Technological Change, Labour Markets, and Economic Performance

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"Many of us who are economists originally started in other disciplines (I started in history, Robin in chemistry). And we fell in love with economics because we believed it offers a coherent worldview that offers real guidelines to making the world a better place. (Yes, most economist are idealists at heart.) But like any powerful tool, economics should be treated with great care. …Students would learn the appropriate use of the models-understand their assumptions and know their limitations as well as their positive uses. Why do we care about this? Because we don't live in a 'one model of the economy fits all' world."

Krugman, P. and Wells, R., "Economics," 2006.

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Summary

This thesis consists of three chapters that offer valuable insights into the roles of technological advancement, labour search, and taxation subsidy schemes in shaping macroeconomic dynamics and performance.

Chapter 1 examines a DSGE model that incorporates labor market frictions to explore the effects of automation-specific technology shocks on productivity and welfare among households. A key feature is the automation channel, which influences productivity, the labor market, and welfare across different types of households. The findings indicate that the shock increases labor market fluctuations and serves as a driving force for economic growth, benefiting the welfare of skilled households. However, it appears to adversely affect the welfare of unskilled households.

Chapter 2 presents a comprehensive Roy model that includes two distinct types of abilities—physical and cognitive—across three sectors: skilled, unskilled, and learning. The primary goal is to assess how technological advancements, such as skill-biased technology and shuffle shocks, impact occupational choice, employment, output, productivity, and skill premiums. The research reveals that both skill-biased technology and shuffle shocks significantly influence decision-making and various economic outcomes. Additionally, the interaction of these shocks explains the J-curve phenomenon in the skill premium observed in the United States, where the skill premium initially declines before rising substantially to a higher level.

Chapter 3 extends the Roy model by incorporating the economic redistribution tools, such as progressive income tax systems, targeted transfer payments, and learning subsidies aimed at promoting skill development. The study finds that the overall output tends to improve with higher tax rates, primarily because individuals often have limited access to capital markets. This limitation forces them to spend their entire income rather than save or invest. Tax and transfer programs help alleviate this issue by redistributing funds from higher-income individuals to lower-income households, thereby enhancing income equality and boosting overall economic activity.

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Chapter 1

Heterogeneous Households, Automation and Welfare

This study presents a dynamic stochastic general equilibrium (DSGE) model incorporating labour market frictions to examine the impact of automation-specific technology shocks on the productivity and welfare of heterogeneous households. In this model, firms are given the choice to operate endogenously between automation and non-automation production processes. The automation channel, a key feature of the model, can affect productivity, labour market, and welfare across different types of households. The results show that the shock increases the labour market fluctuations, the economic growth driving force, and the welfare of skilled households. However, it appears to have a detrimental effect on unskilled households' welfare.

1.1 Introduction

The rapid evolution of technology in recent decades has played a crucial role in driving economic growth in various countries, including the United States, South Korea, and China. This growth is closely tied to technological innovations that have improved productivity and efficiency across different industries. From the widespread adoption of the internet to the integration of artificial intelligence (AI) and machine learning, technological progress has significantly transformed the economic landscape, enabling countries to achieve unprecedented levels of growth. In recent years, one of the most notable trends in this technological revolution has been the increasing use of automation, which is expected to have a profound impact on the global economy.

The rapid advancement of automation, especially in industries with repetitive manual tasks, has transformed processes such as invoice processing. What was once a task requiring human input and verification can now be performed with greater accuracy and speed through automated systems. This not only enhances efficiency but also reduces the risk of human error, improving overall production quality. Firms that embrace automation often experience cost reductions, increased output, and enhanced competitiveness in the global market. However, while the benefits for firms are evident, the impact on the workforce is complex. Automation has shifted the demand for different types of labour, increasing the need for high-skilled workers while decreasing the demand for low-skilled workers whose tasks can be easily automated. This shift has extensive implications for individual workers, households, and the economy as a whole.

The impact of automation on workers of varying skill levels is a key issue in the ongoing discussion about the future of work. High-skilled workers are presented with new opportunities as automation creates a demand for their expertise in designing, implementing, and supervising automated processes. This often translates to higher wages, increased job security, and more prospects for career growth. Conversely, low-skilled workers are at risk of job displacement as automation takes over their roles. This displacement can lead to higher unemployment rates among low-skilled workers, stagnant wages, and a widening income disparity between high- and low-skilled workers.

The literature on the economic impacts of automation offers valuable insights into how automation affects different types of labour. It is observed that automation typically increases the productivity of skilled workers, making them more valuable to employers. Conversely, it may displace jobs for low-skilled workers, contributing to higher unemployment and wage disparities. Automation also significantly influences consumption and welfare among households. The displacement of low-skilled workers can result in reduced income and consumption, while high-skilled workers who benefit from automation may experience increased income and consumption. The overall impact of automation on welfare is multifaceted, with some households benefiting from improved efficiency and productivity, while others struggle with job displacement and income inequality.

The impact of automation extends beyond the labour market to consumption patterns and overall well-being across households. As incomes of high-skilled workers increase, their spending habits may shift towards a greater variety of goods and services, stimulating demand in specific sectors of the economy. Conversely, low-skilled workers experiencing job loss or reduced wages may have less disposable income, resulting in decreased consumption. These changes in spending patterns can have significant implications for economic growth, as shifts in demand for goods and services can shape the trajectory and speed of economic progress.

The relationship between automation and the business cycle is intricate and multifaceted. Automation has the potential to drive economic growth by enhancing productivity, reducing costs, and increasing output. However, its benefits tend to accrue disproportionately, potentially leaving some individuals behind. Understanding the impact of automation-specific technology shocks on the economy, including labour markets, consumption patterns, and the overall business cycle, is crucial. This study aims to explore how these shocks influence labour demand, income, consumption patterns, and broader implications for the business cycle.

1.2 Review of Related Literature

There are some studies that propose that the rapid advancement of technology can have adverse effects. For instance, replacing human labour with machines has decreased labour demand and lower wages (Autor et al., 2015; Acemoglu and Restrepo, 2018c; 2020). The diminishing labour share is a consequence of increasing automation, which is a firm's decision to adopt or not adopt. This, in turn, impacts job vacancies and the unemployment level (Shimer, 2005).

Numerous studies have examined the relationship between automation and labour. Recent research suggests that skilled labour and automation complement production, while unskilled labour and automation act as substitutes. Some studies have indicated that automation replaces unskilled labour, leading to a decrease in the labour force, the labour share, and the wage bargaining power of unskilled labour (Lee, 1999; Autor, 2019; Acemoglu and Restrepo, 2018a). For example, Acemoglu and Restrepo's 2018a study focuses on the competition between humans and machines, emphasizing the implications of technology for economic growth, factor shares, and employment. The study shows that automation could decrease labour share, labour demand, and equilibrium wages. However, increased productivity, labour demand, and labour share could result from creating new tasks in which labour has a comparative advantage (Acemoglu and Restrepo, 2018b).

In a recent study by Leduc and Liu (2020), the authors investigated the sluggish job recovery within a macro model considering search and recruiting intensity. The model incorporates fluctuations in the labour market due to various shocks, such as technology shocks, discount factor shocks, and job-separation shocks, under business cycle fluctuations. These shocks, particularly the technology shock, affect labour market dynamics, including fluctuations in employment, unemployment, vacancies, search intensity, and job-filling rate. The authors assumed exogenous job separation in their model's mechanism, using monthly and quarterly time series for the US labour market. However, the results do not align with real economic fluctuations influenced by endogenous job separation. This discrepancy is highlighted in studies by Hall and Milgrom (2008), Ramey (2008), and Acemoglu and Restrepo (2019). The study also raises questions about the relationship between productivity growth and the tightness of the labour market. Faster growth in businesses not only benefits firms but also increases the share of output for labour, potentially improving household welfare when firms effectively utilize technology to fuel productivity growth.

A discussion paper by Okada (2020) explores the dynamic interplay of education, automation, and economic growth. In this paper, a model is developed to capture the endogenous decision-making processes regarding education and automation. The introduction of automation enhances productivity, while government investment in education is also highlighted as a means to bolster productivity independent of automation. This suggests that firms have various avenues to consider during the production process, particularly given the limitations of human muscle power and the potential for automation to supplant manual tasks. Researchers such as Hémous and Olsen (2014), Atack et al. (2019), and Bergholt et al. (2019) have delved into the effects of automation on production processes. For instance, Atack et al. (2019) studied the automation of manufacturing in the late nineteenth century, focusing on the relationship between manual and machine labour in a task-based context and considering the endogenous effects of automation on production amid assumptions of heterogeneous workers and their complementarity or substitutability. The study reports significant disruptive effects of automation on the labour force and the division of labour, acting as both a job destroyer and creator (Zeira, 1998; Bessen, 2019; Acemoglu and Restrepo, 2020, and Acemoglu et al., 2020). Therefore, a comprehensive understanding of automation's growth is crucial, and models accommodating this dynamic production process are invaluable.

The fluctuation of the labour market caused by technology and automation, as in Blanchard and Diamond (1992), Autor et al. (2003), and Leduc and Liu (2024). In a recent analysis, Leduc and Liu (2024) highlighted the significant impact of automation on labour markets. They found that automation plays a crucial role in driving fluctuations in labour markets, particularly affecting search friction and real wage rigidity. Furthermore, their study emphasized the influence of automation on job creation incentives, as it can displace job vacancies and diminish the bargaining power of unskilled labour lacking the necessary skills to interact with automation (Gertler and Trigari, 2009; Blanchard and Gali, 2010). Consequently, heightened automation poses an increased threat, leading to more significant fluctuations in unemployment, job vacancies, and real wage rigidity.

This chapter aims to examine the effect of automation-specific technology shock on labour market, output market and welfare. To follow these objectives, the study is organised as follows: Section 1.3 illustrates a framework of this study . Section 1.4 constructs a model to study the relationship between automation and welfare. Section 1.5 shows the equilibrium of this model. Section 1.6 provides the values and meanings of the parameters. Section 1.10 shows the simulation results. Finally, Section 1.8 is the conclusion.

1.3 Framework

This study will construct a dynamic stochastic general equilibrium or DSGE model that generalises a standard Diamond-Mortensen-Pissarides model to study the theoretical relationship between the automation-specific shocks on economic growth and welfare. Figure 1.1 is illustrated a framework of a closed-economy model. There are three agents consisting of households, firms and government. Beginning with the households, they are assumed to be two types: skilled and unskilled. Both households supply their labours to firms to gain their income wages as compensation. However, they work in different production sectors. The skilled households work in automated production to get a skilled wage, while the unskilled households supply their labour in non-automated production to gain unskilled wages. Both households have to pay their income for consumption and taxation.

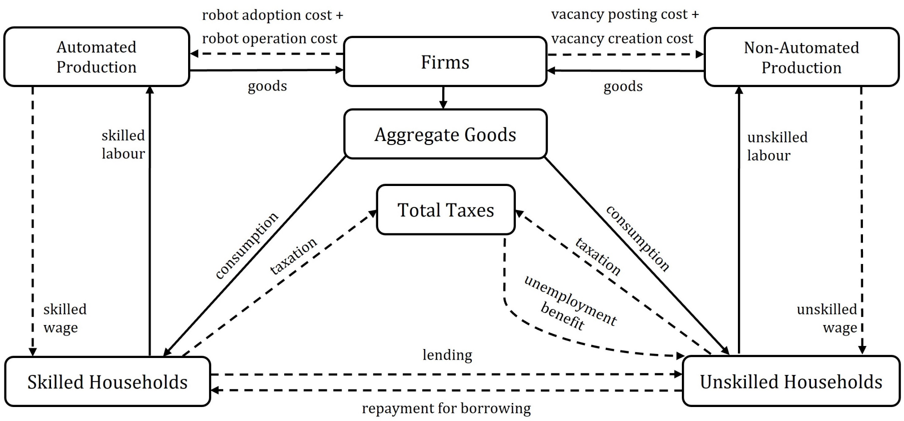

FIGURE 1.1: Study framework
Source: Author

In addition, skilled households are assumed to prefer saving and lending in order to consume more in the future. In contrast, unskilled households are assumed to consume more today by borrowing some money from skilled households. For the firms, they make their decision to operate between automated and non-automated production endogenously. If the benefit of adopted automation is more significant than the cost of automation adoption, firms will choose to operate automation production. Nevertheless, if the benefit of adopted automation is smaller than the cost of automation adoption, the non-automated production will be operated instead. Lastly, government levies income tax from both households and then transfers the unemployment benefit to unskilled unemployment.

1.4 The Model

The framework remodified from Sukkerd's dissertation (2021), based on which Leduc and Liu (2024) present the generalization of Fujita and Ramsey (2007) and Acemoglu and Restrepo (2018a) relatively the equilibrium formation of the firm's decision on automation, wages, and employment determination.

This model is assumed to be the discrete-time and infinite horizon. In the economy, three agents consist of households, firms, and the government. Households maximize their lifetime utility by optimally choosing between consumption and work. In this model, we assume that there are two types of households: skilled and unskilled households. Moreover, we assume that skilled households are more patient than unskilled ones. Therefore, skilled households prefer working and saving more today and then consuming more in the future, while unskilled households like consuming more today and then working more in the future to pay back their borrowing.

The firms try to maximize their benefits by endogenously choosing types of production sectors between automated and non-automated productions and then hiring the level of skilled and unskilled labourers. For the labour market, I assume that unskilled labours encounter market friction while skilled labours do not face friction. Because typically, unskilled labours are more unemployed, while skilled labours are in demand and are less unemployed. Therefore, the Nash bargaining wage is employed in the unskilled labour market while the wage adjustment mechanism is applied in the skilled labour market.

Search and matching are significant role in the unskilled labour market. We also assume that the matching function is the Cobb-Douglas function. The new job matches function m_t is a function of unemployed job seekers u_t and the job vacancies v_t where parameter η is the matching efficiency coefficient, and γ is the job matches elasticity with respect to unemployed job seekers. For the last agent, we assumed that the government try to balance the government budget over time. Lastly, all equations of optimality and definition must satisfy the equilibrium condition. Value functions and market friction satisfy the Bellman equations for households and firms. Search decisions satisfy the optimal search, wage functions satisfy the Nash Bargaining optimality, and the government budget constraint is satisfied. Finally, the distribution of vacancy creation cost and automation adoption cost is consistent with the agent behaviours.

1.4.1 Households

The households are assumed to be heterogeneous households consisting of skilled and unskilled households. The skilled households are assumed to be more patient, while the unskilled households are assumed to be less patient. Thus, skilled households prefer to lend their saving to unskilled households in order to gain the repayment for borrowing.

Skilled households - The skilled households consume consumption goods $C_{s,t}$ and supply their skilled labours $L_{s,t}$ in good producers. So that their utility is given by

$$E\sum_{t=0}^{\infty}\beta_s^t\{\ln C_{s,t}+\omega_s\ln(1-L_{s,t})\}$$

where E, β_s, and ω_s are an expectation operator, the skilled households' subjective discount factor, and skilled labour weight on utility, respectively. The skilled households are more patient than the unskilled one which implies that the skilled discount factor is larger than the unskilled one ($\beta_s > \beta_u$).

The households make their decision to choose the optimal level of consumption, labour, and lending B_t in order to maximise their lifetime welfare $V_{s,t}$. They gain the repayment for borrowing and the skilled wage from firms but their wage must be levied at tax rate τ_s^w. Thus, their maximisation problem is given as follow,

$$V_{s,t}(B_{s,t-1}) \equiv \max_{C_{s,t},L_{s,t},B_{s,t}} \ln C_{s,t}+\omega_s\ln\left(1-L_{s,t}\right)+\beta_s E_t V_{s,t+1}(B_{s,t}) \tag{1.1}$$

subject to their budget constraint,

$$C_{s,t}+B_{s,t}=R_{t-1}B_{s,t-1}+(1-\tau_s^w)W_{s,t}L_{s,t} \quad . \tag{1.2}$$

The optimising decision for consumption, labour and lending are characterised by,

$$\frac{1}{C_{s,t}}=\Lambda_{s,t} \tag{1.3}$$

$$\frac{\omega_s}{1-L_{s,t}}=(1-\tau_s^w)W_{s,t}\Lambda_{s,t} \tag{1.4}$$

$$\beta_s E_t\frac{\Lambda_{s,t+1}}{\Lambda_{s,t}}R_t=1 \tag{1.5}$$

where $\Lambda_{s,t}$ is the Lagrange multiplier which satisfying with their constraint.

The equation (3) states the Lagrangian multiplier of skilled households' budget utility of their consumption. The equation (4) implies that households optimally decide between the amount of consumption and the level of working hours. The equation (5) is well-known as the intertemporal optimal condition or the Euler equation. This condition guarantees that the households can smooth their consumption over time.

Unskilled households - The unskilled households consume the consumption goods $C_{u,t}$ and supply their unskilled labour $L_{u,t}$ in good producers for gaining their utility as follow,

$$E\sum_{t=0}^{\infty}\beta_u^t\{\ln C_{u,t}+\omega_u\ln(1-L_{u,t})\}$$

where β_u and ω_u are the unskilled households' subjective discount factor and the unskilled labour weight in utility, respectively. In this model, we assume the law of motion of unskilled employment as,

$$L_{u,t}=(1-\delta_e)L_{u,t-1}+p_t^u u_t \tag{1.6}$$

where $\delta_e\in(0,1)$ denotes the job separation rate and p_t^u is the job finding probability which is the ratio of new job matches m_t and the unemployed job seekers u_t and $p_t^u = m_t/u_t$. Similarly, the households will choose the optimal level of consumption, labour and borrowing B_t in order to achieve their highest welfare $V_{u,t}$. Thus, they have to repay their previous period debt with interest rate $R_{t-1}B_{u,t-1}$. Moreover, they are levied their wage income tax rate τ_u^w.

Additionally, we assume that they will gain the unemployment benefits at rate ξ. However, they face the borrowing limit as well. Therefore, the unskilled households' consumption problem is formed by,

$$V_{u,t}(L_{u,t-1},B_{u,t-1})\equiv\max_{C_{u,t},L_{u,t},B_{u,t}}\ln C_{u,t}+\omega_u\ln(1-L_{u,t})+\beta_u E_t V_{u,t+1}(L_{u,t},B_{u,t}) \tag{1.7}$$

subject to their budget and borrowing constraints,

$$C_{u,t}+R_{t-1}B_{u,t-1}=B_{u,t}+(1-\tau_u^w)W_{u,t}L_{u,t}+\xi(1-L_{u,t}) \tag{1.8}$$

$$(R_t-1+\kappa)B_{u,t}\leq\psi W_{u,t}L_{u,t} \tag{1.9}$$

where κ and ψ are the debt cost parameter and the exogenous payment-to-income (PTI) ratio, respectively.

The households' optimal decisions for consumption, labour supply, and borrowing are characterised by,

$$\frac{1}{C_{u,t}} = \Lambda_{u,t} \tag{1.10}$$

$$\begin{aligned} ES_t = {} & (1 - \tau_u^w + \psi\mu_{u,t})W_{u,t} - \xi - \frac{\omega_u}{\Lambda_{u,t}(1 - L_{u,t})} \\ & + (1 - \delta_e)\beta_u E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}}(1 - p_{t+1}^u)ES_{t+1} \end{aligned} \tag{1.11}$$

$$\beta_u E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} R_t + \mu_{u,t}(R_t - 1 + \kappa) = 1 \tag{1.12}$$

where $\Lambda_{u,t}, \mu_{u,t}$, and ES_t are the Lagrangian multiplier for their budget constraint, the Lagrangian multiplier for their borrowing constraint and the unskilled employment surplus which is defined by $ES_t = (1/\Lambda_{u,t})(\partial V_{u,t}/\partial L_{u,t})$.

1.4.2 Labour Forces

In the model, there are two types of labour supply: skilled and unskilled labour. Additionally, some unskilled labours are unemployed by firms because of the labour market frictions. However, we assume that the number of the entire labour supply is normalised to be one. Thus, those labours are measured in the unit of each type of labour share per total labours.

Skilled Labour - We assume that the skilled households do not face with the friction in job market. The skilled labour supply is determined by combining the equation (3) and (4) as follows,

$$\frac{\omega_s}{1 - L_{s,t}} = (1 - \tau_s^w)\frac{W_{s,t}}{C_{s,t}} \quad . \tag{1.13}$$

This equation represents the positive relationship between the skilled wages and skilled labour supply and the negative relationship between the skilled wages and skilled consumption.

Unskilled Labour - We assume that unskilled labours will encounter job market friction. Then, the unskilled labour will follow the search and matching mechanism in the labour market. Thus, some unskilled labours are getting jobs while others are still unemployed.

In the beginning period, we can assume that the number of skilled labours is given by $L_{s,t}$ while the amount of unskilled labours who match the job is determined by $L_{u,t-1}$. Thus, the unemployed job seekers are computed from the remaining fraction of the full labour force from skilled and unskilled employment.

$$u_t = 1 - L_{s,t} - (1-\delta_e)L_{u,t-1} \quad . \tag{1.14}$$

The law of motion for vacancies characterises the stock of vacancies v_t are the stream of the remaining previous-period unskilled vacancies that are non-automated, the number of separated employment matches from the last period and the new number of created vacancies ζ_t.

$$v_t = (1-p_{t-1}^v)(1-p_t^a)v_{t-1} + \delta_e L_{u,t-1} + \zeta_t \tag{1.15}$$

where p_t^v and p_t^a are the probability of job filling which is given by the ratio of new job matches m_t and job vacancies v_t, or $p_t^v = m_t/v_t$, and the probability of automation, respectively.

The matching function is determined by the Cobb-Douglas function as,

$$m_t = \eta u_t^\gamma v_t^{1-\gamma} \tag{1.16}$$

where η is the matching efficiency coefficient and $\gamma \in (0,1)$ states the job matches elasticity with respect to unemployment job seekers.

The number of unskilled employment is the combination of the remaining previous-period unskilled employment and the number of new job matches. Therefore, the law of motion of aggregate unskilled employment is given by,

$$L_{u,t} = (1-\delta_e)L_{u,t-1} + m_t \quad . \tag{1.17}$$

For unemployment, we can derive the amount of unemployment from the difference between the unemployed job seekers and the new job matches; in other words, the amount of unemployment is the remaining of the unemployed job seekers who have still unemployed or unmatched job positions. Thus, the amount of unemployment can be characterised by,

$$U_t = u_t - m_t = 1 - L_{s,t} - L_{u,t} \quad . \tag{1.18}$$

Additionally, the labour market tightness is defined as the ratio of job vacancies and available workers, $\theta_t = v_t/u_t$. We note that the tight unskilled labour market puts pressure on a higher unskilled wage.

1.4.3 Firms

The representative firms have two production processes: automation and non-automation productions. However, both production processes produce the same output or identical goods.

Automation Production - The production process with automation produce the consumption goods Y_t^a by using two input factors, automation A_t and skilled labour $L_{s,t}$. For simplicity, we assume the production function as linear form as follows.

$$Y_t^a = Z_t^a(\phi_t A_t)L_{s,t} \tag{1.19}$$

where Z_t^a is the automation total factor productivity (TFP) shock and ϕ_t is the automation-specific technology shock.

$$\ln Z_t^a = (1-\rho_{z^a})\ln \bar{Z}^a + \rho_{z^a}\ln Z_{t-1}^a + \epsilon_{Z^a,t} \quad . \tag{1.20}$$

The automation technology shock is assumed to be the AR(1) process:

$$\ln \phi_t = (1-\rho_\phi)\ln \bar{\phi} + \rho_\phi \ln \phi_{t-1} + \epsilon_{\phi,t} \tag{1.21}$$

where $\bar{\phi}$ is the mean value of automation-specific technology shock, the persistence of the technology shock is $\rho_\phi \in (-1,1)$ and the white noise term is given by $\epsilon_{\phi,t} \sim i.i.d.N(0,\sigma_\phi^2)$. Note that the skilled wage is determined by $W_{s,t} = Z_t^a(\phi_t A_t)$.

Non-automation Production - The production without automation only employ the unskilled labour $L_{u,t}$ as input factor to produce the consumption goods Y_t^e. Then, we also assume the linear production function as follows.

$$Y_t^e = Z_t^e L_{u,t} \tag{1.22}$$

where Z_t^e is the non-automation total factor productivity (TFP) shock.

$$\ln Z_t^e = (1-\rho_{z^e})\ln \bar{Z}^e + \rho_{z^e}\ln Z_{t-1}^e + \epsilon_{z^e,t} \quad . \tag{1.23}$$

1.4.4 Values of vacancy, employment and automation adoption in firms

Value of automation adoption - We assume the cost of automation adoption is fixed at a constant level a and the benefit of adopted automation is a threshold value a_t^*. The production with automation will be applied if the cost is not higher than the benefit.

$$a \leq a_t^* \quad . \tag{1.24}$$

The threshold value is the difference between the automation value and vacancy value,

$$a_t^* = S_t^a - S_t^v \quad . \tag{1.25}$$

We assume the automation cost a is generated from the independent and identically distributed (i.i.d.) distribution $F(a)$. Then, the threshold value a_t^* also is supported by the distribution. Thus, the probability of automation is derived from evaluating a threshold value through the cumulative density function of automation cost.

$$p_t^a = F(a_t^*) \quad . \tag{1.26}$$

The stock of automation consists of the remains of previous-period automation and the rest of previous-period vacancies filled by automation or newly automated positions. Hence, the law of motion of automated positions is given by

$$A_t = (1-\delta_a)A_{t-1} + p_t^a(1-p_{t-1}^v)v_{t-1} \tag{1.27}$$

where $\delta_a \in (0,1)$ is the rate of obsolete automation.

The value of automation is the stochastic discounted streams of difference between the marginal benefit of automation and the marginal cost of automation. Thus, we can characterise the evolution of automation values as the following Bellman's equation,

$$S_t^a = Z_t^a(\phi_t)L_{s,t} - \chi_a + (1-\delta_a)\beta_s E_t \frac{\Lambda_{s,t+1}}{\Lambda_{s,t}} S_{t+1}^a \quad . \tag{1.28}$$

Value of vacancy and employment - The production with labour will be used, and the firms will post the vacancy position to hire unskilled labour when the cost of adopted automation is higher than the benefit of it.

$$a > a_t^* \quad . \tag{1.29}$$

We assume that the entry cost e in units of consumption goods is generated from an i.i.d. distribution $G(e)$. The entry cost will occur if firms create a new vacancy. When the entry cost is non-negative, the firms will post the number of new vacancies ζ_t depending on the cumulative density of the entry cost evaluated at the value of vacancy S_t^v as follow,

$$\zeta_t = G(S_t^v) \quad . \tag{1.30}$$

The value of vacancy is the difference between the product of employment value with a probability of vacancy and the marginal cost of vacancy posts combined with the stochastic discounted streams of automation value and vacancy value in the future.

$$S_t^v = -\chi_v + p_t^v S_t^e + \beta_u (1 - p_t^v) E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \left[p_{t+1}^a S_{t+1}^a + (1 - p_{t+1}^a) S_{t+1}^v \right] \quad . \tag{1.31}$$

Employment value is the difference between the marginal benefit of unskilled labour and its marginal cost combining the stochastic discounted streams of the remaining employment value and the vacancy value in the future.

$$S_t^e = Z_t^e - W_{u,t} + \beta_u E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} \left[(1 - \delta_e) S_{t+1}^e + \delta_e S_{t+1}^v \right] \quad . \tag{1.32}$$

1.4.5 The Nash Bargaining Unskilled Wage

We apply the Nash bargaining optimality in this model to determine the unskilled wage. The wage optimality is the joint negotiation between the unskilled worker's surplus ES_t and the excess return on the firm hiring unskilled labour $S_t^e - S_t^v$. Hence, the bargaining wage optimality can be formed by

$$\max_{W_{u,t}} (ES_t)^{\iota} (S_t^e - S_t^v)^{1-\iota} \tag{1.33}$$

where $\iota \in (0, 1)$ is the weight of unskilled labour's bargaining power.

The total surplus consists of firm's surplus and worker's surplus. We define the total surplus as follows.

$$TS_t \equiv S_t^e - S_t^v + ES_t \quad . \tag{1.34}$$

Hence, the solution of optimal wage bargaining can be characterised by a part ι of the total surplus to be the solution of worker's surplus and a part $1 - \iota$ of the total surplus to be the solution of firm's surplus.

$$\begin{aligned} ES_t &= \iota TS_t, \\ S_t^e - S_t^v &= (1 - \iota) TS_t \quad . \end{aligned} \tag{1.35}$$

Then, we combine both solutions (35) with the optimal decision for unskilled labour supply, equation (11), to become the Bellman's equation,

$$\frac{\iota}{1-\iota}(S_t^e - S_t^v) = (1 + \psi\mu_{u,t} - \tau_u)W_{u,t}^n - \xi - \frac{\omega_u}{\Lambda_{u,t}(1-L_{u,t})} + (1-\delta_e)\beta_u \frac{\iota}{1-\iota} E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}}(1-p_{t+1}^u))(S_{t+1}^e - S_{t+1}^v) \quad (1.36)$$

where $W_{u,t}^n$ is the Nash bargaining wage and we can set $W_{u,t}^n = W_{u,t}$ because the real wage rigidities are assumed to be not existed in this model.

1.4.6 Government

The government's total revenue consists of the revenue from the income tax of households. The government levies the wage income tax on households. The skilled households' tax is defined as follows.

$$T_{s,t} = \tau_s^w W_{s,t} L_{s,t} \quad . \quad (1.37)$$

Similarly, the unskilled households' tax is given by,

$$T_{u,t} = \tau_u^w W_{u,t} L_{u,t} \quad . \quad (1.38)$$

The government budget constraint is assumed to balance every periods. Thus, the aggregate government's revenue is allocated to be the unemployment benefits:

$$\xi(1 - L_{s,t} - L_{u,t}) + G_t = T_t + D_t \quad . \quad (1.39)$$

The government purchase is assumed to be exogenous value,

$$G_t = \bar{G} \quad . \quad (1.40)$$

The government debt is also assumed to be exogenous value,

$$D_t = \bar{D} \quad . \quad (1.41)$$

1.4.7 Aggregations

The total consumption goods are the summation of the skilled households' consumption and the unskilled households' consumption.

$$C_t = C_{s,t} + C_{u,t} \quad . \tag{1.42}$$

Since the outputs are homogeneous in this model, the aggregate outputs are the combination of both-type productions.

$$Y_t = Y_t^a + Y_t^e \quad . \tag{1.43}$$

The total revenue of government consists of the revenue from income tax of both types of household.

$$T_t = T_{s,t} + T_{u,t} \quad . \tag{1.44}$$

1.4.8 Market Clearing

The loan market clearing occurs when we assume that the amount of lending equals the amount of borrowing.

$$B_{s,t} = B_{u,t} \quad . \tag{1.45}$$

The goods market clearing equates the aggregate expenditures and the aggregate outputs. The expenditures consist of the aggregate consumption, the costs of vacancy posting, the cost of automation operating, the costs of vacancy creation and the costs of automation adoption.

We can define the goods market-clearing condition as follows.

$$C_t + \chi_v v_t + \chi_a A_t + (1 - p_t^v) v_{t-1} \int_0^{a_t^*} a dF(a) + \int_0^{S_t^v} e dG(e) + G_t = Y_t \quad . \tag{1.46}$$

For both distributions, we characterise the distribution of vacancy creation cost and the distribution of automation adoption cost to be $G(e) = (e/\bar{e})^{\zeta_e}$ and $F(a) = (a/\bar{a})^{\zeta_a}$, respectively. Then, the goods market-clearing condition becomes,

$$C_t + \chi_v v_t + \chi_a A_t + \frac{\zeta_e}{1+\zeta_e} \zeta_t S_t^v + \frac{\zeta_a}{1+\zeta_a} p_t^a a_t^* (1 - p_{t-1}^v) v_{t-1} + G_t = Y_t \tag{1.47}$$

where $p_t^a = (a_t^*/\bar{a})^{\zeta_a}$ and $\zeta_t = (S_t^v/\bar{e})^{\zeta_e}$ are the automation adoption function and the vacancy creation function, respectively.

1.5 Equilibrium

A Dynamic Stochastic General Equilibrium (DSGE) model combined with a Diamond Mortensen Pissarides (DMP) framework, incorporating automation and heterogeneous households, consists of 40 variables. These variables include: [A_t, a^*_t, $B_{s,t}$, $B_{u,t}$, C_t, $C_{s,t}$, $C_{u,t}$, D_t, G_t, $\Lambda_{s,t}$, $\Lambda_{u,t}$, $L_{s,t}$, $L_{u,t}$, m_t, $\mu_{u,t}$, p^a_t, ϕ_t, p^u_t, p^v_t, R_t, S^a_t, S^e_t, S^v_t, T_t, θ_t, $T_{s,t}$, $T_{u,t}$, U_t, u_t, v_t, $V_{s,t}$, $V_{u,t}$, $W_{s,t}$, $W_{u,t}$, Y_t, $Y_{a,t}$, $Y_{e,t}$, Z^a_t, Z^e_t, ζ_t]. Consequently, this model requires a system of 40 equations (refer to the Table 1.1) to correspond with the number of variables.

In equilibrium, we can calculate the steady-state variables as follows. First, we set the unemployment rate U and the probability of job filling p^v to 0.0595 and 0.71, respectively, following the work of Leduc and Liu (2024). Then, we assume the skilled labor share L_s to be 0.5, corresponding to the average skilled employment share about 50 percent. The government debt, government spending, automation-specific technology shock, skilled labor productivity level, and unskilled labor productivity level are specified as follows: exogenous government debt $\bar{D}$, exogenous government spending $\bar{G}$, and the normalized level of automation-specific productivity $\bar{\phi}$.

Next, the interest rate is calculated from the skilled households' Euler equation: $R = 1/\beta_s$. The Lagrangian multiplier for the unskilled households' borrowing constraint is computed from the unskilled households' Euler equation: $\mu_u = (1-\beta_u R)/(R-1+\kappa)$. The unskilled labor share is determined by the remaining total labor share after accounting for the skilled labor share and unemployment: $L_u = 1-L_s-U$. The number of new job matches is characterized by $m = \delta_e L_u$. To calculate the unemployed job seekers, we use the equation $u = 1 - L_s - (1-\delta_e)L_u$. The probability of finding a job and the number of job vacancies are approximated by $p^u = m/u$ and $v = m/p^v$, respectively.

The value of automation is computed using $S^a = (Z_a \phi L_s - \chi_a)/(1-(1-\delta_a)\beta_s)$. The probability of automation is characterized by $p^a = ((\delta_e L_u - (1-(1-p^v))v)\bar{e} + S^a)/(\bar{a} + (1-p^v)v\bar{e})$. The automation positions are measured by $A = (p^a(1-p^v)v)/\delta_a$. The number of newly created vacancies is estimated by $\zeta = (1-(1-p^v)(1-p^a))v - \delta_e L_u$. The value of a vacancy is given by $S^v = \bar{e}\zeta$. The threshold value for adopting automation benefits is determined by $a^* = \bar{a}p^a$. The value of employment is computed using $S^e = (S^v + \chi_v - (1-p^v)\beta_s((1-p^a)S^v + p^a S^a))/p^v$.

The unskilled and skilled wages are calculated by $W_u = Z_e + \beta_u \delta_e S^v - (1-\beta_u(1-\delta_e))S_e$ and $W_s = Z^a \phi A$, respectively. The skilled consumption is derived from the skilled households' intratemporal optimal condition: $C_s = ((1-\tau^w_s)W_s(1-L_s))/\omega_s$. The Lagrangian multiplier for skilled households is $\lambda_s = 1/C_s$.

The outputs of automated and non-automated firms, as well as aggregate output, are approximated by $Y_a = Z_a(\phi A)L_s$, $Y_e = Z_e L_u$, and $Y = Y_a + Y_e$, respectively. The unskilled consumption is derived from the aggregate consumption and resource constraint as follows: $C_u = Y - C_s - \chi_v v - \chi_a A - \zeta_e/(1+\zeta_e)\zeta_a S^v - \zeta_a/(1+\zeta_a)p^a a^*(1-p^v)v - g$. The Lagrangian multiplier for unskilled households is $\lambda_u = 1/C_u$. The aggregate consumption is the sum of skilled and unskilled consumption: $C = C_s + C_u$.

The skilled and unskilled taxes, as well as aggregate taxation, are given by $T_s = \tau_s^w W_s L_s$, $T_u = \tau_u^w W_u L_u$, and $T = T_s + T_u$, respectively. The welfare of skilled households is given by: $V_s = (ln(C_s) + \omega_s ln(1-L_s))/(1-\beta_s)$ while the welfare of unskilled households is: $V_u = (ln(C_u)+\omega_u ln(1-L_u))/(1-\beta_u)$. The amount of borrowing by unskilled households is derived from their budget constraint as: $B_u = 1/(1-R)(C_u - (1-\tau_u^w)W_u L_u - \xi(1-L_u))$. The amount lent by skilled households equals the amount borrowed by unskilled households: $B_s = B_u$. Finally, labor market tightness is measured by $\theta = v/u$.

1.6 Parameterisation

The parameters used in this model have been gathered from several sources, including Hall and Milgrom (2008), Gertler and Trigari (2009), Blanchard and Gali (2010), Emenugu and Michelis (2019), Rubio and Yao (2020), Leduc and Liu (2024), as well as our own calibration outlined in the Table 1.2, which includes their definitions and values.

For the skilled household, the subjective discount factor β_s is set at 0.995, corresponding to an average quarterly interest rate of about 0.5 percent. The unskilled household's subjective discount factor β_u is set at 0.985, aligning with an average quarterly interest rate of approximately 1.5 percent, as indicated by Rubio and Yao (2020). The debt cost parameter κ is set to 0.024, and the payment-to-income (PTI) limit ψ is established at 0.28 based on findings from Emenugu and Michelis (2019). Following Leduc and Liu (2024), the scale for robot adoption costs $\bar{a}$ is 1.8593, while the scale for vacancy creation costs $\bar{e}$ is 8.3941. The marginal costs for vacancy posting χ_v and automation adoption χ_a are 0.30 and 0.435, respectively. The depreciation rate of automation δ_a is set at 0.03, which translates to an average annual rate of 12 percent. The steady-state quarterly job separation rate δ_e is 0.10, implying an average quarterly rate of 10 percent.

We calibrate a scale parameter that measures matching efficiency η to be 0.5496. The job match elasticity with respect to unemployed job seekers γ and the bargaining weight assigned to unskilled labour ι are both set to 0.50, as per Blanchard and Gali (2010) and Gertler and Trigari (2009). The utility weights for skilled labour ω_s and unskilled labour ω_u are set at 1 and 0.7292, respectively. For the automation shock, we establish the average level of the automation-specific shock $\bar{\phi}$ at 1, the persistence of this shock ρ_ϕ at 0.9, and the standard deviation of the automation-specific shock σ_ϕ at 0.001. The tax rates on the wages of both skilled labour τ_s^w and unskilled labour τ_u^w are set equally at 0.0125, equating to a 5 percent annualized tax rate on wage income. We calibrate the unemployment benefit payments ξ to be 1.1987. In terms of the shape of the distribution function, both the shape parameter of the automation cost distribution ζ_a and the shape parameter of the vacancy creation cost distribution ζ_e are set to 1, indicating a uniform distribution. Finally, the average productivity levels for skilled labour $\bar{Z}^a$ and unskilled labour $\bar{Z}^e$ are normalized to 1.

This model is simulated using the first-order perturbation method through the Dynare application, which is an additional path application of the Matlab program. The results of the impulse response are illustrated over quarterly periods.

1.7 Results

This study examines the impact of a positive automation-specific technology shock on the economy. The shock has both direct and indirect effects on the short-run and long-run economic outcomes. The effects can be explained using Figures 1.2 and 1.3, as detailed below.

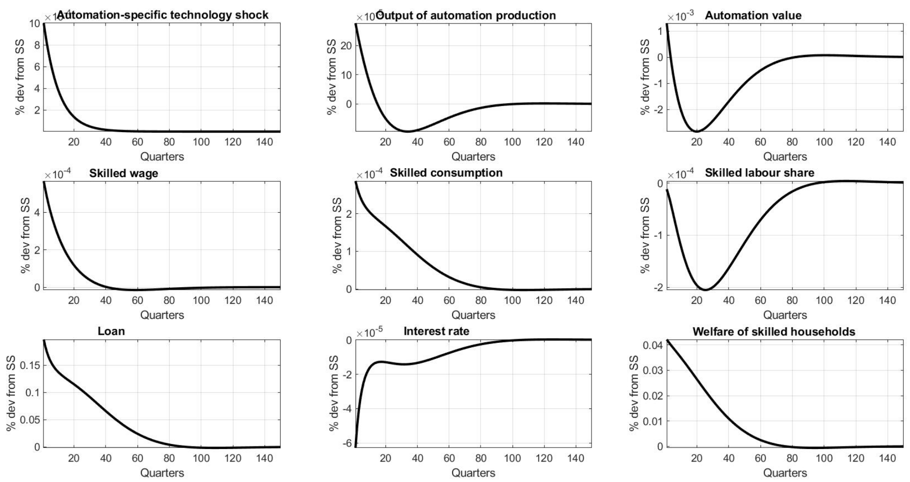

FIGURE 1.2: Impulse response to a one standard deviation of automation-specific technology shock
Source: Author's calculation

First of all, we focus on the Figure 1.2. In the short run, the positive automation-specific technology shock directly influences automated production by increasing output, driven by enhanced automation productivity and skilled workers' efficiency. This leads to an increase in automation value and skilled labour wages. As a result, wealthier skilled households tend to consume more while reducing their labour supply, ultimately increasing their welfare. Furthermore, they save their income to lend more, which significantly reduces the interest rate due to the excess supply of loans.

On the other hand, we now focus on Figure 1.3, the indirect effects of the shock lead to a reduction in employment value but an increase in the value of job vacancies and unemployed job seekers initially. The tightness of the labour market has reached a high level because the rise in job vacancies exceeds the increase in unemployed job seekers. Then, the value of employment starts to dominate due to higher demand for non-automated products. This leads to an increase in job vacancies, new job matches, and unskilled wages. Despite this, the income effect is outweighed by the substitution effect, leading unskilled households to consume less and work more. This initially causes a slight increase in unskilled labour share and non-automated production output, which gradually grows over time. Consequently, unskilled households experience a decrease in welfare due to decreased consumption and higher unskilled labour participation. Additionally, they prefer to borrow more due to the lower interest rates.

Looking at the long-run effects, the marginal product of automation and skilled workers' productivity decreases, leading to a decline in automated production output. This, in turn, leads to reductions in skilled labour wages and automation value. As a result, skilled households readjust by working more and consuming less to return to their previous welfare levels. Moreover, they supply fewer loans until reaching the initial steady-state lending, after which the interest rate begins to increase to its initial steady state.

Additionally, the values of employment, job vacancies, and unemployed job seekers also adjust and converge to their steady-state values in the long run. Then, The labor market tightness has declined and has now reached its steady-state level. This translates to a decline in job vacancies, new job matches, and unskilled wages. However, as the substitution effect becomes dominant, unskilled households shift towards consuming more and working less. Consequently, the share of unskilled labour and non-automated production output decrease and converge to their steady state. Ultimately, higher consumption and lower unskilled labour participation lead to a decrease in unskilled households' welfare, which also converges to a steady-state value in the long run. Furthermore, they reduce their loan to its initial steady-state level as the interest rate increasingly adjusts to its original steady state.

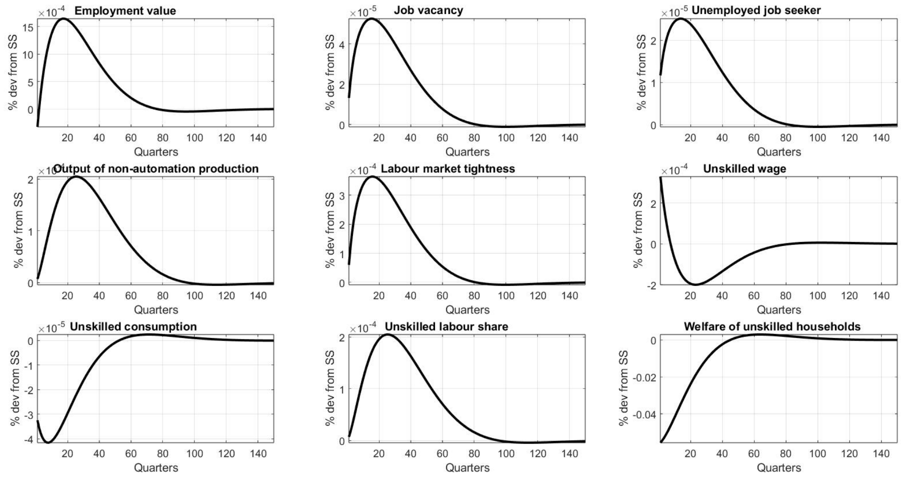

FIGURE 1.3: Impulse response to a one standard deviation of automation-specific technology shock (cont.)
Source: Author's calculation

1.8 Conclusions

The rapid evolution of technology in recent decades has significantly driven economic growth in countries like the United States, South Korea, and China. This growth is closely related to innovations that enhance productivity and efficiency across industries. Notically, the rise of automation, particularly in repetitive tasks, has transformed processes such as invoice processing. Automated systems perform tasks faster and with fewer errors, benefiting firms through cost reductions and increased output. However, the impact on the workforce is complex; automation raises demand for high-skilled workers while reducing opportunities for low-skilled workers, leading to job displacement and growing income inequality. High-skilled workers often find new opportunities and better wages, while low-skilled workers face risks of unemployment.

Research shows that automation increases the productivity of skilled workers but contributes to job loss for low-skilled labour. This affects consumption patterns, as higher incomes for skilled workers lead to increased spending, while low-skilled workers face reduced financial means, impacting overall economic growth. The relationship between automation and the business cycle is complex, as automation can drive growth through increased productivity but may also leave some individuals behind. Understanding how automation affects labour demand, income, and consumption is essential for grasping its broader implications on the economy.

This research creates a model to examine how automation-specific technological advances impact productivity, economic growth, and the well-being of different types of households. The findings indicate that a positive technology shock related to automation suddenly boosts the output of automated production, wages for skilled workers, consumption by skilled households, and overall welfare. However, the share of skilled labour in the workforce decreases in response to the shock. The response to the shock is influenced by the Nash bargaining wage optimality and friction in the unskilled labour market. This leads non-automated producers to anticipate higher future surpluses, resulting in increased wages for unskilled workers, a higher share of unskilled labour, and greater output for non-automated processes. However, the consumption and welfare of unskilled workers decline, underscoring the need for policies that bridge this gap.

Therefore, as we navigate this era of rapid technological advancement, it is imperative to understand how automation affects labor demand, income distribution, and consumption patterns, ensuring that growth benefits all layers of society, not just a select few. Only then can we truly harness the potential of automation to create a more equitable and prosperous future for everyone.

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1.10 Appendix

Systems of Equilibrium Equations, Variable Descriptions, and the Set of Parameters used in this Chapter 1.

1.10.1 Systems of Equilibrium Equations

A standard Diamond-Mortensen-Pissarides (DMP) and a Dynamic Stochastic General Equilibrium (DSGE) models with heterogeneous households consist of 40 equations as follows:

1. Marginal utility of skilled households' consumption

$$\Lambda_{s,t} = \frac{1}{C_{s,t}}$$

2. Marginal utility of unskilled households' consumption

$$\Lambda_{u,t} = \frac{1}{C_{u,t}}$$

3. Skilled households' intratemporal optimal condition

$$(1-\tau_s^w)\Lambda_{s,t}W_{s,t} = \frac{\omega_s}{1-L_{s,t}}$$

4. Unskilled households' intertemporal optimal condition

$$\beta_u E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}} R_t + \mu_{u,t}(R_t - 1 + \kappa) = 1$$

5. Skilled households' intertemporal optimal condition

$$\beta_s E_t \frac{\Lambda_{s,t+1}}{\Lambda_{s,t}} R_t = 1$$

6. Unskilled households' budget constraint

$$C_{u,t} + R_{t-1}B_{u,t-1} = B_{u,t} + (1-\tau_u^w)W_{u,t}L_{u,t} + \xi(1-L_{u,t})$$

7. Matching function

$$m_t = \eta \mu_t^{\gamma} v_t^{1-\gamma}$$

8. Job finding rate

$$p_t^u = \frac{m_t}{u_t}$$

9. Vacancy filling rate

$$p_t^v = \frac{m_t}{v_t}$$

10. Employment dynamics

$$L_{u,t} = (1 - \delta_e)L_{u,t-1} + m_t$$

11. Number of searching workers

$$u_t = 1 - L_{s,t} - (1 - \delta_e)L_{u,t-1}$$

12. Unemployment

$$U_t = 1 - L_{s,t} - L_{u,t}$$

13. Vacancy dynamics

$$v_t = (1 - p_{t-1}^v)(1 - p_t^a)v_{t-1} + \delta_e L_{u,t-1} + \zeta_t$$

14. Labour market tightness

$$\theta_t = \frac{v_t}{u_t}$$

15. Automation dynamics

$$A_t = (1 - \delta_a)A_{t-1} + p_t^a(1 - p_{t-1}^v)v_{t-1}$$

16. Employment value

$$S_t^e = Z_t^e - W_{u,t} + \beta_u E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}}[\delta_e S_{t+1}^v + (1 - \delta_e)S_{t+1}^e]$$

17. Vacancy value

$$S_t^v = p_t^v S_t^e - \chi_v + (1 - p_t^v)\beta_s E_t \frac{\Lambda_{s,t+1}}{\Lambda_{s,t}}[(1 - p_{t+1}^a)S_{t+1}^v + p_{t+1}^a S_{t+1}^a]$$

18. Automation value

$$S_t^a = Z^a(\phi_t)L_{s,t} - \chi_a + (1-\delta_a)\beta_s E_t \frac{\Lambda_{s,t+1}}{\Lambda_{s,t}} S_{t+1}^a$$

19. Automation threshold

$$a_t^* = S_t^a - S_t^v$$

20. Automation adoption

$$p_t^a = \left(\frac{a_t^*}{\bar{a}}\right)^{\zeta_a}$$

21. Vacancy creation

$$\zeta_t = \left(\frac{S_t^v}{\bar{e}}\right)^{\zeta_e}$$

22. Aggregation output

$$Y_t = Y_t^e + Y_t^a$$

23. Production function with automation

$$Y_t^a = Z_t^a(\phi_t A_t)L_{s,t}$$

24. Production function without automation

$$Y_t^e = Z_t^e L_{u,t}$$

25. Skilled wage

$$W_{s,t} = Z_t^a(\phi_t A_t)$$

26. Nash bargaining unskilled wage

$$\begin{aligned}\frac{\iota}{1-\iota}(S_t^e - S_t^v) =& (1+\psi\mu_{u,t} - \tau_u)W_{u,t} - \xi - \frac{\omega_u}{\Lambda_{u,t}(1-L_{u,t})} \\ &+ (1-\delta_e)\beta_u \frac{\iota}{1-\iota} E_t \frac{\Lambda_{u,t+1}}{\Lambda_{u,t}}(1-p_{t+1}^u)(S_{t+1}^e - S_{t+1}^v)\end{aligned}$$

27. Skilled households' income tax

$$T_{s,t} = \tau_s^w W_{s,t} L_{s,t}$$

28. Unskilled households' income tax

$$T_{u,t} = \tau_u^w W_{u,t} L_{u,t}$$

29. Government budget

$$\xi(1 - L_{s,t} - L_{u,t}) + G_t = T_t + D_t$$

30. Government purchase

$$G_t = \bar{G}$$

31. Government debt

$$D_t = \bar{D}$$

32. Aggregate consumption

$$C_t = C_{s,t} + C_{u,t}$$

33. Aggregate taxation

$$T_t = T_{s,t} + T_{u,t}$$

34. Saving borrowing market clearing

$$B_{s,t} = B_{u,t}$$

35. Resource constraint

$$C_t + \chi_v v_t + \chi_a A_t + \frac{\zeta_e}{1+\zeta_e}\zeta_t S_t^v + \frac{\zeta_a}{1+\zeta_a} p_t^a a_t^* (1 - p_{t-1}^v) v_{t-1} = Y_t$$

36. Unskilled households' welfare

$$V_{u,t} = \ln C_{u,t} + \omega_u \ln(1 - L_{u,t}) + E_t \beta_u V_{u,t+1}$$

37. Skilled households' welfare

$$V_{s,t} = \ln C_{s,t} + \omega_s \ln (1 - L_{s,t}) + E_t \beta_s V_{s,t+1}$$

38. Exogenous process: automation-specific technology shock

$$\ln \phi_t = (1 - \rho_\phi) \ln \bar{\phi} + \rho_\phi \ln \phi_{t-1} + \epsilon_{\phi,t}$$

39. Exogenous process: automation total factor productivity shock

$$\ln Z_t^a = (1 - \rho_{z^a}) \ln \bar{Z}^a + \rho_{z^a} \ln Z_{t-1}^a + \epsilon_{z^a,t}$$

40. Exogenous process: non-automation total factor productivity shock

$$\ln Z_t^e = (1 - \rho_{z^e}) \ln \bar{Z}^a + \rho_{z^e} \ln Z_{t-1}^e + \epsilon_{z^e,t}$$

1.10.2 Variable Descriptions

The model includes a total of 40 variables, which are detailed in the table below.

Table 1.1: Variable Descriptions

Variables	Description
A_t	Stock of automation or automation positions
a_t^*	Threshold value of adopted automation benefit
$B_{s,t}$	Amount of skilled households' lending
$B_{u,t}$	Amount of unskilled households' borrowing
C_t	Aggregate consumption
$C_{s,t}$	Skilled households' consumption
$C_{u,t}$	Unskilled households' consumption
D_t	Government debt
G_t	Government purchase
$\Lambda_{s,t}$	Lagrangian multiplier for skilled households
$\Lambda_{u,t}$	Lagrangian multiplier for unskilled households
$L_{s,t}$	Skilled labour share
$L_{u,t}$	Unskilled labour share
m_t	New job matches
$\mu_{u,t}$	Lagrangian multiplier for unskilled households' borrowing constraint
p_t^a	Probability of automation
ϕ_t	Automation-specific technology shock
p_t^u	Probability of job finding
p_t^v	Probability of job filling
R_t	Interest rate for lending/borrowing
S_t^a	Value of automation
S_t^e	Value of employment
S_t^v	Value of vacancy
T_t	Aggregate taxation
θ_t	Unskilled labour market tightness
$T_{s,t}$	Skilled households' income tax
$T_{u,t}$	Unskilled households' income tax
U_t	Unemployment share
u_t	Unemployed job seekers
v_t	Stock of vacancies or job vacancies
$V_{s,t}$	Skilled households' welfare
$V_{u,t}$	Unskilled households' welfare
$W_{s,t}$	Skilled wage
$W_{u,t}$	Unskilled wage
Y_t	Aggregate output
$Y_{a,t}$	Automated firms' output
$Y_{e,t}$	Non-automated firms' output

Variables	Description
Z_t^a	Automation total factor productivity shock
Z_t^e	Non-automation total factor productivity shock
ζ_t	Newly number of created vacancies

1.10.3 The Set of Parameters

The model includes 31 parameters, which are detailed in the following table. These parameters are derived from the work of Hall and Milgrom (2008), Gertler and Trigari (2009), Blanchard and Galí (2010), Emenugu and Michelis (2019), Rubio and Yao (2020), and Leduc and Liu (2024). Additionally, some parameters are obtained from the calibration based on the model's environment setup.

TABLE 1.2: The Set of Parameters

Variables	Description	Value
$\bar{a}$	Scale for robot adoption cost	1.8593
β_s	Skilled household's subjective discount factor	0.9950
β_u	Unskilled household's subjective discount factor	0.9850
χ_v	Vacancy posing cost	0.3000
χ_a	Automation adoption cost	0.4350
δ_a	The robots depreciate rate at an average annual rate	0.0300
δ_e	The job separation rate at the quarterly frequency	0.1000
$\bar{e}$	Scale for vacancy creation cost	8.3941
η	A scale parameter that measures matching efficiency	0.5496
$\bar{D}$	Exogenous government debt	0.2122
γ	The job matches elasticity w.r.t unemployed job seekers	0.5000
$\bar{G}$	Exogenous governments' spending	0.1500
ι	The weight of unskilled labours' bargaining power	0.5000
κ	Debt cost parameter	0.0240
ω_s	The skilled labour weight on utility	1.0000
ω_u	The unskilled labour weight on utility	0.7292
$\bar{\phi}$	Normalizing the level of automation-specific productivity	1.0000
ψ	Payment-to-income (PTI) limit	0.2800
ρ_ϕ	Persistence of automation-specific shock	0.9000
ρ_{z^a}	Persistence of skilled labour productivity shock	0.9000
ρ_{z^e}	Persistence of unskilled labour productivity shock	0.9000
σ_ϕ	Standard deviation of automation-specific shock	0.0010
σ_{z^a}	Standard deviation of skilled labour productivity shock	0.0010
σ_{z^e}	Standard deviation of unskilled labour productivity shock	0.0010
τ_s^w	Tax on skilled labour' wage	0.0125
τ_u^w	Tax on unskilled labour' wage	0.0125
ξ	Unemployment benefits	1.1987
ζ_a	The automation cost follows a uniform distribution	1.0000
ζ_e	The vacancy creation cost follows a uniform distribution	1.0000
$\bar{Z}^a$	The steady-state level of skilled labour productivity	1.0000
$\bar{Z}^e$	The steady-state level of unskilled labour productivity	1.0000

Chapter 2

Multidimensional Ability, Technological Advancement and Distributional Dynamics

The study presents a comprehensive Roy model that incorporates two distinct types of abilities- physical and cognitive- alongside three sectors: skilled, unskilled, and learning sectors. The primary objective is to assess the effects of technological advancements such as skill-biased technology and shuffle shocks on occupational choice, employment, output, productivity, and skill premium. The research highlights that skill-biased technology and shuffle shocks significantly affect decision-making, output, employment, productivity, and skill premium. Additionally, the interaction of both shocks explains the J-curve skill premium in the United States, where the skill premium initially declines before rising significantly to a higher level.

2.1 Introduction

During the late $20th$ and early $21th$ centuries, the United States experienced rapid economic growth driven by technological advancements. The late $20th$ century saw significant progress, especially in information technology, with the widespread use of personal computers, the internet, and telecommunications, which reshaped the economy. In the $21st$ century, there have been rapid advancements in digital technology, artificial intelligence, and automation, greatly influencing the output and labour markets as well as earnings distribution. In Figure 2.1, technological advancements have played a pivotal role in driving productivity growth. The emergence of the information technology sector brought about a further enhancement in economic output. The $1990s$ marked a transformative period in the U.S. economy, as the widespread proliferation of personal computers, the internet, and telecommunications led to a substantial increase in productivity. This digital revolution not only revolutionized the tech industry but also facilitated a remarkable transformation in other sectors, enabling them to boost productivity and output through improved communication and data management. The era from 1998 to 2005 witnessed another substantial surge, with productivity growing at average annual rate of 3.21 percent, largely attributable to the widespread integration and adoption of information technology across various industries. However, the global financial crisis between mid 2007 and early 2009 caused the productivity sharply increase to a rate of 5.75 percent in last quarter of 2009. Recently, the Covid-19 pandemic influenced the productivity reach to a rate of 6.61 percent in third quarter of 2020.

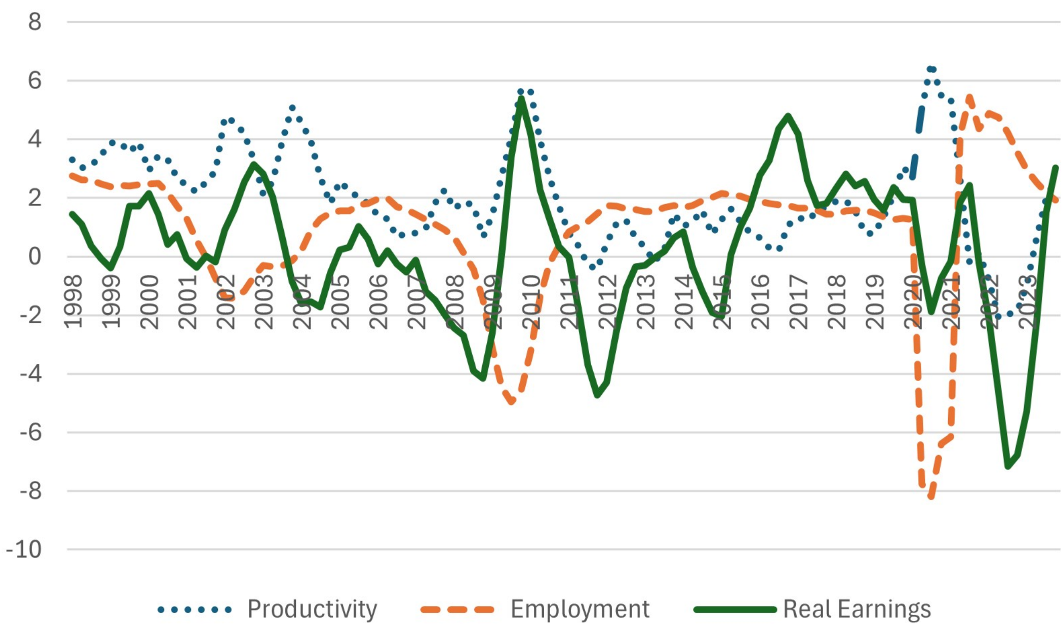

FIGURE 2.1: Change in Productivity, Employment and Earnings (Percentage)
Source: U.S. Bureau of Labour Statistics, Nonfarm Business Sector: Labour Productivity (Output per Hour) for All Workers [OPHNFB], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/OPHNFB, June 17, 2024., U.S. Bureau of Labour Statistics, All Employees, Total Nonfarm [PAYEMS], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/PAYEMS, June 17, 2024., U.S. Bureau of Labour Statistics, Average Hourly Earnings of Production and Nonsupervisory Employees, Manufacturing [CES3000000008], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CES3000000008, June 17, 2024., U.S. Bureau of Labour Statistics, Consumer Price Index for All Urban Consumers: Food at Home in U.S. City Average [CUUR0000SAF11], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CUUR0000SAF11, June 17, 2024.

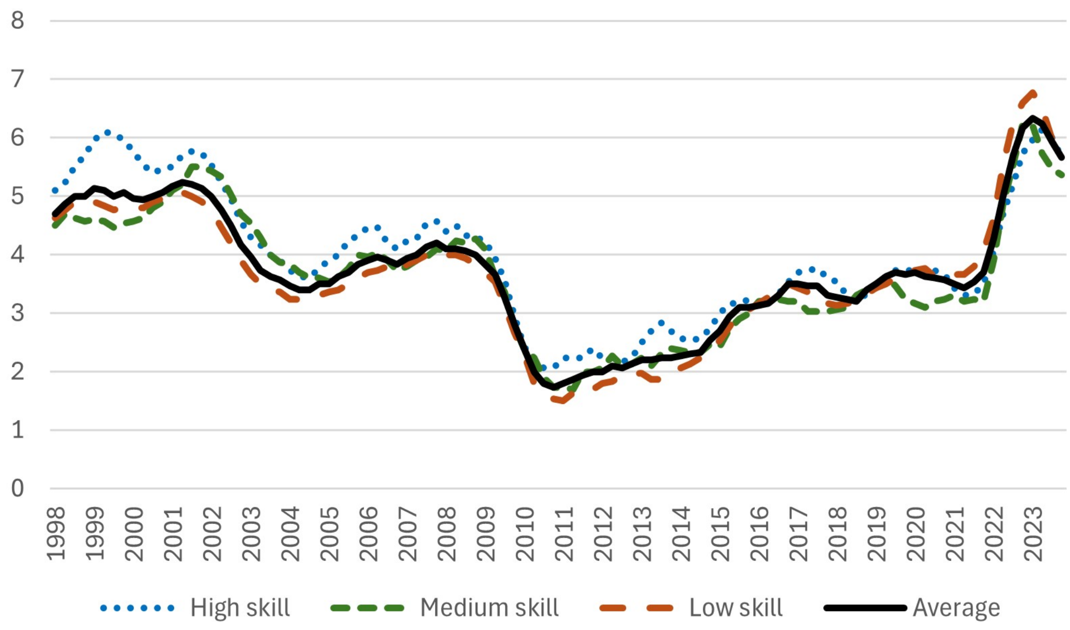

FIGURE 2.2: Wage growth by education (Percentage)
Source: U.S. Current Population Survey (CPS), Wage Growth Tracker by Education, retrieved from FRED, Federal Reserve Bank of Atlanta;
https://www.atlantafed.org/chcs/wage-growth-tracker, June 17, 2024.

The IT revolution of the late $20th$ century brought about a wave of new employment opportunities. Between 1998 and 2000, there was an significantly increase in the growth of employment more than an annual rate of 2.40 percent, reflecting the surge in demand for tech-related skills, particularly in the computer and data processing services sector. Nevertheless, this period also saw job displacement in traditional industries, thereby contributing to a more polarized labour market. However, the employment gradually decrease to a rate of -1.42 percent in first quarter of 2002 because of Dot-Com crisis in 2001. Moreover, the employment collapse to a rate of -4.96 percent in third quarter of 2009 since global financial crisis and to a rate of -8.19 percent in third quarter of 2020 due to Covid-19 pandemic.

The Figure 2.1 also shows the average real earnings grew increasingly to 2.16 percent in first quarter of 2000 and to 3.14 percent in a last quarter of 2002. The Dot-Com crisis in $2001 - 2002$ seem lightly affect the real earnings. However, a decreasingly change took place from the 2003 onwards to 2008. The real earnings was at -4.16 percent in a last quarter of 2008 because of global financial crisis. Recently, the Covid-19 pandemic decrease the real earnings to -1.87 percent in third quarter of 2020. In the period from the 1998 to the 2023, there was a fluctuate growth in hourly real earnings, but the average rate of real earnings was still positive. This noticeable trend can be partly attributed to the increasing value of education and skills associated with technological proficiency. Additionally, the concentration of technological gains among top earners and capital owners has played a pivotal role in this phenomenon. Furthermore, the impact of technological advancements has also been a contributing factor to the growing income inequality.

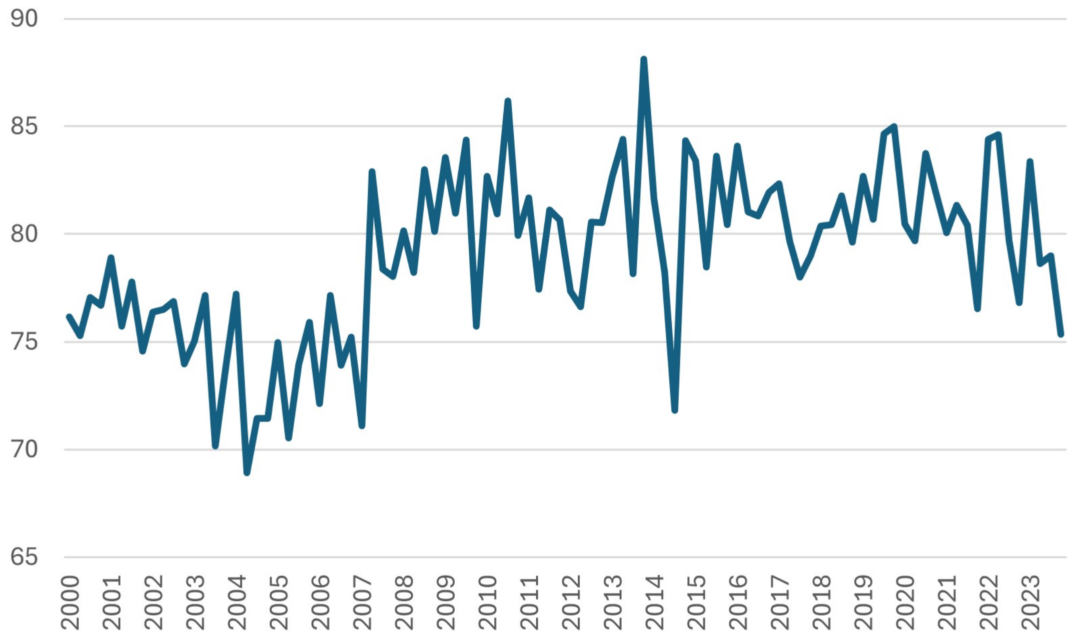

FIGURE 2.3: Difference between skilled and unskilled earnings (Percentage)
Source: U.S. Bureau of Labour Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: Bachelor's degree and higher: 25 years and over [LEU0252918500Q], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/LEU0252918500Q, June 17, 2024., U.S. Bureau of Labour Statistics, Employed full time: Median usual weekly nominal earnings (second quartile): Wage and salary workers: High School graduates, no college: 25 years and over [LEU0252917300Q], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/LEU0252917300Q, June 17, 2024.

The rise of technology has led to a growing demand for skilled workers, resulting in higher wages for those with specialized skills. Heckman et al. (1998) created a model that explains rising wage inequality as a result of skill-biased technical change. The model suggests that an increase in the bias of aggregate technology starting in the early 1960s are consistent with the U.S. wage inequality over the past 35 years. Furthermore, in Figure 2.2, high-skilled workers have experienced significant wage growth since 1998 to 2008 and 2011 to 2018, outpacing medium and low-skilled workers. This trend reflects the increased need for employees who can effectively utilize new technologies.

In Figure 2.2, there is also notable wage dispersion among workers during the periods from 1998 to 2009 and from 2011 to 2018. However, wage growth for all types of workers became more similar during the periods from 2009 to 2010 and from 2019 to 2023. The dot-com crisis in 2001 led to a decline in wage growth, which contributed to increased wage dispersion during that period and the following years. Similarly, the global financial crisis in 2008 also resulted in decreased wage growth, but eventually wages aligned more closely again. In contrast, the Covid-19 pandemic in 2019 initially caused an increase in wage growth, yet this too eventually led to similar wage rates across workers.

Over the past two decades, the gap in average earnings between university-educated workers and those who only hold a high school diploma has significantly widened. This trend is clearly illustrated in Figure 2.3. In the year 2000, empirical data indicated that the average earnings of college graduates were about 76 percent higher than those of high school graduates. This substantial earnings premium for college-educated individuals reflected the increasing value placed on higher education in the workforce. However, this trend experienced a setback due to the Dot-Com crisis, which caused a sharp decline in earnings for recent graduates. By the second quarter of 2004, the premium had fallen to a low of 68.92 percent, marking a significant drop in the financial advantage of obtaining a college degree. Fortunately, the earnings gap began to recover in the following years. The premium steadily increased, ultimately reaching its peak at 88.12 percent in the third quarter of 2013. This surge highlighted a renewed demand for college-educated workers, coinciding with economic recovery and growth in various industries. Since then, the average premium has stabilized and has remained consistently around 80 percent over the last decade. This sustained level suggests that while the market continues to value higher education, the growth rate of earnings for college graduates may be moderating, indicating a potential shift in the dynamics of the job market. Overall, these trends underscore the importance of higher education in enhancing earning potential and career opportunities in today's economy.

The theory of human capital suggests that investments in education and training can improve worker productivity and earnings, shaping wage growth across different occupations in the United States. This theory also emphasizes the importance of both cognitive and non-cognitive abilities in enhancing worker productivity, with each ability playing a distinct role depending on the occupation. There are some recent studies emphasise the importance of both cognitive and non-cognitive capabilities in shaping life outcomes and provide evidence showing that these abilities are crucial for social and economic success (Heckman et al., 2006; Cunha and Heckman, 2009; Cunha et al., 2010; Kautz et al., 2014). However, this study focuses on physical and cognitive abilities and delves into the economic effects of these abilities on job choices in various sectors, especially in response to technological advancements, and how they impact output, employment, productivity, skill premium, and earnings.

This study involves a comprehensive analysis, creating a Roy model that incorporates two types of abilities – cognitive and physical. The model also takes into account three types of sectors – skilled, unskilled, and learning. The primary goal of this study is to thoroughly analyze the impact of technological upheavals on the labour market, output, and income distribution. The study will specifically focus on the skill-biased technology shock and the pure shuffle shock and their effects. Furthermore, this study is also investigating the theoretical relationship between the correlation between the two types of abilities and the impact of technological advancements.

2.2 Review of Related Literature

Extensive research in the field of literature has provided in-depth insights into the significant influence of technological advancements on the allocation of jobs, determination of wages, and the resulting inequality. Through the development of comprehensive frameworks and empirical analyses, researchers have been able to illuminate the shifts in worker-job allocation, the polarization of wages, and the dispersion of wages that occur as a consequence of technological change (Jones and Newman, 1995; Helpman and Rangel, 1999; Krusell et al., 2000; Ordine and Rose, 2009; Lindenlaub, 2017). This body of knowledge offers us a more profound comprehension of the evolving dynamics within the labour market and a clearer understanding of the challenges and opportunities that arise as a result.

The influence of technological advancements on wage disparities in the United States has been studied extensively. Many researchers have observed a strong correlation between capital intensity and non-production worker wage bills in the early 1900s, as well as the surge in wage inequality during the same period (Aghion and Howitt, 1992; Bartel and Lichtenberg, 1987; Bound and Johnson, 1992; Aghion and Howitt, 1994; Eicher, 1996; Andolfatto and MacDonald, 1998; Autor et al., 1998; Goldin and Katz, 1998; Heckman et al., 1998; Bárány and Siegel, 2018). Skilled-labour-biased technical change and unmeasured labour quality were identified as primary drivers of wage differentials in the US during the 1980s. The adaptability of highly educated workers, a crucial factor, to new technologies continues to influence wage dynamics, while job polarization trends have been observed since the 1950s. The advent of computerization heightened the demand for college graduates, resulting in persistent growth in relative demand favouring college workers in the late 20th century. Additionally, the impact of economic growth and new technologies on long-run unemployment has been explored, with researchers highlighting dual effects on unemployment and the deviations from optimal growth due to both favourable and unfavourable effects.

For multidimensional abilities, there are some related studies as follows. Cunha et al. (2010) developed a model to study the impact of parental investments on children's cognitive and non-cognitive skills throughout their life cycle. Kautz et al. (2014) reviewed the literature on the economics and psychology of non-cognitive skills and interventions to develop them. Cunha and Heckman (2009) reviewed recent research on the origins of inequality, emphasizing the importance of both cognitive and non-cognitive capabilities in shaping life outcomes. Heckman et al. (2006) provide new evidence showing that both cognitive and non-cognitive abilities are crucial for social and economic success. These studies emphasize that cognitive ability is not only a factor influencing life success, but non-cognitive ability is also another factor in life's path.

Multiple in-depth studies that offer insight into the intricate dynamics of wage growth and inequality within the United States (Bound and Johnson, 1992; Heckman and Lochner, 1998; Lochner et al., 2018; Adda and Dustmann, 2023; Böhm et al., 2024). These studies highlight significant findings, such as the crucial roles played by both routine-manual and cognitive-abstract skills in determining wage growth, as well as the influence of job amenities on individuals' decisions regarding job mobility. Furthermore, some studies shed light on the changing landscape of unobserved skills' returns and the increasing variability of skills due to life cycle skill growth shifts (Lochner et al., 2018). The study's findings suggest a decline in the returns of unobserved skills, coupled with a rise in the variability of skills, attributed to changes in life cycle skill growth.

Additionally, it explores the seminal work of Bound and Johnson (1992), which has shaped insight into the factors contributing to the rise in wage differentials by educational attainment and the decrease in gender differentials, pointing to skilled-labour-biased technical change and shifts in unmeasured labour quality as significant drivers of these changes. Moreover, the comprehensive model developed by Heckman and Lochner (1998) has provided valuable insights into the effects of skill-biased technical change and human capital supply on wage inequality, highlighting the intricate relationship between schooling, post-school skill investment, and the impact of skill-biased technical change on wage inequality. Furthermore, a study conducted by Böhm et al. (2024) delves into the intricacies of the relationship between occupational employment, wages, and inequality, revealing the impact of transitioning between occupations on earnings and shedding light on the significant influence of changing occupations on the overall landscape of wage inequality.

Numerous studies explore the relationship between technological progress, education, and the labour market. For example, Eicher (1996) suggests that adopting new technologies increases the demand for skilled labour, leading to higher wages. Ma's (2020) study on skilled immigration indicates that skilled immigrants and native workers can complement each other, benefiting the economy. Additionally, several studies examine the impact of factors on workforce income distribution and human capital. Smith (2010) found that assignment frictions affect income distribution. Taber and Vejlin (2020) found that premarket skills account for wage variation. Charlota and Decreuse (2005) argue that search frictions can result in overeducation. Helpman and Rangel (1999) identify varying effects of technological change based on different types of human capital, while Jones and Newman (1995) explore optimal growth through adaptive search investment. Both studies provide insights into the economics of technological advancements, indicating that the initial adoption of new technology may cause an economic downturn.

There is a wealth of literature that not only discusses but also illuminates the profound impact of technology on skills and wage structures throughout history (Goldin and Katz, 1998; Heckman et al., 1998; Lindenlaub, 2017; Donovan and Schoellman, 2023). These works, which are a testament to the depth of research in this field, highlight how technology has influenced industries, worker mobility, and wage inequality. They delve into wage growth, skill-biased technological changes, occupational employment, and the impact of human capital accumulation and technological change on wages and economic growth (Eicher, 1996; Autor et al., 1998; Adda and Dustmann, 2023; Böhm et al., 2024). The need to address rapid skill upgrading to mitigate the widening wage gap and the relationship between occupational changes and surging wage inequality is also explored in these works. Furthermore, this literature discusses how technological shifts have led to changes in worker-job matching based on skills, causing wage polarization and an increase in wage dispersion in the U.S. These factors significantly shape and interact with technological advancements, the development of new skills and capital accumulation, impacting the labour market and the economy in ways that are highly relevant to our current economic landscape.

The Roy model has been widely used in literature related to wage differentials. Several studies have extended the model to incorporate different aspects such as search and human capital accumulation, compensating differentials, self-selection, migration, and talent allocation (Heckman and Honoré, 1990; Funkhouser, 1998; Gardner, 2020; Taber and Vejlin, 2020; Barros et al., 2023; Cubas et al., 2024; Henry et al., 2024). These extensions have provided insights into various aspects of labour market dynamics, including occupational choices and talent allocation. For example, a study by Cubas et al. (2024) found that the absence of insurance markets against permanent earnings shocks significantly impacts aggregate productivity and results in talent misallocation in competitive equilibria.

Additional literature review encompasses a wide range of studies and models in the field of self-selection and occupational choice. For instance, the works of Heckman and

Honoré (1990), Hsieh et al. (2019), Guvenen (2020), Mourifié et al. (2020), Gola (2024), and Henry et al. (2024) provide comprehensive insights into the reasons behind flat wages in certain occupations, the implications of multidimensional skill mismatch on wage growth, and the impact of pursuing comparative advantage on earnings inequality. These studies also delve into the cost of STEM fields, factors influencing occupational choice and wage differentials, and the analysis of the Roy model with a focus on sector-specific unobserved heterogeneity and self-selection based on potential outcomes.

Furthermore, multiple studies shed light on various factors affecting wage growth and talent allocation, with significant implications. For instance, Adda and Dustmann (2023) underscore the role of skills, job amenities, and vocational training in shaping career outcomes, while Costrell and Loury (2004) focus on the impact of technology and ability distribution on earnings. Cubas et al. (2024) highlight how the absence of insurance markets against permanent earnings shocks can lead to a misallocation of talent, and Fan and DeVaro (2019) emphasize that work history serves as a signal of ability that can impact job hoppers' wages negatively. These findings underscore the importance of informed job moves and talent retention, making the research highly relevant and applicable.

This chapter aims to (1) examine the influence of technological disruptions on the job market, productivity, and wealth distribution. The research will concentrate specifically on the impact of skill-biased technological shock and the pure shuffle shock. Additionally, this study aims to (2) explore the theoretical link between the correlation of these two types of ability; physical and cognitive, and the consequences of technological progress. To follow these objectives, the study is organised as follows: Section 2.3 illustrates a framework of this study that describes the relationship between individuals and sectors. Section 2.4 outlines a Roy model that takes into account households with multidimensional abilities to study individuals' decision making. Section 2.5 presents the steady state and transition dynamics of the model to study the relationship between different correlation of abilities and effect of technological advancements on labour and output markets. Section 2.6 provides the values and meanings of the parameters. Section 2.7 shows the simulation results. The study concludes and discusses potential avenues for further research in Section 2.8.

2.3 Framework

The structure of the Roy model is illustrated in Figure 2.4 that consists of the skilled, unskilled, and learning sectors. There are two types of individuals: trained and untrained, each with different physical and cognitive abilities. Initially, trained individuals with higher cognitive potential work in the skilled sector for gaining more skilled earnings, while those with higher physical potential work in the unskilled sector to get more physical earnings. Meanwhile, untrained individuals with low cognitive ability work in the unskilled sector, while those with high cognitive ability choose to train in the learning sector.

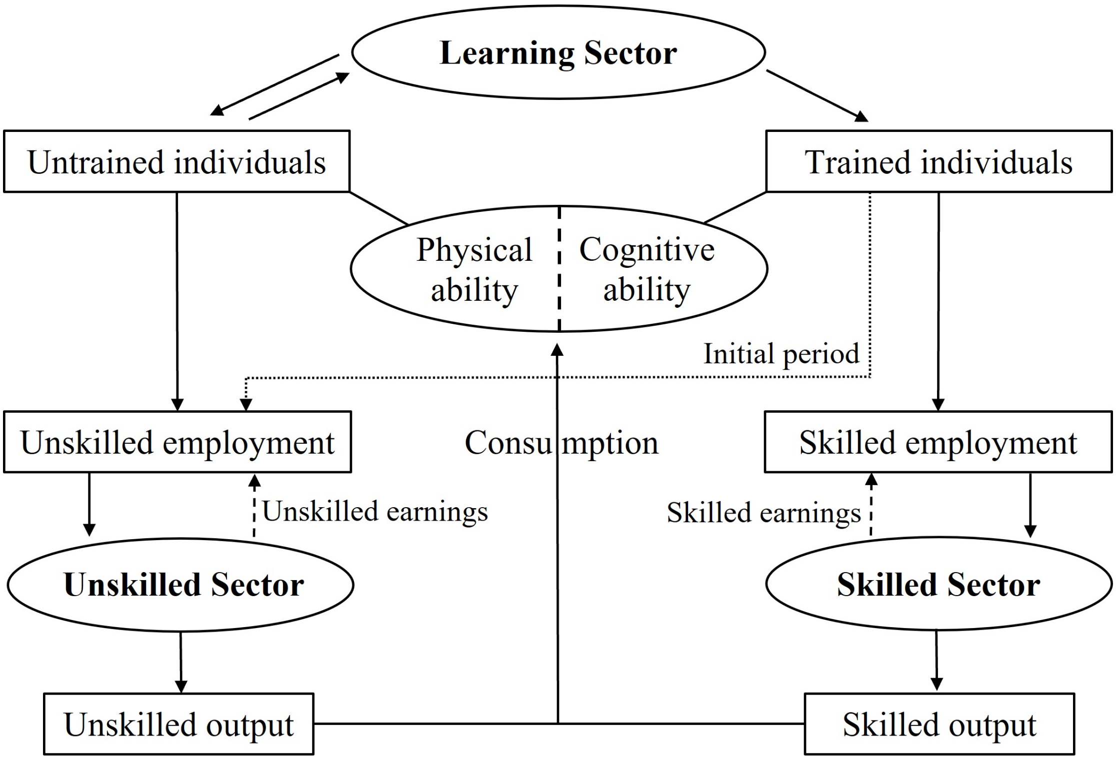

FIGURE 2.4: Model's structure
Source: Author

In subsequent periods, some untrained individuals with high cognitive ability successfully train and transition to the skilled sector. However, some individuals die and are replaced by descendants. Some of the new individuals with high cognitive ability choose to train in the learning sector, while others with low cognitive ability immediately work in the unskilled sector. Those working in the skilled sector earn skilled earnings, while others earn unskilled earnings, all of which are spent on consumption.

Regarding the sectors, the learning sector transforms untrained individuals and descendants with high cognitive ability into skilled workers. The cost of training is the opportunity cost of working in the unskilled sector. Finally, the unskilled sector hires unskilled labour to produce unskilled output, while the skilled sector employs skilled workers to produce skilled output, assuming both outputs are homogeneous goods.

Additionally, the Figure 2.5 illustrates the decision-making processes of both trained and untrained individuals. Firstly, let's examine how trained individuals choose between skilled and unskilled sectors, as shown in the upper part of the figure. Initially, trained individuals aim to maximize their earnings based on their physical and cognitive abilities. If their cognitive earnings exceed the threshold of iso-earning, meaning their cognitive ability is at least equal to the level associated with indifferent earning, they will opt to work in the skilled sector. Conversely, if their physical earnings surpass the iso-earning threshold, indicating that their cognitive ability is below the level associated with indifferent earning, they will choose to work in the unskilled sector. In subsequent periods, some trained individuals may pass away, but the surviving trained individuals will continue to work in their respective sectors.

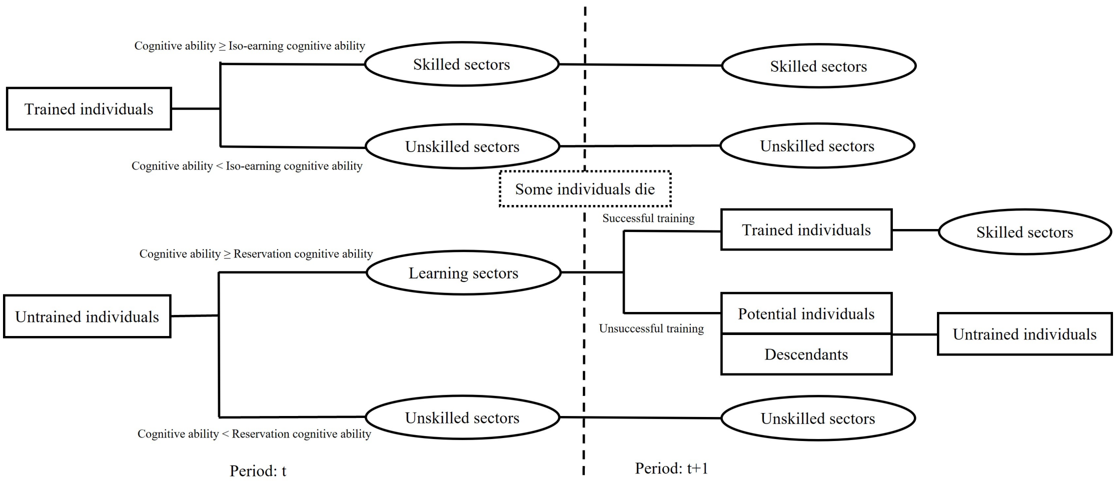

Figure 2.5: Individuals' decision-making processes
Source: Author

Next, we will discuss how untrained individuals decide between pursuing training in learning sectors and working in unskilled sectors, as depicted in the lower part of the figure. Initially, untrained individuals with cognitive abilities lower than the reservation cognitive ability will not pursue training and will immediately begin working in unskilled sectors. In contrast, untrained individuals with cognitive abilities greater than the reservation cognitive ability will choose to undergo training in learning sectors. For these individuals, there is an opportunity cost to consider, which is the potential earnings from unskilled work.

In later periods, some untrained individuals may die. The remaining unskilled individuals with low cognitive abilities will continue working in unskilled sectors. However, those with sufficiently high cognitive abilities will have a chance at successful training. Some of them will successfully train and subsequently become trained individuals, allowing them to enter the skilled sectors. Meanwhile, those who do not succeed in their training will be classified as individuals with potential to train, namely potential individuals. These potential individuals and their descendants will form a new cohort of untrained individuals and will also face decisions about whether to pursue training or remain untrained. This cycle will continue to repeat over time for each new generation of untrained individuals.

2.4 The Model

The framework of the Roy model is highlighted in a study by Andolfatto and Smith (2001), which illustrates the model's heterogeneous cognitive abilities incorporating skilled, unskilled, and learning sectors. This study extends the Roy model by introducing multidimensional abilities consisting of two types of ability—cognitive and physical abilities—to explain how individuals with multidimensional abilities make decisions about training or working in different sectors. The study also examines how these decisions impact the accumulation of skills and the distribution of earnings across households.

2.4.1 The economic environment

The model is assumed to be discrete time and infinite horizon, $t = 0, 1, 2, ..., \infty$. The population is normalise to be one. However, each individual is assumed to have two types of innate abilities, physical ability $a_1 \geq 0$ and cognitive ability $a_2 \geq 0$ which a_2 implies the market values of individual cognitive ability depending on the reservation cognitive ability a_{2R} while a_1 can determine the reservation cognitive ability. For cognitive ability, each level of ability will earn different income level y_t. In each period, their consumption c_t is assumed to consume all their income. They also face with discount factor, $0 < \beta < 1$ and death rate, $0 < \delta < 1$. However, the population is always constant because deaths will be replace by descendants. Therefore, the individual welfare is the discount stream of consumption c_t over time,

$$E \sum_{t=0}^{\infty} \beta^t (1 - \delta)^t c_t \tag{2.1}$$

Then, we can rewrite above equation to recursive form as following value function which consists of present consumption and the discount future value,

$$V_t(a_1, a_2) = c_t + \beta(1 - \delta) E_t V_{t+1}(a_1, a_2)$$

However, the individuals are assumed to consume all output or income y_t in each period, then the value function also can be illustrated as,

$$V_t(a_1, a_2) = y_t + \beta(1-\delta)E_tV_{t+1}(a_1, a_2) \tag{2.2}$$

Following the Andolfatto and Smith (2001), there are three sectors consist of unskilled sector (Sector 1), skilled sector (Sector 2), and learning sector (Sector 3). Trained individuals supply physical ability to unskilled sector for unskilled output w_1a_1 or choose supply cognitive ability to skilled sector to generate skilled output w_2a_2 depending on their relative ability incomes (y_2/y_1). For untrained individuals, they immediately work in unskilled sector to produce unskilled output when cognitive ability is lower than reservation cognitive ability $(a_2 < a_{2R})$ or attend the learning sector to get a chance to become trained individuals working in skilled sector in next period in case that cognitive ability is larger than or equal to reservation cognitive ability $(a_2 \geq a_{2R})$.

2.4.2 Decision-making

In this model, we construct the recursive model because the recursive structure enables us to conceptualize optimal decision-making as a solution to a dynamic programming problem, where decisions are made at various stages over time. Within this framework, we identify three distinct value functions that shed light on the benefits associated with different states of physical ability a_1 and cognitive ability a_2. These are: the value of being a trained individual $V(a_1, a_2)$, the value of being an untrained individual in training within the learning sector $S(a_1, a_2)$, and the value of being an untrained individual working in the unskilled sector $Q(a_1, a_2)$.

For trained individuals, they choose to work in unskilled sector when physical earning is higher than cognitive one $(w_2a_2 < w_1a_1)$ or allocate their time in skilled sector in case that cognitive earning is larger than or equal to physical one $(w_2a_2 \geq w_1a_1)$. Thus, for a given (w_1, w_2), they try to maximize their life-time income or earning as the value function for trained individual with physical ability a_1 and cognitive ability a_2,

$$V(a_1, a_2) = \max\{w_1a_1, w_2a_2\} + \beta(1-\delta)V(a_1, a_2) \tag{2.3}$$

We can rearrange the last equation to get the value of trained individual as the discount value of maximum income earnings between physical and cognitive abilities,

$$V(a_1, a_2) = \frac{\max\{w_1a_1, w_2a_2\}}{1-\beta(1-\delta)} \tag{2.4}$$

To consider the decision-making of trained individuals, we must first define relative wage as the ratio between the additional output from physical ability and the additional output from cognitive ability $\omega \equiv w_1/w_2$. We notice that trained individuals with cognitive ability less than threshold of iso-earning cognitive ability ($a_2 < a_{2I} \equiv \omega a_1$) will choose to work in the unskilled sector, while trained individuals with cognitive ability larger than or equal to the threshold cognitive ability ($a_2 \geq a_{2I} \equiv \omega a_1$) prefer working in skilled sector instead. Thus, trained individuals make decisions by considering their cognitive abilities with the iso-earning cognitive ability ($a_{2I} \equiv \omega a_1$), as illustrated in the left graph of Figure 2.6 and in the lower panel of Figure 2.22 in the Appendix.

For untrained individuals, they make decisions between choosing work in unskilled sector or training in learning sector.

$$K(a_1, a_2) = \max\{(S(a_1, a_2), Q(a_1, a_2)\} \tag{2.5}$$

The value function for untrained individual who chooses training is the difference between benefit of training and cost of training. The benefit is the discount value of individuals' unsuccessful training and individuals' successful training. The cost is the opportunity cost to work in unskilled sector and gain physical earnings.

$$S(a_1, a_2) = \beta(1-\delta)[(1-\theta)K(a_1, a_2) + \theta V(a_1, a_2)] \tag{2.6}$$

The value function for untrained individual who works in unskilled sector is the summation of physical ability earnings and the discount value of decision between working or training.

$$Q(a_1, a_2) = w_1 a_1 + \beta(1-\delta)K(a_1, a_2) \tag{2.7}$$

For stationarity, the value of training is given by the ratio of trained individuals' value and fraction of alive individuals' successful training ξ.

$$S(a_1, a_2) = \frac{V(a_1, a_2)}{\xi} \quad , \quad a_2 \geq a_{2R}, \tag{2.8}$$

where $\xi = \beta(1-\delta)\theta/[1-\beta(1-\delta)(1-\theta)] \in [0, 1]$.

The steady-state value of untraining is the discount physical ability earnings.

$$Q(a_1, a_2) = \frac{w_1 a_1}{1 - \beta(1 - \delta)} \quad , \quad a_2 < a_{2R} \tag{2.9}$$

The reservation cognitive ability level is characterised by the ratio of threshold cognitive ability and fraction of alive individuals' successful training.

$$a_{2R} = \frac{\omega a_1}{\xi} \tag{2.10}$$

Equation 2.10 indicates that the reservation level of cognitive ability is greater than the iso-earning cognitive ability. This is due to the opportunity costs of training for untrained individuals, which are illustrated in the right graph of Figure 2.6 and in the lower panel of Figure 2.22 in the Appendix.

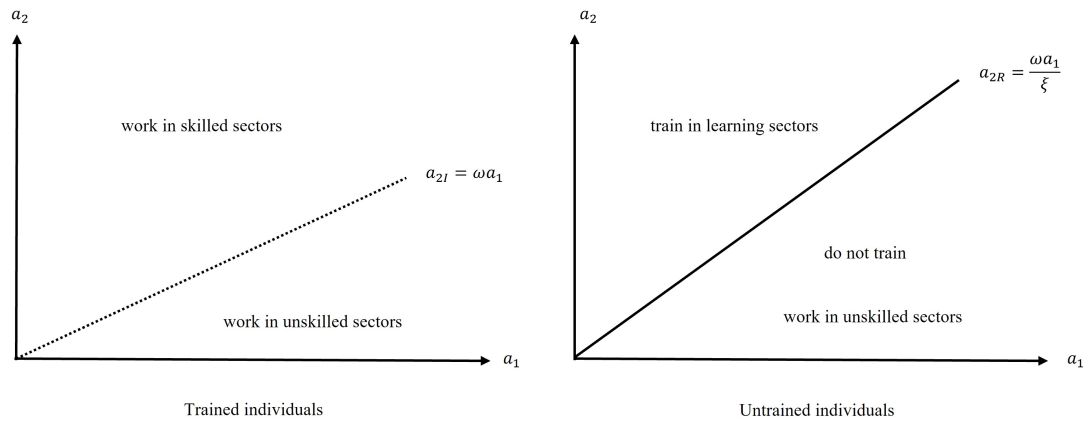

FIGURE 2.6: Comparing decision-making between trained and untrained individuals
Source: Author

To analyse the details further, the Figure 2.6 illustrates the decision-making processes of trained and untrained individuals. We first examine the trained individuals, depicted in the left graph of the figure. Trained individuals compare their earnings based on physical and cognitive abilities. The line representing iso-earning cognitive ability, denoted as a_{2I}, is derived from their comparison of earnings related to both physical and cognitive skills. Trained individuals will choose to work in skilled sectors if their combination of physical and cognitive abilities is at least equal to the threshold indicated by the iso-earning cognitive ability line. Conversely, if their combination falls below this threshold, they will opt to work in unskilled sectors.

Next, we analyse the more complex decision-making processes of untrained individuals, illustrated in the right graph of the figure. Untrained individuals attempt to maximize their earnings based on their physical and cognitive abilities; however, they face opportunity costs associated with training, which involves a period without earnings, as well as the probability of successfully completing training, represented by the fraction of individuals who succeed in training, denoted as ξ. If they choose to train in learning sectors with the aim of qualifying for skilled jobs in the future, they must consider their reservation cognitive ability, represented by a_{2R}, which is based on their relative earnings and the associated opportunity costs. If their combination of physical and cognitive abilities is lower than the reservation cognitive ability line, untrained individuals will work immediately in unskilled sectors. If their combination is equal to or greater than the reservation cognitive ability line, they will decide to train in the learning sectors.

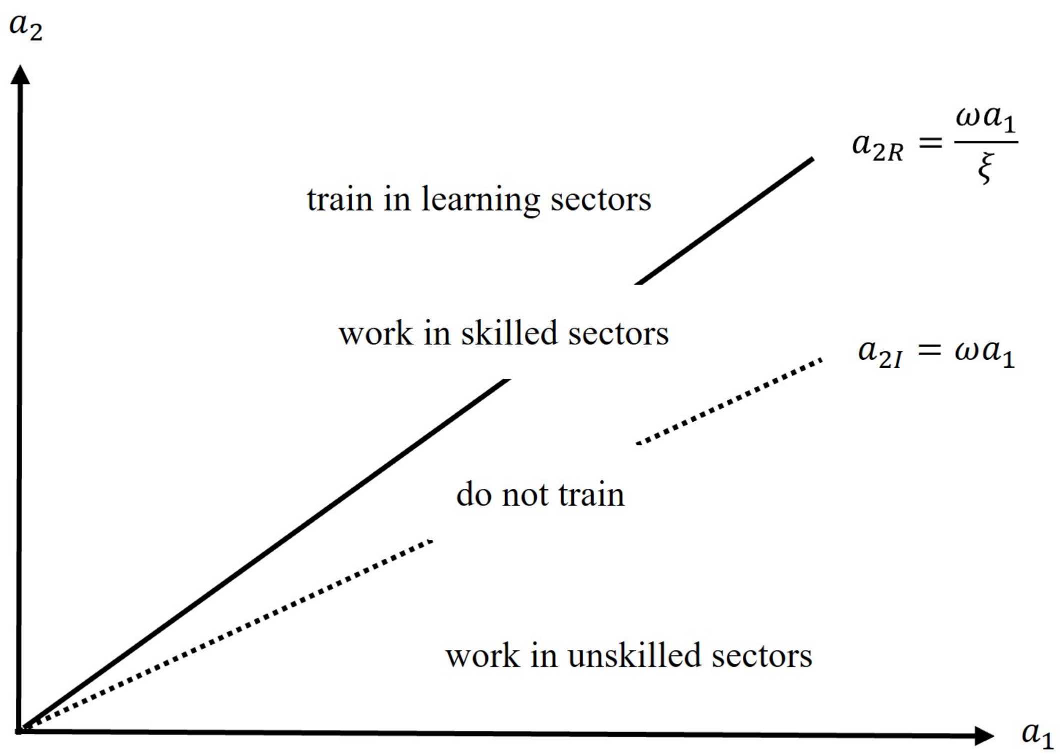

FIGURE 2.7: The overall decision-making process of individuals
Source: Author

Furthermore, we can synthesise both graphs into a single one, as shown in Figure 2.7, to provide a more comprehensive analysis. This new graph presents three distinct areas: the lower area indicates that both trained and untrained individuals work in unskilled sectors, while the upper area signifies that trained individuals work in skilled sectors and untrained individuals engage in training within learning sectors. The middle area reveals that trained individuals are working in skilled sectors while untrained individuals are in unskilled sectors. It is noteworthy that untrained individuals in this middle area have the potential to shift their decision towards training if their opportunity costs decrease or their chances of successful training increase.

Additionally, we observe that if relative wages decrease, both the reservation cognitive ability line and the iso-earning cognitive ability line will also lower. As a result, there will be an increase in trained individuals working in skilled sectors and untrained individuals training in learning sectors, while the number of both trained and untrained individuals working in unskilled sectors will decrease.

2.5 Steady States and Transition Dynamics

2.5.1 Steady States

We define $\lambda_t(a_2|a_1)$ and $\mu_t(a_2|a_1)$ are the densities of trained and untrained individuals with cognitive ability a_2 conditional on physical ability a_1 at date t. We assume these densities are restricted by the densities of population $g(a_2|a_1) = \lambda_t(a_2|a_1) + \mu_t(a_2|a_1)$. In each period, some untrained individuals with $a_2 \geq a_{2R}$ become trained individuals at rate θ, then the stock of trained individuals with $a_2 \geq a_{2R}$ at the end period is determined by $\lambda_t(a_2|a_1) + \theta\mu_t(a_2|a_1)$.

The evolution of trained individuals with cognitive ability a_2 conditional on physical ability a_1 is characterised by:

$$\lambda_{t+1}(a_2|a_1) = \begin{cases} (1-\delta)[(1-\theta)\lambda_t(a_2|a_1) + \theta g(a_2|a_1)] & \text{for } a_2 \geq a_{2R} \\ (1-\delta)\lambda_t(a_2|a_1) & \text{for } a_2 < a_{2R} \end{cases} \tag{2.11}$$

Equation 2.11 states the distribution of trained individuals in future period derived by the present distribution of trained individuals that being still alive when individuals have cognitive ability lower than reservation cognitive ability. In case individuals having cognitive ability at least the reservation cognitive ability, the trained individuals' distribution in next period is the distribution of population that train successfully and the previous period distribution of trained individuals subtracting the distribution with successful training because it is the double count distribution; in other word, the distribution is the remaining of trained individuals and untrained individuals who train successfully.

For simplicity, we can consider some initial distribution $\lambda_0(a_2|a_1)$ and then the density of trained individuals with cognitive ability $a_2 < a_{2R}$ is $\lambda_t(a_2|a_1) = (1-\delta)^t\lambda_0(a_2|a_1)$

while the density for $a_2 \geq a_{2R}$ is given by:

$$\begin{aligned}\lambda_t(a_2|a_1) &= \phi^t \lambda_0(a_2|a_1) + [\phi^{t-1} + \phi^{t-2} + ... + 1](1-\delta)\theta g(a_2|a_1) \\ &= \phi^t \lambda_0(a_2|a_1) + [(1-\phi^t)/(1-\phi)](1-\delta)\theta g(a_2|a_1)\end{aligned}$$

where $\phi \equiv (1-\delta)(1-\theta)$ is a fraction of remaining alive and unnecessary retaining.

Thus, the evolution of distribution function is become:

$$\lambda_t(a_2|a_1) = \begin{cases} \rho(1-\phi^t)g(a_2|a_1) + \phi^t \lambda_t(a_2|a_1) & \text{for } a_2 \geq a_{2R} \\ (1-\delta)^t \lambda_0(a_2|a_1) & \text{for } a_2 < a_{2R} \end{cases} \tag{2.12}$$

where $0 < \rho \equiv (1-\delta)\theta/(\delta+\theta-\delta\theta) < 1$ is a proportion of remaining alive and successful training (per total of new descendants and its numerator). Equation 2.12 shows the distribution of trained individuals at time t is determined by a fraction of remaining alive over time t of the initial distribution of trained individuals when individuals have lower cognitive ability. If individuals have high cognitive ability, the trained individuals' distribution at time t consists of a fraction of distribution of population who are new descendants, still alive, and successful training and a fraction of distribution of trained individuals who remain alive over time t. When time converges to infinity as $t \to \infty$, the distribution will converge to a steady-state distribution as $\lambda_t \to \lambda$. Thus, the steady-state distribution of trained individuals is characterised by:

$$\lambda(a_2|a_1) = \begin{cases} \rho g(a_2|a_1) & \text{for } a_2 \geq a_{2R} \\ 0 & \text{for } a_2 < a_{2R} \end{cases} \tag{2.13}$$

Equation 2.13 intuitively indicates that there is no trained individuals in the long run if all individuals have low cognitive ability. In contrast, if individuals have high enough cognitive ability, the distribution of trained individuals will be a fraction of population distribution that remains alive and trains successfully. Then, follow the restriction of densities, the steady-state distribution of untrained individuals is determined by difference of population's distribution and trained individuals' distribution:

$$\mu(a_2|a_1) = g(a_2|a_1) - \lambda(a_2|a_1) \tag{2.14}$$

For the steady-state employment and output in each sector, we can determine those by using the distributions of trained and untrained individuals. Thus, the long-run measure of untrained individuals working in the unskilled sector is characterised by the summation of untained individuals' distribution over low cognitive ability and all physical ability:

$$\begin{aligned} N_1 &= \int_0^{\infty} \int_0^{a_{2R}} \mu(a_2|a_1) g(a_1) da_2 da_1 \\ &= \int_0^{\infty} \int_0^{a_{2R}} \mu(a_1, a_2) da_2 da_1 \end{aligned} \tag{2.15}$$

and the long-run measure of output in the unskilled sector is given by the summation of untrained individuals' physical product over low cognitive ability and all physical ability:

$$\begin{aligned} Y_1 &= w_1 \int_0^{\infty} \int_0^{a_{2R}} a_1 \mu(a_2|a_1) g(a_1) da_2 da_1 \\ &= w_1 \int_0^{\infty} \int_0^{a_{2R}} a_1 \mu(a_1, a_2) da_2 da_1 \end{aligned} \tag{2.16}$$

In the skilled sector, the measure of trained individuals working in the skilled sector in the long run is determined by the summation of trained individuals' distribution over high cognitive ability and all physical ability:

$$\begin{aligned} N_2 &= \int_0^{\infty} \int_{a_{2R}}^{\infty} \lambda(a_2|a_1) g(a_1) da_2 da_1 \\ &= \int_0^{\infty} \int_{a_{2R}}^{\infty} \lambda(a_1, a_2) da_2 da_1 \end{aligned} \tag{2.17}$$

and the output in the skilled sector in the long run is measured by the summation of trained individuals' cognitive product over high cognitive ability and all physical ability:

$$\begin{aligned} Y_2 &= w_2 \int_0^{\infty} \int_{a_{2R}}^{\infty} a_2 \lambda(a_2|a_1) g(a_1) da_2 da_1 \\ &= w_2 \int_0^{\infty} \int_{a_{2R}}^{\infty} a_2 \lambda(a_1, a_2) da_2 da_1 \end{aligned} \tag{2.18}$$

We notice that N_1 is increasing in a_{2R} while N_2 is decreasing in a_{2R}. Since the effect of change in reservation cognitive ability on unskilled employment dominates the effect of another one, hence the total employment $N \equiv N_1 + N_2$ is increasing in a_{2R}.

For the long-run productivity, it can be determined by the ratio of long-run output to employment in each sector. The productivity in unskilled sector is given by:

$$P_1 = \frac{Y_1}{N_1} \tag{2.19}$$

While the productivity in skilled sector is denoted by the average of the skilled output and employment:

$$P_2 = \frac{Y_2}{N_2} \tag{2.20}$$

The wage differential or skill premium in the long run is measured by the ratio of skilled to unskilled productivity:

$$\Pi = \frac{P_2}{P_1} \tag{2.21}$$

This model is assumed that the wages of both skilled and unskilled workers are directly proportional to their productivity levels, $W_1 = P_1$ and $W_2 = P_2$. This means that each group's compensation aligns with the value of the output they generate. Additionally, their wage growths are assumed to measure as percentage change of their wages between present and previous periods, $\alpha_{1t} = W_{1t}/W_{1t-1} - 1$ and $\alpha_{2t} = W_{2t}/W_{2t-1} - 1$. Under these conditions, there will be no further growth in wages for either skilled or unskilled workers at the steady state, $\alpha_1 = 0$ and $\alpha_2 = 0$.

For the distribution of earnings in the long run, it can be characterised by the ratio of amount of worker with different income level to total employment. Then, we define $H(y)$ is the share of workers with different earnings y to labour force as follow,

$$H(y) = \begin{cases} 0 & \text{for } 0 \leq y < a_1 w_1 \\ [\int_0^\infty \int_0^{a_{2R}} \mu(a_1, a_2) da_2 da_1]/N & \text{for } a_1 w_1 \leq y < a_{2R} w_2 \\ [(1-\rho) \int_0^\infty \int_0^{a_{2R}} \mu(a_1, a_2) da_2 da_1 + & \\ \rho \int_0^\infty \int_{y/w_2}^\infty \lambda(a_1, a_2) da_2 da_1]/N & \text{for } a_{R2} w_2 \leq y < \infty \end{cases} \tag{2.22}$$

Equation 2.22 states that no one desire to work if the level of earnings is lower than physical ability earnings. The share of worker is a proportion of untrained individuals that sum over low cognitive ability and all physical ability per the aggregate employment when the level of earnings is between at least physical ability earnings and less than reservation cognitive ability earnings. If the level of earnings is at least reservation cognitive ability earnings, the share of workers consists of (1) summation of untrained individuals' distribution with proportion of new descendants (per aggregation of new descendants and individuals who remain alive and train successfully) over cognitive ability lower than reservation cognitive ability and all physical ability per total employment as well as (2) summation of trained individuals' distribution with proportion of remaining alive and successful training (per total of new descendants and individuals remaining alive and successful training) over cognitive ability at least earnings in term skilled wage and all physical ability per aggregate employment.

2.5.2 Transition Dynamics

The study is interested in the case that how economic variables adjust themselves over time $t \geq 0$. Suppose there is a technological advancement that improve productivity in unskilled sector by a factor γ_1 or productivity in skilled sector by a factor γ_2 such that $(w_1^{'}, w_2^{'}) = (\gamma_1 w_1, \gamma_2 w_2)$. An interested cases is $\gamma_2 > \gamma_1 \geq 1$ which is known as the skill-biased technological shock.

Then, we are also interested in case neutral technology advancement $\gamma_1 = \gamma_2 = \gamma \geq 1$ and introduce the shuffle shock which is the randomly shuffling on the individuals' abilities across sectors. In the impact period, we assume the shuffle shock occurs at period $t = 0$ by randomly rearranging the trained individual N_2 according to distribution of population. Thus, the distribution of trained individual at $t = 0$ is given by the product of skilled employment and population's distribution:

$$\lambda_0(a_2|a_1) = N_2 g(a_2|a_1) \tag{2.23}$$

Then, the distribution of trained individual for period $t \geq 0$ is characterised by:

$$\lambda_t(a_2|a_1) = \begin{cases} \rho[1 - \phi^t \int_0^\infty \int_0^{a_{2R}} \mu(a_1, a_2) da_2 da_1] g(a_2|a_1) & \text{for } a_2 \geq a_{2R} \\ (1-\delta)^t N_2 g(a_2|a_1) & \text{for } a_2 < a_{2R} \end{cases} \tag{2.24}$$

Equation 2.24 implies that the distribution of trained individuals at time t is determined by the remaining of the product of trained individuals working in skilled sector and distribution of population when the level of cognitive ability is less than the reservation cognitive ability. If the level of cognitive ability is greater than or equal to the reservation cognitive ability, the distribution of trained individuals is derived from distribution of population scaling by the proportion of remaining alive and successful training per new descendants and successful training incorporating with the remaining of aggregate distribution after subtracting distribution of untrained individuals that remain alive and unsuccessfully train integrating over the level of cognitive ability being lower than the reservation cognitive ability and all physical ability.

The measure of untrained individuals working in the unskilled sector is characterised by the summation of trained individuals' distribution over the level of cognitive ability being lower than the threshold cognitive ability combining with the summation of difference between population's distribution and trained individuals' distribution over the level of cognitive ability at least the threshold cognitive ability but less than the reservation cognitive ability and all physical ability:

$$\begin{aligned} N_{1t} &= \int_0^\infty \int_0^{a_{2I}} \lambda_t(a_2|a_1)g(a_1)da_2da_1 + \int_0^\infty \int_{a_{2I}}^{a_{2R}} [g(a_2|a_1) - \lambda_t(a_2|a_1)]g(a_1)da_2da_1 \\ &= \int_0^\infty \int_0^{a_{2I}} \lambda_t(a_1,a_2)da_2da_1 + \int_0^\infty \int_{a_{2I}}^{a_{2R}} [g(a_1,a_2) - \lambda_t(a_1,a_2)]da_2da_1 \end{aligned} \tag{2.25}$$

and the measure of output in the unskilled sector is given by the summation of trained individuals' physical ability output over the level of cognitive ability being less than the threshold cognitive ability and all physical ability incorporating with the summation of untrained individuals' physical ability output over the level of cognitive being not less than the threshold cognitive ability but more than the reservation cognitive ability and all physical ability:

$$\begin{aligned} Y_{1t} &= \gamma_1 w_1 \int_0^\infty \int_0^{a_{2I}} a_1 \lambda_t(a_2|a_1)g(a_1)da_2da_1 + \\ &\gamma_1 w_1 \int_0^\infty \int_{a_{2I}}^{a_{2R}} a_1[g(a_2|a_1) - \lambda_t(a_2|a_1)]g(a_1)da_2da_1 \\ &= \gamma_1 w_1 \int_0^\infty \int_0^{a_{2I}} a_1 \lambda_t(a_1,a_2)da_2da_1 + \\ &\gamma_1 w_1 \int_0^\infty \int_{a_{2I}}^{a_{2R}} a_1[g(a_1,a_2) - \lambda_t(a_1,a_2)]da_2da_1 \end{aligned} \tag{2.26}$$

In the skilled sector, the measure of trained individuals working in the skilled sector is determined by summation of trained individuals' distribution over the level of of cognitive ability being at least the threshold cognitive ability and all physical ability:

$$
\begin{aligned}
N_{2t} &= \int_0^{\infty}\int_{a_{2I}}^{\infty}\lambda_t(a_2|a_1)g(a_1)da_2da_1 \\
&= \int_0^{\infty}\int_{a_{2I}}^{a_{2R}}\lambda_t(a_2|a_1)g(a_1)da_2da_1 + \int_0^{\infty}\int_{a_{2R}}^{\infty}\lambda_t(a_2|a_1)g(a_1)da_2da_1 \\
&= \int_0^{\infty}\int_{a_{2I}}^{a_{2R}}\lambda_t(a_1,a_2)da_2da_1 + \int_0^{\infty}\int_{a_{2R}}^{\infty}\lambda_t(a_1,a_2)da_2da_1
\end{aligned} \tag{2.27}
$$

and the output in the skilled sector is measured by summation of trained individuals' cognitive ability output over the level of cognitive ability being more than or equal to the threshold cognitive ability and all physical ability:

$$
\begin{aligned}
Y_{2t} &= \gamma_2 w_2 \int_0^{\infty}\int_{a_{2I}}^{\infty} a_2\lambda_t(a_2|a_1)g(a_1)da_2da_1 \\
&= \gamma_2 w_2 \int_0^{\infty}\int_{a_{2I}}^{a_{2R}} a_2\lambda_t(a_2|a_1)g(a_1)da_2da_1 + \\
&\gamma_2 w_2 \int_0^{\infty}\int_{a_{2R}}^{\infty} a_2\lambda_t(a_2|a_1)g(a_1)da_2da_1 \\
&= \gamma_2 w_2 \int_0^{\infty}\int_{a_{2I}}^{a_{2R}} a_2\lambda_t(a_1,a_2)da_2da_1 + \\
&\gamma_2 w_2 \int_0^{\infty}\int_{a_{2R}}^{\infty} a_2\lambda_t(a_1,a_2)da_2da_1
\end{aligned} \tag{2.28}
$$

For the productivity, it can be determined by the ratio of long-run output to employment in each sector. The productivity in unskilled sector is given by:

$$
P_{1t} = \frac{Y_{1t}}{N_{1t}} \tag{2.29}
$$

While the productivity in skilled sector is denoted by the average cognitive ability of trained individual and the skilled output:

$$
P_{2t} = \frac{Y_{2t}}{N_{2t}} \tag{2.30}
$$

The wage differential or skill premium is measured by the ratio of skilled to unskilled productivity:

$$
\Pi_t = \frac{P_{2t}}{P_{1t}} \tag{2.31}
$$

The wages of skilled and unskilled workers at any period are directly linked to their productivity levels at that time, reflecting the value they bring to their respective roles in each period, $W_{1t} = P_{1t}$ and $W_{2t} = P_{2t}$. Wage growth is measured as the percentage change in wages over time, comparing the current wage levels to those from previous periods, $\alpha_{1t} = W_{1t}/W_{1t-1} - 1$ and $\alpha_{2t} = W_{2t}/W_{2t-1} - 1$.

For the distribution of earnings, it can be characterised by the ratio of amount of worker with different income level to total employment. Then, we define $H(y)$ is the share of workers with different earnings y to labour force as follow,

$$H_t(y) = \begin{cases} 0 & \text{for } 0 \leq y < a_1 w_1 \\ [\int_0^\infty \int_0^{a_{2R}} \mu_t(a_1, a_2) da_2 da_1 + & \\ (1-\delta)^t N_{2t} (\int_0^\infty \int_{y/w_2}^\infty \lambda_t(a_1, a_2) da_2 da_1 - & \\ \int_0^\infty \int_0^{a_{2R}} \mu_t(a_1, a_2) da_2 da_1)]/N_t & \text{for } a_1 w_1 \leq y < a_{2R} w_2 \\ [\int_0^\infty \int_0^{a_{2R}} \mu_t(a_1, a_2) da_2 da_1 + & \\ \rho(1-(1-\delta-\theta+\delta\theta)^t) \int_0^\infty \int_0^{a_{2R}} \mu_t(a_1, a_2) da_2 da_1 * & \\ (\int_0^\infty \int_{y/w_2}^\infty \lambda_t(a_1, a_2) da_2 da_1 - & \\ \int_0^\infty \int_0^{a_{2R}} \mu_t(a_1, a_2) da_2 da_1)]/N_t & \text{for } a_{R2} w_2 \leq y < \infty \end{cases} \tag{2.32}$$

Equation 2.32 states that individuals do not want to work or share of workers is zero if the level of earnings is less than the physical ability earnings. If the level of earnings is between at least physical ability earnings and less than reservation cognitive ability earnings, the share of workers is proportion of untrained individuals' distribution integrating over lower cognitive ability and all physical ability combining with the remaining of skilled employment that relating with difference between summation of trained individuals' distribution over the level of cognitive ability being more than earnings per skilled wage and all physical ability and summation of untrained individuals' distribution over the level of cognitive ability is less than the reservation cognitive ability and all physical ability per total employment. However, if the level of earnings is at least reservation cognitive ability earnings, the share of workers consists of (1) the summation of untrained individuals' distribution over lower cognitive ability and all physical ability per aggregate employment as well as (2) a fraction that being new descendants, remaining alive and successful training of the summation of untrained individuals' distribution over low cognitive ability and all physical ability incorporating with the difference between the summation of trained individuals' distribution over the level of cognitive ability being at least earnings per skilled wage and all physical ability and the summation of untrained individuals' distribution over low cognitive ability and all physical ability per total employment.

2.6 Parameterisation

This model operates over a yearly period. The parameters used in this model are drawn from Andolfatto and Smith (2001) and our calibration, as shown in Table 2.1, which includes their definitions and values.

Following Andolfatto and Smith (2001), the consumer's subjective discount factor, β, is set to 0.96, reflecting an average yearly risk-free rate of about 4 percent. The consumer's probability of death parameter, δ, is set at 0.10, implying that about 10 percent of individuals die each year, but this is offset by an increase in descendants of approximately 10 percent annually. The probability of absorbed knowledge, θ, is set at 0.35, indicating that untrained individuals pursuing education and training have a success rate of about 35 percent.

The additional output of physical ability, w_1, is set to 1, while the additional output of cognitive ability, w_2, is set to 2. This setup implies that the relative wages of physical ability to cognitive ability, ω, are approximately 0.5. The additional productivity attributable to technology in the unskilled sector, γ_1, is set to 1, whereas the additional productivity in the skilled sector, γ_2, takes on three values: 1.0 (indicating no skill-biased technology shock), 1.2 (indicating a positive skill-biased technology shock of about 20 percent), and 2.0 (indicating a 100 percent increase).

We assume a zero mean for both physical and cognitive abilities (μ_{a_1}, μ_{a_2}) and a unit variance ($\sigma^2_{a_1}, \sigma^2_{a_2}$). However, the covariance between physical and cognitive abilities, ($\sigma_{a_1 a_2}$), varies among three values: 0, 0.9, and -0.9. This variation is used to illustrate three scenarios: no correlation between physical and cognitive abilities, positive correlation, and negative correlation, respectively.

The model is coded and executed using the Matlab program, and the results of the impulse responses are illustrated over yearly periods. To obtain the numerical results, we first generate the probability density function of a bivariate normal distribution representing physical and cognitive abilities. We then calculate the reservation cognitive ability, the iso-earning cognitive ability, the value of trained individuals, and the value of untrained individuals pursuing education.

Next, we use the probability density function to evaluate physical and cognitive abilities against the reservation cognitive ability in order to approximate the value of untrained individuals working in the unskilled sector, as well as the value of their choice between training and working in that sector. Following this, we measure the steady-state distributions of trained and untrained individuals to determine their respective shares. We use these steady-state shares, in conjunction with the reservation cognitive ability, to compute other steady-state variables. Finally, we introduce technology shocks, such as skill-biased technology and shuffle shocks, to simulate the adjustments of these variables around their steady state over time.

2.7 Result

2.7.1 Skill-biased technology and pure shuffle shocks

The effect of skill-biased technology shock

In Figure 2.8, we can see the introduction of a positively skilled-biased technology shock, which does not affect the iso-earning and reservation cognitive ability earning lines. However, this shock causes the iso-earning cognitive ability and reservation cognitive ability lines to shift to the right. As a result, the shock impacts the decision making of trained and untrained individuals, and subsequently affects output, employment, productivity, skill premium, and the distribution of earnings. Additionally, the relationship between physical and cognitive abilities can also influence these variables, which will be illustrated below.

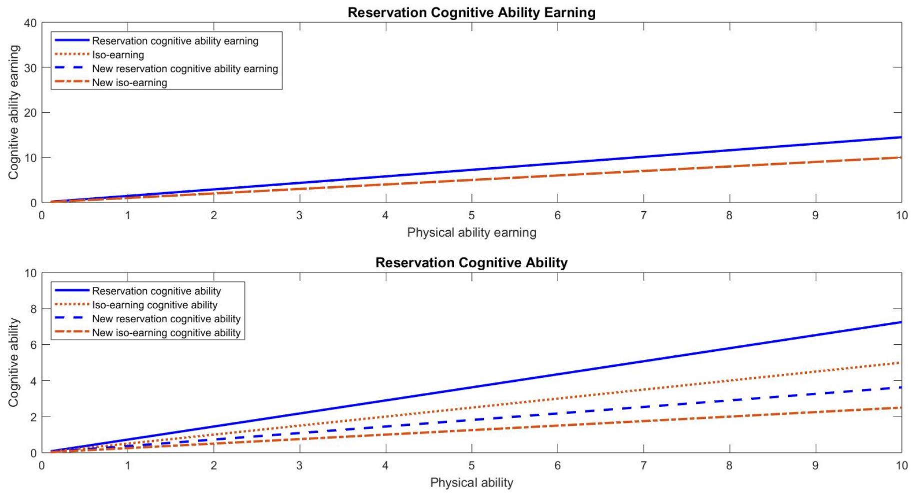

FIGURE 2.8: The skill-bias technology shock and the change in reservation cognitive ability
Source: Author's calculation

The impact of a positive skill-biased technology shock on output, employment, productivity, and the skill premium is depicted in Figure 2.9. We can analyze both its short-run and long-run effects. In the short run, the positive shock suddenly increases the output of skilled labour, the marginal product of skilled labour, skilled productivity, and the skill premium. There is a greater demand for skilled labour, leading to an excess demand for skilled workers, resulting in increased wages and earnings for skilled workers. Moreover, there is a decrease in the relative earnings of unskilled workers compared to skilled workers, leading more untrained individuals to pursue training in learning sectors. Consequently, unskilled employment and output decrease, while unskilled productivity increases due to the decrease in unskilled output being less than the decrease in unskilled employment. Then, aggregate productivity also increases because aggregate output increases while aggregate employment decreases. Moreover, the skill premium is higher because skilled productivity is larger than unskilled productivity. In empirical evidence, the sharp growth in productivity in 2001, 2004, 2010, and 2020 in Figure 2.1, as well as the significantly decreased employment in 2000-2002, 2006-2009, and 2019-2020, support the short-run impact of skill-biased technology shock on productivity and employment.

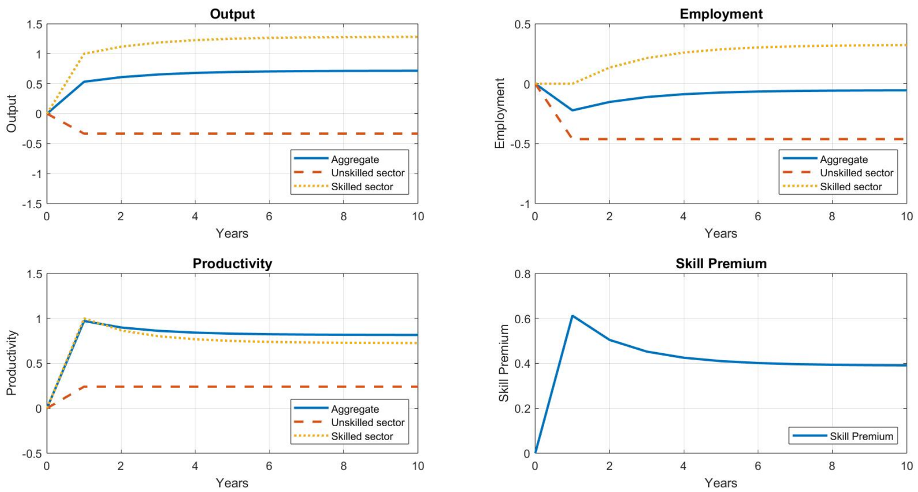

FIGURE 2.9: The effect of skill-biased technology shock on output, employment, productivity, and skill premium
Source: Author's calculation

In the long run, untrained individuals who successfully undergo training and become skilled contribute to increased skilled employment and output. A greater supply of skilled labour in subsequent and steady-state periods partly reduces the excess demand for skilled workers observed in the impact period. As a result, there is a lower level of skilled productivity, aggregate productivity, wages, and skill premium in subsequent and steady-state periods. Figure 2.1 shows productivity grew more than 2 percent from 1998 to 2005, emphasizing that the skill-biased technology shock can persistently drive productivity. The positive change in employment in 2011-2020 in Figure 2.1 also illustrates that employment is better off in the medium and long run due to skill-biased technological upheaval

The effect of a skill-biased technology shock on wage changes is illustrated on the left-hand side of Figure 2.10. The shock causes a sudden positive change in skilled wages during the impact period because it raises the marginal product of the skilled sector and increases the demand for skilled labour. However, the supply of skilled labour cannot adjust immediately in this period, resulting in excess demand for skilled workers. Consequently, skilled wages sharply increase to attract untrained individuals into the training sector. As a result, some unskilled workers voluntarily choose to become unemployed so they can acquire new skills. This creates a shortage in the supply of unskilled labour. Therefore, while the change in unskilled wages is also positive, it is of smaller magnitude than the change in skilled wages. Since both skilled and unskilled wages are rising, the change in average wages is also positive, but to a lesser extent. In subsequent periods, some individuals successfully acquire skills and enter skilled employment. This increases the supply of skilled labour and reduces the excess supply. As the excess supply diminishes, the change in skilled wages shifts from positive to negative. Meanwhile, those who are still untrained become unskilled workers, leading to an excess supply of unskilled labour. This oversupply puts downward pressure on unskilled wages, causing them to drop sharply to zero. Consequently, the change in average wages becomes negative, reflecting the decline in skilled wages. However, over the long run, both skilled and average wages will continue to adjust until their changes stabilize around zero.

On the right-hand side of Figure 2.10, the effect of a positive skill-biased technology shock on the earnings distribution is examined as follows. The short run represents the impact period, while the long run depicts the steady state. In the short run, the shock increases skilled earnings and slightly raises unskilled earnings, resulting in greater income inequality between skilled and unskilled workers. In the long run, the supply of skilled labour increases as a result of successful training, leading to a partial decrease in skilled earnings. Unskilled earnings, however, remain at the same level as in the impact period. As a result, the increase in income inequality is smaller in the long run. Overall, the positive skill-biased technology shock enhances both skilled and unskilled earnings but can also lead to increased income inequality among workers. The initial steady state of wage distribution is stochastically dominated by the new steady state of wage distribution. In empirical evidence, real earnings rapidly grew over 2008-2010 and 2015-2019, which supports the skill-biased technology shock and increased earnings across workers. The significant difference in wage growth by occupation over 1998-2008 in Figure 2.2 also supports the increase in income inequality across workers due to skill-biased technological change, which agrees with Heckman et al. (1998). The data indicates that the growth rate of wages for high-skill workers exceeds the growth rate of average wages. Conversely, the rates of low and medium-skill wages consistently lag behind the average wage rate. This suggests that high-skill workers are better positioned to reap the benefits of ongoing technological advances compared to their counterparts in other skill categories. Consequently, the disparity in wages across different skill levels continues to widen, contributing to a concerning trend of increasing income inequality over time. This effectively explains the impact of the Dot-com crisis illustrated in Figure 2.2. Wage growth reflects greater dispersion in both the short run and long run.

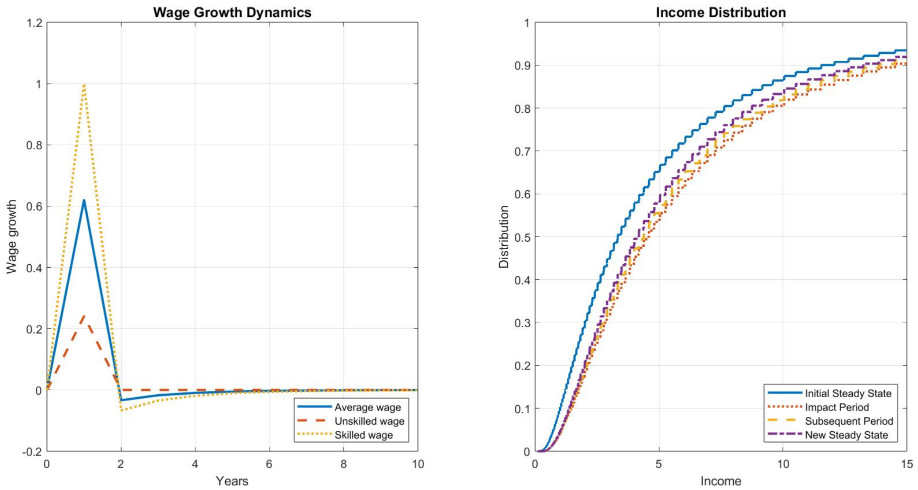

FIGURE 2.10: The effect of skill-biased technology shock on distribution of earning
Source: Author's calculation

The effect of pure shuffle shock

The pure shuffle shock left rotates the iso-earning and reservation cognitive ability earning lines because trained and untrained individuals gain higher earnings from skilled and unskilled employment as well as higher value from training in learning sectors. However, the pure shuffle shock unaffects to the iso-earning cognitive ability and reservation cognitive ability lines. Thus, the pure shuffle shock affects results in the short run but cannot affect the results in the long run.

The impact of a pure shuffle shock on various factors such as output, employment, productivity, and skill premium is illustrated in Figure 2.11. This analysis examines both short-term and long-term effects. In the short term, the shock results in a random redistribution of individuals' skills, with some untrained individuals entering skilled sectors and some trained individuals moving to unskilled sectors. As a result, there is a decrease in skilled employment and output, and a decrease in unskilled employment and output. However, the decrease in skilled employment and output is more significant due to the successful training of unskilled individuals with new high cognitive abilities to work in skilled sectors. Aggregate employment and output decrease in the short run, following the employment decline during the Dot-com crisis, global financial crisis, and Covid-19 pandemic as shown in Figure 2.1. This sudden decrease in skilled productivity is primarily due to a reduction in skilled output being greater than the reduction in skilled employment. In contrast, unskilled productivity experiences a sharp increase because the decrease in unskilled output is less than the decrease in unskilled employment. Then, the aggregate productivity decreases because the change in skilled productivity dominates the change in unskilled productivity. The decrease in productivity does not align with the empirical evidence presented in Figure 2.1. As a result, the skill premium experiences a sudden decrease due to reduced skilled productivity and increased unskilled productivity.

In the long run, untrained individuals who successfully undergo training and become skilled workers contribute to an increase in skilled employment and output. Both skilled employment and output increase over several periods, surpassing the initial steady state, and then stabilize at the same level in the long run. However, the increase in skilled employment outpaces the increase in skilled output, resulting in a gradual decline in skilled productivity in subsequent periods, which eventually stabilizes at the same level in the long run. Conversely, individuals with low cognitive ability immediately enter unskilled sectors, gradually raising unskilled employment and output. Increases in both skilled and unskilled employment drive aggregate employment, similar to the patterns observed during the Dot-com crisis in 2001 and the Covid-19 pandemic. However, unskilled productivity gradually declines because the increase in unskilled output is smaller than the increase in unskilled employment. Then, the aggregate productivity gradually increases because the change in skilled productivity dominates the change in unskilled productivity. Additionally, there is a noticeable increase in productivity following the Covid-19 pandemic, as shown in Figure 2.1. Therefore, the skill premium experiences a slow decline in subsequent periods before eventually increasing and returning to the initial steady state in the long run.

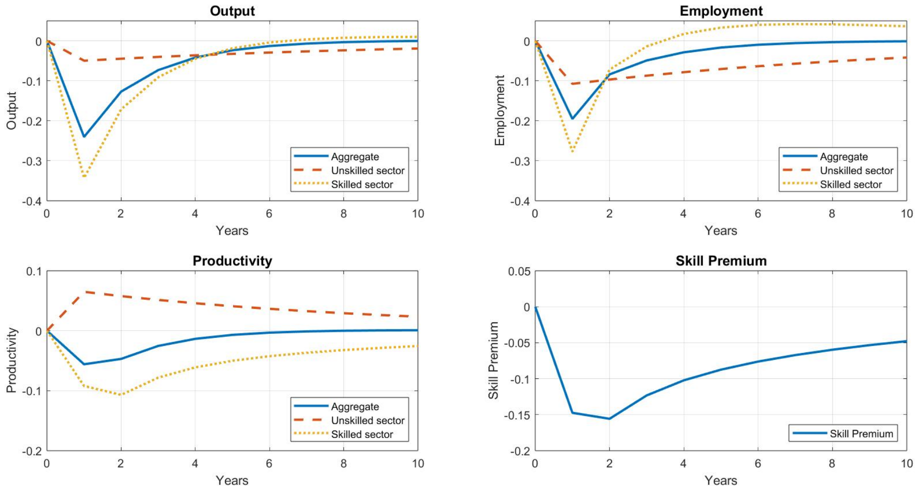

FIGURE 2.11: The effect of pure shuffle shock on output, employment, productivity, and skill premium
Source: Author's calculation

The effect of pure shuffle shock on wage changes is illustrated on the left-hand side of Figure 2.12. During the initial impact period, the shock suddenly decreases skilled productivity because some trained individuals are reallocated to unskilled sectors, while some untrained individuals move to training sectors before working in skilled roles. As a result, the demand for skilled labour in skilled sectors decreases. The shortage of skilled workers leads to a reduction in skilled wages, resulting in a negative change in skilled wages. Conversely, the shock increases unskilled productivity, prompting unskilled sectors to seek more unskilled labour. This excess demand for unskilled labour results in a positive change in unskilled wages. Overall, the change in average wage is negative because the decrease in skilled wages outweighs the increase in unskilled wages. In subsequent periods, some individuals successfully train and secure employment in skilled sectors. Skilled sectors, which have a higher marginal product of skilled labour, then seek to hire more skilled workers. This excess demand for skilled labour drives skilled wages up, resulting in a positive change in skilled wages. Meanwhile, trained individuals working in unskilled sectors pass away and are replaced by new descendants with lower cognitive abilities. Consequently, unskilled productivity declines, leading unskilled sectors to hire fewer unskilled workers. This creates an excess supply of unskilled labour and exerts downward pressure on unskilled wages, resulting in a negative change in unskilled wages. In this context, the average wage change becomes positive as the increase in skilled wages dominates the decrease in unskilled wages. Over the long term, the changes in skilled and average wages gradually approach zero, while the changes in unskilled wages also slowly converge to zero.

On the right-hand side of Figure 2.12, the effect of pure shuffle shock on earnings distribution is demonstrated as follows. The short run represents the impact period, while the long-run corresponds to the steady state. In the short run, the pure shuffle shock can lead to some trained individuals with high earnings working in unskilled sectors and earning less, while some untrained individuals with low earnings may receive training in skilled sectors and consequently earn more. This results in a decrease in the share of individuals with different earnings. As depicted in Figure 2.12, the distribution of earnings in the impact period adjusts to a new pattern more rapidly than the initial steady-state distribution, indicating reduced income inequality. The impact period of wage distribution is stochastically dominated by the initial steady-state of wage distribution. This suggests a reduction in wage dispersion among workers following the global financial crisis and the Covid-19 pandemic, as illustrated in Figure 2.2. However, in the long run, the increase in the supply of skilled labour from newly trained individuals and the supply of unskilled workers with low cognitive abilities gradually increases the share of individuals with different earnings. In Figure 2.12, the distribution of earnings converts to a new pattern at a slower rate in subsequent periods and the steady state, signifying an increase in income inequality and a return to the initial steady state. In conclusion, the pure shuffle shock does not have a lasting impact on earnings distribution in the long run. This effectively explains the impact of the global financial crisis shown in Figure 2.2: in the short run, wage growth exhibits little variation, while in the long run, there is a notable increase in dispersion.

The combined effects of skill-biased technology and shuffle shocks

The impact of both positive skill-biased technology and shuffle shocks on output, employment, productivity, and skill premium is illustrated in Figure 2.13. In this scenario, an unanticipated technological advancement reshuffles on individuals' cognitive ability and raises productivity 20 percent in skilled sector as well. In the short run, these shocks decrease unskilled, skilled and aggregate outputs because the negative impact of shuffle shock outweighs the positive effect of skill-biased technology shock. Employment levels for unskilled, skilled, and aggregate workers decline as both types of shocks negatively affect unskilled and aggregate employment as well as the negative effect of shuffle shock on skilled employment dominate the positive effect of skill-biased technology shock on it. The decline in overall employment can account for the Dot-com crisis, the global

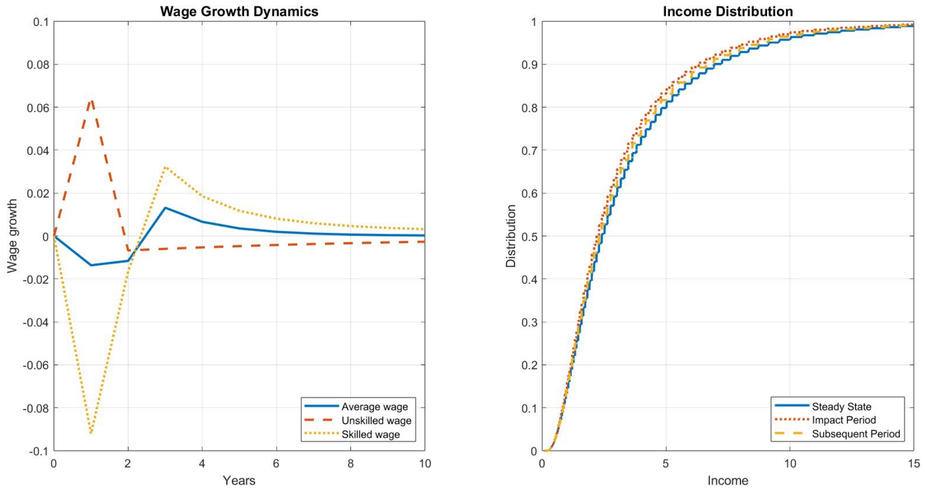

FIGURE 2.12: The effect of pure shuffle shock on wage growth and distribution of earning
Source: Author's calculation

financial crisis, and the Covid-19 pandemic as shown in Figure 2.1. The positive impact of skill-biased technology is overshadowed by the adverse effects of shuffle shocks. Despite the decrease in employment, skilled, unskilled and aggregate productivity increase because skilled, unskilled and aggregate employments decrease more than their outputs. The rise in overall productivity can also explain the Dot-com crisis, global financial crisis, and the Covid-19 pandemic. Additionally, the increased productivity among unskilled workers is due to unskilled employment decreases more significantly than unskilled output. Finally, the skill premium decreases as unskilled productivity rises more than skilled productivity.

In the long run, the positive impact of skill-biased technology shock dominate the negative effect of shuffle shock because the skill-biased technology shock is permanent while the shuffle shock is temporary. Both skilled and aggregate outputs turn to increase at a diminishing rate, ultimately reaching higher steady-state values. In contrast, unskilled output remains permanently at a lower steady-state level. Employment for skilled, unskilled, and aggregate workers tends to increase and settle at new steady-state values. The increase in overall employment can help explain the Dot-com crisis and the impact of the Covid-19 pandemic. Skilled and aggregate productivity also continue to rise at a diminishing rate, reaching higher steady-state values, while unskilled productivity experiences a slight decrease, settling at a new lower steady-state level. The rise in aggregate productivity can account for the Global Financial Crisis. Finally, the skill premium turn to increase and tnen reach to higher steady state because the skilled productivity is higher than the unskilled productivity in the long run. Moreover, the overall skill

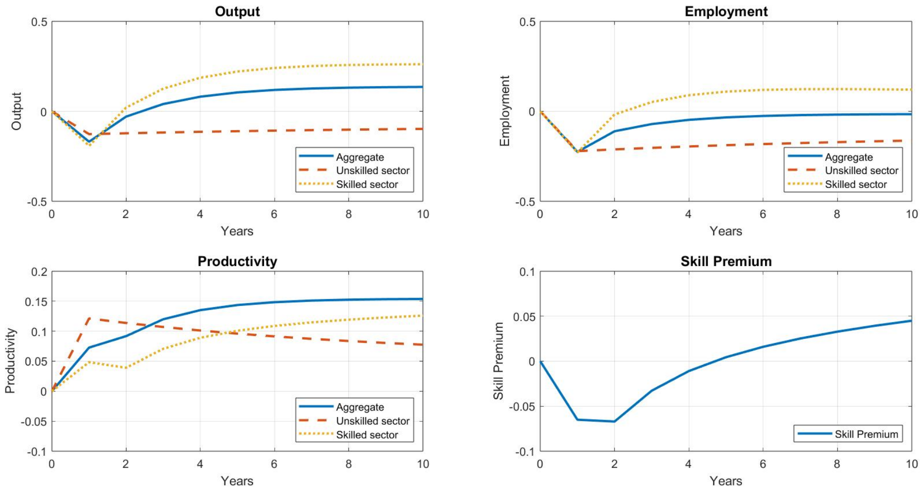

FIGURE 2.13: The combined effects of skill-biased technology and shuffle shocks on output, employment, productivity, and skill premium
Source: Author's calculation

premium graph looks like J-curve and can explain the behavior of skill premium over last two decades in the Figure 2.3.

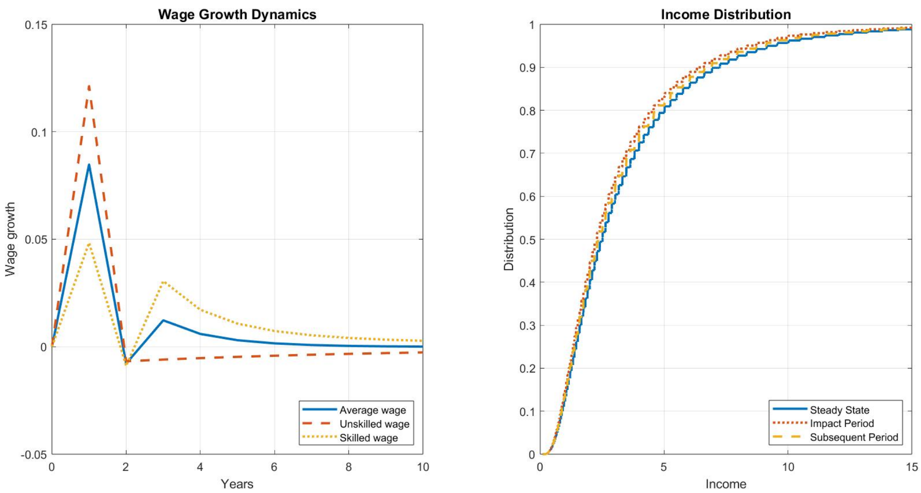

FIGURE 2.14: The combined effects of skill-biased technology and shuffle shocks on wage growth and distribution of earning
Source: Author's calculation

The combined effects of skill-biased technology and shuffle shocks on wage changes are illustrated on the left-hand side of Figure 2.14. During the initial period, these shocks lead to a significant increase in skilled wages because the positive impact of the skill-biased technology shock outweighs the negative effects of the shuffle shock. Meanwhile, the changes in unskilled wages are also significantly positive, as both skill-biased technology and shuffle shocks move in the same direction. As a result, these shocks positively influence the average wage change. In subsequent periods, however, all wage changes sharply drop into negative value. This occurs because the effect of the skill-biased technology shock continues to dominate the shuffle shock in the short term. After the second period, though, all wage changes begin to align with the direction of the shuffle shock, as its influence ultimately prevails over the skill-biased technology shock in the long run.

On the right-hand side of Figure 2.14, we illustrate the impact of both shocks on the distribution of earnings, both immediately and in subsequent periods, as well as in the long run. Overall, the results are similar to those observed with just the pure shuffle shock on the distribution of earnings. However, the positive skill-biased technology shock causes all lines to shift to the right, indicating that this shock leads to increased income inequality. This explains the effect of the Covid-19 pandemic on the growth of wage across occupations in Figure 2.2 well. Wage growth shows slight dispersion in the short run but not in the long run.

2.7.2 Comparing effects of shocks in case different correlation of physical and cognitive abilities

The positive correlation suggests that individuals' physical and cognitive abilities are linked. If an individual's physical ability is weak, then their cognitive ability is also weak. Conversely, if an individual's physical ability is strong, their cognitive ability is also strong. As a result, they are more adaptable in changing jobs between unskilled and skilled sectors. The negative correlation suggests that individuals' physical and cognitive abilities are different. When an individual has weak physical abilities, their cognitive abilities tend to be strong, and vice versa. This means they are less likely to switch between skilled and unskilled sectors.

We study the effects of different correlations of physical and cognitive abilities by employing the multivariate normal probability density function (mvnpdf), which shows results as follows. In Figure 2.15, we illustrate the densities of individuals with three different correlation of physical and cognitive abilities. The first, second, and third columns represent no correlation, positive correlation, and negative correlation, respectively. In the first row, we show the joint normal probability density of physical and cognitive abilities with correlations of 0, 0.9, and -0.9, respectively.

The three-dimensional results in the first row can be represented as two-dimensional graphs in the second row. No correlation is represented as a circle, while positive and negative correlations are represented as ovals. We observe that the blue solid line represents the reservation cognitive ability line, showing how untained individuals choose between training in the learning sector or working in the unskilled sector, while the red dotted line represents the iso-earning cognitive ability line, illustrating how trained individuals choose to work in skilled sectors or unskilled sectors. The blue solid line is higher than the red dotted line because training processes are costly.

Effect of skill-biased technology shock with varying correlation

The positive correlation amplifies the effect of a skill-biased technology shock on output, employment, productivity, skill premium, and distribution of earnings in the second column, while a negative correlation impedes the effect of a skill-biased technology shock on those variables in the third column.

Firstly, we will examine the second row of Figure 2.16. As the correlation shifts from no correlation to positive correlation, the positive impact of skill-biased technology shock on skilled employment increases, while the negative impact on unskilled employment worsens. This is because untrained individuals with strong cognitive abilities are more likely to choose training in sectors that require learning. Consequently, there is a greater

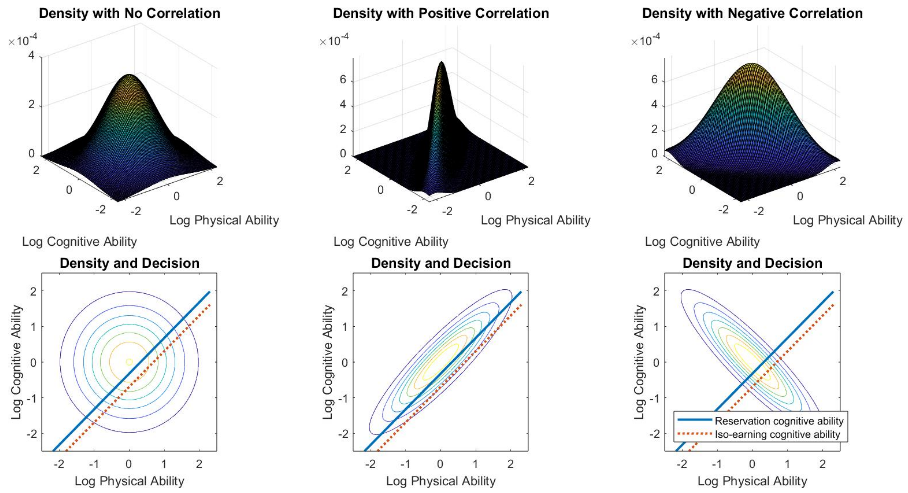

FIGURE 2.15: The change in correlation of abilities and the decision making
Source: Author's calculation

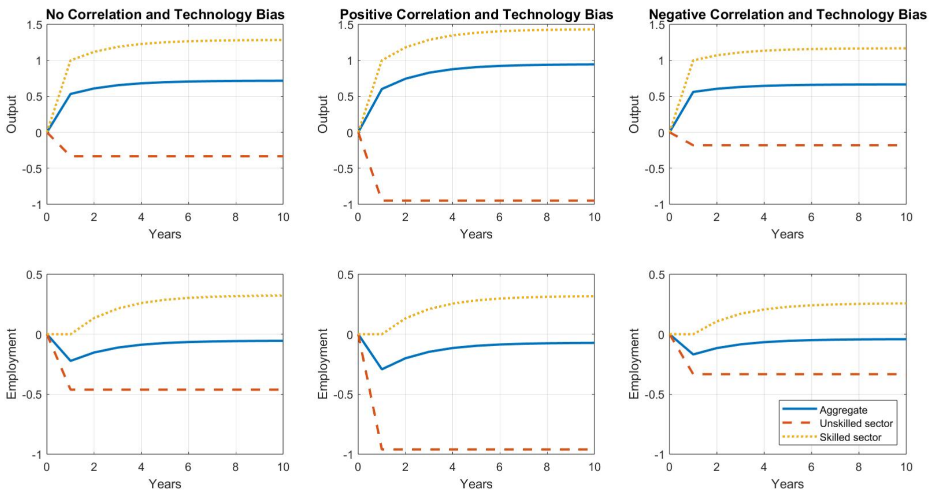

FIGURE 2.16: The change in correlation of abilities and the effect of skilled-biased technology shock on output and employment
Source: Author's calculation

decrease in unskilled employment and a more significant increase in skilled employment, resulting in a slight overall decrease in aggregate employment due to the substantial decrease in unskilled employment. On the other hand, a decrease in correlation from no correlation to negative correlation weakens the positive effect of skill-biased technology shock on skilled employment and mitigates the negative effect on unskilled employment. This is because untrained individuals with stronger physical abilities but weaker cognitive skills are less likely to succeed in training in learning sectors. As a result, there is a smaller increase in skilled employment and a lesser decrease in unskilled employment, leading to a tiny increase in aggregate employment because of the small decrease in unskilled employment.

In addition, let's consider the output in the first row of the Figure 2.16. Positive correlation results in an increase in skilled output and a decrease in unskilled output. This leads to an overall increase in aggregate output because the increase in skilled output outweighs the decline in unskilled output. Conversely, negative correlation impedes an increase in skilled output and moderates a decrease in unskilled output, resulting in a reduction in aggregate output as the decrease in unskilled output is larger than the increase in skilled output.

Moving on to the productivity and skill premium in Figure 2.17, positive correlation enhances both unskilled, skilled, and aggregate productivity. Moreover, the increase in skilled productivity is greater than the increase in unskilled productivity, leading to an increase in skill premium when correlation increases. Alternatively, negative correlation hinders both skilled, unskilled, and aggregate productivity. However, the decrease in skilled productivity outweighs the decrease in unskilled productivity, leading to a decrease in the skill premium in this case.

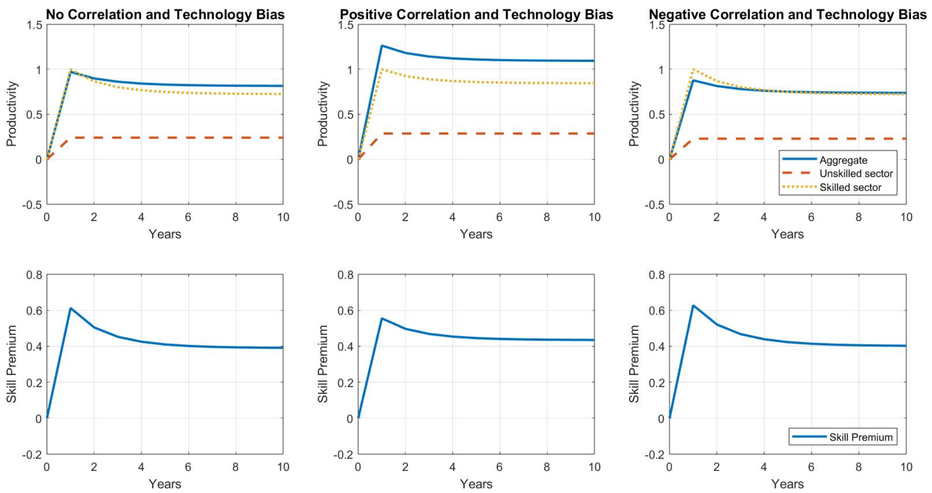

FIGURE 2.17: The change in correlation of abilities and the effect of skilled-biased technology shock on productivity and skill premium
Source: Author's calculation

The effects of a skill-biased technology shock on wage changes are illustrated in the first row of Figure 2.18, considering three scenarios: no correlation, positive correlation, and negative correlation between cognitive and physical abilities. We observe that the effects of a positive skill-biased technology shock on skilled wage changes are similar across all three scenarios. However, the magnitudes of wage changes for unskilled workers and the average worker are larger when there is a positive correlation, while these magnitudes are smaller in the case of a negative correlation.

In second row of Figure 2.18, the impact of a positive skill-biased technology shock, in the absence of correlation between physical and cognitive abilities, suggests that the resulting inequality in income distribution will be greater in subsequent periods, including the new steady state, than in the initial steady state. Skilled workers will see an increase in earnings, while unskilled workers' earnings will remain unchanged. Conversely, a positive correlation between physical and cognitive abilities can reduce income inequality not only in subsequent periods and the new steady state but also in the initial steady state. On the other hand, a negative correlation between physical and cognitive abilities will exacerbate income inequality in both the initial and new steady states as well as in subsequent periods.

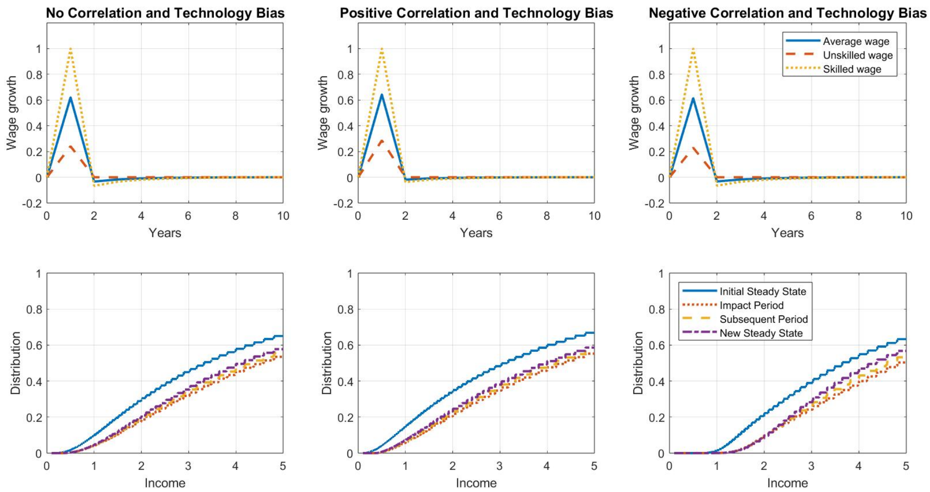

FIGURE 2.18: The change in correlation of abilities and the effect of skilled-biased technology shock on wage growth and distribution of earnings
Source: Author's calculation

Effect of pure shuffle shock with varying correlation

A strong positive correlation increases the impact of a pure shuffle shock on output, employment, productivity, skill premium, and earnings distribution in the second column. Conversely, a negative correlation hinders the impact of a pure shuffle shock on these variables in the third column.

In Figure 2.19, we can see the impact of pure shuffle shock on output and employment. The scenarios include no correlation in the first column, positive correlation in the second column, and negative correlation in the third column. When comparing employments in the second row, we observe that the decrease in skilled, unskilled, and aggregate employments is larger in the case of positive correlation compared to no correlation. This is because individuals with indifferent levels between physical and cognitive abilities can easily switch between skilled and unskilled sectors. On the other hand, the decrease in skilled, unskilled, and aggregate employments is smaller in the case of negative correlation, as individuals with dominant physical or cognitive abilities are less likely to switch sectors.

Moving on to the first row, we find that the decrease in skilled, unskilled, and aggregate outputs is larger in the case of positive correlation compared to no correlation. Conversely, the decrease in skilled, unskilled, and aggregate outputs is smaller in the case of negative correlation. Notably, in the positive correlation case, a substantial decrease in unskilled employment during the impact period results in an over-shooting of skilled employment in subsequent periods. This pattern is also observed in the outputs, where a larger decrease in unskilled output during the impact period leads to an over-shooting of skilled output in subsequent periods.

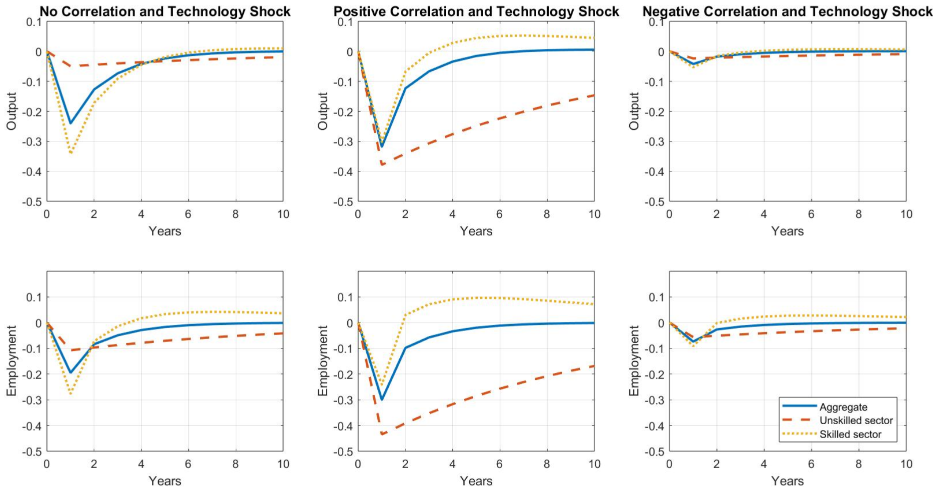

FIGURE 2.19: The change in correlation of abilities and the effect of pure shuffle shock on output and employment
Source: Author's calculation

In Figure 2.20, we can observe the impact of pure shuffle shock on productivity and skill premium under different correlation scenarios. Specifically, the first column represents no correlation, the second column represents positive correlation, and the third column represents negative correlation.

When there is a positive correlation, an increase in unskilled productivity results in a larger increase in unskilled employment compared to the no-correlation scenario. This is because there is a significant increase in unskilled employment despite a decrease in unskilled output. Conversely, a decrease in skilled productivity is smaller in the positive correlation case compared to the no-correlation scenario due to a considerable decrease in skilled output despite a decrease in skilled employment. Then, the decrease in aggregate productivity of the positive correlation case is smaller than the scenario of no correlation.

Conversely, when there is a negative correlation, an increase in unskilled productivity yields a smaller increase in unskilled employment compared to the no-correlation scenario. This is because there is a significant decrease in unskilled output despite a decrease in unskilled employment. Notably, in the case of negative correlation, skilled productivity increases, whereas it decreases in both the no-correlation and positive correlation scenarios, as a decrease in unskilled output is less than a decrease in unskilled employment. Then, the aggregate productivity of negative correlation temporarily increases because a decrease in aggregate employment is larger than a decrease in aggregate output in the short run.

As for skill premium, in the case of negative correlation, it increases in the impact period and decreases in subsequent periods, while in the no-correlation and positive correlation scenarios, it sharply decreases in the impact period, slightly decreases in the following periods, and eventually returns to its initial steady state in the long run.

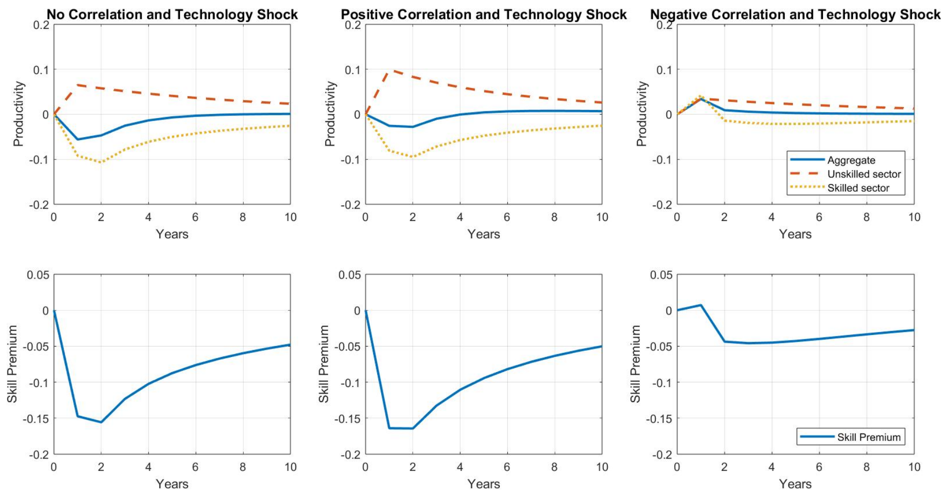

FIGURE 2.20: The change in correlation of abilities and the effect of pure shuffle shock on productivity and skill premium
Source: Author's calculation

The effects of pure shuffle shock on wage changes, considering no correlation, positive correlation, and negative correlation between cognitive and physical abilities, are illustrated in the first row of Figure 2.21. When comparing positive correlation with no correlation, we find that the decrease in skilled wage change is smaller, while the increase in unskilled wage change is larger, leading to an overall increase in average wage change. In the subsequent period, the increase in skilled wage change is smaller, the decrease in unskilled wage change is larger, and average wage change declines to a lower level. Following this, skilled and average wage changes increase to a smaller extent before gradually decreasing to zero, while unskilled wage change also gradually approaches zero. In contrasting negative correlation with no correlation, skilled, unskilled, and average wage changes increase at the same rate during the impact period. In the following period, all three types of wage changes begin to decrease. However, the decrease in skilled wage change is more significant than that of unskilled wage change, whereas the reduction in average wage change is moderate. Notably, skilled, unskilled, and average wage changes quickly approach zero in the next period.

The second row of Figure 2.21 shows the impact of pure shuffle shock on the distribution of earnings in scenarios with different correlations. In the first column, there is no correlation, in the second column there is a positive correlation, and in the third column there is a negative correlation. Even though the distributions of earnings in steady state, impact, and subsequent periods are similar in the no-correlation scenario compared to the positive and negative correlation scenarios, there are differences. In the positive correlation scenario, the distribution of earnings grows faster than in the no-correlation scenario, while in the negative correlation scenario, the distribution of earnings grows slower. This implies that income inequality is less in the positive correlation scenario compared to the no-correlation scenario, while income inequality is greater in the negative correlation scenario compared to the no-correlation scenario.

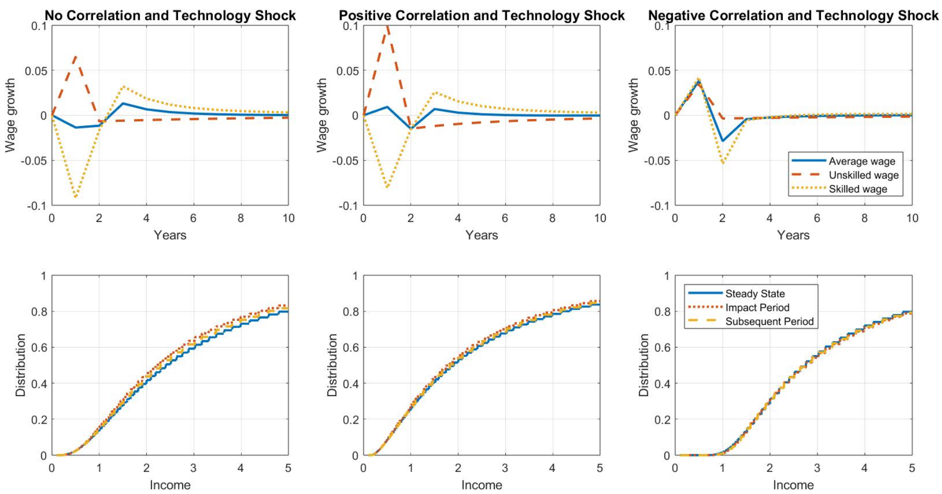

FIGURE 2.21: The change in correlation of abilities and the effect of pure shuffle shock on wage growth and distribution of earnings
Source: Author's calculation

2.8 Conclusions

This study takes a unique approach by presenting a comprehensive analysis of a Roy model that incorporates cognitive and physical abilities, along with skilled, unskilled, and learning sectors. The aim is to analyse the impact of technological upheavals on the labour market, output, and income distribution. It specifically focuses on skill-biased technology shocks and pure shuffle shocks, investigating the theoretical correlation between the two types of abilities and the impact of technological advancements. The main findings reveal that a positively skilled-biased technology shock significantly impacts decision-making, output, employment, productivity, skill premium, and the distribution of earnings. It also influences the relationship between physical and cognitive abilities. In the short run, the positive shock leads to increased demand for skilled labour, resulting in higher wages for skilled workers and a decrease in relative earnings for unskilled workers, thereby prompting more untrained individuals to seek training in learning sectors.

The study underscores the long-term benefits of training unskilled individuals on skilled employment and output. Successful training effectively reduces the excess demand for skilled workers, leading to a sustained level of skilled productivity, wages, and skill premium in subsequent periods. Despite initially boosting income inequality between skilled and unskilled workers in the short run, a positive skill-biased technology shock shows a reassuring trend of diminishing the gap in the long run due to the expanding supply of skilled labour resulting from successful training.

In the short term, a pure shuffle shock results in a random redistribution of skills, which leads to a greater decrease in skilled employment and output than in unskilled employment and output. This results in a sudden decrease in the skill premium. In the long run, trained individuals contribute to an increase in skilled employment and output, while untrained individuals gradually raise unskilled employment and output. However, unskilled productivity gradually declines, leading to a slow decline in the skill premium before eventually increasing and returning to the initial steady state.

Moreover, the impact of pure shuffle shock on earnings distribution is significant. In the short run, the shock leads to a decrease in the share of individuals with different earnings, resulting in reduced income inequality. However, in the long run, the distribution of earnings returns to the initial steady state, indicating that the pure shuffle shock does not have a lasting impact on earnings distribution. These insights provide a comprehensive understanding of the labour market dynamics and the potential impacts of technological shocks. This equips us with valuable knowledge for further study and policy-making efforts.

Additionally, the correlation between individuals' physical and cognitive abilities is a topic of interest to the study's contribution. A positive correlation, for instance, suggests that strong physical abilities are not only beneficial in their own right but also linked to strong cognitive abilities, thereby enabling adaptability between job sectors. Conversely, a negative correlation indicates that weak physical abilities could be accompanied by strong cognitive abilities, which in turn, reduces the likelihood of sector switching. The result vividly illustrates the impact of altering skill-biased technology shock with different correlation scenarios between physical and cognitive abilities.

Finally, the study's three-dimensional results can be conveniently visualized as two-dimensional graphs, with no correlation depicted as a circle and positive and negative correlations as ovals. The influence of a skill-biased technology shock on output, employment, productivity, skill premiums, and earnings distribution intensifies with a positive correlation, while a negative correlation restricts its effects. Overall, these positive shocks lead to an increase in income inequality, favoring skilled workers, but ultimately highlight the importance of training and adaptability in shaping a more equitable labor market. By investing in our workforce and embracing continuous learning, we can pave the way for a more prosperous and balanced economic future.

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2.10 Appendix

2.10.1 Decision-making

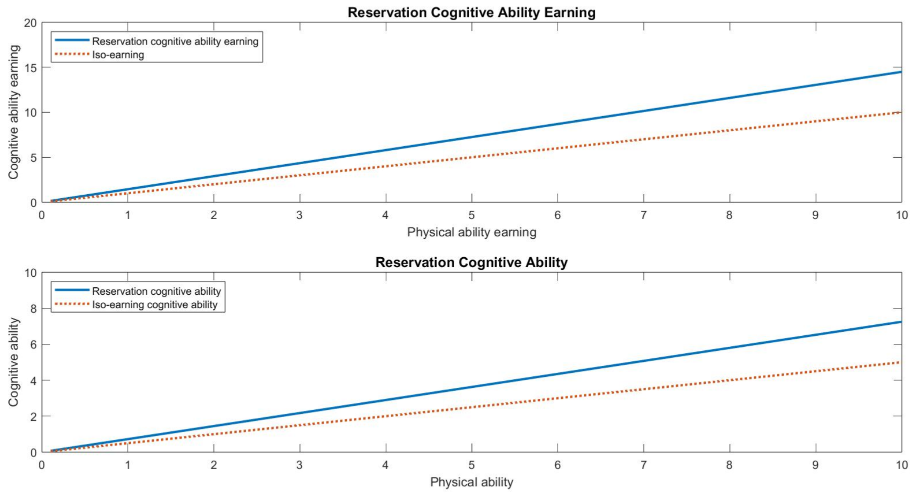

FIGURE 2.22: The reservation cognitive ability earning and the reservation cognitive ability
Source: Author's calculation

Figure 2.22 illustrates the reservation earning levels associated with cognitive abilities, distinguishing between those that fall below a certain threshold and those that exceed it. The red dashed line on the graph marks the point of indifference, where individuals have no preference between earning potential from physical abilities and cognitive abilities, as their earnings are equal at this point. Trained individuals are inclined to pursue careers in skilled sectors when their expected earnings from cognitive abilities meet or exceed those generated from physical labour. This decision highlights the significant impact of cognitive skills in the job market, especially for individuals with advanced training or education, as they seek roles that more closely align with their higher earning potentials.

The red dashed line in the graph below represents a threshold of cognitive ability, which is derived from the red dashed line depicted in the graph above. Similarly, the blue solid line in the graph below reflects the standards set by the blue solid line above. Individuals who have undergone training and possess cognitive abilities that meet or exceed the level indicated by the red dashed line are more inclined to pursue careers in skilled sectors, such as engineering, healthcare, or information technology, where specialized knowledge and expertise are required. Conversely, those whose cognitive abilities fall below this threshold are typically drawn to unskilled sectors, which may include positions in manual labour, hospitality, or retail, where the demands for specialized training are lower. This distinction highlights the significant role cognitive ability plays in career decision-making and labour market outcomes.

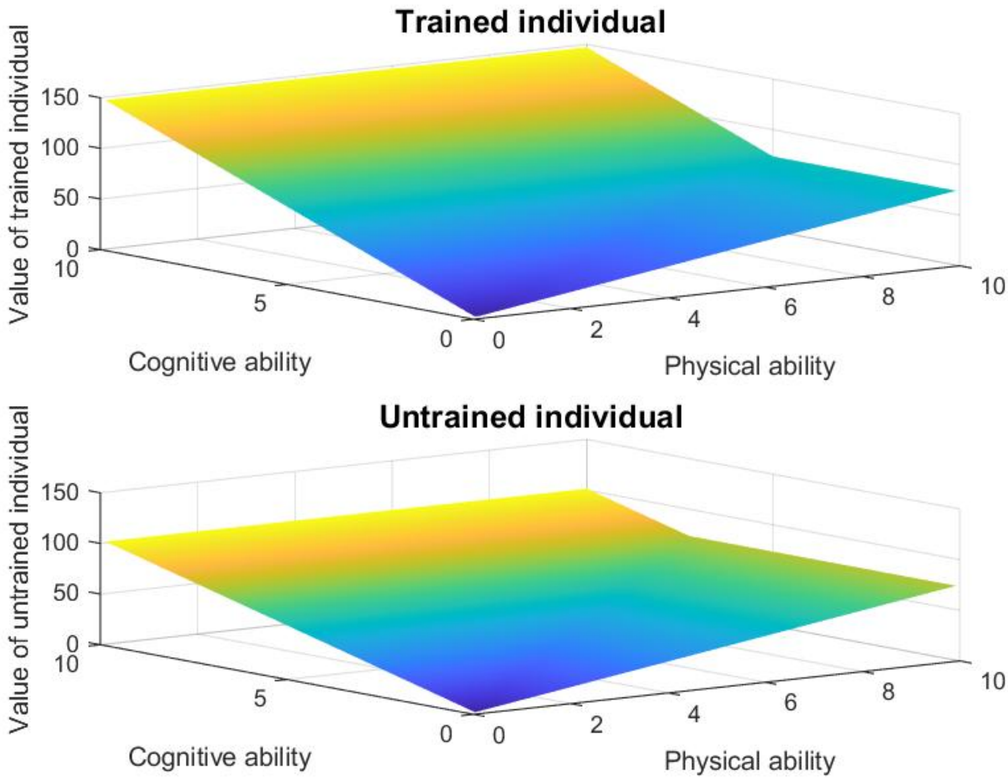

FIGURE 2.23: The values of trained and untrained individuals
Source: Author's calculation

Untrained individuals often choose to pursue training in various learning sectors when they anticipate that their potential earnings will meet or exceed the reservation cognitive ability earning line. This line represents the minimum income level that justifies the time and resources spent on training. Notably, the reservation cognitive ability earning line is set higher than the iso-earning line, which indicates that individuals weigh the opportunity cost associated with training against the risks of failing to achieve the desired outcomes. The opportunity cost involves the wages they could have earned by working in an unskilled job during the training period. Consequently, untrained individuals who expect their potential earnings to fall below this critical threshold, represented by the blue solid line, are more likely to make the decision to enter unskilled sectors immediately. This choice reflects a rational assessment of their current skills relative to the expected benefits of further training.

Figure 2.23 demonstrates the optimal value associated with individuals who have received training compared to those who have not, highlighting the differences across a spectrum of physical and cognitive abilities. The data illustrates that untrained individuals who possess cognitive abilities exceeding the reservation cognitive ability line tend to have a lower optimal value than their trained counterparts with similar abilities. This lower valuation can be attributed to the opportunity costs untrained individuals face; while they possess the potential for higher cognitive capability, they are engaged in unskilled labour and miss out on training opportunities that could enhance their skills and employability. Consequently, this reinforces the importance of training and education in maximizing individual potential in the labour market.

2.10.2 The Set of Parameters

The model consists of parameters collected in the following table. These parameters are contributed by Andolfatto and Smith (2001). Additionally, some parameters are calibrated based on the model environment setup.

Table 2.1: The Set of Parameters

Parameters	**Description**	**Value**
β	Consumer's subjective discount factor	0.96
δ	Consumer's probability of death	0.10
γ_1	Additional productivity of technology in unskilled sector	1.00
γ_2	Additional productivity of technology in skilled sector	1.0, 1.2, 2.0
μ_{a_1}	Mean of physical ability (mvnpdf)	0
μ_{a_2}	Mean of cognitive ability (mvnpdf)	0
$\sigma^2_{a_1}$	Variance of physical ability (mvnpdf)	1.00
$\sigma^2_{a_2}$	Variance of cognitive ability (mvnpdf)	1.00
$\sigma_{a_1 a_2}$	Covariance of physical and cognitive abilities (mvnpdf)	0, 0.9, -0.9
θ	Probability of absorbed knowledge	0.35
w_1	Additional output of physical ability	1.00
w_2	Additional output of cognitive ability	2.00

Chapter 3

Earning Inequality and Redistribution

This study conducts an extended version of the Roy model, which integrates multiple dimensions of physical and cognitive abilities, as discussed in Chapter 2. It includes various economic redistribution tools, such as progressive income tax systems, targeted transfer payments, and learning subsidies aimed at enhancing skill development.The extended Roy model sheds light on how these redistribution mechanisms affect individuals' decisions regarding education, training, and employment across different sectors. The study finds that a nation's overall output tends to improve with higher tax rates. This improvement is primarily attributed to individuals having limited access to capital markets, which forces them to spend all their income rather than save or invest. Tax and transfer programs help mitigate this issue by redistributing funds from higher-income individuals to lower-income households, thereby enhancing income equality, and boosting overall economic activity.

3.1 Introduction

During the late $20th$ century, particularly starting in the $1970s$, income inequality in the United States began to undergo a significant and pronounced rise. This shift was driven by a combination of complex factors. Globalization, characterized by the increasing interconnectedness of economies, played a pivotal role in altering the distribution of wealth. Moreover, rapid technological advancements reshaped industries and labor markets, leading to disparities in earnings. Policy decisions also contributed to the widening income gap, as there was a trend favoring capital over labor. Additionally, the weakening of labor unions, deregulation, and tax policies that predominantly favored the wealthy further exacerbated the situation, magnifying the discrepancy in income distribution.

Income inequality has been a pressing and escalating concern in the United States over the past several decades. According to the U.S. Bureau of Labor Statistics, as illustrated in Figure 3.1, the top 20 percent of earners in the U.S. observed a substantial 333 percent increase in their incomes from 1984 to 2022, whereas the bottom 20 percent experienced a comparatively modest 212 percent of income growth during the same period. This data indicates a glaring rise in income inequality, highlighting the ever-widening disparity between the wealthiest individuals and the lower-income population.

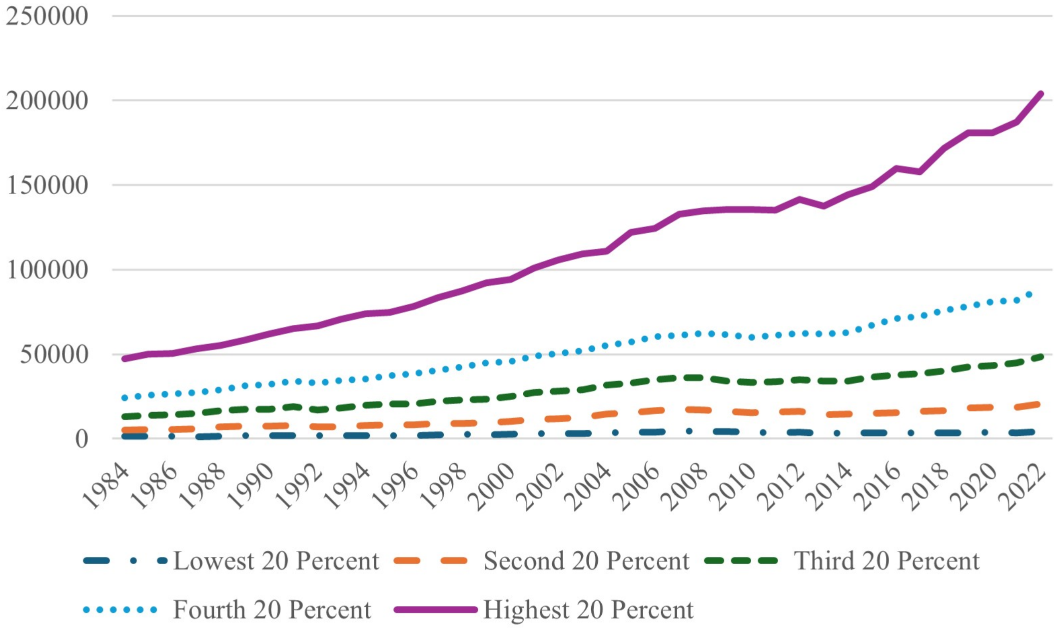

FIGURE 3.1: Wages by Quintiles of Income Before Taxes (Annual U.S. Dollars)
Source: U.S. Bureau of Labor Statistics, Income Before Taxes: Wages and Salaries by Quintiles of Income Before Taxes, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CXU900000LB0102M, June 17, 2024.

Figure 3.2 provides a visual representation of the proportion of income by quintiles before taxes from the year 1984 to 2022. The data analysis reveals a significant income disparity, indicating that the top 20 percent of earners receive an income that is over 30 times greater than that of the bottom 20 percent of earners. This extraordinary difference illustrates a substantial and persistent income gap over the entire duration covered by the data, pointing to persistent inequality in income distribution over the years.

The increasing wealth gap has many negative effects on society. It weakens social unity, adds to political division, and restricts economic advancement. A high level of income inequality is linked to worse health results, lower educational achievements, and higher crime rates. Additionally, it may result in a consolidation of political influence among the rich, making the cycle of inequality even more entrenched.

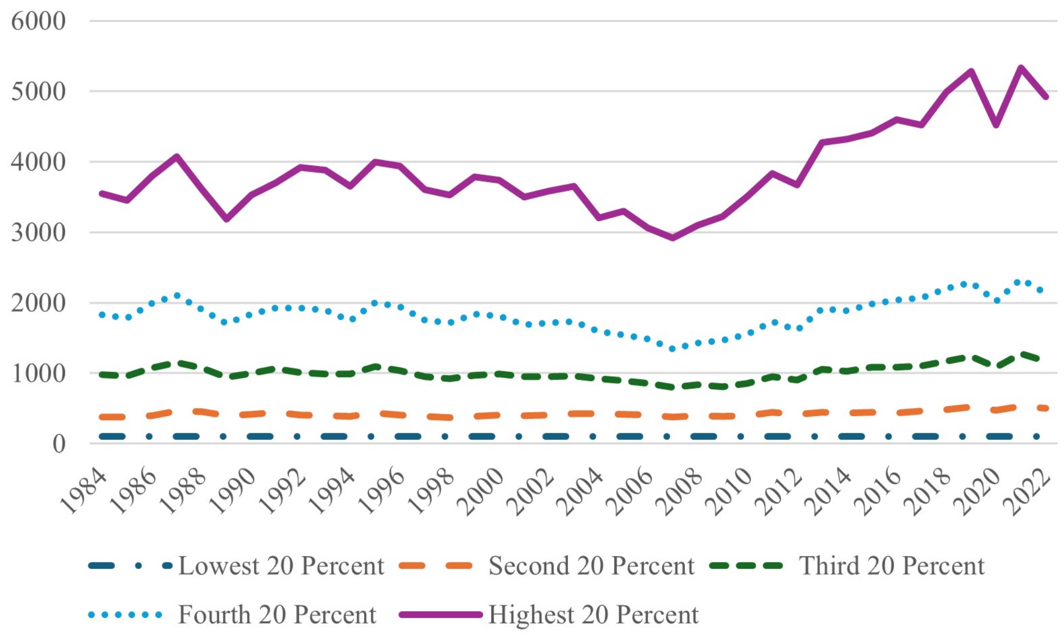

FIGURE 3.2: Comparing Quintiles of Income Before Taxes with Lowest Income (Lowest 20 percent = 100)
Source: U.S. Bureau of Labor Statistics, Income Before Taxes: Wages and Salaries by Quintiles of Income Before Taxes, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CXU900000LB0102M, June 17, 2024.

Income tax policy plays a crucial role in addressing income inequality within a society. A progressive income tax system is characterized by higher income earners paying a larger percentage of their income in taxes, which can effectively contribute to the redistribution of wealth and the reduction of economic disparities. Historically, the United States had a more progressive tax system, evidenced by top marginal tax rates exceeding 90 percent during the 1950s and 1960s. However, over the years, these rates have been significantly reduced, with the current top marginal tax rate standing at 37 percent. Proponents argued that these cuts would stimulate economic growth and benefit all Americans. However, critics argue that the benefits have disproportionately favored the wealthy, exacerbating income inequality.

In response to the issue of income inequality, there has been a growing consensus to increase taxes on the wealthy. This has resulted in the consideration of various policy measures aimed at addressing this disparity. Among the proposed solutions are calls for imposing higher marginal tax rates on top earners, introducing a wealth tax, and closing tax loopholes that predominantly benefit the wealthy. Advocates argue that these measures would ensure a more equitable distribution of contributions from the wealthiest individuals and corporations to finance public goods and services, ultimately promoting a fairer and more balanced economic landscape.

The data presented in Figure 3.3 illustrates the distribution of income quintiles after taxes compared to the lowest income bracket from 1984 to 2022. It reveals that the top 20 percent of earners receive incomes that are roughly $12-16$ times higher than those in the bottom 20 percent. Although income tax plays a significant role in reducing income disparities, the gaps remain quite pronounced. This suggests that further fiscal interventions are needed to effectively mitigate these disparities and ensure more equitable income distribution.

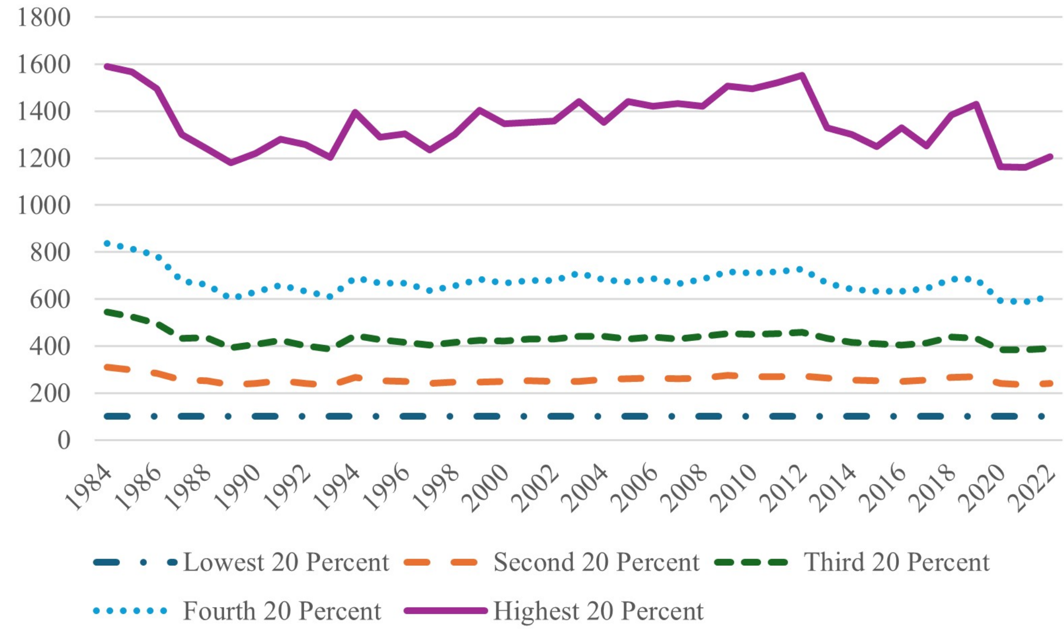

FIGURE 3.3: Comparing Quintiles of Income After Taxes with Lowest Income (Lowest 20 percent = 100)
Source: U.S. Bureau of Labor Statistics, Income After Taxes: Income After Taxes by Quintiles of Income Before Taxes, retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/CXUINCAFTTXLB0102M, June 17, 2024.

Transfer payments are government-initiated financial aids to individuals in need, including Social Security, unemployment benefits, food stamps (SNAP), and housing assistance. These payments hold immense significance in reducing poverty and providing essential support to lower-income households. For instance, Social Security has significantly decreased poverty rates among elderly Americans. However, critics argue that certain transfer programs can foster dependence and discourage workforce participation. To mitigate these concerns, reforms can be focused on ensuring that transfer payments meet the recipients' needs adequately and are structured in a way that motivates individuals to enter or re-enter the workforce.

Training subsidies are a crucial tool in addressing income inequality by promoting economic mobility. These subsidies are designed to provide financial support for education and vocational training programs, with the goal of improving the skills and employability of the workforce. In today's economy, which is increasingly reliant on technology and advanced skill sets, such training is essential for workers to maintain their competitiveness in the job market. The subsidies support job seekers with comprehensive training and employment services to help them secure meaningful employment. By aligning training programs with the specific needs of the labor market, these subsidies can effectively narrow the skills gap and contribute to a reduction in unemployment and underemployment.

Combining income taxation, transfer payments, and training subsidies offers a comprehensive approach to reducing income inequality. Progressive taxation ensures that those who can afford to contribute more to public revenue do so, enabling funding for essential social programs. Transfer payments provide immediate relief to those in need, ensuring a minimum standard of living and reducing poverty. Meanwhile, training subsidies invest in the future workforce, enhancing skills and promoting long-term economic mobility.

To tackle income inequality in the United States, a comprehensive approach is needed. This should involve progressive income taxation, well-planned transfer payments, and specific training subsidies. Each of these elements has a distinct role in redistributing wealth, supporting marginalized communities, and improving economic mobility. By using these strategies together, policymakers can strive for a fairer society where economic opportunities are available to everyone, thus promoting both social stability and economic growth.

This research study encompasses a thorough exploration of creating a Roy model that integrates three key fiscal instruments - income taxation, transfer payments, and training subsidies. The objective of this study is to delve into the impact of fiscal policy on individuals' decision-making regarding their occupations, as well as the redistribution of labor and output markets and earnings.

3.2 Literature Review

There have been several studies discussing optimal taxation and multidimensional abilities. For instance, Scheuer (2014) demonstrates that applying different non-linear tax schedules to profits and labour income can lead to optimal outcomes without causing production distortions. Jacquet and Lehmann (2023) introduce the 'allocation perturbation' method, which reshapes the understanding and implementation of tax policies by determining optimal non-linear income tax schedules based on multidimensional individual characteristics. Additionally, Lindenlaub and Postel-Vinay (2023) examine the sorting process in a labour market and highlight that workers tend to sort into jobs aligning with their specific skill mix rather than just their overall skill level. Lindenlaub (2017) studies the matching of workers with different skills to jobs requiring specific skill combinations and identifies significant technology shifts contributing to changes in worker-job matching and wage disparity. Finally, in the study by Thuemmel (2023), the optimal taxation of robots, other capital, and labour income is examined. The study suggests that distorting robot adoption through a robot tax or subsidy can positively impact welfare by compressing wages and reducing income-tax distortions on labour supply. It is found that the most significant gains come from optimising labour income taxes.

Building on the previous studies, the works of Rothschild and Scheuer in 2013, 2014, and 2016 provide further insights into optimal redistribution and income taxation in various economic models. The 2013 study focuses on the implementation of a single non-linear income tax to achieve the constrained Pareto frontier in the presence of self-selection into occupational sectors. This study offers practical implications for optimal income taxation. In 2014, a framework for optimal income taxation in multi-sector economies with externalities was introduced, along with discussions on the implications for tax schedules in various scenarios. The 2016 study proposes a framework for determining the best way to tax income in situations where individuals can earn money through traditional work and rent-seeking activities, highlighting the identification of optimal income tax rates and the implications of correcting rent-seeking externalities through the income tax code.

Mayr (2025) examines how changes in capital taxes affect overall welfare in the economy. The study finds that lower substitutability between capital and labor in production leads to larger changes in wages and financial returns, while also mitigating the negative effects of raising capital taxes. This suggests that higher capital taxes could be more beneficial, especially when the effects of wealth distribution are more pronounced. Analyzing the U.S. data, the study shows that the bottom two-thirds of the income distribution could actually benefit from higher capital taxes, particularly if the additional revenue is distributed equally among everyone. The research establishes goals within a model to calculate optimal rates for capital and labor taxes. The results indicate that we should implement higher capital taxes and also increase labor taxes, albeit making them less progressive compared to the current system.

In addition, countless studies have delved into the realms of wage subsidies, negative income tax, income inequality, and optimal income tax structures. Notably, the pioneering work of Berglas (1976), Cremer and Gahvari (1996), and Cevik and Correa-Caro (2020) has not only identified and addressed the limitations of current models but has also offered potential solutions, pushing the boundaries of our understanding. Their empirical evidence on income distribution and the impact of fiscal policy in China and other emerging market economies is particularly enlightening. Moreover, their incorporation of tax evasion into the optimum general income tax problem has yielded definitive results, shedding new light on audit and tax structures for high- and low-wage individuals.

Fleurbaey and Maniquet (2006) and Golosov et al. (2013) have not only examined the optimal taxation for income redistribution based on efficiency and fairness principles but have also explored the optimal redistribution of income inequality caused by search and matching frictions in the labour market. With its practical implications, their research delves into the justification for capital income taxation based on goods preferred by those with high abilities. It analyses the forces determining optimal commodity taxation. Findings from these studies reveal that optimal capital income tax rates vary based on the intertemporal elasticity of substitution; however, the overall welfare gains of using optimal capital taxes are relatively small, a crucial insight for policymakers.

The importance of optimal taxation and income redistribution programs is highlighted in the recent literature, which covers various aspects directly related to these topics. For instance, it discusses the role of workfare in optimal tax policy, the impact of screening mechanisms on participation in redistribution programs, and the significance of stigma in program efficiency (Heady, 1993; Hamilton, 2010; and Heathcote and Tsujiyama, 2021). Additionally, it explores the influence of feasibility on tax policy, the optimal shape of income tax and transfer schedules in an environment with distinct roles for public and private insurance, and the relevance of tax progressivity over lump-sum transfers. Furthermore, it discusses the impact of affordable higher education on income inequality and the challenges of implementing tax subsidy schemes for optimal taxation (Mirrlees, 1971; Mirrless, 1976; Hendel et al., 2005). Understanding the impact of commodity taxes in the presence of an optimal income tax is crucial.

The relevance of optimal income taxation in addressing concerns about inequality is also discussed. Recent literature covers various areas, such as multidimensional abilities and endogenous wages, asymmetric information, production and consumption externalities from labour effort, market power and rents, behavioural phenomena related to the income tax schedule, optimal income transfers, and issues relating to social welfare and utility functions (Kaplow, 2024; Støstad and Cowell, 2024). Additionally, this literature underscores the profound impact of economic inequality on societal outcomes, such as crime rates, economic growth, and political polarisation, and proposes treating economic inequality as an externality.

The study by Akcigit et al. (2024) sheds light on the positive impact of combining R&D subsidies with higher education subsidies to support economically disadvantaged individuals in pursuing research careers. Caner and Okten (2013) conducted a study on the distribution of benefits of publicly funded higher education in Turkey, stressing the urgent need for measures to address socioeconomic disparities. Additionally, the theory of the distribution of earnings developed by Becker and Chiswick (1966) emphasises the importance of maximising economic welfare through investments in human capital, pointing towards the potential for positive policy interventions to promote equal opportunities and economic growth.

Blundell (2022) underscores the critical importance of addressing poor wage progression for lower- and middle-educated workers in tackling labour market inequality. To effectively address this issue, a balanced combination of tax and welfare benefit policies, along with human capital policies, minimum wage regulations, and labour market regulations, is recommended. Meanwhile, the study by Caucutt and Kumar (2003) explores the impact of increasing higher education subsidies in the US on inequality, welfare, and efficiency. Their findings suggest that further increases in higher education subsidies may need to be carefully reconsidered based on the conducted experiments.

The US Aid to Families with Dependent Children (AFDC) program and Temporary Assistance to Needy Families (TANF) have been found to decrease the willingness of single mothers to engage in work by 10-50 per cent. Even in the absence of AFDC, non-recipients were already working relatively low hours. According to Hendel et al. (2005), easing financial constraints for postsecondary education may increase wage inequality by affecting job market signalling. This aspect has yet to be thoroughly explored in recent studies on wage inequality.

It may be a good idea to tax relatively poor individuals to help cover the educational costs of the more affluent population. This is because there is likely a collaborative relationship between workers with different skill levels, and the less wealthy benefit from the overall increase in skill levels. This can be achieved through a financing arrangement that benefits both parties (Johnson, 1984). Economists were comfortable discussing the tradeoff between equality and efficiency, including the impact of tax rates on labour supply. Decision makers were reluctant to interfere with wage determination but were open to progressive taxes and income-tested transfers. Efforts have been made to estimate the labour supply costs of equalising through taxes and transfers, but the results still need to be determined (Rivlin, 1975).

A consistent human capital framework can help us find better ways to allocate government expenditures, tax subsidies, and deductions for human capital. Steuerle (1996) has given some helpful tax examples, but we can also use this framework to compare different educational policies, such as grants versus loans, subsidies available by age, subsidy rates for different types of education, and subsidy rates for different types of educational institutions. While education provides valuable skills and information, it is important to consider that it can also contribute to inequality. Finding a balance between efficiency and distribution is crucial, as excessive educational spending may worsen inequality and reduce national income. Moreover, reducing educational screening could lead to on-the-job screening, which might lower national output without addressing inequality (Stiglitz, 1975).

This chapter aims to investigate the effect of fiscal instruments such as income taxation, transfer payments, and training subsidies on the individuals' occupational choice and redistribution of labour and output markets and earnings. To follow this objective, the study is organised as follows: Section 3.3 presents the study model, Section 3.4 presents the steady state and transition dynamics of the model, the parameters' values and meanings are provided in Section 3.5, Section 3.6 shows results and their interpretations, and the study is concluded in Section 3.7.

3.3 The Model

The baseline Roy model of this study was created by Andolfatto and Smith (2001). This study examined the extended Roy model, developed by incorporating multidimensional physical and cognitive abilities in Chapter 2, by introducing redistribution tools such as income tax, transfer payment and learning subsidy. The extended Roy model with these tools investigates how redistribution approaches affect individuals' decisions about training or working in different sectors. The study also explores the role of these tools in relocating employment, output, productivity and earnings across households.

3.3.1 The economic environment

The model assumes a discrete time and infinite horizon, which is denoted as $t = 0, 1, 2, ..., \infty$. The population is normalized to be one, but each individual is assumed to have two types of innate abilities: physical ability $a_1 \geq 0$ and cognitive ability $a_2 \geq 0$. Cognitive ability a_2 implies the market value of an individual's cognitive ability depending on the reservation cognitive ability a_{2R}, while a_1 can determine the reservation cognitive ability. For cognitive ability, each level of ability will earn a different income level denoted as y_t. In each period, their consumption c_t is assumed to consume all their income. Additionally, individuals face a discount factor denoted as $0 < \beta < 1$ and a death rate denoted as $0 < \delta < 1$. However, the population is always constant because deaths are replaced by descendants. Therefore, the individual welfare is represented by the discount stream of consumption c_t over time,

$$E\sum_{t=0}^{\infty}\beta^t(1-\delta)^t log(c_t) \quad . \tag{3.1}$$

Then, we can rewrite above equation to recursive form as following value function after tax,

$$V_t^T(a_1,a_2) = log(c_t) + \beta(1-\delta)E_t V_{t+1}^T(a_1,a_2) \quad .$$

The individuals face with an income tax rate $\tau \in (0,1)$ and then they have to pay income tax τy_t. However, they get transfer payment from government T_t. Thus, the disposable income is given by $y_t^d = (1-\tau)y_t + T_t$. The individuals can consume all disposable income y_t^d for every period, then the value function after tax also can be illustrated as,

$$V_t^T(a_1,a_2) = log\{(1-\tau)y_t + T_t\} + \beta(1-\delta)E_t V_{t+1}^T(a_1,a_2) \quad . \tag{3.2}$$

Based on Andolfatto and Smith (2001), the economy consists of three sectors: the unskilled sector (Sector 1), skilled sector (Sector 2), and the learning sector (Sector 3). Trained individuals can either contribute their physical ability to the unskilled sector, producing unskilled output w_1a_1, or supply their cognitive ability to the skilled sector, generating skilled output w_2a_2, depending on their relative earnings in each sector (y_2/y_1). Untrained individuals will work in the unskilled sector if their cognitive ability is lower than a certain threshold $(a_2 < a_{2R}^T)$, or they can join the learning sector if their cognitive ability meets or exceeds this threshold $(a_2 \geq a_{2R}^T)$, with the aim of becoming trained individuals working in the skilled sector in the next period.

3.3.2 Decision-making

In this model, we develop a recursive structure that allows us to conceptualize optimal decision-making as a dynamic programming problem. This approach involves making decisions at various stages over time. Within this framework, we identify three distinct value functions that illustrate the benefits associated with different levels of physical ability a_1 and cognitive ability a_2 after the implementation of income tax rate τ and lump-sum transfer T. These value functions are as follows: the value of being a trained individual after the scheme, $V^T(a_1,a_2)$; the value of being an untrained individual pursuing an education after taxation and subsidy, $S^T(a_1,a_2)$; and the value of being an untrained individual working in the unskilled sector after tax and transfer scheme, $Q^T(a_1,a_2)$.

For trained individuals, they choose to work in unskilled sector when physical earning is higher than cognitive one ($w_2a_2 < w_1a_1$) or allocate their time in skilled sector in case that cognitive earning is larger than or equal to physical one ($w_2a_2 \geq w_1a_1$). Thus, they try to maximize their life-time income or earning as the value function after tax for trained individual with physical ability a_1 and cognitive ability a_2,

$$V^T(a_1, a_2) = \max\{log[(1-\tau)w_1a_1 + T], log[(1-\tau)w_2a_2 + T]\} + \beta(1-\delta)V^T(a_1, a_2) \tag{3.3}$$

We can rearrange the last equation to get the value of trained individual after tax as the discount value of maximum income earning,

$$V^T(a_1, a_2) = \frac{\max\{log[(1-\tau)w_1a_1 + T], log[(1-\tau)w_2a_2 + T]\}}{1-\beta(1-\delta)} \quad . \tag{3.4}$$

As the income tax and rebate scheme is assumed to be identical in levying and transferring on both physical and cognitive earnings, it cannot impact the decision-making of trained individuals. The trained individuals with cognitive ability less than value of physical ability in term of cognitive ability ($a_2 < a_{2I} \equiv \omega a_1$) will choose to work in the unskilled sector, while trained individuals with cognitive ability larger than or equal to value of physical ability in term of cognitive ability ($a_2 \geq a_{2I} \equiv \omega a_1$) prefer working in skilled sector instead. Thus, the scheme does not affect the iso-earning cognitive ability ($a_{2I} \equiv \omega a_1$).

For untrained individuals, they make decisions between choosing work in unskilled sector or training in learning sector after tax,

$$K^T(a_1, a_2) = \max\{(S^T(a_1, a_2), Q^T(a_1, a_2)\} \quad . \tag{3.5}$$

The value function after tax and subsidy for untrained individual who chooses training,

$$S^T(a_1, a_2) = log(T) + \beta(1-\delta)[(1-\theta)K^T(a_1, a_2) + \theta V^T(a_1, a_2)] \quad . \tag{3.6}$$

The value function after tax for untrained individual who works in unskilled sector,

$$Q^T(a_1, a_2) = log[(1-\tau)w_1a_1 + T] + \beta(1-\delta)K^T(a_1, a_2) \quad . \tag{3.7}$$

For stationarity, the value of training after tax and transfer is

$$S^T(a_1,a_2) = \frac{[1-\beta(1-\delta)]log(T) + \beta(1-\delta)\theta log[(1-\tau)w_2 a_2 + T]}{[1-\beta(1-\delta)][1-\beta(1-\delta)(1-\theta)]} \quad , \quad a_2 \geq a_{2R}^T \; . \tag{3.8}$$

For stationarity, the value of untraining after tax is

$$Q^T(a_1,a_2) = \frac{log[(1-\tau)w_1 a_1 + T]}{1-\beta(1-\delta)} \quad , \quad a_2 < a_{2R}^T \quad . \tag{3.9}$$

The reservation cognitive ability level after tax is characterised by

$$a_{2R}^T = \frac{1}{(1-\tau)w_2}\left[\left(\frac{[(1-\tau)w_1 a_1 + T]^{[1-\beta(1-\delta)(1-\theta)]}}{T^{[1-\beta(1-\delta)]}}\right)^{\frac{1}{\beta(1-\delta)\theta}} - T\right] \quad . \tag{3.10}$$

The equation above shows that the scheme can affect the level of reservation cognitive ability. To analyse the impact of the scheme on reservation cognitive ability, we can apply the technique of partial differentiation. This begins with differentiating the reservation cognitive ability with respect to the income tax rate,

$$\begin{aligned}\frac{\partial a_{2R}^T}{\partial \tau} = & \frac{1}{(1-\tau)^2 w_2}\left(\frac{[(1-\tau)w_1 a_1 + T]^{[1-\beta(1-\delta)(1-\theta)]}}{T^{[1-\beta(1-\delta)]}}\right)^{\frac{1}{\beta(1-\delta)\theta}} \\ & - \frac{T}{(1-\tau)^2 w_2} \\ & - \left[\left(\frac{[1-\beta(1-\delta)(1-\theta)]\, w_1 a_1}{\beta(1-\delta)\theta(1-\tau)w_2 T^{[1-\beta(1-\delta)]}\,[(1-\tau)w_1 a_1 + T]^{\beta(1-\delta)(1-\theta)}}\right) \times \right. \\ & \left.\left(\frac{[(1-\tau)w_1 a_1 + T]^{[1-\beta(1-\delta)(1-\theta)]}}{T^{[1-\beta(1-\delta)]}}\right)^{\frac{1-\beta(1-\delta)\theta}{\beta(1-\delta)\theta}}\right] \quad \lesseqgtr \quad 0 \; .\end{aligned}$$

The equation indicates that a tax changes the relative attractiveness of training in two opposing ways: the scaling (denominator) channel raises the threshold while the outside option erosion channel lowers it. In the first channel, training leads to a skilled wage later, which after tax, is affected by the tax rate. Since the formula divides by the skilled wage, a higher tax rate mechanically increases the threshold; we need a higher ability to justify training when the skilled wage is taxed more. This can be understood as follows: Since the government takes more from the skilled wage, training becomes less rewarding, so we need a higher level of ability to make it worthwhile.

In the second channel, the tax also impacts unskilled labor income, reducing the return on not training. As a result, the option of remaining unskilled becomes less appealing, making training appear more attractive even for individuals with lower abilities. This effectively lowers the threshold. We could say: even though training is taxed, unskilled work is also taxed, so training no longer has to outperform a favorable unskilled alternative — they would train even at lower ability.

Then, we can derive the partial difference with respect to the lump-sum transfer as follows,

$$\frac{\partial a_{2R}^{T}}{\partial T} = \frac{1}{(1-\tau)w_2}\Bigg\{\frac{1}{\beta(1-\delta)\theta}\left(\frac{[(1-\tau)w_1a_1+T]^{[1-\beta(1-\delta)(1-\theta)]}}{T^{[1-\beta(1-\delta)]}}\right)^{\frac{1-\beta(1-\delta)\theta}{\beta(1-\delta)\theta}} \times$$
$$\Bigg[\left(\frac{[1-\beta(1-\delta)(1-\theta)]}{T^{[1-\beta(1-\delta)]}\,[(1-\tau)w_1a_1+T]^{\beta(1-\delta)(1-\theta)}}\right)$$
$$-\left(\frac{[1-\beta(1-\delta)]\,[(1-\tau)w_1a_1+T]^{[1-\beta(1-\delta)(1-\theta)]}}{T^{[2-\beta(1-\delta)]}}\right)\Bigg]$$
$$-1\Bigg\} \quad \leq \quad 0 \ .$$

The equation of partial differentiation indicates that changes in transfer subsidies have a non-positive effect on the reservation cognitive ability level. The transfer acts as a form of insurance and is not linked to an individual's skill choice. It diminishes the risks associated with not pursuing training. This has three key effects: it lowers the opportunity cost of training, reduces the marginal advantage of unskilled work, and neutralizes risk for marginal agents. Firstly, with a steady financial cushion, if training does not succeed or if there is a delay in earning income, an individual's consumption remains secure. Therefore, a person does not need as much cognitive ability to be willing to take risks, effectively lowering the threshold for pursuing training.

Secondly, when a portion of income is guaranteed (denoted as T), the appeal of unskilled work diminishes, particularly for individuals with lower cognitive abilities. This again contributes to a reduction in the threshold for training. Lastly, marginal or borderline agents are typically those who fear the worst-case scenario. A higher transfer amount makes the potential negative outcomes of failure less daunting, which directly decreases the barrier for their willingness to pursue training. Therefore, transfers encourage training by lowering the reservation cognitive ability required, allowing more individuals to cross this threshold.

3.4 Steady States

We define $\lambda_t^T(a_2|a_1)$ and $\mu_t^T(a_2|a_1)$ are the densities afer tax of trained and untrained individuals with cognitive ability a_2 conditional on physical ability a_1 at date t. We assume these densities are restricted by the densities of population $g(a_2|a_1) = \lambda_t^T(a_2|a_1) + \mu_t^T(a_2|a_1)$. In each period, some untrained individuals with $a_2 \geq a_{2R}^T$ become trained individuals at rate θ, then the stock of trained individuals with $a_2 \geq a_{2R}^T$ at the end period is determined by $\lambda_t^T(a_2|a_1) + \theta\mu_t^T(a_2|a_1)$.

The evolution of trained individuals with cognitive ability a_2 conditional on physical ability a_1 is characterised by:

$$\lambda_{t+1}^T(a_2|a_1) = \begin{cases} (1-\delta)[(1-\theta)\lambda_t^T(a_2|a_1) + \theta g(a_2|a_1)] & \text{for } a_2 \geq a_{2R}^T \\ (1-\delta)\lambda_t^T(a_2|a_1) & \text{for } a_2 < a_{2R}^T \end{cases} \tag{3.11}$$

The equation 3.11 describes the distribution of trained individuals in a future period after accounting for taxes. This distribution is derived from the current pool of trained individuals who are still alive but possess cognitive abilities lower than the reservation cognitive ability threshold after tax. For individuals whose cognitive abilities meet or exceed the reservation cognitive ability after tax, the distribution of trained individuals in the next period consists of those who successfully complete their training, adjusted by subtracting the portion of the previous distribution that overlaps with this successful training group to avoid double counting. In other words, the distribution reflects the remaining trained individuals along with those untrained individuals who train successfully.

For simplicity, we can consider some initial distribution $\lambda_0^T(a_2|a_1)$ and then the density of trained individuals with cognitive ability $a_2 < a_{2R}^T$ is $\lambda_t^T(a_2|a_1) = (1-\delta)^t\lambda_0^T(a_2|a_1)$ while the density for $a_2 \geq a_{2R}^T$ is given by

$$\begin{aligned} \lambda_t^T(a_2|a_1) &= \phi^t\lambda_0^T(a_2|a_1) + [\phi^{t-1} + \phi^{t-2} + ... + 1](1-\delta)\theta g(a_2|a_1) \\ &= \phi^t\lambda_0^T(a_2|a_1) + [(1-\phi^t)/(1-\phi)](1-\delta)\theta g(a_2|a_1) \quad , \end{aligned} \tag{3.12}$$

where $\phi \equiv (1-\delta)(1-\theta)$. Thus, the evolution of distribution function is become:

$$\lambda_t^T(a_2|a_1) = \begin{cases} \rho(1-\phi^t)g(a_2|a_1) + \phi^t\lambda_t^T(a_2|a_1) & \text{for } a_2 \geq a_{2R}^T \\ (1-\delta)^t\lambda_0^T(a_2|a_1) & \text{for } a_2 < a_{2R}^T \end{cases} \tag{3.13}$$

where $0 < \rho \equiv (1-\delta)\theta/(\delta+\theta-\delta\theta) < 1$ represents the proportion of individuals who are still alive and successfully trained, relative to the total number of new descendants and their numerator. Equation 3.13 indicates that the distribution of trained individuals at time t after tax is influenced by the fraction of those who remain alive over time t, starting from the initial distribution of trained individuals with lower cognitive ability. Conversely, for individuals with higher cognitive ability, the distribution of trained individuals at time t is composed of both a fraction of the population who are new descendants and who are still alive and successfully trained, as well as a fraction of previously trained individuals who have remained alive over the same time period. When time converges to infinity as $t \to \infty$, the distribution will converge to a steady-state distribution as $\lambda_t^T \to \lambda^T$. Thus, the steady-state distribution of trained individuals after tax is characterised by:

$$\lambda^T(a_2|a_1) = \begin{cases} \rho g(a_2|a_1) & \text{for } a_2 \geq a_{2R}^T \\ 0 & \text{for } a_2 < a_{2R}^T \end{cases} \tag{3.14}$$

Equation 3.14 suggests that if all individuals possess low cognitive ability, there will ultimately be no trained individuals in the long run. Conversely, if individuals have sufficiently high cognitive ability, the proportion of trained individuals will correspond to a fraction of the population that survives and successfully undergoes training. Then, follow the restriction of densities, the steady-state distribution of untrained individuals after tax is determined by:

$$\mu^T(a_2|a_1) = g(a_2|a_1) - \lambda^T(a_2|a_1) \quad . \tag{3.15}$$

For the steady-state employment and output in each sector after tax and transfer, we can determine those by using the distributions of trained and untrained individuals. Thus, the long-run measure of untrained individuals working in the unskilled sector after tax is characterised by:

$$\begin{aligned} N_1^T &= \int_0^\infty \int_0^{a_{2R}^T} \mu^T(a_2|a_1)g(a_1)da_2da_1 \\ &= \int_0^\infty \int_0^{a_{2R}^T} \mu^T(a_1, a_2)da_2da_1 \quad , \end{aligned} \tag{3.16}$$

and the long-run measure of output in the unskilled sector after tax is given by:

$$\begin{aligned} Y_1^T &= w_1 \int_0^\infty \int_0^{a_{2R}^T} a_1\mu^T(a_2|a_1)g(a_1)da_2da_1 \\ &= w_1 \int_0^\infty \int_0^{a_{2R}^T} a_1\mu^T(a_1, a_2)da_2da_1 \quad . \end{aligned} \tag{2.17}$$

In the skilled sector, the measure of trained individuals working in the skilled sector after tax in the long run is determined by:

$$\begin{aligned} N_2^T &= \int_0^\infty \int_{a_{2R}^T}^\infty \lambda^T(a_2|a_1)g(a_1)da_2da_1 \\ &= \int_0^\infty \int_{a_{2R}^T}^\infty \lambda^T(a_1,a_2)da_2da_1 \quad , \end{aligned} \tag{3.18}$$

and the output in the skilled sector after tax in the long run is measured by:

$$\begin{aligned} Y_2^T &= w_2 \int_0^\infty \int_{a_{2R}^T}^\infty a_2\lambda^T(a_2|a_1)g(a_1)da_2da_1 \\ &= w_2 \int_0^\infty \int_{a_{2R}^T}^\infty a_2\lambda^T(a_1,a_2)da_2da_1 \quad . \end{aligned} \tag{3.19}$$

We notice that N_1^T is increasing in a_{2R}^T while N_2^T is decreasing in a_{2R}^T. Since the effect of change in reservation cognitive ability on unskilled employment dominates the effect of another one, hence the total employment after tax $N^T \equiv N_1^T + N_2^T$ is increasing in a_{2R}^T.

For the long-run productivity after tax, it can be determined by the ratio of long-run output to employment after tax in each sector. The productivity in unskilled sector is given by:

$$P_1^T = \frac{Y_1^T}{N_1^T} \quad . \tag{3.20}$$

While the productivity in skilled sector after tax is denoted by the average of the skilled output and employment after tax:

$$P_2^T = \frac{Y_2^T}{N_2^T} \quad . \tag{3.21}$$

The wage differential or skill premium after tax in the long run is measured by the ratio of skilled to unskilled productivity after tax:

$$\Pi^T = \frac{P_2^T}{P_1^T} \quad . \tag{3.22}$$

For long-run government revenue and transfer subsidy in each type of worker, we can determine those by using the distributions of trained and untrained individuals. Thus, the steady-state tax on unskilled workers is measured by tax revenue of untrained individuals:

$$\begin{aligned} R_1 &= \tau w_1 \int_0^{\infty} \int_0^{a_{2R}^T} a_1 \mu^T(a_2|a_1) g(a_1) da_2 da_1 \\ &= \tau w_1 \int_0^{\infty} \int_0^{a_{2R}^T} a_1 \mu^T(a_1, a_2) da_2 da_1 \quad . \end{aligned} \tag{3.23}$$

For the skilled workers, their tax in the long run is determined by levying tax on trained individuals working in the skilled sector:

$$\begin{aligned} R_2 &= \tau w_2 \int_0^{\infty} \int_{a_{2R}^T}^{\infty} a_2 \lambda^T(a_2|a_1) g(a_1) da_2 da_1 \\ &= \tau w_2 \int_0^{\infty} \int_{a_{2R}^T}^{\infty} a_2 \lambda^T(a_1, a_2) da_2 da_1 \quad . \end{aligned} \tag{3.24}$$

The total tax revenue is summation of skilled and unskilled tax $R = R_1 + R_2$.

For simplicity, we assume that all government revenue is used for transfers, with no other government expenditures. Then, the government's break-even or balanced budget requires that tax revenue equals transfer payments.:

$$R = T \quad . \tag{3.25}$$

For the distribution of earnings after tax in the long run, it can be characterised by the ratio of amount of worker with different disposabel income level to total employment. Then, we define $H^T(y^d)$ is the share of workers with different disposable earnings y^d to labor force as follow,

$$H^T(y^d) = \begin{cases} 0 & \text{for } T \leq y^d < (1-\tau)a_1 w_1 + T \\ [\int_0^{\infty} \int_0^{a_{2R}^T} \mu^T(a_1, a_2) da_2 da_1]/N^T & \text{for } (1-\tau)a_1 w_1 + T \leq y^d < (1-\tau)a_{2R}^T w_2 + T \\ [(1-\rho)\int_0^{\infty} \int_0^{a_{2R}^T} \mu^T(a_1, a_2) da_2 da_1 + & \\ \rho \int_0^{\infty} \int_{y/w_2}^{\infty} \lambda^T(a_1, a_2) da_2 da_1]/N^T & \text{for } (1-\tau)a_{R2}^T w_2 + T \leq y^d < \infty \end{cases} \tag{3.26}$$

The equation above suggests that the income tax and rebate scheme can affect the distribution of earnings in the following ways. First, no one is inclined to work if the level of disposable income is lower than what they could earn through physical ability after accounting for taxes and transfers. When the level of disposable income falls between the earnings from physical ability after the scheme and the reservation wage for those with cognitive ability, the share of workers consists of a proportion of untrained individuals, which includes both those with low cognitive ability and those with physical ability, relative to the total aggregate employment. If the level of disposable income is at least equal to the reservation wage for cognitive ability after accounting for taxes and transfers, the distribution of workers is composed of (1) The share of untrained individuals, adjusted for new descendants (which refers to the sum of new descendants plus individuals who are alive and have trained successfully) who possess cognitive abilities lower than the reservation wage, and those with all physical abilities, relative to total employment as well as (2) The share of trained individuals, adjusted for those who remain alive and have successfully completed training (which includes the total number of new descendants and those who have survived and trained successfully) who possess cognitive abilities at least equal to the threshold for skilled wages, alongside individuals with all physical abilities relative to aggregate employment.

3.5 Parameterisation

This model operates on a yearly basis. The set of parameters utilized in this model is collected from Andolfatto and Smith (2001) along with our calibration, which is presented in Table 3.1 along with their definitions and values.

In accordance with Andolfatto and Smith (2001), the consumer's subjective discount factor β is set at 0.96, reflecting an average yearly risk-free rate of about 4 percent. The consumer's probability of death parameter δ is set at 0.10, indicating that approximately 10 percent of individuals die each year. This is balanced by an increase in descendants by roughly the same percentage. The probability of acquiring knowledge θ is set at 0.35, which means that untrained individuals pursuing education successfully train about 35 percent of the time.

The additional output from physical ability ω_1 is set to 1, while the additional output from cognitive ability ω_2 is set to 2, suggesting that the relative wages of physical ability to cognitive ability ω is approximately 0.5. We assume a zero mean for both physical and cognitive abilities (μ_{a_1}, μ_{a_2}), zero covariance $(\sigma_{a_1 a_2})$, and a unit variance for both abilities $(\sigma^2_{a_1}, \sigma^2_{a_2})$.The income tax rate τ varies between 0.01 and 0.99, allowing for an analysis of the effects of changing tax rates.

This model is coded and executed using Matlab program. To obtain the numerical results, we first compute the probability density function of a bivariate normal distribution for physical and cognitive abilities. After incorporating a tax and rebate scheme into the model, we calculate several key values: the reservation cognitive ability, the iso-earning cognitive ability, the value of a trained individual, and the value of an untrained individual pursuing education.

Next, we use the probability density function that reflects the physical and cognitive abilities of individuals in relation to the reservation cognitive ability. This allows us to approximate the value of an untrained individual working in the unskilled sector and evaluate the choices available to untrained individuals between training and working in the unskilled sector.

We then assess the steady-state distributions of trained and untrained individuals to determine the steady-state shares of each group. Using these steady-state shares in conjunction with the reservation cognitive ability, we compute additional steady-state variables. Finally, we vary the income tax rate to calculate new steady-state variables and illustrate the comparative static results.

3.6 Result

The Figure 3.4 illustrates how variations in income tax rates influence individuals' decision-making regarding employment in skilled and unskilled sectors, as well as training in the learning sector. The tax rates are varied from 10 percent (low tax scenario) to 30 percent (medium tax scenario) and then to 50 percent (high tax scenario). The result shows that different tax rates do not affect the iso-earning cognitive ability line. This means that the choice of trained individuals between working in skilled and unskilled sectors remains unchanged. Those positioned above the red dotted line maintain the same proportion of trained individuals, while those below the line also remain at a consistent fraction. However, a higher tax rate causes the reservation cognitive ability line to rotate to the right. This indicates that more untrained individuals above the blue line are willing to pursue training in the learning sector, while there is a decrease in the number of untrained individuals below the blue line who choose to work in the unskilled sector.

Intuitively, an increase in the income tax rate raises government revenue. The government then allocates this revenue to increase transfer payments and provide more subsidies for training costs in the education sector. These additional educational subsidies encourage more untrained individuals to choose to pursue training in this sector. As a result, this leads to a transition from unskilled employment to trainee positions, ultimately allowing individuals to move into skilled employment.

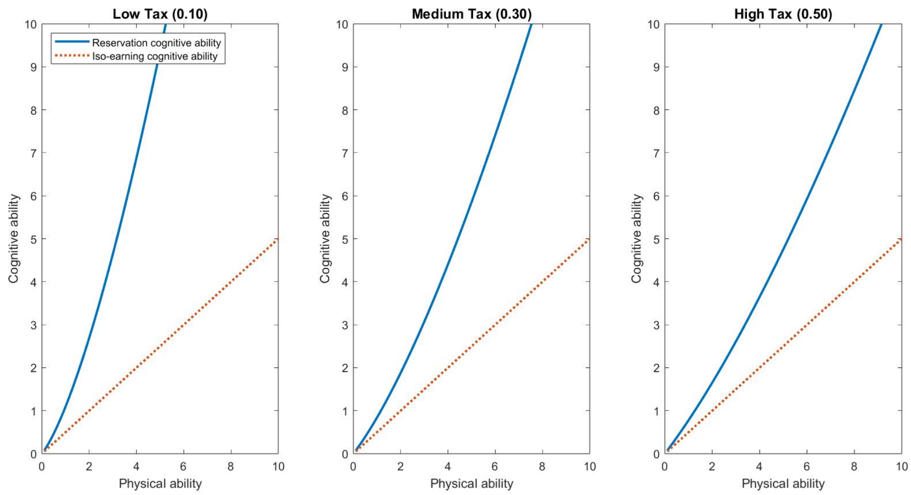

Figure 3.4: The effect of change in income tax on reservation cognitive ability
Source: Author's calculation

The impact of changes in the income tax rate on output, employment, productivity, and skill premium is illustrated in Figure 3.5. First, let's examine the employment graph. An increase in the tax rate leads to a rise in skilled employment but a decline in unskilled and overall employment. This is because higher tax rates provide greater educational subsidies for the training sector. As a result, some unskilled laborers face lower education costs, making them more willing to pursue training opportunities to become skilled workers in the future. However, not all will succeed in their training, potentially leading to an increase in unemployment among unskilled workers. Thus, while skilled employment rises, aggregate employment decreases with an increase in the tax rate. Next, we analyze the output graph. While an increase in the tax rate results in a decrease in unskilled output, it simultaneously boosts skilled and aggregate outputs. The decline in unskilled employment contributes to the drop in unskilled output, whereas the increase in skilled employment drives up skilled output. Overall, despite the decrease in aggregate employment, aggregate output still rises because the growth in skilled output outweighs the fall in unskilled output. This occurs since skilled sectors utilize more advanced technology, enabling them to produce significantly more than unskilled sectors.

The aggregate output of an economy tends to improve when tax rates increase, primarily because individuals are often limited by imperfect access to capital markets. In many cases, individuals lack the ability to effectively save or borrow, which prevents them from smoothing their consumption over time. This inability to manage their finances leads them to consume all of their income as it is received, which contributes to market inefficiencies and failures. In this context, tax and transfer programs play a crucial role in addressing these shortcomings of the capital market. Specifically, the government collects funds from higher-income individuals through taxes and redistributes those funds to lower-income individuals. This redistribution not only provides necessary financial support to those in need but also stimulates overall economic activity by increasing the purchasing power of lower-income households. As a result of these tax and rebate schemes, individuals, particularly those from lower-income brackets, find themselves in a better financial position. This improvement can lead to enhanced welfare, enabling them to invest in health, education, and other opportunities that may have been previously inaccessible. Therefore, the interplay between taxation, government transfers, and individual economic circumstances ultimately contributes to a more equitable and productive economy.

Now, let's consider the effect of the tax rate on productivity, as shown in the second graph. Higher tax rates reduce skilled productivity because the increase in skilled employment surpasses the increase in skilled output. Conversely, they stimulate unskilled productivity since the decline in unskilled employment is greater than the decrease in unskilled output. Nevertheless, the overall productivity increases due to the rise in aggregate output while aggregate employment falls. Finally, higher tax rates adversely affect skill premium. This occurs because skilled productivity declines while unskilled productivity increases.

This result is in accordance with Figure 3.1, which illustrates that wage dispersion is a result of variations in productivity levels, which are deeply influenced by differing individual abilities. This relationship underscores how distinct skill sets and talents contribute to variations in both productivity and corresponding wages. Over the last two decades, wage dispersion has not only persisted but has also widened significantly, largely due to continuous advancements in technology across the United States. These technological improvements have particularly favored skilled workers, creating a phenomenon known as skill-biased technological change. As a result,

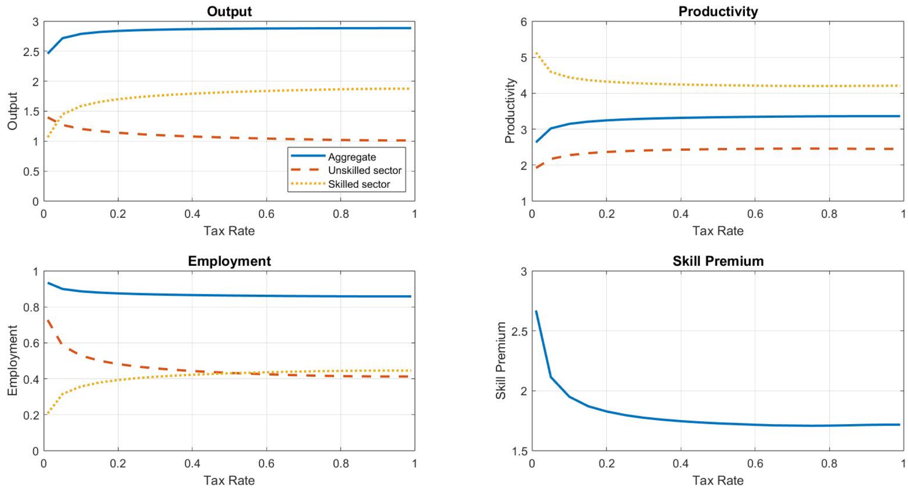

FIGURE 3.5: The effect of change in income tax on output, employment, productivity, and skill premium
Source: Author's calculation

productivity and wages for individuals with higher skill levels have consistently risen, further exacerbating the wage gap between high and low-skilled workers.

In Figure 3.6, we vary the income tax rates across five levels: 1 percent, 10 percent, 30 percent, 50 percent, and 99 percent. The results indicate that the scheme with the lowest tax rate, represented by the blue solid line, shows that individuals have varying income levels. In contrast, the scheme with the highest tax rate, represented by the green dot-dashed line, demonstrates that nearly all individuals have similar income levels. Therefore, this variation allows us to examine how the distribution of earnings changes with different tax rates. We find that higher tax rates increase the initial income distribution, causing the cumulative distribution of earnings to converge more quickly. This demonstrates that raising the tax rate can help reduce income inequality.

This analysis aligns with the comparison of income levels before and after taxes, as illustrated in Figures 3.2 and 3.3. The data indicates that income disparity in the United States experienced a significant reduction following the introduction of income tax policies and rebate programs. Specifically, these reforms aimed to redistribute wealth more equitably across different income groups. The empirical evidence underscores the effectiveness of these initiatives in promoting greater income equality, demonstrating how taxation and fiscal measures can play a crucial role in addressing economic disparities within society.

We observe that a policy package involving an income tax paired with lump-sum transfers and educational subsidies can help mitigate the typical equity-efficiency trade-off. The rationale is that the income tax and transfer scheme work to reduce income inequality. When the additional output from cognitive ability is sufficiently high resulting in the negative effects of the income tax or the negative effect of lump-sum subsidies can overwhelm the positive effects of the income tax, especially in cases where the marginal productivity of cognitive ability is low. This situation encourages some untrained individuals to seek training for skilled sectors, resulting in an

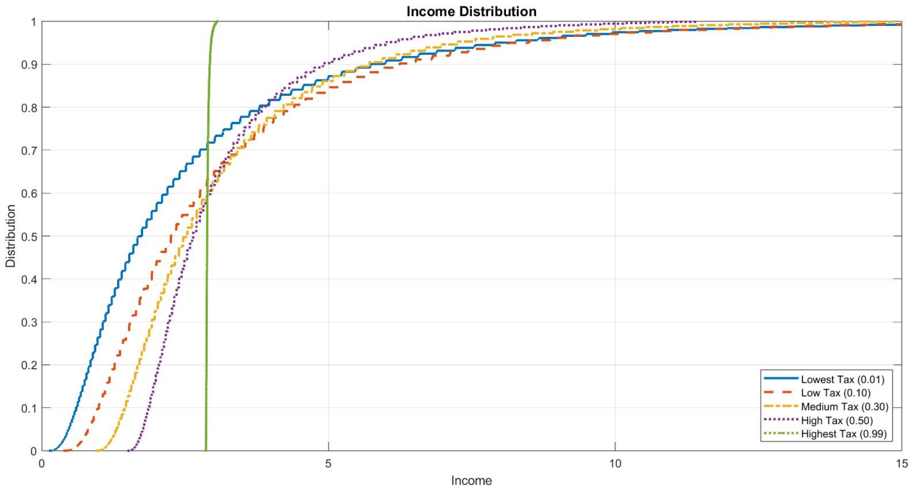

FIGURE 3.6: The effect of change in income tax on distribution of earnings
Source: Author's calculation

increase in skilled output that exceeds the decrease in unskilled output due to reduced unskilled employment. Consequently, aggregate output appears to increase.

Conversely, the equity-efficiency trade-off can be satisfied if the additional output from cognitive ability is low, leading to the positive effects of income tax outweighing the negative effects of transfer subsidies. This scenario results in lower skilled output and discourages untrained individuals from pursuing education in learning sectors, ultimately causing aggregate output to decrease. Thus, while the policy mix promotes income equality, it can simultaneously lead to a reduction in overall output.

3.7 Conclusions

Recently, income inequality in the United States began to rise significantly over last fifty years. This shift was driven by factors such as globalization, technological advancements, and policy decisions that favored capital over labor. The weakening of labor unions and tax policies that benefited the wealthy further widened the income gap. Income inequality has become an escalating concern, resulting in a substantial disparity between the wealthiest individuals and lower-income populations. This growing wealth gap negatively impacts society by weakening social unity, increasing political division, and limiting economic growth. It is also associated with poorer health outcomes, lower educational achievements, and higher crime rates.

A progressive income tax system is important for reducing income inequality because it requires wealthy people to pay a larger share of their income in taxes. There's agreement on raising taxes on the rich through higher tax rates, a wealth tax, and eliminating tax loopholes. These changes aim to distribute wealth more fairly for public services. Programs like Social Security and unemployment benefits help reduce poverty and support low-income families, though some critics say they can lead to dependence. Instead, reforms can encourage people to participate in the workforce. Providing financial support for education and job training can also help reduce income inequality by equipping workers with the skills they need in the job market. A comprehensive approach to income inequality in the U.S. should include progressive taxes, effective transfer payments, and targeted training support. Together, these measures can help redistribute wealth, aid marginalized communities, and improve economic opportunities for everyone.

This study conducts an extended version of the Roy model, which integrates multiple dimensions of physical and cognitive abilities, as outlined in Chapter 2. It incorporates various tools for economic redistribution, including progressive income tax systems, targeted transfer payments, and learning subsidies aimed at enhancing skill development. The extended Roy model offers insights into how these redistribution mechanisms influence individuals' decisions regarding education, training, and employment across sectors. By analyzing these dynamics, the study aims to uncover the interplay between government policies and individual choices, particularly how they affect career paths and sectoral shifts. Furthermore, the research delves into the implications of these tools on broader economic outcomes, including overall employment rates, production levels, labor productivity, and household earnings.

This study shows that when tax rates increase, the overall output of the economy often improves. This happens because many people have limited access to borrowing and saving options. As a result, they cannot manage their finances well and usually spend all their income as soon as they get it. This leads to problems in the market. Tax and transfer programs are important for fixing these issues. The government collects taxes from higher-income people and gives that money to lower-income individuals. This process helps those in need and boosts the economy by increasing the spending power of these lower-income households. Thanks to these tax and rebate programs, people with lower incomes can improve their financial situation. This can enhance their well-being, allowing them to invest in health, education, and other opportunities that they might not have been able to access before. Thus, the relationship between taxes, government support, and personal finances helps create a fairer and more productive economy.

The wage dispersion is a result of variations in productivity levels, which are deeply influenced by differing individual abilities. This relationship underscores how distinct skill sets contribute to variations in both productivity and corresponding wages. Over the last two decades, wage dispersion has not only persisted but has also widened significantly, largely due to continuous advancements in technology across the United States. These technological improvements have particularly favored skilled workers, creating a phenomenon known as skill-biased technological change. As a result, productivity and wages for individuals with higher skill levels have consistently risen, further exacerbating the wage gap between high and low-skilled workers.

The findings from this study highlight a significant contrast between two tax rate schemes. The scheme with the lowest tax rate indicates that individuals experience a wide range of income levels, suggesting a higher degree of economic diversity and opportunity. This disparity points to a system where lower taxation may encourage entrepreneurial activities and allow individuals to retain more of their earnings, leading to varied financial outcomes. On the other hand, the scheme with the highest tax rate reveals that almost all individuals receive similar income levels. This scenario suggests a more equal income distribution, but it may also imply reduced incentives for individual effort and economic activity. This analysis aligns with empirical evidence indicating that income inequality in the United States significantly decreased after the introduction of income tax and rebate schemes. Thus, these schemes were designed to redistribute wealth more equitably among different income groups.

In conclusion, addressing income inequality is not only a moral imperative but a necessary step toward building a more resilient and prosperous society. By implementing a progressive tax system, enhancing access to education and job training, and ensuring robust support for low-income families, we can create an economy that works for everyone, lifts people out of poverty, and fosters sustainable growth. It's time for us to take action and invest in a fairer future.

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3.9 Appendix

3.9.1 Change in income tax and decision making

The impact of changes in income tax rates on earnings and reservation cognitive ability is illustrated in Figure 3.7. In the upper panel of the figure, the red dotted line represents the indifferent earning level between physical and cognitive abilities or the level of iso-earning, while the blue solid line indicates the earnings related to reservation cognitive ability. In the lower panel, which is derived from the upper one, the red dotted line represents the iso-earning cognitive ability, and the blue solid line denotes the reservation cognitive ability. In the upper panel, we observe that an increase in income tax and transfer payment schemes reduces the interval between both lines and narrows the gap between them. This indicates that the earnings of untrained individuals and trained individuals become more similar. In the lower panel, we find that an increase in income tax and subsidy schemes does not affect the iso-earning cognitive ability line but causes a rightward rotation of the line for reservation cognitive ability. This suggests that raising income tax does not change the decision-making of trained individuals but may encourage more untrained individuals to pursue training in the learning sector.

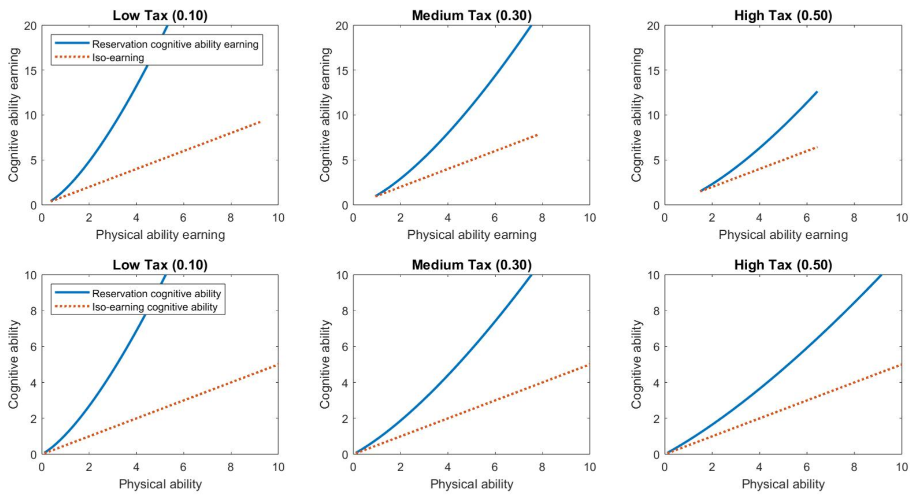

FIGURE 3.7: The effect of change in income tax on earnings and reservation cognitive ability
Source: Author's calculation

In Figure 3.8, we examine the impact of changes in income tax on the values of trained and untrained individuals. The upper panel of the figure illustrates the value of trained individuals, which can be separated into two surfaces by a line indicating iso-earning cognitive ability. The left surface indicates the value of trained individuals with dominant cognitive abilities, while the right surface reflects the value of trained individuals with dominant physical abilities. The lower panel of the figure shows the value of untrained individuals, which is divided into two surfaces by a line representing the reservation cognitive ability. The left surface represents the value of untrained individuals with sufficiently high cognitive abilities, while the right surface denotes the value of untrained individuals with dominant physical abilities. In the upper panel, we observe that an increase in income tax and transfer payment schemes raises the minimum value of trained individuals while pressing down the maximum value of trained individuals. This implies that the earnings gap is decreasing as these schemes are implemented. In the lower panel, we see that an increase in income tax and subsidy schemes also raises the minimum value of untrained individuals. The change in the maximum value of untrained individuals presents an interesting phenomenon: the schemes decrease the maximum value for those with low cognitive abilities, while increasing the maximum value for individuals with sufficiently high cognitive abilities. This suggests that the schemes enhance the value of untrained individuals with higher cognitive abilities and encourage more untrained individuals to pursue training in the learning sector.

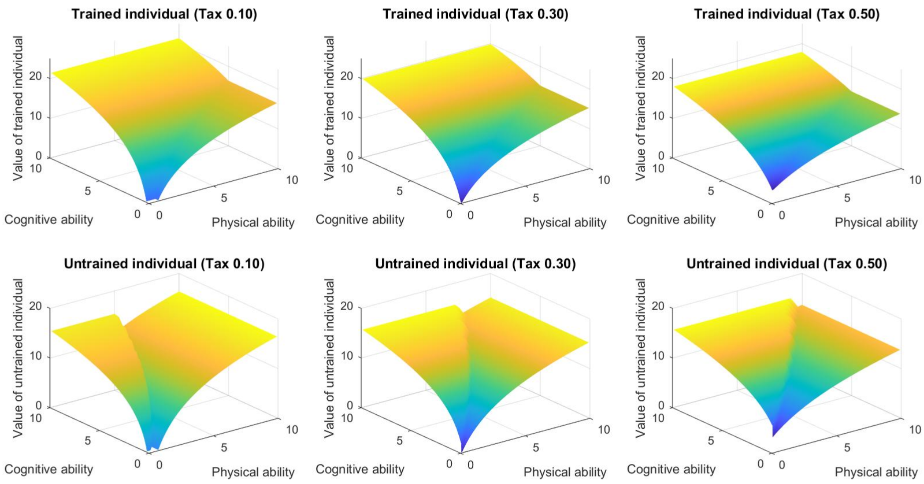

FIGURE 3.8: The effect of change in income tax on values of trained and untrained individuals
Source: Author's calculation

3.9.2 The Set of Parameters

The model includes parameters listed in the table below. These parameters are provided by Andolfatto and Smith (2001). Furthermore, certain parameters are adjusted according to the configuration of the model's environment.

TABLE 3.1: The Set of Parameters

Parameters	**Description**	**Value**
β	Consumer's subjective discount factor	0.96
δ	Consumer's probability of death	0.10
μ_{a_1}	Mean of physical ability (mvnpdf)	0
μ_{a_2}	Mean of cognitive ability (mvnpdf)	0
$\sigma^2_{a_1}$	Variance of physical ability (mvnpdf)	1.00
$\sigma^2_{a_2}$	Variance of cognitive ability (mvnpdf)	1.00
$\sigma_{a_1 a_2}$	Covariance of physical and cognitive abilities (mvnpdf)	0
τ	Income tax rate	[0.01,0.99]
θ	Probability of absorbed knowledge	0.35
w_1	Additional output of physical ability	1.00
w_2	Additional output of cognitive ability	2.00