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Nash equilibrium seeking in non-cooperative heterogeneous multi-robot systems via output regulation ☆

Yi Dong [a], Zhongguo Li [b], Shoaib Ehsan [c], Zhengtao Ding [b], Xiaowei Huang [a],*

[a] Department of Computer Science, University of Liverpool, Ashton Building, Liverpool, L69 3BX, Liverpool, UK
[b] Department of Electrical and Electronic Engineering, University of Manchester, Engineering A, M13 9PL, Manchester, UK
[c] Department of Electronic and Computer Science, University of Southampton, Building 32, Southampton, So17 1BJ, UK

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ABSTRACT

This paper presents a study on Nash equilibrium seeking in noncooperative games within multi-robot systems, a topic of increasing importance in diverse sectors including civil, security, and military. Unlike conventional approaches where players can directly observe the actions of others, our method assumes limited visibility, where players can only communicate through an undirected and connected communication graph. We introduce a novel distributed control approach, integrating gradient play with a consensus protocol. This method facilitates effective Nash equilibrium seeking by leveraging information shared among neighboring robots in heterogeneous linear dynamic systems. The proposed solution employs a high-level distributed Nash equilibrium-seeking algorithm, serving as an optimal reference generator for each robot to track the Nash equilibrium, and an advanced output regulation technique, aiming to regulate the output (e.g., position) of the robots with respect to the obtained references. Theoretical analysis confirms the convergence of our algorithm through Lyapunov stability analysis. The effectiveness and practical applicability of our approach are validated through numerical simulations and empirical testing with physical robots, highlighting its efficacy and utility in real-world scenarios.

1. Introduction

In recent years, the field of multi-robot systems has witnessed significant advancements, driven by the growing need for efficient and intelligent coordination in various sectors such as civil, security, and military operations [1–3]. The complexity of interactions within these systems often mirrors that of noncooperative games, where individual robots (or players) pursue their objectives in a shared environment [4,5]. A fundamental concept in this domain is the Nash equilibrium, a state where no player can benefit by unilaterally changing their strategy, assuming other players' strategies remain constant. Achieving Nash equilibrium in multi-robot systems is crucial for ensuring optimal and stable operation, but it presents unique challenges, particularly in scenarios with limited visibility and communication constraints [6–10].

Traditional approaches to Nash equilibrium seeking in multi-robot systems often rely on the assumption that players have direct observation of each other's actions [11,12]. However, this assumption is not always practical or feasible, especially in environments where communication is restricted or in scenarios involving large numbers of robots. To address this gap, our study introduces a novel approach that operates under the assumption of limited visibility. Similar to [13–15], our algorithm considers that players can only communicate through an undirected and connected communication graph, a scenario more reflective of real-world conditions.

Over the past two decades, the research on Nash equilibrium searching mainly considers multi-agent systems without underlying dynamics [5,16,17]. Yet, it is inherent that multi-agent systems come with underlying dynamics [18,19]. To apply Nash equilibrium seeking algorithms for dynamic systems, methods utilizing hierarchical structures to separate the search and tracking processes into two distinct stages have been proposed [20], which necessitates separated time-scales in reference generation and tracking. In this paper, we argue that it is possible to develop an integrated Nash equilibrium searching-tracking, which consequently eliminates troublesome hierarchical tuning and implementation.

We introduce a distributed control approach that ingeniously combines gradient play with a consensus protocol. This integration is pivotal in addressing the challenge of Nash Equilibrium (NE) seeking in environments where direct observation among players is not feasible.

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* Corresponding author.
E-mail address: xiaowei.huang@liverpool.ac.uk (X. Huang).

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The consensus protocol enables the robots to reach an agreement on certain key parameters or states, despite the inherent limitations in their communication capabilities [18,21]. By leveraging the information shared among neighboring robots through the communication graph, the proposed approach facilitates an effective and efficient path to achieving Nash equilibrium. This method stands out for its ability to adapt to the dynamic and often unpredictable nature of multi-robot interactions, ensuring that collective decision-making leads to optimal outcomes even in the absence of full visibility.

Distinct from the existing literature without considering the underlying dynamics of the agents, in this paper we develop a Nash equilibrium seeking method for heterogeneous linear dynamic systems. These systems are characterized by their diversity in dynamics and roles of individual robots, reflecting a wide array of real-world scenarios in multi-robot operations, e.g., consensus and formation control in classic multi-agent systems research [22]. The heterogeneity of the system adds another layer of complexity to the Nash equilibrium seeking process. We address this problem by employing advanced output regulation techniques and a sophisticated high-level distributed optimization algorithm [23,24]. This algorithm serves as an optimal reference generator for each robot, tailored to its specific dynamics and role within the system. By doing so, it ensures that each robot can independently track the Nash equilibrium while collectively contributing to the system's overall objective. This focus on heterogeneous systems not only enhances the applicability of our approach to a broader range of scenarios but also underscores its versatility in dealing with the intricacies of real-world multi-robot systems.

Compared with the existing works, the main contributions of the paper are summarized as follows:

1. The paper introduces a novel communication model where robots in noncooperative games communicate through an undirected graph, reflecting real-world scenarios with communication constraints.
2. Our approach combines gradient play with a consensus protocol, enhancing Nash equilibrium seeking in multi-robot systems with limited visibility. Meanwhile, the proposed solution specifically targets heterogeneous linear dynamic systems, broadening its applicability in diverse real-world multi-robot scenarios.
3. The proposed algorithm has been successfully validated on a real-world multi-robot system, demonstrating its practical applicability. Furthermore, to promote transparency and facilitate reproducibility, all source codes related to the algorithm are openly available on GitHub,[1] enabling researchers and practitioners to validate and further build upon the findings.

The remainder of this article is organized as follows. In Section 2, some fundamental results are presented together with motivations and formulations of NES for multi-robot systems. In Section 4, a distributed algorithm is presented for NES, together with the convergence analysis of the proposed algorithm. Simulation examples are shown in Section 5 to demonstrate the effectiveness of the proposed algorithm and the possibility to extend the algorithm to the real robot cases. Finally, some concluding remarks are drawn in Section 6.

2. Preliminaries

2.1. Graph theory

Graph theory plays a pivotal role in the design and analysis of consensus algorithms, particularly in the context of distributed systems like multi-robot networks. In these systems, the interaction among agents (robots) can be effectively modeled using graphs, providing a powerful framework for understanding and implementing consensus processes.

A graph $\mathcal{G} = (V, E)$ is used to represent the communication topology of the network, where V is the set of vertices (or nodes) corresponding to the agents, and E is the set of edges representing the communication links between them. An edge $(i, j) \in E$ indicates that agents i and j can directly communicate with each other. The connectivity of the graph is often represented by an adjacency matrix A, where each element $a_{i,j}$ indicates whether there is a direct link between agents i and j. In the context of consensus algorithms, $a_{i,j}$ is typically set to 1 if $(i, j) \in E$ and 0 otherwise. The degree matrix D is a diagonal matrix where each diagonal element $d_{i,i}$ represents the number of neighbors of agent i. The graph Laplacian $L = D - A$ is a key tool in analyzing consensus algorithms, as it captures both the connectivity and the degree information of the graph [25].

Assumption 1. The communication topology $\mathcal{G}$ considered in this paper is undirected and connected.

Remark 1. For Assumption 1, it represents a standard connectivity requirement in multi-agent control and learning problems. If a graph is connected, we can design a communication matrix that satisfies Assumption 1 [26]. Moreover, each agent allocates a positive weight to the information obtained from its neighbors, and assigns a zero weight when no connecting edge exists between agents.

2.2. Nash equilibrium

Nash Equilibrium is an action profile on which no player can gain less cost by unilaterally changing its own action, i.e. an action profile ξ^* is the Nash equilibrium if:

$$f_i(\xi_i^*, \xi_{-i}^*) \le f_i(\xi_i, \xi_{-i}^*), \forall i \in V \tag{1}$$

where $\boldsymbol{\xi}_{-i} = \left[\xi_1, \xi_2, \dots, \xi_{i-1}, \xi_{i+1}, \dots, \xi_N\right]$ represents the aggregated actions of all the agents in the network except that of the ith agent. In the rest of this paper, we use $\boldsymbol{\xi}$ to represent the stacked vector $(\xi_i, \boldsymbol{\xi}_{-i})$.

To maintain the uniqueness and feasibility of the solution, several conditions of the cost function should be satisfied.

Assumption 2. Each player's cost function $f_i(\xi_i, \boldsymbol{\xi}_{-i})$ is convex in ξ_i, $\forall i \in V$.

$$(\boldsymbol{\xi} - \boldsymbol{z})^T(\nabla F_{\xi_i}(\boldsymbol{\xi}) - \nabla F_{z_i}(\boldsymbol{z})) \ge L\|\boldsymbol{\xi} - \boldsymbol{z}\|^2 \tag{2}$$

where $\nabla F_{\xi_i}(\boldsymbol{\xi})$ represents $\frac{\partial f_i(\xi)}{\partial \xi_i}$, and L is a positive constant.

Assumption 3. For each $i \in V$, $f_i(\boldsymbol{\xi})$ is a C^2 function, and $[\partial f_i(\boldsymbol{\xi})]/\partial \xi_i$ is globally Lipschitz with constant l_i.

Remark 2. Assumption 2 is utilized to ensure the existence of a unique Nash equilibrium [27,28]. Assumption 3 ensures the smoothness of the objective function, so that we can use gradient based methods and prove the convergence of the proposed algorithms.

3. Problem formulation

In this non-cooperative game, we consider N players. Each agent is a selfish unit that aims to optimize its own objective function, that is,

$$\min_{y_i \in R^N} f_i(\boldsymbol{y}) \tag{3a}$$

$$\text{s.t. } x_i(k+1) = A_i x_i(k) + B_i u_i(k) \tag{3b}$$

$$y_i(k) = C_i x_i(k) \tag{3c}$$

where x_i denotes the states, y_i denotes the output of the ith agent, and A_i, B_i and C_i are the system matrices of appropriate dimensions,

[1] https://github.com/YD-19/HMNES.

representing the heterogeneous robotic systems, and $\boldsymbol{y} = [y_1, y_2, \ldots, y_N]$ denotes all the outputs of the agents in the network. $f_i(\boldsymbol{y})$ is the objective function of agent i, which is coupled with its own action and also other competitors' actions [29].

Remark 3. The objective for this game is different from the objective in distributed Nash equilibrium-seeking algorithms. In this game, players seek to optimize their individual costs by adjusting their actions based on others' moves. In contrast, agents in distributed Nash equilibrium-seeking problems work together to minimize the combined objective functions that all agents involved.

4. Methods

4.1. Algorithm design

To find the Nash equilibrium in heterogeneous linear systems, we divide the problem into two sub-tasks: seeking the Nash equilibrium and tracking it. In this paper, we aim to find the Nash equilibrium point via distributed Nash equilibrium-seeking techniques, see (4b) and (4c), and employ output regulation methods to precisely guide the robots to their designated equilibrium positions, see (4a).

$$u_i(k) = -K_i x_i(k) + (G_i + K_i\Psi_i)\xi_i(k) \tag{4a}$$

$$\xi_i(k+1) = \xi_i(k) - t\alpha_i \nabla F_{\xi_i}(\xi_i(k), \boldsymbol{y}_{-i}(k)) \tag{4b}$$

$$\begin{aligned} y_{ij}(k+1) = y_{ij}(k) + \beta\Bigg(&\sum_{m=1}^{N} a_{im}(y_{ij}(k) - y_{mj}(k)) \\ &+ a_{ij}(y_{ij}(k) - \xi_j(k))\Bigg) \end{aligned} \tag{4c}$$

where $t > 0$ is the step size, $\xi_i(k)$ is an internal auxiliary variable generating the tracking reference for the ith agent. $\boldsymbol{y}_{-i} = [y_{i,1}, y_{i,2}, \ldots, y_{i,i-1}, y_{i,i+1}, \ldots, y_{i,N}]$, K_i is chosen such that $A_i - B_iK_i$ is Schur stable under assumption that the dynamics (A_i, B_i) are controllable. G_i and Ψ_i are gain matrices, which can be obtained by solving the following

$$\begin{aligned} &(A_i - I)\Psi_i + B_iG_i = 0 \\ &C_i\Psi_i - I = 0. \end{aligned} \tag{5}$$

Here, the gain matrices K_i, G_i, Ψ_i are designed to track the reference according to system dynamics A_i, B_i, C_i. To ensure the solvability of (5), we adopt the following assumption, which is a regulation equation in the output regulation literature [24].

Assumption 4. The pairs $(A_i, B_i), \forall i \in \mathcal{G}$ are controllable, and

$$\text{rank}\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix} = n + q. \tag{6}$$

Remark 4. In Assumption 4, controllability is a typical requirement for control algorithms, and Eq. (6) is essential to ensure the solvability of Eq. (5). The suggested algorithm actually merges techniques of gradient-descent optimization and output regulation. The internal model $\xi_i(k)$ comes from a consensus-driven gradient-descent optimization process, while $\boldsymbol{y}_{-i}$ is used to approximate the observations of other robots in the non-cooperative game. The formulation of the control input u_i is inspired by the traditional output regulation method as outlined in [24].

4.2. Convergence analysis

The convergence analysis of the proposed algorithm (4) will be conducted in two phases. Initially, it is proven that the algorithm will reach the Nash equilibrium for the problem (3a). Subsequently, it is shown that the heterogeneous linear systems will achieve the distinct optimal Nash equilibrium points that have been solved.

4.2.1. Nash equilibrium seeking for non-cooperative game

We start by analyzing the convergence of algorithm (4) in its pursuit of the Nash Equilibrium, which will later act as a basis for generating references in the proof of linear system regulation.

Theorem 1. *If Assumptions 1–3 hold, there exists a positive constant t, where the equilibrium $(\xi^*, 1 \otimes \xi^*)$ is globally exponentially stable under Eq. (4).*

Proof. Let $V(k)$ be the Lyapunov function:

$$V(k) = \frac{1}{4}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*) + \frac{1}{2}\bar{\boldsymbol{y}}(k)^T P\bar{\boldsymbol{y}}(k) \tag{7}$$

where $\alpha = diag\{\alpha_i\}$, $\bar{\boldsymbol{y}}(k) = \boldsymbol{y}(k) - \mathbf{1}_N \otimes \boldsymbol{x}(k)$ and $\boldsymbol{y} = [y_{1,1}, y_{1,2}, \ldots, y_{1,N}, y_{2,1}, \ldots, y_{2,N}, \ldots, y_{N,1}, \ldots, y_{N,N}]$.

Then

$$\begin{aligned} V(k+1) - V(k) =& \frac{1}{4}(\xi(k+1) - \xi^*)^T\alpha^{-1}(\xi(k+1) - \xi^*) \\ &- \frac{1}{4}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*) \\ &+ \frac{1}{2}\bar{\boldsymbol{y}}(k+1)^T P\bar{\boldsymbol{y}}(k+1) \\ &- \frac{1}{2}\bar{\boldsymbol{y}}(k)^T P\bar{\boldsymbol{y}}(k) \end{aligned}$$

Let $\mathcal{M}(\xi) = \frac{1}{4}(\xi(k+1) - \xi^*)^T\alpha^{-1}(\xi(k+1) - \xi^*) - \frac{1}{4}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*)$ and $\mathcal{N}(\bar{\boldsymbol{y}}) = \frac{1}{2}\bar{\boldsymbol{y}}(k+1)^T P\bar{\boldsymbol{y}}(k+1) - \frac{1}{2}\bar{\boldsymbol{y}}(k)^T P\bar{\boldsymbol{y}}(k)$. Then now, we analyze these two parts separately, starting with $\mathcal{M}(\xi)$. It is noticed that the gradient term of the equilibrium point equals zero, $F_\xi(\xi^*) = 0$. Therefore, with Assumption 2, we have:

$$(\xi(k) - \xi^*)^T F_\xi(\xi(k)) \geq L\|\xi(k) - \xi^*\|^2 \tag{8}$$

Then we have:

$$\begin{aligned} \mathcal{M}(\xi) =& \frac{1}{4}(\xi(k+1) - \xi^*)^T\alpha^{-1}(\xi(k+1) - \xi^*) \\ &- \frac{1}{4}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*) \\ =& \frac{c}{2}(\xi(k) + \alpha_i\nabla F_{\xi_i}(\xi_i, \boldsymbol{y}_{-i}) - \xi^*)^T\alpha^{-1} \\ &\times (\xi(k) + \alpha_i\nabla F_{\xi_i}(\xi_i, \boldsymbol{y}_{-i}) - \xi^*) \\ &- \frac{c}{2}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*) \\ =& \frac{1}{4}(\xi(k) - \xi^* - t\alpha\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i}))^T\alpha^{-1} \\ &\times (\xi(k) - \xi^* - t\alpha\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i})) \\ &- \frac{1}{4}(\xi(k) - \xi^*)^T\alpha^{-1}(\xi(k) - \xi^*) \end{aligned}$$

With Cauchy–Schwarz Inequality, Lipschitz Inequality and Eq. (8), we have

$$\begin{aligned} \mathcal{M}(\xi) =& \frac{1}{4}\alpha\|\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i})\|^2 \\ &- \frac{1}{2}\|\xi(k) - \xi^*\|^T\|\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i})\| \\ =& \frac{1}{4}\alpha\|\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i})\|^2 \\ &- \frac{1}{2}\|\xi(k) - \xi^*\|\|\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i}) - \nabla F_\xi(\xi(k))\| \\ &- \frac{1}{2}\|\xi(k) - \xi^*\|\|\nabla F_\xi(\xi(k)) - \nabla F_\xi(\xi^*)\| \\ \leq& \frac{t^2}{4}\alpha\|\nabla F_\xi(\xi(k), \boldsymbol{y}_{-i}(k))\|^2 \\ &- \frac{t}{2}L\|\xi(k) - \xi^*\|^2 + \frac{t}{2}l_{max}\|\xi(k) - \xi^*\|\|\bar{\boldsymbol{y}}(k)\| \\ \leq& -\frac{t}{2}L\|\xi(k) - \xi^*\|^2 + \frac{t}{2}l_{max}\|\xi(k) - \xi^*\|\|\bar{\boldsymbol{y}}(k)\| \\ &+ \frac{\alpha_{max}t^2}{4}\left(l_{max}\|\bar{\boldsymbol{y}}(k)\| + \sqrt{N}l_{max}\|\xi(k) - \xi^*\|\right)^2 \end{aligned}$$

where $\alpha_{max} = \max_{i\in\mathcal{N}}\{\alpha_i\}, l_{max} = \max_{i\in\mathcal{N}}\{l_i\}$.

Then, take look into the second part $\mathcal{N}(\bar{\mathbf{y}})$.

Inspired by the Lemma 3.1 in [30], let $\mathcal{H} = I_{N^2\times N^2} - \beta(\mathcal{L}\otimes I_{N\times N} + A_d)$, where $A_d = diag\{a_{i,j}\}, i, j \in \mathcal{G}$ and n be a positive integer. If $\beta < \min_{i,j\in\mathcal{N}} \frac{1}{\sum_{l=1}^{N} a_{i,l}+a_{i,j}}$, then $\|\mathcal{H}\|^{\infty} < 1$.

$$
\begin{aligned}
\bar{\mathbf{y}}(k+1) =& \mathbf{y}(k+1) - \mathbf{1}_N \otimes \boldsymbol{\xi}(k+1) \\
=& \mathbf{y}(k) - h(\mathcal{L}\otimes I_{N\times N} + A_d)\bar{\mathbf{y}}(k) - \mathbf{1}_N \otimes \boldsymbol{\xi}(k+1) \\
=& \mathbf{y}(k) - \mathbf{1}_N \otimes (\boldsymbol{\xi}(k) - t\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k))) \\
&- \beta(\mathcal{L}\otimes I_{N\times N} + A_d)\bar{\mathbf{y}}(k) \\
=& \bar{\mathbf{y}}(k) - \beta(\mathcal{L}\otimes I_{N\times N} + A_d)\bar{\mathbf{y}}(k) \\
&+ t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k))) \\
=& \mathcal{H}\bar{\mathbf{y}}(k) + t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k)))
\end{aligned}
$$

Let P be a symmetric positive-definite matrix, we have:

$$
\mathcal{H}P\mathcal{H} - P = -Q \tag{9}
$$

$$
\begin{aligned}
\mathcal{N}(\bar{\mathbf{y}}) =& \frac{1}{2}\bar{\mathbf{y}}(k+1)^T P \bar{\mathbf{y}}(k+1) \\
&- \frac{1}{2}\bar{\mathbf{y}}(k)^T P \bar{\mathbf{y}}(k) \\
=& \frac{1}{2}(\mathcal{H}\bar{\mathbf{y}}(k) + t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k))))^T P \\
&(\mathcal{H}\bar{\mathbf{y}}(k) + t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k)))) \\
&- \frac{1}{2}\bar{\mathbf{y}}(k)^T P \bar{\mathbf{y}}(k)
\end{aligned} \tag{10}
$$

Based on Eq. (9), we have

$$
\begin{aligned}
\mathcal{N}(\bar{\mathbf{y}}) =& -\frac{1}{2}\bar{\mathbf{y}}(k)^T Q \bar{\mathbf{y}}(k) + \frac{1}{2}\bar{\mathbf{y}}(k)^T \mathcal{H}^T P \\
&(t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k)))) \\
&+ \frac{1}{2}(t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k))))^T P \\
&(t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k)))) \\
\leq& -\frac{1}{2}\bar{\mathbf{y}}(k)^T Q \bar{\mathbf{y}}(k) + \frac{1}{2}\bar{\mathbf{y}}(k)^T \mathcal{H}^T P \\
&(t\mathbf{1}_N \otimes (\alpha\nabla F_{\xi}(\boldsymbol{\xi}(k), \mathbf{y}_{-i}(k)))) \\
&+ \frac{\alpha_{max}^2 t^2}{2}\|P\|\left(l_{max}\|\bar{\mathbf{y}}(k)\| + \sqrt{N}l_{max}\|\boldsymbol{\xi}(k) - \xi^*\|\right)^2 \\
\leq& -\frac{1}{2}\lambda_{min}(Q)\|\bar{\mathbf{y}}(k)\|^2 + t\alpha_{max}\|\mathcal{H}P\|\sqrt{N}l_{max}\|\bar{\mathbf{y}}(k)\|^2 \\
&+ t\alpha_{max}\|\mathcal{H}P\|\|\bar{\mathbf{y}}(k)\|Nl_{max}\|\boldsymbol{\xi}(k) - \xi^*\| \\
&+ \frac{\alpha_{max}^2 t^2}{2}\|P\|\left(\sqrt{N}l_{max}\|\bar{\mathbf{y}}(k)\| + Nl_{max}\|\boldsymbol{\xi}(k) - \xi^*\|\right)^2
\end{aligned}
$$

Then we can combine $\mathcal{M}(\xi_i)$ and $\mathcal{N}(\bar{\mathbf{y}})$ together, and abstract three terms $\|\bar{\mathbf{y}}(k)\|^2$, $\|\bar{\mathbf{y}}(k)\|\|\boldsymbol{\xi}(k) - \xi^*\|$ and $\|\boldsymbol{\xi}(k) - \xi^*\|^2$:

$$
\begin{aligned}
&V(k+1) - V(k) \\
\leq& \left(\frac{\alpha_{max}^2 t^2}{2}\|P\|Nl_{max}^2 + \frac{\alpha_{max}t^2}{4}l_{max}^2\right. \\
&\left. - \frac{1}{2}\lambda_{min}(Q) + t\alpha_{max}\|\mathcal{H}P\|\sqrt{N}l_{max}\right)\|\bar{\mathbf{y}}(k)\|^2 \\
&+ (\frac{\alpha_{max}^2 t^2}{2}\|P\|N^2 l_{max}^2 - \frac{Lt}{2} + \frac{\alpha_{max}t^2}{4}Nl_{max}^2)\|\boldsymbol{\xi}(k) - \xi^*\|^2 \\
&+ \left(\frac{l_{max}t}{2} + \frac{\alpha_{max}t^2}{2}\sqrt{N}l_{max}^2 + t\alpha_{max}\|\mathcal{H}P\|Nl_{max}\right. \\
&\left. + t^2\alpha_{max}^2\|P\|N^{\frac{3}{2}}l_{max}^2\right)\|\bar{\mathbf{y}}(k)\|\|\boldsymbol{\xi}(k) - \xi^*\|
\end{aligned} \tag{11}
$$

For any positive constant d, we have

$$
\|\bar{\mathbf{y}}(k)\|\|\boldsymbol{\xi}(k) - \xi^*\| \leq \frac{1}{2}\left(\frac{\|\bar{\mathbf{y}}(k)\|^2}{d} + d\|\boldsymbol{\xi}(k) - \xi^*\|^2\right) \tag{12}
$$

Let:

$$
\begin{aligned}
\sigma_1 =& \frac{\lambda_{min}(Q)}{2} - \frac{\alpha_{max}^2}{2}\|P\|Nl_{max}^2 - \frac{\alpha_{max}}{4}l_{max}^2 \\
&- \alpha_{max}\|\mathcal{H}P\|\sqrt{N}l_{max} - \frac{1}{2d}\left(\frac{l_{max}}{2}\right. \\
&+ \frac{\alpha_{max}}{2}\sqrt{N}l_{max}^2 + \alpha_{max}\|\mathcal{H}P\|Nl_{max} \\
&\left. + \alpha_{max}^2\|P\|N^{\frac{3}{2}}l_{max}^2\right)
\end{aligned} \tag{13}
$$

$$
\begin{aligned}
\sigma_2 =& \frac{Lt}{2} - \frac{\alpha_{max}^2 t^2}{2}\|P\|N^2 l_{max}^2 - \frac{\alpha_{max}t^2}{4}Nl_{max}^2 \\
&- \frac{d}{2}\left(\frac{l_{max}t}{2} + \frac{\alpha_{max}t^2}{2}\sqrt{N}l_{max}^2\right. \\
&\left. + t\alpha_{max}\|\mathcal{H}P\|Nl_{max} + t^2\alpha_{max}^2\|P\|N^{\frac{3}{2}}l_{max}^2\right)
\end{aligned} \tag{14}
$$

To make the $V_i(k+1) - V_i(k) \leq 0$, we must have $\sigma_2 > 0, \sigma_1 > 0$, and therefore we can get:

$$
\begin{aligned}
&2L - 2\alpha_{max}^2 t\|P\|N^2 l_{max}^2 - \alpha_{max}tNl_{max}^2 \\
&> d\left(l_{max} + \alpha_{max}t\sqrt{N}l_{max}^2\right. \\
&\left. + 2\alpha_{max}\|\mathcal{H}P\|Nl_{max} + 2t\alpha_{max}^2\|P\|N^{\frac{3}{2}}l_{max}^2\right)
\end{aligned} \tag{15}
$$

Here, t is a constant positive step size, so we can always find a small $t \to 0$ to achieve the following:

$$
d < \frac{2L}{2\|\mathcal{H}P\|Nl_{max}\alpha_{max} + l_{max}} \tag{16}
$$

With Eq. (16), we can obtain $\sigma_2 > 0, \sigma_1 > 0$. Then, substituting Eqs. (13)–(14) to Eq. (11), we have:

$$
V_i(k+1) - V_i(k) \leq -\sigma_1\|\bar{\mathbf{y}}(k)\|^2 - \sigma_2\|\boldsymbol{\xi}(k) - \xi^*\|^2 \quad \square \tag{17}
$$

4.2.2. Heterogeneous linear system tracking

Drawing from the outcome of the Nash equilibrium as detailed in Theorem 1, our subsequent objective is to guide heterogeneous dynamic systems towards following the Nash equilibrium point. It is important to note that current leading research in NES primarily focuses on the initial task. In our study, we expand upon this by developing methods for seeking and maintaining Nash equilibrium in heterogeneous systems, as described by the dynamics in Eqs. (3b) and (3c). To accomplish this goal, we employ output regulation methods [24]. First, we apply a state transformation to (4).

We define the error dynamics as

$$
\bar{x}_i(k) = x_i(k) - x_{i,s}(k) \tag{18}
$$

$$
\bar{u}_i(k) = u_i(k) - u_{i,s}(k) \tag{19}
$$

which lead to

$$
x_{i,s}(k) = \Psi_i\xi_i(k) \tag{20}
$$

$$
u_{i,s}(k) = G_i\xi_i(k) \tag{21}
$$

Invoking the control design in (4) leads to the closed-loop error dynamics

$$
\begin{aligned}
\bar{x}_i(k+1) &= (A_i - B_iK_i)\bar{x}_i(k) - \Psi_i\Delta_i(k) \\
e_i(k) &= C_i\bar{x}_i(k).
\end{aligned} \tag{22}
$$

where $\Delta_i(k) = \xi_i(k) - t\alpha_i\nabla F_{\xi_i}(\xi_i(k), \mathbf{y}_{-i}(k)) - \xi_i(k)$.

Now we are ready to present the overall convergence result of the proposed algorithm in (4).

Theorem 2. *Let Assumptions 1–4 hold. If K_i is chosen such that $A_i - B_iK_i$ is Schur stable and the G_i and Ψ_i are designed by solving the regulation equations in (5), then the proposed algorithm in (4) solves the Nash equilibrium problem in (3).*

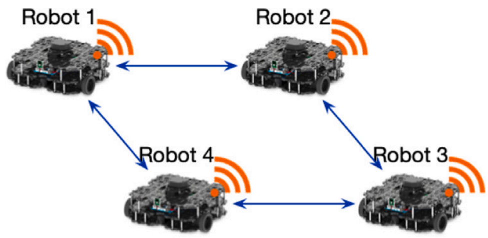

Fig. 1. Communication graphs.

Proof. We first demonstrate that, for any bounded $\Delta_i(k)$, we have

$$\limsup_{k\to\infty}\|e_i(k)\| \leq \alpha \limsup_{k\to\infty}\|\Delta_i(k)\|. \tag{23}$$

Encapsulating (5) into a matrix form leads to

$$\begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix}\begin{bmatrix} \Psi_i \\ G_i \end{bmatrix} = \begin{bmatrix} 0 \\ I \end{bmatrix}. \tag{24}$$

Denoting $\mathcal{O}_i = \begin{bmatrix} A_i - I & B_i \\ C_i & 0 \end{bmatrix}$ and $T_i = \begin{bmatrix} \Psi_i \\ G_i \end{bmatrix}$, by leveraging the property of Kronecker product, $\text{vec}\left(\mathcal{O}_i T_i I\right) = \left(I \otimes \mathcal{O}_i\right)\text{vec}(T_i)$, we can obtain a standard linear algebraic equation

$$\left(I \otimes \mathcal{O}_i\right)\text{vec}(T_i) = \text{vec}\left(\begin{bmatrix} 0 \\ I \end{bmatrix}\right) \tag{25}$$

of which the solvability is guaranteed under (6) in Assumption 4.

For notation convenience, we denote $A_{i,c} = A_i - B_i K_i$ and $B_{i,c} = -\Psi_i$. Then, we have

$$\bar{x}_i(k+1) = A_{i,c}\bar{x}_i(k) + B_{i,c}\Delta_i(k). \tag{26}$$

Recursively iterating (26) results in

$$\bar{x}_i(k) = A_{i,c}^k\bar{x}_i(0) + \sum_{j=0}^{k-1} A_{i,c}^{k-j-1}B_{i,c}\Delta_i(j). \tag{27}$$

With (5), we obtain

$$e_i(k) = C_i\bar{x}(k) = C_i A_{i,c}^k\bar{x}_i(0) - \sum_{j=0}^{k-1} A_{i,c}^{k-j-1}\Delta_i(j) \tag{28}$$

Because $A_{i,c}$ is Schur stable, we can get $\lim_{k\to\infty} C_i A_{i,c}^k\bar{x}_i(0) = 0$. In 4.2.1, the convergence of reference generator has been proved, which implies Δ_i always converges to zero as $k\to\infty$. Denote $\varpi_i := \limsup_{k\to\infty}\|\Delta_i(k)\|$. Therefore, for any small constant $\epsilon > 0$, there exists a positive time index $\zeta > 0$ such that

$$\|\Delta_i(k)\| < \varpi_i + \epsilon,\ \forall k > \zeta. \tag{29}$$

Based on the time index ζ, the second term in (28) can be separated into two parts, written as

$$\sum_{j=0}^{k-1} A_{i,c}^{k-j-1}\Delta_i(j) = \sum_{j=0}^{\zeta} A_{i,c}^{k-j-1}\Delta_i(j) + \sum_{j=\zeta+1}^{k-1} A_{i,c}^{k-j-1}\Delta_i(j). \tag{30}$$

Taking the Euclidean norm of (30) and invoking (29):

$$\begin{aligned} \left\|\sum_{j=0}^{k-1} A_{i,c}^{k-j-1}\Delta_i(j)\right\| &= \left\|\sum_{j=0}^{\zeta} A_{i,c}^{k-j-1}\Delta_i(j) + \sum_{j=\zeta+1}^{k-1} A_{i,c}^{k-j-1}\Delta_i(j)\right\| \\ &\leq \left\|A_{i,c}^{k-\zeta-1}\right\|\left\|\sum_{j=0}^{\zeta} A_{i,c}^{\zeta-j}\Delta_i(j)\right\| + (\varpi_i+\epsilon)\left\|\sum_{j=\zeta+1}^{k-1} A_{i,c}^{k-j-1}\right\|. \end{aligned} \tag{31}$$

Noticing that

$$\sum_{j=\zeta+1}^{t-1}\|A_{i,c}\|^{t-1-j} = \frac{1-\|A_{i,c}\|^{t-\zeta}}{1-\|A_{i,c}\|} < \frac{1}{1-\|A_{i,c}\|} \tag{32}$$

$$\lim_{k\to\infty}\left\|A_{i,c}^{k-\zeta-1}\right\| = 0. \tag{33}$$

Then, we have

$$\limsup_{k\to\infty}\|e_i(k)\| \leq \frac{1}{1-\|A_{i,c}\|}\left(\varpi_i+\varepsilon\right) \tag{34}$$

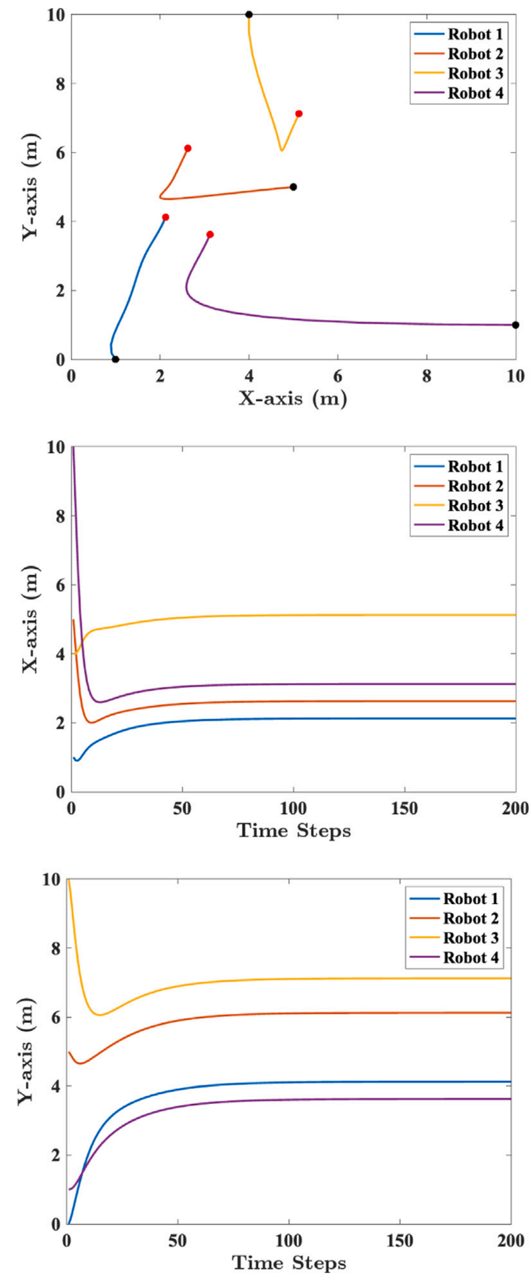

Fig. 2. Numerical simulation results of effectiveness.

As ϵ can be set arbitrarily small, it follows from (34) that

$$\limsup_{k\to\infty}\|e_i(k)\| \leq \alpha \limsup_{k\to\infty}\|\Delta_i(k)\|. \tag{35}$$

where $\alpha = \frac{1}{1-\|A_{i,c}\|}$.

Finally, let $\tilde{x}_i(k) = x_i(k) - \Psi_i y_i^*$. Then, we have

$$\begin{aligned} \tilde{x}_i(k+1) =& A_i x_i(k) + B_i[-K_i x_i(k) + (G_i + K_i\Psi_i)\xi_i(k)] - \Psi_i y_i^* \\ =& (A_i - B_i K_i)\tilde{x}_i(k) + B_i(G_i + K_i\Psi_i)(\xi_i(k) - y_i^*). \end{aligned} \tag{36}$$

It follows from Theorem 1 and (35) that $\xi_i(k)$ converges to $\xi_i^* = y_i^*$. Thus, we can conclude the convergence of the proposed algorithm (4) by treating $B_i(G_i + K_i\Psi_i)(\xi_i(k) - y^*)$ as $\Delta_i(k)$ in (35). □

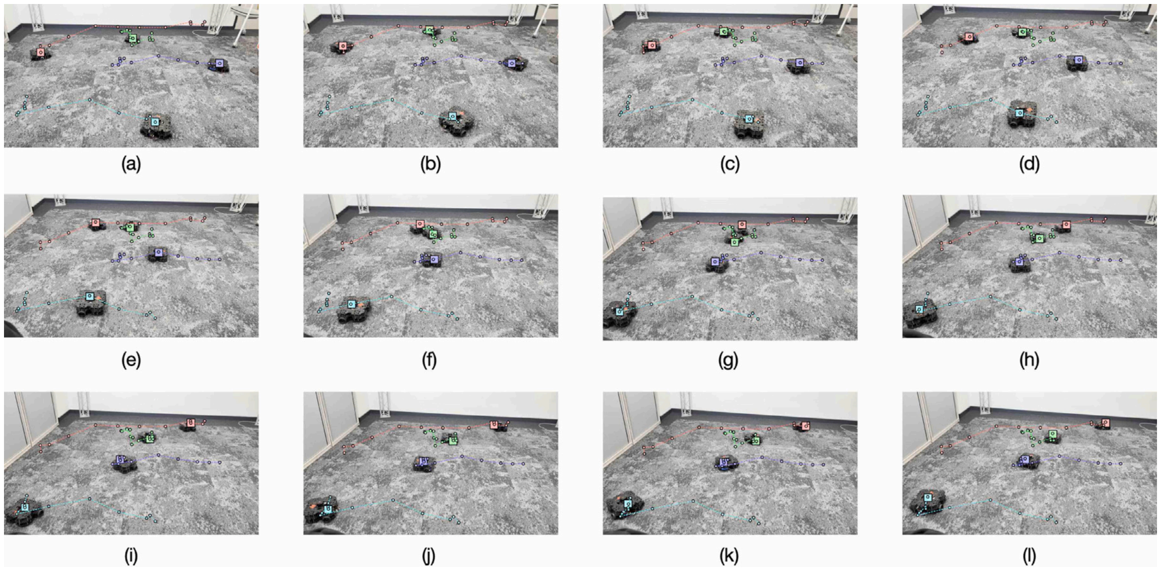

Fig. 3. Trajectory of real robots.

5. Case studies

In this section, the experiment focuses on evaluating the suggested algorithm in two distinct scenarios. Initially, we conduct numerical tests to confirm the algorithm's accuracy and to showcase its adaptability and scalability with various robotic models in simulated environments. Subsequently, we implement the algorithm in real multi-robot systems to determine its real-world viability. The proposed algorithm is validated and applied on both numerical study and real robots, where all source code, distributed algorithm details and environment setting files are publicly available at our project website.[2]

5.1. Numerical simulation

Here, we utilize numerical tests to verify and confirm the accuracy of the proposed algorithm thoroughly. In this case, we construct the experiment with four robots to demonstrate the effectiveness of the proposed algorithm. In this scenario, each robot is governed by its own linear dynamics: $A_{1,2} = [0, 1; 2, 1]$, $B_{1,2} = [1, 0; 0, 1]$, $C_{1,2} = [1, 0; 0, 1]$, $A_{3,4} = [0, -1; 1, -2]$, $B_{3,4} = [1, 0; 3, -1]$, $C_{3,4} = [-1, 0; 0, 1]$. The initial points are $x_0 = [1, 5, 4, 10], y_0 = [0, 5, 10, 1]$, while their targets are $r_x = [1, 2, 7, 3], r_y = [3, 7, 9, 2]$. $\alpha = 0.1, t = 0.1$. For the purpose of simplification, we assume that the robots possess rigid bodies during this experiment. Specifically, the communication topology is designed as depicted in Fig. 1 and the experimental results are depicted in Fig. 2.

Fig. 2 displays the process of achieving Nash equilibrium by the suggested algorithm for a group of four robots. The initial positions of the robots are marked in black on the top-left chart, and their paths are traced by various colored lines, with red markers denoting their final positions. The other two charts track the movements of x_i and y_i for the four robots, with the theoretical Nash equilibrium values indicated by dashed lines. These charts reveal that the values of x_i and y_i stabilize and accurately converge to the ideal Nash equilibrium points. The outcomes of this test confirm the efficiency of our algorithm in guiding the five robots to accurately estimate the desired mean values and successfully attain a state of Nash equilibrium. The results of this experiment illustrate that our algorithm is not only capable of facilitating precise estimations of target values among the robots but also ensures that these values are used strategically to guide the group towards a Nash equilibrium state. In achieving this state, each robot optimally balances its individual objectives with the collective goals of the group, demonstrating the algorithm's effectiveness in a dynamic multi-agent environment.

5.2. Real robots experiments

In this section, the proposed algorithm is implemented on physical robots, Turtlebot Waffle Pi [31], to evaluate its real-world performance. Four turtlebots are deployed and positioned within TACPSLab[3] at University of Liverpool. Each robot has a distinct local objective to represent a variety of goal situations. Throughout the experiment, communication among the robots is restricted, allowing them to only interact with their immediate neighbors. The trajectory taken by the robots are snapped and illustrated in Fig. 3.

In Fig. 3, sub-figures (a) to (l) showcase key instances of real robots in the process of seeking Nash equilibrium, with a complete video accessible at YouTube.[4] For clarity, upon reaching the target location (Nash equilibrium points), the robots will circle in place. These visuals demonstrate that, through the implementation of the proposed algorithm, each robot autonomously navigates towards its unique optimal Nash equilibrium positions. Through the implementation of our algorithm, each robot autonomously and efficiently navigates towards its uniquely determined optimal Nash equilibrium position. This autonomous navigation is underpinned by the algorithm's ability to dynamically adjust each robot's trajectory based on real-time data and interactions with the environment and other robots. Furthermore, we have documented the average cost of real robot experiments to demonstrate the convergence capabilities of our proposed algorithm, as illustrated in 4.

Fig. 4 depicts the cost functions among four real robots. While the algorithm effectively steers the multi-robot system towards its Nash equilibrium point, noticeable performance degradation is observed. This degradation is an inherent characteristic of the Nash equilibrium-seeking process, which is not optimized at every individual step but rather through continuous and iterative adjustments. Each robot adapts its strategy based on both its own state and the estimated states of others. Additionally, environmental variables such as slip rates, uneven terrain, and sensor noise can influence the robots' movements and

[2] Project website: https://github.com/YD-19/HMNES.

[3] https://cgi.csc.liv.ac.uk/~acps/home/.

[4] Demo Video is available at https://youtu.be/LXNBuZvCz2g.

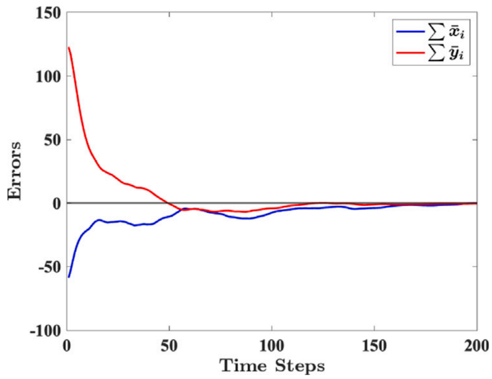

Fig. 4. Costs of real robots.

measurements, thereby introducing further uncertainty into the system. Despite these challenges, our algorithm adeptly guides each robot towards the Nash equilibrium, facilitating a process where each robot strategically selects actions to optimize its utility while maintaining a balanced state with its peers.

6. Conclusion

This paper proposed a distributed control methodology for Nash equilibrium seeking in multi-robot systems, despite limited visibility among players, which is both effective and practical. By integrating gradient play with a consensus protocol and employing advanced optimization techniques, the approach ensures reliable Nash equilibrium tracking in heterogeneous linear dynamic systems. Theoretical and empirical validations confirm its applicability across various sectors, highlighting its potential for enhancing multi-robot cooperation in real-world scenarios.

CRediT authorship contribution statement

Yi Dong: Writing – review & editing, Writing – original draft, Visualization, Software, Resources, Methodology, Investigation, Data curation, Conceptualization. **Zhongguo Li:** Writing – review & editing, Writing – original draft, Validation, Investigation, Conceptualization. **Shoaib Ehsan:** Writing – review & editing, Funding acquisition. **Zhengtao Ding:** Writing – review & editing, Supervision. **Xiaowei Huang:** Writing – review & editing, Writing – original draft, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Xiaowei huang reports financial support was provided by Engineering and Physical Sciences Research Council. Shoaib Eshan reports financial support was provided by Engineering and Physical Sciences Research Council. If there are other authors, they declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Dr. Yi Dong is a Lecturer in AI at the University of Liverpool. He received his Ph.D. degrees in electrical and electronic engineering from the University of Manchester in 2021. He is also a visiting fellow of both University of Southampton and University of Manchester. He has published more than 20 papers in interdisciplinary fields of Trustworthy AI, Responsible AI, Decision Making/Control, and Distributed Optimization for Autonomous Systems (e.g. Robotics, and Energy Systems). Beyond publications, he has secured funding from Innovate U.K., The Alan Turing Institute, and The Royal Society as a principal/co-investigator.

Dr. Zhongguo Li is currently a Lecturer in Robotics, Control, Communication, and AI at the University of Manchester. He received the B.Eng. and Ph.D. degrees in electrical and electronic engineering from the University of Manchester, in 2017 and 2021, respectively. Before re-joining Manchester in September 2023, he worked as a Lecturer at University College London and a Research Associate at Loughborough University. His research interests include optimization and decision-making for advanced control, distributed algorithm development for game and learning in network connected multi-agent systems, and their applications in robotics and autonomous vehicles. In these research fields, he has authored or co-authored more than 30 papers in well-recognized journals and conferences, including IEEE Transactions on Automatic Control, Automatica, IEEE Transactions on Cybernetics, IEEE Transactions on Neural Networks and Learning Systems, etc.

Dr Shoaib Ehsan is an Associate Professor in the Agents, Interaction and Complexity (AIC) Research Group in the School of Electronics and Computer Science at the University of Southampton. He completed his Ph.D. in Computing and Electronic Systems at the University of Essex in 2012. Dr Ehsan obtained his B.Sc. Electrical Engineering degree from the University of Engineering and Technology, Taxila (Pakistan) in 2003. He is an experienced researcher having won major research accolades, including the British Machine Vision Association's Sullivan Doctoral Thesis Prize (2013), and recognition as a 'Researcher with Exceptional Promise' by the Royal Academy of Engineering (2014). He is also a Senior Member of the IEEE. Dr Ehsan has been involved in several EPSRC, ESRC and EU projects and has made significant contributions with his expertise in multiple fields, especially computer vision, embedded systems, robotics and artificial intelligence.

Prof. Zhengtao Ding received the B.Eng. degree from Tsinghua University, Beijing, China, in 1984, and the M.Sc. degree in systems and control and the Ph.D. degree in control systems from the University of Manchester Institute of Science and Technology, Manchester, UK, in 1986 and 1989, respectively. He is a Professor of Control Systems with the Department of Electrical and Electronic Engineering. He is the author of the book Nonlinear and Adaptive Control Systems (IET, 2013) and has published over 300 research papers. His research interests include nonlinear and adaptive control theory and their applications, more recently, network-based control, distributed optimization, and distributed machine learning, with applications to power systems and robotics. Prof. Ding has served as an Associate Editor for the IEEE Transactions On Automatic Control, IEEE Control Systems Letters, and several other journals He is a member of the IEEE Technical Committee on Nonlinear Systems and Control, IEEE Technical Committee on Intelligent Control, and IFAC Technical Committee on Adaptive and Learning Systems. He is a Fellow of The Alan Turing Institute, the UK national institute for data science and artificial intelligence.

Prof. Xiaowei Huang is an expert of AI algorithms and theories, especially on the algorithms to verify, falsify, improve, and explain the robustness and trustworthiness of deep learning, and formal methods and symbolic verification. He published 200+ papers, most of which appear in top venues such as NeurIPS, ICML, CVPR, ICCV, AAAI, IJCAI, ICSE, ASE, CAV, etc. He authored a book "Machine Learning Safety", which was published in 2023 by Springer. His works have been well received, with 6000+ citations and 34 H-index in his Google Scholar profile. Since 2018, XH chairs the AISafety workshop series, which is held twice per year with the top AI conferences AAAI and IJCAI.