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Allocation among multiple selves

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Abstract
We propose a model of individual decision making that separates a central executive system ("Ego") from a sub-self with its own preferences ("Cold") and an impulsive sub-self with different preferences ("Hot"). Ego cares only about the payoffs of the other two sub-selves and allocates available resources between them. For standard neoclassical preferences over joint consumption, we show that in subgame perfect Nash equilibrium, Ego will choose a resource allocation that enables neither Hot nor Cold to attain their most preferred affordable bundle. Moreover, the two sub-selves each choose a more extreme and specialized bundle than they would choose if they unilaterally controlled all available resources.

Keywords Multiple self · Menu dependence · Individual choice

JEL Classification C72 · D11 · D16

1 Introduction

The late Stanford psychologist Walter Mischel was famous for his marshmallow test. Between 1967 and 1973 he offered about 500 preschoolers the choice between a treat now (typically one marshmallow) or a bigger treat (typically two marshmallows) a little later. Mischel and coauthors reported, e.g., in Shoda et al. (1990), that the kids who could hold out for the bigger treat were more successful later in life.[1]

[1] Replication and followup studies generally confirm a positive correlation between broad measures of self control in preschoolers and later life outcomes, but find that by itself the marshmallow test has little to no predictive power; see e.g. Benjamin et al. (2020) or List et al. (2023).

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How should decision theorists think about such self-control problems? We adapt ideas from the multiple self literature of recent decades, and parse the marshmallow test as follows. A typical kid prefers two marshmallows to one, and that preference is captured in a "cold self" who takes a broader perspective. A typical kid also prefers marshmallows now to marshmallows later, and this preference is captured in a "hot self" who responds more strongly to immediate impulses. A central executive system, which we refer to as "ego," allocates resources to the hot and cold sub-selves. For some kids (especially four year olds), more attentional resources go to the hot self, but some other kids (especially five year olds) allocate enough resources to the cold self to pass the marshmallow test.

As explained below, multiple self models can explain phenomena such as menu dependence and time inconsistency. Our multiple self model differs from most predecessors in that it separates the central executive system (Ego) from the carriers of conflicting preferences (Hot and Cold). This disaggregation allows us to focus on how resources can be allocated to meliorate internal conflicts, and to examine the consequences.

In particular, we consider consumption choices of a generic good and a temptation good such as chocolate or a trip to Capri. We allow general preferences — both Hot and Cold may enjoy both goods to some degree — but they conflict in that Hot cares more (has a higher marginal rate of substitution) than Cold for the temptation good. Ego doesn't care about the goods *per se*. It does care about the utility of both Hot and Cold, and chooses an allocation of available resources between Hot and Cold that maximizes its social welfare function over their utilities.

Imposing standard neoclassical assumptions on Hot and Cold preferences and Ego's social welfare function, we solve for subgame perfect Nash equilibrium of the three player game. We find, a bit to our surprise,[2] that in equilibrium both Hot and Cold make more specialized and extreme choices than they would make given dictatorial powers. Indeed, to maximize its own welfare function, Ego will allocate resources such that Hot will acquire only the temptation good and Cold will only acquire the generic good. Ego's equilibrium allocation always leaves both Hot and Cold a bit frustrated, unable to attain their preferred consumption bundles. Our model can thus capture situations where a planner not only caters to the sensible Cold system but also to some degree indulges the needs of Hot. In this sense, the eventual action of an individual human is characterized by the outcome of the strategic interaction among their subselves Hot, Cold and Ego.

Economists have been interested in multiple self models at least since the seminal paper of Gul and Pesendorfer (2001). That paper introduces a model with a present self and a future self. The two selves might agree what to pick from a limited menu but disagree when facing a larger menu. They capture such conflicts in a set betweenness axiom (menus satisfy $A \succcurlyeq B \implies A \succcurlyeq A \cup B \succcurlyeq B$); the main policy implication is that people are happier when temptations are removed from the choice set.

Bénabou and Pycia (2002) propose a related two-self model featuring a planner and a doer. The planner gives instructions to the doer who, with some probability,

[2] Our uninformed initial intuition was that Hot and Cold's choices would either meliorate their differences to some degree or else be unaffected by each other's presence.

may deviate towards indulgence. The policy implication again is that commitment devices (such as a limited menu) can help. Samuelson and Swinkels (2006) obtain a similar implication from a evolutionary model of preference formation, where utility depends directly on actions as well as on outcomes. Fudenberg and Levine (2006) focus on the cost function for the future self (or planner) to control the present self (or doer). Outcomes depend more sensitively on its functional form than one might have supposed.

Our own model is motivated in part by Alonso et al. (2014) who emphasize the role of the Central Executive System in allocating scarce mental resources to other brain subsystems. On a conceptual level our model also shares certain aspects with Freud 's (1923) psychoanalytic theory describing the human psyche as the result of the interaction of three sub-selves, the instinctual and uncoordinated *id*, the critical and moralizing *super-ego*, and *ego* which mediates between the other two sub-selves.

Toussaert (2018) reports a laboratory experiment testing the menu-limitation predictions of models following Gul and Pesendorfer (2001). Another empirical study more directly connected to our own paper is Cherchye et al. (2020) who report that purchases of healthy food decline from January to December each year, and also vary over weekly cycles. They use a dual self model, where the counterpart of Ego in our model maximizes a weighted average of healthy self's utility and unhealthy self's utility. Those weights change arbitrarily over time in their model. Our model is more fully developed in that Ego's weights reflect stable preferences and take the more flexible form of resource allocation. Also, our selves are less specialized, and potentially value all sorts of consumption, albeit to different degrees.

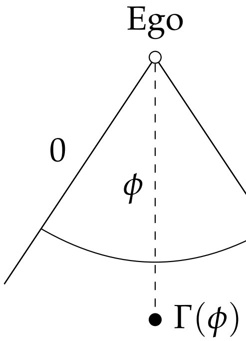
The next section lays out the elements of our model. It then solves the subgame played between the "doer" sub-selves Hot and Cold, given any fixed resource allocation. Points are illustrated and conclusions foreshadowed in a parametric example. Finally, we find the subgame perfect Nash equilibrium allocation choice by the "planner" self Ego. In a concluding discussion, we mention some variants (e.g., a "Stackelberg" version of the doer subgame) and extensions of the model, as well as different interpretations such as a Principal with two Agents and joint consumption.

2 Model

Consider an extensive form game played between "planner" sub-self Ego (*E*) and two "doer" sub-selves Cold (*C*) and Hot (*H*). In the first stage, Ego decides on the fraction $\phi \in [0,1]$ of available resources to be allocated by Cold, with Hot allocating the remaining fraction $1-\phi$. In the second stage, denoted in Fig. 1 below as the subgame $\Gamma(\phi)$, the two sub-selves $j = H, C$ simultaneously choose levels of two activities, $i = A$ and *B*, given their resources.

Our analysis builds on well-known insights from microeconomics covered in standard textbooks such as Hirshleifer et al. (2005) or Varian (1992), so the exposition will use standard microeconomics terminology such as goods, bundles, purchases, income, budget shares, etc. However, there are broader interpretations of the resources and activities, as noted in Sect. 3.1 below.

Fig. 1 The multiple self game tree



In the subgame, x_i^j denotes how much of good *i* sub-self *j* purchases. The vector $x^j = (x_A^j, x_B^j)$ denotes the bundle chosen by sub-self *j*, $X_i = x_i^C + x_i^H$ denotes the combined consumption of good *i*, and $X = (X_A, X_B)$ denotes the combined bundle. Let $p > 0$ denote the (relative) price of good *A*, and normalize income and the price of good *B* to 1 (so *B* serves as numerarire). Then the budget constraints are

$$\begin{aligned} px_A^C + x_B^C &\leq \phi \quad \text{and} \\ px_A^H + x_B^H &\leq 1 - \phi, \end{aligned} \tag{1}$$

for sub-selves Cold and Hot respectively.

Since they inhabit the same body, both sub-selves benefit from the consumption of the combined amount of goods, irrespective of which of them purchased particular units. Thus, the preferences of the two sub-selves are defined over the aggregate levels of consumption X_A and X_B. We assume standard neoclassical preferences over the two goods. Specifically, in terms of utility functions, we impose

Assumption 1 *The utility functions $U^C(X)$ and $U^H(X)$ for the two sub-selves are strictly positive, strictly monotone increasing, strictly quasiconcave and twice continuously differentiable at every strictly positive allocation $X = (X_A, X_B)$.*

Note that, among other things, this assumption implies that $MRS^j(X) = \frac{\partial_A U^j(X)}{\partial_B U^j(X)}$, the marginal rate of substitution of sub-self *j*, is continuously differentiable at every strictly positive $X = (X_A, X_B)$.[3]

While our monotonicity assumption implies that both sub-selves benefit from the consumption of both goods, we are interested in a scenario where they differ in the relative benefit they derive from the two goods. Specifically, we assume that at any allocation the Hot system is willing to trade more units of good *B* to acquire one additional unit of good *A* than is the Cold system. In this sense, good *A* is a *tempta-*

[3] We use the notation $\partial_i U^j(X) = \frac{\partial U^j(X)}{\partial X_i}$ for the partial derivative of *j*'s utility with respect to the aggregate level of consumption of good *i*.

tion good for the Hot system. (Of course, with only two goods, B can then be seen as a temptation good for the Cold system.) The formal definition relies on the single crossing property:

Definition 1 *Good A is a temptation good if* $MRS^H(X_A, X_B) > MRS^C(X_A, X_B)$ *at every allocation* $(X_A, X_B) > 0$.

Often we will also impose Inada conditions, that at least a small amount of each good is essential to both sub-selves. Specifically,

Assumption 2 *Utility functions satisfy* $\lim_{X_A \to 0} MRS^j(X_A, X_B) = \infty$ *and* $\lim_{X_B \to 0} MRS^j(X_A, X_B) = 0$ *for* $j = H, C$.

Finally, it remains to specify the payoff of Ego. We assume that Ego cares only about the happiness of both sub-selves. More specifically, Ego's payoff is just a weighted average of the hot- and the cold- systems utility: for any strictly positive $X = (X_A, X_B)$,

$$U^E(X) = \gamma U^C(X) + (1 - \gamma)U^H(X), \tag{2}$$

for some given $\gamma \in (0, 1)$.[4]

Thus the normal form subgame $\Gamma(\phi)$ in Fig. 1 consists of the three players Ego, Cold and Hot, with the payoff functions U^E, U^C and U^H just specified. Given price $p > 0$ and Ego's action $\phi \in [0, 1]$, the action sets for Cold and Hot are defined in equations (1), together with the usual non-negativity constraints.

2.1 The cold vs. hot subgame

As a first step we analyse the subgame $\Gamma(\phi)$, taking the budget allocation ϕ as given. Cold chooses her purchases (x_A^C, x_B^C) to solve the following problem,

$$\max_{x_A^C, x_B^C} \quad U^C(x_A^C + x_A^H, x_B^C + x_B^H)$$
$$\text{s.t.} \quad px_A^C + x_B^C \leq \phi, \quad x_A^C \geq 0, \quad x_B^C \geq 0.$$

taking the consumption levels (x_A^H, x_B^H) of Hot as given. Similarly, Hot solves

$$\max_{x_A^H, x_B^H} \quad U^H(x_A^C + x_A^H, x_B^C + x_B^H)$$
$$\text{s.t.} \quad px_A^H + x_B^H \leq 1 - \phi, \quad x_A^H \geq 0, \quad x_B^H \geq 0.$$

It turns out to be convenient to rewrite these problems in the following way.

[4] We'll see later that less restrictive welfare functions (e.g., a Cobb-Douglas aggregator function) will suffice, but the linear aggregation in (2) is convenient and intuitive.

$$\max_{X_A, X_B} \quad U^C(X_A, X_B) \qquad\qquad \max_{X_A, X_B} \quad U^H(X_A, X_B) \tag{3}$$

$$\text{s.t.} \quad pX_A + X_B \le px_A^H + x_B^H + \phi \qquad \text{s.t.} \quad pX_A + X_B \le px_A^C + x_B^C + 1 - \phi \tag{4}$$

$$X_A \ge x_A^H \qquad\qquad X_A \ge x_A^C \tag{5}$$

$$X_B \ge x_B^H \qquad\qquad X_B \ge x_B^C \tag{6}$$

Under this alternative formulation, each of the sub-selves chooses the overall amount of the two goods, taking the contributions of the other sub-self as given.

Denote by $\tilde{X}^C = (\tilde{X}_A^C, \tilde{X}_B^C)$ and $\tilde{X}^H = (\tilde{X}_A^H, \tilde{X}_B^H)$ the solutions of problem (3-4), temporarily ignoring constraints (5-6). Because the utility functions are strictly quasi-concave, these solutions are unique. Assumption 2 ensures that, for any *p*, the solutions are interior and characterized by the first order condition, $MRS^j(\tilde{X}^j) = p$ for $j = C, H$.

Returning now to the full problem (3-6), a constraint in (5) or (6) will bind whenever $\tilde{X}_i^j < x_i^{-j}$, i.e., whenever sub-self *j*'s desired consumption of good *i* is less than the given amount purchased by the other sub-self. In that case, *j* will choose the corner purchase $x_i^j = 0$ and spend her entire budget on the other good $-i$. Thus, in terms of final consumption, Cold's best response is

$$(X_A^{C*}, X_B^{C*}) = \begin{cases} (\frac{\phi}{p} + x_A^H, x_B^H) & \text{if } \tilde{X}_A^C \ge x_A^H \text{ and } \tilde{X}_B^C < x_B^H \\ (\tilde{X}_A^C, \tilde{X}_B^C) & \text{if } \tilde{X}_A^C \ge x_A^H \text{ and } \tilde{X}_B^C \ge x_B^H \\ (x_A^H, \phi + x_B^H) & \text{if } \tilde{X}_A^C < x_A^H \text{ and } \tilde{X}_B^C \ge x_B^H \end{cases} \tag{7}$$

We can rule out the remaining case, that the constraints bind for both goods, because that would imply that total expenditure is less than Hot's expenditure. More formally, $\tilde{X}_A^C < x_A^H \;\&\; \tilde{X}_B^C < x_B^H \implies p\tilde{X}_A^C + \tilde{X}_B^C < px_A^H + x_B^H \implies 1 < \phi$, a contradiction.

To rewrite in terms of Cold's choice variables, x^C, we subtract Hot's given purchase (x_A^H, x_B^H), yielding the best response function

$$(x_A^{C*}, x_B^{C*}) = \begin{cases} (\frac{\phi}{p}, 0) & \text{if } \tilde{X}_A^C - x_A^H \ge 0 \text{ and } \tilde{X}_B^C - x_B^H < 0 \\ (\tilde{X}_A^C - x_A^H, \tilde{X}_B^C - x_B^H) & \text{if } \tilde{X}_A^C - x_A^H \ge 0 \text{ and } \tilde{X}_B^C - x_B^H \ge 0 \\ (0, \phi) & \text{if } \tilde{X}_A^C - x_A^H < 0 \text{ and } \tilde{X}_B^C - x_B^H \ge 0 \end{cases} \tag{8}$$

Similarly, Hot's best response function is

$$(x_A^{H*}, x_B^{H*}) = \begin{cases} (\frac{1-\phi}{p}, 0) & \text{if } \tilde{X}_A^H - x_A^C \geq 0 \text{ and } \tilde{X}_B^H - x_B^C < 0 \\ (\tilde{X}_A^H - x_A^C, \tilde{X}_B^H - x_B^C) & \text{if } \tilde{X}_A^H - x_A^C \geq 0 \text{ and } \tilde{X}_B^H - x_B^C \geq 0 \\ (0, 1-\phi) & \text{if } \tilde{X}_A^H - x_A^C < 0 \text{ and } \tilde{X}_B^H - x_B^C \geq 0 \end{cases} \quad (9)$$

As shown in Table 1, we thus potentially have nine cases, according to whether each sub-self has an interior solution or either of the two corner solutions. Before proceeding further, we consider a parametric example that will provide helpful insight.

2.2 Cobb-Douglas example

To illustrate the analysis so far, consider Cobb-Douglas utility functions for two sub-selves,

$$U^j(X_A, X_B) = X_A^{\alpha^j} X_B^{1-\alpha^j}, \quad j = C, H \quad (10)$$

for given budget share parameters $\alpha^j \in (0, 1)$. It is well-known (and easily verified) that these utility functions satisfy Assumptions 1 and 2, and that the marginal rate of substitution is

$$MRS^j(X_A, X_B) = \frac{\partial_A U^j}{\partial_B U^j} = \frac{\alpha^j}{1-\alpha^j} \frac{X_B}{X_A}. \quad (11)$$

The last expression in equation (11) tells us that, according to Definition 1, good A is a temptation for Hot $\iff \frac{\alpha^H}{1-\alpha^H} > \frac{\alpha^C}{1-\alpha^C} \iff \alpha^H > \alpha^C$.

Budget exhaustion follows from Assumption 1, so $px_A^C + x_B^C = \phi$ and $px_A^H + x_B^H = 1 - \phi$. Plugging in those expressions, we see that budget constraint (4) reduces to

$$pX_A + X_B = 1. \quad (12)$$

We previously noted that the relaxed solution $\tilde{X}^j$ (ignoring non-negativity constraints) was characterized by the first order condition $MRS^j(X) = p$. Combining that observation with (12), we obtain the familiar result

Table 1 Possible cases for joint best response

	$x_A^{H*} > 0, x_B^{H*} > 0$	$x_A^{H*} > 0, x_B^{H*} = 0$	$x_A^{H*} = 0, x_B^{H*} > 0$	$x_A^{H*} = 0, x_B^{H*} = 0$
$x_A^{C*} > 0, x_B^{C*} > 0$	①	②	③	×
$x_A^{C*} > 0, x_B^{C*} = 0$	④	⑤	⑥	×
$x_A^{C*} = 0, x_B^{C*} > 0$	⑦	⑧	⑨	×
$x_A^{C*} = 0, x_B^{C*} = 0$	×	×	×	×

$$\tilde{X}_A^j = \frac{\alpha^j}{p}, \quad \tilde{X}_B^j = 1 - \alpha^j, \quad j = H, C. \tag{13}$$

That is, the relaxed expenditure shares for each sub-self coincide with their given Cobb-Douglas parameters.

These expressions for $\tilde{X}$ together with the general best response function (8-9) enable us to find Nash equilibrium of $\Gamma(\phi)$. To begin, suppose that $\phi \in (0, 1-\alpha^H)$. Then $\alpha^H < 1-\phi$, i.e., Hot's relaxed expenditure $p\tilde{X}_A^H = p\frac{\alpha^H}{p} = \alpha^H$ on their temptation good is less than their allocated budget $1-\phi$. Thus Hot will purchase a positive amount of good B as well as a positive amount of A, and so the relevant column in Table 1 is the first (cases 1, 4 or 7). It is intuitively clear that case 4 is not relevant: surely Cold will purchase some of her preferred good to increase her consumption of it beyond that chosen by Hot, who cares less for it. It is also intuitively clear that case 1 is not relevant, since it implies that Cold will not choose to improve on Hot's optimal relaxed bundle $\tilde{X}^H$. (The next subsection will rigorously support these intuitions.) We conclude that case 7 applies, with $x_A^H = \tilde{X}_A^H = \frac{\alpha^H}{p}, x_B^H = 1-\phi-\alpha^H$ and $x_A^C = 0, x_B^C = \phi$.

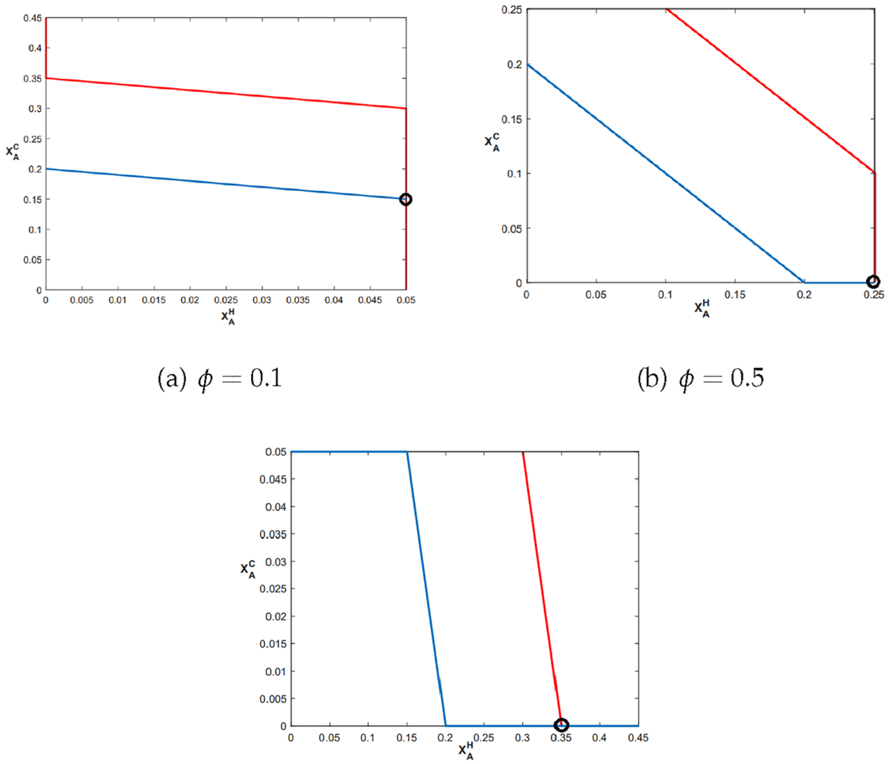
Now suppose that $\phi \in (1-\alpha^C, 1)$. Equation (13) shows that now Cold can purchase more good B than in their optimal relaxed bundle $\tilde{X}^C = (\frac{\alpha^C}{p}, 1-\alpha^C)$, so the top row of Table 1 applies. The same intuition as previously rules out cases 3 (Hot doesn't purchase any of their temptation good) and 1 (Hot is happy with Cold's relaxed choice of total consumption). Thus case 2 applies, with $x_A^H = \frac{1-\phi}{p}, x_B^H = 0$ and $x_A^C = \frac{\alpha^C+\phi-1}{p}, x_B^C = \tilde{X}_B^C = 1-\alpha^C$.

When $\phi \in (1-\alpha^H, 1-\alpha^C)$, neither sub-self will have the purchasing power to unilaterally acquire their desired amount of their preferred good. It is intuitively clear that case 8 applies, where each of them will use their entire budget on their preferred good. That is, now the unique NE is $x_A^H = \frac{1-\phi}{p}, x_B^H = 0$ and $x_A^C = 0, x_B^C = \phi$. Figure 2 shows the best responses and Nash equilibrium for an allocation ϕ in each of the three relevant ranges. Note that one of the selves is completely specialized ($x_A^i = 0$ or $x_B^i = 0$) in NE in panels (a) and (c), and that both selves are completely specialized in panel (b).

To take the analysis a step further, consider the NE payoffs in this parametric example as a function of ϕ:

$$U^{C*} = \begin{cases} \left(\frac{\alpha^H}{p}\right)^{\alpha^C} \left(1-\alpha^H\right)^{1-\alpha^C} & \text{if} \quad 0 \le \phi < 1-\alpha^H \\ \left(\frac{1-\phi}{p}\right)^{\alpha^C} \phi^{1-\alpha^C} & \text{if} \quad 1-\alpha^H \le \phi \le 1-\alpha^C \\ \left(\frac{\alpha^C}{p}\right)^{\alpha^C} \left(1-\alpha^C\right)^{1-\alpha^C} & \text{if} \quad 1-\alpha^C < \phi \le 1 \end{cases} \tag{14}$$

and



(c) $\phi = 0.9$

Fig. 2 Best responses in Cobb-Douglas example with $\alpha^C = 0.4$, $\alpha^H = 0.7$. and $p = 2$. Blue lines show best responses x_A^{C*} to x_A^H (with $x_B^H = 1 - \phi - px_A^H$) and red lines show best response x_A^{H*} to x_A^C (with $x_B^C = \phi - px_A^C$). The black circles indicate Nash Equilibria

$$U^{H*} = \begin{cases} \left(\frac{\alpha^H}{p}\right)^{\alpha^H} (1-\alpha^H)^{1-\alpha^H} & \text{if} \quad 0 \le \phi < 1 - \alpha^H \\ \left(\frac{1-\phi}{p}\right)^{\alpha^H} \phi^{1-\alpha^H} & \text{if} \quad 1 - \alpha^H \le \phi \le 1 - \alpha^C \\ \left(\frac{\alpha^C}{p}\right)^{\alpha^H} (1-\alpha^C)^{1-\alpha^H} & \text{if} \quad 1 - \alpha^C < \phi \le 1 \end{cases} \tag{15}$$

Figure 3 graphs these payoff functions as functions of ϕ for a particular choice of $\alpha^C < \alpha^H$, together with Ego's payoff, $U^E = \gamma U^{C*} + (1-\gamma) U^{H*}$ for $\gamma = 0.5$.

Note that payoff for each of the three sub-selves is constant for $\phi \le 1 - \alpha^H = 0.3$ and for $\phi \ge 1 - \alpha^C = 0.6$. For $\phi \in (0.3, 0.6)$, Hot's payoff decreases, Cold's decreases, while Ego's payoff increases at first but then decreases. Thus Ego's optimal choice ϕ^* will lie in the interval (0.3, 0.6). We will soon see that many qualitative features of this parametric example hold in the general case.

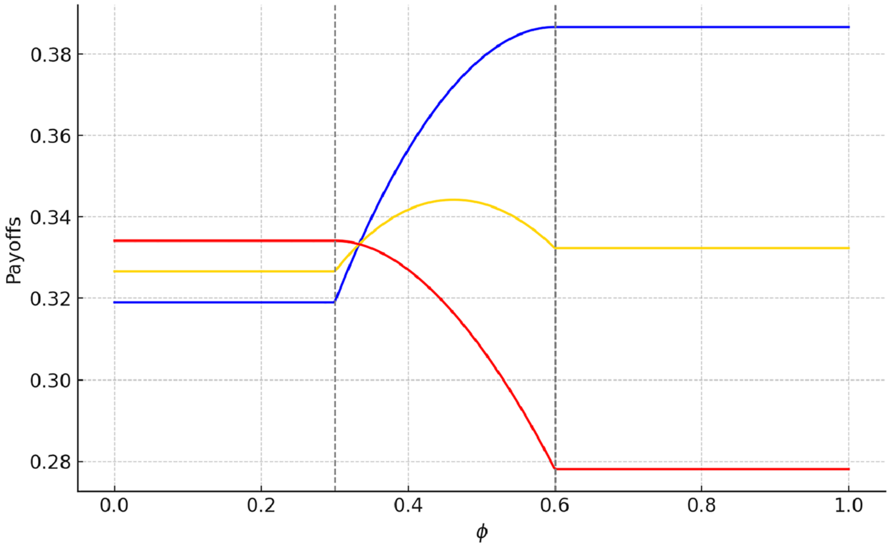


Fig. 3 NE payoffs in Cobb-Douglas example with α^C = 0.4, α^H = 0.7, $\gamma = 0.5$ and p = 2. Red, blue and yellow lines respectively show payoffs for Hot, Cold and Ego

2.3 Subgame nash equilibrium

In the Cobb-Douglas example, only three of the nine possible cases arose in NE. We will now see that the same is true in our general model. We first eliminate what might seem to be the most natural case, where both sub-selves choose interior bundles.

Lemma 1 *Let A be a temptation good, and let preferences satisfy Assumption 1. Then case 1 can not occur in Nash equilibrium.*

Proof Suppose to the contrary, that case (1) holds at $X = x^H + x^C$, where both components of x^H and of x^C are strictly positive. It follows from Assumption 1 and the standard first order conditions that $MRS^H(X) = \frac{\partial_A U^H(X)}{\partial_B U^H(X)} = p$. For the same reason, $MRS^C(X) = p$. Hence $MRS^H(X) = MRS^C(X)$, contradicting the assumption that A is a temptation good. □

Lemma 2 *Let preferences satisfy Assumptions 1 and 2, and fix $\phi \in [0, 1]$ and $p > 0$. Then at any Nash equilibrium of $\Gamma(\phi)$, total consumption is positive for both goods: $X_A > 0$ and $X_B > 0$.*

Proof Suppose to the contrary that $X_A = 0$ at the NE of $\Gamma(\phi)$. If $\phi > 0$, then Cold could purchase some positive amount of A, and by Assumption 2 this would increase her utility. Thus $X_A = 0$ is not compatible with Cold choosing a best response, a contradiction. If $\phi = 0$, then the same contradiction is obtained by applying the

Inada condition (Assumption 2) to Hot's choice. The same argument establishes that $X_B > 0$ at the NE of $\Gamma(\phi)$ for every $\phi \in [0, 1]$. □

We now show that, not surprisingly, neither sub-self will purchase only her less preferred good.

Lemma 3 *Let A be a temptation good, and let preferences satisfy Assumptions 1 and 2. Then cases 3, 4, 5, 6 and 9 can not occur in Nash equilibrium.*

Proof Suppose to the contrary that, at NE total allocation $X = x^H + x^C$, Hot purchases only good B, so $x^H = (0, \phi)$, as in cases 3, 6 and 9. At NE, the usual Kuhn-Tucker conditions will hold for Hot, and these imply that $MRS^H(X) \leq p$. It follows from Lemma 2 that Cold will purchase a positive amount of good A, so $X_A = x_A^C > 0$. Therefore, by Cold's first-order conditions, $MRS^C(X) \geq p$. Hence $MRS^C(X) \geq MRS^H(X)$, contradicting the assumption that A is a temptation good for Hot. A parallel argument eliminates the possibility that Cold purchases only good A, as in cases 4, 5, and 6. □

As a benchmark, it turns out to be useful to consider the situations where either of the two sub-selves has the entire purchasing power. Let $\hat{X}^C(p) = (\hat{X}_A^C(p), \hat{X}_B^C(p))$ be Cold's most preferred affordable bundle at price p and $\phi = 1$, and let $\hat{X}^H(p) = (\hat{X}_A^H(p), \hat{X}_B^H(p))$ be Hot's most preferred affordable bundle when $\phi = 0$. Then,

Lemma 4 *Let A be a temptation good. Then $\hat{X}_A^H > \hat{X}_A^C$ and $\hat{X}_B^C > \hat{X}_B^H$.*

Proof Since A is a temptation good, we have $MRS^H(X) > MRS^C(X)$ for all allocations, including $X = \hat{X}^C \equiv (\hat{X}_A^C, \hat{X}_B^C)$. Thus $MRS^C(\hat{X}^C) = p < MRS^H(\hat{X}^C)$. By a basic property of MRS, it follows that U^H will increase as we move from $\hat{X}^C$ to $\hat{X}^C + (\epsilon, -p\epsilon)$ for sufficiently small positive ϵ. Since $\hat{X}^C$ and $\hat{X}^H$ lie on the same budget line of slope $-p$ and, by quasi-concavity, $MRS^H(X)$ decreases monotonically as we move along that budget line from $\hat{X}^C$ towards higher X_A, we see that U^H continues to increase until we reach $X = \hat{X}^H$, where $MRS^H(X) = p$. Thus indeed $\hat{X}_A^H > \hat{X}_A^C$ and $\hat{X}_B^H > \hat{X}_B^C$. □

We are now ready to fully characterize the subgame Nash equilibria for general preferences and any relative price $p > 0$. It turns out that, as the wealth allocation ϕ to Cold increases, the NE always transitions from one where Hot purchases both goods but Cold purchases only B (case 7) to both purchasing only their preferred good (case 8) to Cold purchases both goods but Hot purchases only A (case 2). More precisely,

Proposition 1 *Let A be a temptation good, and let preferences satisfy Assumptions 1 and 2. Then, for each* $\phi \in [0, 1]$ *and* $p > 0$ *, the subgame* $\Gamma(\phi)$ *has a unique NE* $x^*(\phi, p)$*. Moreover, there are threshold values* $0 \leq \underline{\phi}(p) < \overline{\phi}(p) \leq 1$ *such that*

- $x^*(\phi, p)$ is case 7 for $\phi < \underline{\phi}(p)$
- $x^*(\phi, p)$ is case 8 for $\underline{\phi}(p) \leq \phi \leq \overline{\phi}(p)$
- $x^*(\phi, p)$ is case 2 for $\phi > \overline{\phi}(p)$.

Proof When Hot has unilateral power, $\phi = 0$, both players have trivial dominant strategies, so $x^H = \hat{X}^H(p), x^C = 0$ is the unique NE. It then follows from Lemma 2 that both components of $\hat{X}^H(p)$ are positive.

Set $\underline{\phi}(p) = 1 - p\hat{X}^H_A(p)$. Since both components of $\hat{X}^H(p)$ are positive and (by virtue of our normalizations) expenditure on that bundle is 1, it follows that $0 < \underline{\phi}(p) < 1$. For $\phi \in [0, \underline{\phi}(p))$, Hot's budget $1 - \phi$ suffices to purchase the temptation component $\hat{X}^H_A(p)$ of his preferred bundle with some left over for the other component. This eliminates cases 2 and 8 from the possible NE, and the Lemmas remove all other cases except 7. Thus the best response equations (7-8) tell us that NE is uniquely defined by

$$x^C = (0, \phi) \text{ and } x^H = (\hat{X}^H_A(p), \hat{X}^H_B(p) - \phi) \tag{16}$$

when $\phi \in [0, \underline{\phi}(p))$. Of course, the NE payoff vector consists of the three players' utilities at allocation $X = \hat{X}^H(p)$.

Similarly, consider Cold's most preferred affordable bundle when $\phi = 1$, denoted $\hat{X}^C(p) = (\hat{X}^C_A(p), \hat{X}^C_B(p))$. Set $\overline{\phi}(p) = \hat{X}^C_B(p)$, the expenditure (with price p_B normalized to 1) required to purchase the second component of that bundle. An argument parallel to that above establishes that there is a unique NE, which is case 2, when $\phi \in (\overline{\phi}(p), 1]$. It takes the form

$$x^C = \left(\hat{X}^C_A(p) - \frac{1-\phi}{p}, \hat{X}^C_B(p)\right) \text{ and } x^H = \left(\frac{1-\phi}{p}, 0\right); \tag{17}$$

the payoff vector consists of the players' utilities at allocation $X = \hat{X}^C(p)$.

Note that by Lemma 4 we have $\hat{X}^H_A(p) > \hat{X}^C_A(p)$ and $\hat{X}^C_B(p) > \hat{X}^H_B(p)$. It follows that $\underline{\phi}(p) = 1 - p\hat{X}^H_A(p) = \hat{X}^H_B(p) < \hat{X}^C_B(p) = \overline{\phi}(p)$.

To complete the proof, suppose that $\underline{\phi}(p) \leq \phi \leq \overline{\phi}(p)$. Case 7 is not possible since, with $\underline{\phi}(p) \leq \phi$, Hot no longer has the purchasing power to acquire a positive amount of good B while purchasing $\hat{X}^H_A$. Likewise, with $\phi \leq \overline{\phi}(p)$, Case 2 is not possible. Having ruled out all other cases, it must be for this range of ϕ, the NE are case 8. It follows from the BR equations (7-8) that the NE is unique with

$$x^C = (0, \phi) \text{ and } x^H = \left(\frac{1-\phi}{p}, 0\right). \tag{18}$$

□

Considering the joint consumption vector in Nash equilibrium $X^*(\phi, p) = x^{C*}(\phi, p) + x^{H*}(\phi, p)$ we obtain the following corollary.

Corollary 1

$$X^*(\phi, p) = \begin{cases} \left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right) & \text{if } 0 \leq \phi \leq \underline{\phi} \\ \left(\frac{1-\phi}{p}, \phi\right) & \text{if } \underline{\phi} < \phi < \overline{\phi} \\ \left(\frac{1-\overline{\phi}}{p}, \overline{\phi}\right) & \text{if } \overline{\phi} \leq \phi \leq 1. \end{cases} \tag{19}$$

Proof The proof follows from adding up x^C and x^H in equations (16)-(18) and noting that $\hat{X}^H(p) = \left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right)$ and $\hat{X}^C(p) = \left(\frac{1-\overline{\phi}}{p}, \overline{\phi}\right)$. □

2.4 The full game

Figure 3 is for a particular numerical example, but Proposition 1 and Corollary 1 tell us that it nicely illustrates the general case. The kinks in NE payoff at the α's in the example generalize to kinks at $\underline{\phi}(p)$ and $\overline{\phi}(p)$. The NE consumption bundles (and therefore NE payoffs) are constant below $\underline{\phi}(p)$ and above $\overline{\phi}(p)$. In the range $\phi \in [\underline{\phi}(p), \overline{\phi}(p)]$, NE payoff is decreasing for Hot and increasing for Cold. However, as we increase ϕ^* slightly starting at $\underline{\phi}(p)$, the kink in Cold's payoff indicates that their payoff increases faster than the (non-kinked) payoff for Hot decreases. Therefore Ego's payoff increases here for any weighting $\gamma \in (0, 1)$. Similarly, Fig. 3 suggests that Ego's payoff is decreasing in ϕ slightly below $\overline{\phi}(p)$. Otherwise put, the player getting her preferred bundle loses utility only gradually with a small deviation from that bundle, while the other player gets a more direct boost. Thus it would seem that Ego's optimum must occur in that middle range of ϕ. More precisely, we have

Proposition 2 *Let A be a temptation good, and let preferences satisfy Assumptions 1 and 2. Then for any $\gamma \in (0, 1)$ and $p > 0$, Ego will choose $\phi \in (\underline{\phi}(p), \overline{\phi}(p))$, where both sub-selves completely specialize their purchases, in any subgame perfect Nash equilibrium (SPNE) of the full game.*

Proof Fix $p > 0$ and streamline notation by omitting it. Let $X(\phi)$ represent the NE consumption vector characterized in the previous Proposition, and let $V^H(\phi) = U^H(X(\phi))$ and $V^C(\phi) = U^C(X(\phi))$ denote the corresponding NE payoffs for the two sub-selves. Also let $V^E(\phi) = \gamma V^C(\phi) + (1-\gamma)V^H(\phi)$ be the corresponding payoff for Ego.

In SPNE, Ego maximizes the continuous function $V^E(\phi)$ over the compact set $\phi \in [0, 1]$. Such a maximum must exist. The key to completing the proof is the following

Claim.For$\epsilon > 0$sufficientlysmall,$V^E(\underline{\phi}) < V^E(\underline{\phi} + \epsilon)andV^E(\overline{\phi}) < V^E(\overline{\phi} - \epsilon)$.

To prove the claim, note by Corollary 1 that the right derivative of $V^i(\phi)$ is given by

$$\partial_+ V^i(\phi) = -\frac{1}{p}\partial_A U^i\left(\frac{1-\phi}{p}, \phi\right) + \partial_B U^i\left(\frac{1-\phi}{p}, \phi\right) \tag{20}$$

$$= \left(p - MRS^i\left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right)\right)\frac{\partial_B U^i\left(\frac{1-\phi}{p}, \phi\right)}{p} \tag{21}$$

in the interval $[\underline{\phi}, \overline{\phi})$ and is 0 otherwise. Note that at $\hat{X}^H = \left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right)$ we have

$$MRS^H\left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right) = p > MRS^C\left(\frac{1-\underline{\phi}}{p}, \underline{\phi}\right)$$

where the inequality comes from Assumption 1. Thus, we have $\partial_+ V^H(\underline{\phi}) = 0$ and $\partial_+ V^C(\underline{\phi}) > 0$. Since $\partial_+ V^E(\underline{\phi}) = \gamma\partial_+ V^C(\underline{\phi}) + (1-\gamma)\partial_+ V^H(\underline{\phi})$ it follows that $\partial_+ V^E(\underline{\phi}) > 0$. A similar argument establishes that the left derivative of ego's payoff at $\overline{\phi}$ is negative, $\partial_- V^E(\overline{\phi}) = \gamma\partial_- V^C(\overline{\phi}) + (1-\gamma)\partial_- V^H(\underline{\phi}) < 0$, and the claim follows.

To complete the proof of the Proposition, take a first order Taylor expansion for $V^E(\underline{\phi} + \epsilon)$. For ϵ sufficiently small, we have $V^E(\underline{\phi} + \epsilon) = V^E(\underline{\phi}) + \epsilon\partial_+ V^E(\underline{\phi}) + o(\epsilon^2) > V^E(\underline{\phi})$. By the same token, $V^E(\overline{\phi} - \epsilon) \approx V^E(\overline{\phi}) - \epsilon\partial_- V^E(\overline{\phi}) > V^E(\overline{\phi})$.

It follows that the maximum of $V^E(\phi)$ can't occur in [0, $\underline{\phi}$] nor in [$\overline{\phi}$, 1], after noting that $V^E(\phi)$ is constant over each of those intervals. By elimination, the maximum (i.e., Ego's best response to the subgame NE strategy profiles in $\Gamma(\phi)$) must occur at some $\phi \in (\underline{\phi}(p), \overline{\phi}(p))$. □

3 Discussion

Proposition 1 tells us that "doer" sub-self Cold (resp. Hot) will obtain her most preferred affordable bundle in any Nash equilibrium of the subgame whenever the resource allocation ϕ is above a threshold $\overline{\phi}$ (resp. below a threshold $\underline{\phi}$). However, Proposition 2 tells us that Ego will ultimately choose a resource allocation $\phi^* \in (\underline{\phi}, \overline{\phi})$ such that neither of the "doer" sub-selves gets their preferred bundle. Taken together, the Propositions tell us that in SPNE both of the doer sub-selves overspecialize: Hot purchases only the temptation good and Cold purchases only the generic good, even though both of them include both goods in their most preferred bundles.

3.1 Alternative interpretations

To make use of standard terminology, we framed our model as consumer choice with multiple sub-selves sharing a bundle of public goods, with their budgets controlled by a benevolent principal. Of course, in thinking about the process behind a person's observed behavior, we have in mind something more general. The resource that the principal sub-self ("Ego") divides into shares $(\phi, 1-\phi)$ for the other sub-selves might be time or attention rather than purchasing power per se. The other sub-selves might allocate their shares to activities other than consumption goods. In a "Robinson Crusoe" setting, for example, activity A might be napping (a temptation good for the impulsive sub-self) while activity B might be weaving gill nets to improve fishing productivity. The given price P then would reflect the opportunity cost, so $1/P$ would be the foregone hours of napping to weave a gill net.

Some readers might wonder whether Hot is capable of best-responding to the other sub-selves. By informally describing Hot as "impulsive" we do not mean to suggest that Hot is incoherent, but rather that its preferences are less aligned with the individual's long term health and well-being than sub-self Cold's preferences. Neither Hot nor Cold need to have a high degree of rationality for our conclusions to be valid: an extensive literature in game theory and biology obtains mutual best response as a typical outcome with sub-rational myopic adaptive agents,[5] [6]

The limitations of Ego may also deserve further discussion. If Ego cares directly about the total bundle X and has the power to choose it unilaterally, then our setup would be equivalent to the standard rational choice model for a unified self; the payoffs to Hot and Cold would be of no consequence. If one unifies Ego and Cold into a single sub-self, then our setup again would reduce to the standard model with Cold's preferences. Following classic psychological literature from Freud (1933) to Baumeister et al. (2018) and beyond, we prefer to model Ego as a principal (or an executive function) that can act only indirectly via allocating scarce internal resources to agents who act on their own preferences. That approach seems useful when some sort of inner conflict affects observed individual choices.

Variants of our model could potentially have applications beyond conflicted individual choice. For example, consider private provision of public goods or, more specifically, an international body that allocates budgets ϕ to its members. The members may purchase public goods (e.g., education or defense or pollution abatement) but differ in how much they benefit from those goods. In a similar vein, our model might be used to analyze altruism, where agents benefit from the wellbeing of their peers,

[5] See Kuhn et al. (1996) who emphasize the "mass-action" interpretation introduced in John Nash's 1950 dissertation, and writers following Selten (1983) or Binmore (1987) who favor "eductive" over "deductive" interpretations of Nash equilibrium. Recent textbooks on evolutionary game theory include (Sandholm, 2010) and Friedman and Sinervo (2016).

[6] There is a separate, more specialized literature on the evolution of preferences, e.g., Frank (1988), Güth and Yaari (1992), Samuelson and Swinkels (2006), Friedman and Singh (2009). It provides a starting point for researchers interested in investigating how a choice architecture like our three subselves might evolve.

or to analyze family dynamics where each member chooses some portion of joint consumption.

Variants of our model may also speak to the allocation of resources among multiple divisions or departments within a firm.[7] Suppose that the CEO decides on the allocation of resources between two divisions, each one having an advantage in one aspect of production x_i (but also being able to provide the other). Then U^i would simply be divisional profits and U^E the sum of these. In this scenario, our model predicts that, no matter the relative importance of the divisions or their ability to diversify, the CEO will allocate resources so that each division specializes on production of just one good.

3.2 Extensions

The model can be extended in various ways. For example, Eq. (2) assigns a linear social welfare function to Ego. However, the proof of Proposition 2 still goes through when Ego's payoff is any smooth social welfare function strictly increasing in both doer sub-self payoffs. One could also explore the width of the range $[\underline{\phi}, \overline{\phi}]$ where Ego's SPNE choice lies. We conjecture that, for some sensible metric on the space of preferences, that width is an increasing function of the distance between Hot's preferences and Cold's preferences.

Sequential subgame In our main model we assume that Hot and Cold move simultaneously in the subgame $\Gamma(\phi)$. However, it seems equally plausible that they move sequentially. To capture the idea that Hot is more impulsive and Cold can anticipate what will happen, we might postulate that Cold moves before Hot, while maintaining the assumption that Ego moves first. Appendix B analyzes this "Stackelberg" game and shows that, with the exception of certain sub-games that are not reached in sub-game perfect equilibrium, outcomes are essentially the same as in the benchmark version where the sub-selves move simultaneously. In standard oligopoly games, by contrast, the sequential and simultaneous move equilibria yield very different predictions.[8]

Dynamic extensions Our model is a one-shot extensive form game in which the first mover, Ego, freely chooses the resource allocation ϕ. Perhaps more realistically, the central executive system might be able to adjust that variable only gradually. According to some existing psychological studies such as Baumeister et al. (2018), and Oaten and Cheng (2006), the power of self-control can be strengthened over time via a form of investment (or presumably weakened via disinvestment). If Ego faces adjustment costs (e.g., quadratic, possibly asymmetric), then dynamic programming methods may help the analysis. Such a model might provide insight into behavior over time, as a person matures or just struggles to quit smoking.

[7] Alonso et al. (2008) emphasize the importance of decentralized decision making in organizations when there is a conflict between superior information of local branch managers and a need to coordinate at the company level.

[8] Dato et al. (2022) use a more sophisticated notion of equilibrium to accommodate a self with possibly irrational beliefs.

Appendices

A Connection to NBS

There is a connection between our SPNE and the Nash bargaining solution associated with a well-chosen subgame $\Gamma(\phi)$. Shift (i.e., add constants to) the Hot and Cold utility functions so that they are zero at some baseline consumption levels that are taken to be the threat point. Rewrite Ego's social welfare function so that, instead of being a weighted average of Hot and Cold utility, it is a weighted average of the *log utility gains* above the baseline level, and set the weight $\gamma = 0.5$. Suppose that the set of feasible overall consumption vectors $X = x^C + x^H$ are those that are affordable at income 1.0 and relative price *p*. Then it can be shown that the NBS of the bargaining problem between Hot and Cold gives them the same (originally scaled) utilities that, for some $\phi \in [0, 1]$, they would get in the NE of subgame $\Gamma(\phi)$.

B Sequential game among selves

We proceed to identify the subgame perfect equilibria using backwards induction. In the last stage the hot system will choose a best response, as characterized in (9). Now consider the cold system which can anticipate Hot's best response.

First, consider the case where $\phi \geq \overline{\phi}(p)$. In this case Cold can buy $\hat{X}_B^C(p) = \overline{\phi}(p)$ units of good *B* and spend the remainder to buy $\frac{\phi - \hat{X}_B^C(p)}{p}$ units of good *A*. Now the hot system will simply spend all of her money on good *A* and buy $\frac{1-\phi}{p}$ units of it. It follows that total consumption is $X(p) = \left(\hat{X}_B^C(p), \hat{X}_B^C(p)\right)$ which is the same bundle Cold would choose if it had unilateral purchasing power, $\phi = 1$. Note that choosing a bundle that contains more of good *B* would result in a final bundle with $X_B^C(p) > \hat{X}_B^C(p)$ and choosing a bundle that contains less of good *B* would result in a final bundle with $X_B^C(p) < \hat{X}_B^C(p)$. Neither is optimal for the Cold system. Thus, for $\phi \geq \overline{\phi}$ the equilibrium in the sub-game among selves is unique with

$$x^C = (0, \phi) \text{ and } x^H = \left(\frac{1-\phi}{p}, 0\right). \tag{22}$$

Second, consider the case where where $\phi \leq \underline{\phi}(p)$. Assume that the cold system chooses some bundle that fulfills its budget constraint $px_A^C + x_B^C = \phi$. Note that $x_B^C \leq \phi \leq \underline{\phi}(p)$. If the hot system chooses the bundle $(\hat{X}_A^H - x_A^C, \hat{X}_B^H - x_B^C)$, it can assure itself the same payoff as if it had unilateral purchasing power $\phi = 0$. Note that the hot system can afford this bundle since $p(\hat{X}_A^H - x_A^C) + \hat{X}_B^H - x_B^C = 1 - \phi$. So, no matter the choice of the cold system, the hot system has the budget to ensure the final bundle $\left(\hat{X}_B^H(p), \hat{X}_B^H(p)\right)$ where it obtains the maximal level of utility. It follows that for $\phi \leq \underline{\phi}$ there exists a continuum of subgame perfect Nash equilibria characterized by

$$x^C \in \left\{(x_A^C, x_B^C) | px_A^C + x_B^C = \phi\right\} \text{ and } x^H = \left(\hat{X}_A^H - x_A^C, \hat{X}_B^H - x_B^C\right) \quad (23)$$

Finally, consider the case $\underline{\phi} < \phi < \overline{\phi}$. Consider the cold system and assume that it only buys its preferred good, $x^C = (0, \phi)$. Since $\phi > \underline{\phi}$ we have $p\hat{X}_A^H = 1 - \underline{\phi}(p) > 1 - \phi$, the hot system will only buy its temptation good A, $x^H = (\frac{1-\phi}{p}, 0)$. By choosing less of good B and buying a positive amount of good A, the cold system would strictly decrease its own payoff. It follows that for $\underline{\phi} < \phi < \overline{\phi}$

$$x^C = (0, \phi) \text{ and } x^H = \left(\frac{1-\phi}{p}, 0\right) \quad (24)$$

is the only subgame perfect NE.

It follows that equilibrium payoffs and, thus, also the payoffs of the ego are the same as in the simultaneous move game. Consequently, ego choice of ϕ will again have to lie between $\underline{\phi}$ and $\overline{\phi}$.

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Conflict of interest The authors have not disclosed any conflict of interest.

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