MIMO-SCMA Codebook Design: Joint Optimization of Sum-Rate and Error Performance

Minerva Priyadarsini*, Kuntal Deka†, Zilong Liu‡, and Sanjeev Sharma§

*School of Electrical Sciences, Indian Institute of Technology Goa, India

†Department of Electronics & Electrical Engineering, Indian Institute of Technology Guwahati, India

‡School of Computer Science and Electrical Engineering, University of Essex, Colchester CO4 3SQ, U.K.

§Department of Electronics Engineering, Indian Institute of Technology (BHU) Varanasi, India

emails: minerva183212005@iitgoa.ac.in, kuntaldeka@iitg.ac.in, zilong.liu@essex.ac.uk, sanjeev.ece@iitbhu.ac.in

***Abstract*—Codebook design for overloaded multiple-input multiple-output (MIMO) sparse code multiple access (SCMA) systems is critical, as reliability and spectral efficiency are inherently coupled. Existing approaches often optimize either error performance or spectral efficiency in isolation. In this paper, we propose a joint framework that balances both objectives using a weighted-sum hybrid objective function. To ensure computational efficiency, we adopt a reduced-dimensional parameterization to optimize the generator points of the joint MIMO-SCMA constellations. We show that codebooks designed for single-antenna systems are suboptimal in MIMO settings, highlighting the need for MIMO-aware design. Simulation results demonstrate that the proposed codebooks improve error performance while maintaining competitive sum rate.**

***Index Terms*—MIMO-SCMA, Sum-rate, SER.**

I. Introduction

Non-orthogonal multiple access (NOMA), particularly sparse code multiple access (SCMA) [1], improves spectral efficiency by allowing multiple users to share the same resource elements through sparse multidimensional codebooks and message-passing algorithm (MPA)-based detection [2]. Extending SCMA to multiple-input multiple-output (MIMO) systems provides additional spatial degrees of freedom and can further enhance overloaded connectivity. However, in MIMO-SCMA, spatial multiplexing, sparse code-domain superposition, and multi-user interference are jointly coupled, making the overall performance highly dependent on the codebook structure. Hence, codebooks designed for single-antenna SCMA may not directly provide a balanced reliability–throughput tradeoff in MIMO-SCMA systems.

Existing SCMA codebook designs can be broadly viewed as reliability-oriented or capacity-oriented. Reliability-oriented methods improve error performance by maximizing distance-related metrics; for example, Deka *et al.* [3] used differential evolution (DE) to improve error performance and minimum Euclidean distance (MED). Other works exploit cross-entropy-based optimization [4] or combine minimum product distance (MPD) with constellation rotations [5] to enhance spatial diversity. In addition, graph-based detectors [6, 7] and partially active MPA receivers [8] have been developed to reduce the detection complexity of MIMO-SCMA systems. In contrast, capacity-oriented designs [9] focus on improving spectral efficiency through inter-user rotations and rate-oriented constellation shaping.

Although these approaches improve either reliability or achievable rate, the two objectives are inherently coupled in overloaded MIMO-SCMA systems. A distance-oriented design may improve codeword distinguishability at the MPA receiver but may not preserve the finite-alphabet achievable sum rate under spatial signal mixing and multi-user interference. Conversely, a rate-oriented design may improve throughput but can reduce the codeword separation required for reliable detection. This motivates a hybrid MIMO-SCMA codebook design that jointly accounts for error-rate performance and finite-alphabet sum-rate behavior.

In this work, we propose a weighted hybrid codebook design framework for MIMO-SCMA. Unlike Gaussian-approximation-based approaches, the proposed method explicitly accounts for the finite-alphabet structure of the desired and interfering SCMA codewords together with the sparse code-domain mapping. The proposed framework introduces a tunable objective that jointly captures SER and finite-alphabet sum-rate performance, develops a finite-alphabet sum-rate metric for overloaded MIMO-SCMA, and employs a structured reduced-dimensional parameterization to reduce the search space of the resulting high-dimensional non-convex optimization problem. Simulation results for the overloaded $(K, J) = (4, 6)$ MIMO-SCMA system show that the proposed design improves error performance compared with benchmark codebooks while maintaining a competitive finite-alphabet sum rate.

II. MIMO-SCMA System Model

Consider a MIMO-SCMA transmission model with N_t transmit antennas and N_r receive antennas. Each transmit antenna supports J SCMA layers over K orthogonal resource elements. Therefore, the total number of transmitted SCMA layers, also referred to as virtual users, is $J_{\text{tot}} = N_t J$. A virtual user is indexed by the pair (n, j), where $n \in \{1, \ldots, N_t\}$ denotes the transmit antenna and $j \in \{1, \ldots, J\}$ denotes the SCMA layer associated with that antenna. Let $\mathbf{C}_{n,j}$ denote the SCMA codebook of virtual user (n, j) with cardinality M. Each codeword $\mathbf{c}_{n,j} \in \mathbf{C}_{n,j}$ is a sparse vector $\mathbf{c}_{n,j} \in \mathbb{C}^{K\times 1}$ defined over the K resource elements. The nonzero positions of $\mathbf{c}_{n,j}$ are determined by the underlying SCMA factor-graph structure.

The transmit signal from the n-th antenna is obtained by superimposing the codewords of its J SCMA layers as

$$\mathbf{x}_n = \sum_{j=1}^{J} \sqrt{p_{n,j}}\, \mathbf{c}_{n,j}, \qquad n = 1, \ldots, N_t, \tag{1}$$

where $p_{n,j}$ denotes the transmit power allocated to virtual user (n, j). Stacking the transmit signals from all transmit antennas gives the aggregate transmit vector

$$\mathbf{x} = \left[\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_{N_t}^T\right]^T \in \mathbb{C}^{N_t K \times 1}. \quad (2)$$

In this work, we consider an equivalent joint-detection model for MIMO-SCMA, where the received observations across all receive antennas and resource elements are jointly processed. This model is used to evaluate the codebook structure and the achievable performance of MIMO-SCMA under joint MPA-based detection. Hence, the virtual-user index should not be interpreted as a separate physical downlink receiver; rather, it denotes one transmitted SCMA layer in the equivalent MIMO-SCMA factor graph.

1) Extended Factor Graph Representation: The sparse spreading structure of SCMA is described by a binary factor graph that maps SCMA layers to resource elements. For the n-th transmit antenna, the factor graph is represented by $\mathbf{F}_n \in \{0,1\}^{K \times J}$, where each column corresponds to an SCMA layer and each row corresponds to a resource element. An entry $[\mathbf{F}_n]_{k,j} = 1$ indicates that the j-th layer of the n-th transmit antenna occupies the k-th resource element, whereas $[\mathbf{F}_n]_{k,j} = 0$ indicates no connection.

For simplicity, identical factor graphs are assumed across all transmit antennas, i.e., $\mathbf{F}_1 = \mathbf{F}_2 = \cdots = \mathbf{F}_{N_t} = \mathbf{F}$. Let d_f and d_v denote the factor-node degree and variable-node degree, respectively. Due to the MIMO channel, each receive antenna observes the superposition of the signals transmitted from all transmit antennas. Therefore, joint detection across the $N_t J$ virtual users is performed by constructing an extended factor graph. The extended factor graph can be represented by the binary indicator matrix

$$\mathbf{F}_{\text{joint}} = \mathbf{1}_{N_r \times N_t} \otimes \mathbf{F} \in \{0,1\}^{N_r K \times N_t J}, \quad (3)$$

where $\mathbf{1}_{N_r \times N_t}$ denotes the all-one matrix and $\otimes$ represents the Kronecker product. In this construction, each block row corresponds to one receive antenna, while each block column corresponds to one transmit antenna. Hence, the joint factor graph consists of $N_r K$ factor nodes and $N_t J$ variable nodes.

Fig. 1 illustrates the extended factor graph of a 2×2 MIMO-SCMA system with $J = 6$ SCMA layers per transmit antenna and $K = 4$ resource elements. The corresponding underlying SCMA factor matrix is given by

$$\mathbf{F} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}. \quad (4)$$

For this example, the equivalent joint factor graph contains $N_t J = 12$ virtual users and $N_r K = 8$ received resource observations.

2) Received Signal Model: For notational convenience, we index the transmitted virtual users by $u \in \{1, \dots, N_t J\}$, where each virtual user corresponds to a unique pair (n, j). Here, n denotes the transmit antenna and j denotes the SCMA layer associated with that antenna. In this paper, the term virtual user refers to an SCMA layer generated by a transmit-antenna–user pair (n, j); it does not imply a separate receiver observation for each virtual user.

In the considered MIMO-SCMA model, the received observations across all N_r receive antennas and K resource elements are stacked for joint MPA-based detection. Let $\mathbf{H}_k \in \mathbb{C}^{N_r \times N_t}$ denote the MIMO channel matrix on the k-th resource element, where the (r, n)-th entry denotes the channel coefficient from transmit antenna n to receive antenna r on resource element k. The channel matrix across all resource elements is written as

$$\mathbf{H} = \text{blkdiag}\left(\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_K\right) \in \mathbb{C}^{N_r K \times N_t K}. \quad (5)$$

Equivalently, $\mathbf{H}$ can be expressed as

$$\mathbf{H} = \begin{bmatrix} \text{diag}(\mathbf{h}_{1,1}) & \cdots & \text{diag}(\mathbf{h}_{1,N_t}) \\ \vdots & \ddots & \vdots \\ \text{diag}(\mathbf{h}_{N_r,1}) & \cdots & \text{diag}(\mathbf{h}_{N_r,N_t}) \end{bmatrix}, \quad (6)$$

where $\mathbf{h}_{r,n} \in \mathbb{C}^{K \times 1}$ denotes the channel vector from transmit antenna n to receive antenna r across the K resource elements.

Using the aggregate transmit vector $\mathbf{x}$ defined in (2), the stacked received signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (7)$$

where $\mathbf{y} \in \mathbb{C}^{N_r K \times 1}$ is the stacked received observation vector and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_{N_r K})$ is the additive white Gaussian noise vector. For Rayleigh fading simulations, the channel coefficients are modeled as $h_{r,n,k} \sim \mathcal{CN}(0, 1)$, and are independently generated over receive antennas, transmit antennas, resource elements, and channel realizations. The proposed codebook design is performed under statistical channel conditions and does not require instantaneous CSI at the transmitter.

A. Finite-Alphabet Sum-Rate for MIMO-SCMA

The maximum finite-alphabet achievable sum rate is obtained when all MIMO-SCMA virtual users are jointly decoded from the aggregate received vector. This quantity is characterized by the mutual information between the aggregate transmit vector $\mathbf{x}$ and the stacked received vector $\mathbf{y}$. Since each virtual user employs a codebook $\mathbf{C}_u$ of cardinality M, the total number of possible aggregate transmit vectors is $M_{\text{sys}} = M^{N_t J}$.

Let $\{\mathbf{x}_m\}_{m=1}^{M_{\text{sys}}}$ denote the set of all possible realizations of the aggregate transmit vector $\mathbf{x} \in \mathbb{C}^{N_t K \times 1}$. Assuming equiprobable signaling, the exact finite-alphabet mutual information is given by [10, 11]

$$\begin{aligned} R^{\text{exact}} = \log_2 M_{\text{sys}} - \frac{1}{M_{\text{sys}}} \sum_{m=1}^{M_{\text{sys}}} \mathbb{E}_{\mathbf{n}} \Bigg[\log_2 \Bigg(\sum_{\ell=1}^{M_{\text{sys}}} \\ \exp\left(-\frac{\|\mathbf{H}(\mathbf{x}_m - \mathbf{x}_\ell) + \mathbf{n}\|^2 - \|\mathbf{n}\|^2}{N_0} \right) \Bigg) \Bigg]. \end{aligned} \quad (8)$$

Direct evaluation of (8) is computationally prohibitive due to the exponential growth of M_{sys} and the expectation over the noise distribution. Therefore, a tractable finite-alphabet rate metric is required for codebook optimization.

To obtain a lower-complexity expression, we evaluate the contribution of a given virtual user while treating the remaining virtual users as finite-alphabet discrete interference. Let $u \in \{1, \dots, N_t J\}$ denote the desired virtual user. The aggregate received signal can be decomposed as

$$\mathbf{y} = \mathbf{H}\mathbf{x}_u + \boldsymbol{\zeta}_{\text{int},u} + \mathbf{n}, \quad (9)$$

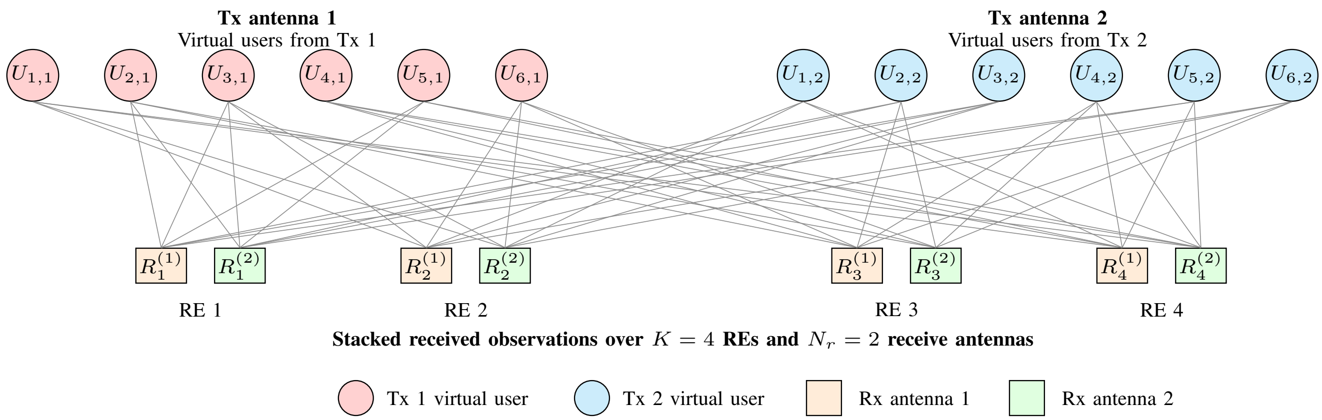


Fig. 1: Equivalent joint bipartite factor graph for the considered 2×2 MIMO-SCMA joint detection model. The 12 variable nodes denote the transmitted virtual users generated by the two transmit antennas, where $U_{j,n}$ represents the j-th SCMA user associated with transmit antenna n. The 8 factor nodes denote the stacked received observations over $K = 4$ resource elements and $N_r = 2$ receive antennas, where $R_k^{(r)}$ represents the observation on the k-th resource element at the r-th receive antenna. The underlying SCMA factor matrix is shown in (4).

where $\mathbf{x}_u \in \mathbb{C}^{N_t K \times 1}$ denotes the contribution of virtual user u to the aggregate transmit vector, and

$$\boldsymbol{\zeta}_{\text{int},u} = \sum_{u' \neq u} \mathbf{H}\mathbf{x}_{u'} \in \mathbb{C}^{N_r K \times 1} \tag{10}$$

denotes one aggregate interference from the remaining users.

Let $\mathcal{Z}_{\text{int},u}$ denote the set of all possible realizations of $\boldsymbol{\zeta}_{\text{int},u}$. Under independent equiprobable codeword selection, the interference-generating codeword combinations are equiprobable. Hence, for $\boldsymbol{\zeta} \in \mathcal{Z}_{\text{int},u}$, the corresponding PMF is written as $P(\boldsymbol{\zeta}) = \frac{1}{|\mathcal{Z}_{\text{int},u}|}$. For two candidate codewords $\mathbf{c}_a, \mathbf{c}_b \in \mathbf{C}_u$, let $\mathbf{x}_a^{(u)}$ and $\mathbf{x}_b^{(u)}$ denote the corresponding aggregate transmit vectors obtained by placing $\mathbf{c}_a$ and $\mathbf{c}_b$ at the position of virtual user u, respectively. The desired-user difference vector is defined as $\Delta\mathbf{x}_{ab}^{(u)} = \mathbf{x}_a^{(u)} - \mathbf{x}_b^{(u)}$. Since the interfering virtual users also employ finite-alphabet codebooks, two candidate aggregate transmit vectors may correspond to different interference realizations. Therefore, the interference contribution in the pairwise distance is represented by the difference between two finite-alphabet interference states. Specifically, for $\boldsymbol{\zeta}, \boldsymbol{\zeta}' \in \mathcal{Z}_{\text{int},u}$, we define $\Delta\boldsymbol{\zeta} = \boldsymbol{\zeta} - \boldsymbol{\zeta}'$. Thus, $\Delta\boldsymbol{\zeta}$ represents the variation in the finite-alphabet interference between two candidate aggregate transmit vectors. When the same interference realization is assumed for both candidates, this term becomes zero; otherwise, it captures the effect of different interfering codeword combinations on the pairwise distance.

Applying Jensen's inequality yields the following finite-alphabet rate metric for virtual user u:

$$\begin{aligned} R_u = {} & \log_2 M - \kappa \\ & - \frac{1}{M} \sum_{\mathbf{c}_a \in \mathbf{C}_u} \log_2 \Bigg(\sum_{\mathbf{c}_b \in \mathbf{C}_u} \sum_{\boldsymbol{\zeta}, \boldsymbol{\zeta}' \in \mathcal{Z}_{\text{int},u}} P(\boldsymbol{\zeta}) P(\boldsymbol{\zeta}') \\ & \times \exp\left[- \frac{\left\| \mathbf{H}\Delta\mathbf{x}_{ab}^{(u)} + (\boldsymbol{\zeta} - \boldsymbol{\zeta}') \right\|^2}{2N_0} \right] \Bigg). \end{aligned} \tag{11}$$

where $\kappa = N_r K \left(\frac{1}{\ln 2} - 1 \right)$.

Finally, the finite-alphabet sum-rate metric used for codebook optimization is obtained by summing the contributions of all virtual users:

$$R_{\text{sum}} = \sum_{u=1}^{N_t J} R_u. \tag{12}$$

It is noted that (11) is used as a Jensen-based finite-alphabet rate metric for codebook optimization. It preserves the discrete-alphabet nature of both the desired virtual user and the interfering virtual users, while avoiding the direct evaluation of the exact mutual information in (8).

III. Formulation of the Codebook Design as an Optimization Problem

In overloaded MIMO-SCMA systems, error rate and sum rate are inherently coupled due to multi-user interference across both spatial and code domains. Building on the system model in Section II, the superposition of sparse codewords across all virtual users forms a multidimensional constellation governed by the codebooks $\{C_{n,j}\}$ and the factor-graph matrix $\mathbf{F}_{\text{joint}}$. The system performance is dictated by the geometry of this composite constellation: the error rate depends on the pairwise Euclidean distances between superimposed codewords, while the achievable sum rate depends on the efficiency of signal space utilization, as characterized by the mutual information. This leads to a fundamental trade-off between reliability and spectral efficiency.

To balance this trade-off, the MIMO-SCMA codebook design is formulated as the following optimization problem:

$$\begin{aligned} \{C_{n,j}\}_{\text{opt}} = \arg \min_{\{C_{n,j}\}} \quad & f(\{C_{n,j}\}) \\ \text{s.t.} \quad & \|\mathbf{c}_{n,j}\|_2 = 1, \quad \forall\, \mathbf{c}_{n,j} \in C_{n,j},\ \forall n, j, \end{aligned} \tag{13}$$

where the constraint enforces unit-energy codewords to ensure fair transmit power normalization. The objective function $f(\cdot)$ is defined as a weighted combination of SER and achievable sum-rate:

$$f(\{C_{n,j}\}) = \gamma \log_{10}(f_{\mathrm{SER}}(\{C_{n,j}\})) - (1-\gamma)\,\frac{f_{\mathrm{SR}}(\{C_{n,j}\})}{R_{\mathrm{ref}}}, \tag{14}$$

where $\gamma \in [0,1]$ controls the trade-off between reliability and spectral efficiency, $f_{\mathrm{SER}}(\cdot)$ denotes the SER evaluated via Monte Carlo simulations with a MIMO-MPA detector, and $f_{\mathrm{SR}}(\cdot)$ denotes the achievable sum-rate R_{sum} (12) derived in Section II-A.

The logarithmic SER term is used because the SER varies over several orders of magnitude. The sum-rate term is normalized by R_{ref} to keep both terms numerically comparable during optimization. For the considered $(K,J) = (4,6)$ MIMO-SCMA configuration with $N_t = 2$ and $M = 4$, the reference rate is chosen as $R_{\mathrm{ref}} = N_t J \log_2 M = 24$ bits/cu. The sensitivity of the proposed design to γ is further examined in the simulation results.

A. Framework for Codebook Parameterization

The MIMO-SCMA codebooks consist of sparse complex-valued codewords whose nonzero entries are generated from a small set of complex parameters. Instead of directly optimizing all sparse codewords, we optimize a reduced-dimensional real-valued parameter vector and then map it to the structured MIMO-SCMA codebooks.

1) Parameter Vector and Generator Points: Let Q denote the number of complex generator points per transmit antenna. For the n-th antenna, the generator vector is defined as $\mathbf{a}_n = [a_{1,n}, a_{2,n}, \dots, a_{Q,n}]^T \in \mathbb{C}^Q$. The corresponding real-valued optimization vector is

$$\mathbf{p} = [\mathbf{p}_1^T, \mathbf{p}_2^T, \dots, \mathbf{p}_{N_t}^T]^T \in \mathbb{R}^{2QN_t}, \tag{15}$$

where $\mathbf{p}_n = [\Re(a_{1,n}), \Im(a_{1,n}), \dots, \Re(a_{Q,n}), \Im(a_{Q,n})]^T$. Thus, the total search dimension is $D = 2QN_t$.

2) Construction of 1D Constellation Blocks: For each antenna n, $\mathbf{p}_n$ is optimized. For simplified description, we remove the antenna index n and use a_q for $a_{q,n}$. To obtain a zero-mean constellation with a large MED a $1 \times M$ constellation block $\mathbf{c}_q^o$ is constructed, where M is typically even, as follows:

$$\mathbf{c}_q^o = [a_q, a_{q+1}, \dots, a_{q+\frac{M}{2}-1}, -a_{q+\frac{M}{2}-1}, \dots, -a_q]. \tag{16}$$

This formulation ensures that the constellation points are symmetric around the origin, which simplifies the optimization process.

3) Structural Mapping and Parameterization: For the considered $(K,J) = (4,6)$ SCMA configuration, $Q = 6$ generator points per transmit antenna are used to form three blocks $\{\mathbf{c}_1^o, \mathbf{c}_2^o, \mathbf{c}_3^o\}$. These blocks are assigned to the active resource positions according to

$$\mathbf{S}_{4\times 6} = \begin{bmatrix} \mathbf{c}_1^o & 0 & \mathbf{c}_2^o & 0 & \mathbf{c}_3^o & 0 \\ 0 & \mathbf{c}_2^o & \mathbf{c}_3^o & 0 & 0 & \mathbf{c}_1^o \\ \mathbf{c}_2^o & 0 & 0 & \mathbf{c}_1^o & 0 & \mathbf{c}_3^o \\ 0 & \mathbf{c}_1^o & 0 & \mathbf{c}_3^o & \mathbf{c}_2^o & 0 \end{bmatrix}. \tag{17}$$

For $N_t = 2$, the number of real-valued optimization variables becomes $D = 2QN_t = 24$.

4) Codeword Normalization and Optimization: Each generated sparse codeword is normalized to satisfy a unit-energy constraint,

$$\mathbf{x}_{n,j,m}^{\mathrm{norm}} = \frac{\mathbf{x}_{n,j,m}}{\sqrt{\sum_{k=1}^{K} |x_{n,j,m,k}|^2}}, \tag{18}$$

where $\mathbf{x}_{n,j,m}$ denotes the m-th codeword of virtual user (n,j). This prevents the optimizer from improving the objective through unequal power scaling.

The parameter vector $\mathbf{p}$ is optimized using differential evolution. At generation G, a trial vector $\mathbf{u}_i^{(G)}$ is generated from the current population through mutation and crossover following [3]. Since the objective includes Monte Carlo-based SER evaluation, both the incumbent $\mathbf{p}_i^{(G)}$ and the trial candidate $\mathbf{u}_i^{(G)}$ are evaluated using the same channel and noise realization set, denoted by Ξ_G. The selection rule is

$$\mathbf{p}_i^{(G+1)} = \begin{cases} \mathbf{u}_i^{(G)}, & f(\mathbf{u}_i^{(G)}|\Xi_G) < f(\mathbf{p}_i^{(G)}|\Xi_G), \\ \mathbf{p}_i^{(G)}, & \text{otherwise}, \end{cases} \tag{19}$$

where $f(\cdot)$ is the hybrid objective function defined in (14). Using the same realization set Ξ_G for both candidates ensures a fair stochastic comparison at each generation. The process continues until the maximum number of generations is reached, and the best vector $\mathbf{p}_{\mathrm{opt}}$ is used to construct the final MIMO-SCMA codebooks.

B. Computational Complexity

The receiver complexity is mainly determined by the MPA updates on the extended factor graph $\mathbf{F}_{\mathrm{joint}}$ in (3). For an $N_t \times N_r$ MIMO-SCMA system, the graph has $N_t J$ variable nodes and $N_r K$ factor nodes. Since each factor node combines the contributions from $N_t d_f$ connected virtual users, the per-iteration complexity of MPA detection is

$$\mathcal{C}_{\mathrm{MPA}} = \mathcal{O}\big(T\left[N_r K (N_t d_f) M^{N_t d_f} + N_t J d_v M\right]\big), \tag{20}$$

where T is the number of MPA iterations. Thus, the exponential term is governed by the local joint factor-node degree $N_t d_f$.

In contrast, exhaustive ML detection jointly searches over all $N_t J$ virtual-user symbol combinations, resulting in

$$\mathcal{C}_{\mathrm{ML}} = \mathcal{O}\big(N_r K M^{N_t J}\big). \tag{21}$$

Hence, the MPA detector avoids the full ML search space by exploiting the sparse factor-graph structure, leading to a substantially lower detection complexity for the considered overloaded MIMO-SCMA setting.

IV. Simulation Results

The considered SCMA configuration has $J = 6$ users sharing $K = 4$ resources per transmit antenna, corresponding to a 150% overloading factor. TABLE I summarizes the optimization and simulation parameters. Unless mentioned otherwise, the weighting factor is set to $\gamma = 0.6$, which provides a balanced trade-off between detection reliability, measured by SER, and spectral efficiency, measured by the finite-alphabet sum-rate. For a fair comparison, all benchmark codebooks are evaluated under the same MIMO-SCMA

TABLE I: MIMO-SCMA optimization and simulation parameters.

Category	Parameter	Value
System	SCMA configuration (K, J)	$(4, 6)$
	Antenna size $(N_t \times N_r)$	2×2, 4×4, 8×8
	Virtual users $(J_{\text{tot}} = N_t J)$	12
	Constellation size (M)	4
	Overloading factor $(\lambda = J/K)$	150%
	Reference rate (R_{ref})	24 bits/cu
	Receiver	Joint MPA detector
	MPA iterations	10
Optimization	Search dimension (D)	24
	Population size (N_p)	30
	Max. generations $(G_{\max})$	30
	Mutation factor	0.6
	Crossover rate	0.95
	Optimization SNR	20 dB
	Weighting factor (γ)	0.6
Evaluation	Power allocation	Equal across virtual users
	Normalization	Unit-norm codewords
	Benchmark setting	Same channel, power, detector

transmission model, Rayleigh fading channel, transmit-power normalization, and MPA detector. Since several benchmark codebooks were originally designed for single-antenna SCMA, they are embedded into the considered MIMO-SCMA structure using the same factor graph and normalized to the same average transmit-power constraint. Consequently, the comparison reflects the effect of codebook geometry rather than differences in channel model, power scaling, or receiver implementation. The designed MIMO-SCMA codebooks are publicly available at **https://github.com/kuntaliitg/MIMO-SCMA-Codebook-Design**.

As shown in Fig. 2, the proposed codebook provides a clear reliability improvement over the benchmark designs. At a target SER of 10^{-4}, it achieves an approximate coding gain of 2.2 dB compared with the next best benchmark [5]. While some existing designs [3, 4, 9] perform competitively at low SNR, their SER curves decay more slowly at high SNR. This indicates that the proposed joint reliability-sum-rate design yields a more robust composite constellation under MIMO fading.

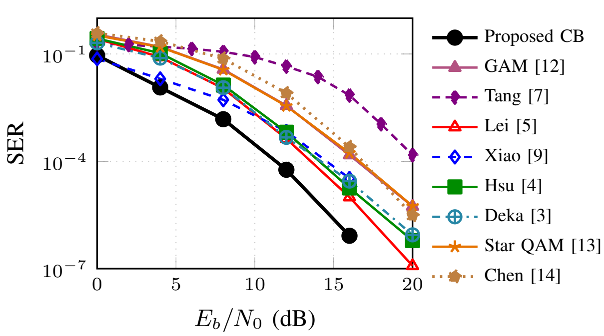


Fig. 2: SER performance for $J = 6,\ K = 4$ for 2×2 MIMO-SCMA system with different codebooks.

Fig. 3 compares the sum-rate performance of the proposed and benchmark codebooks. The proposed design achieves a sum-rate close to the best throughput-oriented benchmarks [4, 14], while also providing the SER gain observed in Fig. 2. It is worth noting that the capacity-oriented codebook in [9], originally designed for SISO-SCMA, does not retain the same rate advantage after being embedded into the MIMO-SCMA setting. Similarly, reliability-oriented MIMO designs such as [5] may improve error performance but show a lower sum-rate.

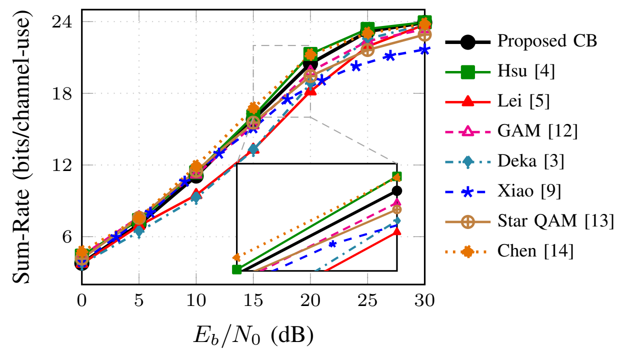


Fig. 3: Comparison of sum-rate for $J = 6,\ K = 4$ for 2×2 MIMO-SCMA system with different codebooks.

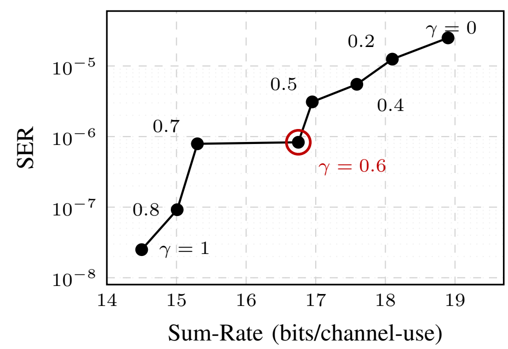


Fig. 4: SER-sum-rate Pareto tradeoff of the proposed for $(K, J) = (4, 6)$ 2×2 MIMO-SCMA codebook design for different values of the weighting factor γ.

Fig. 4 shows the SER–sum-rate trade-off of the proposed 2×2 MIMO-SCMA codebook for different values of the weighting factor γ. Since γ controls the relative emphasis on reliability, decreasing γ shifts the design toward higher spectral efficiency, but at the cost of increased SER. In particular, the achievable sum-rate increases from about 14.5 to 18.9 bps/Hz, while the SER increases from 2.5×10^{-8} to 2.5×10^{-5}. The Pareto curve shows a clear knee around $\gamma = 0.6$. Beyond this point, further improvement in SER requires a noticeable loss in sum-rate, whereas smaller values of γ provide only moderate rate improvement with a rapid degradation in reliability. Therefore, $\gamma = 0.6$ is used as the default operating point, as it provides a balanced compromise between error-rate performance and finite-alphabet sum-rate.

To compare the geometric properties of different codebooks, we evaluate the MED and MPD of the composite superposition constellation. For fairness, all codebooks are embedded into the same $(K, J) = (4, 6)$ MIMO-SCMA structure and normalized to satisfy a common average transmit-power constraint, $\mathbb{E}[\|\mathbf{x}\|_2^2] = 1$, where $\mathbf{x}$ is defined in (2). The MED is computed as

$$d_{\min} = \min_{\mathbf{x}_i \neq \mathbf{x}_j} \|\mathbf{x}_i - \mathbf{x}_j\|_2 , \tag{22}$$

where $\mathbf{x}_i$ and $\mathbf{x}_j$ denote two distinct composite transmit vectors. As shown in TABLE II, the proposed design achieves an MED of 0.76, which is close to the highest value of 0.77 obtained

TABLE II: MED and MPD comparison for different SCMA codebook designs in the $(K, J) = (4, 6)$ scenario.

Metric	**Hsu** [4]	**GAM** [12]	**Star QAM** [13]	**Lei** [5]	**Deka** [3]	**Chen** [14]	**Xiao** [9]	**Proposed**
MED	0.40	0.65	0.54	0.56	0.57	0.58	**0.77**	0.76
MPD	0.38	0.32	0.14	0.39	0.28	0.33	0.21	**0.42**

by Xiao [9]. More importantly, it achieves the highest MPD of 0.42, indicating improved multidimensional separation of the composite constellation. Although Xiao [9] gives a slightly higher MED, its lower MPD suggests that MED alone is insufficient to characterize MIMO-SCMA performance. This supports the need for a hybrid design criterion that jointly considers reliability and finite-alphabet sum-rate behavior.

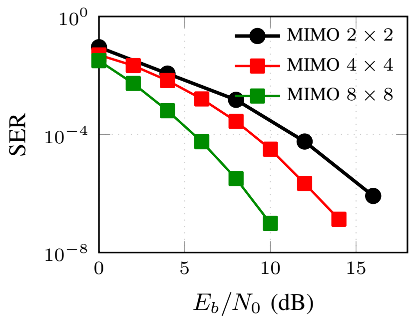
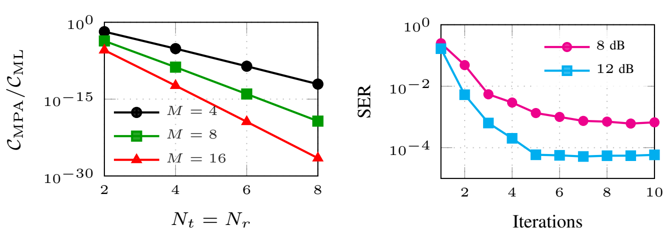


Fig. 5: SER performance of the proposed MIMO-SCMA system for different MIMO antenna configurations with $(K, J) = (4, 6)$.

Fig. 5 shows the SER performance of the proposed MIMO-SCMA system for different antenna configurations. As the MIMO dimension increases from 2×2 to 4×4 and 8×8, a clear improvement in SER performance is observed due to the enhanced spatial diversity. In particular, the 8×8 configuration achieves the steepest SER decay, demonstrating that the proposed codebook design remains effective when the number of transmit and receive antennas is increased. These results further address the scalability of the proposed design beyond the basic 2×2 setup.



(a) Complexity ratio

(b) Detector convergence

Fig. 6: Complexity and convergence behavior of the proposed MIMO-SCMA framework for the overloaded $(K, J) = (4, 6)$ system.

Fig. 6 jointly illustrates the complexity and convergence behavior of the proposed detector. As shown in Fig. 6(a), the complexity ratio $\mathcal{C}_{\mathrm{MPA}}/\mathcal{C}_{\mathrm{ML}}$ decreases rapidly with the number of antennas and constellation size, confirming that the sparse factor-graph structure substantially reduces the detection burden compared with exhaustive ML search. Fig. 6(b) shows that the SER decreases sharply with the number of MPA iterations and reaches a steady value within approximately 5–6 iterations for both SNR values. This indicates that the proposed codebook structure enables efficient message propagation over the joint factor graph with a small number of iterations.

V. Conclusion

This paper presents a weighted-sum hybrid objective framework for MIMO-SCMA codebook design that jointly optimizes reliability and throughput. Our results demonstrate that the proposed design achieves significant SER gains under iterative MPA detection while maintaining competitive throughput, outperforming traditional single-metric benchmarks in 150% overloaded scenario. The analysis confirms that effective MIMO-SCMA design requires a balance of joint constellation structure and spatial diversity, as distance metrics alone fail to capture the complexities of detector convergence in fading channels.

Acknowledgment

The work of Z. Liu was supported in part by the UK Engineering and Physical Sciences Research Council (EPSRC) under Grants EP/Y037243/1 (“TITAN/REVOL6G” and “TITAN/LEGEND6G”), EP/X040569/1 (“HASC/RETHIN6G”), EP/X035352/1 (“DRIVE”), and EP/Y000986/1 (“SORT”), and in part by the Royal Society under Grants IEC/NSFC/233292 and IES/R1/241212.

References

[1] H. Nikopour and H. Baligh, “Sparse Code Multiple Access,” in *2013 IEEE 24th Annu. Int. Symp. on Pers., Indoor, and Mobile Radio Commun. (PIMRC)*, 2013, pp. 332–336.

[2] S. Chaturvedi, Z. Liu, V. A. Bohara, A. Srivastava, and P. Xiao, “A Tutorial on Decoding Techniques of Sparse Code Multiple Access,” *IEEE Access*, vol. 10, pp. 58 503–58 524, 2022.

[3] K. Deka, M. Priyadarsini, S. Sharma, and B. Beferull-Lozano, “Design of SCMA Codebooks using Differential Evolution,” in *2020 IEEE Int. Conf. on Commun. Workshops (ICC Workshops)*, June 2020, pp. 1–7.

[4] Y.-M. Chen, Y.-C. Hsu, M.-C. Wu, R. Singh, and Y.-C. Chang, “On Near-Optimal Codebook and Receiver Designs for MIMO-SCMA Schemes,” *IEEE Trans. on Wireless Commun.*, vol. 21, no. 12, pp. 10 724–10 738, 2022.

[5] T. Lei, Q. Luo, S. Ni, S. Chen, L. Yu, and P. Xiao, “Novel Sparse Codebook Design for MIMO-SCMA,” *IEEE Trans. on Veh. Technol.*, vol. 74, no. 5, pp. 8407–8412, 2025.

[6] Y. Du, B. Dong, Z. Chen, P. Gao, and J. Fang, “Joint Sparse Graph-Detector Design for Downlink MIMO-SCMA Systems,” *IEEE Wireless Commun. Lett.*, vol. 6, no. 1, pp. 14–17, 2017.

[7] S. Tang, L. Hao, and Z. Ma, “Low Complexity Joint MPA Detection for Downlink MIMO-SCMA,” in *2016 IEEE Global Commun. Conf. (GLOBECOM)*, 2016, pp. 1–4.

[8] J. Dai, G. Chen, K. Niu, and J. Lin, “Partially Active Message Passing Receiver for MIMO-SCMA Systems,” *IEEE Wireless Commun. Lett.*, vol. 7, no. 2, pp. 222–225, 2018.

[9] K. Xiao, B. Xia, Z. Chen, B. Xiao, D. Chen, and S. Ma, “On Capacity-Based Codebook Design and Advanced Decoding for Sparse Code Multiple Access Systems,” *IEEE Trans. on Wireless Commun.*, vol. 17, no. 6, pp. 3834–3849, 2018.

[10] Y. Wu, M. Wang, C. Xiao, Z. Ding, and X. Gao, “Linear Precoding for MIMO Broadcast Channels With Finite-Alphabet Constraints,” *IEEE Trans. on Wirel. Commun.*, vol. 11, no. 8, pp. 2906–2920, 2012.

[11] W. Zeng, C. Xiao, and J. Lu, “A Low-Complexity Design of Linear Precoding for MIMO Channels with Finite-Alphabet Inputs,” *IEEE Wirel. Commun. Lett.*, vol. 1, no. 1, pp. 38–41, 2012.

[12] Z. Mheich, L. Wen, P. Xiao, and A. Maaref, “Design of SCMA Codebooks Based on Golden Angle Modulation,” *IEEE Trans. on Veh. Technol.*, vol. 68, no. 2, pp. 1501–1509, 2019.

[13] L. Yu, P. Fan, D. Cai, and Z. Ma, “Design and Analysis of SCMA Codebook Based on Star-QAM Signaling Constellations,” *IEEE Trans. on Vehicular Technology*, vol. 67, no. 11, pp. 10 543–10 553, 2018.

[14] Y.-M. Chen and J.-W. Chen, “On the Design of Near-Optimal Sparse Code Multiple Access Codebooks,” *IEEE Trans. on Commun.*, vol. 68, no. 5, pp. 2950–2962, 2020.