www.nature.com/scientificreports

scientific reports

OPEN

Bet-hedging via Kelly betting in a limited environment leads to logistic growth in the Game of Fitness

Fatih Gulec[1], Takhmina Iliiasova[2], Nigel Wallbridge[3] & Andrew W. Eckford[2]✉

In finance and population dynamics, it is well known that achieving the optimal logarithmic growth rate in an uncertain environment is accomplished by a bet-hedging strategy known as Kelly betting. Such bets assume exponential growth with an unlimited upper bound on population size and a known success probability to determine the bet size. However, practical systems are constrained by finite resources and space, in which case populations instead follow logistic growth with a carrying capacity, defined as the maximum population size that the environment can sustain. In this paper, we introduce a population-dependent Kelly betting strategy that explicitly incorporates environmental limits and show analytically that it converges to the logistic growth function when a carrying capacity is present. To illustrate this, we propose a spatially explicit simulation model, called the *Game of Fitness*, which models growth as the outcome of local reproductive and dormancy decisions in a finite environment. Within the Game of Fitness, the success probability is not assumed *a priori*, but instead is predicted from the evolving spatial interactions of the population. This model reveals how departures from idealised Kelly assumptions arise when spatial and informational constraints are taken into account. In particular, the results reveal the effects of spatial reproduction mobility and environmental information availability, showing that the former is more dominant than the latter. These findings demonstrate how Kelly-based growth is modified in realistic, constrained environments, providing insight into biological populations and structurally analogous growth processes in economic and financial systems.

Keywords Kelly criterion, Logistic growth, Bet-hedging, Carrying capacity, Probabilistic cellular automaton, Population dynamics, Information theory

Organisms acquire and process information to improve their evolutionary fitness[1–3]. The living world is the emergent result of countless information-processing decisions over billions of years, such as a bacterium deciding which nutrient to metabolise[4] or a plant's seeds deciding when to germinate[5]. In cases where information processing mechanisms gave 'successful' results, particularly by achieving a higher growth rate than competitors, those mechanisms were passed to offspring and displaced less proficient mechanisms. Thus, 'optimal' information processing can be viewed as a goal of adaptive organisms, which can be studied by information-theoretic analysis[6,7]. In realistic settings, however, both information acquisition and the ability to act upon that information are constrained by environmental, spatial, and resource limitations. One framework for understanding how organisms make decisions under such constraints is bet-hedging.

Bet-hedging is a key concept in the study of biological information processing[8–10] and in evolutionary biology more broadly, where phenotypes diversify or vary growth strategies to spread risk across generations in the face of environmental uncertainty[11–13]. Two main approaches are conservative and diversified bet-hedging. Dormancy, e.g., seed dormancy in plants, and phenotype switching are considered diversified bet-hedging, whereas storing resources such as water instead of using them for growth exemplifies conservative bet-hedging[14,15]. Notably, the conditions that favour dormancy are not exogenous but arise from the spatial and competitive structure of the environment itself. As local population density increases and resources are locally depleted, reproductive success declines and dormancy becomes increasingly advantageous[16]. Furthermore, it is shown that bet-hedging is favorable in spatially fluctuating environments[17].

[1]School of Computer Science and Electronic Engineering, University of Essex, Colchester CO4 3SQ, UK. [2]Department of Electrical Engineering and Computer Science, York University, Toronto M3J 1P3, Canada. [3]Vivent SA, Gland 1196, Switzerland. ✉email: aeckford@yorku.ca

Kelly betting can be interpreted as a form of adaptive bet-hedging in this broader evolutionary context, unifying the perspectives described above under an information-theoretic growth criterion[18,19]. Kelly betting, since deeply woven into many fields such as quantitative finance, maximises the *logarithm* of the player's expected return, rather than its simple value. This method has two features: first, a true Kelly bet will always avoid bankruptcy or extinction, as $\lim_{x\to 0}\log 0 = -\infty$ (i.e., ending up with a fortune of zero is infinitely penalized); and second, since the logarithm is sublinear, going from fortune *f* to $f + x$ has a larger effect for small *f*, compared with large *f*. Both properties are important to biological systems, where logarithmic growth rates are related to evolutionary fitness, sometimes called *Malthusian fitness*[20]. Throughout this paper, fitness is interpreted operationally in terms of realised survival and growth outcomes, rather than as an intrinsic genotypic or phenotypic trait. Accordingly, the optimal bid in Kelly betting combines the proximate wealth of the gambler with the success probability, linking immediate resource availability to long-term survival and growth outcomes. This makes Kelly betting a natural candidate framework for modelling growth decisions under uncertainty in biological systems.

Kelly growth is focused on an organism's exponential phase of growth. However, a population cannot continue to grow forever, and populations will generally approach a *carrying capacity*, representing the maximum number of their species that the environment can support given finite resources and space[21]. Observed growth curves are sigmoidal, following the logistic function. Such a limitation makes eventual extinction a likely event, even though extinction should be impossible in a pure Kelly framework under unconstrained conditions. (To understand why, consider that starting from a low carrying capacity, a lost bet may result in a fractional 'population' between 0 and 1, effectively extinct; this case is very unlikely to occur if the population is enormous and exponentially growing.) These limitations have been addressed from several complementary perspectives. First, carrying capacity has been incorporated into Kelly betting frameworks through stochastic processes with multiplicative noise and nonlinear drift terms that decrease as a function of the ratio between current population and carrying capacity[22]. Moreover, a game-theoretic perspective demonstrates that log-optimal strategies represent evolutionarily stable equilibria that remain robust under finite carrying capacity and extinction risk[23]. More recently, statistical estimators for population-size-dependent branching processes have been developed that model logistic growth through density-dependent mean offspring rates[24].

A key challenge is that classical Kelly formulations implicitly assume unlimited growth potential and independence of bets, assumptions that are violated in spatially constrained and resource-limited environments. A further limitation is that the success probability in Kelly betting is typically treated as fixed and known *a priori*. In biological populations subject to spatial and resource constraints, however, reproductive success is neither stationary nor directly accessible, but instead depends on the evolving population density and the environment. Despite the advances reviewed above, a critical gap remains: no existing framework establishes an analytical connection between Kelly-based reproductive decisions and logistic growth in which the success probability depends on the evolving population state. In addition, no study has shown through spatially explicit simulation that such logistic dynamics emerge when the success probability is generated endogenously. Specifically, three core assumptions of classical Kelly betting are systematically violated in spatially constrained biological systems: (i) the success probability is not fixed but decreases endogenously as local population density approaches carrying capacity; (ii) bets are not independent but are coupled through spatial resource competition and neighbour effects; and (iii) growth is not unlimited but bounded by finite resources that are locally and irreversibly depleted. To date, no study has shown how spatially distributed Kelly-optimal agents, each responding to local density-dependent cues, collectively generate the canonical logistic growth equation with an emergent, rather than imposed, carrying capacity.

The main contribution of this paper is to give a Kelly-based growth strategy for games with an explicit carrying capacity in a limited environment. We show that when the betting strategy adapts to the distance from the carrying capacity, the resulting population dynamics naturally converge to logistic growth, rather than unbounded exponential growth. To test how this theoretical connection persists under spatial constraints, we introduce the Game of Fitness, a spatially explicit, probabilistic cellular automaton in which population growth emerges from dormancy and local reproductive decisions. Crucially, within this framework, the success probability is not imposed externally, but emerges endogenously from spatial interactions and can be predicted from the evolving population state. By explicitly incorporating both information availability, which shows the environmental awareness of organisms, and spatial reproduction mobility, this model enables us to explore how deviations from idealised assumptions affect population growth. Our results show that while increased information generally improves fitness outcomes, spatial reproduction mobility plays a dominant role in determining the achievable carrying capacity and growth rate. In particular, strong spatial constraints introduce deviations from classical logistic growth, even under optimal Kelly-based strategies. Considering the significance of both exponential growth and carrying capacities in biological (and, by structural analogy, economic and financial) growth models, our results provide a unifying perspective on Kelly betting and logistic growth in constrained environments.

The rest of the paper is organised as follows. First, we introduce the theoretical foundations underlying the study, reviewing the classical approaches to growth modelling, including logistic growth and Kelly betting. We then establish a mathematical bridge between these two frameworks through analytical derivations. Next, we present the proposed Game of Fitness model and describe its spatially explicit simulation framework. Subsequently, we present the results, including validation of the model and an analysis of the observed deviations from logistic growth. Finally, we conclude with a discussion of the results and the broader implications of the proposed framework.

Theoretical foundations

In this section, the theory behind the logistic growth and Kelly betting models is presented. Then, a mathematical bridge between these two models is established through analytical derivations.

Logistic growth

This subsection reviews the logistic growth equation and its discrete form, which serve as the target model for population growth throughout this paper.

Several different sigmoidal curves are used as models of empirical biological growth limited by a carrying capacity[25]. As an example, one of the most common was proposed by Pierre-Francois Verhulst in 1838, which he called the Logistic Equation, shown below in its continuous form, where the population ($S(t)$) is expressed as a function of time (t), the carrying capacity (K), and a measure of the aggressiveness of growth (r).

$$\frac{dS(t)}{dt} = rS(t)\left(1 - \frac{S(t)}{K}\right). \tag{1}$$

The solution to this Logistic Equation is

$$S(t) = \frac{KS(0)}{\left(K - S(0)\right)e^{-rt} + S(0)}, \tag{2}$$

where $S(0)$ is the population at time $t = 0$.

The Verhulst equation can also be expressed in its discrete form, known as the Logistic Map[26] as follows.

$$S_{n+1} = S_n + rS_n\left(1 - \frac{S_n}{K}\right). \tag{3}$$

It is the last term, $rS_n(1 - S_n/K)$), which provides the characteristic asymptotic flattening towards the carrying capacity. Both forms of Verhulst's equation, and most other growth sigmoids, such as the Gompertz equation[27], explicitly assume a value for the carrying capacity. Our question is how a sigmoid of this kind can be produced using a Kelly betting strategy. To this end, we introduce the basics of Kelly betting in the next subsection.

Kelly betting

The following subsection introduces the Kelly betting framework. Additionally, it illustrates the comparisons of bets in stationary and non-stationary environments.

Kelly betting is a strategy that maximises the logarithm of the growth rate in the long term. Initially, it was proposed as a gambling strategy by drawing parallels between the transmission of symbols in a communication channel and playing bets[9]. The core idea of Kelly betting is to optimally determine how the main capital should be divided into different bets to reduce risk and achieve the maximum growth in the long term, i.e., optimal bet-hedging. Here, risk is defined as the probability of ruin, i.e., the probability that the capital reaches zero (bankruptcy) following an unfavourable outcome. In the biological context, bankruptcy is equivalent to extinction. Kelly betting depends on the probability of success (or winning), which is exemplified below.

An illustrative application of Kelly, and perhaps the best known, is betting on the toss of a biased coin[28]. This bet can represent gambling in a game with different outcomes in biology or finance. For instance, a plant can choose to reproduce in a stochastic environment with different phenotypes. In finance, it can be a decision-making problem between investing in government funds and/or technology stocks. The player can bid (b) any amount up to their current capital or wealth (C), which will be lost if the guess is incorrect, and gained if correct. The expected outcome (ξ), that is, the average of many repeated experiments, is therefore:

$$\xi = p(C + b) + (1 - p)(C - b) \tag{4}$$

where p is the probability of guessing correctly, i.e., success probability. To calculate b, using Kelly, we maximise the log of the expected value by differentiating and setting it to zero:

$$\frac{d}{dC}\left(p\log(C + b) + (1 - p)\log(C - b)\right) = 0 \tag{5}$$

which solves to give the optimal bid at the n^{th} time step for each round of gambling b_n as follows:

$$b_n = C_n(2p_n - 1) \tag{6}$$

where p_n is the success probability at the n^{th} time step.

This simple and elegant result illustrates key features of the Kelly strategy: The size of the optimal bid depends only upon the success probability and current wealth of the gambler. Thus, the wealth is derived for each gamble as given by

$$\begin{aligned} C_{n+1} &= p_n[C_n + C_n(2p_n - 1) + (1 - p_n)[C_n - C_n(2p_n - 1)] \qquad (7) \\ &= 2C_n(2p_n^2 - 2p_n + 1), \qquad (8) \end{aligned}$$

where n shows the time step for each round of gambling. If the current probability at each gamble remains constant over time, i.e., $p_n = p$, the expected value after n recursive gambles, with bids calculated according to Eq. (6), can be visualised as shown in Fig. 1 (a). In this figure, blue circles are the expected result of a Kelly betting strategy for 15 recursive biased coin tosses in a limitless environment by setting $C_0 = 0.01$ and $p = 0.7$ in Eq. (8). The red crosses show the maximum the gambler could theoretically win with perfect foresight by betting the entire wealth at each game with the same initial capital. In this case, however, a single incorrect outcome would result in bankruptcy (i.e., ruin, where the gambler's wealth reaches zero and no further bets are possible). Although returns from Kelly betting are much lower than the maximum potential, they are achieved with substantially reduced risk, understood here as reduced sensitivity to ruin.

On the other hand, if we suppose that the success probability decreases over time, for example, due to an increasing limitation of the environment, then the size of the bids progressively decreases and returns approach zero as the probability falls to 0.5. As shown in Fig. 1 (b), this behaviour produces a sigmoidal growth curve, reflecting how the system responds to diminishing rewards as it approaches an upper bound or carrying capacity. In this figure, p is modelled as a piecewise function: $p = 0.6$ for the first 20 rounds, then decreasing linearly to $p = 0.5$ over the next 20 rounds, where it remains constant thereafter. It is important to emphasise that the constant-p and time-varying-p cases correspond to different environmental assumptions rather than competing strategies. While a stationary environment with constant p allows sustained exponential-like growth, a non-stationary environment with decreasing p naturally limits growth and increases uncertainty, with $p = 0.5$ representing the highest-risk regime.

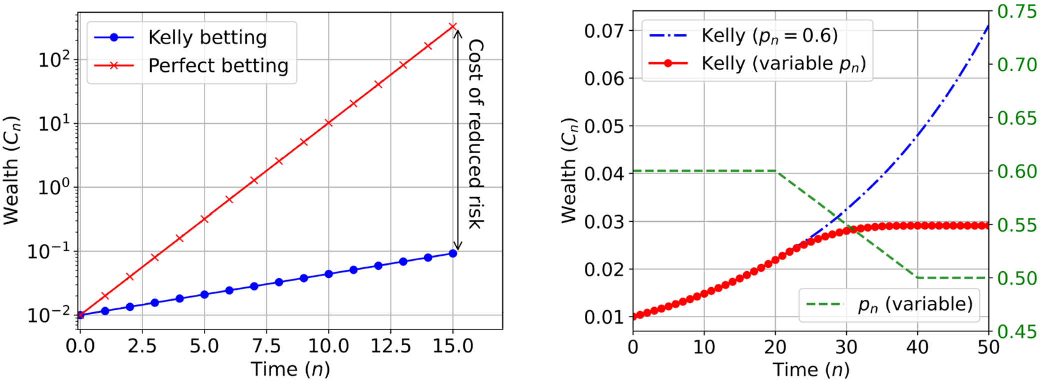
It should be noted that the classical optimality of the Kelly bid in (6), established for a fixed p[28], does not extend directly to the non-stationary case where p_n varies with the evolving population state. In this paper, the Kelly betting is therefore applied in a myopic sense: at each step, p_n represents the current estimate of the local success probability, and the bid is calculated accordingly. This is consistent with adaptive Kelly practice[29,30], though without a formal global optimality guarantee.

Given that real-world environments are constrained by carrying capacity and characterised by variable success probabilities, a more realistic approach is to place bets according to variable probabilities derived from reasonable environmental predictions. At this point, we can consider whether the carrying capacity is a special phenomenon in biological growth or, alternatively that it is simply a long-term environmental change like any other[31,32]. As of the next section, we explore the connection between the Kelly betting and logistic growth.

Mathematical bridge: Kelly betting as an interpretation of logistic growth

In this subsection, we hypothesise that the Kelly betting strategy provides a generative mechanism underlying logistic growth. We derive an analytical equivalence between the discrete-time Kelly update and the logistic map.

To construct a mathematical bridge between logistic growth and Kelly betting frameworks, we explicitly identify the gambler's wealth in the Kelly process with population size in a biological growth model, i.e., we set $C_n \equiv S_n$. This identification is motivated by the information-theoretic interpretation of Kelly betting in biological systems, where capital represents the accumulated count of surviving and reproducing individuals



(a) **(b)**

Fig. 1. (**a**) A comparison of recursive Kelly betting and perfect betting scenarios with the same initial wealth ($C_0 = 0.01$). For Kelly betting, $p = 0.7$. Kelly betting significantly reduces risk, defined here as sensitivity to ruin, i.e., the probability of bankruptcy following an incorrect bet. While perfect betting achieves the maximum possible return under perfect foresight, it is maximally fragile, as any single mistake results in immediate bankruptcy. In contrast, Kelly betting reduces risk by minimising the probability of ruin while maximising expected logarithmic growth. Note that the y-axis is logarithmic. (**b**) Growth in wealth under Kelly betting for two different environmental assumptions: a stationary environment with constant success probability ($p = 0.6$), and a non-stationary environment where the success probability decreases over time ($p = 0.6$ until bet 20, then decreases linearly to $p = 0.5$ by bet 40 and remains there). The initial wealth is $C_0 = 0.01$ in both cases. The two curves represent the same Kelly strategy operating under different probability dynamics. (**a**) Kelly betting versus perfect betting. (**b**) Kelly betting in stationary and non-stationary environments.

rather than monetary wealth[2]. Under this identification, the discrete-time Kelly update in Eq. (8) and the discrete logistic map in Eq. (3) can be directly compared.

With this hypothesis, it is straightforward to equate the expressions for the next-step values S_{n+1} and C_{n+1} arising from Eq. (8) and (3), respectively. We evaluate this equivalence analytically by matching the discrete-time Kelly update with the logistic growth equation, and subsequently verify its implications numerically through simulations in the next section. This analytical equivalence is expressed as

$$S_{n+1} = 2S_n(2p_n^2 - 2p_n + 1) = S_n + rS_n\left(1 - \frac{S_n}{K}\right). \tag{9}$$

where p_n denotes the success probability of the population at time step *n*, defined operationally as the probability that a reproductive attempt succeeds under the changing environmental conditions, thereby contributing to population survival and growth, which we refer to as fitness in the sense of realised outcomes. When the population size S_n is small relative to the carrying capacity *K*, resources and space are abundant, and reproductive attempts are more likely to succeed, which results in higher values of p_n. As S_n approaches *K*, increased competition and spatial limitation reduce the likelihood that reproduction succeeds, which is reflected by a decreasing p_n. In this sense, p_n captures how environmental constraints dynamically modulate reproductive success.

Via the manipulation of (9), the success probability can be derived as

$$p_n = \frac{1}{2} \pm \frac{1}{2}\sqrt{1-c} \tag{10}$$

where $c = (1-r) + r\frac{S_n}{K}$. Here, there are two roots for the solution of p_n, i.e., $p_n^{(1)}$ and $p_n^{(2)}$. In fact, these two roots are complementary to each other, and their addition is equal to one, i.e., $p_n^{(1)} = 1 - p_n^{(2)}$. In addition, either root gives the same result due to the $2p_n^2 - 2p_n$ term in (8) for Kelly betting. This term also ensures that S_{n+1} is still a real number even if the p_n is a complex number. Hence, the success probability at the n^{th} time step is given by

$$p_n = \frac{1}{2} \pm \frac{1}{2}\sqrt{r\left(1 - \frac{S_n}{K}\right)}. \tag{11}$$

Based on (11), the results of Kelly betting for different carrying capacities and the corresponding probabilities are given in Fig. 2. In this analysis, we use only the root of p_n that includes the positive sign before the square root, as given in (11). The figure illustrates that Kelly betting can be used to model an idealised scenario in which the parameters *r* and *K* are known *a priori*. Furthermore, it is observed that the probability of success decreases when the population approaches the carrying capacity, reflecting increasing environmental limitation.

Additionally, p_n depends on the parameter *r* and *K* in (11). However, these parameters may not be known *a priori*. To remove this dependency, we can use the following equation by manipulating (3) as follows.

$$r\left(1 - \frac{S_n}{K}\right) = \frac{S_{n+1} - S_n}{S_n}. \tag{12}$$

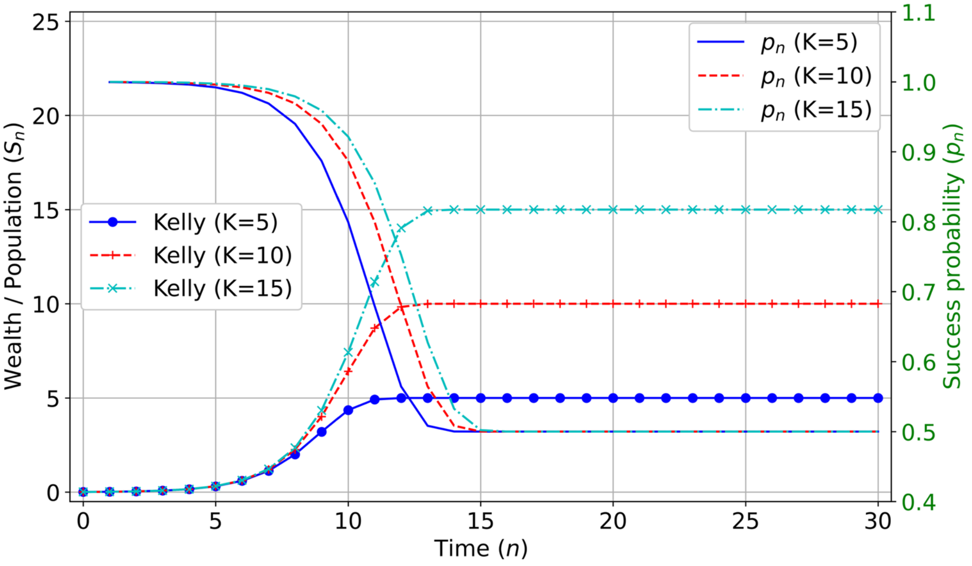


Fig. 2. Expected behaviour of Kelly betting for the wealth/population via logistic growth interpretation (left y-axis) and the success probability (right y-axis) for different carrying capacity values, $S_0 = 0.01$, and $r = 1$.

By substituting (12) into (11), we can obtain the success probability as given by

$$p_n = \frac{1}{2} \pm \frac{1}{2}\sqrt{\left(\frac{S_{n+1}}{S_n} - 1\right)}. \tag{13}$$

which can be used to calculate p_n when the current and next population numbers are known. It should be noted that (13) is only meaningful in the growth phase where $S_{n+1} \geq S_n$. When $S_{n+1} < S_n$, the expression under the square root becomes negative, yielding a complex value. This simply indicates that the equation is being evaluated outside its domain of applicability, and has no consequence for the simulation results since p_n is always computed via (15) in the simulation, which is real and bounded in [0, 1] by construction, as applied in the next section.

The Game of Fitness model: a simulation to show the relation between Kelly betting and logistic growth

Revealing emerging patterns with simple rules has always been fascinating for scientists. In this quest, cellular automata are widely used in physics, biology, chemistry, and computer science. One famous example is the Game of Life developed by John Conway in 1970[33]. This simple zero-player game or a simulation has only four rules for the survival, reproduction, and death of cells on a 2-D grid. The only intervention of the user is to determine the initial conditions of the cells, which are the number and position of cells. These simple rules can lead to spectacular patterns, such as moving spaceships, or oscillators. However, the deterministic structure of this cellular automaton can sometimes fail to produce nature's patterns. Hence, probabilistic cellular automata, in which the transition probabilities between states are random rather than deterministic, are also used in the literature[34]. Recent work has shown that probabilistic cellular automata can produce logistic growth and density-dependent dynamics from local interaction rules. For example, logistic probabilistic cellular automata have been used to model spatial pattern formation under local propagation and competition[35], while totalistic cellular automata have been shown to approximate logistic dynamics under mean-field conditions[36]. Probabilistic cellular automata have also been applied to study bacterial competition and coexistence driven by antagonistic neighbourhood interactions[37]. However, in these models the transition rules either impose logistic growth locally or use fixed transition probabilities. None of them embed a bet-hedging strategy such as Kelly betting into the local decision rule.

Inspired by the Game of Life, we propose the Game of Fitness model, which is a probabilistic cellular automaton in essence, to explore and validate the growth of a population via Kelly betting. Comparable work in evolutionary game theory involves strategic interaction between competing agents[38–41]; however, in this work, each organism independently applies a Kelly-optimal reproductive decision in response to local environmental conditions. As shown in Fig. 3, organisms, such as cells or plants, represented by a yellow tile, can reproduce their offspring on another tile based on some set of rules in the Game of Fitness as detailed in the next subsection.

Model setup and assumptions

This subsection describes the spatial environment, the organism states and strategies, the key parameters and assumptions governing the Game of Fitness.

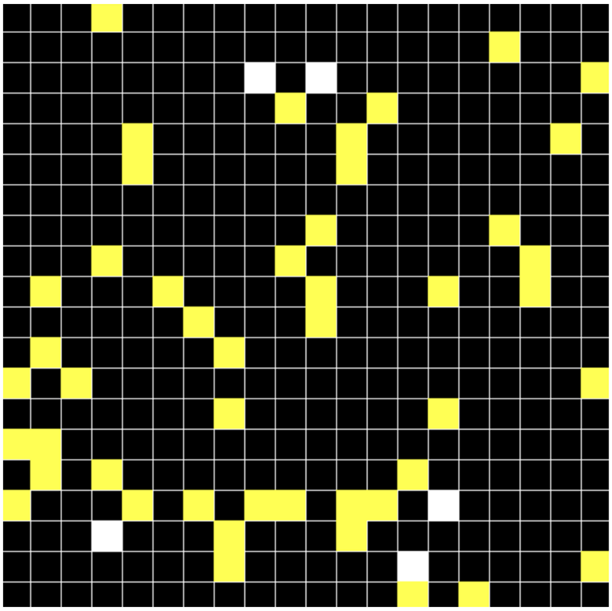


Fig. 3. A snapshot of the 'Game of Fitness' simulation. The 20 × 20 grid displays living organisms (yellow, e_2), available tiles (black, e_1), and depleted tiles (white, e_3).

As illustrated in Fig. 3, the simulation environment is represented as a two-dimensional square grid. Each grid tile can exist in one of three possible environmental states:

$$\begin{aligned} e_1 &: \text{Unoccupied tile,} \\ e_2 &: \text{Occupied tile,} \\ e_3 &: \text{Depleted tile.} \end{aligned}$$

Here, we assume that tiles in state e_1 contain food and are available for occupation. When a tile becomes occupied (e_2) and is subsequently vacated, it transitions to the depleted state (e_3). This transition only occurs when the resident organism dies as a result of a failed reproductive attempt. Tiles in state e_3 are permanently unavailable for occupation and are excluded from further interactions in the simulation. The permanent unavailability of depleted tiles (e_3) is motivated by irreversible local resource exhaustion observed across biological organisms. For instance, in plant communities, root-mediated soil resource exhaustion makes previously occupied patches unavailable for recolonisation[42].

As for the reproduction, each organism can adopt one of the following two strategies:

$$\begin{aligned} s_1 &: \text{Reproduce,} \\ s_2 &: \text{Do not reproduce and stay dormant.} \end{aligned}$$

At any time step *n*, the fraction of the population to reproduce (b_n), i.e., which organism chooses the strategy s_1, is determined by the bid size of Kelly betting via

$$b_n = \text{round}(S_n(2p_n - 1)), \tag{14}$$

where p_n is the success probability and $\text{round}(\cdot)$ denotes rounding to the nearest integer using the round-half-to-even convention (also known as Banker's rounding) to ensure that the result is an integer. To estimate p_n, we assume that the population's success to survive decreases as the environment gets more populated, i.e., food gets more scarce and less space is left in the environment. For the n^{th} time step, it is estimated by

$$p_n = \frac{E_{1,n}}{E_T - E_{3,n}}, \tag{15}$$

where E_T is the number of total tiles, $E_{1,n}$ and $E_{3,n}$ show the number of tiles at state e_1 and e_3, respectively. (15) estimates the success probability from the current spatial configuration of the grid, under the assumption that the fraction of unoccupied habitable tiles reflects the likelihood of a randomly chosen target being available. The quantity p_n therefore captures the instantaneous success probability of reproduction given the current spatial configuration, while the outcomes of individual reproductive and survival decisions are defined in a fitness matrix (*W*). As introduced earlier, fitness is understood here in terms of realised survival and reproduction outcomes, rather than as an intrinsic trait of the organism. *W* is given by

$$W = \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix}, \tag{16}$$

where the rows correspond to the environmental states e_1 and e_2, and the columns correspond to the strategies s_1 and s_2. Each coefficient in *W* denotes the final number of organisms present at the next time step resulting from the action of a single organism under the given state and strategy. For example, $w_{22} = 1$ indicates that an organism remains alive when it chooses not to reproduce and stays dormant, while $w_{21} = 0$ represents that both parent and offspring die in a reproduction attempt to an occupied tile. The state e_3 is excluded from *W* since it represents permanently unavailable tiles and therefore does not contribute to reproductive or survival dynamics. Additionally, the probability of the environmental states at the n^{th} time step is a function of several parameters ($h(\cdot)$), as given by

$$P(e_{j,n}) = h_j(\rho, \alpha, S_n), \tag{17}$$

where *j* is the environmental state index, ρ is the Moore radius (which determines the reproduction mobility of organisms)[43], and α is the information parameter, which determines the environmental awareness of the organisms. It should be noted that it is non-trivial to write a closed-form expression for (17) due to the spatial complexity of the Game of Fitness. Instead, an explanation about its parameters is as follows.

ρ defines the extent of the neighbourhood around each cell where it can reproduce, and therefore directly controls its reproduction mobility. For instance, if $\rho = 1$, an organism can reproduce into any of the 8 tiles surrounding it in the 3×3 square it occupies. The parameter α determines the degree to which organisms are guided by environmental information when selecting reproduction sites:

- $\alpha = 0$: The population is not informed, and the reproduction is purely random among all habitable tiles (occupied or unoccupied), with no regard to environmental state.
- $0 < \alpha < 1$: The population is partially informed and the reproduction has a bias toward unoccupied tiles proportional to α. With probability α, an organism selects its reproduction target uniformly from unoccupied tiles within its neighbourhood; with probability $1 - \alpha$, it selects uniformly from all habitable tiles, including occupied ones.

- $\alpha = 1$: The population is fully informed, such that organisms always restrict their choice to unoccupied tiles, selecting uniformly at random when multiple options are available.

The parameter α is inspired by the varying capacity of organisms to sense local population density. In bacterial populations, quorum sensing is a well-known mechanism by which cells estimate local density through diffusing signalling molecules, enabling graded collective responses to crowding[44–46]. In plant populations, similar awareness arises through above-ground phytochrome-mediated detection of neighbouring vegetation via changes in the red-to-far-red light ratio[47], and through below-ground root-secreted signalling chemicals that scale with neighbour density[42,48]. Thus, α allows us to model a continuum between uninformed random choice and perfectly informed decision-making. Even if $\alpha = 1$, there is still randomness in the simulation, since a random empty tile is chosen when multiple are available. Next, the extent of this randomness and the implementation algorithm are detailed.

Algorithm

This subsection details the step-by-step procedure by which the selection, reproduction, and evaluation rules are executed at each time step of the simulation.

The simulation algorithm flowchart is shown in Fig. 4. The simulation starts with initialising parameters: α, ρ, and N, which is the number of maximum time steps. Initially, all tiles are unoccupied (at state e_1), and an initial organism is placed at the coordinates (x_0, y_0). For each time step, the simulation consists of 3 distinct phases:

- **Selection:** First, p_n is calculated by using Eq. (15) and b_n organisms that will participate in reproduction (using strategy s_1) are selected based on Eq. (14). We select b_n organisms at random from the grid, with each organism having equal probability of being chosen, i.e., according to a uniform distribution over all occupied sites.
- **Reproduction:** Each selected organism chooses a target tile randomly for reproduction according to the information parameter α within the region determined by ρ.
- **Evaluation:** The outcomes of the reproduction step is evaluated based on the target tile's state: Choosing a tile at state e_1 leads to successful reproduction, and the tile the offspring occupies transitions to state e_2. If a tile at state e_2 is chosen, then both the offspring and parent organisms die, and their tiles become uninhabitable for the rest of the simulation, i.e., the states of these tiles are updated as e_3.

The simulation stops if there are no live organisms left on the grid, or there are no available tiles to reproduce into, or the maximum time step has been reached. In addition, the population stops making bets if p_n is equal to or smaller than 0.5 since b_n becomes 0 or less according to Eq. (14). It is important to note that W is implicitly utilised in the "Reproduction", "Failed Bet", and "Successful Bet" stages of the algorithm depicted in Fig. 4.

Results

In this section, the numerical results are presented in two subsections. First, we present validation results showing that population growth with Kelly betting leads to logistic growth. Second, we present population growth results from the Game of Fitness simulation for various ρ and α values. We also analyse changes in carrying capacity and spatial heatmaps of final population density.

The simulations use a 20×20 spatial grid and start from a single organism placed at the centre $(x_0, y_0) = (10, 10)$. Each run is evolved for at most $N = 100$ time steps, and all results are averaged over $10,000$ independent simulation runs. The fitness matrix is fixed as $W = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$. In the fitness matrix W, under strategy s_1, a parent organism that reproduces will result in two organisms if the bet is successful ($w_{11} = 2$), or both parent and offspring die if reproduction is attempted on an occupied tile ($w_{21} = 0$). $w_{12} = w_{22} = 1$ indicates that an organism at state e_1 or e_2 does not reproduce and stays dormant.

The fitness matrix W reflects a dormancy-reproduction trade-off observed across resource-limited biological systems. The dormancy entries $w_{12} = w_{22} = 1$ model the well-established strategy of forgoing reproduction under competitive pressure to maintain survival[49,50]. A successful reproductive attempt into an unoccupied tile ($w_{11} = 2$) produces two surviving organisms. A failed attempt into an occupied tile ($w_{21} = 0$) penalises both parent and offspring, as the offspring cannot establish and the parent incurs the full cost of a failed reproductive attempt with no compensating gain.

Unless otherwise stated, the Moore radius is set to its maximum value $\rho = 19$, so that organisms can reproduce anywhere on the grid, and the information parameter is fixed at $\alpha = 0$, indicating that the randomness in the selection of tiles is at its maximum.

Validation

This subsection validates that Kelly-based population growth in the spatially limited Game of Fitness environment produces logistic growth. It also shows that the theoretical and simulated success probabilities derived in the Theoretical Foundations section are valid.

The validation results are shown in Figure 5 (a)-(c) with their fitting values given in the caption. Figure 5(a) shows the population growth, defined as the number of organisms over time, while Fig. 5(b) presents the normalised growth curve together with a logistic fit. The red line represents the average population size across simulations, whereas the grey points indicate the scatter over $10,000$ independent runs in Fig. 5 (a). The logistic fit, obtained using the closed-form solution of the logistic equation given in Eq. (2), overlaps almost perfectly

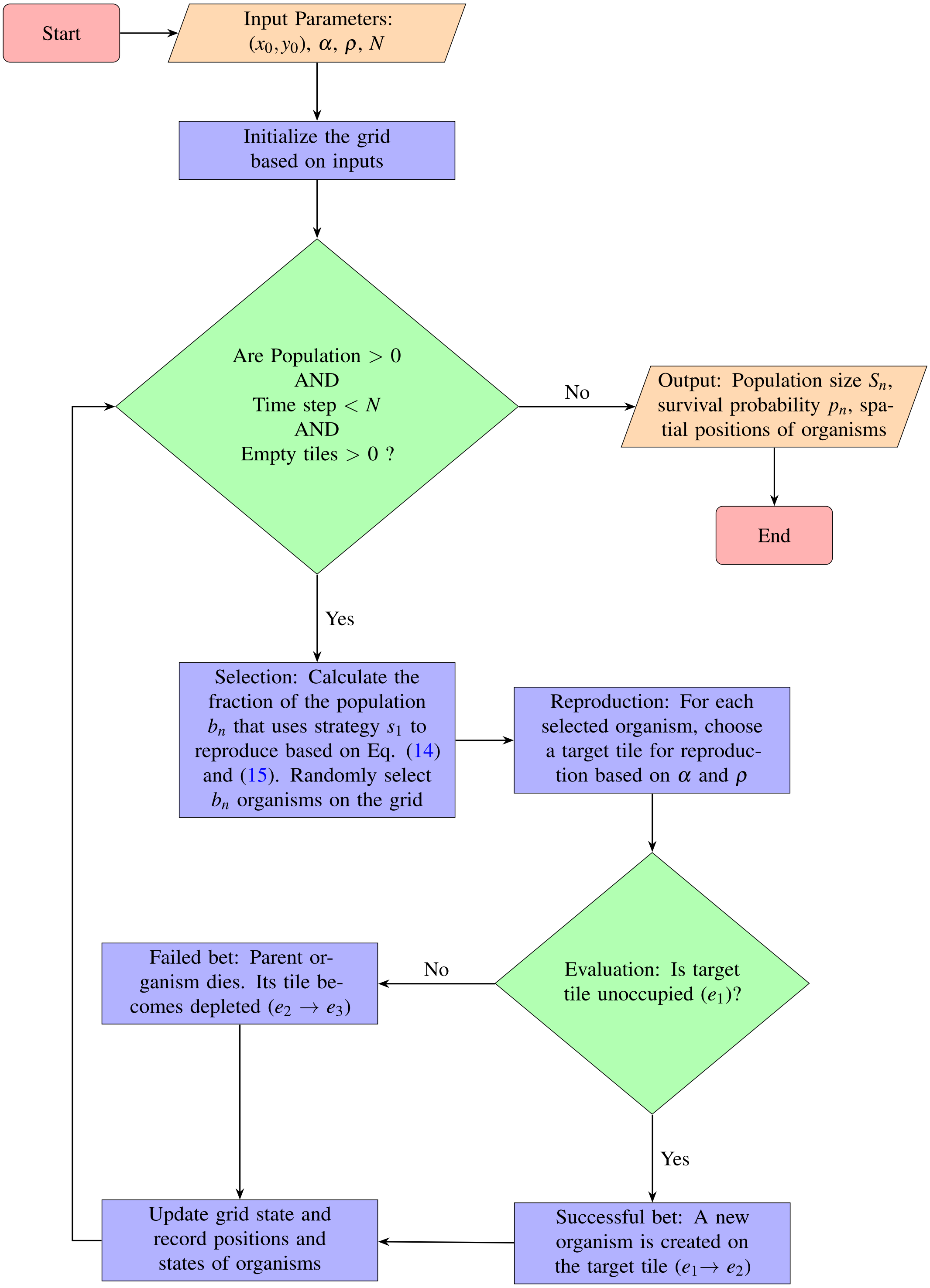


Fig. 4. Flowchart of the simulation algorithm.

with the simulation results, yielding a mean squared error (MSE) of 2.79×10^{-6}. The fit was computed using a standard nonlinear least-squares optimisation based on the Levenberg–Marquardt algorithm[51].

The plots in Fig. 5 (a) and (b) clearly show that logistic growth emerges from the growth of organisms playing bets according to the Kelly criterion in a spatially limited environment. This is further validated by Fig. 5 (c), which shows the course of success probabilities for theoretical values derived in Eq. (13) and simulated values proposed in Eq. (15). For Eq. (13), we adopt the root $p_n = 0.5 + 0.5\sqrt{(S_{n+1}/S_n)} - 1$, as it yields the physically meaningful solution. The overlapping values in this plot verify the mathematical bridge between Kelly betting and logistic growth in a limited environment.

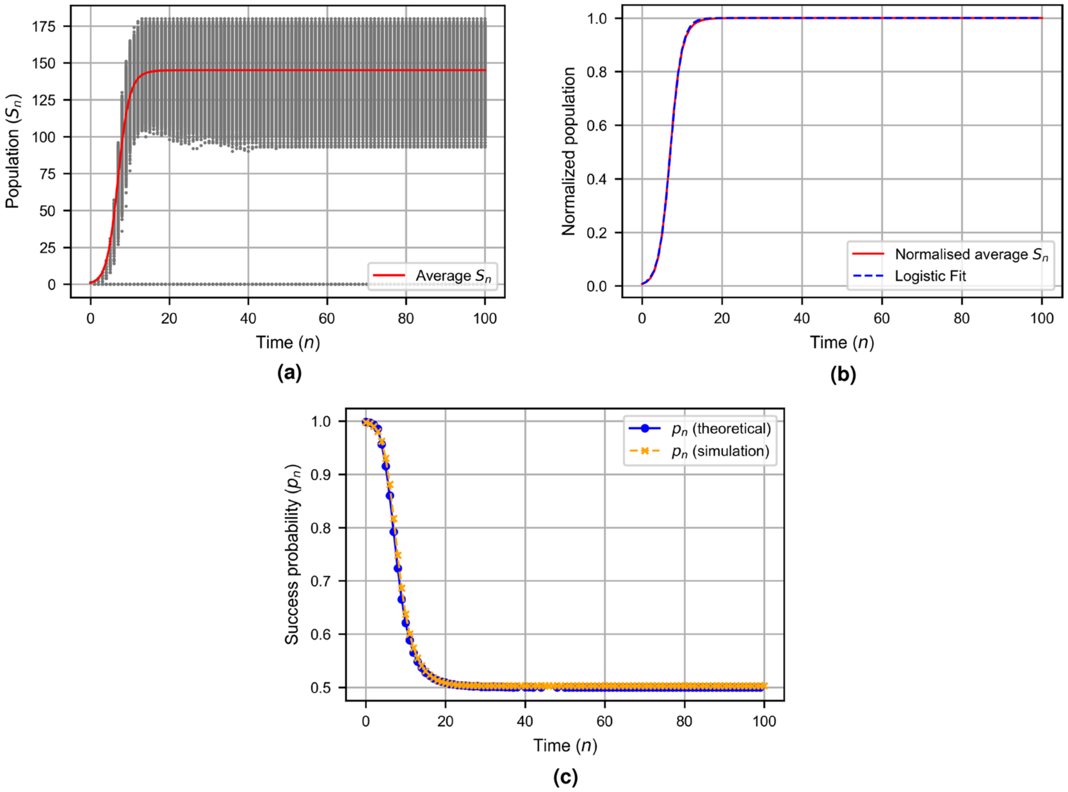


Fig. 5. Validation results showing that Kelly betting in a limited environment leads to logistic growth. (a) Population growth with scatter plot and the mean value over 10000 runs. (b) Normalised mean population growth in (a) together with its logistic fit, yielding parameters $K = 1, r = 0.69, t_0 = 7.06$, and mean squared error (*MSE*) $= 2.79 \times 10^{-6}$. (c) Mean success probabilities calculated by Eq. (13) as theoretical results and Eq. (15) as simulation results over 10000 runs, $MSE = 3.68 \times 10^{-5}$. (**a**) Population growth trajectories. (**b**) Logistic fit of normalised population. (**c**) Success probability comparison.

An important outcome of the Game of Fitness simulation is that the success probability governing Kelly betting does not need to be specified externally. Instead, it is predicted dynamically from the spatial configuration of the population via Eq. (15), which depends on the availability of unoccupied tiles. As the population expands and space becomes limited, the predicted success probability decreases, reflecting increasing environmental constraints. This emergent probability directly determines the effective Kelly bid size and links local spatial interactions to population-level growth dynamics.

Simulation results

This subsection illustrates how varying the reproduction mobility ρ and information parameter α affects population growth, carrying capacity, and spatial distribution, and compares Kelly betting against alternative bet-hedging strategies.

We first analyse the effect of reproduction mobility (via Moore radius ρ) and information (via α) on population growth and carrying capacity. In Fig. 6 (a), the mean population growth for different values of ρ is given. Here, $\rho = 19$ corresponds to the scenario simulated in Fig. 5. As observed in Fig. 6 (a), the growth pattern diverges from the logistic function as ρ decreases. This stems from the fact that the increase in the spatial limitation decreases the likelihood of finding unoccupied tiles. Hence, they have more failed bets even if they can predict the environment successfully, i.e., p_n is high. Moreover, it is interesting to note that the population almost dies out in the long term for $\rho = 1$, meaning that when the constraint on their first reproduction move is most restrictive, they cannot use the carrying capacity of the environment efficiently. Additionally, the effect of α is shown in Fig. 6 (b). It is revealed that the population can achieve a higher carrying capacity as they are more informed about the environment.

Fig. 6 (c) shows how the carrying capacity (*K*) changes with ρ and α. Here, *K* denotes the converged population size at the end of the simulation, as illustrated in Fig. 6 (a)-(b). The colour gradient in the figure provides a visual encoding of *K*, highlighting regions of low and high carrying capacities across the parameter space. The results in Fig. 6 (c) for a constant α value imply that the spatial constraint on the reproduction limits the carrying capacity significantly. When the range of organisms that they can reproduce is small, the population cannot exploit the environment efficiently, although there is plenty of space available. We can make an analogy of the reproduction of plants with their seeds dispersed by the wind. Suppose there is more wind to disperse seeds further, i.e., ρ is higher. In that case, they can benefit from the environment more efficiently, whereas in a less windy

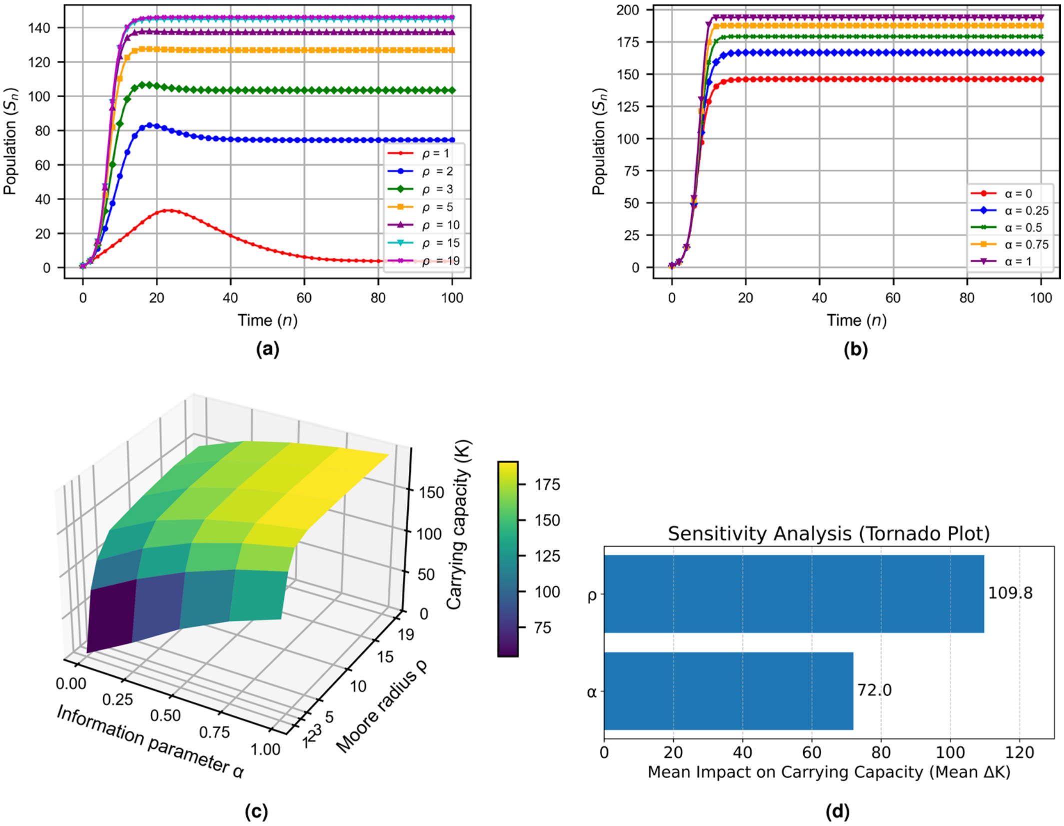


Fig. 6. Mean population over time for various values of (**a**) Moore radius $\rho = \{1, 2, 3, 5, 10, 15, 19\}$ and (**b**) information parameter $\alpha = \{0, 0.25, 0.5, 0.75, 1\}$. (**c**) Three-dimensional representation of the carrying capacity (K) as a function of α and ρ. The colour scale indicates the magnitude of K, with warmer colours corresponding to higher carrying capacity. (**d**) Tornado plot of the sensitivity of carrying capacity K to the parameters α and ρ. Bar length represents the mean ΔK when each parameter is varied across its full range while the other is held fixed, averaged across all values of the fixed parameter. (**a**) Population growth for varying ρ and constant α. (**b**) Population growth for varying α and constant ρ. (**c**) Carrying capacity K as a function of ρ and α. (**d**) Sensitivity analysis.

season (smaller ρ), the population remains smaller. In addition to the ability to spread into the environment, information about the environment is also effective in determining the carrying capacity as shown in Fig. 6 (c). Therefore, we argue that the population's capability in exploiting environmental space depends on the availability of environmental information, which in turn recursively influences the success probability of the bets placed. In this sense, higher values of K–highlighted by warmer colours–correspond to more effective exploitation of the available environmental space. Furthermore, while both parameters (α and ρ) have high values, the population maximises the use of the carrying capacity, although the weight of the mobility is higher. This plot also reveals that the effect of being highly informed about the environment is limited when the reproduction mobility is low. Moreover, we quantify the relative influence ofα and ρ on the carrying capacity K with a sensitivity analysis using a tornado plot, as shown in Fig. 6 (d). For each parameter, the mean range of K (mean ΔK) was computed by averaging the peak-to-peak variation in K across all values of the other parameter. The results show that ρ produces a mean ΔK of 109.8 compared to 72 for α, confirming quantitatively that reproduction mobility has a more dominant effect on carrying capacity than environmental information availability.

Next, we examine the spatial distribution of the final population density. Fig. 7 illustrates the heatmap of the mean occupancy density of the organisms' final positions for various α and ρ values. When the population is not informed, i.e., $\alpha = 0$, as given in Figs. 7 (a), (d), and (g), the spatial spread of the population increases proportionally with the reproduction range, leaving the centre of the grid as the least occupied. As the population becomes more informed, the density concentrates on a circle surrounding the centre for limited reproduction mobility, as illustrated in Figs. 7 (b), (c), (e), and (f). However, for high reproduction mobility ($\rho = 19$), the dispersion of organisms gets more homogenous except for the centre tile where the first organism is deployed initially, as denoted in Figs. 7 (h) and (i). The population can use the environment more effectively by spreading as they have more information when ρ is constant. As with carrying capacity, reproduction mobility is the dominant factor for spatial dispersion, while information acts more as a modulating factor.

Finally, we compare Kelly betting with alternative bet-hedging strategies. In Fig. 8, simulation results for the Kelly (referred to as Full Kelly in Fig. 8) and other bet-hedging methods, such as fractional Kelly, and fixed fraction bets are compared for various α and ρ values. In fractional Kelly, Eq. (14) becomes

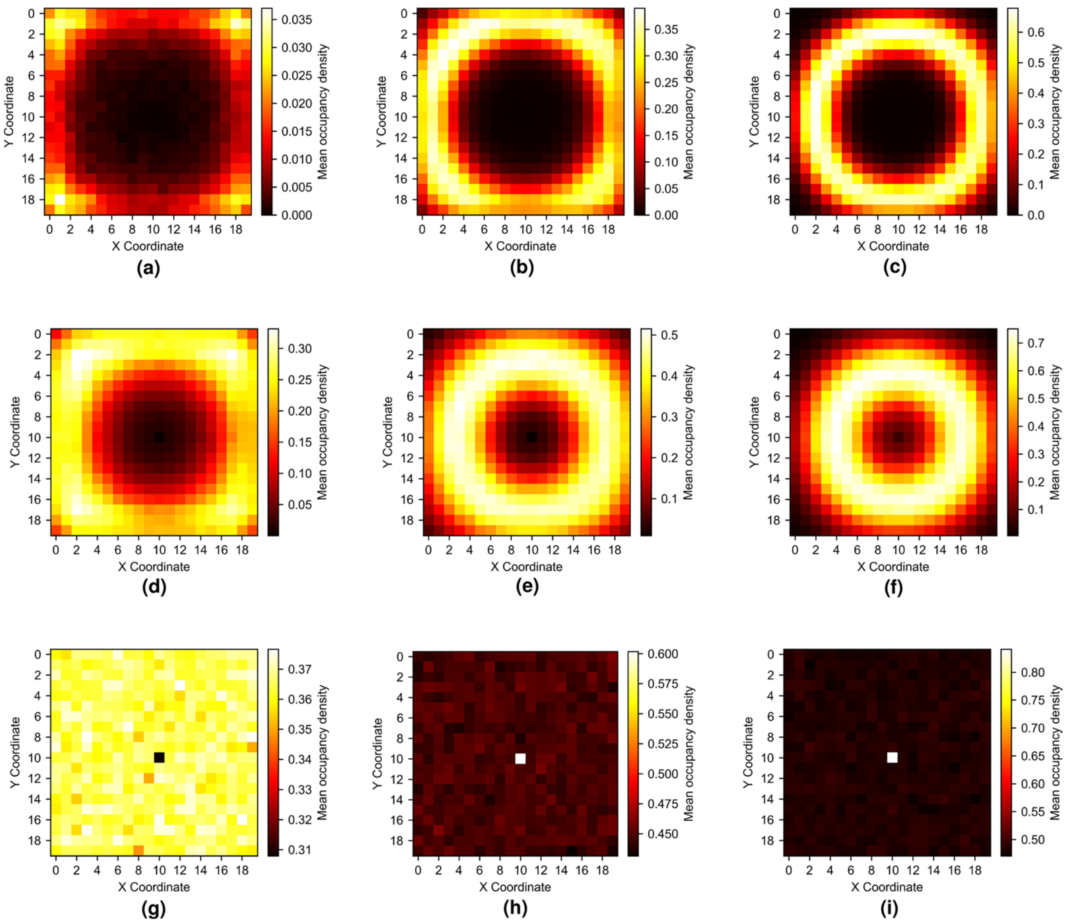


Fig. 7. Heatmaps showing the mean spatial occupancy density of the population under different combinations of the information parameter α and Moore radius ρ. (**a**) α = 0, ρ = 1. (**b**) α = 0.5, ρ = 1. (**c**) α = 1, ρ = 1. (**d**) α = 0, ρ = 2. (**e**) α = 0.5, ρ = 2. (**f**) α = 1, ρ = 2. (**g**) α = 0, ρ = 19. (**h**) α = 0.5, ρ = 19. (**i**) α = 1, ρ = 19.

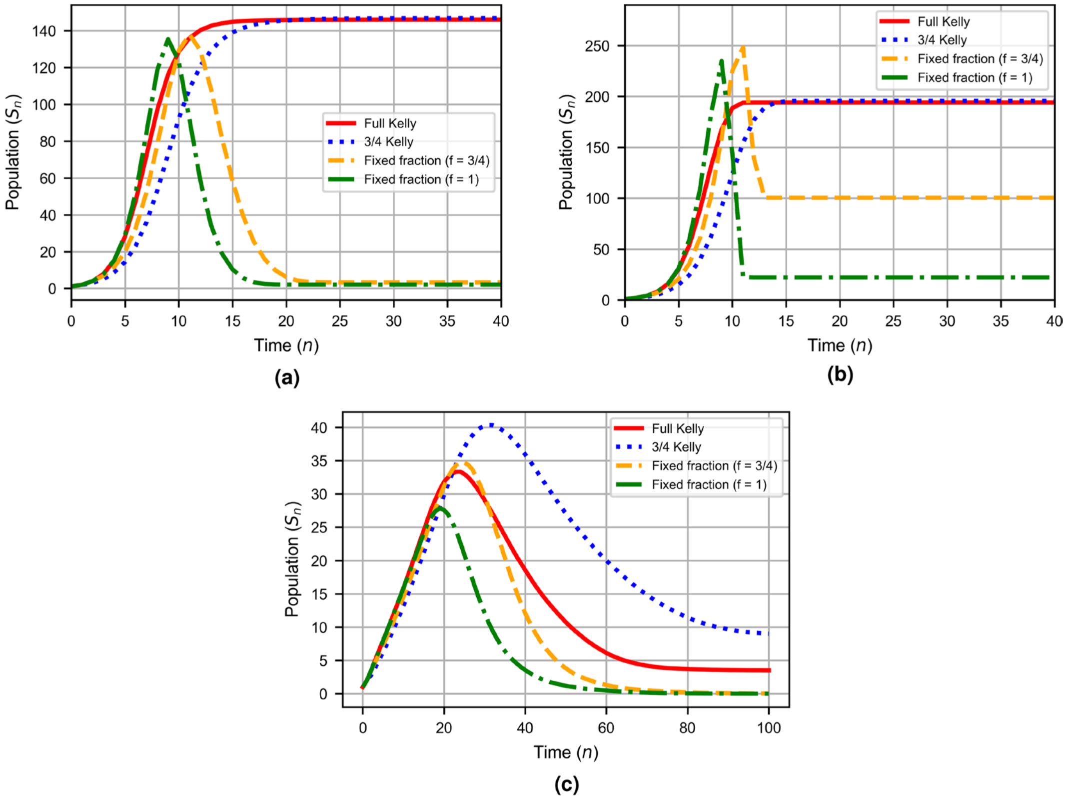
$$b_n^{(FK)} = \text{round}(aS_n(2p_n - 1)), \tag{18}$$

where *a* is the fraction coefficient, set to 0.75 for the dashed blue line in Fig. 8. For $a = 1$, Eq. (18) reduces to the standard Kelly betting strategy given in Eq. (14). For the fixed fraction strategy, the bid size is independent of the success probability as given by

$$b_n^{(FF)} = \text{round}(fS_n), \tag{19}$$

where *f* is the fraction coefficient, which is 0.75 and 1 for the results shown in Fig. 8 with blue and green lines, respectively.

The results in Fig. 8 (a) show that using any of the three bet-hedging strategies (Kelly, fractional Kelly, and fixed fraction betting) provides a rapid growth in the short-term. However, Kelly betting provides a clear advantage, maintaining stability around a high carrying capacity in the long term. In the fractional Kelly betting strategy, it takes a longer time to reach the carrying capacity, but it still ensures that the population eventually reaches the carrying capacity and stabilises, while keeping the logistic growth pattern. However, using a fixed fraction strategy is risky as it ultimately leads to extinction. Therefore, Kelly betting is a robust strategy for organisms in the long term, even in a limited environment. Additionally, when the population is fully informed about the environment, fixed fraction betting gives a short-term advantage over Kelly betting, producing higher peak values and faster growth curves as shown in Fig. 8 (b). However, Kelly betting strategy still keeps its advantage in the long-term growth with a higher carrying capacity. Furthermore, decreasing the reproduction mobility causes a pattern with a slower growth rate and lower carrying capacity as illustrated in Fig. 8 (c). Yet, Kelly and fractional Kelly betting save the population from extinction, whereas fixed fraction betting populations die out. Interestingly, the fractional Kelly betting strategy gives the highest carrying capacity values in Fig. 8 (c). This implies that more conservative Kelly bets in uninformed and low mobility reproduction scenarios give a population a better chance to exploit the environment.



(a)

(b)

(c)

Fig. 8. Comparison of mean population growth over time for Kelly-based strategies (Full Kelly corresponding to Eq. (14), and fractional Kelly with $a = 0.75$ in Eq. (18)) versus fixed-fraction strategies ($f = 0.75$ and $f = 1$ in Eq. (19)). (**a**) Comparison of Kelly-based strategies for $\alpha = 0$, $\rho = 19$. (**b**) Comparison of Kelly-based strategies for $\alpha = 1$, $\rho = 19$. (**c**) Comparison of Kelly-based strategies for $\alpha = 0$, $\rho = 1$.

Discussion

In this paper, we propose the idea that bet-hedging using the Kelly betting strategy and logistic growth are two different paths leading to the same destination in a limited environment. To demonstrate this, we designed a simulation, the Game of Fitness, which validates this connection. We approach the problem to model the growth of a population from a mathematical biology perspective and show the viability of the Game of Fitness to simulate the growth of a population by betting on the decision to reproduce or stay dormant.

In our study, we adopt a scenario with diversified bet-hedging using dormancy as an alternative strategy to reproduction. More importantly, our focus is to explore the spatial limitations of the environment in a bet-hedging scenario, specifically by using Kelly betting. A key implication of our results is that the success probability required by Kelly betting need not be treated as an externally known parameter. In the Game of Fitness, this probability emerges from spatial structure and population density, effectively allowing organisms to infer reproductive success from local environmental conditions. This provides a mechanistic interpretation of how Kelly-optimal strategies could operate in realistic biological systems, where success probabilities are neither stationary nor directly observable. With the mathematical framework we propose, the convergence of population growth in a limited environment is actually a logistic growth, which is a well-known phenomenon for limited environments such as the growth of bacteria in a petri dish.

Beyond the convergence to logistic growth, our results highlight the distinct roles of information and reproduction mobility in determining fitness in limited environments. Information, which is an essential feature of living things, plays a significant role in the fitness of organisms[1]. In essence, the more information a population has about the environment, the better it fits that environment[2]. However, neither resources nor space are unlimited in an ecosystem; hence, a carrying capacity is imposed in a limited environment. Therefore, the effect of being more informed about the environment provides the ability to use the environment with a higher carrying capacity (see Fig. 6 (c)).

The reproduction mobility (ρ), i.e., the range to reproduce offspring, has an even more pronounced effect on population dynamics. Even if a population is not informed about its environment, but has a long range to reproduce offspring, it can still reach a high carrying capacity (see Fig. 6 (c)). This suggests that spatial reach can compensate, to some extent, for informational limitations. Considering both parameters, α and ρ, our model also captures scenarios in which increased information about the environment does not necessarily lead to maximal exploitation of available resources. This occurs when reproduction mobility is limited, indicating that spatial constraints can play a dominant role relative to information in determining long-term population capacity. The causal relationship between ρ and α can be understood as follows. When ρ is large, the neighbourhood contains many empty tiles, so even an uninformed organism has a high probability of landing on an empty tile by chance, limiting the additional benefit of environmental awareness. When ρ is small, the neighbourhood is more restricted and crowded, making α relatively more valuable for avoiding occupied tiles within the limited range. However, despite this relative advantage of α at low ρ, the absolute carrying capacity K remains substantially lower than at high ρ regardless of α. Therefore, ρ dominates not because it renders α irrelevant, but because the absolute scale of K is overwhelmingly determined by the size of the accessible neighbourhood. In this sense, α acts as a modulator of performance within the space that ρ opens up, but cannot compensate for a fundamentally restricted neighbourhood.

The comparison with alternative bet-hedging strategies further illustrates the robustness of Kelly-based approaches. While fixed fraction strategies may offer short-term advantages under full information, they are prone to extinction in the long term, particularly under spatial constraints or limited information. In contrast, Kelly and fractional Kelly strategies consistently maintain stable populations. Notably, fractional Kelly betting performs particularly well under low-information and low-mobility conditions, indicating that more conservative betting can be advantageous in uncertain and spatially constrained environments.

Our study has its limitations, such as the use of a single phenotype and a static environment. Our future work will be to further explore the effects of phenotypic diversification in a limited and changing environment. We expect that Kelly betting strategy may lead to a generalised logistic growth function as observed in limited environments such as the algae population with two phenotypes having an S-shaped growth curve[52].

On the other hand, the equivalence of the logistic growth and Kelly betting framework can have useful applications in finance and economics to analyse the effects of the Kelly betting strategy in a limited economic environment, where resources or opportunities are finite. For instance, capital growth and Kelly-based strategies are traditionally studied in idealised settings, where betting opportunities are assumed to be repeated, statistically independent, and unconstrained, leading to asymptotic optimality in terms of long-term growth rate[28,53]. In practice, however, financial decision-making is subject to estimation errors, risk constraints, and structural limitations, which often motivate deviations from full Kelly strategies, such as fractional Kelly betting[54]. Interpreting information availability (α) as the quality of market knowledge and reproduction mobility (ρ) as access to investment opportunities or diversification pathways suggests a possible analogy to constrained economic environments. While our model is not intended as a direct representation of financial markets, it highlights how asymmetries between information and access can shape long-term growth and stability under finite-resource conditions.

Data availability

The simulation code and analysis scripts used to generate the results reported in this study are publicly available via a permanent repository at https://github.com/takhmina-iliiasova/game-of-fitness. An interactive web-based implementation of the model, allowing readers to explore the dynamics described in this paper, is accessible at https://www.gameoffitness.net/.

Received: 3 February 2026; Accepted: 31 March 2026

References

1. Bartlett, S. et al. Physics of life: Exploring information as a distinctive feature of living systems. *PRX Life* **3**, 037003 (2025).
2. Donaldson-Matasci, M. C., Bergstrom, C. T. & Lachmann, M. The fitness value of information. *Oikos* **119**, 219–230 (2010).
3. Rivoire, O. & Leibler, S. The value of information for populations in varying environments. *J. Stat. Phys.* **142**, 1124–1166 (2011).
4. Mitchell, A. et al. Adaptive prediction of environmental changes by microorganisms. *Nature* **460**, 220–224 (2009).
5. Abley, K., Goswami, R. & Locke, J. C. Bet-hedging and variability in plant development: Seed germination and beyond. *Philosophical Transactions of the Royal Society B: Biological Sciences* **379**, 20230048 (2024).
6. Endler, J. A. Some general comments on the evolution and design of animal communication systems.. *Philosophical Transactions of the Royal Society of London Series B: Biological Sciences* **340**, 215–225 (1993).
7. Bialek, W. *Biophysics: searching for principles* (Princeton University Press, 2012).
8. Okabe, T. & Yoshimura, J. A new long-term measure of sustainable growth under uncertainty. *PNAS Nexus* **1**, pgac228 (2022).
9. Kelly Jr, J. A new interpretation of information rate. *Bell Labs Technical Journal* (1956).
10. Moffett, A. S. & Eckford, A. W. Kelly bets and single-letter codes: Optimal information processing in natural systems. *IEEE Trans. Mol. Biol. Multi-Scale Commun.* https://doi.org/10.1109/tmbmc.2025.3581468 (2025).
11. Philippi, T. & Seger, J. Hedging one's evolutionary bets, revisited. *Trends Ecol. Evol.* **4**, 41–44 (1989).
12. Veening, J. W., Smits, W. K. & Kuipers, O. P. Bet-hedging and epigenetic inheritance in bacterial cell development. *Proc. Natl. Acad. Sci. U.S.A.* **105**, 4393–4398 (2008).
13. Grimbergen, A. J., Siebring, J., Solopova, A. & Kuipers, O. P. Microbial bet-hedging: The power of being different. *Curr. Opin. Microbiol.* **25**, 67–72 (2015).
14. Childs, D. Z., Metcalf, C. & Rees, M. Evolutionary bet-hedging in the real world: Empirical evidence and challenges revealed by plants. *Proc. R. Soc. B Biol. Sci.* **277**, 3055–3064 (2010).
15. de Groot, D. H., Tjalma, A. J., Bruggeman, F. J. & van Nimwegen, E. Effective bet-hedging through growth rate dependent stability. *Proc. Natl. Acad. Sci. U. S. A.* **120**, e2211091120 (2023).
16. Rossetti, V., Filippini, M., Svercel, M., Barbour, A. & Bagheri, H. C. Emergent multicellular life cycles in filamentous bacteria owing to density-dependent population dynamics. *J. R. Soc. Interface* **8**, 1772–1784 (2011).
17. Villa Martín, P., Muñoz, M. A. & Pigolotti, S. Bet-hedging strategies in expanding populations. *PLoS Comput. Biol.* **15**, e1006529 (2019).
18. Piñero, J., Sowinski, D. R., Ghoshal, G., Frank, A. & Kolchinsky, A. Information bounds production in replicator systems. *Communications Physics* (2026).
19. Dinis, L., Unterberger, J. & Lacoste, D. Pareto-optimal trade-off for phenotypic switching of populations in a stochastic environment. *J. Stat. Mech. Theory Exp.* **2022**, 053503 (2022).
20. Wu, B., Gokhale, C. S., van Veelen, M., Wang, L. & Traulsen, A. Interpretations arising from wrightian and malthusian fitness under strong frequency dependent selection. *Ecol. Evol.* **3**, 1276–1280 (2013).

21. Chapman, E. J. & Byron, C. J. The flexible application of carrying capacity in ecology. *Glob. Ecol. Conserv.* **13**, e00365 (2018).
22. Caravelli, F., Sindoni, L., Caccioli, F. & Ududec, C. Optimal growth trajectories with finite carrying capacity. *Phys. Rev. E* **94**, 022315 (2016).
23. Tal, O. & Tran, T. D. Adaptive bet-hedging revisited: Considerations of risk and time horizon. *Bull. Math. Biol.* **14**, 342–351 (2020).
24. Braunsteins, P., Hautphenne, S. & Minuesa, C. Consistent least squares estimation in population-size-dependent branching processes. *Journal of the American Statistical Association* 1–13 (2025).
25. Bacaër, N. *A Short History of Mathematical Population Dynamics* (Springer, 2011).
26. May, R. M. Simple mathematical models with very complicated dynamics. *Nature* **261**, 459–467. https://doi.org/10.1038/261459a0 (1976).
27. Tsoularis, A. & Wallace, J. Analysis of logistic growth models. *Math. Biosci.* **179**, 21–55 (2002).
28. Breiman, L. Optimal gambling systems for favorable games. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics*, vol. 4, 65–79 (University of California Press, 1961).
29. Orabona, F. & Pál, D. Coin betting and parameter-free online learning. *Advances in Neural Information Processing Systems* **29** (2016).
30. Despons, A., Peliti, L. & Lacoste, D. Adaptive strategies in kelly's horse races model. *J. Stat. Mech. Theory Exp.* **2022**, 093405 (2022).
31. Mote, S., Rivas, J. & Kalnay, E. A novel approach to carrying capacity: From a priori prescription to a posteriori derivation based on underlying mechanisms and dynamics. *Annu. Rev. Earth Planet. Sci.* **48**, 657–683 (2020).
32. McLeod, S. R. Is the concept of carrying capacity useful in variable environments?. *Oikos* https://doi.org/10.2307/3546897 (1997).
33. Conway, J. The game of life. *Scientific American* **223**, 4 (1970).
34. Agapie, A., Andreica, A. & Giuclea, M. Probabilistic cellular automata. *J. Comput. Biol.* **21**, 699–708 (2014).
35. Sun, J., Li, X., Chen, N., Wang, Y. & Song, G. Regular pattern formation regulates population dynamics: Logistic growth in cellular automata. *Ecol. Modell.* **418**, 108878 (2020).
36. Bagnoli, F. Emulating the logistic map with totalistic cellular automata. arXiv preprint arXiv:2512.04140 (2025).
37. Ogawa, A. et al. Extracellular dnases facilitate antagonism and coexistence in bacterial competitor-sensing interference competition. *Appl. Environ. Microbiol.* **88**, e01437-22 (2022).
38. Feng, M., Pi, B., Deng, L.-J. & Kurths, J. An evolutionary game with the game transitions based on the markov process. *IEEE Trans. Syst. Man Cybern. Syst.* **54**, 609–621 (2023).
39. Cimpeanu, T., Santos, F. C., Pereira, L. M., Lenaerts, T. & Han, T. A. Artificial intelligence development races in heterogeneous settings. *Sci. Rep.* **12**, 1723 (2022).
40. Pi, B., Deng, L.-J., Feng, M., Perc, M. & Kurths, J. Dynamic evolution of complex networks: A reinforcement learning approach applying evolutionary games to community structure. *IEEE Trans. Pattern Anal. Mach. Intell.* https://doi.org/10.1109/TPAMI.2025.3579895 (2025).
41. Kumar, A., Capraro, V. & Perc, M. The evolution of trust and trustworthiness. *J. R. Soc. Interface* https://doi.org/10.1098/rsif.2020.0491 (2020).
42. Gottlieb, R. & Gruntman, M. Belowground plant competition: Uncoupling root response strategies of peas. *Proc. R. Soc. B Biol. Sci.* https://doi.org/10.1098/rspb.2024.0673 (2024).
43. Kari, J. Theory of cellular automata: A survey. *Theor. Comput. Sci.* **334**, 3–33 (2005).
44. Rattray, J. B. et al. Bacterial quorum sensing allows graded and bimodal cellular responses to variations in population density. *MBio* **13**, e00745-22 (2022).
45. Gulec, F. & Eckford, A. W. Stochastic modeling of biofilm formation with bacterial quorum sensing. In *ICC 2023-IEEE International Conference on Communications*, 4470–4475 (IEEE, 2023).
46. Moreno-Gámez, S., Hochberg, M. E. & Van Doorn, G. Quorum sensing as a mechanism to harness the wisdom of the crowds. *Nat. Commun.* **14**, 3415 (2023).
47. Gautrat, P. et al. Lights, location, action: Shade avoidance signalling over spatial scales. *J. Exp. Bot.* **76**, 695–711 (2025).
48. Kong, C.-H. et al. Plant neighbor detection and allelochemical response are driven by root-secreted signaling chemicals. *Nat. Commun.* **9**, 3867 (2018).
49. Lennon, J. T. & Jones, S. E. Microbial seed banks: The ecological and evolutionary implications of dormancy. *Nat. Rev. Microbiol.* **9**, 119–130 (2011).
50. Blath, J. & Tóbiás, A. Invasion and fixation of microbial dormancy traits under competitive pressure. *Stoch. Process. Their Appl.* **130**, 7363–7395 (2020).
51. Virtanen, P. et al. Scipy 1.0: fundamental algorithms for scientific computing in python. *Nat. methods* **17**, 261–272 (2020).
52. Tang, S. et al. Bet hedging in a unicellular microalga. *Nat. Commun.* **15**, 2063 (2024).
53. Hakansson, N. H. & Ziemba, W. T. Capital growth theory. *Handb. Oper. Res. Manag. Sci.* **9**, 65–86 (1995).
54. MacLean, L. C., Thorp, E. O. & Ziemba, W. T. *The Kelly capital growth investment criterion: Theory and practice* Vol. 3 (world scientific, 2011).

Author contributions

F.G., N.W., and A.W.E. conceptualised the study. F.G., T.I., and A.W.E. developed the methodology. F.G. performed the formal analysis and led the investigation. T.I. and F.G. developed the simulation software, with T.I. leading the implementation. F.G., N.W., A.W.E., and T.I. wrote the original manuscript. All authors reviewed and approved the final version.

Funding

Fatih Gulec was supported by EU Horizon Europe under the Marie Skłodowska-Curie COFUND grant No 101081327 YUFE4Postdocs. Andrew Eckford and Takhmina Iliiasova were supported by Discovery grant RGPIN-2023-05064 from the Natural Sciences and Engineering Research Council (NSERC).

Declarations

Competing interests

The authors declare no competing interests.

Additional information

Correspondence and requests for materials should be addressed to A.W.E.