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The Institute of Electronics, Information and Communication Engineers
Kikai-Shinko-Kaikan Bldg., 5-8, Shibakoen 3chome, Minato-ku, TOKYO, 105-0011 JAPAN

PAPER

Sum-Rate Analysis of Slotted ALOHA with Buffered Inter- and Intra-Slot SIC under Mixture Gamma Fading

Daiki FUKUI^{†a)}, Yuhei TAKAHASHI^{†b)}, Guanghui SONG^{††c)}, Tomotaka KIMURA^{†d)},
Zilong LIU^{†††e)}, and Jun CHENG^{†f)},

SUMMARY This paper presents an analytical framework for Buffered-SIC (successive interference cancellation) slotted ALOHA (BSIC-SA) systems with real-time feedback under composite fading channels. The BSIC-SA employs buffered inter-slot SIC to utilize the residual signals from previous slots and intra-slot SIC under random fading for enhanced packet recovery. The received signal-to-noise ratio (SNR) is modeled by a mixture gamma (MG) distribution, which generalizes the exponential SNR model of Rayleigh fading by incorporating both exponential and power terms in its probability density function (PDF). By exploiting the series representations of the lower and upper incomplete gamma functions, closed-form expressions for the state transition probabilities in the Markov process of a two-device system are derived. The transmission probability and coding rate are jointly optimized to maximize the achievable sum rate. Our proposed analysis accommodates various fading environments without requiring separate derivations for individual channel models. Numerical analyses and simulation results confirm the validity of the proposed framework and demonstrate the achievable maximum sum rate for multi-device BSIC-SA systems.
key words: slotted ALOHA, general composite fading channel, mixture gamma distribution, sum-rate, Markov model

1. Introduction

This paper examines a K_T -device slotted ALOHA (SA) communication system, in which K_T devices communicate with an access point (AP). SA, as a random access protocol, has been widely adopted in various commercial communication systems and standards [1]. Conventional performance analyses of SA used a collision channel model, which assumes negligible noise and fading and allows successful packet decoding only in single-packet slots.

Power-domain collision models inspired by non-

orthogonal multiple access (NOMA) and successive interference cancellation (SIC) have been considered in [2, 3], where multiple packets can be decoded if they occupy sufficiently separated power levels, providing an abstracted representation of multi-packet reception. However, these models rely on simplified physical-layer assumptions and do not explicitly capture fading-induced power variations.

The throughput of SA has been significantly improved by using transmission diversity [4] and SIC across slots in recent studies, leading to the development of irregular repetition slotted ALOHA (IRSA) [5] [6]. In fading channels, intra-slot SIC further enhances decoding capacity. For the Rayleigh channel model, Clazzer *et al.* derived a closed-form expression of the probability $\Pr\{E_m^n\}$ that, among n colliding packets, the first m strongest packets are successfully decoded. The expression enables analytical evaluation of the average normalized throughput of IRSA systems [7]. Takahashi *et al.* [8] further examined the converse bound and the maximum a posteriori (MAP) threshold for throughput. More recently, throughput analyses have been extended to coded SA systems over Rayleigh block-fading channels [9]. The coded SA scheme [6] [9] generalizes IRSA by applying linear block coding instead of simple packet repetition. While the IRSA scheme improves throughput by employing inter-slot SIC using packet replicas, it typically operates without real-time feedback. In other words, the AP sends acknowledgments (ACKs) to devices only after completing an entire frame of slots, rather than after each slot.

In addition, Gupta *et al.* [10] consider a two-slot transmission diversity scheme, where each device transmits a packet and its replica over independently Rayleigh-fading slots. Closed-form throughput expressions are derived under a simplified decoding model that does not explicitly account for noise, leading to analytically tractable but approximate results.

While these works focus on diversity and SIC, they do not consider real-time feedback. Such feedback refers to the operation in which ACKs are transmitted from the AP to devices after each slot, rather than after an entire frame. Such slot-by-slot feedback can further enhance spectral and energy efficiency in SA systems by reducing unnecessary retransmissions. Moreover, after

[†]Department of Intelligent Information Engineering and Sciences, Doshisha University, Kyoto 610-0321, Japan

^{††}The State Key Laboratory of Integrated Services Networks, Xidian University, Xi'an 710071, China

^{†††}School of Computer Science and Electronic Engineering, University of Essex, Colchester, CO4 3SQ, UK

a) E-mail: daifukui@ieee.org

b) E-mail: cyjk1101@mail4.doshisha.ac.jp

c) E-mail: songguanghui@xidian.edu.cn

d) E-mail: tomkimur@mail.doshisha.ac.jp

e) E-mail: zilong.liu@essex.ac.uk

f) E-mail: jcheng@ieee.org

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intra-slot SIC, the remaining undecoded portion of the received signal, referred to as the residual signal, can be stored in a buffer for subsequent inter-slot SIC processing.

Several studies have analyzed SA systems with feedback by modeling iterative decoding as a Markov process [11] [12]. For two- and three-device SA systems with a collision channel model [11], the closed-form solution for the optimal transmission probability maximizes the SA throughput. In Rayleigh fading channels [7] [12], the exponential distribution of the received signal-to-noise ratio (SNR) enables closed-form derivations of state transition probabilities through a double integral over two independent exponentially distributed random variables (RVs). For the two-device case, the SA throughput can be obtained analytically, and an exhaustive search by simulations can determine the optimal transmission probability and the coding rate that maximize the sum rate [12]. This framework provides a useful basis for our analysis. However, extending this approach to other fading channels is challenging, primarily because closed-form expressions for the required probabilities are difficult to derive. Therefore, the existing analysis in [12] is limited to simplified Rayleigh fading assumptions and cannot be directly extended to more general fading models.

Multipath fading and shadowing are essential phenomena in radio signal propagation [13]. Multipath fading arises from the interference of signal components that are randomly delayed, reflected, scattered, and diffracted, and it is characterized by a probability density function (PDF) of the signal amplitude. Various statistical models, including Nakagami- q (Hoyt), Nakagami- n (Rician), Nakagami- m , and Rayleigh models, describe the characteristics of multipath fading depending on the radio environment. Among these models, the Rayleigh channel model represents a typical non-line-of-sight (NLOS) condition, simulating multipath fading without a direct propagation path. In addition to multipath fading, shadowing caused by large terrains and/or high-rise buildings induces slow variations in signal power and is commonly modeled by a log-normal distribution of the received signal power (or SNR). The combined effects of multipath fading and shadowing in wireless environments are represented by composite channel models such as the Nakagami/log-normal (NL) channel model [13] [14].

The SNR distribution of the NL channel model follows a gamma/log-normal (GL) distribution [13], whose PDF generally requires numerical methods for evaluation, resulting in high computational complexity. Atapattu, Tellambura, and Jiang [15] demonstrated that a mixture gamma (MG) distribution, which is defined as a linear combination of gamma distributions, can closely approximate the GL PDF. Moreover, the MG model is versatile for various multipath fading scenarios, as it can approximate or exactly represent the SNR

distributions of many fading models through proper parameter selection [16, Table II]. Hence, the MG distribution effectively alleviates the analytical difficulties inherent in conventional composite fading models. This motivates the extension of SA performance analysis to MG fading channels. Prior MG-based works [15, 16] do not consider buffered SIC-based SA systems with real-time feedback or the associated Markov modeling and decoding structure.

However, extending the SA analysis of [12] to MG fading is non-trivial, as the MG fading model leads to a fundamentally different characterization of decoding probabilities. In particular, the received power distribution becomes a weighted sum of gamma components, which prevents the direct application of existing analytical results and requires a reformulation of the decoding analysis. Therefore, the proposed analysis is not a straightforward extension of [12], but requires a new treatment of the decoding probabilities under generalized fading.

In this paper, we consider a general composite fading channel model in which the SNR of the received signal follows a MG distribution. We investigate a slotted ALOHA system referred to as Buffered-SIC SA (BSIC-SA). In the transmission phase, each device transmits its packet in a slot with a given transmission probability. At the receiver side, the AP performs intra-slot SIC under random fading and inter-slot SIC using the signals stored in the buffer. The AP also provides real-time feedback at the end of each slot to acknowledge successfully decoded packets, while the residual signal containing undecoded packets is stored in the buffer for subsequent inter-slot SIC processing.

Building upon the BSIC-SA analysis under Rayleigh fading in [12], we generalize the framework to a composite fading environment characterized by the MG distribution. Unlike the exponential SNR distribution in the Rayleigh channel model [12], the PDF of the MG distribution involves products of exponential and power terms, which significantly complicates decoding probability derivation. By exploiting the integrable property of the lower and upper incomplete gamma functions, which can be represented as series expansions, we derive closed forms for the state transition probabilities in the associated Markov process for a two-device BSIC-SA system. The transmission probability and coding rate are jointly optimized using a sequential quadratic programming (SQP) algorithm to maximize the sum rate of the two-device system. Although the analytical derivations are for the MG distribution, the results are applicable to general composite fading models, reducing the need for case-by-case analyses of individual channel types. Numerical results confirm that the proposed analytical framework accurately characterizes the performance of various fading environments, including NL, Nakagami- m , and Rayleigh. In addition, simulation results for K_T -device BSIC-SA

systems demonstrate the achievable maximum sum rate for a given coding rate.

The main contributions of this paper are summarized as follows:

- We generalize the analytical framework of two-device BSIC-SA systems from Rayleigh fading to composite fading channels modeled by the MG distribution.
- We derive closed-form expressions for state transition probabilities under MG fading.
- We optimize the transmission probability and coding rate to maximize the sum rate and demonstrate, through numerical and simulation results, the impact of fading characteristics on system performance.

This paper is organized as follows. Section 2 reviews the BSIC-SA system and its asymptotic performance analysis using a Markov process for the two-device case. In Section 3, we derive explicit expressions for the relevant event probabilities under the MG distribution, which in turn enable the calculation of state transition probabilities. Section 4 presents numerical and simulation results that demonstrate the achievable maximum sum rate of the BSIC-SA systems. Finally, Section 5 concludes the paper.

2. Buffered-SIC SA with Real-time Feedback

This section summarizes the K_T -device BSIC-SA system with real-time feedback over a fading channel and presents an asymptotic performance analysis using a Markov process for $K_T = 2$ [12].

2.1 Random Uplink Transmission and Re-Transmission

We study a K_T -device BSIC-SA system consisting of K_T devices and an AP. The devices and the AP, each equipped with a single antenna, use a shared code $\mathcal{C}$ with rate R , where R denotes the information-theoretic transmission rate (spectral efficiency), measured in bits per channel use. In practice, such a rate, particularly when $R > 1$, can be realized by combining a channel code with higher-order modulation, although no specific coding scheme is assumed in this paper. At the receiver side, the AP performs iterative decoding between intra- and inter-slot SICs and transmits real-time confirmation messages (ACKs) to the devices at the end of each slot. Such a system has been studied in [11] [12] and is referred to as a K_T -device BSIC-SA system with feedback in this paper.

Each device is assumed to have its own message stream. Messages are encoded into codewords, termed packets, using code $\mathcal{C}$. The entire transmission period is divided into slots, each corresponding to one packet duration. Upon successful packet transmission, the device updates its message stream and prepares a new packet

for transmission. The devices are assumed to be synchronized in both symbol timing and slot boundaries.

Within the BSIC-SA framework, during slot t , each device transmits or retransmits its packet to the AP with a probability p and power P . The AP decodes packets from both the superimposed signal in slot t and the residual signals stored in the buffer from previous slots j ($< t$). It then sends ACKs to devices whose packets were fully decoded and stores the residual signal containing undecoded packets in the buffer for subsequent inter-slot SIC processing. Devices that receive ACKs introduce new packets in slot $t + 1$.

To further clarify the transmission procedure, Fig. 1(a) illustrates a two-device BSIC-SA system in which packets are randomly transmitted on a slot-by-slot basis. As an example, in slot $t - 1$, devices v_1 and v_2 independently and randomly transmit their packets $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. Since no ACK is received by the devices at the end of this slot, device v_2 retransmits its packet $\mathbf{x}_2$ in slot t .

2.2 SIC-based Decoding over Fading Channels

At the AP, decoding is performed across both the intra-slot and inter-slot domains.

2.2.1 Decoding within a Slot: Intra-Slot SIC

Consider a generic slot, for example, slot t . Suppose a set of K ($\leq K_T$) devices, denoted by $\mathcal{K}$, is active and transmits packets in the slot. For notational simplicity, the slot index t is omitted hereafter. The active device k accesses the slot, with its codeword vector $\mathbf{x}_k \in \mathcal{C}$ corresponding to the code $\mathcal{C}$ of a rate R and length n . Each device satisfies the power constraint $\|\mathbf{x}_k\|^2/n \leq P$. In the AP, the superimposed received signal in the slot is expressed as

$$\mathbf{y} = \sum_{k \in \mathcal{K}} h_k \mathbf{x}_k + \mathbf{z}.$$

Here, the noise vector $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I})$ represents additive circularly symmetric complex Gaussian (CSCG) noise with zero mean and covariance matrix $\sigma^2 \mathbf{I}$, σ^2 denotes noise power, and $\mathbf{I}$ is the identity matrix. The channel coefficient h_k describes the channel fading between device k and the AP, assumed constant within a slot and varying independently across slots. The instantaneous SNR of device k is given by $\gamma_k = P|h_k|^2/\sigma^2$.

The channel magnitudes $|h_k|$ for $k \in \mathcal{K}$ are independent and identically distributed (i.i.d.) random variables (RVs) [12]. Accordingly, the instantaneous SNRs $\gamma_k = P|h_k|^2/\sigma^2$, which are proportional to the squared channel amplitudes, follow a certain SNR distribution. In this work, the PDFs of γ_k are uniformly and accurately approximated by an MG distribution

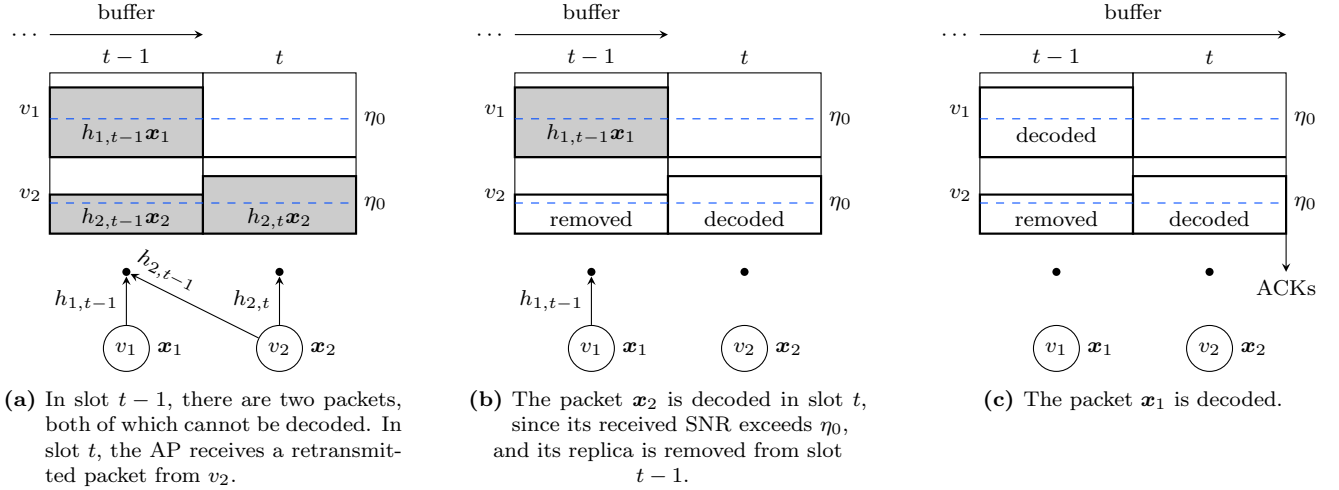


Fig. 1: In the two-device BSIC-SA system, packets are randomly transmitted slot-by-slot by devices v_1 and v_2 , and buffered inter- and intra-slot SIC are performed at the AP.

[15, 16]:

$$p(\gamma_k) = \sum_{i=1}^N a_i \gamma_k^{b_i-1} e^{-c_i \gamma_k}, \quad \gamma_k > 0, \quad (1)$$

where N denotes the number of mixture components, and a_i , b_i , and c_i are the parameters of the i -th gamma component. The expected value of each RV γ_k is given by $\bar{\gamma} \triangleq \mathbb{E}(\gamma_k)$. Two special cases of the MG distribution are the gamma distribution ($N = 1$, $b_1 = m$) and the exponential distribution ($N = 1$, $b_1 = 1$). Their corresponding channel amplitudes $|h_k|$ follow the Nakagami- m and Rayleigh distributions, respectively.

As a common detection scheme, threshold-based decoding assumes that a packet (codeword) $\mathbf{x}_k$ is successfully decoded when the signal-to-interference-plus-noise ratio (SINR) η_k exceeds a specified threshold η_0 , i.e.,

$$\Pr\{\text{decode packet } \mathbf{x}_k\} = \begin{cases} 1, & \eta_k > \eta_0, \\ 0, & \eta_k \leq \eta_0, \end{cases}$$

where the SNR threshold satisfies $R = \log_2(1 + \eta_0)$ for the given coding rate R [17].

Within a slot, interference is sequentially removed from the received signal $\mathbf{y}$, a process referred to as *intra-slot SIC*. Assuming $\gamma_1 > \gamma_2 > \dots > \gamma_K$, the AP first decodes $\mathbf{x}_1$ and subtracts $h_1 \mathbf{x}_1$ from $\mathbf{y}$ upon successful decoding. After successfully decoding $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{k-1}$, the SINR at step k of SIC is given by

$$\eta_k \triangleq \frac{P|h_k|^2}{\sigma^2 + \sum_{u=k+1}^K P|h_u|^2} = \frac{\gamma_k}{1 + \sum_{u=k+1}^K \gamma_u}. \quad (2)$$

Packet $\mathbf{x}_k$ is successfully decoded if $\eta_k > \eta_0$; otherwise, the SIC process terminates. The AP is assumed to have perfect channel state information.

A slot in which the AP successfully decodes κ

packets from $\mathbf{y}$ using intra-slot SIC is referred to as a κ -recoverable slot. The remaining signal, which contains the $K - \kappa$ unrecovered packets, is referred to as a $(K - \kappa)$ -collided residual signal for that slot. This residual signal is subsequently stored in the buffer for future inter-slot processing.

2.2.2 Decoding between Slots: Inter-Slot SIC

After performing intra-slot SIC in slot t , if a recovered packet corresponds to a retransmission, the interference from its replicas is removed from the associated residual signal(s) stored in earlier slots j ($j < t$) within the buffer. This enables further intra-slot SIC operations on these residual signals. The procedure is repeated until no additional packets can be decoded from the buffer. Once all possible packets are successfully recovered, the AP sends ACKs to the corresponding devices.

SIC Algorithm summarizes the overall decoding procedure, where a “recoverable slot” indicates that at least one packet in the slot can be decoded by intra-slot SIC.

To provide further insight, Figs. 1(b) and (c) illustrate an example of how the decoding process operates across intra- and inter-slot stages. From the received signal in slot t , the AP first recovers packet $\mathbf{x}_2$, removes its replica $h_{2,t-1} \mathbf{x}_2$ from the residual signal in slot $t-1$, and then decodes packet $\mathbf{x}_1$. Finally, at the end of slot t , the AP sends ACKs to the devices to inform them that their packets have been successfully decoded.

2.3 Markov Process of Two-Device BSIC-SA [11] [12]

This subsection explores the modeling of two-device BSI-SA as a Markov process, as depicted in Fig. 2, where the event probabilities are summarized in Table 1.

SIC Algorithm: Buffered Inter-Slot and Intra-Slot SIC Processing in Slot t

```

1: Initialize: the current slot  $t$ 
2: if slot  $t$  is recoverable then
3:   execute the intra-slot SIC in slot  $t$ .
4:   store the residual signal into the buffer.
5:   remove any replicas of recovered packets in buffer
6:   while any buffer slot  $j(\leq t)$  is recoverable do
7:     execute the intra-slot SIC in slot  $j$ .
8:     remove any replicas of the recovered packets
9:   end while
10:  ACK feedback for recovered packets.
11: else
12:   store the received signal into the buffer.
13: end if
    
```

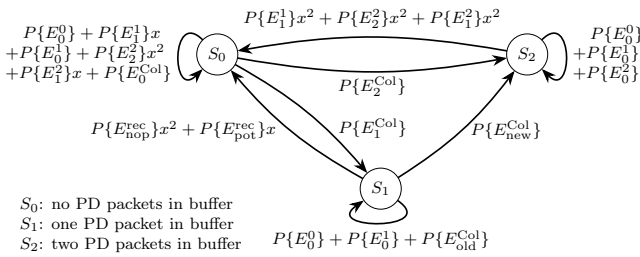


Fig. 2: State transition diagram of the Markov process for the two-device case [12].

First, we define a *potentially-decodable* (PD) packet. For a residual signal, an unrecovered packet x_k is considered PD if its SNR fulfills $\gamma_k > \eta_0$. In two-collision cases, the residual signal can contain zero, one, or two PD packets. An example of two PD packets is shown in slot $t-1$ in Fig. 1(a). In single-packet slots (one-collision), storing a non-decodable packet in the buffer is unnecessary, thus no PD packets exist.

The decoding process for intra- and inter-slot SICs can be modeled as a Markov process. The residual-signal buffer for two devices has three possible states, denoted by $\mathcal{S} = \{S_0, S_1, S_2\}$. State S_0 signifies an empty buffer, while states S_1 and S_2 indicate that one or two PD packet(s) remain, respectively. A representative instance of state S_2 in the buffer is illustrated in Fig. 1(a), while Fig. 1(c) shows a representative instance of state S_0 .

Let $\Pr\{s_t, d_t | s_{t-1}\}$ denote the probability of transition from state s_{t-1} to s_t with d_t packets recovered. We define a state-transition probability polynomial as [11]

$$p_{s_{t-1}, s_t}(x) \triangleq \sum_{d=0}^2 \Pr\{s_t, d_t = d | s_{t-1}\} x^d.$$

State transitions are initiated by decoding in slot t , with related events detailed in Table 1.

Let us explain the state transitions, for example, from $s_{t-1} = S_2$ to $s_t = S_0$. During the decoding of slot t , three scenarios can initiate this transition. Firstly, if two packets are decodable in a two-collision (event E_2^2), their replicas are eliminated from the previous residual

signals, and no residual signal exists at slot t . Consequently, the buffer state updates to $s_t = S_0$. Secondly, if one of two packets in a two-collision event (event E_1^2) is decodable, its replica is removed from the previous residual signals. The other packet is then decoded due to PD in residual signals, leading to a buffer state of $s_t = S_0$. The third scenario occurs when a packet is decodable in a one-collision (event E_1^1) (see Fig. 1(a)), resulting in the buffer state transitioning to $s_t = S_0$. The probabilities for these transitions are given by

$$\Pr\{s_t = S_0, d_t = 0 | s_{t-1} = S_2\} = 0$$

$$\Pr\{s_t = S_0, d_t = 1 | s_{t-1} = S_2\} = 0$$

$$\Pr\{s_t = S_0, d_t = 2 | s_{t-1} = S_2\} = P\{E_2^2\} + P\{E_1^2\} + P\{E_1^1\}.$$

The polynomial representing the state-transition probability (also illustrated in Fig. 2) is

$$p_{s_{t-1}=S_2, s_t=S_0}(x) = P\{E_2^2\}x^2 + P\{E_1^2\}x^2 + P\{E_1^1\}x^2.$$

State-transition polynomial matrix is then expressed in Eq. (3) [12].

The first-order derivative $dp_{s',s}(x)/dx$ evaluated at $x = 1$ indicates the expected number of packets retrieved during the transition from state s' to state s . Consequently, matrix $P'(x=1) = dP(x)/dx|_{x=1}$, which measures number of packets during transition, is

$$P'(1) = \begin{pmatrix} P\{E_1^1\} + P\{E_1^2\} + 2P\{E_2^2\} & 0 & 0 \\ 2P\{E_{\text{nop}}^{\text{rec}}\} + P\{E_{\text{pot}}^{\text{rec}}\} & 0 & 0 \\ 2P\{E_1^1\} + 2P\{E_2^2\} + 2P\{E_1^2\} & 0 & 0 \end{pmatrix}. \quad (4)$$

Let a stationary distribution of the Markov process be $\mathbf{w} = (w_0, w_1, w_2)^T$, such that $\mathbf{w}^T P(x=1) = \mathbf{w}^T$ and $\mathbf{w}^T \mathbf{1} = 1$, where $\mathbf{1} = (1, 1, 1)^T$. The normalized throughput of the two-device SA with SICs and feedback, as $t \rightarrow \infty$, is given by

$$T(p, R, \bar{\gamma}) = \mathbf{w}^T P'(1) \mathbf{1} \text{ packets/slot} \quad (5)$$

which represents the average number of packets decoded successfully per time slot.

Furthermore, we define the sum rate

$$R_s \triangleq RT(p, R, \bar{\gamma}) \text{ bits/channel use} \quad (6)$$

which is the expected amount of information recovered from each received symbol.

We have outlined the Markov process and its associated state transition probabilities. The next section derives these probabilities in closed form.

3. Event Probabilities Using MG distributions

In this section, we provide explicit expressions for the event probabilities in Table 1 using the MG distributions with b_i ($i = 1, 2, \dots, N$) being a positive integer, as commonly noted in the literature.

$$P(x) = \begin{pmatrix} P\{E_0^0\} + P\{E_1^1\}x + P\{E_0^1\} + P\{E_2^2\}x^2 + P\{E_1^2\}x + P\{E_0^{\text{Col}}\} & P\{E^{\text{Col}}\} & P\{E_2^{\text{Col}}\} \\ P\{E_{\text{nop}}^{\text{rec}}\}x^2 + P\{E_{\text{pot}}^{\text{rec}}\}x & P\{E_0^0\} + P\{E_0^1\} + P\{E_{\text{old}}^{\text{Col}}\} & P\{E_{\text{new}}^{\text{Col}}\} \\ P\{E_1^1\}x^2 + P\{E_2^2\}x^2 + P\{E_1^2\}x^2 & 0 & P\{E_0^0\} + P\{E_0^1\} + P\{E_0^2\} \end{pmatrix} \quad (3)$$

Table 1: Events Occurring in the Current Slot [12]

event	interpretation
E_m^n	n active devices with m -recovery, ($m \leq n$, $n = 0, 1, 2$)
$E_{\text{nop}}^{\text{rec}}$	a packet is recovered, whose replica in the buffer is not PD
$E_{\text{pot}}^{\text{rec}}$	only a packet is recovered, whose replica in the buffer is PD
E_j^{Col}	j PD packets in 2-collision ($j = 0, 1, 2$)
$E_{\text{new}}^{\text{Col}}$	two-collision occurs with a new PD packet
$E_{\text{old}}^{\text{Col}}$	two-collision occurs without a new PD packet

3.1 Preliminary

For the PDF of Eq. (1), by applying the gamma function defined in Eq. (9), it follows that

$$\int_0^\infty p(\gamma_j) d\gamma_j = \sum_{i=1}^N a_i c_i^{-b_i} (b_i - 1)! = \sum_{i=1}^N d_i = 1,$$

where, for compact notation,

$$d_i \triangleq a_i c_i^{-b_i} (b_i - 1)!.$$

For the computation of the event probabilities, it is necessary to evaluate the lower and upper incomplete gamma functions as series expansions [18, Eq. 8.352.1] [18, Eq. 8.352.4], considering m as a positive integer.

$$\gamma(m, x) \triangleq \int_0^x t^{m-1} e^{-t} dt = (m-1)! \left(1 - e^{-x} \sum_{k=0}^{m-1} \frac{x^k}{k!}\right). \quad (7)$$

$$\Gamma(m, x) \triangleq \int_x^\infty t^{m-1} e^{-t} dt = (m-1)! e^{-x} \sum_{k=0}^{m-1} \frac{x^k}{k!}. \quad (8)$$

In addition, the (complete) gamma function defined below is related to the factorial by [18, Eqs. (8.310.1) and (8.321.1)]

$$\Gamma(m) \triangleq \int_0^\infty t^{m-1} e^{-t} dt = (m-1)!. \quad (9)$$

3.2 Probabilities of E_m^n

We are now ready to compute the probabilities of events in Table 1.

3.2.1 Probability of E_2^2

Initially, we evaluate $P\{E_2^2\}$. Considering a superimposed signal from two packets, the probability of both packets being successfully decoded is $\Pr\{\frac{\gamma_1}{1+\gamma_2} > \eta_0, \gamma_2 > \eta_0\}$ under the assumption that $\gamma_1 > \gamma_2$. There

is also a scenario where $\gamma_1 < \gamma_2$. Furthermore, the probability of both devices are active in the same slot is p^2 . Consequently, the probability for event E_2^2 can be described as follows:

$$\begin{aligned} P\{E_2^2\} &= 2p^2 \Pr\left\{\frac{\gamma_1}{1+\gamma_2} > \eta_0, \gamma_2 > \eta_0\right\} \\ &= 2p^2 \int_{\eta_0}^\infty p(\gamma_2) \int_{\eta_0(1+\gamma_2)}^\infty p(\gamma_1) d\gamma_1 d\gamma_2. \end{aligned} \quad (10)$$

where the integral with respect to γ_1 is

$$\begin{aligned} \int_{\eta_0(1+\gamma_2)}^\infty p(\gamma_1) d\gamma_1 &= \sum_{n=1}^N a_n c_n^{-b_n} \Gamma(b_n, c_n(\eta_0(1+\gamma_2))) \\ &= \sum_{n=1}^N a_n c_n^{-b_n} (b_n - 1)! e^{-c_n \eta_0(1+\gamma_2)} \sum_{k_1=0}^{b_n-1} \frac{(c_n \eta_0)^{k_1} (1+\gamma_2)^{k_1}}{k_1!} \\ &\stackrel{(a)}{=} \sum_{n=1}^N d_n e^{-c_n \eta_0} e^{-c_n \eta_0 \gamma_2} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \gamma_2^{k_2} \right). \end{aligned} \quad (11)$$

In (a) we use binomial expansion $(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k$. In Eq. (10), we integrate with respect to γ_2 and obtain

$$\begin{aligned} &\int_{\eta_0}^\infty p(\gamma_2) e^{-c_n \eta_0(\gamma_2)} \gamma_2^{k_2} d\gamma_2 \\ &= \sum_{m=1}^N a_m \int_{\eta_0}^\infty \gamma_2^{b_m+k_2-1} e^{-(c_n \eta_0 + c_m) \gamma_2} d\gamma_2 \\ &= \sum_{m=1}^N a_m (c_n \eta_0 + c_m)^{-(b_m+k_2)} \Gamma(b_m+k_2, (c_n \eta_0 + c_m) \eta_0). \end{aligned} \quad (12)$$

Thus, the probability of the event E_2^2 is:

$$\begin{aligned} P\{E_2^2\} &= 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \right) \\ &\cdot \sum_{m=1}^N a_m (c_n \eta_0 + c_m)^{-(b_m+k_2)} \Gamma(b_m+k_2, (c_n \eta_0 + c_m) \eta_0). \end{aligned}$$

Based on the above derivation, we obtain a closed-form expression for $P\{E_2^2\}$ involving series expansions. This result is derived from double nested integrals over two-variable MG distributions by evaluating the upper incomplete gamma functions via series expansions, enabling efficient numerical evaluation and significantly reducing the computational burden compared to direct numerical integration.

3.2.2 Probability of E_1^2 and E_0^2

Likewise, the subsequent probabilities are as follows:

$$\begin{aligned}
 P\{E_1^2\} &= 2p^2 \Pr\left\{\frac{\gamma_1}{1+\gamma_2} > \eta_0, \gamma_2 < \eta_0\right\} \\
 &= 2p^2 \int_0^{\eta_0} p(\gamma_2) \int_{\eta_0(1+\gamma_2)}^{\infty} p(\gamma_1) d\gamma_1 d\gamma_2 \\
 &= 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \right. \\
 &\quad \cdot \left. \sum_{m=1}^N a_m (c_n \eta_0 + c_m)^{-(b_m+k_2)} \gamma(b_m+k_2, (c_n \eta_0 + c_m) \eta_0) \right). \\
 P\{E_0^2\} &= 2p^2 \Pr\{\gamma_2 < \gamma_1 < \eta_0(1+\gamma_2), 0 < \gamma_2 < \infty\} \\
 &= 2p^2 \int_0^{\infty} p(\gamma_2) \int_{\gamma_2}^{\eta_0(1+\gamma_2)} p(\gamma_1) d\gamma_1 d\gamma_2. \\
 &= 2p^2 \sum_{n=1}^N d_n \sum_{k_1=0}^{b_n-1} \frac{(c_n)^{k_1}}{k_1!} \sum_{m=1}^N a_m (c_n + c_m)^{-(b_m+k_1)} \Gamma(b_m+k_1) \\
 &\quad - 2p^2 \sum_{n=1}^N \left\{ d_n e^{-c_n \eta_0} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \right. \right. \\
 &\quad \cdot \left. \left. \sum_{m=1}^N (c_n \eta_0 + c_m)^{-(b_m+k_2)} a_m \Gamma(b_m+k_2) \right) \right\}.
 \end{aligned}$$

3.2.3 Probabilities of E_1^1 , E_0^1 , and E_0^0

When at most one packet is received in the current slot, the corresponding probabilities are given by:

$$\begin{aligned}
 P\{E_1^1\} &= 2p(1-p) \Pr\{\gamma_1 > \eta_0\} \\
 &= 2p(1-p) \sum_{n=1}^N a_n c_n^{-b_n} \Gamma(b_n, c_n \eta_0) \\
 P\{E_0^1\} &= 2p(1-p) \Pr\{\gamma_1 \leq \eta_0\} \\
 &= 2p(1-p) \sum_{n=1}^N a_n c_n^{-b_n} \gamma(b_n, c_n \eta_0) \\
 P\{E_0^0\} &= (1-p)^2.
 \end{aligned}$$

3.3 Probabilities of E_j^{Col}

In a similar manner, we determine the probabilities $P\{E_j^{\text{Col}}\}$ for the existence of j PD packet(s) during a two-collision scenario, where $j = 2, 1, 0$.

$$\begin{aligned}
 P\{E_2^{\text{Col}}\} &= 2!p^2 \Pr\{\gamma_2 < \gamma_1 < \eta_0(1+\gamma_2), \eta_0 < \gamma_2\} \\
 &= 2!p^2 \int_{\eta_0}^{\infty} p(\gamma_2) \int_{\gamma_2}^{\eta_0(1+\gamma_2)} p(\gamma_1) d\gamma_1 d\gamma_2
 \end{aligned}$$

$$\begin{aligned}
 &= 2p^2 \sum_{n=1}^N d_n \sum_{k_1=0}^{b_n-1} \frac{c_n^{k_1}}{k_1!} \\
 &\quad \cdot \sum_{m=1}^N a_m (c_n + c_m)^{-(b_m+k_1)} \Gamma(b_m+k_1, (c_n + c_m) \eta_0) \\
 &\quad - 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \right. \\
 &\quad \cdot \left. \sum_{m=1}^N a_m (c_n \eta_0 + c_m)^{-(b_m+k_2)} \Gamma(b_m+k_2, (c_n \eta_0 + c_m) \eta_0) \right). \\
 P\{E_1^{\text{Col}}\} &= 2!p^2 \Pr\{\eta_0 < \gamma_1 < \eta_0(1+\gamma_2), 0 < \gamma_2 < \eta_0\} \\
 &= 2!p^2 \int_0^{\eta_0} p(\gamma_2) \int_{\eta_0}^{\eta_0(1+\gamma_2)} p(\gamma_1) d\gamma_1 d\gamma_2 \\
 &= 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \left(\sum_{k_1=0}^{b_n-1} \frac{(c_n \eta_0)^{k_1}}{k_1!} \right) \\
 &\quad \cdot \left(\sum_{m=1}^N a_m c_m^{-b_m} \gamma(b_m, c_m \eta_0) \right) \\
 &\quad - 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \sum_{k_1=0}^{b_n-1} \left(\frac{(c_n \eta_0)^{k_1}}{k_1!} \sum_{k_2=0}^{k_1} \binom{k_1}{k_2} \right. \\
 &\quad \cdot \sum_{m=1}^N a_m (c_n \eta_0 + c_m)^{-(b_m+k_2)} \\
 &\quad \cdot \left. \gamma(b_m+k_2, (c_n \eta_0 + c_m) \eta_0) \right). \tag{13}
 \end{aligned}$$

$$\begin{aligned}
 P\{E_0^{\text{Col}}\} &= 2!p^2 \Pr\{\gamma_2 < \gamma_1 < \eta_0, 0 < \gamma_2 < \eta_0\} \\
 &= 2p^2 \int_0^{\eta_0} p(\gamma_2) \int_{\gamma_2}^{\eta_0} p(\gamma_1) d\gamma_1 d\gamma_2 \\
 &= 2p^2 \sum_{n=1}^N d_n \sum_{k_1=0}^{b_n-1} \frac{c_n^{k_1}}{k_1!} \\
 &\quad \cdot \sum_{m=1}^N a_m (c_n + c_m)^{-(b_m+k_1)} \gamma(b_m+k_1, (c_n + c_m) \eta_0) \\
 &\quad - 2p^2 \sum_{n=1}^N d_n e^{-c_n \eta_0} \left(\sum_{k_1=0}^{b_n-1} \frac{(c_n \eta_0)^{k_1}}{k_1!} \right) \\
 &\quad \cdot \left(\sum_{m=1}^N a_m c_m^{-b_m} \gamma(b_m, c_m \eta_0) \right).
 \end{aligned}$$

3.4 $P\{E_{\text{pot}}^{\text{rec}}\}$, $P\{E_{\text{nop}}^{\text{rec}}\}$, $P\{E_{\text{new}}^{\text{Col}}\}$, $P\{E_{\text{old}}^{\text{Col}}\}$

To illustrate, take the event $E_{\text{pot}}^{\text{rec}}$ as an example. This event happens when a) a packet from the current slot is successfully recovered, and b) this recovered packet turns out to be a PD replica within the buffer. If condition (a) is met, then condition (b) will occur with a

probability of $1/2$. Consequently, as shown in [12],

$$P\{E_{\text{pot}}^{\text{rec}}\} = (P\{E_1^1\} + P\{E_1^2\})/2.$$

Likewise, the other probabilities listed in Table 1 are detailed in the same reference [12]

$$\begin{aligned} P\{E_{\text{nop}}^{\text{rec}}\} &= (P\{E_1^1\} + P\{E_1^2\})/2 + P\{E_2^2\} \\ P\{E_{\text{new}}^{\text{Col}}\} &= P\{E_1^{\text{Col}}\}/2 + P\{E_2^{\text{Col}}\} \\ P\{E_{\text{old}}^{\text{Col}}\} &= P\{E_1^{\text{Col}}\}/2 + P\{E_0^{\text{Col}}\}. \end{aligned}$$

Remark 1: It is known that the MG distribution, as shown in Eq. (1), simplifies to a gamma distribution when $N = 1$, and further deduces to an exponential distribution when $N = 1$ and $b_1 = 1$. Consequently, the event probabilities calculated in this work correspond to those reported in [12], where the channel fading is a Rayleigh distribution, that is, $N = 1$ and $b_1 = 1$, for the channel amplitude $|h_k|$.

However, unlike the exponential SNR distribution in the Rayleigh model, the MG PDF involves products of exponential and power terms (see Eq. (1)), resulting in weighted sums of gamma components. This fundamentally alters the structure of the decoding probabilities and prevents a direct analytical extension from the Rayleigh case. Consequently, the extension to MG fading is non-trivial and requires a reformulation of the decoding probability analysis. By leveraging the series representations of the incomplete gamma functions in Eqs. (7) and (8), closed-form expressions for the Markov state transition probabilities are obtained. $\square$

Remark 2: It is noted that the intra-slot SIC is typically utilized for managing, for example, two-collision scenarios in most systems [19], primarily due to increased interference from multiple devices, increased cumulative error in the SIC, and extensive computational complexity. Consequently, in configurations involving multiple devices, it is advisable to focus solely on two-collision decoding. Our analysis can be adapted to accommodate a multi-device configuration that incorporates two-collision decoding in the current slot while disregarding, for example, five or more collisions in residual signal. The resultant number of states is moderate and acceptable. $\square$

3.5 Optimization of Probability p and Coding Rate R

Prior to concluding this section, we aim to optimize transmission probability p and coding rate R . This optimization ensures that the sum rate of Eq. (6) is maximized for the given average received SNR $\bar{\gamma}$.

$$\begin{aligned} &\underset{0 < p \leq 1, 1 \leq R}{\text{maximize}} && R_s = RT(p, R, \bar{\gamma}). \\ &\text{subject to} && \bar{\gamma} \text{ is constant.} \end{aligned} \quad (14)$$

The joint optimization of p and R is performed offline for given system conditions using the SQP algorithm. The resulting parameters are applied during operation without real-time recomputation and updated only when system conditions change. The AP broadcasts these parameters to all devices via control signaling with limited overhead. Reliable control signaling is assumed, i.e., parameter sharing is error-free, as commonly adopted in physical-layer analyses. The impact of imperfect sharing is beyond the scope of this work.

4. Numerical and Simulation Results

In this section, we present numerical results for the sum rate of BSIC-SA and its maximum value achieved by optimizing the transmission probability and coding rate. Additionally, we conduct computer simulations to verify our analytical results. The number of slots in the simulation is set to 10^6 slots, unless otherwise specified.

For the multi-device simulations in Section 4.3, the coding rate is set, for example, to $R = 5.4854$.

4.1 Composite Nakagami/log-normal channel model

In wireless communications, environments with combined multipath fading and shadowing are characterized by a composite channel model of Nakagami- m fading and log-normal shadowing, illustrated by the Nakagami/log-normal (NL) channel model. Aside from fading amplitude, the model's SNR distribution is obtained by average a gamma distributed SNR over the conditional density of a log-normally distributed average SNR, given the following PDF [14] [15] [13]

$$p_{\text{GL}}(\gamma) = \int_0^\infty \frac{\gamma^{m-1} e^{-\frac{m\gamma}{\rho y}}}{\Gamma(m)} \left(\frac{m}{\rho y}\right)^m \frac{e^{-\frac{(\ln y - \mu)^2}{2\lambda^2}}}{\sqrt{2\pi\lambda^2}} dy \quad (15)$$

known as a gamma/log-normal (GL) distribution. Here ρ is the mean of unfaded SNR, and μ and λ are the mean and standard deviation of the log-normal distribution. The expected value of the GL distribution is $\bar{\gamma} = \mathbb{E}(\gamma) = \rho \exp(\mu + \frac{\lambda^2}{2})$. A closed-form expression for the PDF of the GL distribution is not available in the literature.

The PDF of Eq. (15) is approximated by the MG distribution of Eq. (1) with parameters [15] [16, Table II] $N = 20$ and

$$\begin{aligned} a_i &= \frac{\theta_i}{\sum_{j=1}^N \theta_j \Gamma(b_j) c_j^{-\beta_j}}, \quad b_i = m, \quad c_i = \frac{m}{\rho} e^{-(\sqrt{2}\lambda t_i + \mu)}, \\ \theta_i &= \left(\frac{m}{\rho}\right)^m \frac{w_i e^{-m(\sqrt{2}\lambda t_i + \mu)}}{\sqrt{\pi} \Gamma(m)}. \end{aligned} \quad (16)$$

Here t_i and w_i are the roots of a Hermite polynomial and their corresponding weight factors [20] [21, p. 924].

For an average SNR $\bar{\gamma} = 25$ dB and the

Nakagami/log-normal channel's parameters $(m, \lambda, \mu) = (2, 1, 0.25)$, we determine the parameters of the MG distribution in Eq. (16) with $N = 20$, needed to calculate the sum rate of Eq. (6). Utilizing the SQP algorithm [22], with Eq. (14) we find the maximum sum rate $R_s^* = 3.2018$ at the optimal values of $p^* = 0.5991$ and $R^* = 5.4854$, indicated by the square in Fig. 3.

We also show the sum rates of Eq. (6) in our analysis for the intervals $R \in [1, 10]$ and $p \in [0, 1]$ in Fig. 3, where the highest point corresponds to the optimal point noted by the square.

In Fig. 4, we provide a cross section of Fig. 3 at the optimal coding rate of $R^* = 5.4854$. In addition, we also give the sum rate at $R = 1.0, 2.0, 3.0$, and 5.0 for comparison. Moreover, we conducted two types of simulations to compute the sum rates: 1) The random numbers are generated from the MG distribution, denoted by $*$ in the figure. Specifically, among the N random numbers sampled from their corresponding gamma distributions, respectively, select one with weighted probabilities; 2) the values of SNR are sampled from the original gamma/log-normal distribution of Eq. (15), denoted by $\circ$. The findings indicate that the analysis and two simulation curves closely align with each other.

Also in Fig. 4, for comparison, we give the sum rates of two-device SA with the intra-slot SIC only. Without the inter-slot SIC, the throughput is represented as $T = dp_{S_0, S_0}(x)/dx|_{x=1} = P\{E_1^1\} + P\{E_2^1\} + 2P\{E_2^2\}$, which is entry $(P'(1))_{(0,0)}$ of Eq. (4). The optimized coding rate is $R_{w/o}^* = 5.4732$ and the optimized transmission probability is $p_{w/o}^* = 0.5116$. Obviously, the inter-slot SIC improves throughput.

Figure 5 shows the sum rate for various values of λ . Here, λ is the standard deviation of the logarithm of y in the log-normal distribution Eq. (15), affecting the severity of the shadow. A smaller λ indicates a more stable channel with reduced shadowing effects. The sum rate increases as λ decreases from 1.00 to 0.00.

4.2 Nakagami- m and Rayleigh [12] channel models

When $N = 1$, $a_1 = \frac{m}{\Gamma(m)\bar{\gamma}^m}$, $b_1 = m$, and $c_1 = \frac{m}{\bar{\gamma}}$, the MG distribution reduces to a gamma distribution, and aligns with the SNR distribution of a Nakagami- m fading channel model with average SNR $\bar{\gamma}$. In Figs. 6 and 7, we give the sum rates of Eq. (6) from our analysis with the Nakagami- m fading model with $m = 5$ and $m = 1$, where the maximum point is consistent with the optimal point. Note that Rayleigh corresponds to $m = 1$ in Nakagami- m . Our analytic results include the previous ones [12, Fig. 3(b)], as a special case of $m = 1$.

In Fig. 8, we show the analysis results of Nakagami- m with $m = 1$ and 5, at corresponding optimal coding rates $R^* = 6.1267$ and 7.0247. The curve of the Nakagami- m model ($m = 5$) is close to that of the colli-

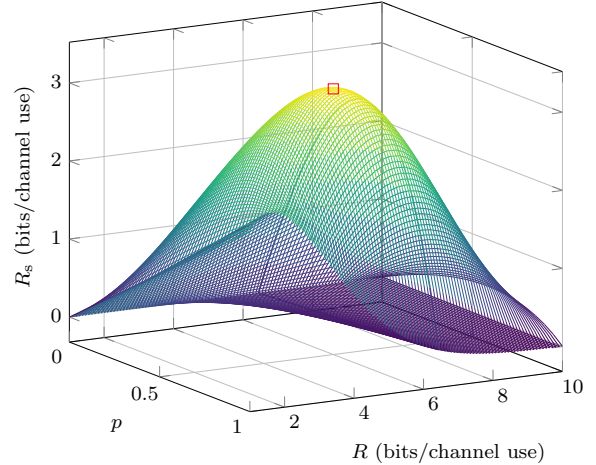


Fig. 3: NL channel with $(m, \lambda, \mu) = (2, 1, 0.25)$: The sum rates at SNR $\bar{\gamma} = 25$ dB are shown. The maximum sum rate $R_s^* = 3.2018$ is obtained by the SQP algorithm at the optimal values of $p^* = 0.5991$ and $R^* = 5.4854$, marked by $\square$.

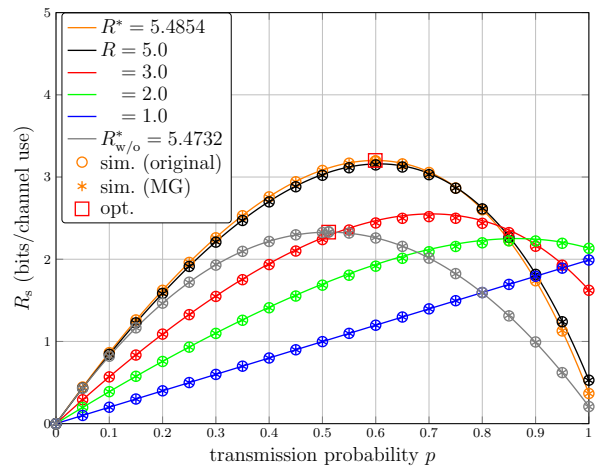


Fig. 4: NL channel with $(m, \lambda, \mu) = (2, 1, 0.25)$: The sum rates are shown at SNR $\bar{\gamma} = 25$ dB. Analysis and simulation with MG, as well as simulation with the original gamma/log-normal distribution of Eq. (15). The maximum sum rates are marked by $\square$.

sion channel model [11]. This is expected, as Nakagami- m fading approaches the behavior of a non-fading additive white Gaussian noise (AWGN) channel as m increases along with an elevated $\bar{\gamma}$ [13].

Analogously to Fig. 4, Fig. 8 presents the sum rates for two-device SA without inter-slot SIC over Nakagami- m and Rayleigh ($m = 1$). Clearly, employing inter-slot SIC enhances the sum rate.

4.3 Multiple Devices: Simulation Results

Modeling a K_T -device BSIC-SA system using Markov processes becomes challenging as the state count increases for $K_T > 2$.

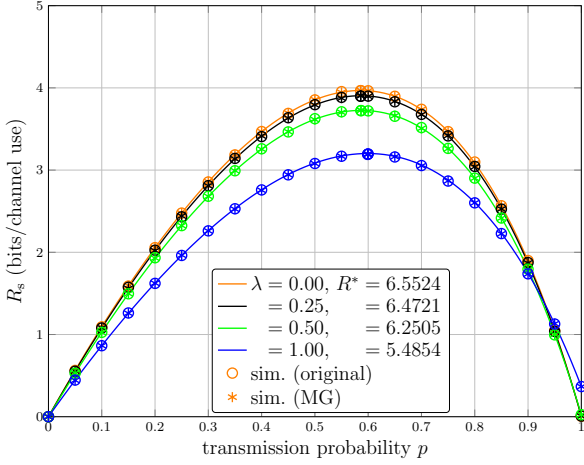


Fig. 5: NL channel with $(m, \mu) = (2, 0.25)$: The sum rates are shown at $\bar{\gamma} = 25$ dB and different values of λ .

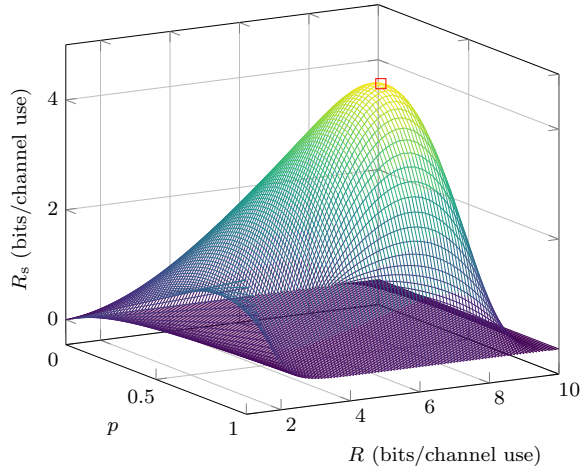


Fig. 6: Nakagami- m channel with $m = 5$: The sum rates are shown at average SNR $\bar{\gamma} = 25$ dB. The SQP algorithm provides the maximum sum rate $R_s^* = 4.548$ at $p^* = 0.58579$ and $R^* = 7.0247$, marked by $\square$.

This is because the number of Markov states grows combinatorially with the number of devices, as each state must represent the decoding status and residual signal configuration of multiple packets across slots. In particular, for $K_T > 2$, the receiver must account for all possible combinations of successfully decoded packets, remaining interfering packets, and their corresponding power ordering under fading. As a result, the state space becomes prohibitively large, and the computation of state transition probabilities requires high-dimensional integrals over multiple random variables, which makes closed-form analysis intractable. Therefore, for multi-device systems, we resort to Monte Carlo simulations to evaluate system performance.

Figure 9 illustrates the peak sum rates along with the corresponding optimal transmission probability for

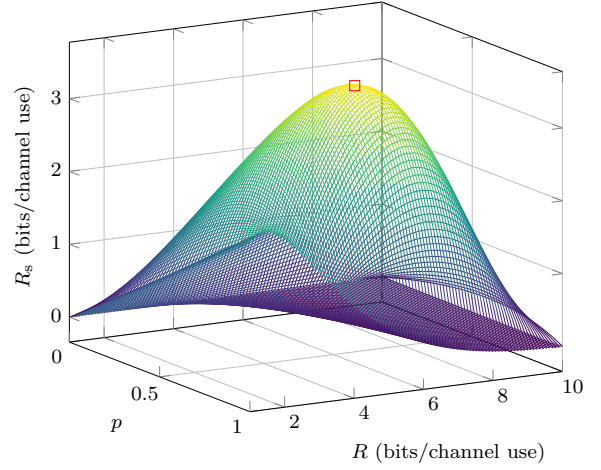


Fig. 7: Rayleigh channel ($m = 1$) [12]: The sum rates at average SNR $\bar{\gamma} = 25$ dB. When $p^* = 0.59429$ and $R^* = 6.1267$, the sum rate $R_s^* = 3.4309$ is maximum, marked by $\square$.

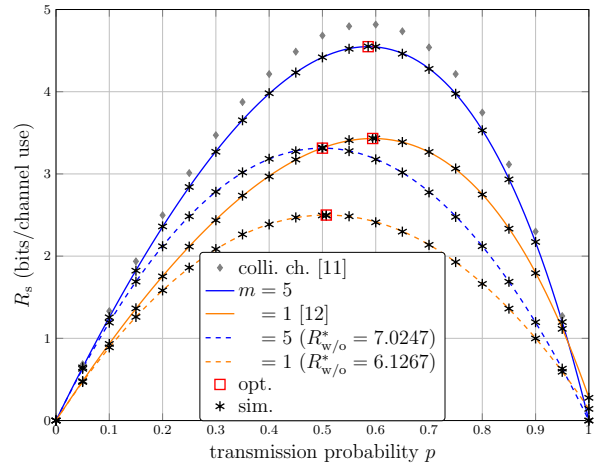


Fig. 8: Nakagami- m channel with $m = 5$ and $m = 1$ (Rayleigh): The sum rates are shown at average SNR $\bar{\gamma} = 25$ dB.

the coding rate of $R = 5.4854$ at $\bar{\gamma} = 25$ dB over NL channel model. The number of slots n_s is 1000 and 10000. The results are averaged over 100 simulations.

Finally, regarding implementation complexity, the iterative decoding process follows the same operational principles as in prior BSIC-SA system [12]. The more general channel model only affects the statistical characterization of the channel gains and does not require any modification to the decoding procedure. The SIC operations remain sequential, and their complexity scales linearly with the number of decodable packets per slot, as in [12].

5. Conclusion

We have derived an analytical sum-rate expression for two-device BSIC-SA systems with real-time feedback.

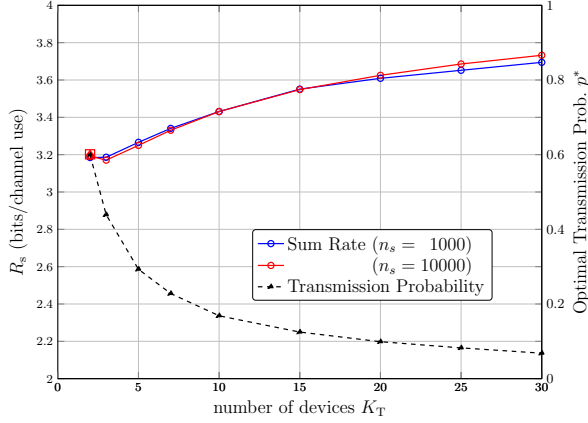


Fig. 9: Simulation results of K_T -device system over **NL** channel at average SNR $\bar{\gamma} = 25$ dB with $R = 5.4854$ and $K_T = 2, 3, 5, 7, 10, 15, 20, 25, 30$.

The integrable properties of lower and upper incomplete gamma functions, expressed as series expansions, allow us to provide closed forms for the state transition probabilities in the Markov process for SNR of MG distribution. By optimizing the transmission probability and coding rate, we maximize the sum rate. Our performance analysis applies to the general composite fading model, reducing the need for specific channel model analyses. The numerical results have confirmed that our analysis is robust in various fading models, including NL, Nakagami- m and Rayleigh. Furthermore, we have presented the simulation results for SA systems with multiple devices and their maximum sum rate for a specified coding rate.

In practical systems, the feedback overhead associated with slot-by-slot ACK can be reduced by techniques such as group acknowledgment (GACK), where multiple devices are acknowledged simultaneously [23]. Such mechanisms complement the present BSIC-SA system and offer an effective approach to reducing feedback overhead in practical implementations.

The present paper focuses on the single-antenna BSIC-SA case and provides a detailed analytical foundation under MG fading; the extension to multi-antenna reception, which introduces additional spatial dimensions and analytical complexity, is addressed in our subsequent work.

Future work includes deriving an analytical performance characterization for multi-device SA systems with real-time feedback, which is challenging, particularly when the AP is equipped with multiple antennas.

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Daiki Fukui received B.S. degree from the Department of Intelligent Information Engineering and Sciences, Doshisha University, Kyoto, Japan in 2023. He is now a graduate student at the same institution. His research interests are in the areas of communication theory and wireless communications.



Yuhei Takahashi received B.S. and M.S. degrees from the Department of Intelligent Information Engineering and Sciences, Doshisha University, Kyoto, Japan in 2021 and 2023. Currently, he works for the Rail Systems Business Unit, Japan LoB Rail Control & Signaling at Hitachi, Ltd., while pursuing his Ph.D. at the Graduate School of Science and Engineering, Doshisha University. His research interests are in the areas of coding theory,

wireless sensor networks, and radio communication systems.



Guanghui Song received the B.E. degree in communication and engineering from Henan Normal University, Xinxiang, China, in 2006. He received the M.S. degree in telecommunications engineering from Xidian University, Xi'an, China, in 2009 and the Ph.D. degree in the department of intelligent information engineering and sciences, Doshisha University, Kyoto, Japan, in 2012. From 2013 to 2021, he did postdoctoral research in Doshisha

University, Kyoto, Japan, University of Western Ontario, London, Canada, and Singapore University of Technology and Design, Singapore. Currently, he is an Associate Professor with Xidian University, Xi'an, China. His research interests are in the areas of channel coding, multi-user coding, and coding for data storage systems.



Tomotaka Kimura received the B.Eng., M.Eng., and Ph.D. degrees in communications engineering from Osaka University, Japan, in 2008, 2010, and 2015, respectively. From April 2015 to March 2018, he was an Assistant Professor with the Department of Electrical Engineering, Faculty of Engineering, Tokyo University of Science. From April 2018 to March 2020, he was an Assistant Professor with the Department of Intelligent Information Engineering and Sciences, Faculty of Science and Engineering, Doshisha University, Kyoto, Japan, where he is currently an Associate Professor. His research interests include performance analysis and designs of communication networks. He is a member of IEICE, IEEEJ, and JSAI.



Zilong Liu received the bachelor's degree from the School of Electronics and Information Engineering, Huazhong University of Science and Technology (HUST), China, in 2004, the master's degree from the Department of Electronic Engineering, Tsinghua University, China, in 2007, and the Ph.D. degree from the School of Electrical and Electronic Engineering, Nanyang Technological University (NTU), Singapore, in 2014.

He was a Visiting Ph.D. Student with the Hong Kong University of Science and Technology and the University of Melbourne. From January 2018 to November 2019, he was a Senior Research Fellow with the Institute for Communication Systems (ICS), Home of the 5G Innovation Centre (5GIC), University of Surrey, during which he studied the air-interface design of 5G communication networks. Prior to his career in the U.K., he spent nine and half years at NTU, first as a Research Associate, from July 2008 to October 2014 and then as a Research Fellow, from November 2014 to December 2017. He has been with the School of Computer Science and Electronic Engineering, University of Essex, since December 2019, first as a Lecturer and then as a Senior Lecturer since October 2023. His Ph.D. thesis on "Perfect- and Quasi- Complementary Sequences," focusing on fundamental limits, algebraic constructions, and applications of complementary sequences in wireless communications, has settled a few long-standing open problems in the field. His research interests include the interplay of coding, signal processing, and communications, with a major objective of bridging theory and practice. Recently, he has developed an interest in advanced 6G V2X communication, sensing, and localization technologies for future connected autonomous vehicles and machine learning for enhanced communications and networking.

Dr. Liu was the Hosting General Co-Chair of the 12th Sequences and Their Applications (SETA'2024, <https://seta-2024.github.io/>) and the Tenth IEEE International Workshop on Signal Design and its Applications in Communications (iwsda2022.github.io). He was named the 2024 Outstanding Mid-Career Researcher at the Faculty of Science and Health, University of Essex, in May 2024. He was a Consultant of Japanese Government on 6G assisted autonomous driving in January 2023. He was the Track Co-Chair of Networking and MAC in IEEE PIMRC'2023. He is an Associate Editor of IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY, IEEE WIRELESS COMMUNICATIONS LETTERS, and Advances in Mathematics of Communications. Details of his research can be found

at: <https://sites.google.com/site/zilongliu2357>.



Jun Cheng received B.S. and M.S. degrees in telecommunications engineering from Xidian University, Xi'an, China, in 1984 and 1987, and a Ph.D. degree in electrical engineering from Doshisha University, Kyoto, Japan in 2000. Currently, he is a professor in the Department of Intelligent Information Engineering and Sciences, Doshisha University, Kyoto.

From 1987 to 1994, he was an assistant professor and lecturer in the Department of Information Engineering, Xidian University. From 1995 to 1996, he was an associate professor in the National Key Laboratory on Integrated Service Network, Xidian University. In April 2000, he joined ATR Adaptive Communications Research Laboratories, Kyoto, where he became a visiting researcher. From August 2002 to June 2003, he was a staff engineer at the R&D Center, Panasonic Mobile Communications Co., Ltd. (formerly Wireless Solution Labs., Matsushita Communication Industrial Co., Ltd.), Yokosuka. From July 2003 to March 2004, he was a staff engineer at the Next-Generation Mobile Communications Development Center, Matsushita Electric Industrial Co., Ltd., Yokosuka. In April 2004, he joined Doshisha University, Kyoto. From October 2016 to September 2017, he was a visiting scholar with the Department of Electrical and Computer Engineering, Texas A&M University.

His research interests are in the areas of communications theory, information theory, coding theory, array signal processing, and radio communication systems.