A Novel Design of Sparse Vector Coding Aided URLLC With Periodic Quasi-Complementary Pairs

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Abstract—Recently, sparse vector coding (SVC) has emerged as a promising coding scheme for short-packet transmission in ultra-reliable and low-latency communications (URLLCs). The encoding and decoding of SVC are jointly formulated as a standard compressed sensing (CS) problem, in which a codebook with low mutual coherence is required to achieve reliable decoding performance. In this paper, we propose the concept of periodic quasi-complementary pairs (PQCPs), based on which we present a novel design of a deterministic spreading matrix with low mutual coherence for efficient SVC encoding. Furthermore, a modified multipath matching pursuit (MMP) decoder for the proposed SVC scheme is developed. Simulation results demonstrate that the proposed designs significantly improve the block error rate (BLER) performance of SVC compared with existing approaches.

Index Terms—Sparse vector coding (SVC), periodic quasi-complementary pair (PQCP), error correcting code, short packet transmission, ultra-reliable low-latency communications (URLLC).

I. INTRODUCTION

Ultra-reliable low-latency communication (URLLC) is one of the key service pillars for next-generation communication systems, targeting scenarios with stringent requirements on both reliability (typically with packet delivery success rate of at least 99.999%) and latency (typically with end-to-end latency of no more than 1 ms). To meet the latency constraints and due to the bursty nature of certain critical messages, short packet transmission has emerged as an effective means for URLLC services. This is challenging because achieving high reliability generally requires sufficient long codeword lengths with respect to the Shannon capacity.

In 2018, a novel coding technique called sparse vector coding (SVC) was proposed for short packet URLLC transmission [1]. Specifically, during the encoding process, the information bits are mapped to certain nonzero elements in a sparse vector. A codeword is then generated by multiplying a predesigned spreading matrix with this sparse vector. At the receiver,

the sparse vector can be efficiently recovered using sparse recovery algorithms such as multipath matching pursuit (MMP) [2] and orthogonal matching pursuit (OMP) [3]. In [4], [5], enhanced SVC (ESVC) and constellation rotation-based SVC (CR-SVC) were proposed, respectively. These schemes map the transmitted information bits onto both the nonzero positions and their corresponding nonzero elements of a sparse vector. The index redefinition-based SVC (IR-SVC) scheme was subsequently proposed in [6], in which the positions of the nonzero elements in the sparse vector are determined based on an index redefinition technique. Various sparse vector mapping designs have been proposed towards improved decoding performance [7]–[10].

In contrast to the aforementioned works, there are studies focusing on the design of spreading matrix for enhancing the block error rate (BLER) performance [11], [12]. A key optimization objective is to minimize the mutual coherence of the spreading matrix in SVC. However, for a fixed matrix size, in general, it is challenging to bound the mutual coherence value of the resulting spreading matrix, even though their objective is to minimize it. To make a difference, this work seeks a coding approach by studying novel pairs of sequences.

In the literature, periodic complementary pairs (PCPs) were first introduced in [13], characterized by the property that the sum of the periodic autocorrelation functions of the two constituent sequences is zero at every nonzero time-shift. Subsequently, the concept of periodic Z-complementary pairs (PZCPs) was proposed in [14], where the periodic autocorrelation sums take zero values within a zero correlation zone (ZCZ). In [15], periodic almost-complementary pairs (PACPs) were proposed, each having maximum ZCZ width and minimal out-of-zone periodic auto-correlation sum magnitudes. Driven by the need for more flexible SVC encoding and unlike the existing pairs of sequences, we aim for a systematic study of periodic quasi-complementary pairs (PQCPs), whereby every periodic autocorrelation sum of the two constituent sequences is bounded by a small constant magnitude. We show that PQCPs can be utilized to construct efficient SVC spreading matrices whose mutual coherence values can be precisely calculated. Furthermore, based on the SVC framework, a modified MMP decoder is proposed. Simulation results show that the proposed

SVC scheme based on PQCPs achieves significantly enhanced BLER performance compared to that from optimization-based methods.

II. PRELIMINARIES AND DEFINITIONS

The notations used throughout this paper are as follows:

- $\lfloor \cdot \rfloor$ denotes the floor operation;
- $\lceil \cdot \rceil$ denotes the ceiling operation;
- $(\cdot)^T$ denotes the transpose operation;
- $\text{Re}\{x\}$ denotes the real part of the complex number x;
- $\text{Im}\{x\}$ denotes the imaginary part of the complex number x;
- $\|\cdot\|_2$ denotes the l_2-norm;
- $(\cdot)_{\bmod L}$ denotes the modulo operation with respect to L;
- "+" and "−" denote 1 and −1, respectively;
- $\tilde{\boldsymbol{a}}$ denotes the reverse of the sequence $\boldsymbol{a}$;
- $\boldsymbol{a}^{(u)}$ is the u-th downward cyclicly shifted version of $\boldsymbol{a}$.

A. Sparse Vector Coding

The encoding and decoding process of the SVC scheme is illustrated in Fig. 1. In this scheme, the message $\boldsymbol{m}$ is mapped to a sparse vector $\boldsymbol{s} = (s_0, s_1, \ldots, s_{N-1})^T$. By choosing K out of N symbol positions, we can encode b_t bits of information. Hence, the number of choices $\binom{N}{K}$ allows us to encode at least

$$\left\lfloor \log_2 \binom{N}{K} \right\rfloor \geq b_t. \tag{1}$$

For example, when $N = 5$, $K = 2$, and $b_t = 3$, the corresponding sparse mapping rules are listed in Table I, where the entries "1" and "j" represent nonzero elements in the sparse vector. Afterward, the transmitted codeword $\boldsymbol{x}$ for message $\boldsymbol{m}$ is generated by the codebook spreading. Specifically, we let $\boldsymbol{C} = [\boldsymbol{c}_0, \boldsymbol{c}_1, \cdots, \boldsymbol{c}_{N-1}]$ be a spreading matrix where $\boldsymbol{c}_i = (c_{i,0}, c_{i,1}, \cdots, c_{i,V-1})^T$ is the i-th spreading sequence with length V. For practical applications, the spreading matrix $\boldsymbol{C}$ is typically designed as a bipolar matrix whose entries are constrained to $+1$ and -1. Consequently, when the codebook matrix $\boldsymbol{C}$ is employed, the transmitted codeword of SVC is given by

$$\boldsymbol{x} = \boldsymbol{C}\boldsymbol{s} = \sum_{i=0}^{N-1} \boldsymbol{c}_i s_i. \tag{2}$$

For $K = 2$, the transmitted codeword $\boldsymbol{x}$ is mapped to the QPSK symbol since the nonzero entries of the sparse vector are 1 and j. When a higher sparsity level is employed, the mapping can be readily extended to high order modulations. For example, $K = 4$ and $K = 6$ correspond to 16-QAM and 64-QAM, respectively. Specifically, when $K = 4$, two nonzero entries of the sparse vector are set to 1 and 2, while the remaining two nonzero entries are set to j and $2j$. Similarly, when $K = 6$, three nonzero entries are assigned to 1, 2, and 3, and the other three are assigned to j, $2j$, and $3j$. The normalization factor for M-QAM modulation is given by $\alpha = \sqrt{\frac{2(M-1)}{3}}$.

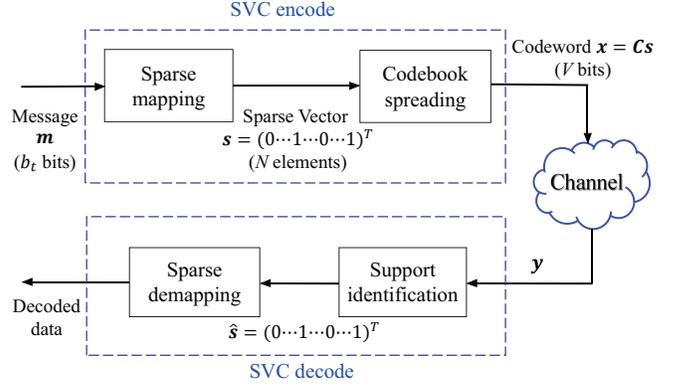

Fig. 1. The encoding and decoding process of the SVC scheme.

TABLE I
AN EXAMPLE OF SPARSE MAPPING WITH $b_t = 3$, $N = 5$, AND $K = 2$.

Message $\boldsymbol{m}$	Sparse Vector $\boldsymbol{s}$
000	$(1, j, 0, 0, 0)^T$
001	$(1, 0, j, 0, 0)^T$
010	$(0, 1, j, 0, 0)^T$
011	$(1, 0, 0, j, 0)^T$
100	$(0, 1, 0, j, 0)^T$
101	$(0, 0, 1, j, 0)^T$
110	$(1, 0, 0, 0, j)^T$
111	$(0, 1, 0, 0, j)^T$

In the receiver, the received signal $\boldsymbol{y}$ after the channel fading can be represented as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{w} = \boldsymbol{H}\boldsymbol{C}\boldsymbol{s} + \boldsymbol{w} \triangleq \boldsymbol{\Phi}\boldsymbol{s} + \boldsymbol{w} \tag{3}$$

where $\boldsymbol{H}$ is a diagonal matrix formed from the channel vector $\boldsymbol{h} = (h_0, h_1, \ldots, h_{V-1})$ and $\boldsymbol{w} = (w_0, w_1, \ldots, w_{V-1})$ is the complex additive white Gaussian noise (AWGN), i.e., $w_i \sim \mathcal{CN}(0, \sigma^2)$.

The MMP decoder is employed as the baseline for SVC decoding to recover the sparse vector $\boldsymbol{s}$ [1], [2]. The MMP decoding process explores multiple candidate paths. Let $\hat{\Omega}_{k,l}$ denote the estimated set of nonzero position indices obtained at the k-th iteration along candidate path l, and let $\boldsymbol{r}_{k,l}$ be the corresponding residual vector. Each candidate path l is determined by a distinct branching pattern $\{d_1, d_2, \ldots, d_K\}$ where $d_k \in \{1, 2, \ldots, D\}$ specifies the branch taken at the k-th iteration. The mapping between the path index l and the branching pattern is expressed as $l = 1 + \sum_{k=1}^{K}(d_k - 1)D^{k-1}$. For example, when $D = 2$ and $K = 2$, there are four possible branching patterns: $(d_1, d_2) = (1, 1), (2, 1), (1, 2)$, and $(2, 2)$, corresponding to $l = 1, 2, 3$, and 4, respectively. Based on the above labeling, each candidate path l follows the procedure below to estimate its set of nonzero position indices up to the K-th iteration.

1) Initialization: The initial conditions are set as $\hat{\Omega}_{0,l} = \emptyset$ and $\boldsymbol{r}_{0,l} = \boldsymbol{y}$.

2) Correlation and Expansion: At each iteration $k = 1, 2, \cdots, K$, every candidate path expands into D child paths $\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^D$ by selecting the indices of columns that are maximally correlated with the current residual. For odd iterations ($k = 1, 3, 5, \ldots$), the set of indices is estimated as

$$\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^D\} = \arg \max_{|\lambda|=D} \left\| \left(\mathrm{Re} \left\{ \widetilde{\boldsymbol{\Phi}}^T \boldsymbol{r}_{k-1} \right\} \right)_\lambda \right\|_2^2 \tag{4}$$

where $\widetilde{\boldsymbol{\Phi}}$ denotes the normalized matrix of $\boldsymbol{\Phi}$. For even iterations ($k = 2, 4, 6, \ldots$), the set of indices is estimated as

$$\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^D\} = \arg \max_{|\lambda|=D} \left\| \left(\mathrm{Im} \left\{ \widetilde{\boldsymbol{\Phi}}^T \boldsymbol{r}_{k-1} \right\} \right)_\lambda \right\|_2^2. \tag{5}$$

Each selected column corresponds to a newly generated child path, and the estimated set of nonzero position indices is updated as $\hat{\Omega}_{k,l} = \hat{\Omega}_{k-1,l} \cup \{\lambda_k^{d_k}\}$.

3) Residual Update: The residual vector is then updated by $\boldsymbol{r}_{k,l} = \boldsymbol{y} - \boldsymbol{\Phi}_{\hat{\Omega}_{k,l}} s_{k,l}$ where $\boldsymbol{\Phi}_{\hat{\Omega}_{k,l}}$ denotes the submatrix of $\boldsymbol{\Phi}$ formed by the columns indexed by $\hat{\Omega}_{k,l}$, and $\boldsymbol{s}_{k,l}$ is known to the receiver as it corresponds to the nonzero values in the original sparse vector.

Finally, the decoder selects the candidate path with the minimum residual $\arg \min_l \|\boldsymbol{r}_{K,l}\|_2^2$ and the corresponding estimated set of nonzero position indices $\hat{\Omega}$ is obtained.

Definition 1: The mutual coherence of $\boldsymbol{C}$ is defined as

$$\mu(\boldsymbol{C}) = \max_{0 \le i \ne j \le N-1} \frac{\boldsymbol{c}_i^T \boldsymbol{c}_j}{\|\boldsymbol{c}_i\|_2 \cdot \|\boldsymbol{c}_j\|_2}. \tag{6}$$

Note that the mutual coherence for SVC employed in [11], [12] involves the absolute value of the inner product, i.e., $|\boldsymbol{c}_i^T \boldsymbol{c}_j|$. In this paper, the absolute operation is omitted to retain the sign information.

B. Sequences

Let $\boldsymbol{a} = (a_0, a_1, \ldots, a_{L-1})^T$ and $\boldsymbol{b} = (b_0, b_1, \ldots, b_{L-1})^T$ be two binary sequences of length L. The periodic cross-correlation function (PCCF) of sequences $\boldsymbol{a}$ and $\boldsymbol{b}$, denoted by $\phi(\boldsymbol{a}, \boldsymbol{b}; u)$, is defined as

$$\phi(\boldsymbol{a}, \boldsymbol{b}; u) = \begin{cases} \sum_{t=0}^{L-1} a_{(t+u) \bmod L} b_t^*, & 0 \le u \le L-1; \\ \sum_{t=0}^{L-1} a_t b_{(t-u) \bmod L}^*, & -L+1 \le u < 0. \end{cases} \tag{7}$$

If $\boldsymbol{a} = \boldsymbol{b}$, $\phi(\boldsymbol{a}, \boldsymbol{a}; u)$ is the *periodic autocorrelation function* (PACF) of $\boldsymbol{a}$ and simply denoted by $\phi(\boldsymbol{a}; u)$.

Definition 2 (Periodic Complementary Pair [13]): A pair $(\boldsymbol{a}, \boldsymbol{b})$ is called a PCP if

$$\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u) = 0, \quad \text{for } u \ne 0. \tag{8}$$

Definition 3 ([13]): A sequence pair $(\boldsymbol{c}, \boldsymbol{d})$ is referred to as a mate of $(\boldsymbol{a}, \boldsymbol{b})$, if

$$\phi(\boldsymbol{a}, \boldsymbol{c}; u) + \phi(\boldsymbol{b}, \boldsymbol{d}; u) = 0, \text{ for all } u. \tag{9}$$

A typical mate of $(\boldsymbol{a}, \boldsymbol{b})$ can be obtained as as $(\boldsymbol{c}, \boldsymbol{d}) = (\tilde{\boldsymbol{b}}^*, -\tilde{\boldsymbol{a}}^*)$ [13].

Definition 4 (Periodic Z-Complementary Pair [14]): For integer Z with $Z \le L$, the pair $(\boldsymbol{a}, \boldsymbol{b})$ is called a PZCP if

$$\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u) = 0, \quad \text{for } 1 \le u < Z. \tag{10}$$

If $Z = L$, then the pair $(\boldsymbol{a}, \boldsymbol{b})$ reduces to a PCP [13].

Definition 5 (Optimal Binary Periodic Almost-Complementary Pair [15]): A pair $(\boldsymbol{a}, \boldsymbol{b})$ of length L is said to form an optimal PACP if
1) For even L, it possesses a ZCZ of width $L/2$, and the sum of its PACFs at the shift $u = L/2$ has a magnitude of 4;
2) For odd L, all of its out-of-phase PACF sums have the same magnitude of 2.

For integer shift u, the aperiodic cross-correlation function between sequences $\boldsymbol{p}$ and $\boldsymbol{g}$ is given by

$$\rho(\boldsymbol{p}, \boldsymbol{g}; u) = \sum_{t=0}^{L-1-u} p_{t+u} g_t^*, \quad 0 \le u \le L-1. \tag{11}$$

Likewise, when $\boldsymbol{p} = \boldsymbol{g}$, the function $\rho(\boldsymbol{p}, \boldsymbol{p}; u)$ reduces to the aperiodic autocorrelation function of $\boldsymbol{p}$, denoted by $\rho(\boldsymbol{p}; u)$.

Definition 6 (Golay Complementary Pair [16]): A pair $(\boldsymbol{p}, \boldsymbol{g})$ is called a GCP of length L if

$$\rho(\boldsymbol{p}; u) + \rho(\boldsymbol{g}; u) = 0, \quad \text{for } 1 \le u < L. \tag{12}$$

III. PROPOSED CODEBOOKS OF SVC BASED ON PERIODIC QUASI COMPLEMENTARY PAIRS

In this section, we present a novel method for constructing a spreading matrix based on PQCPs.

First, we will provide the definition of the PQCP and then propose a generic construction of PQCPs by leveraging the GCPs.

Definition 7 (Periodic Quasi-Complementary Pairs): A pair $(\boldsymbol{a}, \boldsymbol{b})$ is called an (L, ϵ)-PQCP, if

$$\left| \phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u) \right| \le \epsilon, \quad \text{for } 1 \le u < L. \tag{13}$$

When $\epsilon = 0$, the $(L, 0)$-PQCP reduces to a PCP of length L. Moreover, when $\epsilon = 2$ and L is odd, the $(L, 2)$-PQCP is referred to as an optimal PACP. Therefore, a PQCP can be regarded as a more general definition that does not take the concept of a ZCZ into account. The relationship between PQCPs and PZCPs is illustrated in Fig. 2.

An example of a PQCP is given below.

Example 1: Let $\boldsymbol{a} = (+ - + - - - + + +)$ and $\boldsymbol{b} = (+ - - + - - - - -)$, the pair $(\boldsymbol{a}, \boldsymbol{b})$ is a $(9, 2)$-PQCP since

$$\left(\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u) \right)_{u=0}^8 = (18, 2, 2, 2, -2, -2, 2, 2, 2).$$

We then present a construction of PQCPs based on GCPs.

Theorem 1: Suppose that $(\boldsymbol{p}, \boldsymbol{g})$ is a binary GCP of length L. Then, the pair $(\boldsymbol{a}, \boldsymbol{b})$ is an $(L+1, 2)$-PQCP where

$$\begin{aligned} \boldsymbol{a} &= (p_0, p_1, \ldots, p_{L-1}, x_0), \\ \boldsymbol{b} &= (g_0, g_1, \ldots, g_{L-1}, y_0), \end{aligned} \tag{14}$$
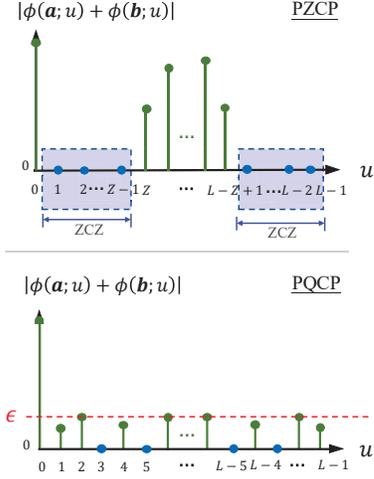
Fig. 2. The correlation properties of PQCPs and PZCPs.

and $x_0, y_0 \in \{+1, -1\}$.

Proof: The proof is provided in Appendix A. ∎

Example 2: Let us consider the GCP of length 10 given by $\boldsymbol{p} = (+ - - + - + - - - +)$ and $\boldsymbol{g} = (+ - - - - - - + + -)$. From *Theorem 1*, the pair $(\boldsymbol{a}, \boldsymbol{b})$ is a $(11, 2)$-PQCP where $\boldsymbol{a} = (+ - - + - + - - - + -)$ and $\boldsymbol{b} = (+ - - - - - - + + - +)$. We list the PACF sums of $\boldsymbol{a}$ and $\boldsymbol{b}$ as follows:

$$\left(\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u)\right)_{u=0}^{10} = (18, -2, 2, 2, -2, -2, -2, -2, 2, 2, -2). \tag{15}$$

Now, we can generate spreading matrices based on PQCPs for SVC.

Theorem 2: For an (L, ϵ)-PQCP $(\boldsymbol{a}, \boldsymbol{b})$, we can construct a spreading matrix of size $2L \times 4J$ given in (17) where $1 \leq J \leq L$, $\boldsymbol{c} = \tilde{\boldsymbol{b}}^*$, and $\boldsymbol{d} = -\tilde{\boldsymbol{a}}^*$. Then, the coherence of $\boldsymbol{C}$ satisfies

$$\mu(\boldsymbol{C}) = \frac{\epsilon}{2L}. \tag{16}$$

Proof: We decompose the matrix $\boldsymbol{C}$ into four $2L \times L$ submatrices, denoted as $\boldsymbol{C} = [\boldsymbol{C}_1 \| \boldsymbol{C}_2 \| \boldsymbol{C}_3 \| \boldsymbol{C}_4]$. We first consider the columns in $\boldsymbol{C}_1$ or $\boldsymbol{C}_3$. Each column in $\boldsymbol{C}_1$ (or $\boldsymbol{C}_3$) is generated from the (L, ϵ)-PQCP $(\boldsymbol{a}, \boldsymbol{b})$. The inner product between any two distinct column vectors is equal to the sum of the PACFs of $(\boldsymbol{a}, \boldsymbol{b})$ under different shifts, i.e.,

$$\phi(\boldsymbol{a}^{(l)}, \boldsymbol{a}^{(m)}; 0) + \phi(\boldsymbol{b}^{(l)}, \boldsymbol{b}^{(m)}; 0) = \phi(\boldsymbol{a}; m - l) + \phi(\boldsymbol{b}; m - l). \tag{18}$$

According to (13), we have $|\phi(\boldsymbol{a}; m - l) + \phi(\boldsymbol{b}; m - l)| \leq \epsilon$ when $l \neq m$ and $0 \leq l, m \leq L - 1$. This implies that the column vectors in $\boldsymbol{C}_1$ (or $\boldsymbol{C}_3$) are quasi mutually orthogonal. Similarly, the same property holds for $\boldsymbol{C}_2$ and $\boldsymbol{C}_4$ because the pair $(\boldsymbol{c}, \boldsymbol{d}) = (\tilde{\boldsymbol{b}}^*, -\tilde{\boldsymbol{a}}^*)$ also forms an (L, ϵ)-PQCP. We now consider the inner product between one column taken from $\boldsymbol{C}_i$ and another from $\boldsymbol{C}_j$ where $i, j \in \{1, 2, 3, 4\}$ and $i \neq j$. Here, (i, j) denotes the indices of the submatrix pair under

consideration. For the case $(i, j) = (1, 2)$, since the pairs $(\boldsymbol{a}, \boldsymbol{b})$ and $(\boldsymbol{c}, \boldsymbol{d})$ are mates of each other, the inner product between any two distinct column vectors from $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ is

$$\phi(\boldsymbol{a}^{(l)}, \boldsymbol{c}^{(m)}; 0) + \phi(\boldsymbol{b}^{(l)}, \boldsymbol{d}^{(m)}; 0) = \phi(\boldsymbol{a}, \boldsymbol{c}; m - l) + \phi(\boldsymbol{b}, \boldsymbol{d}; m - l) = 0, \tag{19}$$

for $0 \leq m, l \leq L - 1$. This demonstrates that the columns from $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are mutually orthogonal. Hence, the same argument also holds for the other pairs $(3, 4)$, $(2, 3)$, and $(1, 4)$. Finally, we examine the remaining pairs $(1, 3)$ and $(2, 4)$. For the case $(i, j) = (1, 3)$, the inner products between their corresponding columns can be expressed as

$$\phi(\boldsymbol{a}^{(l)}, -\boldsymbol{a}^{(m)}; 0) + \phi(\boldsymbol{b}^{(l)}, -\boldsymbol{b}^{(m)}; 0) = -(\phi(\boldsymbol{a}; m - l) + \phi(\boldsymbol{b}; m - l)), \text{ for } l \neq m \tag{20}$$

and

$$\phi(\boldsymbol{a}^{(l)}, -\boldsymbol{a}^{(m)}; 0) + \phi(\boldsymbol{b}^{(l)}, -\boldsymbol{b}^{(m)}; 0) = -(\phi(\boldsymbol{a}; 0) + \phi(\boldsymbol{b}; 0)) = -2L < \epsilon, \text{ for } l = m. \tag{21}$$

These inner products have magnitudes no greater than ϵ since the pair $(\boldsymbol{a}, \boldsymbol{b})$ is an (L, ϵ)-PQCP. Similarly, for the case $(i, j) = (2, 4)$, the inner products between their corresponding columns can be derived in the same manner, and their magnitudes are also bounded by ϵ.

We therefore conclude that the magnitude of the inner product between any two distinct columns in $\boldsymbol{C}$ does not exceed ϵ. Consequently, the coherence of $\boldsymbol{C}$ is given by

$$\mu(\boldsymbol{C}) = \max_{0 \leq i \neq j \leq N - 1} \frac{\boldsymbol{c}_i^T \boldsymbol{c}_j}{\|\boldsymbol{c}_i\|_2 \cdot \|\boldsymbol{c}_j\|_2} = \frac{\epsilon}{2L}. \tag{22}$$

This completes the proof. ∎

Remark 1: When a spreading matrix of size $V \times N$ is employed to encode a message of b_t bits, we can set $J = \lceil N/4 \rceil \leq L$ in *Theorem 2*. The first N columns of $\boldsymbol{C}$ are then selected to form the spreading matrix $\boldsymbol{C}$ for the SVC scheme.

Remark 2: Since PQCPs can be regarded as a generalized form of PCPs and PACPs, the existing PCPs and PACPs in the literature can also be employed in *Theorem 2* to generate the spreading matrix for the SVC scheme.

Remark 3: From the structure of $\boldsymbol{C}$ in (17), each column in the third and fourth submatrices is the negated version of its corresponding column in the first and second submatrices. Therefore, $\boldsymbol{C}$ contains $2J$ pairs of antipodal columns, whose vectors have the maximum possible Euclidean distance.

Example 3: Let $(\boldsymbol{a}, \boldsymbol{b})$ be the $(11, 2)$-PQCP obtained in *Example 2*. According to *Theorem 2*, the spreading matrix $\boldsymbol{C}$ of size 22×36 can be constructed with $J = 9$. Therefore, the mutual coherence of $\boldsymbol{C}$ can be calculated as

$$\mu(\boldsymbol{C}) = \frac{\epsilon}{2L} = \frac{2}{22} = 0.0909. \tag{23}$$

Therefore, the spreading matrix $\boldsymbol{C}$ of size 22×33 can be employed for the SVC scheme to encode messages of $b_t = 9$ bits.

$$C = \begin{bmatrix} \boldsymbol{a}^{(0)} & \boldsymbol{a}^{(1)} & \cdots & \boldsymbol{a}^{(J-1)} \\ \boldsymbol{b}^{(0)} & \boldsymbol{b}^{(1)} & \cdots & \boldsymbol{b}^{(J-1)} \end{bmatrix} \begin{matrix} \boldsymbol{c}^{(0)} & \boldsymbol{c}^{(1)} & \cdots & \boldsymbol{c}^{(J-1)} \\ \boldsymbol{d}^{(0)} & \boldsymbol{d}^{(1)} & \cdots & \boldsymbol{d}^{(J-1)} \end{matrix} \begin{matrix} -\boldsymbol{a}^{(0)} & -\boldsymbol{a}^{(1)} & \cdots & -\boldsymbol{a}^{(J-1)} \\ -\boldsymbol{b}^{(0)} & -\boldsymbol{b}^{(1)} & \cdots & -\boldsymbol{b}^{(J-1)} \end{matrix} \begin{matrix} -\boldsymbol{c}^{(0)} & -\boldsymbol{c}^{(1)} & \cdots & -\boldsymbol{c}^{(J-1)} \\ -\boldsymbol{d}^{(0)} & -\boldsymbol{d}^{(1)} & \cdots & -\boldsymbol{d}^{(J-1)} \end{matrix} \end{bmatrix}_{2L \times 4J}$$

$$= \begin{bmatrix} a_0 & a_{L-1} & \cdots & a_{L-J+1} & c_0 & c_{L-1} & \cdots & c_{L-J+1} & -a_0 & -a_{L-1} & \cdots & -a_{L-J+1} & -c_0 & -c_{L-1} & \cdots & -c_{L-J+1} \\ a_1 & a_0 & \cdots & a_{L-J+2} & c_1 & c_0 & \cdots & c_{L-J+2} & -a_1 & -a_0 & \cdots & -a_{L-J+2} & -c_1 & -c_0 & \cdots & -c_{L-J+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{L-1} & a_{L-2} & \cdots & a_{L-J} & c_{L-1} & c_{L-2} & \cdots & c_{L-J} & -a_{L-1} & -a_{L-2} & \cdots & -a_{L-J} & -c_{L-1} & -c_{L-2} & \cdots & -c_{L-J} \\ b_0 & b_{L-1} & \cdots & b_{L-J+1} & d_0 & d_{L-1} & \cdots & d_{L-J+1} & -b_0 & -b_{L-1} & \cdots & -b_{L-J+1} & -d_0 & -d_{L-1} & \cdots & -d_{L-J+1} \\ b_1 & b_0 & \cdots & b_{L-J+2} & d_1 & d_0 & \cdots & d_{L-J+2} & -b_1 & -b_0 & \cdots & -b_{L-J+2} & -d_1 & -d_0 & \cdots & -d_{L-J+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ b_{L-1} & b_{L-2} & \cdots & b_{L-J} & d_{L-1} & d_{L-2} & \cdots & d_{L-J} & -b_{L-1} & -b_{L-2} & \cdots & -b_{L-J} & -d_{L-1} & -d_{L-2} & \cdots & -d_{L-J} \end{bmatrix}. \tag{17}$$

IV. Modified MMP Decoder for SVC

In this section, a modified MMP decoder is developed for the SVC scheme, using the conventional MMP algorithm proposed in [1] as the baseline.

The MMP decoder selects the most correlated columns by maximizing the squared magnitude of the correlation, as expressed in (4) and (5). This approach is suitable for general compressed sensing problems, where the nonzero entries of the sparse vector can take both positive and negative (or complex-valued) amplitudes. However, in the SVC encoding scheme, the nonzero elements are restricted to either positive real or positive imaginary values. Under this condition, the sign of the correlation between the residual vector and each column of the spreading matrix carries directional significance. A negative correlation indicates that the corresponding column vector is oriented in the opposite direction to the residual.

To better match the SVC encoding structure, the correlation and expansion steps of MMP in (4) and (5) are modified as follows:

$$\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^D\} = \arg \max_{|\lambda|=D} \left(\mathrm{Re} \left\{ \widetilde{\boldsymbol{\Phi}}^T \boldsymbol{r}_{k-1} \right\} \right)_\lambda, \tag{24}$$
$$\text{for } k = 1, 3, \cdots,$$

and

$$\{\lambda_k^1, \lambda_k^2, \ldots, \lambda_k^D\} = \arg \max_{|\lambda|=D} \left(\mathrm{Im} \left\{ \widetilde{\boldsymbol{\Phi}}^T \boldsymbol{r}_{k-1} \right\} \right)_\lambda, \tag{25}$$
$$\text{for } k = 2, 4, \cdots,$$

respectively.

V. Simulation Results

In this section, we evaluate the BLER performance of the proposed PQCP-based SVC over AWGN and Rayleigh fading channels using the modified MMP decoder, and compare it with different spreading matrices generated from Bernoulli [1], optimized partial Hadamard matrix (OPHM) [11], and optimized column augmentation (OCA) [11]. The proposed spreading matrix C of size 22×33, constructed in *Example 3*, is employed for comparison with those based on Bernoulli, OPHM, and OCA matrices of the same size. In addition, the normal approximation of the finite blocklength capacity [17] is provided as a benchmark. The BLER comparison is conducted with a message length of $b_t = 9$ bits, $N = 33$, $K = 2$,
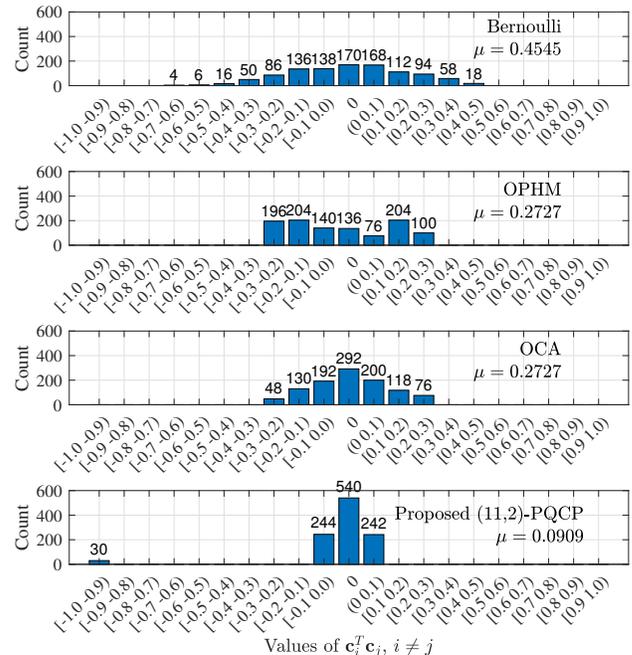

Fig. 3. Histogram of the normalized cross-correlation values of the 22×33 spreading matrices C for Bernoulli, OPHM, OCA, and the proposed $(11, 2)$-QCP, where the mutual coherence values are indicated as μ.

$C \in \{-1, +1\}^{22 \times 33}$, and $D = 2$. In the simulations, the channel matrix H is assumed to be known at the receiver.

Fig. 3 shows the histograms of the normalized cross-correlation values, calculated as $\boldsymbol{c}_i^T \boldsymbol{c}_j$ for $i \neq j$, among the columns of different spreading matrices C. The proposed $(11, 2)$-PQCP-based matrix exhibits a highly concentrated distribution around zero, where nearly all correlation values lie within ± 0.1. A few correlation values at -1 correspond to antipodal column pairs, indicating that these column vectors have the maximum possible Euclidean distance. Consequently, the proposed matrix achieves a mutual coherence of $\mu = 0.0909$, which is considerably lower than those of OCA, OPHM, and Bernoulli matrices.

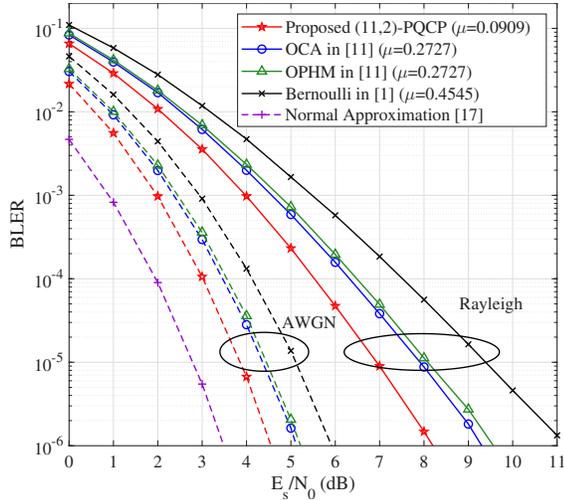Fig. 4 demonstrates the BLER performance of SVC using

Fig. 4. BLER comparison of different codebooks under the modified MMP decoder over AWGN and Rayleigh channels, where $\boldsymbol{C} \in \{-1, +1\}^{22 \times 33}$.

different spreading matrices under the modified MMP decoder over AWGN and Rayleigh fading channels. We can observe that the proposed spreading matrix based on the $(11, 2)$-PQCP outperforms the others, owing to its lowest mutual coherence among the considered matrices, resulting from the PQCP property. For a target BLER of 10^{-5}, the proposed PQCP-based SVC achieves 0.51 dB and 0.98 dB SNR gains over OCA under AWGN and Rayleigh channels, respectively.

VI. CONCLUSION

In this paper, we have presented a novel method for generating spreading matrices based on (L, ϵ)-PQCPs for SVC-aided URLLC. By exploiting the correlation property of PQCPs, a deterministic spreading matrix has been constructed, achieving a mutual coherence of $\epsilon/(2L)$. Moreover, a modified MMP decoder tailored to the SVC encoding has been developed. Simulation results demonstrate that the proposed PQCP-based SVC outperforms existing methods under both AWGN and Rayleigh fading channels. A potential topic for future research is to develop new constructions of (L, ϵ)-PQCPs with flexible L.

APPENDIX A
PROOF OF THEOREM 1

Before proceeding to *Theorem 1*, we state the following lemma that will be useful in the proof.

Lemma 1: [18] Let $(\boldsymbol{p}, \boldsymbol{g})$ be a binary GCP of length L. For $r = 0, 1, \cdots, L/2 - 1$, the following condition holds

$$p_r \, p_{L-1-r} \, g_r \, g_{L-1-r} = -1. \tag{26}$$

Proof of Theorem 1: Without loss of generality, we consider the configuration $p_r = g_r$ and $p_{L-1-r} = -g_{L-1-r}$, which satisfies the condition in *Lemma 1*. This assumption simplifies the derivation while preserving the generality of the result. For $u = 1, 2, \cdots, L - 1$, according to *Lemma 1*, the PACF sums of $\boldsymbol{a}$ and $\boldsymbol{b}$ can be written as

$$\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u) = \rho(\boldsymbol{p}; u) + \rho(\boldsymbol{g}; u) + \rho(\boldsymbol{p}; L + 1 - u)$$
$$+ \rho(\boldsymbol{g}; L + 1 - u) + x_0(p_{u-1} + p_{L-u}) + y_0(g_{u-1} + g_{L-u})$$
$$= p_{u-1}(x_0 + y_0) + p_{L-u}(x_0 - y_0)$$
$$= \begin{cases} 2p_{u-1}, & \text{for } x_0 = y_0 = 1; \\ -2p_{u-1}, & \text{for } x_0 = y_0 = -1; \\ 2p_{L-u}, & \text{for } x_0 = 1, y_0 = -1; \\ -2p_{L-u}, & \text{for } x_0 = -1, y_0 = 1. \end{cases} \tag{27}$$

Since $p_{u-1}, p_{L-u} \in \{\pm 1\}$, we have $|\phi(\boldsymbol{a}; u) + \phi(\boldsymbol{b}; u)| = 2$ for all $1 \le u < L$. Therefore, we can conclude that $(\boldsymbol{a}, \boldsymbol{b})$ is an $(L + 1, 2)$-PQCP. ∎

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