as Colombia, which shows predictive regimes at $m = 15$ but none afterwards.

Overall, Table 39 elucidates the nuanced distinctions between the SEQ and MAX methods in identifying predictive regimes across various exchange rates and training periods. The comparative analysis suggests that both the Brent oil and futures oil cases exhibit detectable predictive regimes.

Tables 40 to 42 delineate the first month when the SEQ identifies a predictive regime. Similar to the findings for Brent crude oil, the exchange rates of oil-exporting countries in

Table 41: First month where a predictive regime is detected by SEQ and MAX with $\pi = 0.05$ for ner case

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	03/20	0.35514	02/15	0.12195	01/15	0.22581
Canada	NaN	NaN	01/15	0.076923	NaN	NaN	01/15	0.11475	NaN	NaN
Colombia	04/23	0.36334	02/23	0.39785	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	NaN	NaN	03/23	0.4	NaN	NaN	12/14	0.10744	01/15	0.22581
Mexico	06/18	0.21739	03/19	0.27586	11/19	0.34286	06/16	0.2	NaN	NaN
Norway	12/14	0.061611	06/23	0.40636	NaN	NaN	NaN	NaN	NaN	NaN
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	NaN	NaN	NaN	NaN	02/20	0.35211	11/14	0.1	12/14	0.21311
Canada	NaN	NaN	12/14	0.071823	12/14	0.086093	12/14	0.10744	NaN	NaN
Colombia	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	NaN	NaN	NaN	NaN	NaN	NaN	01/15	0.11475	11/14	0.2
Mexico	12/14	0.061611	NaN	NaN	10/19	0.33971	12/15	0.18797	07/16	0.4
Norway	10/14	0.052632	12/14	0.071823	01/15	0.092105	01/15	0.11475	NaN	NaN

our sample demonstrate the ability to predict Brent future prices during periods when these countries are impacted by a global economic crisis.

For example, as shown in Table 40, both Brazil and Canada's predictive regime was caught with an SEQ detection with a critical value set at 0.05 for SEQ around February 2015 when $m = 60$ in Table 40. This early detection highlights the impact of global economic shocks on oil-dependent economies, coinciding with volatile commodity market phases. Similarly, in Table 41, Norway's predictive regime was detected at the end of 2014 with SEQ (with a critical value of 0.05) and MAX detection. Mexico's economy signalled a predictive regime in December 2014 with an SEQ detection with a 0.05 critical value. Furthermore, Table 41 shows that, except for Colombia, the other five sample countries have identified predictive regimes around 2015 for $m = 60$. Table 42 also reveals Canada's predictive regime in December 2015 and February 2016 with an SEQ detection, a critical value of 0.05 for SEQ for $m = 60$ and $m = 90$, separately. Additionally, Brazil's predictive regime was identified in April 2019 in

Table 42 with an SEQ detection, aligning with the onset of the COVID-19 pandemic and its impact on global markets. More examples are from Tables 40 to 42.

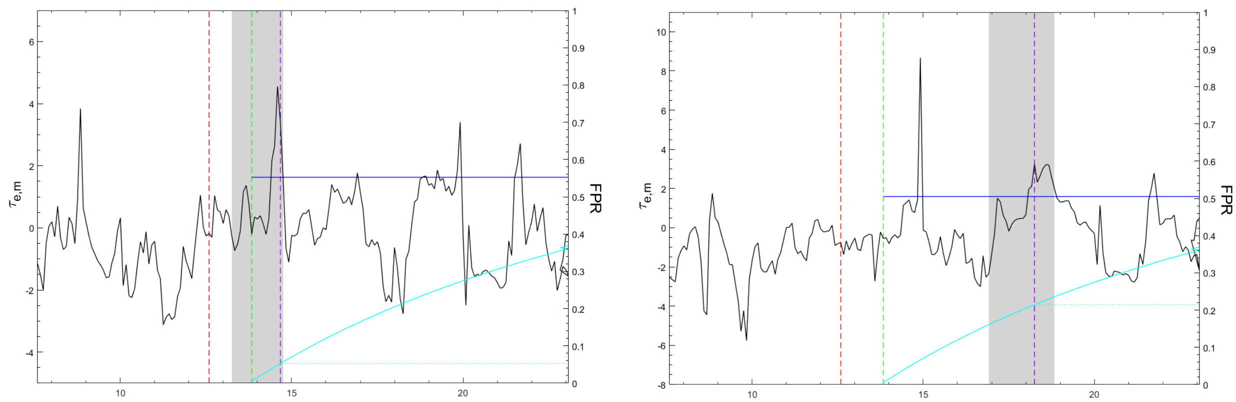
Table 42: First month where a predictive regime is detected by SEQ and MAX with $\pi = 0.05$ for reer case

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}	SEQ	FPR_{SEQ}
Brazil	NaN	NaN	NaN	NaN	04/19	0.3202	NaN	NaN	09/15	0.31429
Canada	NaN	NaN	NaN	NaN	NaN	NaN	12/15	0.18797	02/16	0.36
Colombia	NaN	NaN	04/23	0.40214	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	03/19	0.24427	12/16	0.18049	03/15	0.1039	01/15	0.11475	01/17	0.44186
Mexico	06/18	0.21739	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Norway	01/15	0.066038	NaN	NaN	NaN	NaN	01/16	0.19403	09/14	0.17241
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}	MAX	FPR_{MAX}
Brazil	NaN	NaN	NaN	NaN	02/20	0.35211	12/14	0.10744	11/15	0.33333
Canada	NaN	NaN	01/15	0.076923	11/14	0.08	11/14	0.1	01/16	0.35135
Colombia	NaN	NaN	04/23	0.40214	NaN	NaN	NaN	NaN	NaN	NaN
Indonesia	NaN	NaN	NaN	NaN	02/16	0.16364	11/14	0.1	11/16	0.42857
Mexico	03/18	0.208	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Norway	NaN	NaN	11/14	0.066667	11/14	0.08	11/14	0.1	09/14	0.17241

10 Appendices II : Figure and Tables

Figure 23: The location of the predictive regimes for the sample countries' NEER, SEQ procedure, m=15

(a) Indonesia (b) Mexico



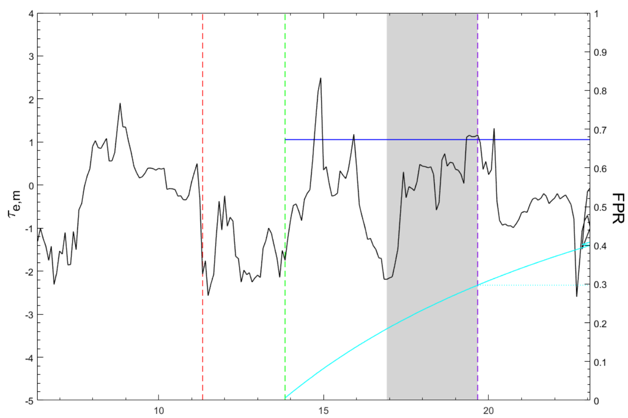
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Table 43: Data sources and collection for Chapter 4

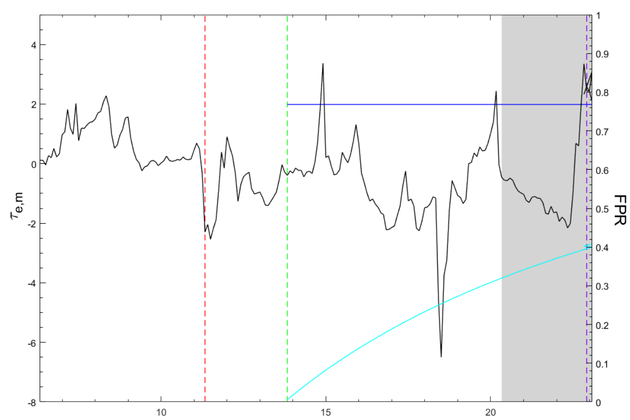
Variables name	period	sources	website
Brazil NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/EXBZUS
Canada NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/EXCAUS
Colombia NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/COLCCUSMA02STM
Indonesia NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/CCUSSP02IDM650N
Mexico NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/DEXMXUS
Norway NER	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/EXNOUS
NEER and REER	Monthly:1995:01-2023:07	Darvas (2021)	https://www.bruegel.org/datasets
Brent oil	Monthly:1995:01-2023:07	Fred	https://fred.stlouisfed.org/series/POILBREUSDM
Brent futures	Monthly:1995:01-2023:07	ICE	https://uk.investing.com/commodities/brent-oil

Figure 24: The location of the predictive regimes for the sample countries' NEER, SEQ procedure, m=30

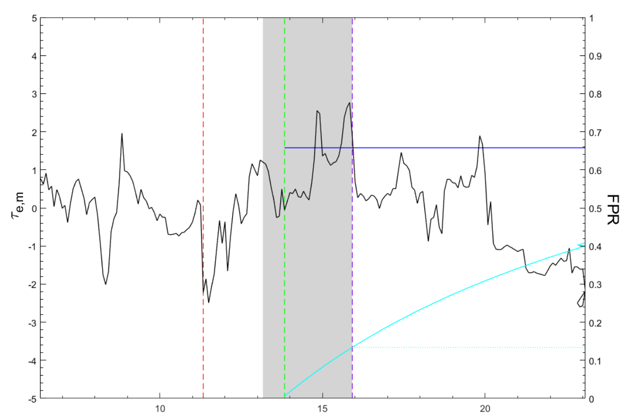
(a) Canada



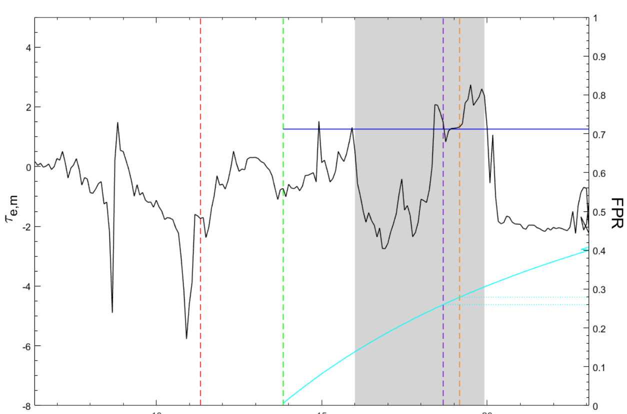
(b) Colombia



(c) Indonesia



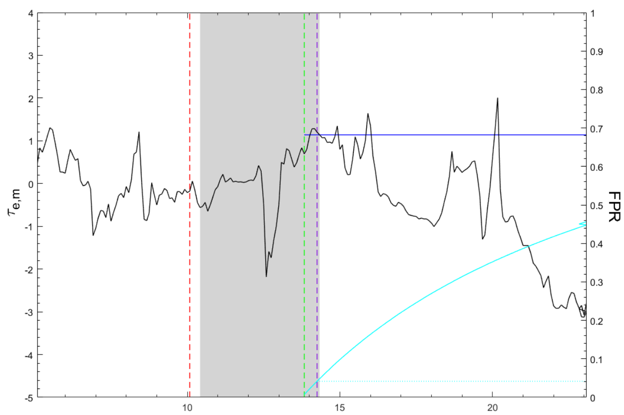
(d) Mexico



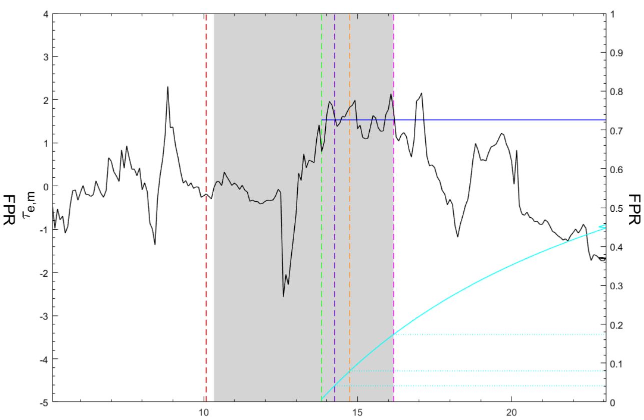
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ▪ is the weak set of dates, — is the false positive rate.

Figure 25: The location of the predictive regimes for the sample countries' NEER, SEQ procedure, m=45

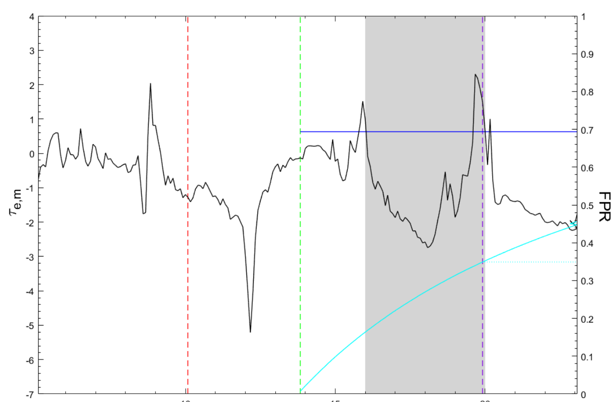
(a) Brazil



(b) Indonesia



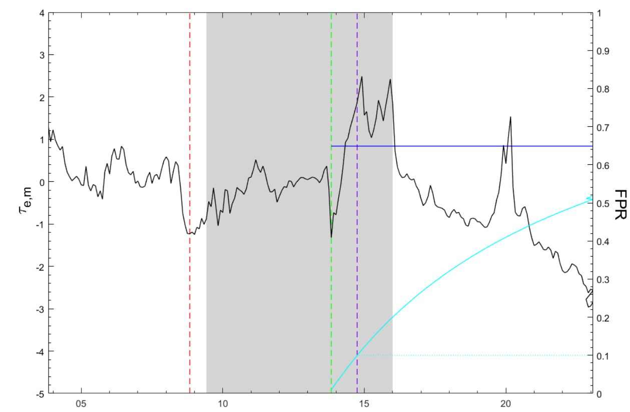
(c) Mexico



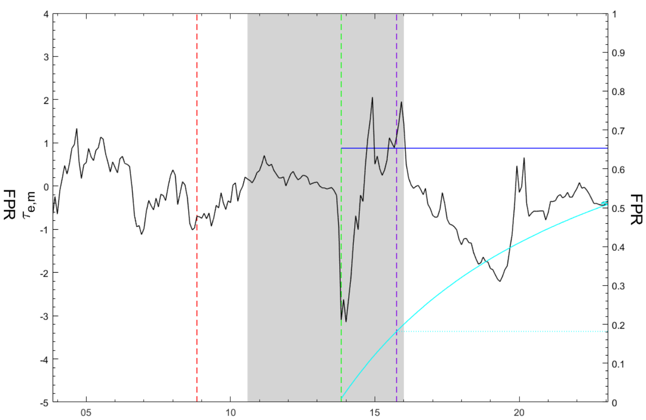
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^{*}),— is the T^{*} +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 26: The location of the predictive regimes for the sample countries' NEER, SEQ procedure, m=60

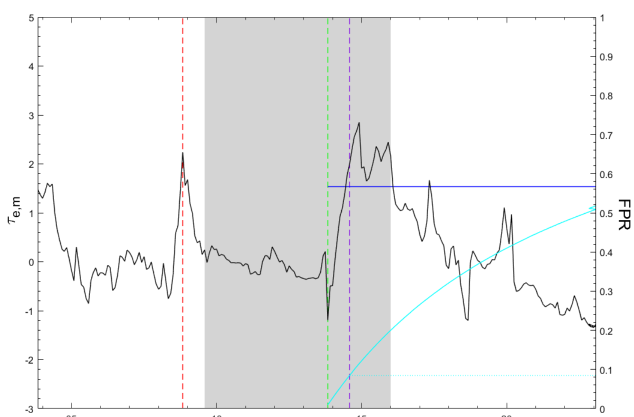
(a) Brazil



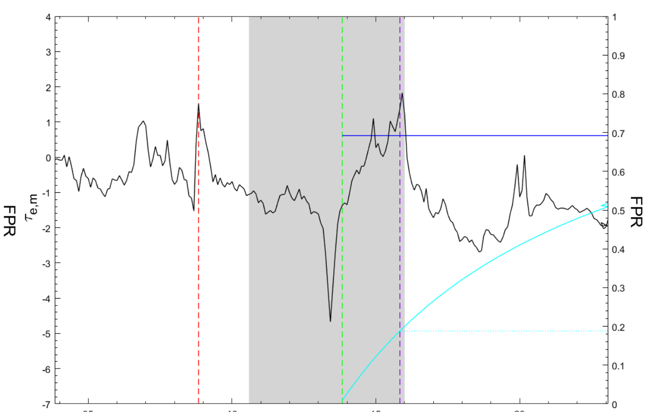
(b) Canada



(c) Indonesia



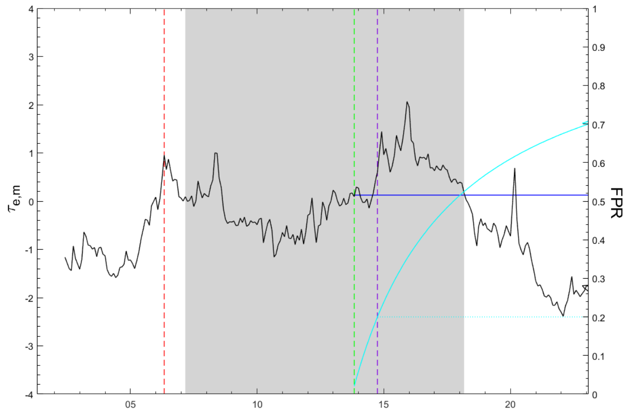
(d) Mexico



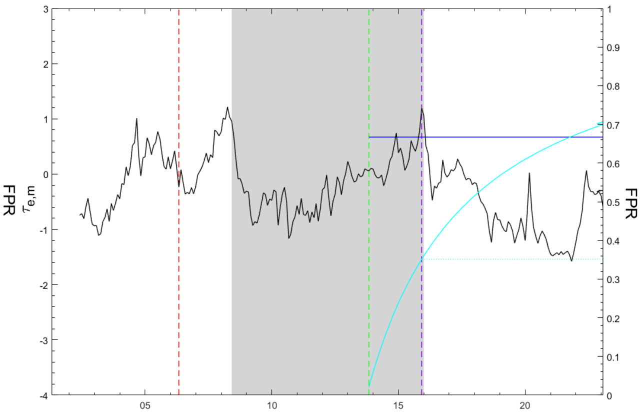
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 27: The location of the predictive regimes for the sample countries' NEER, SEQ procedure, m=90

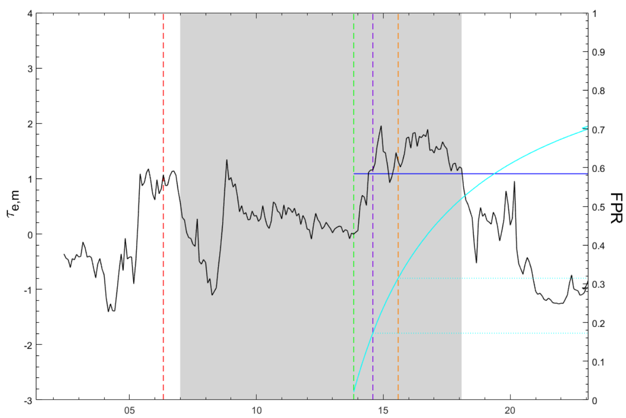
(a) Brazil



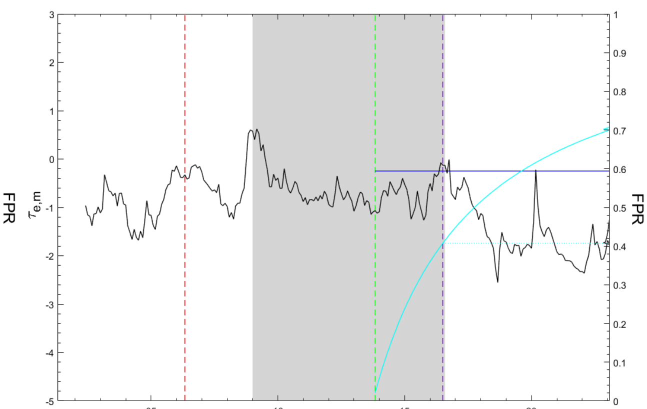
(b) Canada



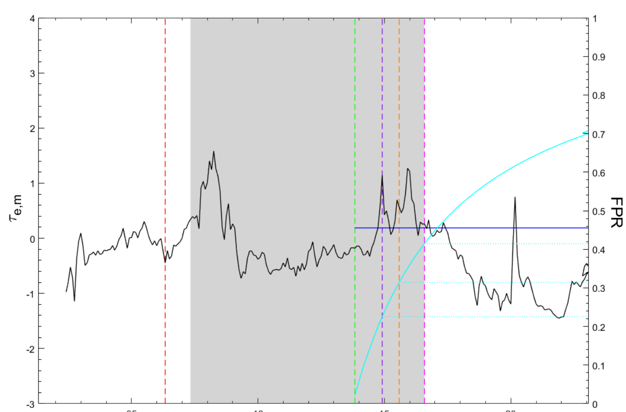
(c) Indonesia



(d) Mexico



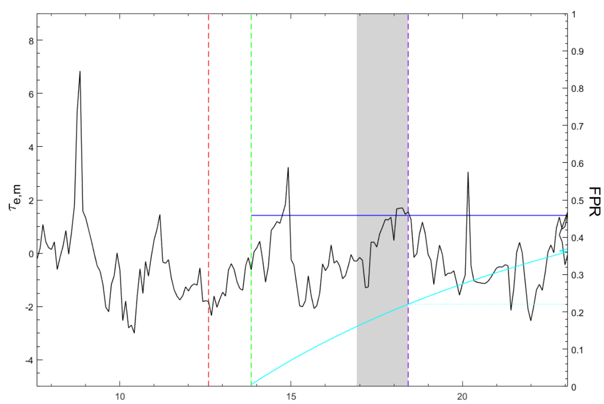
(e) Norway



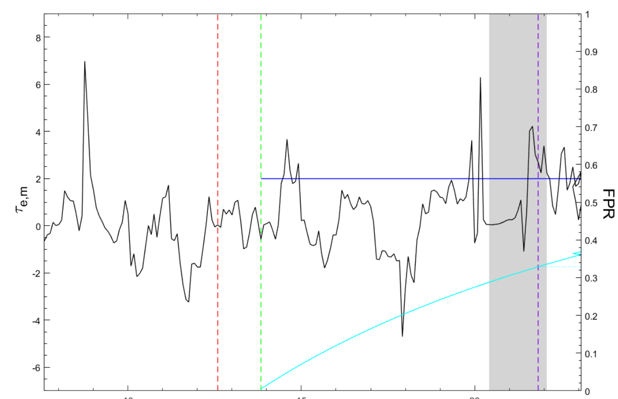
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^{*}),— is the T^{*} +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 28: The location of the predictive regimes for the sample countries' NER, SEQ procedure, m=15

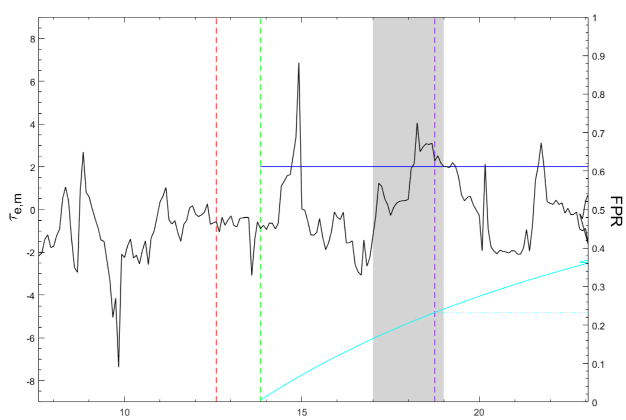
(a) Colombia



(b) Indonesia



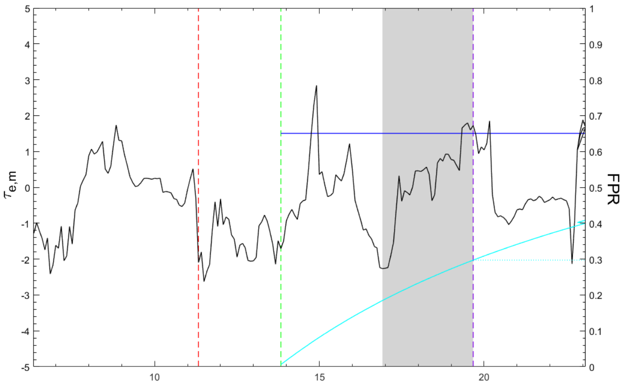
(c) Mexico



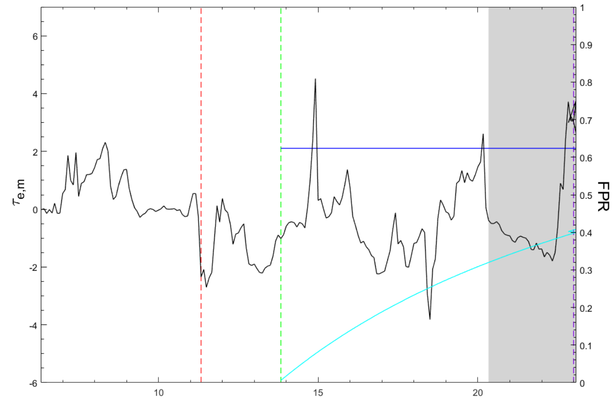
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 29: The location of the predictive regimes for the sample countries' NER, SEQ procedure, m=30

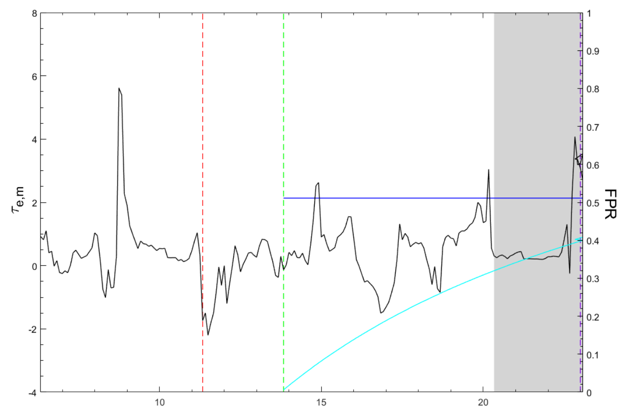
(a) Canada



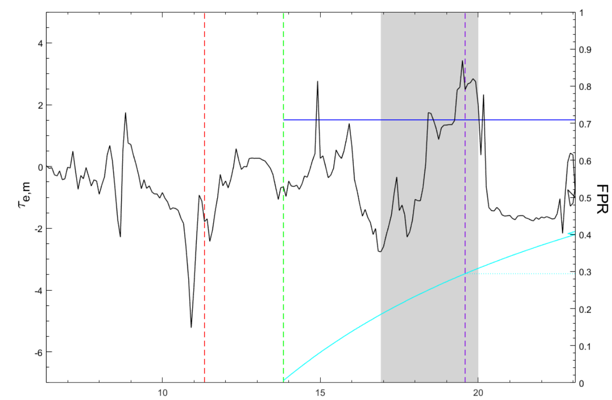
(b) Colombia



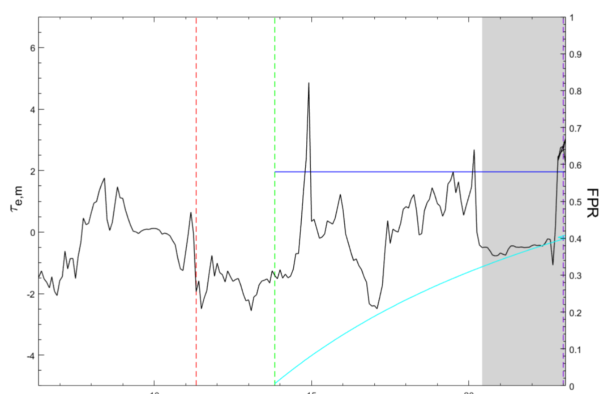
(c) Indonesia



(d) Mexico



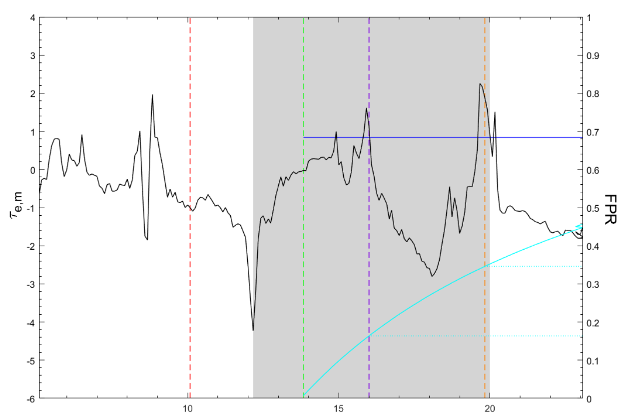
(e) Norway



Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^{*}),— is the T^{*} +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 30: The location of the predictive regimes for the sample countries' NER, SEQ procedure, m=45

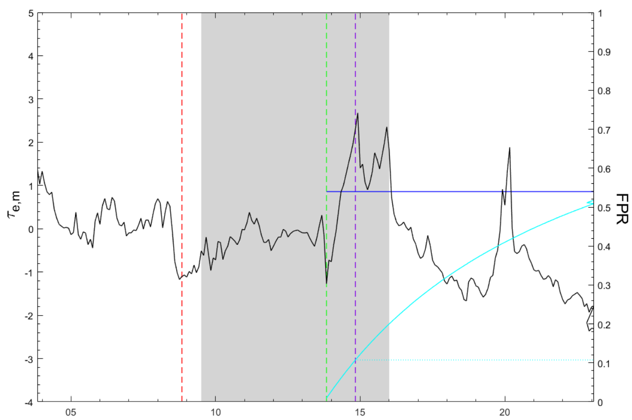
(a) Mexico



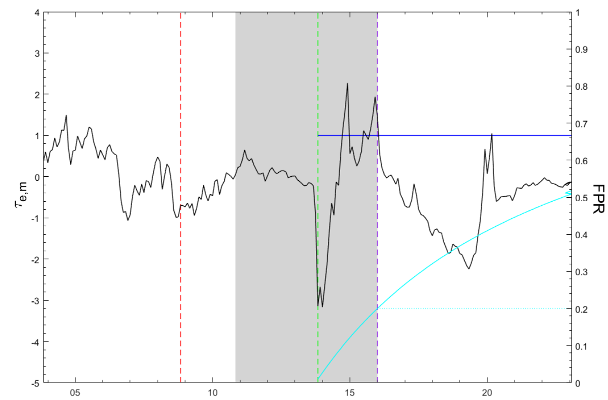
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 31: The location of the predictive regimes for the sample countries' NER, SEQ procedure, m=60

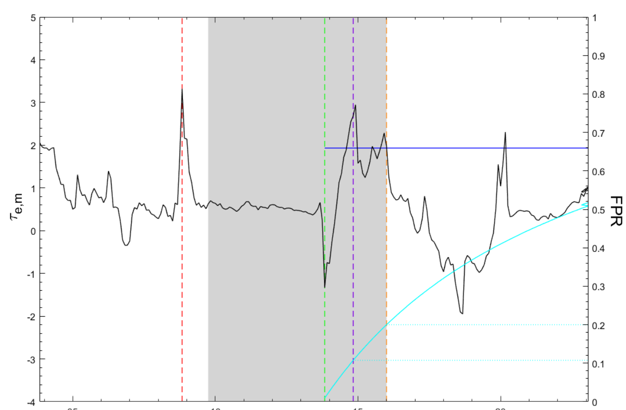
(a) Brazil



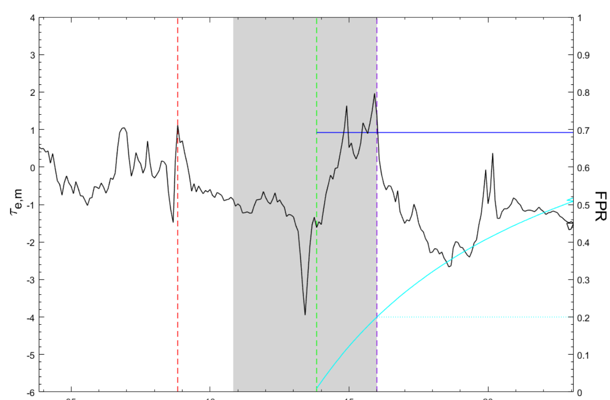
(b) Canada



(c) Indonesia

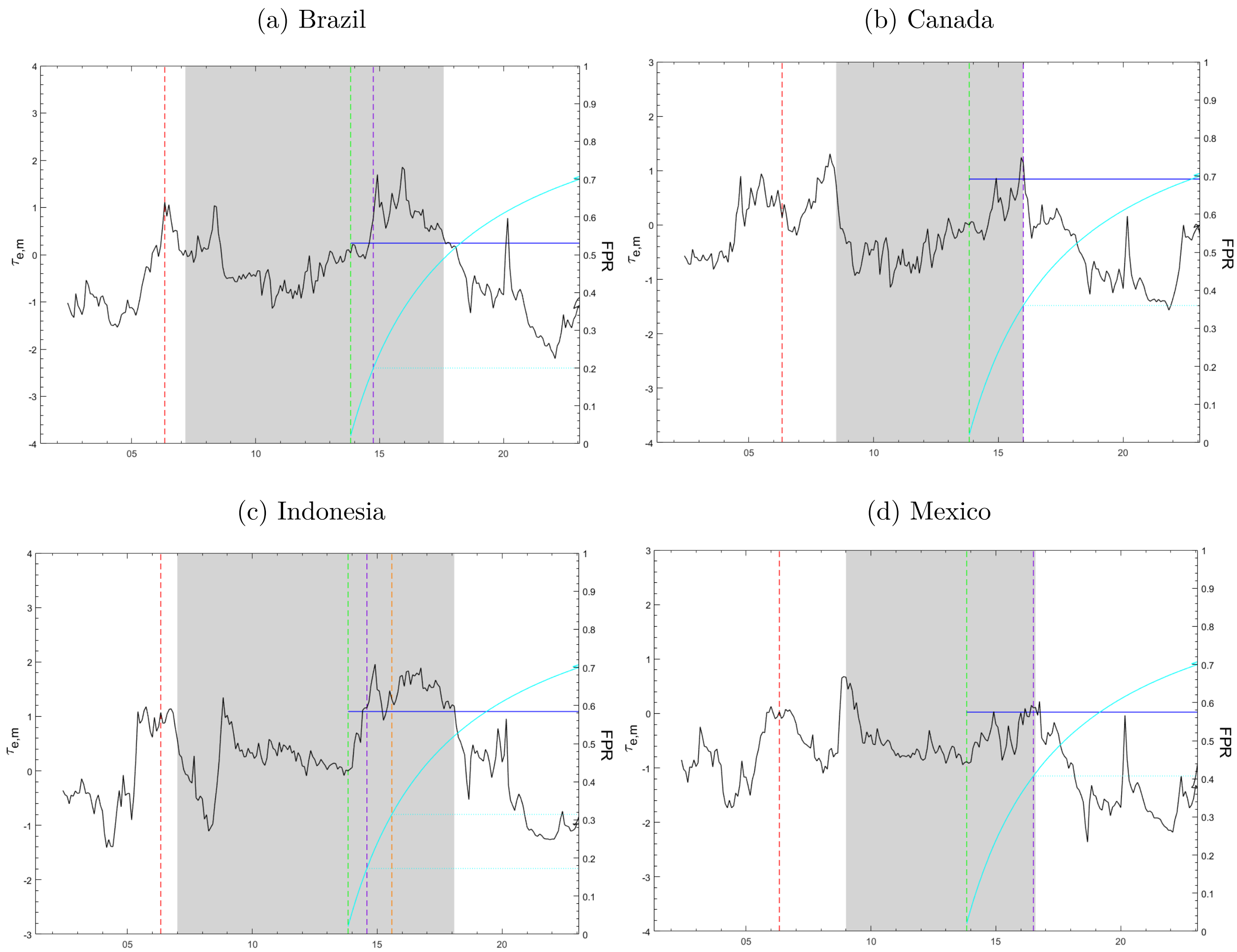


(d) Mexico



Note:— is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^{*}),— is the T^{*} +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

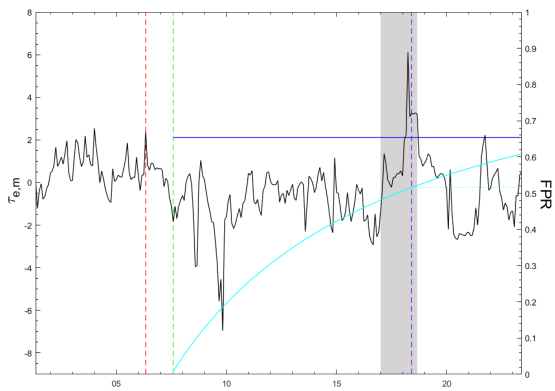
Figure 32: The location of the predictive regimes for the sample countries' NER, SEQ procedure, m=90



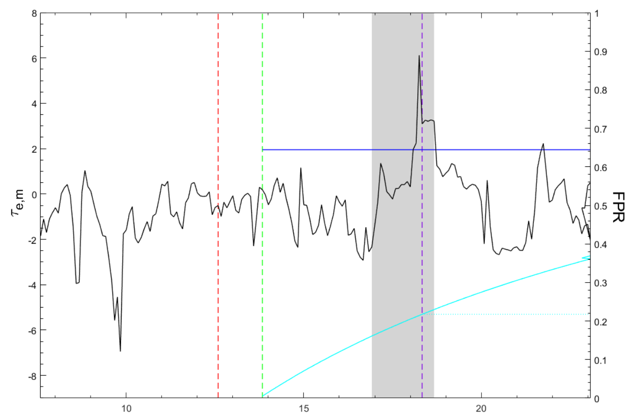
Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 33: The location of the predictive regimes for the sample countries' REER, SEQ procedure, m=15

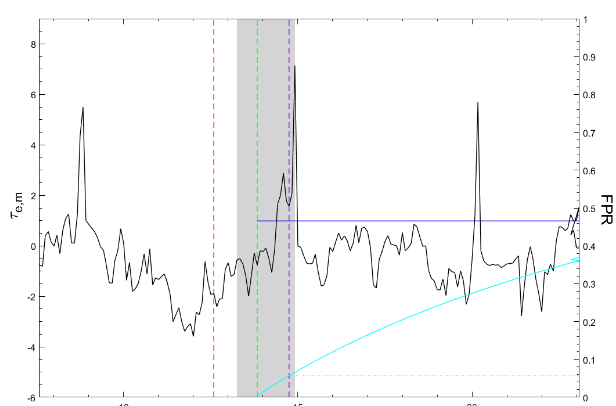
(a) Indonesia



(b) Mexico



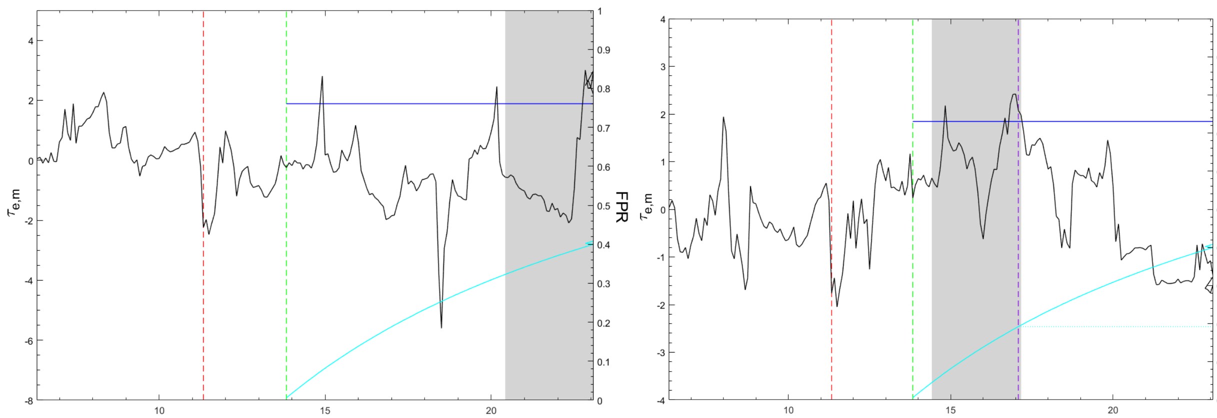
(c) Norway



Note: — is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 34: The location of the predictive regimes for the sample countries' REER, SEQ procedure, m=30

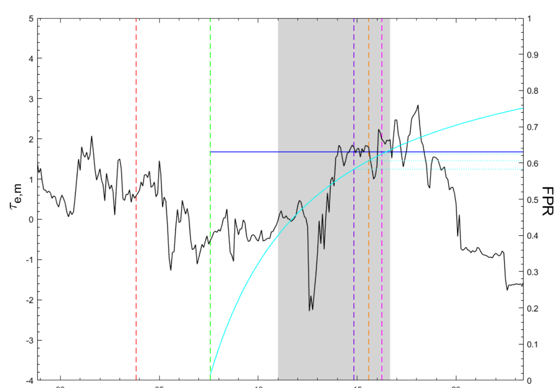
(a) Canada (b) Colombia



Note:— is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 35: The location of the predictive regimes for the sample countries' REER, SEQ procedure, m=45

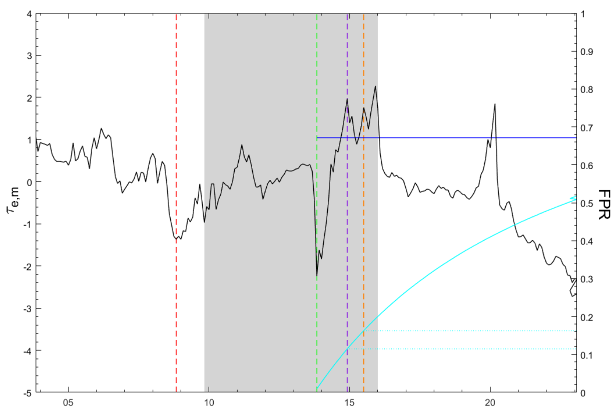
(a) Indonesia



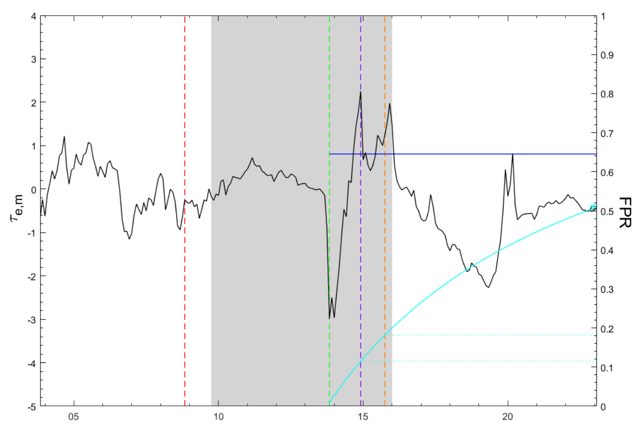
Note:— is the $\tau_{e,m}$, — is the $cv_{0.05}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 36: The location of the predictive regimes for the sample countries' REER, SEQ procedure, m=60

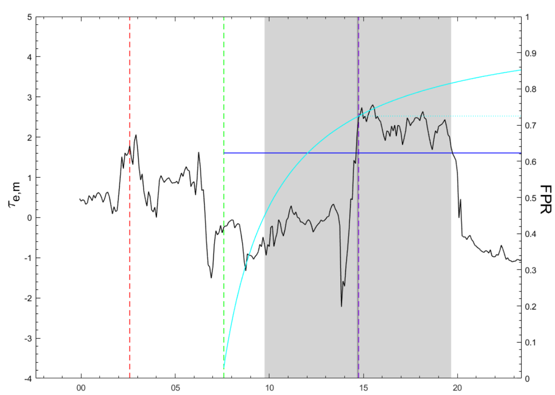
(a) Brazil



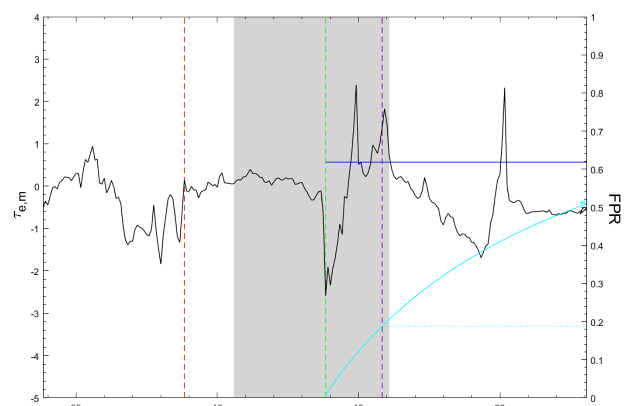
(b) Canada



(c) Indonesia



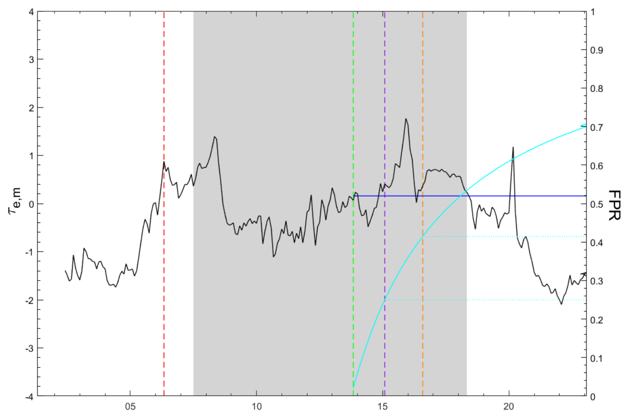
(d) Norway



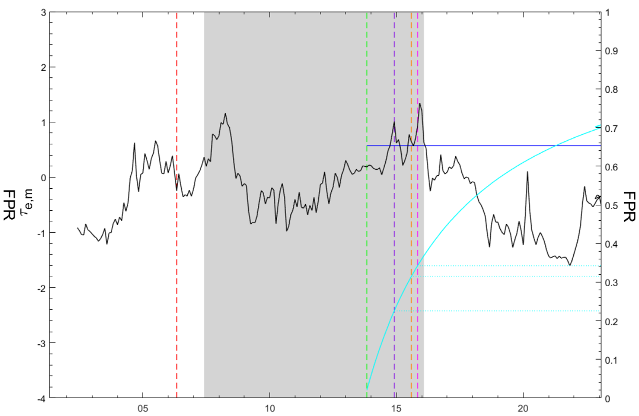
Note:— is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 37: The location of the predictive regimes for the sample countries' REER, SEQ procedure, m=90

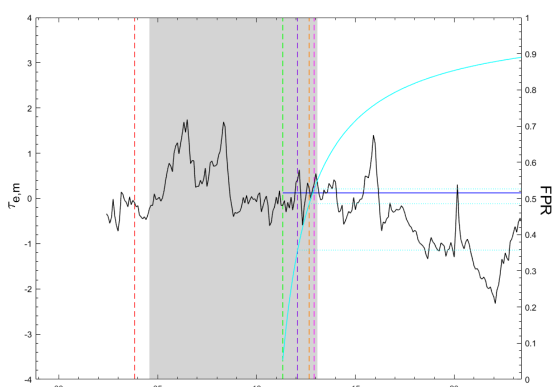
(a) Brazil



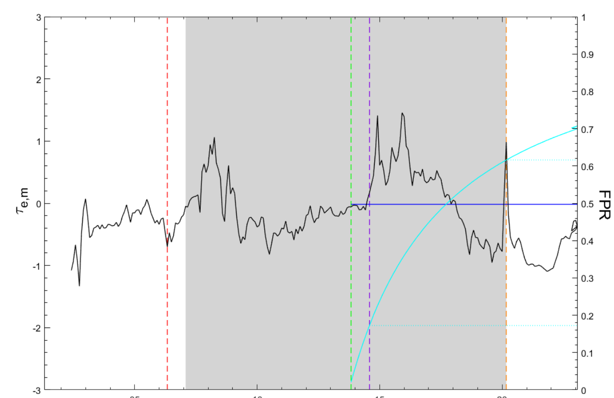
(b) Canada



(c) Colombia



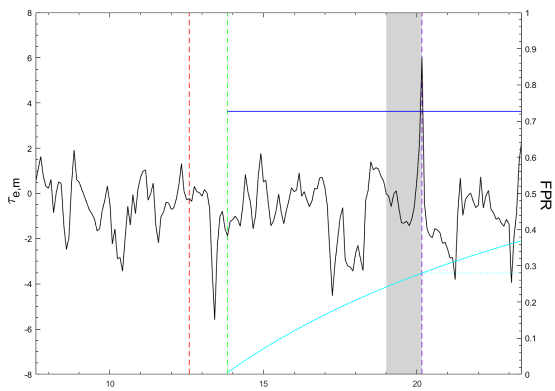
(d) Norway



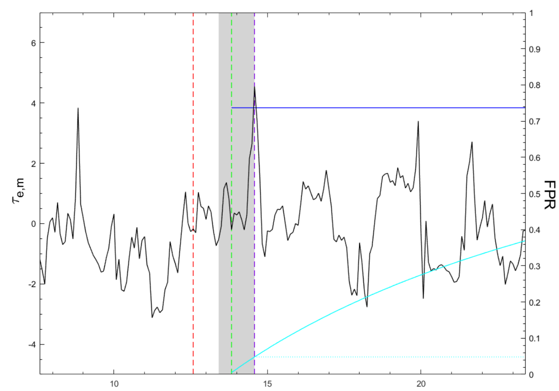
Note:— is the $\tau_{e,m}$, — is the $cv_{0.05}$, ⋯ (T^{*}),— is the T^{*} +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 38: The location of the predictive regimes for the sample countries' NEER, MAX procedure, m=15

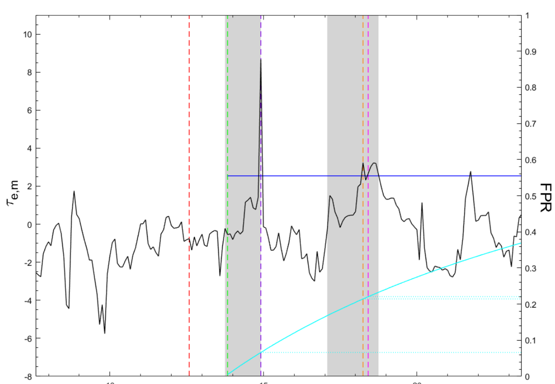
(a) Brazil



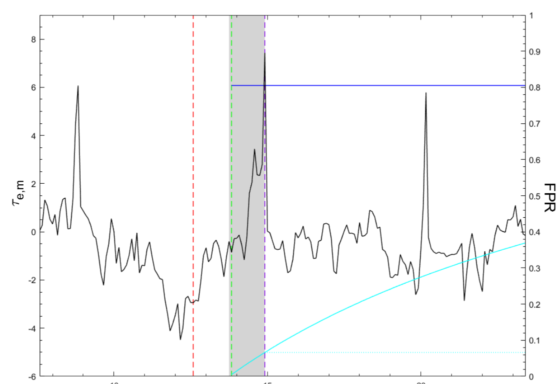
(b) Indonesia



(c) Mexico



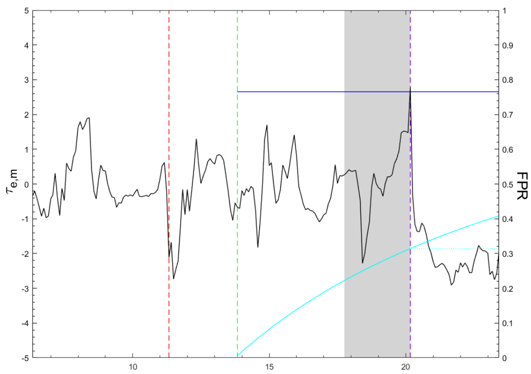
(d) Norway



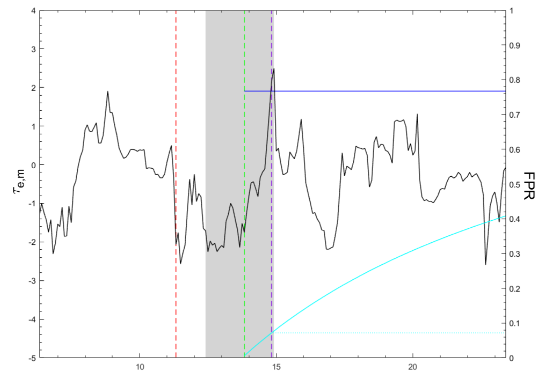
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ⋯ ($T^*$),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 39: The location of the predictive regimes for the sample countries' NEER, MAX procedure, m=30

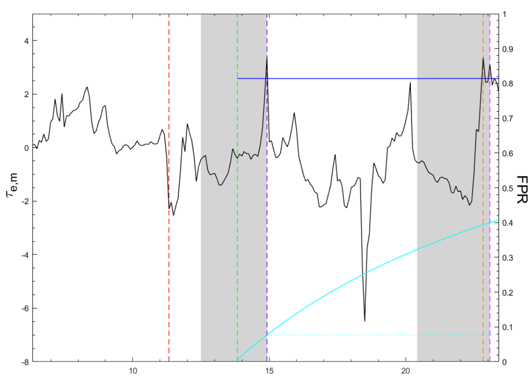
(a) Brazil



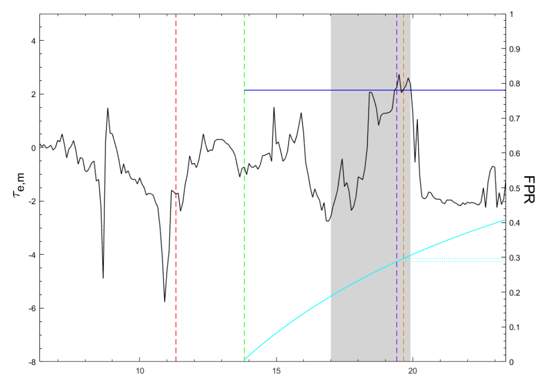
(b) Canada



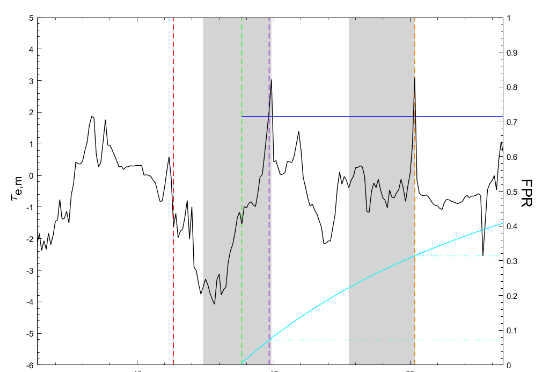
(c) Colombia



(d) Mexico



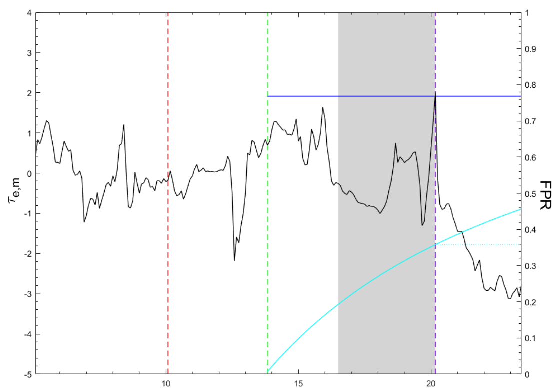
(e) Norway



Note:— is the $\tau_{e,m}$, — is the $max_{e \in [m+1,T^*]} \tau_{e,m}$, ⋯ ($T^*$),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

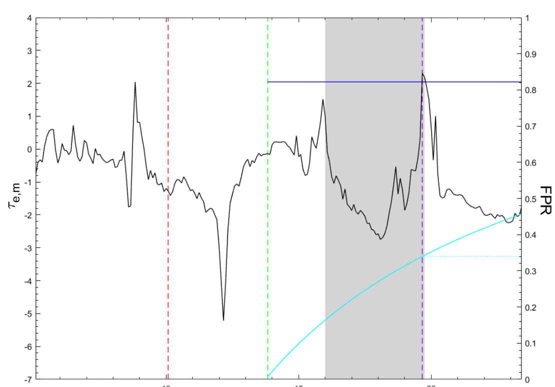
Figure 40: The location of the predictive regimes for the sample countries' NEER, MAX procedure, m=45

(a) Brazil

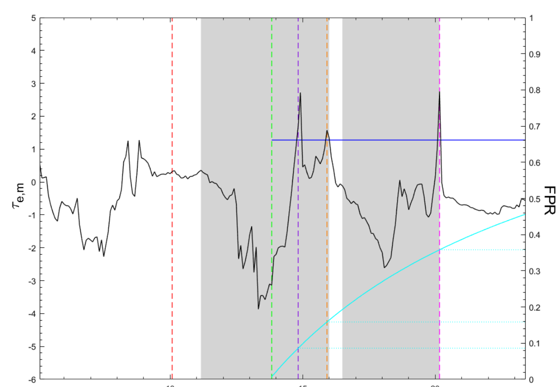


(b) Canada

(c) Mexico



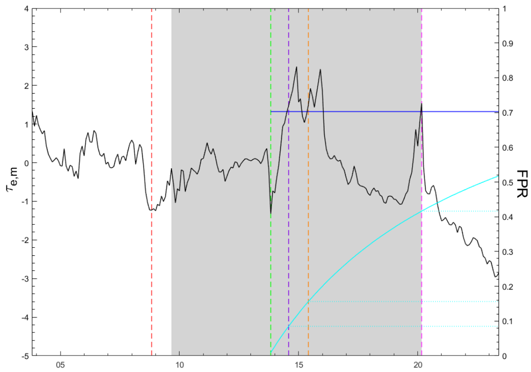
(d) Norway



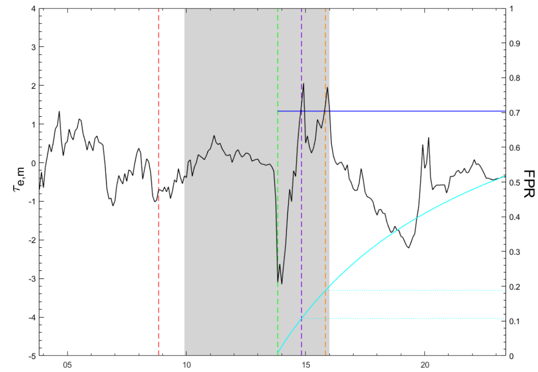
Note: — is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]\tau_{e,m}}$, ··· ($T^*$),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 41: The location of the predictive regimes for the sample countries' NEER, MAX procedure, m=60

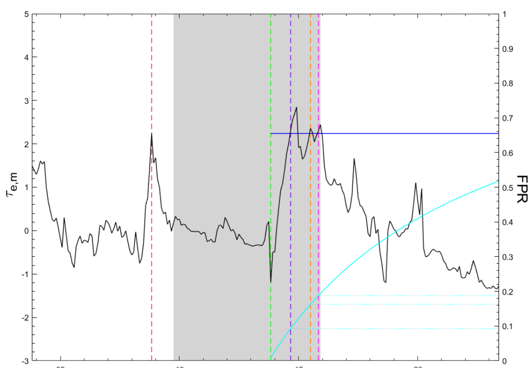
(a) Brazil



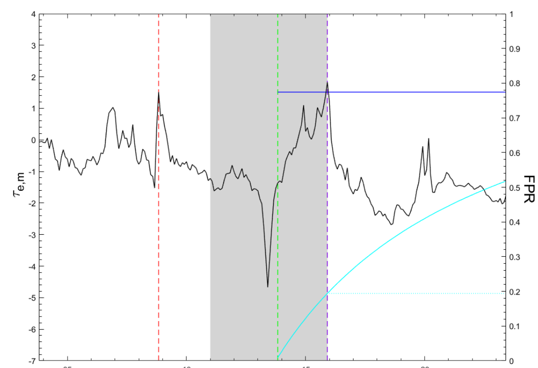
(b) Canada



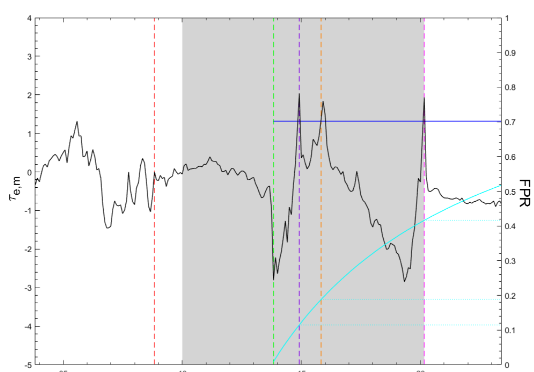
(c) Indonesia



(d) Mexico



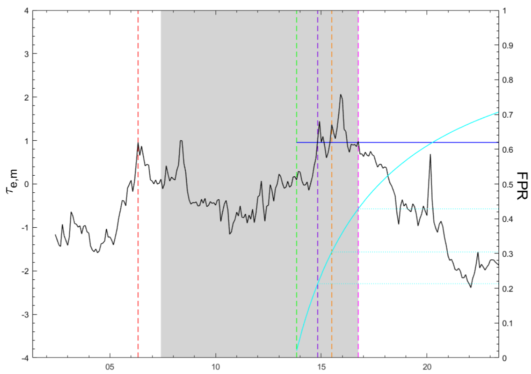
(e) Norway



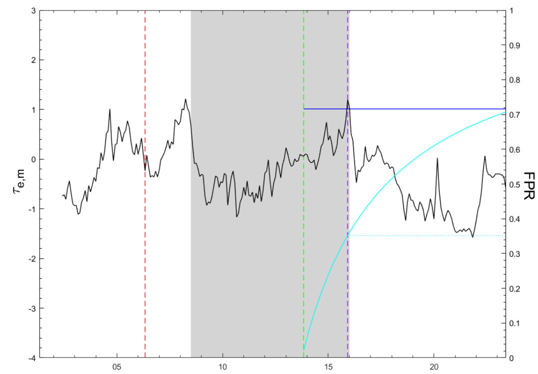
Note:— is the $\tau_{e,m}$, — is the $max_{e \in [m+1,T^*]} \tau_{e,m}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 42: The location of the predictive regimes for the sample countries' NEER, MAX procedure, m=90

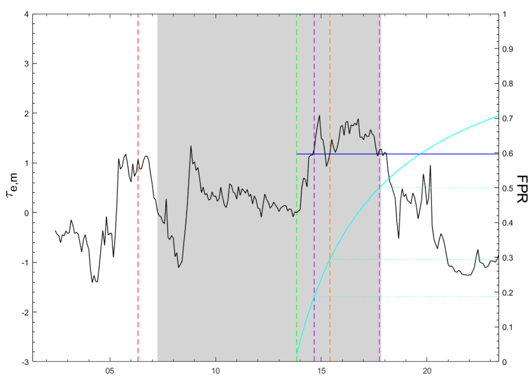
(a) Brazil



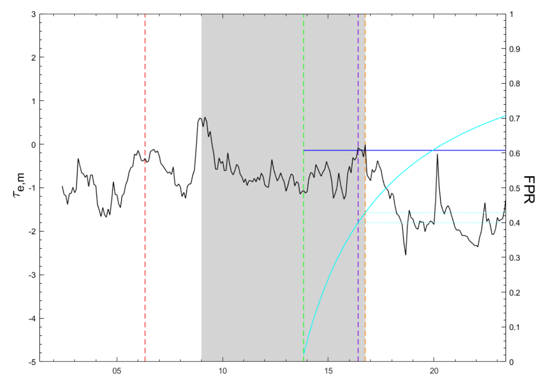
(b) Canada



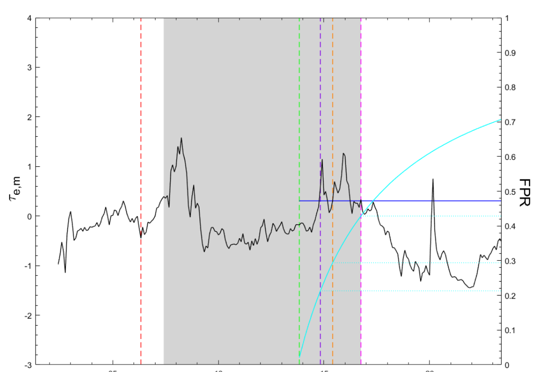
(c) Indonesia



(d) Mexico



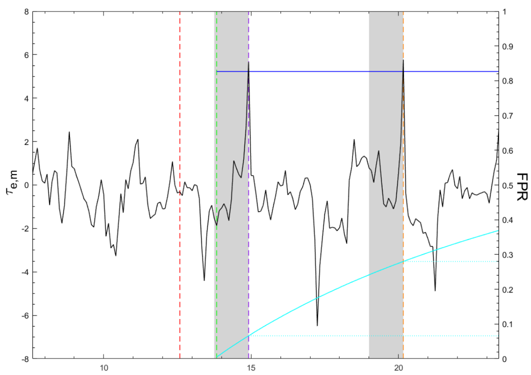
(e) Norway



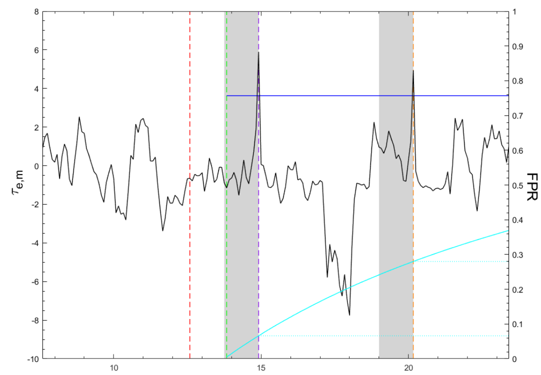
Note: — is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]\tau_{e,m}}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 43: The location of the predictive regimes for the sample countries' NER, MAX procedure, m=15

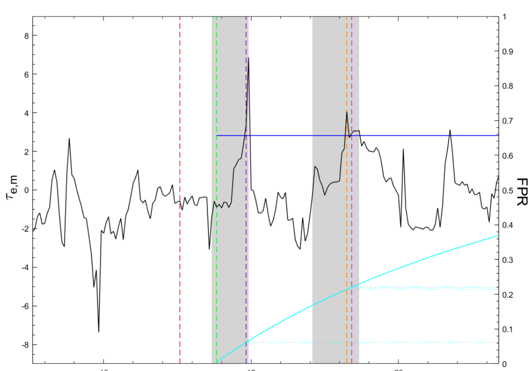
(a) Brazil



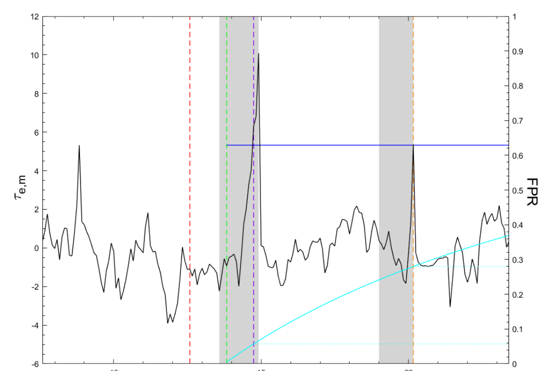
(b) Colombia



(c) Mexico



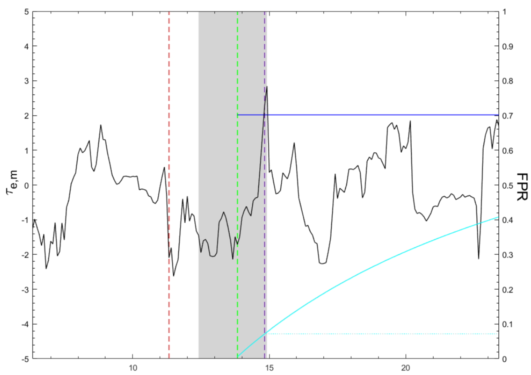
(d) Norway



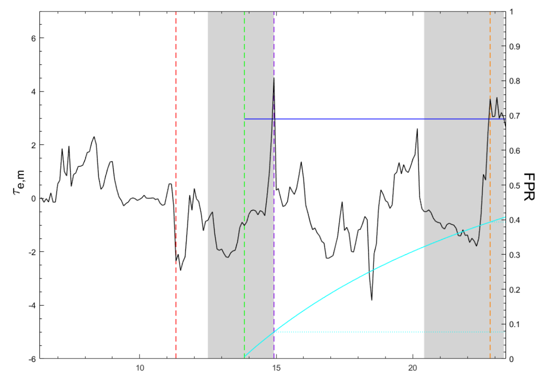
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ⋯ (T^*),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 44: The location of the predictive regimes for the sample countries' NER, MAX procedure, m=30

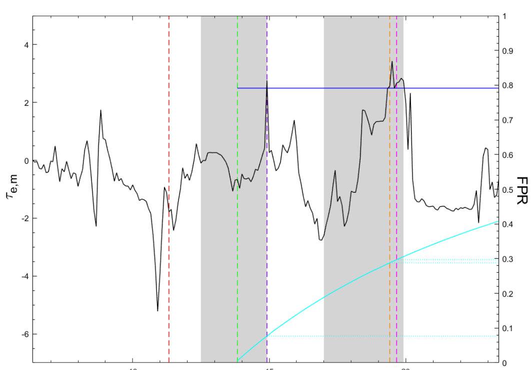
(a) Canada



(b) Colombia



(c) Mexico

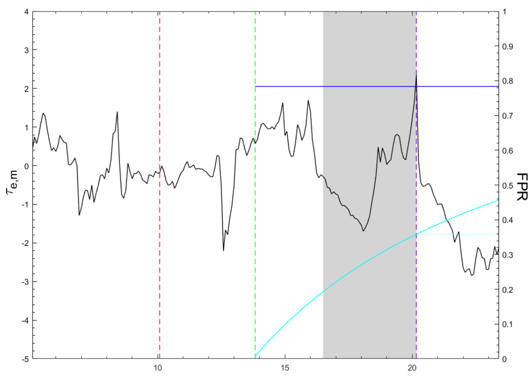


(d) Norway

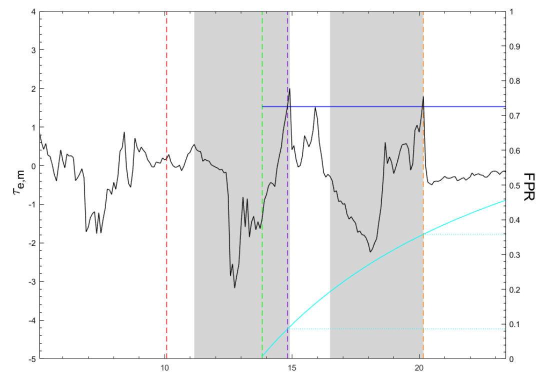
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ⋯ ($T^*$),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 45: The location of the predictive regimes for the sample countries' NER, MAX procedure, m=45

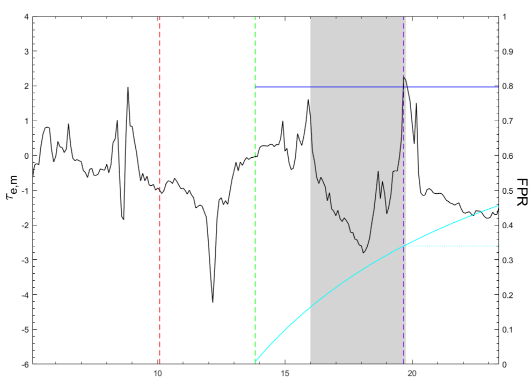
(a) Brazil



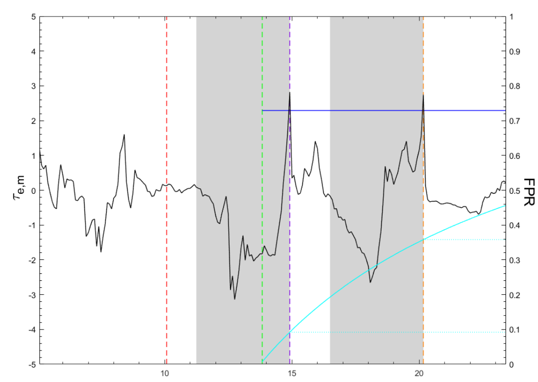
(b) Canada



(c) Mexico



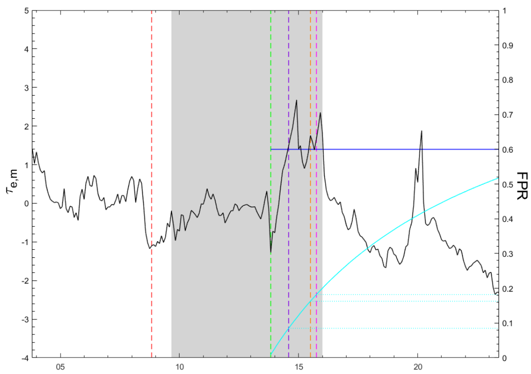
(d) Norway



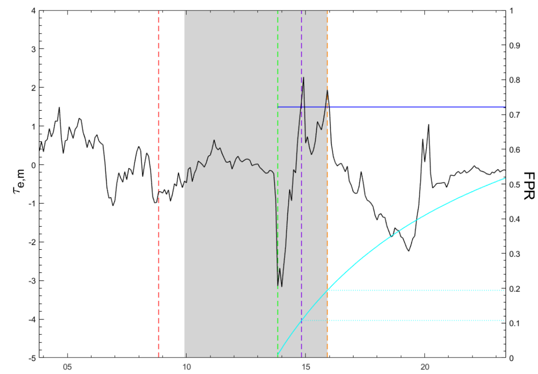
Note: — is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ··· ($T^*$),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 46: The location of the predictive regimes for the sample countries' NER, MAX procedure, m=60

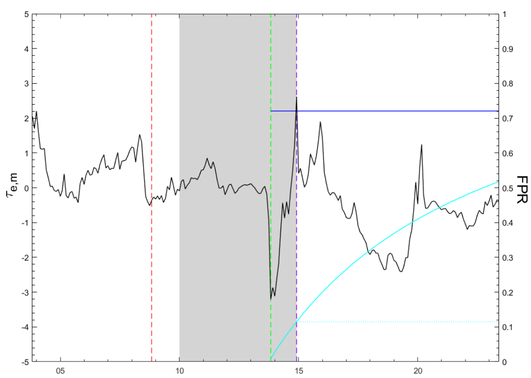
(a) Brazil



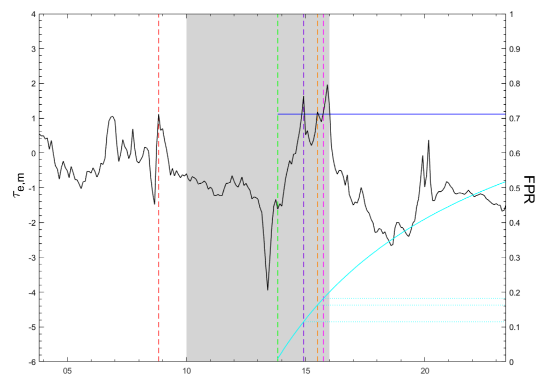
(b) Canada



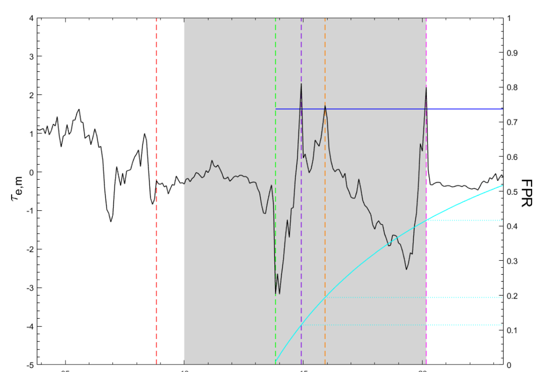
(c) Colombia



(d) Mexico



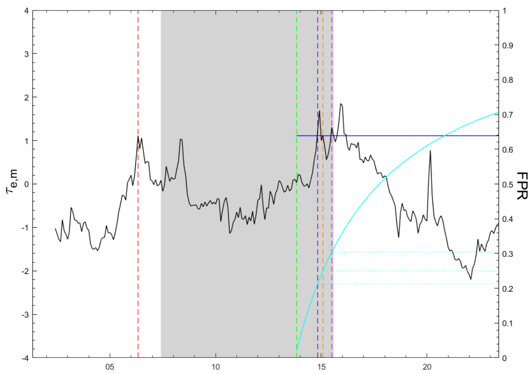
(e) Norway



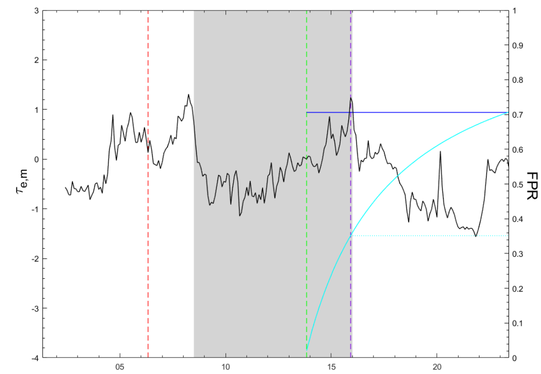
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ··· ($T^*$),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 47: The location of the predictive regimes for the sample countries' NER, MAX procedure, m=90

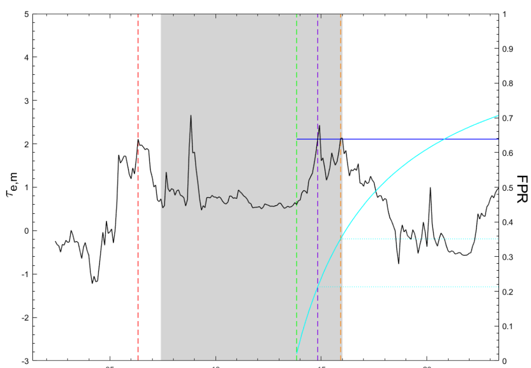
(a) Brazil



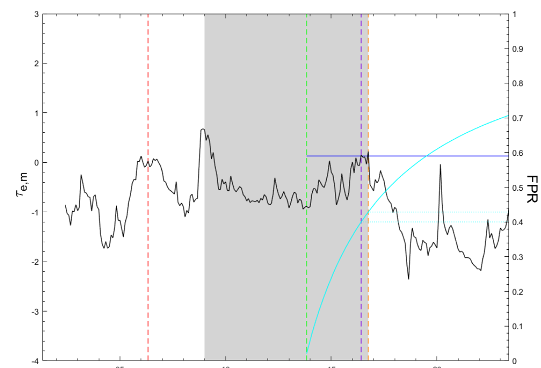
(b) Canada



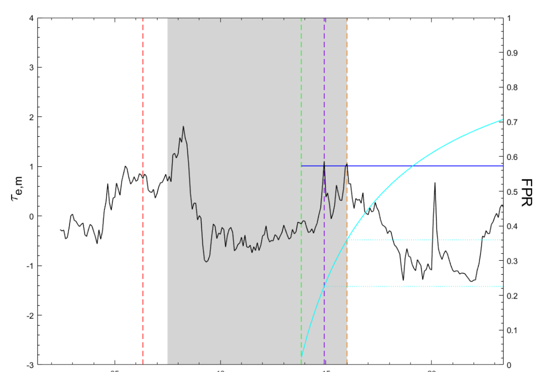
(c) Indonesia



(d) Mexico



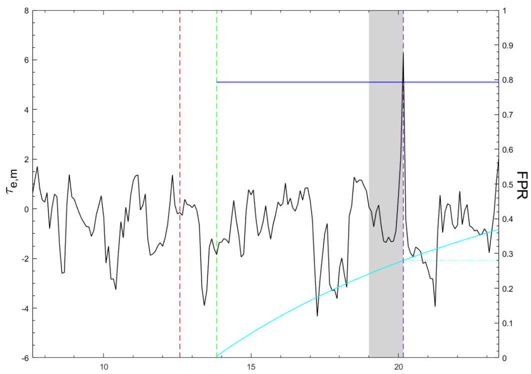
(e) Norway



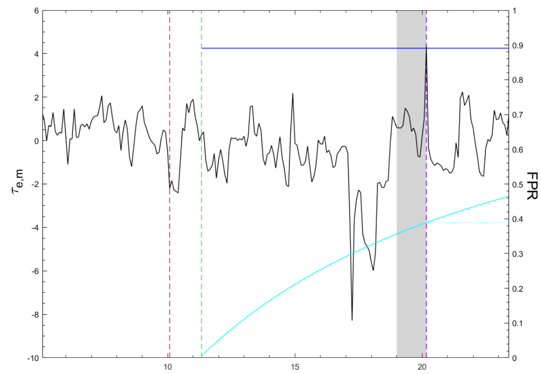
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ··· (T^*),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 48: The location of the predictive regimes for the sample countries' REER, MAX procedure, m=15

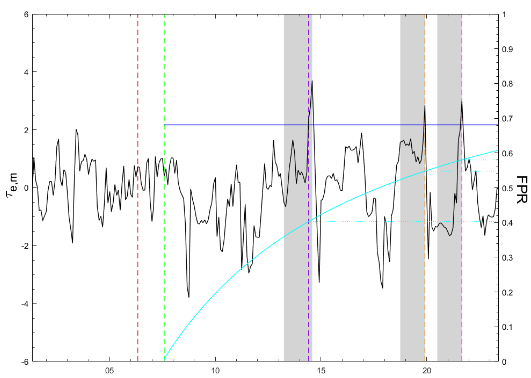
(a) Brazil



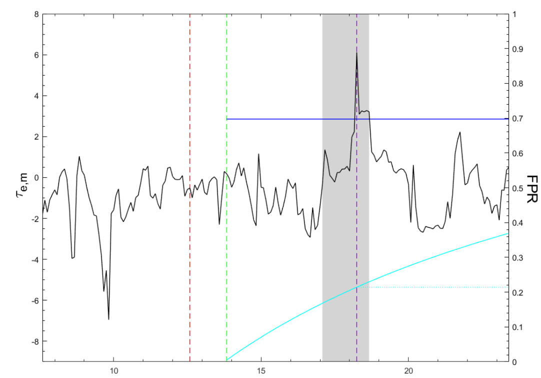
(b) Colombia



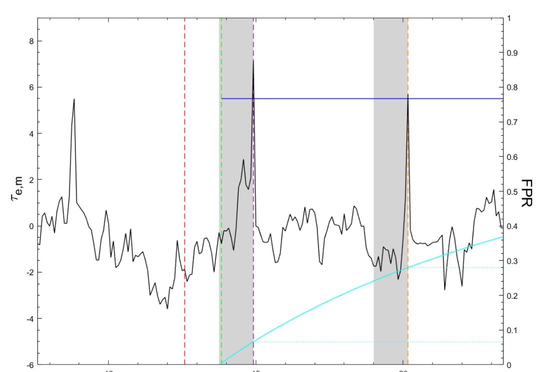
(c) Indonesia



(d) Mexico



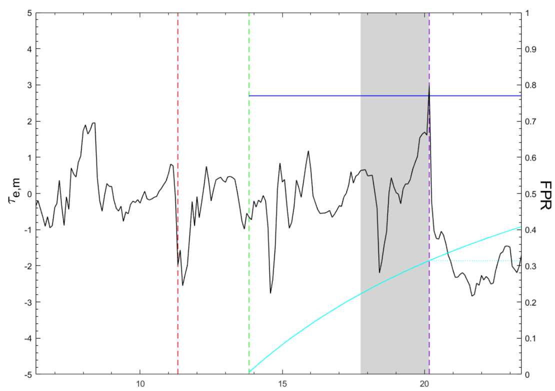
(e) Norway



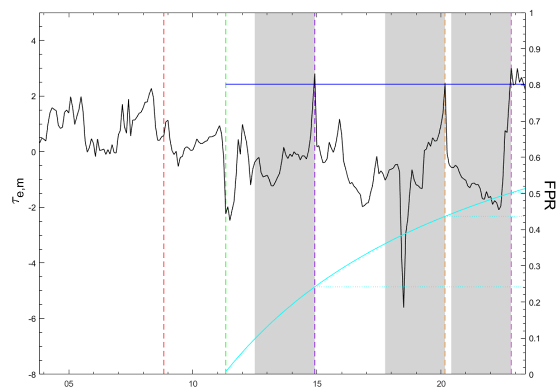
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ⋯ ($T^*$),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 49: The location of the predictive regimes for the sample countries' REER, MAX procedure, m=30

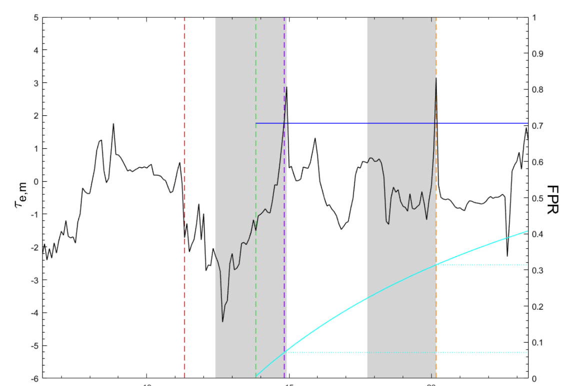
(a) Brazil



(b) Colombia



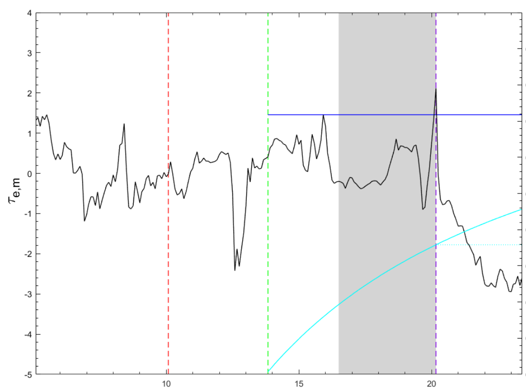
(c) Norway



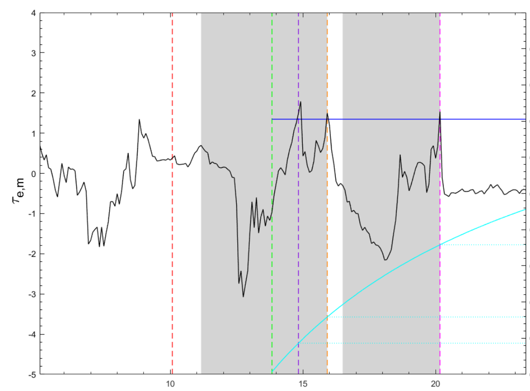
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ⋯ ($T^*$),— is the T^* +m, ⋯ shows the first rejection, ⋯ shows the second rejection, ⋯ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 50: The location of the predictive regimes for the sample countries' REER, MAX procedure, m=45

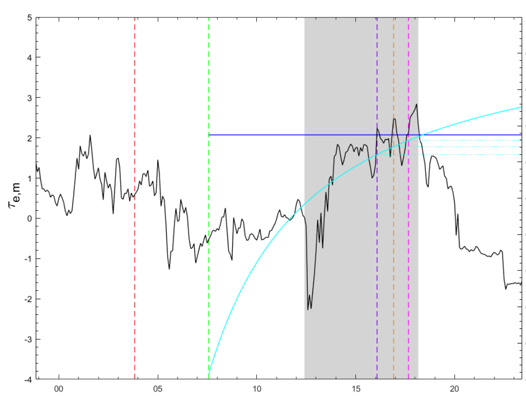
(a) Brazil



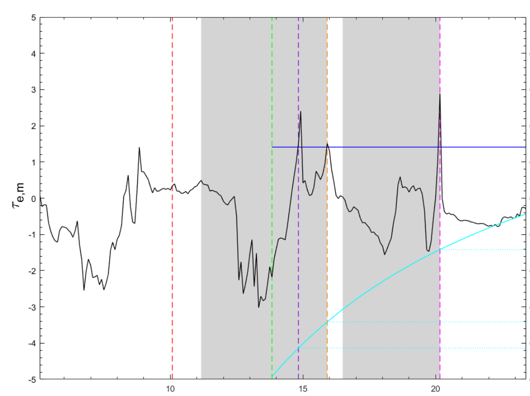
(b) Canada



(c) Indonesia



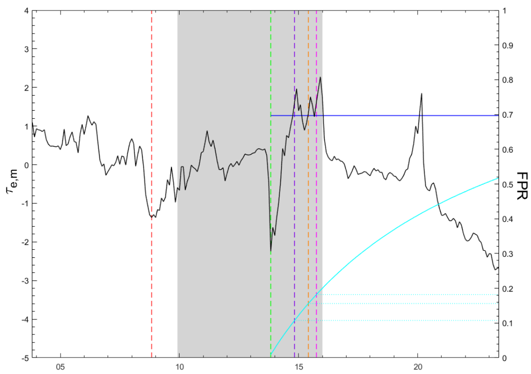
(d) Norway



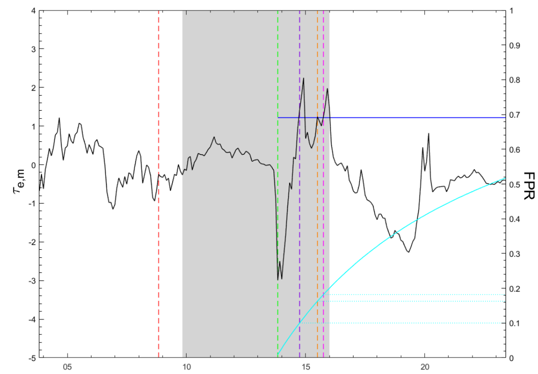
Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ··· ($T^*$),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 51: The location of the predictive regimes for the sample countries' REER, MAX procedure, m=60

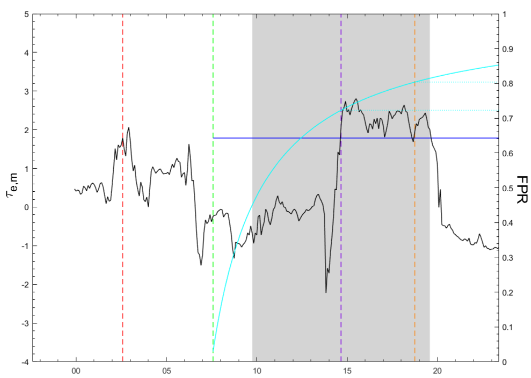
(a) Brazil



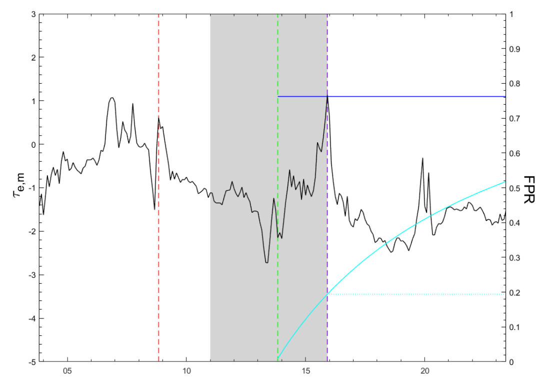
(b) Canada



(c) Indonesia



(d) Mexico

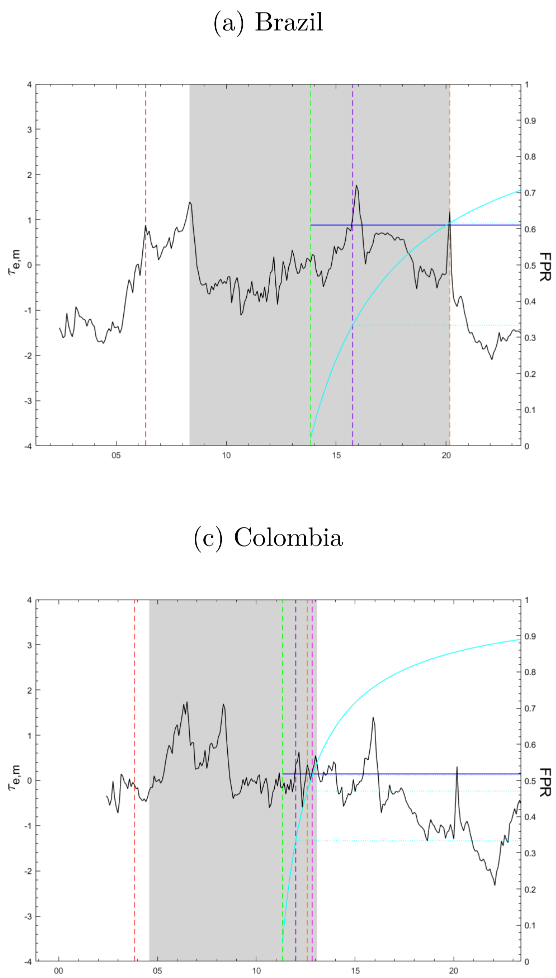


(e) Norway

Note: — is the $\tau_{e,m}$, — is the $max_{e \in [m+1,T^*]} \tau_{e,m}$, $\cdots$ (T^*),— is the T^* +m, $\cdots$ shows the first rejection, $\cdots$ shows the second rejection, $\cdots$ shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Figure 52: The location of the predictive regimes for the sample countries' REER, MAX procedure, m=90

(a) Brazil (b) Canada (c) Colombia (d) Norway



Note:— is the $\tau_{e,m}$, — is the $max_{e\in[m+1,T^*]}\tau_{e,m}$, ··· ($T^*$),— is the T^* +m, ··· shows the first rejection, ··· shows the second rejection, ··· shows the third rejection, ■ is the weak set of dates, — is the false positive rate.

Table 44: IV_{comb} test results for each the training sample for Brent oil returns when m= 15, 30, 45, 60, 90 for NEER, NER and REER

	$m = 15$			$m = 30$			$m = 45$			$m = 60$			$m = 90$		
NEER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.065211	-0.2556	0.83104	-0.064252	-0.49317	0.83452	-0.066805	-0.45134	0.9252	-0.057959	-0.060638	0.73822	-0.068879	-1.2627	1.4237
Canada	0.0069933	0.9372	0.0024367	0.036742	1.2843	0.057815	0.012547	1.215	0.0053224	-0.044041	1.0315	0.06047	0.11459	0.49903	0.20088
Colombia	-0.041658	0.20861	0.23015	-0.041093	-0.097278	0.23218	-0.040863	-0.10274	0.23672	-0.022499	0.28299	0.075024	-0.055411	-1.1338	0.58944
Indonesia	-0.016262	0.12577	0.14089	-0.019661	-0.095596	0.20711	-0.016827	0.031519	0.15157	-0.003274	0.51326	0.0057349	-0.024662	-0.48484	0.40092
Mexico	-0.066414	-0.36168	0.41416	-0.094483	-0.57233	0.73093	-0.089222	-0.36603	0.57526	-0.015567	0.23007	0.01287	-0.14751	-1.0761	1.1938
Norway	-0.021731	1.117	0.0043384	0.038545	1.398	0.012109	0.0073127	1.3165	0.00039601	-0.031698	1.0653	0.0073977	0.034753	0.016399	0.007301
NER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.047387	-0.12742	0.70717	-0.046855	-0.37444	0.71533	-0.04808	-0.34988	0.775	-0.04029	0.038672	0.57627	-0.049426	-1.2564	1.1832
Canada	-0.0024731	1.3087	0.000366	0.023615	1.6056	0.028637	-0.0020329	1.5407	0.00016833	-0.049895	1.5403	0.093903	0.066651	0.27915	0.083003
Colombia	-0.031898	0.26512	0.25268	-0.031107	-0.033177	0.24937	-0.031223	-0.050066	0.25912	-0.019248	0.33017	0.10355	-0.038373	-1.1474	0.54774
Indonesia	-0.010811	0.54675	0.071918	-0.012673	0.28011	0.10127	-0.010888	0.32908	0.076108	-0.00064359	0.72899	0.00027268	-0.017269	-0.42371	0.25416
Mexico	-0.067514	-0.23453	0.42022	-0.090919	-0.45433	0.68896	-0.084778	-0.27351	0.55089	-0.018111	0.30771	0.019878	-0.12015	-1.1134	1.0076
Norway	-0.005107	1.5382	0.0013798	0.016252	1.6318	0.013092	0.0012379	1.512	7.1119e-05	-0.018687	1.5632	0.016156	0.030376	-0.33715	0.03442
REER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.072688	0.2561	0.71826	-0.069519	0.086595	0.61536	-0.09592	-0.027779	1.04211	-0.11597	0.064278	1.5315	-0.10365	-1.1089	1.5457
Canada	-0.0002335	1.1404	1.8519e-06	0.033271	1.4346	0.03273	0.0014108	1.3709	4.6787e-05	-0.046901	1.2972	0.04939	0.053778	0.16724	0.035613
Colombia	-0.026467	1.7856 *	0.031982	-0.0049322	1.7459 *	0.00093973	-0.041859	1.5386	0.056144	-0.082656	1.4519	0.21253	-0.069271	-0.24959	0.17168
Indonesia	0.029074	2.1696 *	0.095589	0.039142	2.2109*	0.17015	0.032091	2.1211 *	0.10953	0.023877	2.0516 *	0.063689	0.046244	1.5322	0.31157
Mexico	0.0033668	1.077	0.00047329	0.0011132	1.1104	5.2957e-05	0.0030474	1.0253	0.00040884	0.041409	0.70317	0.07757	0.046468	1.127	0.13172
Norway	-0.069578	1.1085	0.02955	-0.010369	1.3079	0.00061835	-0.04583	1.2589	0.010892	-0.059625	1.0627	0.018078	-0.079971	-0.18448	0.032302

Note: The critical value used for IV_{comb} is ± 1.645. $P_{CY_{test}}$ present the P value of CY test. For this table, our selection of training periods includes 1995:01–2012:09 (for m = 15), 1995:01–2011:06 (for m = 30), 1995:01–2010:03 (for m = 45), 1995:01–2008:12 (for m = 60) and 1995:01–2006:06 (for m = 90).

Table 45: KMS and CY tests' result for each the training sample for Brent oil returns when m= 15, 30, 45, 60, 90

	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
NEER	KMS	CY	KMS	CY	KMS	CY	KMS	CY	KMS	CY
Brazil	-1.1796	[-0.144, 0.025]	-1.0404	[-0.144, 0.027]	-1.2703	[-0.150, 0.025]	-1.5507	[-0.155, 0.022]	-1.38	[-0.164, 0.007]
Canada	0.1022	[-0.130, 0.210]	0.2846	[-0.119, 0.259]	0.1367	[-0.187, 0.257]	-0.0511	[-0.237, 0.254]	0.399	[N/A]
Colombia	-0.5276	[-0.161, 0.043]	-0.3362	[-0.164, 0.045]	-0.5377	[-0.165, 0.049]	-0.7147	[-0.167, 0.054]	-0.492	[-0.212, 0.018]
Indonesia	-0.3392	[-0.062, 0.040]	-0.2580	[-0.067, 0.038]	-0.2824	[-0.065, 0.044]	-0.2434	[-0.062, 0.052]	-0.177	[-0.079, 0.035]
Mexico	-0.8549	[-0.172, 0.079]	-0.9238	[-0.216, 0.064]	-0.9131	[-0.223, 0.088]	-0.4910	[-0.248, 0.189]	-0.968	[-0.380, 0.030]
Norway	-0.1411	[-0.294, 0.494]	0.0175	[-0.274, 0.600]	0.0063	[-0.372, 0.585]	-0.1940	[-0.487, 0.540]	-0.196	[-0.465, 0.724]
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
NER	KMS	CY	KMS	CY	KMS	CY	KMS	CY	KMS	CY
Brazil	-1.0164	[-0.107, 0.028]	-0.8773	[-0.109, 0.029]	-1.0706	[-0.113, 0.028]	-1.13234	[-0.115, 0.026]	-1.180	[-0.250, 0.419]
Canada	0.0708	[-0.140, 0.174]	0.2859	[-0.132, 0.215]	0.1095	[-0.200, 0.210]	-0.0393	[-0.256, 0.202]	0.461	[-0.250, 0.419]
Colombia	-0.5065	[-0.112, 0.035]	-0.3140	[-0.113, 0.037]	-0.5033	[-0.117, 0.038]	-0.7491	[-0.120, 0.040]	-0.420	[-0.162, 0.004]
Indonesia	-0.863	[-0.051, 0.042]	-0.0300	[-0.054, 0.041]	-0.0426	[-0.053, 0.045]	-0.1000	[-0.051, 0.051]	0.0438	[-0.064, 0.035]
Mexico	-0.7019	[-0.166, 0.090]	-0.7075	[-0.191, 0.080]	-0.7285	[-0.193, 0.099]	-0.4126	[-0.220, 0.206]	-0.790	[-0.321, 0.043]
Norway	-0.1186	[-0.185, 0.164]	0.3150	[-0.171, 0.199]	0.2070	[-0.213, 0.185]	0.1147	[-0.229, 0.188]	0.492	[-0.213, 0.275]
	$m = 15$		$m = 30$		$m = 45$		$m = 60$		$m = 90$	
REER	KMS	CY	KMS	CY	KMS	CY	KMS	CY	KMS	CY
Brazil	-1.0008	[-0.171, 0.030]	-0.8376	[-0.173, 0.040]	-1,1651	[-0.216, 0.019]	-1.6404	[-0.237, 0.006]	-1.31	[-0.231, 0.015]
Canada	0.0921	[-0.181, 0.237]	0.2938	[-0.171, 0.288]	0.1286	[-0.257, 0.283]	-0.0810	[-0.322, 0.272]	-0.296	[-0.344, 0.479]
Colombia	-0.1793	[-0.201, 0.153]	0.0385	[-0.201, 0.194]	-0.2187	[-0.281, 0.171]	-0.5690	[-0.316, 0.175]	-0.316	[-0.323, 0.216]
Indonesia	0.6763	[-0.075, 0.145]	0.8418	[-0.068, 0.162]	0.7150	[-0.086, 0.159]	-0.5001	[-0.094, 0.155]	1.13	[-0.067, 0.174]
Mexico	-0.2775	[-0.115, 0.266]	-0.4513	[-0.126, 0.262]	-0.3238	[-0.138, 0.262]	0.490	[-0.120, 0.293]	-0.370	[-0.135, 0.248]
Norway	0.2317	[-0.514, 0.499]	-0.0715	[-0.482, 0.592]	0.1026	[-0.614, 0.565]	0.079	[-0.718, 0.534]	-0.3093	[-0.701, 0.631]

*Note: The critical value used for KMS is ±1.645. $P_{CY_{test}}$ present the P value of CY test. For this table, our selection of training periods includes 1995:01–2012:09 (for m = 15), 1995:01–2011:06 (for m = 30), 1995:01–2010:03 (for m = 45), 1995:01–2008:12 (for m = 60) and 1995:01–2006:06 (for m = 90). The CY columns report the 90% Bonferroni confidence intervals for $\hat{\beta}$ using the Q-test. Confidence intervals that reject the null are shown with a * mark. As mentioned in Section 3.2, the CY test is not suitable for data with insufficient persistence because it does not meet the assumptions required for the DF-GLS test to be valid. We use (N/A) to indicate that the assumptions of the CY test are not met.*

Table 46: Preliminary results for Brent future with each training period used when monitoring with $m = \{15, 30, 45, 60, 90\}$

	$m=15$			$m=30$			$m=45$			$m=60$			$m=90$		
NEER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.068676	-0.59189	0.86053	-0.066739	-0.74812	0.84277	-0.068598	-0.69979	0.92495	-0.062703	-0.40826	0.81955	-0.071033	-1.4207	1.3559
Canada	-0.0022438	0.80472	0.0002342	0.025443	1.1218	0.025949	0.009561	1.1419	0.0029303	-0.040802	0.93925	0.04923	0.16443	0.81765	0.37038
Colombia	-0.043175	0.15599	0.23081	-0.042371	-0.12962	0.23106	-0.042643	-0.16563	0.24443	-0.027543	0.13289	0.10665	-0.057777	-1.1958	0.57386
Indonesia	-0.017288	-0.12365	0.14866	-0.020702	-0.33153	0.21494	-0.018701	-0.23374	0.1775	-0.0080636	0.13897	0.032998	-0.027954	-0.79158	0.46123
Mexico	-0.063216	-0.32822	0.35034	-0.088411	-0.53025	0.59906	-0.086667	-0.36736	0.51466	-0.032617	0.10762	0.053598	-0.14912	-1.0719	1.0923
Norway	-0.046805	0.95174	0.01879	0.011382	1.251	0.00098837	4.6506e-05	1.2675	1.5186e-08	-0.06936	0.89617	0.033598	0.078963	0.26003	0.033751
NER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.05065	-0.55053	0.75431	-0.04926	-0.7013	0.74006	-0.049898	-0.64725	0.79144	-0.044174	-0.33069	0.65709	-0.051268	-1.412	1.1399
Canada	-0.013861	1.0512	0.010733	0.010331	1.3691	0.0051305	-0.0081465	1.4146	0.002563	-0.051322	1.332	0.09424	0.09952	0.35877	0.16571
Colombia	-0.034101	0.095182	0.26964	-0.032899	-0.15447	0.26108	-0.032959	-0.17565	0.27378	-0.023037	0.13228	0.14071	-0.040318	-1.2283	0.54146
Indonesia	-0.014314	0.11953	0.11772	-0.016018	-0.10178	0.15145	-0.014573	-0.045484	0.12928	-0.0063613	0.28785	0.025269	-0.02122	-0.77359	0.34367
Mexico	-0.067359	-0.24838	0.39054	-0.088042	-0.45421	0.60471	-0.084566	-0.30867	0.51972	-0.035739	0.15248	0.073421	-0.12327	-1.1615	0.94975
Norway	-0.025843	1.0315	0.032988	-0.0043975	1.2466	0.00089721	-0.012446	1.2207	0.0068157	-0.035525	1.1206	0.055384	0.025742	-0.76748	0.022136
REER	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$	$\hat{\beta}$	IV_{comb}	$R^2(\%)$
Brazil	-0.079585	-0.1914	0.80389	-0.075779	-0.27126	0.68439	-0.098907	-0.34763	1.0506	-0.11965	-0.36279	1.5466	-0.10377	-1.2289	1.3871
Canada	-0.014856	0.93507	0.006999	0.015802	1.2151	0.0069112	-0.007256	1.248	0.0011735	-0.051969	1.0999	0.057521	0.10082	0.37638	0.11208
Colombia	-0.032798	1.6751	0.045854	-0.012703	1.6281	0.0058351	-0.044548	1.4504	0.060293	-0.079793	1.3203	0.18787	-0.059274	-0.013838	0.11256
Indonesia	0.01713	1.3343	0.030981	0.026137	1.3452	0.071013	0.020926	1.2936	0.044159	0.012151	1.1625	0.015645	0.038637	0.82111	0.19476
Mexico	0.0096419	1.3244	0.0036241	0.01128	1.3661	0.0050901	0.013885	1.3081	0.0080474	0.04347	0.9011	0.081087	0.061801	1.4416	0.20863
Norway	-0.10924	0.9022	0.068012	-0.052152	1.1336	0.014641	-0.064155	1.1825	0.020237	-0.13363	0.77468	0.086133	-0.061202	-0.10436	0.016941

Note: The critical value used for IV_{comb} is ± 1.645. $P_{CY_{test}}$ present the P value of CY test. For this table, our selection of training periods includes 1995:01–2012:09 (for m = 15), 1995:01–2011:06 (for m = 30), 1995:01–2010:03 (for m = 45), 1995:01–2008:12 (for m = 60) and 1995:01–2006:06 (for m = 90).

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