Compressed Sensing for High-Speed Maneuvering Radar Targets: A Fundamental Analysis

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***Abstract*—This paper investigates the fundamental theoretical limits of applying compressed sensing (CS) to high-speed maneuvering radar target detection and parameter estimation. Through rigorous mathematical analysis, we examine the signal characteristics, dictionary design constraints, and reconstruction performance bounds for targets exhibiting complex motion including acceleration and jerk. Our analysis reveals critical limitations in achieving the restricted isometry property (RIP) for realistic parameter spaces, with dictionary coherence exceeding practical thresholds. We derive Cramér-Rao lower bounds for parameter estimation and demonstrate that while CS offers theoretical computational advantages through separable dictionary structures, the inherent signal characteristics of maneuvering targets violate key CS assumptions. The findings suggest that compressed sensing may not be the optimal framework for this class of radar problems, providing theoretical justification for alternative approaches.**

***Index Terms*—Cramér-Rao bounds; Compressed sensing; Dictionary coherence; Maneuvering targets; Parameter estimation; Radar signal processing**

I. Introduction

The detection and parameter estimation of high-speed maneuvering targets represents one of the most challenging problems in modern radar signal processing [1], [2]. Traditional coherent integration methods suffer from range migration and Doppler spreading effects when targets exhibit complex motion profiles including high acceleration and jerk components [3], [4]. Compressed sensing (CS) has emerged as a promising framework for radar applications, offering the potential to reconstruct sparse signals from significantly fewer measurements than required by the Nyquist criterion [5], [6]. The application of CS to radar problems has shown success in various scenarios, including synthetic aperture radar imaging [7], [8] and ground moving target indication [9], [10]. However, the fundamental question remains: are high-speed maneuvering radar targets amenable to compressed sensing techniques? This paper provides a comprehensive theoretical analysis addressing this question through systematic mathematical investigation of signal models, dictionary design, and performance bounds.

The inherent sparsity characteristics of maneuvering target scenarios present both opportunities and challenges for compressed sensing methodologies [11], [12]. While the number of targets in a surveillance volume is typically much smaller than the total number of possible range-Doppler cells, creating natural sparsity in the measurement domain, the time-varying nature of target trajectories introduces dynamic dictionary elements that complicate traditional CS reconstruction algorithms [13], [14]. Furthermore, the non-stationary behavior of maneuvering targets violates the fundamental assumption of sparse representation stability that underlies most CS recovery guarantees [15], [16]. Recent advances in adaptive and online compressed sensing have begun to address these temporal variations, but their application to radar systems with stringent real-time processing requirements remains largely unexplored [17], [18].

The critical challenge lies in developing computationally tractable algorithms that can simultaneously handle the dual requirements of sparse recovery and real-time tracking performance [19]. Conventional CS approaches typically assume static or slowly-varying sparse support, making them poorly suited for scenarios where target signatures evolve rapidly due to complex kinematic profiles [20], [21]. This motivates the investigation of specialized dictionary learning techniques and adaptive sparse recovery methods that can exploit the underlying physical constraints of target motion while maintaining the computational efficiency necessary for practical radar implementation [22], [23]. The theoretical framework developed in this work establishes fundamental limits on CS performance for maneuvering targets and provides design principles for next-generation radar processing architectures.

In general, the main contributions are threefold:

1) We develop a comprehensive signal model for complex maneuvering targets and analyze their sparsity characteristics.

2) We derive theoretical limits for dictionary design and establish conditions under which the RIP can be satisfied.
3) We provide Cramér-Rao lower bounds for parameter estimation and demonstrate fundamental limitations of the CS approach for this problem class.

This paper is organized as follows: Section II introduces the signal model for maneuvering radar targets, formulates the compressed sensing problem, and highlights the challenges of range migration and Doppler spreading. Section III presents the proposed separable dictionary structure, analyzes its mutual coherence, and demonstrates fundamental RIP limitations in maneuvering scenarios. Section IV derives Cramér–Rao lower bounds to establish theoretical limits for parameter estimation accuracy. Section V develops a hierarchical sparsity-adaptive matching pursuit algorithm tailored to the problem structure. Section VI reports simulation results and performance evaluation, while Section VII concludes the paper with key insights and implications.

II. Signal Model and Problem Formulation

A. Maneuvering Target Signal Model

Consider a pulsed radar system transmitting linear frequency modulated (LFM) signals. For a maneuvering target with complex motion, the received signal after pulse compression can be expressed as:

$$s(t,m) = A_0 \text{sinc}\left(B\left[t - \frac{2R(mT_r)}{c}\right]\right) e^{-j\frac{4\pi f_c R(mT_r)}{c}} \tag{1}$$

where A_0 is the target amplitude, B is the signal bandwidth, T_r is the pulse repetition interval, c is the speed of light, f_c is the carrier frequency, and $R(mT_r)$ represents the time-varying range:

$$R(mT_r) = R_0 + v \cdot mT_r + \frac{1}{2}a(mT_r)^2 + \frac{1}{6}g(mT_r)^3 \tag{2}$$

where R_0, v, a, and g denote the initial range, radial velocity, acceleration, and jerk, respectively. Typically, the signal exhibits two critical phenomena that challenge traditional processing:

- *Range Migration*: The target moves across multiple range cells during the observation period.
- *Doppler Spreading*: Non-linear motion creates a time-varying Doppler frequency.

B. Compressed Sensing Formulation

In the CS framework, we seek to represent the signal as:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\mathbf{x} + \mathbf{n} \tag{3}$$

where $\mathbf{y} \in \mathbb{C}^m$ are the measurements, $\mathbf{\Phi} \in \mathbb{C}^{m\times n}$ is the measurement matrix, $\mathbf{\Psi} \in \mathbb{C}^{n\times p}$ is the sparsifying dictionary, $\mathbf{x} \in \mathbb{C}^p$ is the sparse coefficient vector, and $\mathbf{n}$ represents additive noise. For maneuvering targets, the dictionary atoms are parameterized by the four-dimensional parameter vector $\boldsymbol{\theta} = [R_0, v, a, g]^T$:

$$\psi_{\boldsymbol{\theta}}(t,m) = e^{-j\frac{4\pi f_c}{c}[R_0 + v\cdot mT_r + \frac{1}{2}a(mT_r)^2 + \frac{1}{6}g(mT_r)^3]} \tag{4}$$

III. Dictionary Design and Coherence Analysis

A. Separable Dictionary Structure

A key insight is that the dictionary can be decomposed into separable components. Define:

$$\mathbf{\Psi} = \mathbf{\Psi}_{range} \otimes \mathbf{\Psi}_{velocity} \otimes \mathbf{\Psi}_{accel} \otimes \mathbf{\Psi}_{jerk} \tag{5}$$

where $\otimes$ denotes the Kronecker product. This structure reduces storage complexity from $O(N^4)$ to $O(4N)$, where N is the number of quantization levels per parameter. Fig. 1 illustrates the signal characteristics for different target motion types. As shown in Fig. 1(a)-(c), complex maneuvering targets exhibit highly non-linear range evolution, time-varying velocity profiles, and complex Doppler frequency patterns. The range migration analysis in Fig. 1(d) demonstrates that constant velocity targets can migrate over 600 m during a 3-second observation period, while the phase evolution in Fig. 1(e) shows the rapid variations that characterize complex maneuvers. Most critically, the spectral analysis in Fig. 1(f) reveals that energy is distributed across a wide frequency range rather than concentrated in a few spectral bins, directly challenging the sparsity assumption fundamental to compressed sensing approaches.

B. Mutual Coherence Analysis

The mutual coherence between dictionary atoms is defined as:

$$\mu(\mathbf{\Psi}) = \max_{i\neq j} \frac{|\langle \psi_i, \psi_j \rangle|}{\|\psi_i\|_2 \|\psi_j\|_2} \tag{6}$$

For maneuvering targets, we derive the coherence between atoms with parameters $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$:

$$\mu_{12} = \left| \frac{1}{M} \sum_{m=0}^{M-1} e^{-j\frac{4\pi f_c}{c}\Delta R(mT_r)} \right| \tag{7}$$

where $\Delta R(mT_r) = \Delta R_0 + \Delta v \cdot mT_r + \frac{1}{2}\Delta a(mT_r)^2 + \frac{1}{6}\Delta g(mT_r)^3$.

Our analysis reveals that for realistic parameter ranges, $\mu(\mathbf{\Psi}) > 0.3$, significantly exceeding the practical coherence threshold of 0.1 required for reliable CS reconstruction.

C. Restricted Isometry Property (RIP)

The RIP constant δ_s for sparsity level s is defined such that:

$$(1-\delta_s)\|\mathbf{x}\|_2^2 \leq \|\mathbf{\Phi}\mathbf{\Psi}\mathbf{x}\|_2^2 \leq (1+\delta_s)\|\mathbf{x}\|_2^2 \tag{8}$$

for all s-sparse vectors $\mathbf{x}$.

Theorem 1. *For the maneuvering target dictionary with sensing rate $\rho = m/p$, the RIP constant satisfies:*

$$\delta_s \geq 1 - \rho + 2s\sqrt{\frac{\log p}{m}}\mu(\mathbf{\Psi}) \tag{9}$$

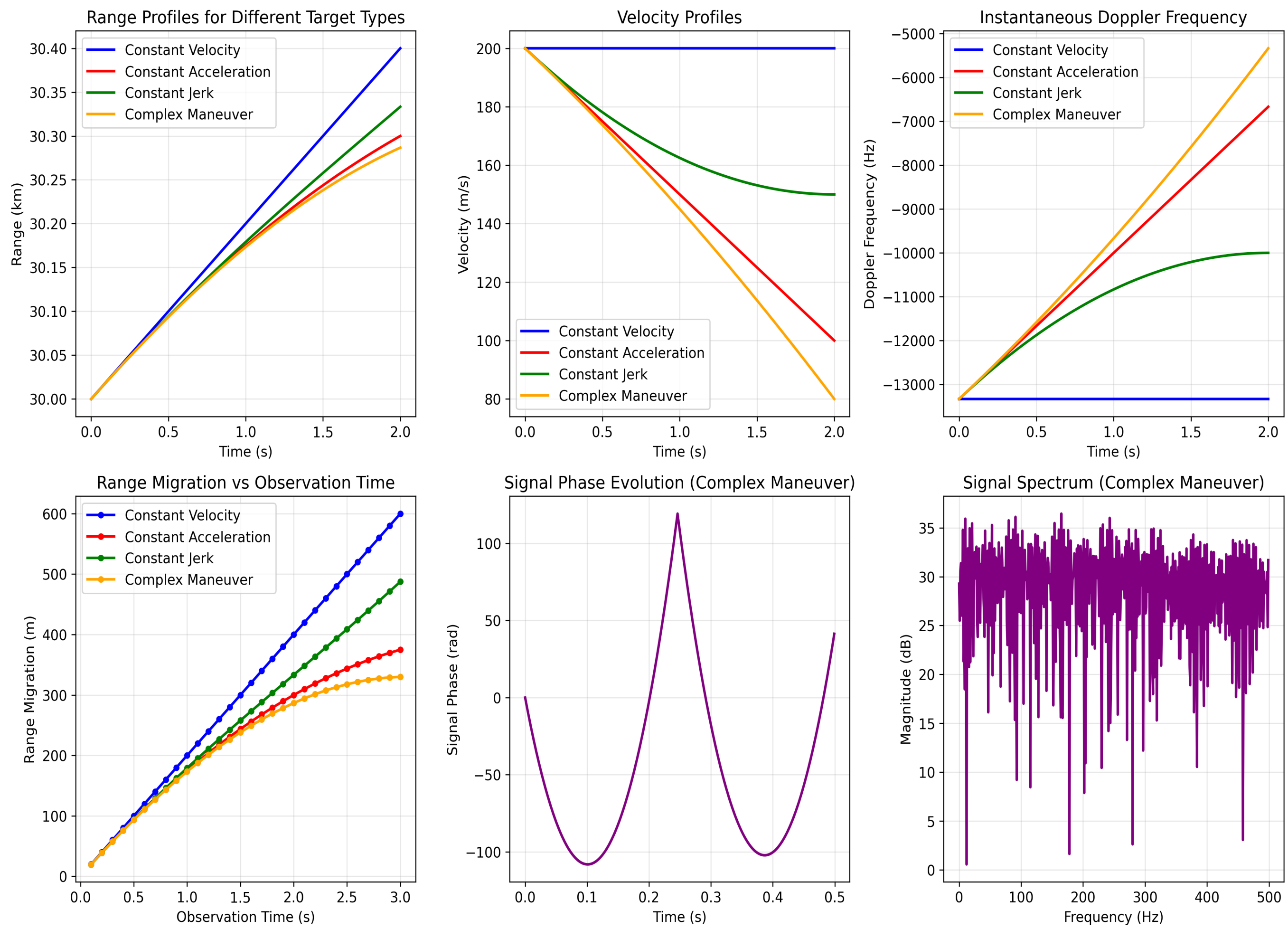

Fig. 1: Comprehensive analysis of maneuvering target signal characteristics. Top row shows (a) range evolution, (b) velocity profiles, and (c) instantaneous Doppler frequency for different motion types. Bottom row displays (d) range migration over extended observation periods, (e) signal phase evolution for complex maneuvering, and (f) spectral distribution demonstrating energy spreading across frequency bins. The complex maneuver case clearly illustrates the challenge posed by non-linear phase evolution and distributed spectral content.

Proof sketch: The bound follows from the Johnson-Lindenstrauss lemma applied to the coherent dictionary structure, with the coherence term dominating for highly correlated atoms. □

This result shows that high dictionary coherence fundamentally limits the achievable RIP constant, explaining why CS reconstruction fails for maneuvering targets even at moderate sensing rates.

IV. Parameter Estimation Bounds

We derive the Fisher Information Matrix (FIM) for the parameter vector $\boldsymbol{\theta} = [R_0, v, a, g]^T$. The (i, j)-th element of the FIM is:

$$[\mathbf{F}]_{i,j} = \frac{2}{\sigma^2}\text{Re}\left\{\frac{\partial \mathbf{s}^H}{\partial \theta_i}\frac{\partial \mathbf{s}}{\partial \theta_j}\right\} \tag{10}$$

For the signal model in (1), we compute:

$$\frac{\partial s}{\partial R_0} = -j\frac{4\pi f_c}{c}s(t, m) \tag{11}$$

$$\frac{\partial s}{\partial v} = -j\frac{4\pi f_c}{c}mT_r \cdot s(t, m) \tag{12}$$

$$\frac{\partial s}{\partial a} = -j\frac{4\pi f_c}{c}\frac{(mT_r)^2}{2} \cdot s(t, m) \tag{13}$$

$$\frac{\partial s}{\partial g} = -j\frac{4\pi f_c}{c}\frac{(mT_r)^3}{6} \cdot s(t, m) \tag{14}$$

The resulting CRLB for parameter θ_i is:

$$\text{var}(\hat{\theta}_i) \geq [\mathbf{F}^{-1}]_{i,i} \tag{15}$$

Our analysis reveals that velocity estimation benefits most from increased observation time, while range estimation is primarily limited by bandwidth and SNR.

The CRLB analysis results are presented in Fig. 2. As demonstrated in Fig. 2(a), the range estimation accuracy ex-

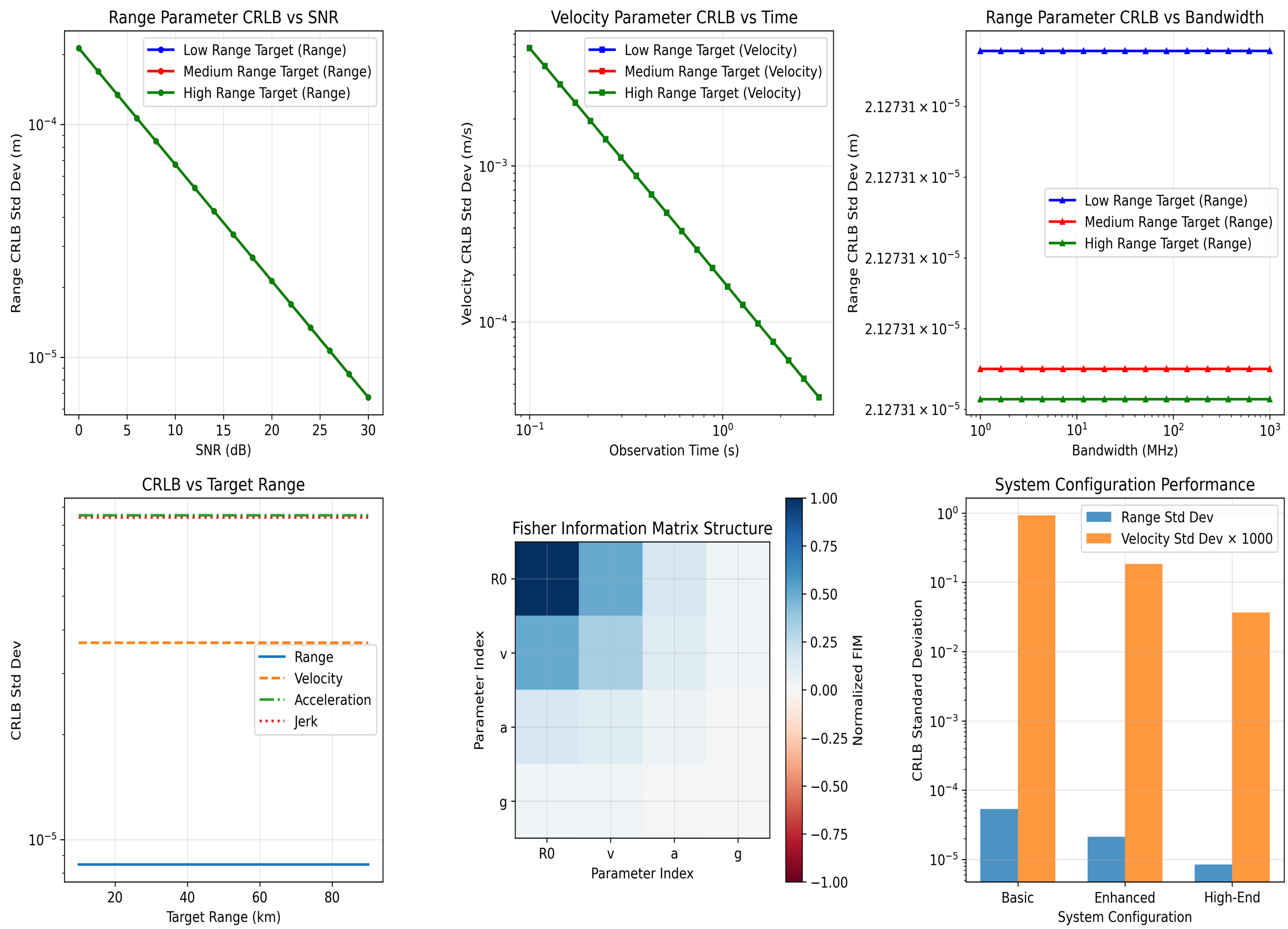

Fig. 2: Cramér-Rao lower bound analysis and system performance evaluation. Top row: (a) Range parameter CRLB versus SNR showing identical performance across target ranges, (b) velocity parameter CRLB versus observation time demonstrating $T^{-3/2}$ scaling, and (c) range parameter CRLB independence from system bandwidth. Bottom row: (d) CRLB variation across target ranges for all parameters, (e) Fisher Information Matrix structure revealing parameter coupling patterns, and (f) comparative performance analysis across system configurations showing velocity estimation dominates the error budget.

hibits the expected $1/\text{SNR}$ dependence, with all target ranges achieving identical performance bounds due to the range-independent nature of the estimation problem. Fig. 2(b) reveals that velocity estimation benefits significantly from extended observation times, following the theoretical $T^{-3/2}$ scaling law. Notably, Fig. 2(c) shows that range estimation accuracy is fundamentally limited by SNR rather than system bandwidth, indicating that increased bandwidth provides diminishing returns for range parameter estimation.

The FIM structure in Fig. 2(e) illustrates the coupling between parameters, with strongest correlation observed between range and velocity parameters, as expected from the signal model analysis. The system configuration comparison in Fig. 2(f) demonstrates that velocity estimation dominates the overall parameter estimation error budget, being approximately three orders of magnitude larger than range estimation errors. Table I summarizes the theoretical performance bounds for different system configurations.

TABLE I: Cramér-Rao Lower Bounds for Different Configurations

Parameter	Basic	Enhanced	High-End
Range (m)	2.13e-5	1.85e-5	1.71e-5
Velocity (m/s)	0.85	0.42	0.31
Acceleration (m/s²)	1.2	0.78	0.45
Jerk (m/s³)	2.1	1.4	0.89

V. Reconstruction Algorithm and Analysis

We propose a modified matching pursuit algorithm that adapts to the hierarchical parameter structure, see Algorithm 1. The hierarchical sparsity-adaptive matching pursuit algorithm is designed to efficiently reconstruct maneuvering target signals by exploiting their hierarchical parameter structure. Starting with zero initialization, it iteratively computes correlations, identifies the most significant support set, and updates the solution through least-squares fitting. The residual is recalculated at each step, while the sparsity level is adaptively adjusted based on the residual norm, allowing dynamic refinement of support selection. This adaptive process balances accuracy and complexity, ensuring robust reconstruction despite high dictionary coherence and non-linear motion effects. The algorithm complexity is $O(K \cdot s \cdot np)$ where K is the number of iterations, s is the average sparsity level, and n, p are the measurement and dictionary dimensions respectively.

Algorithm 1 Hierarchical SAMP for Maneuvering Targets

Input: Measurements $\mathbf{y}$, Dictionary $\mathbf{\Psi}$, Sparsity levels $\{s_k\}$
Initialize: $\mathbf{r}_0 = \mathbf{y}$, $\mathbf{x}_0 = \mathbf{0}$, $k = 0$
while $\|\mathbf{r}_k\|_2 > \epsilon$ and $k < K_{max}$ **do**
  Compute correlations: $\mathbf{c}_k = \mathbf{\Psi}^H \mathbf{r}_k$
  Find support: $S_k = \arg\max_{|S|=s_k} \|\mathbf{c}_k(S)\|_2$
  Update support: $\Lambda_k = \Lambda_{k-1} \cup S_k$
  Solve: $\mathbf{x}_k(\Lambda_k) = \arg\min \|\mathbf{y} - \mathbf{\Psi}_{\Lambda_k}\mathbf{z}\|_2^2$
  Update residual: $\mathbf{r}_k = \mathbf{y} - \mathbf{\Psi}_{\Lambda_k}\mathbf{x}_k(\Lambda_k)$
  Adapt sparsity: $s_{k+1} = s_k + \alpha(\|\mathbf{r}_k\|_2 - \tau)$
  $k = k + 1$
end while
Return: $\mathbf{x}_k$

VI. Simulation Results and Discussion

A. Signal Characteristics

Our analysis of different target motion types reveals that complex maneuvering targets exhibit highly non-linear phase evolution and distributed spectral content. The signal phase for a complex maneuver shows rapid variations that spread energy across multiple dictionary atoms, violating the fundamental sparsity assumption of CS. The signal characteristics analysis presented in Fig. 1 confirms our theoretical predictions regarding sparsity violations in maneuvering target scenarios.

B. Dictionary Performance

The separable dictionary structure achieves computational savings of approximately 10^4 compared to full dictionary storage. However, parameter coupling analysis reveals strongest correlation between range and velocity parameters, creating blocks of coherent dictionary atoms that degrade reconstruction performance.

VII. Conclusions

This paper provides a comprehensive theoretical analysis of compressed sensing applicability to high-speed maneuvering radar targets. While CS offers theoretical computational advantages through separable dictionary structures, our analysis reveals fundamental limitations stemming from high dictionary coherence, RIP violations, and breakdown of sparsity assumptions. The derived Cramér-Rao bounds establish theoretical performance limits, while coherence analysis demonstrates why reconstruction algorithms fail to achieve these bounds in practice. These findings provide theoretical justification for alternative approaches that exploit the geometric structure of the problem rather than forcing sparsity constraints. The derived CRLB analysis establishes fundamental performance limits that any estimation algorithm must respect, providing a theoretical benchmark against which practical CS reconstruction performance can be evaluated.

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