Contracts and on-the-job search

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Contents

Introduction

This thesis is comprised of three papers which address a common issue. This is the issue of contracting in the presence of search frictions, focusing in particular on the case of on-the-job search. My intention is to analyse equilibrium behaviour in long term relations formed through search, but allowing economic agents to look for new possibilities even when they are in a given relationship (referred to as on-the-job search). A typical example is that of a worker looking for better employment while working on a firm; or associated professionals looking for better options. A socially less acceptable example would be spouses looking for better partners while married. Hence, the general framework explicitly allows for the possibility of separations; the worker may want to quit his work in order to move to better employment; a lawyer may want to leave its current partner to form a more profitable partnership with another lawyer, or a spouse may want to divorce its partner to engage in a new relationship.

The main concern of the research is on the (incentives and possibility of) forward looking contracting behaviour of agents when they know that this possibility exists; i.e. that at least one agent in the relationship will or may contact other prospective partners. In particular, I show how do certain labour market institutions, which exist presumably with the purpose of protecting the workers's interests, may in some cases work against this same objective. In many cases, which will be detailed below and throughout the document, the law negates the agents the possibility of contracting on some relevant aspects of their relationship, therefore not allowing them to extract all benefits of trade. In practice, agents are not able to commit themselves to actions which would be beneficial for all parties involved, therefore decreasing the potential surplus created in the contracts.

The two elements in which I focus are the worker's inability to commit to a working relationship, either by renouncing search for other work possibilities or directly by committing not to leave or break a contract. One could, mistakenly, analyse these two phenomena as increasing the worker's freedom in the labour market and therefore working to their benefit. Amongst other things, what this paper shows is that in certain cases, the worker would prefer to be able commit not to leave a contract or commit not to engage in search for other work possibilities, and receive in exchange some reward from the employer. Where labour provides freedom to the worker in terms of ending a contract or looking for alternative job options without legal consequences, the employer will never agree to reward an "unbelievable" promise of loyalty.

In addition, the second paper touches on the possible implications of another labour market institution, the minimum wage, for the optimal contracting between firms and workers. The paper will describe an intuitive mechanism whereby the firm allows the worker to extract rents from future possible employers, and in exchange the worker accepts a lower wage from the firm. Being binding, the existence of minimum wages can work as an impediment for the wage to decrease as it would be needed for the mechanism to be implemented. There are, of course, other reasons why the wage could not decrease sufficiently, like imperfections in the financial market which negate the worker the possibility of borrowing against a future higher income, which they would need if the determined wage falls below a subsistence level.

The first two papers ask how can agents better use the possibility of future alternative productive relationships for their own benefit. If third agents will be met in the future, and turnover may occur, Can and should a contract be designed to maximise the joint value of any given relationship, using this future turnover to their own benefit? Many papers study search environments with on-the-job search, but the issue of agents contracting to exploit this for their own benefit has been somewhat side-stepped, with very few examples (notably Diamond and Maskin, 1979). This point will be addressed after briefly introducing the papers.

The first paper looks at the problem in a partnership/matching framework with heterogenous agents. In the tradition of the matching literature, the rent-sharing is determined by bargaining. In the absence of search frictions, agents would match immediately in such a way that the match configuration is stable. With frictions but without on-the-job search, third agents are never met, so turnover as above would never occur. But in this last case, since entering a partnership denies future possibilities, agents may be selective and accept partnerships only with agents that are attractive enough. Hence, agents behave cooperatively and aim to maximise the joint value of their contract. Further, they use their contract to obtain full bargaining power vis-a-vis third agents. This is done by contractually committing the partner that contacts a third agent to make a take it or leave it offer. This allows them to extract all rents from third agents if and when turnovers occur. Hence, as single agents search for potential partners, their best option is to find other single agent, with whom utility is shared and with whom a contract can be signed to extract all rents from future turnovers. In fact, if single agents do meet an agent who is already in contract with another, they will not receive any surplus if turnover occurs, since this other agent holds all the bargaining power. The paper analyses, in addition, the implications for the "selectiveness" of the agents, (i.e. Will they reject unattractive partnerships?) and for the welfare of different types of agents (which can also be interpreted as the returns to quality).

The second paper focuses not on partnership formation, but on a labour market characterised by search frictions, homogeneous workers and heterogenous firms (high and low productivity). It shifts from the matching tradition to the search tradition in two respects: firms are able to absorb as many workers as they contact (and who accept their offer); and the rent-sharing is not decided by bargaining. Rather, employers post wages before any contact occurs. I look at two environments already existing in the literature, and I argue that in neither of those do firms fully exploit their contractual possibilities to maximise their profits. Then I propose a contract that forward looking firms would offer the workers in order to maximise their own profits, contract which exploits the fact that the workers will meet better firms in the future. The two models I look at are the models presented in Burdett-Mortensen (1998) and in Postel-Vinay (2002a and 2002b). In Burdett-Mortensen (1998), firms post a wage offered to (unidentified) workers, and are constrained to paying this wage to the workers they hire for as long as the relation remains. Firms never react to outside offers received by their workers; offers which they find through search on the job. In Postel-Vinay, the set-up is extended by allowing firms to identify the employment status of a worker upon contact, and to react to outside offers received by their workers. This is an important innovation since now firms enjoy a wider set of strategies: first, they can recognise unemployed workers in order to hire them by offering only their reservation value[1]; second, they may try to retain their workers when these are tempted by better employment elsewhere. As a consequence, firms compete (offering wages) a la Bertrand for the worker; and an employed worker who meets another firm will continue employment in the firm where it is more productive, but at a wage equal to its productivity in the less productive firm (since the latter stops competing when the wage reaches the worker's productivity). In this set up, although firms react optimally when their workers meet other firms, they do not fully exploit the opportunities given by worker's search on the job. I argue that forward looking firms would

[1] The same would be achieved by assuming that firms and workers bargain upon contact, but that firms hold all the bargaining power.

offer contracts which allow them to extract benefits from firms their workers may meet in the future. They can do so by committing to bid up to the worker's productivity in the most productive of the two firms. Hence, when employed workers meet more productive firms, their new wage will be equal to its productivity in the high productive firm, not the low productive firm. Ceteris paribus, this is a more attractive deal for unemployed workers, so firms that offer this contract can give them their reservation value by offering lower wages compared to the Postel-Vinay set up, therefore making higher profits per worker.

Other contributions to the literature address equilibrium behaviour in the presence of on-the-job search, but do not address the issues considered here. Burdett and Coles (2000) study a matching market with on-the-job search but without transferable utility, and hence there is no cooperation to maximise joint value of matches using contracts. Hence, turnover will occur whenever it is *individually* rational for the partner involved. They focus on trust issues in the presence of on-the-job search, so agents decisions to match and/or accept other proposals while matched depends on their belief about their partner's behaviour. Webb (1998) presents a matching model where workers and firms both search on the job for better employment relationships, but in her set up utility is not transferable either; her focus is on the equilibrium quit rates and lay-off rates generated by on-the-job search. Burdett and Mortensen (1998) was already introduced above. Given the restrictions imposed on firms strategies, and the lack of bargaining, contracting to exploit the possibilities that can arise through on-the-job-search are not considered. Their focus is on the consequences for the equilibrium wage distribution, and for the relationship between wage paid and firm's size [2]. Postel-Vinay and Robin (2002a, 2002b) were also introduced above[3]. Pissarides (1994) studies a matching model with firms and workers where workers search on the job and utility is transferable, but he rules out the study of long-term contracts. As a result, there is what he calls "continuous renegotiation". The consequence is that an employed worker's only credible threat is unemployment. Hence, employed status does not affect the bargaining position with firms met through search on the job.

Diamond and Maskin (1979) study an economy with on-the-job search and transferable utility. They do allow matched agents to contract strategically in order to gain full bargaining power vis-a-vis other agents they can meet. This model differs from Chapter I in this document in that they consider homogeneous workers, so search on the job is justified by match heterogeneity. When two agents realise a poor match, it may be in their (joint) interest to keep searching for better matches. Hence, Diamond and Maskin (1979) cannot look at the consequences for turnover behaviour and welfare in the presence of heterogenous

[2] They find that the outcome is one of wage dispersion where firms loose their workers when the latter find higher paying jobs. This outcome results from two opposing effects: higher wages imply lower profits per worker, but also a higher ability to retain existing workers and hire new ones; thus higher supply of workers in steady state. There exists therefore a set of wages, -the support of the wage distribution-, that imply equal and maximum profits for firms. These wages balance the loss in profits per worker and the inscrease in total profits given by the increase in workers supply when higher wages are offered.

[3] In their set up, a policy of matching outside offers allows the firm to retain their workers so long as they do not have to offer a wage higher than the workers' productivity. But matching firms run into a moral hazard problem, since their wage matching policy encourages search by their workers, increasing therefore the probability of loosing them to other firms.

agents.

The third paper leaves aside the above issue, but is still concerned about contracting in the presence of on-the-job search. It also presents a labour market with search frictions, homogenous workers and heterogeneous firms. Nevertheless, it returns to the matching tradition: wages are determined by bargaining, and firms can absorb at most one worker. I introduce search costs, so workers face the decision to search on the job or not. I also make the (natural) assumption that worker's search behaviour is not contractible. The paper centers on the consequences of search costs for the search behaviour of workers, in particular employed workers; and for the wage determination process. Because firms are heterogeneous, workers in bad firms may want to search on the job for better firms. The fact that this is not a contractible decision, but one taken individually by the worker given the wage agreed, is what is at the heart of the model. Another issue is that of the private efficiency of the equilibrium contracts in the presence of this imperfection.

Chapter 1

Contracts and on-the-job search I

1.1 Introduction

The present paper addresses a partnership model in which agents must search for other agents and match in pairs in order to engage in production. The focus will be on the optimal behavior of single and matched agents and the welfare properties given three particular conditions: (a) both agents in a partnership keep contacting other agents even while their partnership or match is in place, (b) utility between any two partnered or matched agents is perfectly transferable through bargaining; and (c) agents enjoy total freedom of contract and are able to fully commit contractually to remain within the partnership.

An interesting point in the presence of condition (a) is that it allows for agents to leave existing partnerships in order to form new ones with agents they may meet; they may want to "divorce" their current partner. Hence, matched agents may set up rules to govern their decisions to "divorce" or separate, rules which will be determined by the economic environment they face. In this paper, conditions (b) and (c) above are the key features that describe the relevant economic environment; and their coexistence is what drives the results of the model.

Condition (c) deserves some words, since it is particular to the analysis of partnerships as opposed to relationships of the "firm-worker" type, more prevalent in the literature. In typical labour markets, it is not possible for workers to commit themselves to remain in a contractual relationship; on the basis that such a commitment is illegal. The worker is always free to leave the firm without any liability (maybe upon the previous placement of a notice). Such a contractual restriction does not exist in a partnership relationship, and it is this in particular what I mean by complete freedom of contract.

One main concern of the paper is to determine which is the best contract two agents can design in this environment. The answer to that question is very intuitive. Imagine and economy populated by teams and players, where teams with better resources (better teams) are more able to exploit players' qualities. Consider a player and his team as the two agents in a partnership. They know that in the future there will be other (better) teams interested in "buying" the player. Can they use their own contract to extract benefits from these other teams, provided that trade occurs, i.e. the player changes teams? If this is the case, will they agree to accept the player changing teams? The answer to the first question is a definite yes. They could write a contract such that the player is legally required to make a take it or leave it offer to third teams as a condition for changing teams, which allows him to extract all the rents from other teams. In a way, the original partnership are using their own contract as a way of gaining full bargaining power vis-a-vis other teams. Turn now to the second question, will they agree to accept the player changing teams? Notice that because utility is transferable and agents bargain over their contract, partnerships behave cooperatively and worry about *joint* utility. When the player changes teams, the original team would suffer because it is loosing a player; the player, on the other hand, would win by enjoying all the benefits from the new partnership, including the extracted rents. Is the loss of the team higher or lower than the gain of the player? If it is higher, then there is a net loss in utility by letting the player go, so the team-player partnership should not accept a move. If it is lower, then there is an overall net gain by letting the player go. Then they should accept the player moving to the other team.

An interesting point is that, provided the player is expected to change teams in the future, the value of any partnership includes not only the value created as long as it remains, but also the value created in new partnerships when the player changes teams.

An important angle of this analysis is to highlight that, in typical labour markets, where workers cannot be legally tied to the current employer as in the previous example, the law may be working against the workers interests rather than in their favour. As will be seen, the degree to which this happens will depend on the relative qualities of the agents involved.

The analysis will be carried out in what is referred to as a "partnership" model, so the economy will be populated by agents of the same kind -for example lawyers setting up a joint office- although they will be of either of two types: high and low productivity.

I will claim that pairs of agents will aim to maximise the value of their relationship through the design of what I call a separation policy; i.e. a policy that states when to separate and under which conditions. In equilibrium, contracts are designed such that separations occur if and only if they yield a positive total surplus; and the existing match will keep all this surplus by making non-negotiable proposals to other agents. To simplify the analysis, assumptions will be made such that meetings between two matched agents are ruled out, avoiding the potential complications of two agents both making non-negotiable proposals.

The other question asked in the paper regards the welfare consequences of such contracts, compared to the case where, in the above example, the player cannot be tied to his current team in any way. Hence, he cannot be legally required to make a take it or leave it offer as a condition to change teams. The answer is that the returns to quality are diminished. The third team will never obtain any surplus from "buying" the player, since any ex-ante surplus will be appropriated by the player through the take it or leave it offer. Hence, even though players are more valuable with them, better teams do not benefit from there higher quality as they could when they buy players, they just see a higher ex-ante surplus being extracted from them. Hence, an important conclusion from the paper will be that, in an environment of heterogenous agents, such freedom of contract benefits less productive agents in detriment of more productive agents. This conclusion is important since it has implications for the agents's incentives to invest in quality.

To stress these point, I analyse at the end of the paper a numerical example with a slightly different set-up. In it, I consider a legislation with restricted freedom of contract. As a consequence, agents in a match are not able to make non-negotiable proposals to single agents, and must therefore bargain with them. So now single agents may keep part of the gains of trade when turnover occurs. This exercise, addressed in section 1.8, allows not only to compare the effects on equilibrium strategies but also to compare the asymmetric welfare effects across types of agents.

As already said above, the paper assumes that agents differ in productivity levels. In the language used in the literature, there is ex-ante heterogeneity. An important dimension of the problem in the presence of ex-ante heterogeneity is the possibility of elitist behavior: if being in a partnership or match implies a decrease in the efficiency with which new agents are met, partnerships with unattractive agents may be turned down in favor of continued search for attractive ones. A characteristic of the model presented here is that, even though search efficiency is lower for matched agents compared to single agents, such a rejection never occurs. In *all* equilibria in the model presented, agents match *always* if they contact each other while single, regardless of their types. I call this non-elitist matching. With the search technology assumed, the ability to extract all surplus from third agents allows to compensate for the loss in arrival rate when forming a match[1,2].

Section 1.2 presents the framework, section 1.3 addresses in detail the contracts designed by the agents, section 1.4 addresses derives the values of the contracts signed, section 1.5 looks at the sets of strategies available to agents given the joint-value maximising contracts, section 1.6 deals with the steady state equilibria, section 1.7 analyses the sorting effects of equilibrium matching and turnover behaviour, section 1.8 addresses the externalities and the efficiency of the equilibria. Finally, section 1.9 looks at the effects of the freedom of contract on equilibrium (matching and turnover) strategies, as well as on the welfare of the different types of agents. Section 1.10 concludes. The appendix contains proofs not included in the main body of the paper

1.2 The Framework

Time is continuous and there is a fixed number of agents normalized to 1. Agents have a discount rate $r > 0$, are infinitely lived and are expected utility maximiz-

[1] This result is particular to the search technology used, and has to do with the relationship between the loss in search efficiency and the bargaining power of single agents vis-a-vis single agents.

[2] Several related models have equilibria with elitist matching. Smith and Shimer (2000) use a continuum of agents and find existence of assortative matching under special conditions of the production function. Their framework has transferable utility but not on-the-job-search. Burdett and Coles (2000) describe a partnership model with ex-ante heterogeneity and find existence of equilibria with elitist matching; it allows for on-the-job search but not for transferable utility. Burdett and Coles (1998) uses a continuum of types and find conditions for the existence of a sorting equilibrium, but it does not allow for transferable utility and agents only contact others while single. The key is that in neither of those papers are matched agents able to keep all the surplus from a separation, and this is what drives my result.

ers. All agents are risk neutral. The population is composed of two types of agents, a proportion γ are Hs and the remaining $1-\gamma$ are Ls. If two Hs are matched, they produce $2Q_H$ per instant; two Ls produce $2Q_L$, and an H and an L produce $2Q_M$. We will be interested in the case $Q_H > Q_M > Q_L > 0$. Agents produce 0 flow output while single, and therefore their objective is to match in order to produce output. At any moment in time, any agent can be in one of three states denoted by $s:$ matched to an H $(s=h)$, matched to an L $(s=l)$, or single $(s=u)$. Let N_{is} denote the number of agents of type i in state s, where $i \in \{H, L\}$. Then adding up implies:

$$\begin{aligned} N_{Hh} + N_{Hl} + N_{Hu} &= \gamma \\ N_{Lh} + N_{Ll} + N_{Lu} &= 1-\gamma \end{aligned}$$

Agents meet other agents both while matched and while single, in a way specified below. When two single agents meet, they see each others' type and state, and decide wether to form a match or not. If two single agents decide to match they engage in a bargaining process to split the match surplus. Single agents vis-a-vis single agents will be assumed to reach the symmetric Nash bargaining solution, where each keeps $\frac{1}{2}$ of the match surplus. Without any loss of generality, I will assume that each agent in the match enjoys half of the flow product of the match, which is not transferable. The outcome of the bargaining process will therefore be settled through lump-sum transfers, paid at the start of the relationship, and assuming that each agent will enjoy one half of the flow product during the existence of the match. This assumption is made to ease exposition.

Only steady state situations will be considered, where individuals know the constant distribution of agents in the market as well as the rate at which they meet others. The steady state is sustained by assuming that matches break at an exogenous rate ∂, and the two agents return to single search when their match breaks up in this way[3]. To focus on pure strategies, I will assume a tie breaking rule, whereby a match will not separate if the agents in the match are indifferent between separation or not. Further, if two single agents are indifferent between matching or not, I will assume they form a match. I will call U_{is} the present lifetime expected discounted value (or discounted value) of an agent of type i in state s.

Finally, I will assume that agents are able to write binding contracts, without legal restrictions. Anything that is technologically possible can be included in a contract between two agents. In other words, agents enjoy complete freedom of contract. It is important to be aware that, at the moment of signing a contract, it is signed in terms of discounted values, which may not be realised in the future. Therefore, when contracts are signed, what agents are maximising is their *expected* utility. Contracts are re-negotiable only if there is mutual

[3]As in Shimer and Smith (2000). Other ways to sustain a steady state are used in the literature. Burdett and Coles (1998) assume an exogenous inflow of agents into the market, while those who form matches (all stable without on-the-job search) leave the market for good. Other studies assume that agents leave the market when they form stable matches and are replaced by identical agents or clones (Webb, 2000).

agreement to do so.

The search technology To focus on essentials, a search technology with the two following properties will be used:

(i) Matched agents will not be allowed to meet matched agents

(ii)The rate at which matched agents meet single agents is lower than the rate at which single agents meet single agents.

Denote $\alpha_{iu}(\beta_{iu})$ the rate at which single (matched) agents meet agents of type i in state u. An example of a search technology with such properties is the following: Only when *single*, agents are able to make phone calls to other agents at rate ε, but *all* agents may receive a phone call, regardless of their state. If S denotes the total number of single agents, εS is the total number of phone calls made. Everybody receives a call at rate $\frac{\varepsilon S}{1} = \varepsilon S$ since the total number of agents is normalized to 1. Hence, single agents can contact other agents by making a call or by receiving one. If they make a call, it is received by a matched agent with probability $(1 - S)$ and by a single agent with probability S. If they receive a call, it can only come from a single agent. This implies that the rate at which single agents meet new agents (in any state) is $\varepsilon(1 - S) + \varepsilon S + \varepsilon S = \varepsilon + \varepsilon S$. This total rate can be broken down into $\varepsilon(1 - S)$, the rate at which a singles meets matched agents; and $2\varepsilon S$, the rate at which singles meet single agents. Matched agents can only receive phone calls from single agents, at rate εS. Hence, matched agents contact single agents at rate εS and contact matched agents at rate 0. Phone calls are random, and this implies that given a contact with a single agent, the probability that it is of type i is $\frac{N_{iu}}{S}$. Given a matched agent is contacted, the probability it is of type i in state $s = h, l$ is $\frac{Nis}{1-S}$. This allows to define the following contact rates:

(a) $2\varepsilon S\frac{N_{iu}}{S} = 2\varepsilon N_{iu}$ is the rate at which singles meet single agents of type i.

(b) $\varepsilon S\frac{N_{iu}}{S} = \varepsilon N_{iu}$ is the rate at which matched agents meet single agents of type i.

(c) $\varepsilon(1-S)\frac{Nis}{1-S} = \varepsilon N_{is}$ is the rate at which single agents meet agents of type i in state $s = h, l$.

Summarising:

$$\alpha_{iu} = 2\varepsilon S\frac{N_{iu}}{S} = 2\varepsilon N_{iu} \qquad \beta_{iu} = \varepsilon S\frac{N_{iu}}{S} = \varepsilon N_{iu}$$

$$\alpha_{is|_{s\neq u}} = \varepsilon(1-S)\frac{N_{iu}}{(1-s)} = \varepsilon N_{_{is|_{s\neq u}}} \quad \beta_{is|_{s\neq u}} = 0$$

Hence, meetings between matched agents are ruled out; and matched agents meet single agents at half the rate single agents meet single agents[4]. The results obtained in the paper are particular to these properties.

1.3 Contracts

A contract is the set of rules that govern the behaviour of the agents within the match. In what follows, the two words match and contract will be used interchangeably referring to the economic unit that two partnered agents form. Given the search technology, there can be contacts between two matched agents

[4]Single agents also meet matched agents, so their unconditional arrival rate is more than twice that of matched agents.

and between a single and a matched agent. Given the different states of the agents involved in each case, the decision problems differ.

Consider first the case where two single agents meet. They must decide whether to form a match or continue searching as single agents. Because utility is transferable, they will behave cooperatively and match if and only if the total surplus generated is positive. This total surplus is given by their joint discounted value should they match, less of their values value in single search (their outside option). For two single agents of types $i \in (H, L)$ and $j \in (H, L)$, their match surplus is given by $U_{ji} + U_{ij} - U_{ju} - U_{iu}$. Because only the joint discounted value matters for this decision, the outcome of the bargaining process between two single agents is irrelevant, since lump-sum transfers only transfer value within the match and do not affect the joint discounted value. I will therefore ignore them in what follows.

In order to make the decision, two single agents must consider the maximum joint discounted value in their potential match. What follows addresses this maximisation strategy. As will be seen, this strategy determines the contract signed when a single agent and a matched agent meet.

Definition 1.1 *Two agents in any given match will be said to separate when either of them contacts a third agent, and leaves her current partner in order to form a new match with the agent just contacted.*

Definition 1.2 *Assume $i \in (H, L)$, $j \in (H, L), k \in (H, L)$. S_{ij}^k is the total surplus of a separation when a type j in an ij match contacts a single type k agent and a separation occurs, creating a $jk = kj$ match, destroying the ij match* *and trading type i for type k into singleness. It is the net of the discounted values created and destroyed in the process, given by* $S_{ij}^k = (U_{jk} + U_{kj}) - (U_{ij} + U_{ji}) + (U_{iu} - U_{ku})$

The model has only two types of agents; so there are only three types of matches $-HH, HL = LH$ and $LL-$. Each agent in a match has the possibility of meeting agents of any of the two types. I will address the decision problem faced by a single H and a single L, and then easily generalise the results to the case where both agents are of type H and the case where both agents are of type L.

The joint discounted value of agents in an HL match (and any match) depends on four things: (i) The flow product they generate during the match (ii) The exogenous destruction rate δ (iii) Their choice of action if either of them contacts another agent and, (iv) The rate at which other agents are contacted. The agents have no discretion on $(i), (ii)$ and (iv), but they have discretion on (iii). Whenever either of them meets a third agent, there are two possible outcomes: either the match separates and a new match is formed, or it does not separate and the third agent is rejected. They must decide, for each possible future contact with a third agent, wether they want a separation to occur and under which conditions; *i.e.* they must design a separation policy.

One must keep in mind that this separation policy is designed at the moment of signing their own contract, and therefore before any possible meeting with a third agent occurs. Because the separation policy is the only choice in their discretion that affects the joint discounted value, it constitutes the contract between the two agents. This policy will depend on the total surplus of a separation as in Definition 1.2.

Remark 1.1 *The following contract maximises the joint discounted value of two agents in an HL match: A vector $X = \left[X^H_{HL}, X^L_{HL}, X^H_{LH}, X^L_{LH}\right]$, where each entry X^k_{ij} is the value of a lump-sum transfer that type j must charge a single type k in order to separate the existing ij match and form a new kj match. $X^k_{ij} = 0$ denotes that the ij match will not accept a separation to occur. X^k_{ij} are not negotiable.*

$$\begin{aligned} X^H_{LH} &= U_{Hh} - U_{Hu} \text{ if } S^H_{LH} > 0, X^H_{LH} = 0 \text{ if } S^H_{LH} \leq 0 \\ X^L_{HL} &= U_{Ll} - U_{Lu} \text{ if } S^L_{HL} > 0, X^L_{HL} = 0 \text{ if } S^L_{HL} \leq 0 \\ X^L_{LH} &= 0, X^H_{HL} = 0 \end{aligned}$$

The above contract characterises the separation policy that maximises the joint discounted value for agents in an HL match. It says that a separation should be implemented upon the meeting of a third agent if the corresponding total surplus is positive ($S^k_{ij} > 0$). If so, the third agent must pay a lump-sum transfer equal to the discounted value of the surplus she will enjoy if the separation occurs and the new match is formed. This means that the third agent enjoys 0 surplus on discounted terms at the moment of the separation, and the surplus accrues fully to the agents in the existing match. If the separation yields negative total surplus ($S^k_{ij} \leq 0$), the agent contacted must be rejected and the separation must not occur.

Consider first the case where the HL match contemplate the possibility that the H contacts another H: (i) They can use their contract to ex-ante govern their behaviour, foreseeing that the H will in the future meet a single H. They can commit contractually not to enter negotiations with other agents, and this allows the H in the HL match to charge a non-negotiable lump-sum transfer in order to form a new match with the single H. The single H must either accept to form the new match under those conditions, or reject it, but cannot start a bargaining process[5].(ii) If the agents in the existing HL match do not want a separation to occur, they should only reject any proposal made by the single H. (iii) If they do want the separation to occur, all that is needed is for type H to charge a lump sum transfer to the single H as in Remark 1.1. The single H will be indifferent to accepting or rejecting the proposal, so she will accept it and the separation will occur, forming a new HH match and destroying the HL match.

If the HL match will allow the separation to occur, the best that they can do is to make to the single H a proposal as described in (iii). Such proposal would allow them to jointly keep the whole surplus generated by the separation. This is because they are directly suffering the change in discounted values because of their change in state, and they are *jointly* appropriating the surplus of the single H, by means of the lump sum transfer she pays. The surplus accruing to the agents (jointly) in the HL match if the separation occurs is given by

[5]To make this commitment strategy work, they must set high enough punishments so that the matched agent involved in the contact never wants to breach the non-bargaining commitment.

$$\begin{aligned} & (U_{Lu} - U_{Lh}) + (U_{Hh} - U_{Hl}) + X_{LH}^{H} \\ = \quad & (U_{Lu} - U_{Lh}) + (U_{Hh} - U_{Hl}) + (U_{Hh} - U_{Hl}) = S_{LH}^{H} \end{aligned}$$

If a higher lump sum transfer is charged, the single H would reject the proposal and the (profitable) separation would not occur. If a lower lump sum transfer is charged, this would allow the single H to keep part of the surplus generated by the separation, therefore reducing the surplus to the agents in the HL match.

This reduces the choice of a separation policy regarding this particular contact to a binary choice, limited to (a) Rejecting a separation, which implies a surplus equal to 0, or (b) Implementing the separation by making a proposal that leaves the single H indifferent, and allows them to keep the whole surplus S_{LH}^{H}. They will implement a separation if $S_{LH}^{H} > 0$.

The previous reasoning argued that the choices are either to reject the corresponding separation, or to charge the single H a lump sum transfer of $X_{LH}^{H} = (U_{Hu} - U_{Hu})$ and implement the separation. In S_{LH}^{H} above, the first term is the utility loss imposed by the separation on the agent that would be left single, L. The last two brackets would be the utility change imposed on H, who would be part of the new HH match and would receive the lump sum transfer X_{LH}^{H}.

If $S_{LH}^{H} > 0$, the H expects to receive the lump sum transfer X_{LH}^{H} upon a separation, which increases her discounted value. But this is only as a consequence of being part of the HL contract, and this should be considered when the bargaining originally takes place to form the HL match. Such a boost in utility that results from the contract would then be recognised, in discounted value, in the lump-sum transfer that results from the bargaining process.

If $S^H_{LH} \leq 0$, it must be the case that the loss that would be imposed on the L (because she is left single) is higher than the utility gain that the H would enjoy (because she will move to the new HH match and receive the lump sum transfer). Hence, the H would like to implement a separation, because it implies a positive surplus for her; and the L would prefer not to implement a separation. The L can transfer value to the H to convince her not to implement the separation when the opportunity arises. This should be reflected in the lump-sum transfer that results form their original bargaining process, given that the H gives up a potential surplus when the opportunity comes. The H would then commit contractually not to separate in the future when the single H is met.

Of course if a single H is actually met in the future, the H in the HL match would like to separate, but she cannot since she is contractually bound, and contracts can only be renegotiated upon mutual agreement. The maximisation is therefore done in expected discounted terms at the moment the original HL match starts.

Consider now the new HH match that would arise from the separation just described. It is not true that the two agents in this new match aim to maximise their joint discounted value, since the H that comes from singleness is not playing any active role. She is just accepting a proposal to form a new match because she is being left indifferent. But the agent (H) that comes from the HL match has an incentive to maximise the joint discounted value in the new HH match, since this allows her to maximise her own surplus. Hence, the contract governing this new match is also written to maximise the joint discounted value; with only one agent deciding on the separation policy and the other playing a passive role.

An adaptation of the above completes the analysis of Remark 1.1. Consider the possibility that the L in the HL match contacts a single L. The separation will be implemented if the total surplus $S_{HL}^{L} = (U_{Hu} - U_{Hl}) + (U_{Ll} - U_{Lh}) + (U_{Ll} - U_{Lu}) > 0$, and in that case she will make the single L a proposal to form a new LL upon the payment of a lump sum transfer with value $X_{HL}^{L} = U_{Ll} - U_{Lu}$. If $S_{HL}^{L} \leq 0$, then the best is to reject the single L, case in which $X_{LL} = 0$.

Consider now the possibility that the H meets a single L. Because the matches that would be created and destroyed by a separation are of the same type, and the agents being traded into singleness are also of the same type, the surplus generated is $S_{LH}^{L} = 0$. Therefore, the separation is not implemented, $X_{HL} = 0$ and the single L is rejected. For an analogue reason, if the L in the HL match meets an H, the surplus generated $S_{HL}^{H} = 0$. This implies that $X_{LH} = 0$ and the H is rejected.

Remark 1.2 *The following contract maximises the joint discounted value of two agents in an HH match: A vector $Y = \left[Y_{HH}^{L}, Y_{HH}^{H}\right]$, where each entry Y_{ij}^{k} is the value of a lump sum transfer that type j must charge a single type k in order to separate the existing ij match and form a new kj match. $Y_{ij}^{k} = 0$ denotes that the ij match will not accept a separation to occur. Y_{ij}^{k} are not negotiable.*

$$
\begin{aligned}
Y^H_{HH} &= 0 \\
Y^L_{HH} &= U_{Lh} - U_{Lu} \text{ if } S^L_{HH} > 0, Y^L_{HH} \text{ if } S^L_{HH} \leq 0
\end{aligned}
$$

Remark 1.3 *The following contract maximises the joint discounted value of two agents in an LL match: A vector* $Z = \left[Z^H_{LL}, Z^L_{LL}\right]$*, where each entry* Z^k_{ij} *is the value of a lump-sum transfer that type* j *must charge a single type* k *in order to separate the existing* ij *match and form a new* kj *match.* $Z^k_{ij} = 0$ *denotes that the* ij *match will not accept a separation to occur.* Z^k_{ij} *are not negotiable.*

$$
\begin{aligned}
Z^L_{LL} &= 0 \\
Z^H_{LL} &= U_{Hl} - U_{Hu} \textit{ if } S^H_{LL} > 0, Z^L_{LL} = 0 \textit{ if } S^H_{LL} \leq 0
\end{aligned}
$$

The reasoning for Remarks 1.2 and 1.3 are just an adaptation of that for Remark 1.1. The only difference is that with two agents of the same type, only two types of contacts must be considered.

When two agents form a match expected to separate in the future, their joint discounted value includes not only the discounted value of the total product generated while the match lasts, but also the discounted value in the match formed after the separation (net of the continuation values), since they jointly keep the total surplus of any future separation.

One way of interpreting this is that forming a match involves two benefits compared to single search: (*i*) A flow product is generated at each instant while the match exists (*ii*) It gives complete bargaining power vis-a-vis single agents[6]. On the other hand, the (potential) cost of matching is the loss in search efficiency as described in the previous section. This loss has two components: (*iii*) The halving of the rate at which single agents are contacted and (*iv*) The complete loss of the rate at which matched agents are contacted. Notice that (*iv*) is irrelevant; the expectation of contacting matched agents while single does not increase single agents' discounted value, since any surplus is lost by single agents after paying the lump-sum transfer to the matched agent. Loosing this arrival rate is therefore not a cost. Hence, the contractual freedom enjoyed by all agents increases the benefits of forming a match because of (*ii*), and it also decreases the cost of doing it because of (*iv*). This increased incentive to form a match will be key in understanding the nature of the equilibria described in section V.

1.4 Discounted Values of Contracts

The previous section made a strong case that what matters for the matching and turnover decisions is:

(*i*) The joint discounted value of a contract

(*ii*)The sign of the surplus of a separation.

There are four possible separations. It will prove useful later to spell out the surplus created by each of them in terms of all U_{ts}. The first two are when an

[6]The bargaining power of *singles* vis-a-vis *singles* is just $\frac{1}{2}$

H leaves and L for another H (S^H_{LH}), which is the opposite to when an H leaves an H for an L (S^L_{HH}).

$$S^H_{LH} = -S^L_{HH} = 2U_{Hh} - U_{Hl} - U_{Lh} + U_{Lu} - U_{Hu}$$

When an H leaves his contract with an L to form an HH match, the value of the new contract is $2U_{Hh}$. The value of the old contract, $U_{Hl} + U_{Lh}$, is lost. Further, an H that was single move into the new HH contract, so U_{Hu} is lost. The L in the old HL contract moves into singleness, so U_{Lu} is created.

The other two are when an L leaves an H for another L (S^L_{HL}), which is the opposite to when an L leaves an L for an H (S^H_{LL}) :

$$S^L_{HL} = -S^H_{LL} = 2U_{Ll} - U_{Hl} - U_{Lh} + U_{Hu} - U_{Lu}$$

The interpretation of the above surplus is analogue to that of $S^H_{LH} = -S^L_{HH}$.

This section makes full use of the above in order to describe the joint discounted value of contracts. From here on, I will ignore the cases that occur with probability 0, i.e. cases in which $S^k_{ij} = 0$

I will start with an HH contract. This type of contracts can be stable (it does not allow for any separation) or separate when either agent meets a single L, which I call unstable-l. Hence, the joint discounted value is given by:

$$r2U_{Hh} = 2Q_H + 2\alpha_{Lu}\left[s^L_{HH}(S^L_{HH})\right] + \delta(2U_{Hu} - 2U_{Hh}) \qquad (1.1)$$

where

$$\begin{aligned} S^L_{HH} &= U_{Hl} + U_{Lh} - 2U_{Hh} + U_{Hu} - U_{Lu} \\ s^L_{HH} &= 1 \text{ if } S^L_{HH} > 0 \\ s^L_{HH} &= 0 \text{ if } S^L_{HH} < 0 \end{aligned}$$

There are two agents in the contract, and the total production is $2Q_H$. The joint rate at which an L will be contacted by either of the agents in the contract is $2\alpha_{Lu}$. The contact will lead to a separation if the corresponding total surplus (S^L_{HH}) is positive, case in which $s^L_{HH} = 1$, and the agents in the original HH match keep all this surplus to themselves. The possible types of an HH contract are:

$$\begin{aligned} &\text{If } s^L_{HH} = 0: \quad \text{stable contract, and } Y^L_{HH} = 0 \\ &\text{If } s^L_{HH} = 1: \quad \textit{unstable} - l \text{ contract, and } Y^L_{HH} = U_{Lh} - U_{Lu} \end{aligned}$$

Regarding LL contracts, they can either be stable or separate when either agent meets an single H, which I call unstable-h. The joint lifetime discounted value is given by

$$r2U_{Ll} = 2Q_L + 2\alpha_{H,u}\left[s^H_{LL}(S^H_{LL})\right] + \delta(2U_{Lu} - 2U_{Ll}) \tag{1.2}$$

where

$$\begin{aligned} S^H_{LL} &= U_{Hl} + U_{Lh} - 2U_{Ll} + U_{Lu} - U_{Hu} \\ s^H_{LL} &= 1 \text{ if } S^H_{LL} > 0 \\ s^H_{LL} &= 0 \text{ if } S^H_{LL} < 0 \end{aligned}$$

The joint rate at which an H will be contacted by either of the agents in the contract is $2\alpha_{H,u}$. The contact will lead to a separation if the corresponding total surplus is positive, $S^L_{HH} > 0$, case in which $s^L_{HH} = 1$, and the agents in the original LL match keep all this surplus to themselves. The possible types of an LL contract are:

$$\begin{aligned} &\text{If } s^H_{LL} = 0: && \text{stable contract, and } Z^H_{LL} = 0 \\ &\text{If } s^H_{LL} = 1: && \textit{unstable} - h \text{ contract, and } Z^H_{LL} = U_{Hl} - U_{Hu} \end{aligned}$$

The case for HL contracts is a bit more lengthy, since these contracts can either be stable, break only when the H meets an H (unstable-h), break only when the L meets an L (unstable-l), or break upon either of the previous cases (unstable-hl). The joint lifetime discounted value is given by

$$r(U_{Hl}+U_{Lh}) = 2Q_M+\alpha_{Lu}\left[s^L_{HL}(S^L_{HL})\right]+\alpha_{Hu}\left[s^H_{LH}(S^H_{LH})\right] \qquad (1.3)$$
$$+\delta(U_{Hu}+U_{Lu}-[U_{Hl}+U_{Lh}])$$

$$S^L_{HL}=-S^H_{LL} \qquad S^H_{LH}=-S^L_{HH}$$
$$s^L_{HL}=1 \text{ if } S^L_{HL}>0 \quad s^H_{LH}=1 \text{ if } S^H_{HL}>0$$
$$s^L_{HL}=0 \text{ if } S^L_{HL}\leq 0 \quad s^H_{LH}=0 \text{ if } S^H_{HL}\leq 0$$

At a rate $\alpha_{L,u}$, the L will contact a single L . A separation will occur if the corresponding surplus S^L_{HL} is positive, case in which $s^L_{HL}=1$, and the agents in the original HL contract keep all this surplus to themselves. At a rate $\alpha_{H,u}$, the H will contact a single H type. A separation will occur if the corresponding surplus S^H_{HL} is positive, case in which $s^H_{LH}=1$, and the agents in the original HL match keep all this surplus to themselves. The possible types of an HL contract are:

If $s^L_{HL}=1, s^H_{LH}=0:$ unstable-l contract, and $X^L_{HL}=U_{Ll}-U_{Lu}, X^H_{LH}=0$

If $s^L_{HL}=0, s^H_{LH}=1:$ unstable-h contract, and $X^L_{HL}=0, X^H_{LH}=U_{Hh}-U_{Hu}$

If $s^L_{HL}=1, s^H_{LH}=1:$ unstable-hl contract, and $X^L_{HL}=U_{Ll}-U_{Lu}, X^H_{LH}=U_{Hh}-U_{Hu}$

If $s^L_{HL}=0, s^H_{LH}=0:$ stable contract, and $X^L_{HL}=0, X^H_{LH}=0$

Notice that if LL matches will separate when one of them meets an H, then HL matches will not separate when the L meets an L. Formally, $S^H_{LL}>0 \Leftrightarrow S^L_{HL}<0$. Hence, LL contracts are *unstable* $-h$ if and only if HL contracts are not $unstable - l$ nor $unstable - hl$. Likewise, if HH matches will not separate when one of them meets an L, then HL contracts will separate when the H meets an H. Formally, $S^L_{HH} < 0 \Leftrightarrow S^H_{LH} > 0$. Hence, HH contracts are stable if and only if HL contracts are $unstable - h$ or $unstable - hl$.

1.5 Strategies

The decision faced by two *single* agents is wether not to sign a contract, to sign an unstable contract or to sign a stable contract. Instead, *matched* agents do not have a strategy as such, since their behavior is given by the contracts they have previously signed when forming their current partnership. Further, the behaviour of single agents vis-a-vis matched agents is determined by the ability of matched agents to make non-negotiable proposals. Singles will accept to form a new match (which implies the existing match separates) if they receive a proposal from a matched agent which will extract all of its surplus.

Hence, two single Hs face a decision between not signing a contract, signing an $unstable - l$ contract or signing a stable contract. Two single Ls must decide between not signing a contract, signing a stable contract or signing an $unstable - h$ contract. A single H and a single L must decide between a not signing a contract, signing a stable contract, signing an unstable-h contract, signing an unstable-l contract, or signing an unstable-hl contract.

The following four Propositions address the decision on **the separation policy for existing matches**. Hence, it is not considered in them wether the matches should have been signed on the first place. This analysis is postponed to further sections. In these Propositions, when I say that contracts are of a given type, it must be understood that this type maximises the joint discounted value of the match, and is therefore the contract type signed.

Proposition 1.1 *Assume HL contracts are not unstable-l nor unstable-hl. Then HH contracts are stable (and HL contracts are unstable-h) if*

$$2(Q_H - 2Q_M) > r(U_{Hu} - U_{Lu})$$

HH contracts are unstable-l (and HL contracts are stable) otherwise.

We first enquire when are stable HH contracts preferred to $unstable-l$ HH contracts. The discounted value of stable HH contracts is represented by (1) with $s^L_{HH} = 0$. Solving, this is given by

$$2\frac{Q_H + \delta U_{Hu}}{r + \delta}$$

The discounted value of $unstable-l$ HH contracts is represented by (1) with $s^L_{HH} = 1$. Solving, this is given by

$$2\frac{Q_H + \alpha_{L,u}(U_{Hl} + U_{Lh} + U_{Hu} - U_{Lu}) + \delta U_{Hu}}{\delta + r + 2\alpha_{Lu}}$$

Stable HH contracts are signed if

$$2\frac{Q_H + \delta U_{Hu}}{r+\delta} > 2\frac{Q_H + \alpha_{Lu}(U_{Hl} + U_{Lh} + U_{Hu} - U_{Lu}) + \delta U_{Hu})}{\delta + r + 2\alpha_{Lu}}$$

which can be reduced to:

$$U_{Hl} + U_{Lh} + U_{Hu} - U_{Lu} < 2\frac{Q_H + \delta U_{Hu}}{r+\delta} \tag{1.4}$$

This is equivalent to the condition $S^L_{HH} < 0$ (or $S^H_{LH} > 0$) evaluated using the value of stable HH contracts, i.e. $U_{Hh} = 2\frac{Q_H+\delta U_{Hu}}{r+\delta}$. If the above condition holds, $S^L_{HH} < 0$ ($S^H_{LH} > 0$) and $s^L_{HH} = 0$ ($s^H_{LH} = 1$).

Because the Proposition assumes that HL contracts are not $unstable - l$ nor $unstable - hl$, we must use also $s^L_{HL} = 0$.This means that HL contracts are $unstable - h$, with value given by (3) with $s^H_{LH} = 1, s^L_{HL} = 0$

$$\begin{aligned} r(U_{Hl} + U_{Lh}) &= 2Q_M + \alpha_{Hu}(S^H_{LH}) + \delta(U_{Hu} + U_{Lu} - [U_{Hl} + U_{Lh}]) \\ U_{Hl} + U_{Lh} &= \frac{2Q_M + \alpha_{Hu}(2U_{Hh} + U_{Lu} - U_{Hu}) + \delta(U_{Hu} + U_{Lu})}{r + \delta + \alpha_{Hu}} \end{aligned}$$

Using $2U_{Hh} = 2\frac{Q_H+\delta U_{Hu}}{r+\delta}$ into $U_{Hl} + U_{Lh}$, and plugging the result in condition (4) above yields

$$2(Q_H - Q_M) > r(U_{Hu} - U_{Lu})$$

This condition is highly intuitive. When an HL contract is destroyed in favour of an HH contract, there is a gain in terms of flow product equal to $2(Q_H - Q_M)$. But at the same time, there is a loss in value equal to $r(U_{Hu} - U_{Lu})$; since an H stops being single and moves into the new HH match, while the L that was in the HL match moves into singleness.

This is equivalent to the condition $S^L_{HH} < 0$, or $S^H_{LH} > 0$. Using the values of a stable HH contract and an *unstable* $- h$ HL contract into $S^H_{LH} > 0$, this condition yields

$$S^H_{LH} = \frac{r(U_{Lu} - U_{Hu}) + 2(Q_H - Q_M)}{r + \alpha_{H,u} + \delta} = \frac{D}{r + \alpha_{Hu} + \delta}$$

and

$$S^H_{LH} > 0 \Leftrightarrow 2(Q_H - Q_M) > r(U_{Hu} - U_{Lu})$$

Notice S^H_{LH} is the discounted value of the surplus created by the corresponding separation at the moment the HL contract is originally signed.

It follows from the above analysis that if HL contracts are not unstable-l nor unstable-hl, $2(Q_H - 2Q_M) < r(U_{Hu} - U_{Lu})$ implies that $S^H_{LH} < 0$ and $S^L_{HH} > 0$, so HH contracts are *unstable* $- l$ and HL contracts are stable.

Proposition 1.2 *Assume HL contracts are not unstable-h nor unstable-hl.*

Then LL contracts are unstable-h (and HL contracts are stable) if

$$2(Q_M - 2Q_L) > r(U_{Hu} - U_{Lu})$$

LL contracts are stable (and HL are unstable-l) otherwise.

The first concern is when are $unstable - h$ LL contracts preferred to stable LL contracts. The discounted value of an unstable-h LL contracts is represented by (2) with $s^H_{LL} = 1$. Solving, this is given by

$$2\frac{Q_L + \alpha_{Hu}(U_{Hl} + U_{Lh} + U_{Lu} - U_{Hu}) + \delta U_{Lu}}{\delta + r + 2\alpha_{Hu}}$$

The discounted value of a stable LL contracts is given by (2) with $s^H_{LL} = 0$. Solving, this is given by

$$2\frac{Q_L + \delta U_{Hu}}{r + \delta}$$

Unstable-h LL contracts are signed if

$$2\frac{Q_L + \alpha_{Hu}(U_{Hl} + U_{Lh} + U_{Lu} - U_{Hu}) + \delta U_{Lu}}{\delta + r + 2\alpha_{Hu}} > 2\frac{Q_L + \delta U_{Hu}}{r + \delta}$$

which can be reduced to:

$$U_{Hl} + U_{Lh} + U_{Hu} - U_{Lu} > 2\frac{Q_L + \delta U_{Hu}}{r + \delta} \tag{1.5}$$

This is equivalent to the condition $S^H_{LL} > 0$ (or $S^L_{HL} < 0$) evaluated at $2U_{Ll} = 2\frac{Q_L+\delta U_{Hu}}{r+\delta}$.

If the above condition holds, $S^H_{LL} > 0$ ($S^L_{HL} < 0$) and $s^H_{LL} = 1$ $\left(s^L_{HL} = 0\right)$.

Because the Proposition assumes that HL contracts are not *unstable* $-h$, we must use also $s^H_{LH} = 0$. This means that HL contracts are stable, with value given by (3) with $s^H_{LH} = s^L_{HL} = 0$

$$\begin{aligned} r(U_{Hl} + U_{Lh}) &= 2Q_M + \delta(U_{Hu} + U_{Lu} - [U_{Hl} + U_{Lh}]) \\ U_{Hl} + U_{Lh} &= \frac{2Q_M + \delta(U_{Hu} + U_{Lu})}{r+\delta} \end{aligned}$$

Using $2U_{Ll} = 2\frac{Q_L+\delta U_{Hu}}{r+\delta}$ into $U_{Hl} + U_{Lh}$, and plugging the result in condition (5) above yields

$$2(Q_M - Q_L) > r(U_{Hu} - U_{Lu})$$

The intuition is analogue to that of Proposition 1.4. When an LL contract is destroyed in favour of an HL contract, there is a gain in terms of flow product equal to $2(Q_M - Q_L)$. But at the same time, there is a loss in value equal to $r(U_{Hu} - U_{Lu})$; since an H stops being single and moves into the new HL match, while the L that was in the LL match moves into singleness.

This is equivalent to the condition $S^H_{LL} > 0$ (or $S^L_{HL} < 0$). Using the values of an *unstable* $-$ h LL contract and a stable HL contract into $S^H_{LL} > 0$, this condition yields

$$S_{LL}^{H} = \frac{r(U_{Lu} - U_{Hu}) + 2(Q_M - Q_L)}{r + \alpha_{H,u} + \delta}$$

and

$$S_{LL}^{H} > 0 \Leftrightarrow 2(Q_M - 2Q_L) > r(U_{Hu} - U_{Lu})$$

As in Proposition 1.4, S_{LL}^{H} is the discounted value of the surplus created by the corresponding separation at the moment the LL contract is originally signed.

It follows from this analysis that if HL matches are not unstable-h not unstable-hl, then $2(Q_M - Q_L) < r(U_{Hu} - U_{Lu})$, implies $S_{HL}^{L} > 0$ and $S_{LL}^{H} < 0$, so LL contracts are stable and HL contracts are $unstable - l$.

Proposition 1.3 *Assume HL contracts are unstable-h or unstable-hl. Then LL contracts are unstable-h (and HL contracts are unstable-h) if*

$$2(Q_M - Q_L) + \frac{\alpha_{Hu} D}{r + \alpha_{Hu} + \delta} > r(U_{Hu} - U_{Lu})$$

LL contracts are stable (and HL are unstable-hl) otherwise.

Proposition 1.6 shows that LL contracts are $unstable - h$ if condition (5) holds:

$$U_{Hl} + U_{Lh} + U_{Hu} - U_{Lu} > 2\frac{Q_L + \delta U_{Hu}}{r + \delta}$$

If the above holds, $S_{LL}^{H} > 0$, $s_{LL}^{H} = 1$ and $S_{HL}^{L} < 0$, $s_{HL}^{L} = 0$

The Proposition assumes that HL contracts are *unstable−h* or *unstable−hl*. This means that $S^H_{LH} > 0, s^H_{LH} = 1$, and also that HH matches are stable, so $S^L_{HH} < 0, s^L_{HH} = 0$

The value of an HL contract is therefore given by (3) with $s^L_{HL} = 0$ and $s^H_{LH} = 1$, and

$$U_{Hl} + U_{Lh} = \frac{2Q_M + \alpha_{Hu}(2U_{Hh} + U_{Lu} - U_{Hu}) + \delta(U_{Hu} + U_{Lu})}{r + \delta + \alpha_{Hu}}$$

The value of an HH contract is given by (1) with $s^L_{HH} = 0$, and

$$2U_{Hh} = 2\frac{Q_H + \delta U_{Hu}}{r + \delta}$$

Using $2U_{Hh}$ in $U_{Hl} + U_{Lh}$, and this in condition (5) yields

$$2(Q_M - Q_L) + \frac{\alpha_{Hu} D}{r + \alpha_{Hu} + \delta} > r(U_{Hu} - U_{Lu})$$

The intuition is analogue to that of previous propositions. But now the benefit of destroying an LL match in favour of an HL match is not only the gain in flow values $2(Q_M - Q_L)$. It includes also the discounted value of the surplus created when the new HL match separates to form an HH match, as derived in Proposition 1.4 above. This must be higher than the value loss $r(U_{Hu} - U_{Lu})$ in order to an LL match to separate in favour of an HL match.

1.6 Steady State Equilibria

A set of contracts will be defined as a triple that describes the type of contract signed by each possible pair of single agents. Hence contract set $C = (c_{HH}, c_{LL}, c_{HL})$, where $c_{HH} \in$*{stable , unstable-l , no contract}*, $c_{LL} \in$*{ stable,unstable-h, no contract}* and $c_{HL} \in$*{stable , unstable-h , unstable-l , unstable-hl , no contract}*.

By describing the strategy chosen by any pair of single agents, a given set of contracts determines:

(a)The joint discounted values for matched agents and the discounted values for single agents in terms of the arrival rates.

(b)The steady state distribution of agents given by all N_{is} -and therefore the relevant arrival rates- since it defines the matching and separation dynamics.

Because of (a) and (b) above, a steady state equilibrium can be described by a set of contracts such that no agent has an incentive to write a contract that does not belong to the set.

I only consider the two equilibrium configurations that are parametrically relevant. In the first, referred to as *Equilibrium A*, two single Hs sign stable HH contracts; a single H and single L sign $unstable-h$ contracts and two single Ls sign $unstable-h$ contracts. In *Equilibrium B*, single Hs sign $unstable-l$ contracts; a single H and a single L sign stable HL contracts and two single Ls sign $unstable-h$ contracts.

Theorem 1.1 *Equilibrium A obtains if*

$$\frac{Q_H - Q_M}{Q_M - Q_L} > \frac{\alpha^A_{Lu}}{r + \delta + 2\alpha^A_{Hu} + \alpha^A_{Lu}}$$

Proof. Assume all agents follow the strategies described in Equilibrium A, the contracts set is given by C_A= *(stable, unstable-h , unstable-h)*. Then the equilibrium equations are given by:

(1) with $s^L_{HH} = 0$, (2) with $s^H_{LL} = 1$ and (3) with $s^H_{LH} = 1, s^L_{HL} = 0$

$$\begin{aligned} r2U_{Hh} &= 2Q_H + \delta(U_{Hu} - U_{Hh}) \\ r(U_{Hl} + U_{Lh}) &= 2Q_M + \alpha^A_{Hu}(S^H_{HL}) + \delta(U_{Hu} + U_{Lu} - [U_{Hl} + U_{Lh}]) \\ r2U_{Ll} &= 2Q_L + 2\alpha^A_{Hu}(S^H_{LL}) + \delta(U_{Lu} - U_{Ll}) \end{aligned}$$

Because HL contracts are not $unstable - l$, we follow Proposition 1.4 in writing the condition for HH matches to be stable and HL matches to be $unstable - h$:

$$(i) \; - S^L_{HH} = S^H_{LH} > 0 \Leftrightarrow 2(Q_H - 2Q_M) > r(U_{Hu} - U_{Lu})$$

Because HL contracts are $unstable{-}h$ and HH contracts are stable, we follow Proposition 1.6 in writing the condition for LL matches to be $unstable - h$

$$(ii) \quad -S^L_{HL} = S^H_{LL} > 0 \Longleftrightarrow 2(Q_M - Q_L) + \frac{\alpha^A_{Hu} D}{r + \alpha^A_{Hu} + \delta} > r(U_{Hu} - U_{Lu})$$

These two conditions are evaluated using the lifetime discounted values of single H and L types:

$$\begin{aligned} rU_{Lu} &= 2\alpha^A_{Hu}\left[\frac{U_{Hl} + U_{Lh} - U_{Hu} - U_{Lu}}{2}\right] + 2\alpha^A_{Lu}\left[\frac{2U_{Ll} - 2U_{Lu}}{2}\right] \\ rU_{Hu} &= 2\alpha^A_{Lu}\left[\frac{U_{Hl} + U_{Lh} - U_{Hu} - U_{Lu}}{2}\right] + 2\alpha^A_{Hu}\left[\frac{2U_{Hh} - 2U_{Hu}}{2}\right] \end{aligned}$$

In the Bellman equation for a single H, if she contacts a single L, which happens at rate $2\alpha^A_{Lu}$, they sign the equilibrium *unstable* $- h$ contract and share its surplus equally. If a single H is met, which happens at rate α^A_{Hu}, a stable contract is signed and the surplus is again shared equally. The possibility of contacting matched agents does not increase the discounted value, since any surplus if a separation occurs is extracted from the single agent by means of the lump sum transfer charged by the matched agent. The interpretation of the equation for a single L is analogue, with the difference that equilibrium LL contracts are *unstable* $- h$

Conditions (i) and (ii) can be respectively reduced to

$$(i) \ \frac{Q_H - Q_M}{Q_M - Q_L} > \frac{\alpha^A_{Lu}}{r + \delta + 2\alpha^A_{Hu} + \alpha^A_{Lu}}$$

$$(ii)\ \ (Q_M - Q_L) > 0$$

We need three more conditions for the configuration to represent an equilibrium, which are that the contracts conjectured are preferred to not signing a contract at all:

$$\begin{aligned}
(iii) \qquad 2U_{Hh} &> 2U_{Hu} \\
(iv) \qquad 2U_{Ll} &> 2U_{Lu} \\
(v)\ \ U_{Hl} + U_{Lh} &> U_{Hu} + U_{Lu}
\end{aligned}$$

These three conditions are evaluated using the solution to the equilibrium equations and the equations for single Hs and single Ls. Their reduced form is, respectively

$$(iii) Q_H \geq \frac{2Q_M\alpha_{Lu}(r+\partial+2\alpha^A_{Hu}+2\alpha_{Lu}) - 2Q_L\left(\alpha^A_{Lu}\right)^2}{(r+\partial+2\alpha^A_{Hu}+2\alpha^A_{Lu})(r+\partial+2\alpha^A_{Hu}+\alpha^A_{Lu})}$$

$$(iv)\ Q_L \geq 0$$

$$(v)\ Q_M \geq \frac{Q_L\alpha^A_{Lu}}{r+\partial+2\alpha^A_{Hu}+2\alpha^A_{Lu}}$$

Conditions $(ii), (iv)$and (v) always hold. Condition (iii) can be proven to hold if condition (i) holds. Hence, (i) is the only binding condition. Given Q_H this conditions holds if Q_M is low and/or Q_L is high. The intuition for this condition is based on this and on the intuition given in Proposition 1.4. If Q_M is low compared to Q_H, then the gain in flow output is very high when an HL match destroys in favour of an HH match. Further, if Q_L is high, then the value loss $r(U_{Hu} - U_{Lu})$ addressed in Proposition 1.4 is low enough, since a high Q_L increases the value of U_{Lu}.

Given that forming a match implies a loss in the arrival rate, one would expect to find parameter values for which such a loss is too high, and the agents prefer not to sign a contract and continue looking for better options. The reason it does not happen here is that the ability to keep all the surplus from a separation compensates for the loss in arrival rate. Consider single agents of types H and L. The ultimate goal for the H is to contact another single H, which occurs at rate $2\alpha_{Hu}^{A}$ if she remains single, and create a stable HH match. She keeps $\frac{1}{2}$ of the surplus created if she does meet one. Alternatively, the arrival rate halves to α_{Hu}^{A} if she accepts to match with the L. Ceteris paribus, this just halves the expected gain from meeting another H. But signing a temporary contract with an L also allows the H to keep whole surplus of the match formed upon a separation, instead of only $\frac{1}{2}$ if she does while single given the bargaining process. These two effects cancel on expected terms, and the loss in arrival rate is compensated. Further, the L will always let the H keep all the surplus from the expected HH match. The reason is that the expectation of an HH match is not part of her continuation value as a single agent. Hence, giving it up completely will never make her utility after signing the contract lower than her continuation value.

LL matches form always for an analogue reason. When single, each meets a high type at rate $2\alpha^A_{Hu}$ and share equally with her the surplus created. Forming an LL match halves the individual arrival rate, but if they do find an H, they are able to keep the whole surplus arising from the new HL match, rather than just $\frac{1}{2}$, and these two effects cancel out.

The arrival rates α^A_{iu} are obtained from the solution of the adding up and identity constraints and the flow equations:

$$\begin{aligned} N_{Hl} &= N_{Lh} \\ N_{Hh} + N_{Hl} + N_{Hu} &= \gamma \\ N_{Lh} + N_{Ll} + N_{Lu} &= 1 - \gamma \end{aligned}$$

$$N_{Hu}\left[2\varepsilon N_{Hu}\right] + N_{Hu}\left[\varepsilon N_{Hl}\right] + N_{Hl}\left[\varepsilon N_{Hu}\right] = \delta N_{Hh}$$

In the equation above, Hs can get into HH matches if: (a) A single H contacts another single H. (b) A single H contacts an H in state l. (c) An H in state l contacts a single H. Hs fall out of HH matches only if the exogenous shock occurs.

$$N_{Hu}\left[2\varepsilon N_{Lu}\right] + N_{Hu}\left[\varepsilon N_{Ll}\right] = \delta N_{Hl} + N_{Hu}\left[\varepsilon N_{Hl}\right]$$

Above, Hs get into HL matches if: (a) A single H contacts a single L. (b) A single H contacts an L in state l. Hs fall out of HL matches if: (a) The exogenous shock occurs. (b) An H in state l contacts a single H.

$$N_{Lu}\left[2\varepsilon N_{Lu}\right] = \delta N_{Ll} + N_{Ll}\left[2\varepsilon N_{Hu}\right]$$

Finally, Ls get into LL matches if: (a) A single L contacts another single L. Ls fall out of LL matches if: (a)The exogenous shock occurs (b) An L in state l or his partner contacts a single H.

Theorem 1.2 *Equilibrium B obtains if*

$$\frac{Q_H - Q_M}{Q_M - Q_L} < \frac{\alpha_{Lu}^B}{r + \delta + 2\alpha_{Hu}^B + \alpha_{Lu}^B}$$

The proof of Theorem 1.2 follows the same lines of the proof of Theorem 1.1, and is therefore relegated to the appendix. Because HL contracts are not $unstable - l$, we follow Proposition 1.4 in writing the condition such that HL contracts are stable and HH contracts are unstable-l. Because HL contracts are not $unstable - h$, we follow Proposition 5 in writing the condition such that LL contracts are $unstable - l$ and HL contracts are stable.

Given Q_H, the inequality in Theorem 1.2 is likely to hold for high Q_M and/or low Q_L. Opposite to Equilibrium A, if Q_M is high relative to Q_H, (assume Q_M is high enough that $Q_M \approx Q_H$), then the loss in flow product is not very high when a separation of this kind occurs. If in addition, Q_L is very low, this decreases the value of single Ls. Then the separation is profitable by employing an L in its relatively more productive role: in a match with an H where it is almost as good as another H, rather than single with very low U_{Lu}. The single H that results from the separation has a much higher discounted value than a single L. This is because the single H can hope to produce Q_H upon meeting an H and $Q_M \approx Q_H$ upon meeting an L. On the other hand, a single L can hope to produce $Q_M \approx Q_H$ upon meeting an H, but only a low Q_L upon meeting an L..

Again, if matching reduces the individual arrival rate, one would expect temporary matches not to be profitable for some parameter values which would make this reduction too high a loss. The argument for single Ls to always sign $unstable-h$ contracts is the same as in Equilibrium 1. With HH matches being temporary as well, two single Hs always find it worth it to form a $unstable-l$ HH match for an analogue reason.

1.7 Sorting Effects

The conditions for Equilibria A and B to obtain can be rewritten respectively as:

$$
\begin{aligned}
Q_H - Q_M &> F(\alpha^e_{Hu}, \alpha^e_{Lu})(Q_M - Q_L) \\
Q_H - Q_M &< F(\alpha^e_{Hu}, \alpha^e_{Lu})(Q_M - Q_L)
\end{aligned}
$$

where

$$F(\alpha^e_{H,u}, \alpha^e_{L,u}) = \frac{\alpha_{\alpha^e_{Lu}}}{r + \delta + 2\alpha^e_{Hu} + \alpha^e_{Lu}}$$

Theorem 1.3 *Parameter configurations exist such that there is no set of contracts that represent a pure strategy steady state equilibrium.*

Proof. The arrival rates $\alpha^e_{Hu}, \alpha^e_{Lu}$ differ across the two equilibria. In particular, $\alpha^A_{Lu} > \alpha^B_{Lu}$ and $\alpha^A_{Hu} < \alpha^B_{Hu}$. That is because in Equilibrium A, LH matches break to give rise to HH matches, which decreases the number of single Hs and increases the number of single Ls. In Equilibrium B, the opposite is true. Other than this, the matching and separation behavior is equal across these two equilibria. Further, it is easy to show that:

$$\begin{aligned} F_{\alpha^e_{Lu}} &> 0 \\ F_{\alpha^e_{Hu}} &< 0 \end{aligned}$$

As a consequence of that:

$$\begin{aligned} F(\alpha^A_{Hu}, \alpha^A_{Lu})(Q_M - Q_L) &> F(\alpha^B_{Hu}, \alpha^B_{Lu})(Q_M - Q_L) \text{ if } Q_M > Q_L \\ F(\alpha^A_{Hu}, \alpha^A_{Lu})(Q_M - Q_L) &= F(\alpha^B_{Hu}, \alpha^B_{Lu})(Q_M - Q_L) = 0 \text{ if } Q_M = Q_L \end{aligned}$$

so the lower limit of $Q_H - Q_M$ for Equilibrium A to obtain is higher than the upper limit of $Q_H - Q_M$ for Equilibrium B to obtain[7]. Given the values of the

[7]For $Q_M > Q_L$. They are both 0 if $Q_M = Q_L$.

GRAPH 1

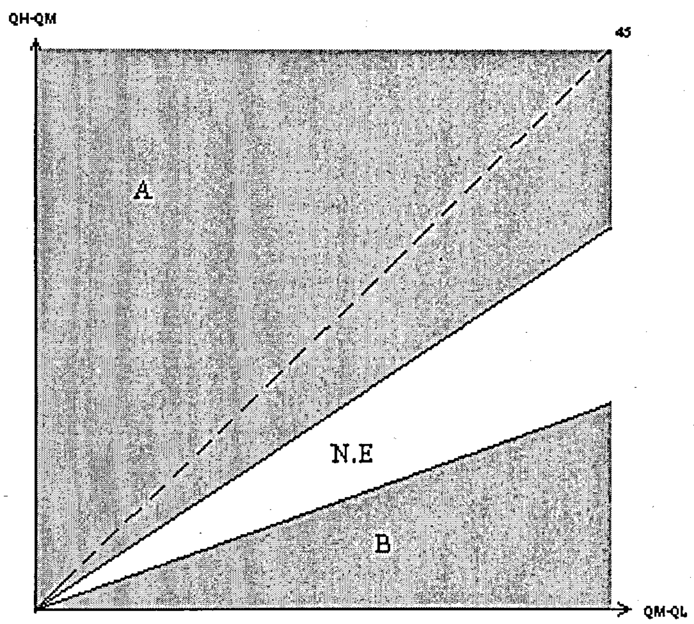

Figure 1-1:

other parameters, there is a range of Q_H values such that neither Equilibrium A nor Equilibrium B exist.

In Graph 1, the two dark lines represent both $F(\alpha^e_{H,u}, \alpha^e_{L,u})(Q_M - Q_L)$. The solid line that limits zone A from below corresponds to $F(\alpha^A_{H,u}, \alpha^A_{L,u})(Q_M - Q_L)$. Zone A represents parameter configurations such that Equilibrium A obtains. The solid line that limits zone B from above corresponds to $F(\alpha^B_{H,u}, \alpha^B_{L,u})(Q_M - Q_L)$, hence, Zone B represents parameter configurations such that Equilibrium B obtains. Zones A and B exclude the lower and upper limit respectively, which belong to the zone of no equilibrium ($N.E.$). The 45^o line in the Graph will be considered when addressing the efficiency of the equilibria. Notice that

$$\frac{\partial(Q_H - Q_M)}{\partial(Q_M - Q_L)} = F(\alpha^e_{H,u}, \alpha^e_{L,u})$$

so $F(\alpha^e_{H,u}, \alpha^e_{L,u})$ represents the slope of the limits in Graph 1. Using this leads also to the construction of Graph 1.

1.8 Externalities and Efficiency of the Equilibria

The amount of search increases with the number of single agents, since it is only single agents who search actively (by making phone calls). On that regard, there is a thick market externality, whereby a decision to match imposes costs on other agents because it decreases the number of singles and therefore of active searchers (callers); which makes it more difficult to contact new people. Notice that U_{is} do not depend on the number of matched agents, but only on the number of single agents and the total number of agents in the market, 1. That is because single agents do not obtain any surplus from a separation that forms a new match, should there be one. Hence, a decision to match imposes an externality on other agents *only* because the number of single agents decreases, and *not* because the number of matched agents increases. Matching decisions affect not only the amount of search but also the composition of active searchers, which is affected differently depending on the types involved. Hence, matching decisions impose both a thick market and a composition externality.

When a separation occurs, the total number of searchers remains constant, so there is no thick market externality. The externality in these cases has to do *only* with changing the composition of searchers. Hence, the sign of the externality is determined by the direction of the change in composition.

All decisions taken regarding matching and separations (the type of contract signed) are *privately efficient*, by considering the costs of the three agents involved in any possible separation. Because new agents must be offered their continuation value to agree to form a new match (after the existing match separates), their cost is being considered as well, although they are not benefiting from the separation. Nevertheless, because all matching and separation decisions impose an externality on other agents, the equilibria need not be socially efficient. The notion of efficiency used is the same as in Diamond and Maskin (1979), where efficiency is measured by total output net of search costs in steady state[8]. Because both equilibria exhibit non-elitist matching, I will address only the separation dynamics, and say that an equilibrium is efficient if the separation dynamics are efficient.

Separations just trade one agent for another into a match and into singleness. This means that both equilibria exhibit the same number of matches and they only differ in the match composition. Hence, by comparing the efficiency of the separation dynamics across equilibria, we are actually comparing the efficiency across the equilibria.

[8]Search costas are 0 here.

Taking the number of matches as given, the condition $Q_H + Q_L > 2Q_M$ implies that an assortative (elitist) matching configuration is efficient. $Q_H + Q_L < 2Q_M$ implies that a negative-assortative matching configuration is efficient (see Roth and Sotomayor, 1990). In Graph 1, the condition $Q_H + Q_L = 2Q_M \Longrightarrow Q_H - Q_M = Q_M - Q_L$ is represented by the zone above the 45^0 line.

In all points above the 45^0, Equilibrium A dynamics are constrained efficient, but not efficient because it would still be possible to improve upon them by not allowing LL matches to destroy in favor of HL matches. In all points below the 45^0, separation dynamics in Equilibrium B are efficient, because they maximize the number of HL matches by destroying HH and LL matches in favor of HL matches

Theorem 1.4 *If the separation dynamics in Equilibrium A are constrained efficient, Equilibrium A obtains.*

Proof.

$$\begin{aligned} Q_H + Q_L &> 2Q_M \Longrightarrow Q_H - Q_M > Q_M - Q_L \Longrightarrow \\ Q_H - Q_M &> \frac{\alpha^A_{Lu}}{r + \delta + 2\alpha^A_{Hu} + \alpha^A_{Lu}}(Q_H - Q_L) \end{aligned}$$

and therefore Equilibrium A obtains (Claim 4). The slope of the lower limit of Zone A is given by $\frac{\alpha^A_{L,u}}{(r+\delta+2\alpha^A_H+\alpha^A_{L,u})} < 1$, so it lies always below the 45^0 line. It can still be the case that Equilibrium A obtains, but Equilibrium B dynamics are efficient. This would happen if $Q_H - Q_M$ lies in Zone A below the 45^0 line.

Theorem 1.5 *If Equilibrium B obtains, the separation dynamics in Equilibrium B are efficient.*

Proof. Theorem 1.2 states that Equilibrium B obtains if

$$
\begin{aligned}
Q_H - Q_M &< \frac{\alpha^B_{Lu}}{r+\delta+2\alpha^B_{Hu}+\alpha^B_{Lu}}(Q_M - Q_L) \\
&\Longrightarrow Q_H - Q_M < Q_M - Q_L \Longrightarrow Q_H + Q_L < 2Q_M
\end{aligned}
$$

and the last inequality implies that Equilibrium B dynamics are efficient. In Graph 1, notice that all points that lie in Zone B also lie below the 45^0 line.

1.9 Asymmetric effects of the Freedom of Contract

Equilibrium A is the case that best reflects the spirit of the model. In this configuration, HH matches are stable and only single H types are able to break existing matches. Separations only occur when an L in an LL match or an H in an HL match contact a *single* H. The problem for single H types is that, upon a separation, their surplus is extracted by means of the lump sum transfer, so they do not benefit from the ability to break existing matches.

I will use a benchmark model to illustrate this point. Assume that matched agents are not able to commit not to bargain with third agents. Thus, matched agents are unable to make non-negotiable proposals, and therefore must bargain with their single counterpart for the surplus of a separation that accrues to them. I will then assume that, upon a separation, the agent leaving a match must compensate her partner for the capital loss imposed on her[9] The surplus for the two agents in the new contract is equal to the sum of their changes in discounted values minus the compensation that must be paid to the agent left single; and this is the surplus they split through bargaining. Therefore, if matching implies the loss of arrival rate of matched agents, now there *is* a positive cost to entering a match in terms of not meeting matched agents.

In the new setting, the separation rule is analogue for existing contracts. Since the agents in the new match must compensate the one that will be left single after the separation, it is still true that the costs and benefits of *all* agents involved are being considered. In that sense, separation decisions are *still privately efficient.* As before, if the total surplus generated by a separation (as defined in Definition 2) is not positive, the separation will not occur.

This new setting limits the "rent extraction" abilities of matched agents. This implies that, whenever single Hs break existing matches, they will profit from such separations. The following investigates how does this affect the discounted value of a single H.

I will rely on a numerical example to address the issue, and find the equilibria for parameter sets that would yield an equilibrium of the A type in the main model, in the sense that HH contracts form and are stable (i.e. when Q_H is high

[9] As in Burdett and Coles, 1999. This decreases the surplus of the separation accruing to the potential match.

enough). In no way does this numerical exercise aim to be an analysis of the matching and turnover behaviour of the alternative model, which can be proven to exhibit a much richer set of equilibria.

Assume the following arbitrary values for the parameters:

$$r = 0.1, \delta = 0.1, Q_L = 1, Q_M = 2, \varepsilon = 1$$

With these parameter values, the steady state stocks and arrival rates are:

$$\begin{aligned} N_{Hu} &= 0.0582, N_{Hl} = 0.1726, N_{Hh} = 0.2690 \\ N_{Lu} &= 0.1417, N_{Lh} = 0.1726, N_{Ll} = 0.1855 \end{aligned}$$

and

$$\alpha^A_{t,s} = \varepsilon N_{ts} = N_{ts}$$

Equilibrium A obtains in the main model if

$$\frac{Q_H - Q_M}{Q_M - Q_L} > \frac{\alpha^A_{L,u}}{r + \delta + 2\alpha^A_{Hu} + \alpha^A_{Lu}} \Rightarrow Q_H > 2.31$$

The discounted value of a single H type results in

$$U_{Hu} = 6.89 + 3.68 Q_H$$

Let us go back to the model with compensation payments. With matched agents unable to make non-negotiable proposals, single agents expect to receive only $\frac{1}{2}$ of the surplus of a separation as a result of the bargaining process (as opposed to nothing in the main model). I report the results for two equilibria that are possible for high values of Q_H in order to illustrate the point.

Equilibrium C exhibits the same contracts set as Equilibrium A, and therefore the same dynamics, the same steady state equations and arrival rates. The difference is given only by the sharing of the separation surpluses. The corresponding Bellman Equations are:

$$rU_{Hh} = Q_H + \delta(U_{Hu} - U_{Hh})$$

$$r\left(U_{Hl} + U_{Lh}\right) = 2Q_M + \alpha^C_{H,u}\frac{S^H_{LH}}{2} + \delta(U_{Hu} + U_{Lu} - U_{Hl} - U_{Lh})$$

$$rU_{Ll} = Q_L + \alpha^C_{H,u}\frac{S^H_{LL}}{2} + \delta(U_{Lu} - U_{Ll})$$

$$rU_{Lu} = 2\alpha^C_{Hu}\left[\frac{U_{Hl}+U_{Lh}-U_{Hu}-U_{Lu}}{2}\right] + 2\alpha^C_{Lu}\left[\frac{2U_{Ll}-2U_{Lu}}{2}\right]$$

$$rU_{Hu} = 2\alpha^C_{Lu}\left[\frac{U_{Hl}+U_{Lh}-U_{Hu}-U_{Lu}}{2}\right] + 2\alpha^C_{Hu}\left[\frac{2U_{Hh}-2U_{Hu}}{2}\right] + \alpha^C_{Hl}\frac{S^H_{LH}}{2} + \alpha^C_{Ll}\frac{S^H_{LL}}{2}$$

The stocks and arrival rates in Equilibrium C are equal to those in Equilibrium A.

The equations for U_{Hu} and U_{Lu} reflect that single agents keep $\frac{1}{2}$ of the surplus of separation when matched agents are contacted, and this is reflected in the Bellman Equations for the discounted value of the contracts. Solving the system given above the U_{ts} of the candidate equilibrium. The equilibrium conditions are analogue to those in Equilibrium A, since the private separation rules are the same. These use the U_{ts} obtained from above. I report the results obtained using the parameter values assumed:

$$\begin{aligned} S_{LH}^{H} &> 0 \Longrightarrow Q_H > 2.47 \\ S_{LL}^{H} &> 0 \Longrightarrow Q_H < 5.23 \\ U_{Ll} &> U_{Lu} \Longrightarrow Q_H > -18.11 \\ U_{Hh} &> U_{Hu} \Longrightarrow Q_H > 1.06 \\ U_{Hl} + U_{Lh} &> U_{Hu} + U_{Lu} \Longrightarrow Q_H < 7.74 \end{aligned}$$

The difference is that now two of the conditions are relevant. Condition $S_{LH}^{H} > 0$ says that Q_H must be high enough that it is profitable to break HL matches when the H contacts a single H; and therefore HH matches are stable contracts. This is analogue to the condition for Equilibrium A to obtain. But now condition $S_{LL}^{H} > 0$ is also binding. It must also be the case that, although high enough, Q_H is not very high. The way of looking at it is the following. If Q_H is very high, then U_{Hu} is very high, and it is more efficient to have him searching for another H than in a contract with an L. If this is the case, $S_{LL}^{H} < 0$ and the separation is inefficient when an L in an LL match contacts an H.

Hence, Equilibrium C obtains for $2.47 < Q_H < 5.23$ and

$$U_{Hu}^{C} = 5.25 + 5.05 Q_H$$

In Equilibrium D, two single Ls sign stable contracts, two single Hs sign stable contracts, and a single H and a single L sign a *unstable* $-$ *hl* contract[10]. In this equilibrium, Q_H is high and U_{Hu} therefore. Hence, the surplus created by the separation $S^l_{Hl} = U_{Hu} + 2U_{LL} - 2U_{Hl} - U_{Lu}$ is positive and the separation is efficient.

The Bellman equations are functions satisfy

$$rU_{Hh} = Q_H + \delta(U_{Hu} - U_{Hh})$$

$$r(U_{Hl} + U_{Lh}) = 2Q_M + \alpha^D_{Hu}\frac{S^H_{LH}}{2} + \alpha^D_{L,u}\frac{S^L_{HL}}{2} + \delta(U_{Hu} + U_{Lu} - U_{Lh} - U_{Hl})$$

$$rU_{Ll} = Q_L + \delta(U_{Lu} - U_{Ll})$$

$$rU_{Hu} = 2\alpha^D_{Lu}\left[\tfrac{1}{2}\left(U_{Hl} + U_{Lh} - U_{Hu} - U_{Lu}\right)\right] + 2\alpha^D_{H,u}\left[\tfrac{2U_{Hh} - 2U_{Hu}}{2}\right] + \alpha^D_{Hl}\frac{S^H_{LH}}{2}$$

$$rU_{Lu} = 2\alpha^D_{Hu}\left[\tfrac{1}{2}\left(U_{Hl} + U_{Lh} - U_{Hu} - U_{Lu}\right)\right] + 2\alpha^D_{Lu}\left[\tfrac{2U_{Ll} - 2U_{Lu}}{2}\right] + \alpha^D_{Lh}\frac{S^L_{HL}}{2}$$

The steady state stocks and arrival rates are given by the solution to the steady state equations in the Appendix:

$$\begin{aligned} N_{Hu} &= 0.1, N_{Hl} = 0.6666, N_{Hh} = 0.3333 \\ N_{Lu} &= 0.1, N_{Lh} = 0.6666, N_{Ll} = 0.3333 \end{aligned}$$

and

$$\alpha^D_{t,s} = \varepsilon N_{ts} = N_{ts}$$

[10] The match will break when the H meets an single H or the L meets a single L.

Solving the system above yields the candidate U^*_{ts}. The Equilibrium conditions are:

$$\begin{aligned} S^H_{LH} &> 0 \Longrightarrow Q_H > 2.28 \\ S^L_{HL} &> 0 \Longrightarrow Q_H > 6.60 \\ U_{Ll} &> U_{Lu} \Longrightarrow Q_H > -20.40 \\ U_{Hh} &> U_{Hu} \Longrightarrow Q_H > 2.28 \\ U_{Hl} + U_{Lh} &> U_{Hu} + U_{Lu} \Longrightarrow Q_H < 9.97 \end{aligned}$$

The condition on $S^L_{HL} > 0$, where $S^L_{HL} = -S^H_{LL}$, is the complement to the condition on S^H_{LL} for equilibrium C. Now, because is high Q_H enough, it is not efficient to break LL matches in favour of HL matches. It is therefore efficient to break HL matches in favour of LL matches. The intuition was given when addressing Equilibrium C. Notice that now we need $Q_H < 9.97$ for HL contracts to form. If Q_H was very high, then H types prefer to remain single, searching more efficiently for another H, rather than signing a contract with an L. Hence, Equilibrium D obtains if $6.60 < Q_H < 9.97$ and

$$U^D_{Hu} = 2.81 + 5.43 Q_H$$

The reader may be puzzled by the big difference between the upper limit for Equilibrium C to obtain and the lower limit for Equilibrium D to obtain. This is due to sorting effects of the type described in section (1.7).

Finally, it is easy to show that

$$
\begin{aligned}
U^C_{Hu} &> U^A_{Hu} \text{ if } 2.47 < Q_H < 5.23 \\
U^D_{Hu} &> U^A_{Hu} \text{ if } 6.60 < Q_H < 9.97
\end{aligned}
$$

This implies that single Hs are better off if matched agents are *not* able to commit not to bargain with single agents. Of course, two agents who have just met would like to sign contracts to commit their further bargaining behaviour, but this is because they have already contacted. Before any contact has occurred, single Hs prefer a system where such contracts are not possible, so that already existing matches cannot extract surplus from them.

1.10 Conclusion

This paper has analised equilibrium contracting behaviour in a partnership model with on-the-job search, and the matching and turnover behaviour implied, as well as the welfare consequences for heterogenous agents. I have argued that in the presence of *i*) bargaining and *ii*) complete freedom of contract, agents will, on the one hand, aim to maximise the joint value of their relationship; and, on the other, use their relationship to gain full bargaining power vis-a-vis agents they may contact in the future. Given this, partnerships between high and low productive agents will separate when the former meets another high productive agent -and a new high-high match will be formed-, provided that *i)* Ceteris paribus, the productivity in the high-low match is low enough, or *ii)*Ceteris paribus, the productivity in the high-high match is high enough, or *iii)* Ceteris paribus, the productivity in a low-low match is high enough. Either of this three conditions implies that such turnover would be privately efficient for the agents in the high-low match, who will then force it to occur. I also showed how the composition effects of agents equilibrium turnover behaviour creates regions where no pure strategy steady state equilibrium exists.

I also argued that, although the turnover behaviour implied is privately efficient, the market outcome need not be socially efficient. When agents decide to form a match or implement a turnover, they do not consider the externality imposed on the other agents in the market. This is a well known feature of search and matching models. Finally, I showed through a numerical example that such equilibrium contracting behaviour decreases the returns to quality for single agents. Because rents are extracted by matched agents, when a turnover occurs, if the third agent is a high productive agent he just sees a higher rent extracted from him, rather than benefit from his own high productivity.

1.11 Appendix

Proof of Theorem 1.2 The contract set conjectured for Equilibrium B is given by C_B= *(unstable-l , unstable-h , s)*. In words, single *H*s sign *unstable-l*

HH contracts; single Ls sign *unstable-h* LL contracts; and single Hs and single Ls sign stable HL contracts. The equilibrium equations are given by:

(1) with $s^L_{HH} = 1$, (2) with $s^H_{LL} = 1$ and (3) with $s^L_{HL} = s^H_{LH} = 0$

$$\begin{aligned}
r2U_{Hh} &= 2Q_H + 2\alpha^B_{L,u}\left[s^L_{HH}(S^L_{HH})\right] + \delta(U_{Hu} - U_{Hh}) \\
r2U_{Ll} &= 2Q_L + 2\alpha^B_{H,u}(S^H_{LL}) + \delta(U_{Lu} - U_{Ll}) \\
r(U_{Hl} + U_{Lh}) &= 2Q_M + \delta(U_{Hu} + U_{Lu} - [U_{Hl} + U_{Lh}])
\end{aligned}$$

Because HL contracts are not $unstable - l$, Proposition 1.4 is followed in writing the condition such that HL contracts are stable and HH contracts are $unstable - l$:

$$(i)\ 2(Q_H - Q_M) < r(U_{Hu} - U_{Lu})$$

Because HL contracts are *not* $unstable - h$, Proposition 1.4 is followed in writing the condition such that HL contracts are stable and LL contracts are $unstable - l$:

$$(ii)\ 2(Q_M - 2Q_L) > r(U_{Hu} - U_{Lu})$$

These conditions are evaluated respectively using the discounted value of single Hs and single Ls:

$$
\begin{aligned}
rU_{Lu} &= 2\alpha_H^B\left[\frac{U_{Hl}+U_{Lh}-U_{Hu}-U_{Lu}}{2}\right]+2\alpha_{L,u}^B\left[\frac{2U_{Ll}-2U_{Lu}}{2}\right] \\
rU_{Hu} &= 2\alpha_{L,u}^B\left[\frac{U_{Hl}+U_{Lh}-U_{Hu}-U_{Lu}}{2}\right]+2\alpha_H^B\left[\frac{2U_{Hh}-2U_{Hu}}{2}\right]
\end{aligned}
$$

Using the solution to the above equations and three equilibrium equations above, conditions (i) and (ii) can be reduced to:

$$(i)\ \frac{Q_H-Q_M}{Q_M-Q_L}<\frac{\alpha_{L,u}^B}{r+\delta+2\alpha_{H,u}^B+\alpha_{L,u}^B}$$

$$(ii)\ \frac{Q_H-Q_M}{Q_M-Q_L}<\frac{(r+\partial+2\alpha_{L,u}^B+\alpha_{H,u}^B)}{\alpha_{L,u}^B}$$

We need three more conditions for the proof, which are that the equilibrium contracts are preferred to not forming a match:

$$
\begin{aligned}
(iii) \qquad 2U_{Hh} &> 2U_{Hu} \\
(iv) \qquad 2U_{Ll} &> 2U_{Lu} \\
(v)\ \ U_{Hl}+U_{Lh} &> U_{Hu}+U_{Lu}
\end{aligned}
$$

These three conditions are evaluated using the solution to the equilibrium equations. There reduced form is, respectively

$$\begin{aligned}
(iii)\ Q_H &\geq 0 \\
(iv)\ Q_L &\geq 0 \\
(v)\ \ Q_H &\leq \frac{Q_M(r + \partial + 2\alpha^B_{L,u} + 2\alpha^B_{H,u}) - Q_L\alpha^B_{L,u}}{\alpha^B_{H,u}}
\end{aligned}$$

Conditions (iii) and (iv) always hold. Conditions (ii) and (v) can be shown to hold if (i) holds. Hence, (i) is the only binding condition.

The arrival rates α^B_i solve the system of steady state equations:

$$\begin{aligned}
N_{Hl} &= N_{Lh} \\
N_{Hh} + N_{Hl} + N_{Hu} &= \varepsilon \\
N_{Lh} + N_{Ll} + N_{Lu} &= 1 - \varepsilon
\end{aligned}$$

$$N_{Hu}\left[2\varepsilon N_{Hu}\right] = \delta N_{Hh} + N_{Hh}\left[2\varepsilon N_{Lu}\right]$$

Hs can get into HH matches if: (a) A single H contacts another single H. Hs fall out of HH matches only if: (a) the exogenous shock occurs. (b) An H in state h or her partner contacts a single L.

$$N_{Hu}\left[2\varepsilon N_{Lu}\right] + N_{Hu}\left[\varepsilon N_{Ll}\right] + N_{Hh}\left[\varepsilon N_{Lu}\right] = \delta N_{Hl}$$

Hs agents get into HL matches if: (a) A single H contacts a single L. (b) A single H contacts an L in state l (c) A single L contacts an H in state h. Hs fall out of HL matches if the exogenous shock occurs.

$$N_{Lu}\left[2\varepsilon N_{Lu}\right] = \delta N_{Ll} + N_{Ll}\left[2\varepsilon N_{Hu}\right]$$

Ls get into LL matches if: (a) A single L contacts another single L. Ls fall out of LL matches if: (a)The exogenous shock occurs (b) An L in state l or her partner contacts a single H.

Steady State Equations for Equilibrium D The arrival rates α_i^D are obtained from the solution of the adding up and identity constraints and the flow equations:

$$\begin{aligned} N_{Hl} &= N_{Lh} \\ N_{Hh} + N_{Hl} + N_{Hu} &= \gamma \\ N_{Lh} + N_{Ll} + N_{Lu} &= 1 - \gamma \end{aligned}$$

$$N_{Hu}\left[2\varepsilon N_{Hu}\right] + N_{Hu}\left[\varepsilon N_{Hl}\right] + N_{Hl}\left[\varepsilon N_{Hu}\right] = \delta N_{Hh}$$

Hs can get into HH matches if: (a) A single H contacts another single H. (b) A single H contacts an H in state l. (c) An H in state l contacts a single H. Hs fall out of HH matches only if the exogenous shock occurs.

$$N_{Hu}\left[2\varepsilon N_{Lu}\right] = \delta N_{Hl} + N_{Hl}\left[\varepsilon N_{Hu}\right] + N_{Hl}\left[\varepsilon N_{Lu}\right]$$

Hs agents get into HL matches if: (a) A single H contacts a single L. Hs fall out of HL matches if: (a) The exogenous shock occurs. (b) An H in state l contacts a single H (c) The partner of an H in state l contacts a single L.

$$N_{Lu}\left[2\varepsilon N_{Lu}\right] + N_{Lu}\left[\varepsilon N_{Lh}\right] + N_{Lh}\left[\varepsilon N_{Lu}\right] = \delta N_{Ll}$$

Ls can get into LL matches if: (a) A single L contacts another single L. (b) A single L contacts an L in state h. (c) An L in state h contacts a single L. Ls fall out of LL matches only if the exogenous shock occurs.

Chapter 2

Contracts and on-the-job search II

2.1 Introduction

Assume a labour market with homogenous workers and heterogenous firms: low and high productivity. Now assume this is a frictionless labour market. This is certainly a set of assumptions you would not like to include in your job market paper, or any other paper for that matter. In a frictionless market, high productivity firms would attract all the workers, condemning low productivity firms to disappear. But assume instead a frictional market, say search frictions, and you are back in business. Given the frictions, every time a firm (high or low productivity) and a worker meet through search, monopoly rents are generated; it is the existence of these rents what allows the low productivity firms to survive in the market.

The present paper addresses, amongst other things, the existence of low productivity firms alongside high productivity firms in a model with search frictions, focusing on the case known as on-the job search. The latter implies that workers will be searching for firms both while unemployed and employed. Two other phenomena are of particular interest when there exists on-the-job search. One is the nature of the turnover dynamics, i.e. how do workers move from firm to firm; the other is the shape of the distribution of wages being offered and paid in the market. In the presence of on-the-job search, the market wage distribution is not degenerate, while the actual shape of the distribution depends on the particular environment. The previous two phenomena are particularly interesting when one considers environments with heterogenous firms. The paper will consider two types: low productivity (LP) and high productivity (HP). Three cases will be considered, briefly described in what follows:

(I) The Burdett-Mortensen environment (the $bm - case$). In Burdett-Mortensen (1998), firms are assumed to post wages to attract workers. Since they have imperfect information about the employment status of the workers they meet, each firm posts only one wage, good for any worker they meet. Further, firms are assumed not to counter the outside offers received by their workers, or in any way change the wage earned by their workers. The posted wage will be earned by any worker who accepts it for as long as he stays with the firm[1], and

[1]The firm could offer wage-tenure contracts, as in Burdett-Coles 2003. The firm increases the worker's original wage as a function of the worker's tenure in the firm, considering that, as time ellapses, some (unidentified) workers will in fact receive better (unobserved) outside

he will stay until he finds a better wage offer in another firm. In other words, better wages imply better employment opportunities, and workers accept them. A non-degenerate continuous wage offer distribution is generated, whereby firms offering higher wages make less profits per worker, but have more workers in steady state[2]. The wages in the support of the distribution are such that these two effects balance each other, therefore implying equal profits for equal firms in steady state.

(II)The Postel-Vinay and Robin environment (the $m - case$). In contrast with Burdett-Mortensen, Postel-Vinay and Robin (2002a and 2002b) consider models in which firms are assumed to: (i) Observe the employment status of any workers met, and (ii) Match the outside offers received by their workers (therefore the $m - case$ designation). As a result, firms offer all unemployed workers their reservation wage[3], and pay this wage until the worker meets another firm. At this moment, the incumbent and poaching firm engage in a Bertrand competition for the worker. The incumbent firm can bid up to a wage equal to the worker's productivity in the firm. The wage that results from this competition depends on the productivity of the two firms involved, but is limited by the productivity of the worker in the incumbent firm. Hence, the distribution of wages paid is not continuous, but has mass points determined by the reservation wage and the productivities of the firms in the market. In this case, workers move from firm to

offers. This is not allowed in Burdett-Mortensen (1998)

[2]Firms attract workers from more firms and loose workers to less firms as the wage offered increases; resulting in a higher number of workers in strady state.

[3]The minimum wage needed for the worker to accept employment in the firm

firm as better employment opportunities come along. Interestingly enough, and different from Burdett and Mortensen, higher wages do not necessarily imply better employment opportunities. As more productive firms are able to bid up to higher wages in a Bertrand competition with other firm, they represent higher future wage increases for the worker, and therefore better employment opportunities for any given wage. The implication is that HP firms can attract workers from LP firms by offering lower wages in some cases.

This is an innovation from the $bm - case$, since it allows firms to react optimally when their workers receive offers from other firms. But, as will be made clear in case III below, firms are still not fully exploiting all possibilities, which they can do by contracting strategically.

(III) A contract approach (the $c - case$). A third environment is introduced in the paper, in which the information structure is equal to that of the Postel-Vinay and Robin environment. The novelty is that firms and workers sign strategic contracts, whereby firms commit to bid up to a wage equal to the worker's productivity in the poaching (maybe more productive) firm. This type of contract is forward-looking, as opposed to those offered in the previous two cases, in the sense that they provide contingencies for possible future events, i.e. the worker contacting another firm through search on the job. The basic difference compared to Postel-Vinay and Robin is that the wage increase that results from Bertrand competition is not limited by the productivity in the incumbent firm, but by the productivity in the poaching firm. Hence, it is no longer true that HP offer higher future wage increases; and as a result they are not able to attract workers by offering lower wages. In terms of worker turnover, this implies that workers will never move from LP to HP firms accepting a wage cut, and this is different from Postel-Vinay and Robin.

In particular, the focus of the paper will be on the implications of each of the previous environments for (i) The coexistence of HP and LP firms in the market; exogenous and endogenous proportions of HP and LP firms will be considered, (ii) The worker turnover dynamics, for exogenously given proportions of HP and LP firms, and (iii) The distribution of wages paid and offered, for given proportions of HP and LP firms.

One important question is the following: If the contract approach described above is privately efficient for firm and worker, because they are extracting rents from future employers, why don't we see them more often in reality? As will be seen, and as is intuitive, the LP firm is in fact benefiting the worker by promising him a high wage increase when a more productive firm is found, and as a compensation for this, the worker accepts a lower wage from the LP firm. Hence, all reasons why the worker will not or cannot accept a sufficiently lower wage will imply that such contract cannot be implemented. In particular, two important reasons can be the existence of minimum wages and the imperfection of financial markers. The former case is clear, since it impedes the wage from decreasing from a determined level. The latter case would occur if the wage needed for the contract approach to be implemented falls below a minimum subsistence level. If imperfections in the financial market impede the worker from borrowing for subsistence against its (expected) increased wage, then the contract approach will not be implemented.

Section 2.2 presents the general environment in which each particular case will be embedded, as well as a summary of the results obtained. Section 2.3 addresses, for each case considered, the equilibrium outcomes regarding the wage distribution and turnover dynamics. In each case, the results of endogenising the proportions of HP and LP firms is given, by allowing firms to become HP firms at a positive investment cost, or LP at 0 investment cost. Section 2.4 looks at the survival of LP firms in each case, and Section 2.5 compares the endogenous proportion of HP and LP firms across the three cases considered. Section 2.6 concludes.

2.2 General environment and summary of the results

There are a large fixed number of workers and firms and equal to 1. Workers are homogenous. Focusing on essentials, assume there are two types of firms; high productivity (HP) and low productivity (LP). In particular, assume a proportion γ_j, $j = bm, m, c$, of firms are HP firms with productivity p_h, whereas a proportion $1 - \gamma_j$ are LP firms, with productivity $p_l < p_h$. A firm with productivity p_i generates revenue p_i per unit of time for each worker it employs, $i = h$ for HP firms, $i = l$ for LP firms. All workers, regardless of their employment status, contact firms at Poisson rate λ. Each worker obtains unemployment benefit b per unit of time while unemployed

Any worker's life in the market can be described by an exponential random variable with parameter δ. Any worker who leaves the market is replaced by another who is initially unemployed. Firms live forever. Finally, keeping things as simple as possible, assume that agents do not discount the future, i.e. we will work in the limit were the discount rate $r \to 0$. Workers maximise there expected lifetime income, whereas firms maximise steady state profits.

Summary of results

Wages Paid for given proportions of HP and LP firms

The following sequence shows the values of extreme wages paid by LP and HP firms in the $bm - case$, compared to relevant parameter values in the environment. $\underline{w}^i$ ($\bar{w}^i$) is the minimum (maximum) wage offered by a firm of type i in the $bm - case$. $i = l$ for LP firms, $i = h$ for HP firms.

$$\underline{w}^l = b < \bar{w}^l = \underline{w}^h < p_l < \bar{w}^h < p_h$$

The minimum wage in the market is paid by an LP firm, and it is equal to the unemployment benefit b. The maximum wage is paid by an HP firm, and it is strictly lower than the productivity of HP firms. The distribution of wages paid in the market is a convex combination of the distribution of wages paid by LP and HP firms, where the weights are given by the respective proportions. Because the arrival rate of firms is independent of the employment status, unemployed workers accept employment if they are offered at least what they are currently "earning" in unemployment, i.e. benefit b. This also implies that the firm offering the lowest wage in the market is also equal to b.

Notice that the existence of a wage distribution, where $\underline{w}^l = b$, implies that workers always share part of the monopoly rents created in any firm-worker contact, because they are offered more than their reservation wage.

I now add to that sequence the values of the wages paid in the $m - case$. These are those values sub-indexed by m, plus p_l and p_h. Again, super-index $i = h, l$ refers to which type of firm pays the wage.

$$w_m^h < w_m^l = \underline{w}^l = b < \bar{w}^l = \underline{w}^h < x_m^h < p_l < \bar{w}^h < p_h{}^4$$

The minimum wage in the $m - case$, w_m^h, is paid by an HP firm to workers met while unemployed; and it is lower than the minimum wage paid in the $bm - case$, $\underline{w}^l = b$ (paid by an LP firm). When firms of either type meet unemployed workers, it represents for the latter the expectation of a large wage increase when they contact another firm through on-the-job search, and both firms compete for him. This expectation allows firms to offer these workers less than what they are "earning" while unemployed, b. Because HP firms offer higher wage increase when their workers meet another firm, they are able to pay unemployed workers lower wages than LP firms do, $w_m^h < w_m^l$. The maximum wage in the $m - case$, p_h, is also paid by HP firms. Imagine a worker earning

[4] $x_m < \bar{w}^l = \underline{w}^h$ is also a possibility. But this is not central to any argument in the paper. What is important is that $b < \bar{w}^l = \underline{w}^h < p_l$ and $b < x_m < p_l$

a wage other than p_h in an HP firm. When that worker contacts another HP firm, his wage is increased to p_h through the Bertrand competition in which the firms engage. Wage p_l arises in a similar situation, but when the firms involved are both LP. Finally, wage x_m^h arises from the competition of an HP and an LP firm for a worker. The LP is willing to bid up to p_l, but the HP firm is able to attract him for a lower wage; for any given wage, the HP firm is more attractive for the worker because it offers a higher future wage increase. Therefore $x_m^h < p_l$.

In this case, notice that firms keep all the monopoly rents in any firm-unemployed worker contact, since they offer unemployed workers only their reservation wage.

I now add to the sequence the wages paid in the $c-case$. These are those sub-indexed with c, plus p_l and p_h.

$$w_c^h = w_c^l = w_m^h < w_m^l < b < \bar{w}^l = \underline{w}^h < x_m^h < x_c^h = p_l < \bar{w}^h < p_h$$

Notice that now the wages paid by HP and LP firms to workers that come from unemployment are equal, $w_c^h = w_c^l$. This is because in this environment, all firms offer the same expectation of wage increase when the worker meets other firms[5]. For this same reason, the wage that arises from the competition between an HP and an LP is $x_c^h = p_l > x_m^h$. The HP firm is not more attractive than the LP in terms of expected wage increases. Wages p_l and p_h arise for reasons analogue to those in the $m-case$. In this case firms also keep all the monopoly

[5]And this is also the reason they are equal to to w_m^h.

rents in any firm-unemployed worker meeting.

As a consequence, the distribution of wages paid becomes more spread as we move from the $bm - case$ to the $m - case$ to the $c - case$.

Worker Turnover, exogenously given proportions of HP and LP firms

In the $bm - case$, workers move from lower to higher wages. In terms of movements across firms, they can be:

(i) From an LP to an LP, provided the second offers a higher wage

(ii) From an HP to an HP, provided the second offers a higher wage.

(iii) Always from an LP to an HP, given that $\bar{\mathrm{w}}^l = \underline{\mathrm{w}}^h$

(iv) Never from and HP to an LP, for the same reason as in (iii)

In terms of social efficiency, turnovers as in (i) and (ii) are neutral[6], because they imply no change in total output. Turnovers as in (iii) are efficient, because they imply an increase in total output. Inefficient turnovers do not occur, case (iv).

In both the $m - case$ and the $c - case$, turnover never occurs from like firm to like firm[7]. In these two cases, it is also true that turnover occurs always from and LP to an HP firm, and never from an HP to an LP firm.

This implies that all the three cases considered share the feature of efficient turnover.

But there is one distinction. In the $bm - case$, workers always move to higher

[6] This is because both search and turnover are costless. It would be a different case if either was costly, but it also would be a different model.

[7] Assuming they prefer to stay with current firm if both firms offered same wage. In any case, this type of turnover would be neutral efficiency-wise

wages. In the $c-case$, workers move from lower to higher wages always, except if they move form an LP where they are earning p_l to an HP that offers $x_c^h = p_l$. In the $m - case$, it is possible the workers change jobs accepting a wage cut. This is the case if they were earning p_l in an LP firm and an HP firm attracts them by offering $x_m^h < p_l$.

Survival of LP firms in the market Assume LP firms can either remain in the market or exit the market, where π_j^l are the profits of LP firms in case $j = bm, m, c$. It is shown in the paper that:

$$\pi_c^l > \pi_m^l > \pi_{bm}^l$$

For given proportions of HP and LP firms, LP firms do better under the $c - case$ than under the $m - case$ than under the $bm - case$. This is very intuitive:

$\pi_m^l > \pi_{bm}^l$ is always true, as Postel-Vinay (2002a) put it, because in the $m-case$ all firms have a wider set of strategies than in the $bm-case$, as they can both (i) Identify unemployed workers and offer them only their reservation wage, and (ii) Match the outside offers received by their workers. Nevertheless, notice that (ii) is irrelevant for LP firms. When they enter the process of Bertrand competition with other firm by matching outside offers, they either (a) keep the worker at wage p_l if competing with another LP firm; the worker is not profitable anymore, or (b) loose the worker to a HP firm. This is relevant when addressing the equilibrium proportion of HP firms (section 2.4)

$\pi_c^l > \pi_m^l$ is always true because, as LP firms offer workers a higher wage increase when they meet an HP firm, they can exploit that to attract unemployed workers by offering lower wages ($w_c^l < w_m^l$) and make a higher profits.

Hence, given an outside option, it is more likely that LP firms survive in the $c - case$ than in the $m - case$ than in the $bm - case$.

Equilibrium proportions of HP and LP firms Assume that firms can be either LP firms at 0 cost of investment, or become HP upon a positive investment cost (k in the paper), where γ_j^* is the equilibrium proportion of HP firms in case $j = bm, m, c$. In the paper, it is shown that:

$$0 \leq \gamma_c^* \leq \gamma_{bm}^* \leq \gamma_m^* \leq 1$$

The previous subsection noted that $\pi_m^l > \pi_{bm}^l$, because in the $m - case$ firms can recognise unemployed workers and therefore offer them only their reservation wage. It was also noted that, although firms can match the outside offers received by their workers, this is not really to the advantage of LP firms, since whenever they enter a Bertrand competition with other firm they will always loose a profitable worker. This is not true for HP firms. If they are competing for a worker with a LP firm, they will keep the worker at wage $x_c^h < p_h$: The worker is still profitable for the HP firm. This implies that, as one moves from the $bm - case$ to the $m - case$, HP firms benefit more than LP firms. Hence, for a given investment level to become HP, the equilibrium proportion of HP firms is higher under the $m-case$: $\gamma_{bm} \leq \gamma_m \leq 1$.

It was also noted in the previous section that $\pi_c^l > \pi_{bm}^l$. The paper also shows that $\pi_c^h < \pi_{bm}^h$: HP firms do better under the $bm - case$ than under the $c - case$. In this latter case, they pay very low wages to unemployed workers they meet, but they loose profitable workers every time their workers meet any firm (HP or LP). Both $\pi_c^l > \pi_{bm}^l$ and $\pi_c^h < \pi_{bm}^h$ imply unambiguously that, for a given investment level to become HP, the equilibrium proportion of HP firms is higher under the $bm - case$: $\gamma_c \leq \gamma_{bm} \leq 1$.

Finally, it is shown in the paper that, in each case, there exists investment costs so high (low) that, in equilibrium, all firms are LP (HP). The levels and ranking of these investment costs are used to show formally that $0 \leq \gamma_c^* \leq \gamma_{bm}^* \leq \gamma_m^* \leq 1$.

2.3 Three different environments with on-the-job search

2.3.1 Burdett-Mortensen Environment ($bm - case$)

In the $bm - case$, each firm posts a wage it will offer any workers who contact it. The market then works as follows: When an unemployed worker contacts a firm, he either accepts the wage offer of that firm, or rejects it and continues to search unemployed. If he accepts the offer, he will earn this wage for as long as he stays with the firm. If an employed worker contacts another firm, he changes firms if the wage offered by the contacted firm is higher than his current wage. The firm is assumed not to counter the outside offers received by the worker.

The following are results derived in Burdett-Mortensen (1998):

(i)The market wage offer distribution is given by $F(w) = (1 - \gamma_{bm})F^l(w) + \gamma_{bm}F^h(w)$

where $F^i(w)$ is the distribution of wages offered by type i firms, $i = l$ for LP forms. $i = h$ for HP firms. γ_{bm} is the fraction of HP firms in the market.

$(ii)\underline{\mathrm{w}}^l = b, \bar{w}^l = \underline{\mathrm{w}}^h$,

where $\underline{\mathrm{w}}_i$ $(\bar{w}_i)$ is the lowest (highest) wage in the support of $F_i(w)$ distribution.

(iii) $F\left(\bar{w}^l\right) = F\left(\underline{\mathrm{w}}^h\right) = 1 - \gamma_{bm}$

$(iv)F^l(w)$ and $F^h(w)$ are continuous over their support, which implies that $F(w)$ is continuous at all $w \neq \bar{w}^l = \underline{\mathrm{w}}^h$, and it has a "kink" at $w = \bar{w}^l = \underline{\mathrm{w}}^h$.

(v) The distribution of wages paid in the market is given by $G(w) = \frac{\lambda}{\delta+\lambda}\frac{F(w)}{(1-F(w))}$.

(vi) Given (iv), the supply of workers for a firm offering wage w is given by:

$$l(w \mid R, F(w)) = \frac{\lambda\delta}{[\delta + \lambda(1 - F(w))]^2}$$

With these "tools", we can derive the shape of $F(w), F^l(w)$ and $F^h(w)$; the values of the minimum and maximum wages on their support: $\underline{\mathrm{w}}^l = b, \bar{w}^l = \underline{\mathrm{w}}^h$ and $\bar{w}^h$, as well as the profits obtained by firms of type $i = l, h$. This is of course just an application of results derived in Burdett-Mortensen (1998) The derivation is based on the condition that all firms of same type must earn the same profits in equilibrium.

LP **Firms** Let us commence with *LP* firms. The profits of a firm offering wage w is given by the profit per worker $(p_L - w)$ times the supply of workers in steady state, given by $l(w \mid R, F(w))$ above. $F\left(\underline{\mathrm{w}}^l = b\right) = 0$, since $F^l\left(\underline{\mathrm{w}}^l\right) = F^h\left(\underline{\mathrm{w}}^l\right) = 0:$ No lower wages can be found in the market. Hence, the profit of firms offering $w = b$ are given by

$$\begin{aligned}
\pi^l_{bm}(b) &= (p_l - b)\left[l(b \mid R, F(w))\right] \\
&= (p_l - b)\frac{\lambda\delta}{(\lambda+\delta)^2}
\end{aligned}$$

For any other firm offering a wage $w > \underline{\mathrm{w}}^l$ in the support of $F^l(w)$ it is true that $F^h(w) = 0$. Hence, for any such wage

$F(w) = (1-\gamma_{bm})F^l(w)$. The profits of such firm are given by:

$$\begin{aligned}
\pi^l_{bm}(b &\leq w \leq \bar{w}^l) = (p_l - w)\left[l(w \mid R, F(w))\right] \\
&= (p_l - w)\frac{\lambda\delta}{\left[\delta + \lambda(1-(1-\gamma_{bm})F_l(w))\right]^2}
\end{aligned}$$

Since firms of the same productivity must earn the same profits in equilibrium, $\pi^l_{bm}(b) = \pi^l_{bm}(w) = \pi^l_{bm}$, and this equation allows to solve for the shape of the distribution $F^l(w)$. Setting $\pi^l_{bm}(b) = \pi^l_{bm}(w)$ and solving implies:

$$F^l(w) = \frac{\lambda + \delta}{\lambda(1-\gamma_{bm})}\left(1 - \sqrt{\frac{p_l - w}{p_l - b}}\right)$$

Notice that when $\gamma_{bm} = 0$ (only LP firms exist), this collapses to the result in Burdett-Mortensen (1998) for only one type of firms. When $\gamma_{bm} = 1$, no LP firms exist, this expression is undetermined.

For the maximum wage offered by LP firms, $F^l(\bar{w}^l) = 1$. Substituting this in the above equation implies:

$$b < \bar{w}^l = \frac{\lambda(1-\gamma_{bm})(\lambda(1+\gamma_{bm}) + 2\delta)p_l + (\delta + \lambda\gamma_{bm})^2 b}{(\lambda+\delta)^2} < p_l$$

Again, this collapses to the result reported in Burdett-Mortensen (1998) for only one type of firm when $\gamma_{bm} = 0$. It is a matter of algebra to check that $\pi^l_{bm}(\bar{w}^l) = \pi^l_{bm}$. Hence, the profits of LP firms are given by:

$$\pi^l_{bm} = (p_l - b)\frac{\lambda\delta}{(\lambda+\delta)^2}$$

***HP* Firms** Let us now turn to HP firms. By (*ii*) and (*iii*) above, $\underline{w}^h = \bar{w}^l$ and $F\left(\bar{w}^l\right) = F\left(\underline{w}^h\right) = 1 - \gamma_{bm}$. This implies that the profits of firms offering the lowest wage in the support of $F_h(w)$ are given by:

$$\pi^h_{bm}(\underline{w}^h) = (p_h - \underline{w}^h)\frac{\lambda\delta}{(\delta + \lambda\gamma_{bm})^2}$$

Using $\underline{w}^h = \bar{w}^l$ in the above expression implies $\pi^h_{bm}(\underline{w}^h) =$

$$\frac{\lambda\delta}{(\delta+\lambda\gamma_{bm})^2}p_h - \frac{\lambda^2\delta(1-\gamma_{bm})(\lambda(1+\gamma_{bm})+2\delta)}{(\delta+\lambda\gamma_{bm})^2(\delta+\lambda)^2}p_l - \frac{\lambda\delta}{(\delta+\lambda)^2}b$$

For any other wage $w > \underline{w}^h$, (offered by HP firms), we have that $F_l(w) = 1$, so $F(w) = (1-\gamma) + \gamma F_h(w)$. This implies that the profits of any HP firm offering wage $w > \underline{w}^h$ are given by:

$$\pi^h_{bm}(\underline{w}^h \leq w \leq \bar{w}^h) = (p_h - w)\frac{\lambda\delta}{\left[\delta+\lambda\gamma_{bm}(1-F_h(w))\right]^2}.$$

Setting the equilibrium condition $\pi_h(\underline{w}^h) = \pi_h(w > w^h) = \pi_h$ allows to solve for the shape of $F_h(w)$

$$\begin{aligned} F^h(w) &= \frac{(\delta+\lambda\gamma_{bm})}{\lambda}\left(1-\sqrt{\frac{(\delta+\lambda)^2\,(p_h-w)}{\Psi}}\right) \\ \Psi &= (\delta+\lambda)^2 p_h - \lambda(1-\gamma_{bm})(\lambda(1+\gamma_{bm})+2\delta)p_l - (\delta+\lambda\gamma_{bm})^2 b \end{aligned}$$

Again, if $\gamma = 1$ (only HP firms exist), then the shape of $F^h(w)$ collapses to the result found in Burdett-Mortensen (1998) for only one type of firm in the market. When $\gamma_{bm} = 0$ the above expression is not determined, since $\gamma_{bm} = 0$ implies no HP firms in the market.

Setting $F^h(w) = 1$ allows to find the highest wage offered by HP firms, which implies $\bar{w}^h =$

$$\frac{\lambda\gamma_{bm}(\lambda\gamma_{bm}+2\delta)}{(\delta+\lambda\gamma_{bm})^2}p_h+\frac{\lambda\delta^2(1-\gamma_{bm})(\lambda(1+\gamma_{bm})+2\delta)}{(\delta+\lambda)^2(\delta+\lambda\gamma_{bm})^2}p_l+\frac{\delta^2}{(\delta+\lambda)^2}b$$

which is strictly lower than p_H.

Proposition 2.1 *Suppose firms can be either LP at no investment cost or HP at investment cost k. A proportion $0 \leq \gamma^*_{bm} \leq 1$ are HP firms in equilibrium.*

Notice that:

$$\begin{aligned}
\pi^l_{bm} &< \pi^h_{bm} \vee \gamma_{bm} \geq 0 \\
\frac{\delta\pi^l_{bm}}{\delta\gamma_{bm}} &= 0 \vee \gamma_{bm} \geq 0 \\
\frac{\delta\pi^h_{bm}}{\delta\gamma_{bm}} &< 0 \vee \gamma_{bm} \geq 0
\end{aligned}$$

This implies that, as the measure of HP firms increases (γ_{bm} increases), the profits of LP firms is unchanged and the profits of good firms decreases. Therefore, the incentive to pay cost k and become an HP decreases. For values of k within a suitable range, an interior solution can be found where $0 < \gamma^*_{bm} < 1$. One can find the critical values of k such that:

$$\begin{aligned}
(i)\ \gamma^*_{bm} &= 0 \text{ if } k \geq \bar{k}_{bm} \\
(ii)\ \gamma^*_{bm} &= 1 \text{ if } k \leq \underline{k}_{bm}
\end{aligned}$$

$$(iii)\ 0 \leq \gamma^*_{bm} \leq 1 \text{ if } \bar{k}_{bm} \leq k \leq \underline{k}_{bm}$$

To obtain the values of $\underline{k}_{bm}$ and $\bar{k}_{bm}$, note that γ^*_{bm} solves $\pi^l_{bm} = \pi^h_{bm} - k$, which implies:

$$k = (p_h - p_l)\frac{\lambda\delta}{(\lambda\gamma^*_{bm} + \delta)^2} \tag{2.1}$$

From where

$$\frac{\delta\gamma^*_{bm}}{\delta k} = (p_l - p_h)\frac{(\lambda\gamma^*_{bm} + \delta)^3}{2\lambda^2\delta} < 0.$$

Hence, as the cost of becoming an HP increases, the equilibrium proportion of HP firms decreases.

To obtain $\bar{k}_{bm}$, use $\gamma^*_{bm} = 0$ into equation (2) above. The result is:

$$\pi^l_{bm} \mid_{\gamma_{bm}=0} > \pi^h_{bm} \mid_{\gamma_{bm}=0} -k \text{ if } k > \bar{k}_{bm} = (p_h - p_l)\frac{\lambda}{\delta}.$$

Hence, if $k > \bar{k}_{bm}$ no firm has the incentive to become and HP firm even if there are no HP firms in the market.

To obtain $\underline{k}_{bm}$, use $\gamma^*_{bm} = 1$ in equation (2) above. The result is:

$$\pi^h_{bm} \mid_{\gamma_{bm}=1} -k > \pi^l_{bm} \mid_{\gamma_{bm}=1} \text{ if } k < \underline{k}_{bm} = (p_h - p_l)\frac{\lambda\delta}{(\lambda + \delta)^2}$$

Hence, if $k < \underline{k}_{bm} = (p_h - p_l)\frac{\lambda\delta}{(\lambda+\delta)^2}$ it is profitable to become an HP firm at cost k even if all firms are HP firms.

It is easy to show that $\underline{k}_{bm} < \bar{k}_{bm}$. Hence, $\underline{k}_{bm} \leq k \leq \bar{k}_{bm} \Rightarrow 0 \leq \gamma^{*}_{bm} \leq 1$.

2.3.2 Postel Vinay-Robin Environment ($m - case$)

The basic difference with respect to the $bm - case$ is that now firms may match the outside offers received by their current workers , and firms contacted by a worker observe the employment status of these worker. Given this, (i) Each firm posts a wage it will offer unemployed workers, (ii)When an unemployed worker contacts a firm, he accepts the wage offer of that firm, or rejects it and continues to search unemployed, and (iii) When an employed worker contacts another firm, the two firms involved will start a bid competition for the services of the worker.

With little loss of generality, assume that if two equally productive firms bid the same wage for the worker, the worker remains with the current employer. If a worker is indifferent between working with an HP or an LP firm, he chooses the HP firm.

In this $m - case$, $w^h_m(w^l_m)$ denotes the wage that an $HP(LP)$ firm offers a worker that contacted the firm while unemployed. x^h_m will denote the wage at which an HP firm attracts a worker that contacts her while working in an LP firm, or keeps a worker who, while working with her, contacts an LP firm. V_m denotes the return to an unemployed worker, while $V^h_m(y)$ $\left[V^l_m(y)\right]$denotes the return to a worker in an HP $[LP]$ firm earning wage y . The respective proportions of firms are now denoted γ_m and $1 - \gamma_m$. Then:

(i) Suppose a worker is unemployed, enjoying return V_m. He is receiving unemployment benefit b. If he contacts an $HP(LP)$ firm, he will accept the wage offer $w_m^h \left[w_m^l\right]$ provided that $V_m^h(w_m^h) \geq V_m \left[V_m^l(w_m^l) \geq V_m\right]$. The worker will reject the offer and continue search otherwise. Assuming the previous two conditions hold, V_m solves:

$$rV_m = b + \lambda\gamma_m \left[V_m^h(w_m^h) - V_m\right] + \lambda(1-\gamma_m)\left[V_m^l(w_m^l) - V_m\right] + \delta\left[0 - V_m\right]$$

(ii) Suppose a worker is employed at an LP firm at wage w_m^l. He is enjoying a return $V_m^l(w_m^l)$. If he contacts an HP firm, both firms bid for his services. The LP firm is willing to bid up to a wage p_l. Being more productive, the HP firm is able to attract the worker, and she does this by offering a wage at which the worker is indifferent between staying with the LP firm or moving to the HP firm. This wage we denoted x_m^h. The worker then enjoys $V_m^h(x_m^h)$. If he contacts another LP firm, the two firms bid up to p_l for his services. The worker then remains with the current firm, with an increased wage p_l, and starts enjoying $V_m^l(p_l)$. Hence, $V_m^l(w_m^l)$ solves:

$$\begin{aligned} rV_m^l(w_m^l) &= w_m^l + \lambda\gamma_m\left[V_m^h(x) - V_m^l(w_m^l)\right] + \\ &\quad \lambda(1-\gamma_m)\left[V_m^l(p_l) - V_m^l(w_m^l)\right] + \delta\left[0 - V_m^l(w_m^l)\right] \end{aligned}$$

(iii) Suppose a worker is at an LP firm at wage p_l. He is enjoying $V_m^l(p_l)$. If he contacts an LP firm, he stays with the current employer at the same wage, since both firms bid up to a wage p_l. He continues enjoying $V_m^l(p_l)$. If he contacts an HP firm, the HP firm attracts him by offering wage x_m^h. The worker changes firms and starts enjoying $V_m^h(x_m^h)$. $V_l(p_l)$ solves

$$rV_m^l(p_l) = p_l + \lambda\gamma_m \left[V_m^h(x) - V_m^l(p_l)\right] + \delta\left[0 - V_m^l(p_l)\right]$$

(iv) Suppose the worker is employed at an HP firm at wage w_m^h. He is enjoying $V_m^h(w_m^h)$. If he contacts an HP firm, both firms bid for him so the worker stays with the current employer at a wage bid up to p_h. He starts enjoying $V_m^h(p_h)$. If he contacts an LP firm, the two firms bid for him, and the worker stays with the HP firm at the wage x_m^h that makes him indifferent between firms. He starts enjoying $V_m^h(x_m^h)$. Hence

$$\begin{aligned} rV_m^h(w_m^h) &= w_m^h + \lambda\gamma_m \left[V_m^h(p_h) - V_m^h(w_m^h)\right] + \\ &\quad \lambda(1-\gamma_m)\left[V_m^h(x_m^h) - V_m^h(w_m^h)\right] + \delta\left[0 - V_m^h(w_m^h)\right] \end{aligned}$$

(v) Suppose a worker is employed at an HP firm at wage x_m^h. He is enjoying $V_m^h(x_m^h)$. If he contacts an LP firm, the HP firm can keep him at the same wage. The worker remains enjoying $V_m^h(x_m^h)$. If he contacts an HP firm, both firms bid a wage up to p_h and the worker stays with the current employer. He starts enjoying $V_m^h(p_h)$. Hence, $V_m^h(x_m^h)$ solves:

$$rV_m^h(x_m^h) = x_m^h + \lambda\gamma_m \left[V_m^h(p_h) - V_m^h(x_m^h)\right] + \delta\left[0 - V_m^h(x_m^h)\right]$$

(iv) Suppose a worker is employed at an HP firm at wage p_h. He is enjoying $V_m^h(p_h)$. In this case he ignores all alternative offers and he earns p_h until he leaves the market. Hence, $V_m^h(p_h)$ solves:

$$rV_m^h(p_h) = p_h + \delta\left[0 - V_h(p_h)\right]$$

Each of the above equations can be solved individually. To focus on the case where workers do not discount the future, we take the limit as $r \longrightarrow 0$. The results are:

$$V_m = \frac{b + \lambda\gamma_m V_m^h(w_m^h) + \lambda(1-\gamma_m)V_m^l(w_m^l)}{\delta + \lambda} \tag{2.2}$$

$$V_m^l(w_m^l) = \frac{w_m^l + \lambda\gamma_m V_m^h(x_m^h) + \lambda(1-\gamma_m)V_m^l(p_l)}{\delta + \lambda} \tag{2.3}$$

$$V_m^l(p_l) = \frac{p_l + \lambda\gamma_m V_m^h(x_m^h)}{\lambda\gamma_m + \delta} \tag{2.4}$$

$$V_m^h(w_m^h) = \frac{w_m^h + \lambda\gamma_m V_m^h(p_h) + \lambda(1-\gamma_m)V_m^h(x_m^h)}{\delta + \lambda\gamma_m + \lambda(1-\gamma_m)} \tag{2.5}$$

$$V_m^h(x_m^h) = \frac{x_m^h + \lambda\gamma_m V_m^h(p_h)}{\delta + \lambda\gamma_m} \tag{2.6}$$

$$V_m^h(p_h) = \frac{p_h}{\delta} \tag{2.7}$$

Equilibrium behaviour pins down the endogenous wages w_m^h, w_m^l, x_m^h. In particular, when a firm of either type contacts an unemployed worker, she will offer him a wage such that he is indifferent to accepting it or not. A lower wage implies that the worker will not accept. A higher wage just decreases the profit of the firm unnecessarily. Hence, the first two equilibrium conditions are:

$$V_m^l(w_m^l) = V_m \tag{2.8}$$

$$V_m^h(w_m^h) = V_m \tag{2.9}$$

We need one more equilibrium condition to pin down the value of x_m^h. This comes from the bidding game between an LP and an HP firm. LP firms are willing to bid up to a wage equal to p_l in order to keep the worker. This would leave the worker enjoying $V_m^l(p_l)$. The HP firm must offer a wage to the worker such that he prefers to work in the HP firm at wage x_m^h to working in an LP firm at wage p_l. Hence, the last equilibrium condition is given by

$$V_m^h(x_m^h) = V_m^l(p_l) \tag{2.10}$$

This determines a system of 9 equations (2-10) and 9 unknowns: the six values corresponding to the returns to the different status:

$$V_m^h(p_h), V_m^h(x_m^h), V_m^h(w_m^h), V_m^l(p_l), V_m^l(w_m^l), V_m$$

and the three wages endogenous wages:

$$w_m^l, w_m^h, x_m^h$$

The solution to the system is simple, mainly because the wages w_l, w_h are such that the unemployed workers see no profit when moving into employment, regardless the type of the firm. Hence,

$$\begin{aligned}
V_m^h(p_h) &= \frac{p_h}{\delta} \\
V_m^h(x_m^h) &= \frac{p_l(\delta + 2\lambda\gamma_m) + \lambda^2\gamma_M^2\frac{p_h}{\delta}}{(\lambda\gamma_m + \delta)^2} \\
V_m^l(p_l) &= \frac{p_l(\delta + 2\lambda\gamma_m) + \lambda^2\gamma_m^2\frac{p_h}{\delta}}{(\lambda\gamma_m + \delta)^2} \\
V_m^h(w_m^h) &= V_m^l(w_m^l) = V_m = \frac{b}{\delta} \\
w_m^l &= \frac{\lambda + \delta}{\delta}b - \frac{\lambda(2\lambda\gamma_m + \delta)}{(\lambda\gamma_m + \delta)^2}p_l - \frac{\lambda^3\gamma_m^2}{(\lambda\gamma_m + \delta)^2}p_h \\
w_m^h &= \frac{\lambda + \delta}{\delta}b - \frac{\lambda(1 - \gamma_m)}{\lambda\gamma_m + \delta}p_l - \frac{\lambda\gamma_m(\lambda + \delta)}{\delta(\lambda\gamma_m + \delta)}p_h < w_l \\
x_m^h &= \frac{2\lambda\gamma_m + \delta}{\lambda\gamma_m + \delta}p_l - \frac{\lambda\gamma_m}{\lambda\gamma_m + \delta}p_h < p_l
\end{aligned}$$

Notice two things in particular. First, $w_m^h < w_m^l$. HP Firms can offer unemployed workers a lower wage in order to make them indifferent between unemployment and accepting the offer. This is due to the prospect of meeting an HP firm in the future. If the worker is in an HP firm, and contacts another HP firm, both firms will bid up to a wage p_h, and the worker will remain with the current employer at wage p_h. On the other hand, if the worker is in an LP firm and contacts and HP firm, the LP firm will bid up to a wage only equal to p_l, and the HP firm will attract him at wage $x_m^h < p_h$. This implies that, for a given wage, it is more attractive to work in an HP firm than in an LP firm, since working in HP firms yields a wage increase when an HP is met. HP firms use this to offer unemployed workers a wage $w_m^h < w_m^l$.

Second, $x < p_l$. The argument is exactly the same. When HP and LP firms bid for a worker, the LP firm is willing to bid up to a wage p_l. But because HP firms yield a higher wage increase as above, so the HP firm can attract the worker at a wage lower than p_l, this wage is $x_m^h < p_l$

In addition to the three endogenous wages, there are other two wages observed in the market, p_h and p_l.

Two other facts are worth noticing. As in Burdett and Coles (2003), some workers obtain a wage increase without changing jobs. As in Postel-Vinay and Robin (2002), some workers change jobs accepting a wage decrease. These are those that move from LP firms earning p_l to HP firms earning x_m^h.

It is not surprising that the equilibrium outcome resembles the Diamond Paradox (in the sense that unemployed workers are offered just their reservation wage) even in the presence of on-the-job search. After all, the wage posting game only occurs vis-a-vis unemployed workers. Notice that, although the wages posted are just high enough to make unemployed workers indifferent to accepting the job offer, those wages are not the only wages paid in the market. The dispersion of wages posted is reduced, but the dispersion of wages paid is increased compared to the $bm - case$. In the $bm - case$, the wages posted (and paid) range from the lowest wage in the market $w_m^l = b$, to a highest wage which is strictly lower than p_h. In the $m - case$ considered in this section, the wages posted are more concentrated at low values $w_m^h < w_m^l < b$. But the wages paid range from the low value of $w_m^h < b$ up to a highest wage equal to the productivity in HP firms, p_h.

An interesting issue is the analysis not only of the market wage dispersion, but the dispersion of wages within firm types. Notice that in the $bm - case$, the lowest wage in the market is offered and paid by LP firms, while the maximum wage is offered and paid by HP firms. In fact, the minimum wage paid by HP firms is equal to the maximum wage paid by LP firms. In contrast, in this $m - case$, the lowest and highest wage in the market are paid by HP firms ($w_m^h < b$ and $p_h > \bar{w}^h$ respectively). The distribution of wages paid by HP firms is more dispersed in the $m - case$. The same is true for LP firms. In the $m - case$ they pay a lowest wage lower than the minimum wage under the $bm - case$ ($w_m^l < b$), and a highest wage higher than the maximum wage paid by LP firms under the $bm - case$ ($p_l > \bar{w}^l$).

Steady States Considering the above dynamics, we can characterise the steady state stocks of workers in each possible state in the market.

(i) Steady state stock of unemployed workers: μ_m

Unemployed workers contact a firm at rate λ, so $\mu_m\lambda$ represents the flow out of unemployment. Employed workers loose employment if they leave the market, which occurs at rate δ. $(1-\mu_m)\delta$ represents the flow into unemployment:

$$\mu_m\lambda = (1-\mu_m)\delta$$

$$\mu_m = \frac{\delta}{\lambda+\delta}$$

(ii) Steady state stock of workers in LP firms: e^l_m

Unemployed workers contact HP firms at rate $\lambda(1-\gamma_m)$, so $\mu_m\lambda(1-\gamma_m)$ represents the flow of workers into LP firms. Workers in LP firms leave those firms either because they leave the market (at rate δ) or because they contact an HP firm and leave the LP firm (at rate $\lambda\gamma_m$). The flow of workers out of LP firms is then $e^l_m(\delta+\lambda\gamma_m)$:

$$\mu_m\lambda(1-\gamma_m) = e^l_m(\delta+\lambda\gamma_m)$$

$$e^l_m = \frac{\lambda(1-\gamma_m)}{\delta+\lambda\gamma_m}\frac{\delta}{\delta+\lambda}$$

(iii) Steady state stock of workers at HP firms: e^h_m

Unemployed workers and workers in LP firms contact HP firms at rate $\lambda\gamma_m$. In both cases, the workers move to the HP firm, so the flow of workers into HP firms is given by $(\mu_m+e^l_m)\lambda\gamma_m$. Workers loose employment on HP firms only if they leave the market, so the flow of workers out of HP firms is given by $e_m^h\delta$:

$$(\mu_m + e_m^l)\lambda\gamma_m = e_m^h\delta$$

Using e_m^l and μ_m from above,

$$e_m^h = \frac{\lambda\gamma_m(\lambda+\delta)}{(\delta+\lambda\gamma_m)\delta}\frac{\delta}{\delta+\lambda}$$

(iv) Steady state stock of workers in LP firms earning w_m^l: e_m^1

Workers in LP firms earn wage w_m^l only if they contacted the firm while unemployed, and the firm offered them only the reservation wage w_m^l. Since unemployed workers contact LP firms at rate $\lambda(1-\gamma_m)$, the flow of worker into LP firms earning w_m^l is given by $\mu_m\lambda(1-\gamma_m)$. Workers in LP firms earning w_m^l loose that status either because they meet another LP firm, and their wage is bid up to p_l, or because they contact an HP firm and change firms at wage x_m^h. They also loose the status if they leave the market. Hence, the flow of workers out of stock e_m^1 is given by $(\delta+\lambda)e_m^1$:

$$\mu_m\lambda(1-\gamma_m) = (\delta+\lambda)e_m^1$$

$$e_m^1 = \frac{\lambda\delta(1-\gamma_m)}{(\lambda+\delta)^2}$$

(v) Steady state stock of workers in HP firms earning w_m^h : e_m^2

Workers in HP firms earn wage w_m^h only if they contacted the firm while unemployed, and the firm offered only the reservation wage w_m^h. Since unemployed workers contact HP firms at rate $\lambda\gamma_m$, the flow of worker into HP firms earning w_m^h is given by $\mu_m\lambda\gamma_m$. Workers in HP firms earning w_m^h loose that status either because they meet another HP firm, and their wage is bid up to p_h, or because they meet an LP firm and their is bid up to x_m^h. Hence, the flow of workers out of stock e_m^2 is given by $(\delta + \lambda)e_m^2$

$$\mu_m\lambda\gamma_m = e_m^2(\delta + \lambda)$$

$$e_m^2 = \frac{\lambda\delta\gamma_m}{(\lambda + \delta)^2}$$

(vi) Steady state stock of workers in HP firms earning $p_h : e_m^3$

Workers flow into the stock of HP firms earning p_h if they are working in an HP, not earning p_h, and contact an HP firm, so their wage is bid up to p_h. This flow is then given by $(e_m^h - e_m^3)\lambda\gamma_m$. Workers in HP firms earning p_h loose their status only if they leave the market, so the flow out of stock e_m^3 is represented by $e_m^3\delta$.

$$(e_m^h - e_m^3)\lambda\gamma_m = e_m^3\delta$$

Using e_m^{hp} from above,

$$e_m^3 = \frac{\lambda^2\gamma_m^2}{(\lambda\gamma_m + \delta)^2}$$

(*vii*) Steady state stock workers in HP firms earning $x_m^h : e_m^4$

Workers in HP firms are earning either wage w_l^h, p_h or x_m^h. Hence, the stock of workers in HP firms earning x_m^h is just equal to the total stock of workers in HP firms (e_m^h) minus those earning w_l^h or $p_h : (e_m^2 + e_m^3)$ Hence:

$$e_m^4 = e_m^h - e_m^2 - e_m^3$$

$$e_m^4 = \frac{\lambda^2\gamma_m\delta(1-\gamma_m)(2\delta+\lambda(\gamma_m+1)}{(\lambda+\delta)^2(\delta+\lambda\gamma_m)^2}$$

(*viii*) Steady state stock of workers in LP firms earning $p_l : e_m^5$

This stock is fueled by workers in LP firms earning w_m^l that find LP firms $e_m^1\lambda(1-\gamma_m)$, and is decreased by those in LP firms earning p_l that die or find an HP firm $(e_m^5(\delta+\lambda\gamma_m))$. Hence:

$$\begin{aligned} e_m^1\lambda(1-\gamma_m) &= e_m^5(\delta+\lambda\gamma_m) \\ e_m^5 &= \frac{\delta\lambda^2(1-\gamma_m)^2}{(\delta+\lambda\gamma_m)^2(\delta+\gamma_m)^2} \end{aligned}$$

LP firms LP firms have workers that earn a wage p_l and workers that earn a wage w_m^l. They only make profits on the second group. If the measure of workers on LP firms earning w_m^l is given by e_m^1, then the measure of workers per firm is given by $\frac{e_m^1}{1-\gamma_m} = \frac{\lambda\delta}{(\delta+\lambda)^2}$. Hence, the profits of individual LP firms is given by, where

$$\pi_m^l = (p_l - w_m^l)\frac{\lambda\delta}{(\delta+\lambda)^2}$$

Using w_m^l from above, this reduces to:

$$\pi_m^l = \frac{-\lambda}{\delta+\lambda}b + \frac{\lambda\delta(\lambda^2\gamma_m^2 + (\delta+\lambda)(\delta+2\lambda\gamma_m)}{(\lambda\gamma_m+\delta)^2(\delta+\lambda)^2}p_l + \frac{\lambda^4\gamma_m^2}{(\lambda\gamma_m+\delta)^2(\delta+\lambda)^2}p_h$$

HP **firms** *HP* firms have workers earning w_m^h, workers earning x_m^h and workers earning p_h. They only make profits on the first two groups. The measure of workers on *HP* firms earning w_m^h is given by e_m^2; and of those earning x_m^h is given by e_3. The respective measures per firm are thus $\frac{e_m^2}{\gamma_m} = \frac{\lambda\delta}{(\lambda+\delta)^2}$ and $\frac{e_m^4}{\gamma_m} = \frac{\lambda^2\delta(1-\gamma_m)(2\delta+\lambda(\gamma_m+1)}{(\lambda+\delta)^2(\delta+\lambda\gamma_m)^2}$. Hence, the profit of each individual *HP* firm is given by:

$$\pi_m^h = (p_h - w_m^h)\frac{\lambda\delta}{(\lambda+\delta)^2} + (p_h - x_m^h)\frac{\lambda^2\delta(1-\gamma_m)(2\delta+\lambda(\gamma_m+1)}{(\lambda+\delta)^2(\delta+\lambda\gamma_m)^2}$$

The use of w_m^h and x_m^h from above implies

$$\begin{aligned}\pi_h &= \frac{-\lambda}{\lambda+\delta}b + \frac{\lambda^2\delta\left[-1+\gamma_m\right]\left[\lambda\gamma_m(\lambda\gamma_m+2\lambda+3\delta)+\delta(\lambda+\delta)\right]}{(\delta+\lambda\gamma_m)^3(\lambda+\delta)^2}p_l + \\ &\quad \frac{\lambda\gamma_m\left[\lambda^2\gamma_m(\lambda\gamma_m+2\delta)+\delta(\lambda+2\delta)(2\lambda+\delta)\right]+\delta^2(\lambda+\delta)^2}{(\delta+\lambda\gamma_m)^3(\lambda+\delta)^2}\lambda p_h\end{aligned}$$

Proposition 2.2 *Suppose firms can be either LP at no investment cost or HP at investment cost k. A proportion* $0 \le \gamma_m^* \le 1$ *are HP firms in equilibrium.*

The conjecture is easy to prove formally, by noting that

$$\begin{aligned}
\pi_m^h &> \pi_m^l \\
\frac{\partial \pi_m^l}{\partial \gamma_m} &= \frac{2\lambda^4 \delta (p_h - p_l)\gamma_m}{(\lambda\gamma_m + \delta)^3(\delta+\lambda)^2} > 0 \\
\frac{\partial \pi_m^h}{\partial \gamma_m} &= \frac{\lambda^2\delta\left[\lambda^3(\gamma_m - 4)\gamma_m - \delta(\delta\lambda + 4\delta\lambda\gamma_m + 6\lambda^2\gamma_m + \lambda^2 + \delta^2)\right]}{(\delta+\lambda\gamma_m)^4(\lambda+\delta)^2}(p_h - p_l) \\
\frac{\partial \pi_m^h}{\partial \gamma_m} &< 0 \text{ for } 0 < \gamma_m < 1
\end{aligned}$$

This implies that as the stock of HP firms increases, (γ_m increases), it is both true that the profits of individual HP firms decrease and the profits of individual LP firms increase, and therefore the incentives to invest k and become an HP firm decrease. This ensures that, for a high enough k, and interior solution will be found where the equilibrium proportion of HP firms is such that $0 < \gamma_m^* < 1$. [8]

One can find the values of investment cost k such that:

[8] Regarding the profits of HP firms: As γ_m increases, $\frac{e_m^2}{\gamma_m}$ remains constant since the flow out of stock e_m^2 is independent of γ_m, $\frac{e_m^3}{\gamma_m}$ decreases since the rate at which a worker on an HP firm contacts an HP is higher. Both wages x_m and w_m^h decrease, precisely because the rate at which HPs are met is higher and this is attractive for workers because their wage is bid up to p_h. On the whole, this four effects are such that $\frac{\partial \pi_m^h}{\partial \gamma_m} < 0$.

Regarding the profits of LP firms: As γ_m increases, $\frac{e_m^1}{1-\gamma_m}$ remains constant since the flow out of stock $e_m^1$is independent of γ_m. As γ_m increases, the wage w_m^l decreases and this increases the profits of LP firms. w_m^l decreases because, as w_m^h decreases, the return V_m to an unemployed worker decreases, and therefore the wage w_m^l to make an unemployed worker indifferent also decreases.

$$
\begin{aligned}
(i)\gamma^*_m &= 0 \text{ if } k \geq \bar{k}_m \\
(ii)\gamma^*_m &= 1 \text{ if } k \leq \underline{k}_m \\
(iii)0 &< \gamma^*_m < 1 \text{if } \underline{k}_m < k < \bar{k}_m
\end{aligned}
$$

First, γ^*_m solves $\pi^h_m - k = \pi^l_m$. Using π^h_m and π^l_m from above, this implies

$$
k = -\frac{\lambda\delta(2\lambda\delta^2\gamma^*_m + 5\lambda^2\delta\gamma^*_m + \lambda^3\left[\gamma^*_m\right]^2 + 2\lambda^3\gamma^*_m + 2\lambda\delta^2 + \delta\lambda^2 + \delta^3)}{(\lambda\gamma^*_m + \delta)^3(\delta + \lambda)^2}(p_l - p_h) \tag{2.11}
$$

from where

$$
\frac{\partial\gamma^*_m}{\partial k} = \frac{(\lambda\gamma_m + \delta)^4(\delta + \lambda)^2}{\lambda^2\delta(p_l - p_h)(4\lambda\delta^2\gamma_m + \delta^3 + 8\lambda^2\delta\gamma_m + \lambda\delta^2 + \lambda^3\gamma^2_m + 4\lambda^3\gamma_m + \delta\lambda^2)} < 0
$$

This implies that, as the cost becoming HP increases, the equilibrium proportion of HP firms decreases.

For case (i), we use $\gamma^*_m = 0$ in equation 11. This yields

$$
\pi^l_m \mid_{\gamma_{m=0}} > \pi^h_m \mid_{\gamma_{m=0}} -k \text{ if } k \geq \bar{k}_m = \frac{\lambda}{\delta}(p_h - p_l)
$$

In other words, if $k \geq \bar{k}_m$ there is no incentive for any firm to become and HP even if no firms are HP. There is a corner solution where $\gamma^*_m = 0$

For case (ii), we use $\gamma^*_m = 1$ in equation for 11. This yields:

$$\pi_m^h \mid_{\gamma_m=1} -k > \pi_m^l \mid_{\gamma_m=1} \Longrightarrow k \leq \underline{k}_m = \frac{\lambda\delta\left[3\lambda(\lambda+\delta)+\delta^2\right]}{(\lambda+\delta)^4}(p_h - p_l)$$

Hence, if $k \leq \underline{k}_m$ firms have an incentive to become HP even when all firms are HP. There is a corner solution where $\gamma_m^* = 1$.

It is easy to show that $\underline{k}_m < \bar{k}_m$.

For $\underline{k}_m < k < k_m$, case (iii) holds and $0 < \gamma_m^* < 1$

2.3.3 A contract approach: The $c-case$

Consider the $m-case$ with one exception. Assume that LP firms offer a contract to unemployed workers that specifies:

(i) The wage initially paid to the worker

(ii) A guarantee that it will bid strategically, up to p_h (not p_l) should the worker contact an HP firm.

The contracts in (ii) are forward-looking contracts, as opposed to those offered in the previous cases, in the sense that they consider the future possibilities presented by the workers on-the-job search effort.

With this contract, the increased attractiveness for workers of HP firms compared to LP firm in terms of expected future wages is lost. Now any employed workers' wage will be bid up to p_h when they contact an HP, regardless of the type of the current employer. Hence, the returns to each possible state must be slightly modified. All endogenous variables are subindexed c (for contract) instead of m.

The return to an unemployed worker remains as before in terms of the corresponding endogenous and exogenous variables:

$$V_c = \frac{b + \lambda\gamma_c V_c^h(w_c^h) + \lambda V_c^l(w_c^l) - \lambda\gamma_c V_c^l(w_c^l)}{\delta + \lambda} \tag{2.12}$$

The return to a worker earning wage p_l on an LP firm changes. Given the new contract, a worker in an LP firm earning wage p_l moves to HP firms but at wage equal to p_h, not x_c^h.

$$V_c^l(p_l) = \frac{p_l + \lambda\gamma_c V_c^h(p_h)}{\lambda\gamma_c + \delta} \tag{2.13}$$

The return to a worker earning w_c^l at an LP firm changes, for the same reason. Hence,

$$V_c^l(w_c^l) = \frac{w_c^l + \lambda\gamma_c V_c^h(p_h) + \lambda(1-\gamma_c)V_c^l(p_l)}{\delta + \lambda} \tag{2.14}$$

The remaining returns are unchanged in terms of the corresponding endogenous and exogenous variables:

$$\begin{aligned}
V_c^h(w_c^h) &= \frac{w_c^h + \lambda\gamma_c V_c^h(p_h) + \lambda(1-\gamma_c)V_c^h(x_c^h)}{\delta + \lambda\gamma_c + \lambda(1-\gamma_c)} && (2.15)\\
V_c^h(x_c^h) &= \frac{x_c^h + \lambda\gamma V_c^h(p_h)}{\delta + \lambda\gamma_c} && (2.16)\\
V_c^h(p_h) &= \frac{p_h}{\delta} && (2.17)
\end{aligned}$$

Again, equilibrium behaviour pins down the endogenous wages w_c^h, w_c^l, x_c^h. The equilibrium conditions are analogue to the previous case:

$$
\begin{aligned}
V_c^l(w_c^l) &= V_c && (2.18)\\
V_c^h(w_c^h) &= V_c && (2.19)\\
V_c^h(x_c^h) &= V_c^l(p_l) && (2.20)
\end{aligned}
$$

This determines a system of 9 equations (12-20) and 9 unknowns: the six values corresponding to the returns to the different status:

$$V_c^h(p_h), V_c^h(x_c^h), V_c^h(w_c^h), V_c^l(p_l), V_c^l(w_c^l), V_c$$

and the three wages endogenous:

$$w_c^l, w_c^h, x_c^h$$

The solution to the system yields:

$$
\begin{aligned}
V_c^h(p_h) &= \frac{p_h}{\delta}\\
V_c^h(x_c^h) &= \frac{x_c^h + \lambda\gamma_c \frac{p_h}{\delta}}{\lambda\gamma_c + \delta}\\
V_c^l(p_l) &= \frac{p_l + \lambda\gamma_c \frac{p_h}{\delta}}{\lambda\gamma_c + \delta}
\end{aligned}
$$

$$\begin{aligned}
V_c^h(w_c^h) &= V_c^l(w_c^l) = V_c = \frac{b}{\delta} \\
w_c^h &= w_c^l = \frac{\delta+\lambda}{\delta} b - \frac{\lambda(1-\gamma_c)}{\lambda\gamma_c+\delta} p_l - \frac{\lambda\gamma_c(\delta+\lambda)}{\delta(\lambda\gamma_c+\delta)} p_h \\
x_c^h &= p_l
\end{aligned}$$

Different from the previous case, now the wages posted by HP firms for unemployed workers is equal to the wage posted by LP firms. Further the wage at which HP firms attract workers from LP firms (x_c^h) is equal to the wage LP firms are willing to bid for them (p_l). The intuition was already advanced. With the new contract, the increased attractiveness of HP firms compared to LP firms in terms of expected future wages is lost. While it is still true that some workers see their wage increased without changing jobs, it is now not the case that some workers change jobs and accept a wage decrease. The discussion on the effect on the wage distribution is relegated to the next sub-section of steady states stocks, since it is the change in this values what determines the change in the market wage distribution.

Steady States The $c-case$ represents a slightly modified dynamics. Although the flows into and out of each type of firms remains, the flows into and out of some wage status changes slightly.

The flows in and out the stocks of unemployment, workers in LP firms, workers in LP firms earning w_c^l, workers in LP firms earning p_l, workers in HP firms and workers in HP firms earning w_c^h remains. This is because the new contract does not affect them directly. Hence

$\mu_c = \mu_m \quad e_c^1 = e_m^1$

$e_c^l = e_m^l \quad e_c^2 = e_m^2$

$e_c^h = e_m^h \quad e_c^5 = e_m^5$

(i) Steady state stock of workers in HP firms earning $x_c^h = p_l : e_c^4$

The flow into the stock of workers in HP firms earning $x_c^h = p_l$ changes. Now workers move into stock e_c^4 only if, while working in an HP firm earning w_c^h, they contact an LP firm and their wage is bid up to x_c^h. Hence, the flow into stock e_c^4 is given by $e_c^2\lambda(1-\gamma_c)$. Workers flow out of stock e_c^4 if they leave the market or if they contact an HP firm and their wage is bid up to p_h. This is given by $e_c^4(\delta + \lambda\gamma_c)$.

$$e_c^2\lambda(1-\gamma_c) = e_c^4(\delta + \lambda\gamma_c)$$

and using e_c^2

$$e_c^4 = \frac{\lambda^2\gamma_c\delta(1-\gamma_c)}{(\lambda+\delta)^2(\delta+\lambda\gamma_c)}$$

(ii) Steady state stock of workers in HP firms earning p_h : e_c^3

Workers flow into stock e_c^3 if they contact an HP firm either while they are in an LP firm or when they are in an HP firm not earning p_h. The flow is therefore given by $(e_c^2 + e_c^4 + e_c^l)\lambda\gamma_c$. Workers flow out of stock e_c^4 only if they leave the market.

$$(e_c^2 + e_c^4 + e_c^l)\lambda\gamma_c = e_c^3\delta$$

$$e_c^3 = \frac{\lambda^2\gamma_c}{(\delta + \lambda\gamma_c)(\delta + \gamma_c)}$$

Notice what this means in terms of the dispersion of wages earned compared to the $m - case.$ The distribution of wages paid by LP firms remains intact, since all that the strategic bidding does is affect the wages paid by HP firms. In fact, the measure of workers earning p_h is higher in the $c - case$ than in the $m - case$ $(e_c^3 > e_m^3)$. For the same reason, the measure of workers earning middle wages in HP firms decreases $e_c^4 < e_m^4$. In fact, it is easy to show that $e_c^3 - e_m^3 = e_m^4 - e_c^4$., i.e. those workers who are not earning middle wages in HP firms are now earning p_h at HP firms. This is just one of the effect of the strategic bidding on the dispersion of wages earned in HP firms. The other is that the middle wages paid by HP firms increase in the $c - case$: $x_c^h = p_h > x_m^h$. The distribution has become more spread, as the measure of high earners in HP firms has increased.

LP **firms** Again, LP firms only make profits on the workers earning p_l, and not on those earning w_c^l. With the total measure of workers in LP firms earning w_c^l equal to e_c^1, the measure of workers per firm is given by $\frac{e_c^1}{1-\gamma_c} = \frac{\lambda\delta}{(\lambda+\delta)^2}$. The profits of an individual LP firm are

$$\pi_c^l = (p_l - w_c^l)\frac{\lambda\delta}{(\lambda+\delta)^2}$$

Which, using w_c^l from above, reduces to

$$\pi_c^l = -\frac{\lambda}{\delta+\lambda}b + \frac{\lambda\delta}{(\delta+\lambda)(\lambda\gamma_c+\delta)}p_l + \frac{\lambda^2\gamma_c}{(\delta+\lambda)(\lambda\gamma_c+\delta)}p_h$$

HP **firms** HP firms make profits only on those workers earning w_c^h or $x_c^h = p_l$. The corresponding measures of workers per firm are given by $\frac{e_c^2}{\gamma_c}$ and $\frac{e_c^3}{\gamma_c}$. The profits of an HP firm are given by:

$$\pi_c^h = (p_h - w_c^h)\frac{e_c^2}{\gamma_c} + (p_h - p_l)\frac{e_c^3}{\gamma_c}$$

Using w_c^h, e_c^2 and e_c^3 implies

$$\pi_c^h = \frac{\lambda(p_h - b)}{\delta+\lambda}$$

Proposition 2.3 *Suppose firms can be either LP at no investment cost or HP at investment cost k.A proportion* $0 \leq \gamma_c^* \leq 1$ *are HP firms in equilibrium.*

Formally, the proof follows the same lines as that of Propositions 2.1 and 2.2.

$$\pi_c^h > \pi_c^l$$

$$\frac{\partial \pi_c^l}{\partial \gamma_c^*} = \frac{\lambda^2 \delta}{(\delta+\lambda)(\lambda\gamma_c^*+\delta)^2}(p_h - p_l) > 0$$
$$\frac{\partial \pi_c^h}{\partial \gamma_c^*} = 0$$

This implies that as the stock of HP firms increases, (γ_c increases), the profits of individual HP firms is unchanged and the profits of individual LP firms increase, and therefore the incentives to invest k and become an HP firm decrease. This ensures that, for a high enough k, and interior solution will be found where the equilibrium proportion of HP firms is such that $0 < \gamma_c^* < 1$.[9] One can find the values of investment cost k such that

$$\begin{aligned}(i)\gamma_c^* &= 0 \text{ if } k \geq \bar{k}_c \\ (ii)\gamma_c^* &= 1\text{if } k \leq \underline{k}_c \\ (iii)\ 0 &< \gamma_c^* < 1\text{if } k_c < k < \bar{k}_c\end{aligned}$$

First, γ_c^* solves $\pi_c^h - k = \pi_c^l$. Using π_c^h and π_c^l from above, this implies

$$k = \frac{\lambda\delta}{(\delta+\lambda)(\lambda\gamma_c+\delta)}(p_h - p_l) \tag{2.21}$$

[9]Regarding the profits of HP firms: As γ increases, $\frac{e_2}{\gamma}$ remains constant since the flow out of stock e_2 is independent of γ, $\frac{e_3}{\gamma}$ decreases since the rate at which a worker on an HP firm contacts an HP is increases with γ. Wage w_h decreases, precisely because the rate at which HPs are met is higher, and this is attractive for workers because their wage is bid up to p_h. One can prove that the positive effect on profits of the decrease in w_h exactly offsets the negative effec on profits of the decrease in $\frac{e_3}{\gamma}$.

from where

$$\frac{\partial \gamma_c^*}{\partial k} = \frac{(\delta+\lambda)(\lambda\gamma_c+\delta)^2}{\lambda^2\delta}\frac{1}{(p_l-p_h)} < 0$$

Hence, as the cost to become an HP increases, the equilibrium proportion of HP firms decreases.

For case (i), we use $\gamma_c^* = 0$ into equation 21. This yields

$$\pi_c^l \mid_{\gamma_c=0} > \pi_c^h \mid_{\gamma_c=0} -k \text{ if } k \geq \bar{k}_c = \frac{\lambda}{\delta+\lambda}(p_h - p_l)$$

In other words, if $k \geq \bar{k}_c$ there is no incentive for any firm to become and HP even if no firms are HP.

For case (ii), we use $\gamma_c^* = 1$ into equation 21. This yields:

$$\pi_c^h \mid_{\gamma_c=1} -k > \pi_c^l \mid_{\gamma_c=1} \text{ if } k < \underline{\mathrm{k}}_c = \frac{\lambda\delta}{(\delta+\lambda)^2}(p_h - p_l)$$

Hence, if $k <\underline{\mathrm{k}}_c$ firms have an incentive to become HP even when all firms are HP.

It is easy to show that $\underline{\mathrm{k}}_c < k_c$. For $\underline{\mathrm{k}}_c < k < \bar{k}_c$, case (iii) holds and $0 < \gamma_c^* < 1$

2.4 Survival of low productivity firms.

Proposition 2.4 $\pi_c^l > \pi_m^l > \pi_{bm}^l$. *For given proportions of HP and LP firms in the market, LP firms do better under the $c-case$ than under the $m-case$. In turn, they do better under the $m-case$ than under the $bm-case$.*

$\pi_m^l > \pi_{bm}^l$ is always true, as Postel-Vinay (2002a) put it, because in the $m-case$ all firms have a wider set of strategies than in the $bm-case$, as they can both (i) Identify unemployed workers and offer them only their reservation wage, and (ii) Match the outside offers received by their workers. Nevertheless, notice that (ii) is irrelevant for LP firms. When they enter the process of Bertrand competition with other firm by matching outside offers, they either (a) keep the worker at wage p_l if competing with another low productivity firm, the worker is not profitable or (b) loose the worker to a high productivity firm. This is relevant when addressing the equilibrium proportion of high productivity firms (next section)

$\pi_c^l > \pi_m^l$ is always true because, as low productivity firms offer workers a higher wage increase when they meet a high productivity firm, they can exploit that to attract unemployed workers by offering lower wages ($w_c^l < w_m^l$) and make a higher profits.

Hence, given an outside option, it is more likely that low productivity firms survive in the $c-case$ than in the $m-case$ than in the $bm-case$.

2.5 Equilibrium proportion of low productivity firms.

This section highlights the fact that, while frictions are necessary LP firms to survive in the market, the degree to which it occurs is directly affected by the type of contracts signed. Propositions 2.5,2.6 and 2.7 below compare the equilibrium proportion of HP firms, assuming firms can be either LP at 0 investment cost or become HP at investment cost k, across the three environments considered in the previous section. In all cases, it can be unambiguously determined under which environment the proportion of HP firms will be higher. As expected, the environment with the lowest equilibrium proportion of HP firms is the $c-case$, for two reasons: (i) Direct competition between two firms can be triggered through on-the-job search. This feature is shared with the $m-case$. (ii) Most importantly, the strategic bidding allows workers and LP firms to extract rents from HP firms whenever this competition is triggered. This is dealt with in Proposition 2.5.

Proposition 2.5 $\gamma_c^* \leq \gamma_{bm}^*$. *The equilibrium proportion of HP firms in the $bm-case$ is never lower than the equilibrium proportion of HP firms in the $c-case$.*

Proposition 2.1 shows that $\pi_c^l > \pi_{bm}^l$. Through simple algebra, it is also easy to prove that $\pi_c^h < \pi_{bm}^h$: HP firms do better under the $bm-case$ than under the $c-case$. In this latter case, they pay very low wages to unemployed workers they meet, but they loose profitable workers every time their workers meet any firm, HP or LP. Both $\pi^l_c > \pi^l_{bm}$ and $\pi^h_c < \pi^h_{bm}$ imply unambiguously that, for a given investment level to become HP, the equilibrium proportion of HP firms is higher under the $bm-case$: $\gamma_c \leq \gamma_{bm} \leq 1$.

More formally, note that:

$$\begin{aligned}
\underline{k}_c &= (p_H - p_L)\frac{\lambda\delta}{(\lambda+\delta)^2} = \underline{k}_{bm} \\
\frac{\delta\gamma^*_c}{\delta k} &< \frac{\delta\gamma^*_{bm}}{\delta k}
\end{aligned}$$

and as a consequence

$$\bar{k}_c = (p_H - p_L)\frac{\lambda}{\delta+\lambda} < \bar{k}_{bm} = (p_H - p_L)\frac{\lambda}{\delta}$$

Therefore,

$$\underline{k}_c = \underline{k}_{bm} < \bar{k}_c < \bar{k}_{bm}$$

Consider then the following values of k:

(i) $k < \underline{k}_c = \underline{k}_{bm} < \bar{k}_c < \bar{k}_{bm}$. In the $bm-case$ $\gamma^*_{bm} = 1$ because $k < \underline{k}_{bm}$; in the $c-case$ $\gamma^*_c = 1$ because $k > \underline{k}_c$. Hence $\gamma^*_c = \gamma^*_{bm}$

(ii) $\underline{k}_c = \underline{k}_{bm} < k < \bar{k}_c < \bar{k}_{bm}$. In the $bm-case$ $0 < \gamma^*_{bm} < 1$ because $\underline{k}_{bm} < k < \bar{k}_{bm}$, in the $c-case$ $0 < \gamma^*_c < 1$ because $\underline{k}_c < k < \bar{k}_c$. But because $\frac{\delta\gamma^*_c}{\delta k} < \frac{\delta\gamma^*_{bm}}{\delta k}$,

the decrease in γ^*_c is higher than the decrease in γ^*_{bm} as k increases from $\underline{k}_c = \underline{k}_{bm}$. Hence $\gamma^*_{bm} > \gamma^*_c$.

(iii) $\underline{k}_c = \underline{k}_{bm} < \bar{k}_c < c < \bar{k}_{bm}$. In the $bm-case$ $0 < \gamma^*_{bm} < 1$ because , in the $c-case$ $\gamma^*_c = 0$ because $\underline{k}_c < \bar{k}_c < c$. Hence $\gamma^*_{bm} > \gamma^*_c$.

(iv) $\underline{k}_c = \underline{k}_{bm} < \bar{k}_c < \bar{k}_{bm} < k$. In the $bm-case$ $\gamma^*_{bm} = 0$ because $\bar{k}_{bm} < k$, in the $c-case$ $\gamma^*_c = 0$ because $\bar{k}_c < k$.

Proposition 2.6 $\gamma^*_{bm} \leq \gamma^*_m$. *The equilibrium proportion of HP firms in the $m-$ case is never lower than the equilibrium proportion of HP firms in the $bm-case$*

Proposition 2.1 noted that $\pi^l_m > \pi^l_{bm}$, because in the $m-case$ firms can recognise unemployed workers and therefore offer them only their reservation wage, which they cannot do in the $bm-case$. It was also noted that, although firms can match the outside offers received by their workers, this is not really to the advantage of LP, since whenever they enter Bertrand competition with other firm they will always loose a profitable worker. This is not true for HP firms. If they are competing for a worker with an LP firm, they will keep the worker at wage $x^h_c < p_h$: the worker is still profitable for the HP firm. This implies that, as one moves from the $bm-case$ to the $m-case$, HP firms benefit more than LP firms. This intuition implies the following relation, which is easy to prove through simple algebra: $\pi^h_m - \pi^h_{bm} > \pi^l_m - \pi^l_{bm}$

More formally, note that:

$$\begin{aligned}
\underline{k}_{bm} &= \frac{\lambda\delta}{(\lambda+\delta)^2}(p_h - p_l) < \underline{k}_m = \frac{\gamma_m \delta(3\lambda[\lambda+\delta]+\delta^2)}{(\lambda+\delta)^4}(p_h - p_l) \\
\bar{k}_{bm} &= \bar{k}_m = \frac{\lambda}{\delta}(p_h - p_l)
\end{aligned}$$

And this implies that:

$$\underline{k}_{bm} < \underline{k}_m < \bar{k}_{bm} = \bar{k}_m$$

Hence, consider the following values of k:

(i) $k < \underline{k}_{bm} < \underline{k}_m < \bar{k}_{bm} = \bar{k}_m$. In the $bm - case$, $\gamma^*_{bm} = 1$ since $k < \underline{k}_{bm}$. In the $m - case$, $\gamma^*_m = 1$ since $k < \underline{k}_m$. Hence $\gamma^*_{bm} = \gamma^*_m$.

(ii) $\underline{k}_{bm} < k < \underline{k}_m < \bar{k}_{bm} = \bar{k}_m$. In the $bm - case$, $0 < \gamma^*_{bm} < 1$ since $\underline{k}_{bm} < k < \bar{k}_{bm}$. In the $m - case$, $\gamma^*_m = 1$ since $k < \underline{k}_m$. Hence $\gamma^*_{bm} < \gamma^*_m$.

(iii) $\underline{k}_{bm} < \underline{k}_m < k < \bar{k}_{bm} = \bar{k}_m$. In the $bm - case$, $0 < \gamma^*_{bm} < 1$ since $\underline{k}_{bm} < k < \bar{k}_{bm}$. In the $m - case$, $0 < \gamma^*_m < 1$ since $\underline{k}_m < k < \bar{k}_m$. The relation $\gamma^*_{bm} < \gamma^*_m$remains true[10] until:

[10] So far, we have interpreted the proportion of HP firms as a function of the cost c, as implied by equations (2) and (22), for obvious reasons of interpretation. Nevertheless, for analytical purposes one can reverse the emphasis and write:
$k_m = -\frac{\lambda\delta(2\lambda\delta^2\gamma+5\lambda^2\delta\gamma+\lambda^3[\gamma]^2+2\lambda^3\gamma+2\lambda\delta^2+\delta\lambda^2+\delta^3)}{(\lambda\gamma_m+\delta)^3(\delta+\lambda)^2}(p_l - p_h)$
$k_{bm} = \frac{\lambda\delta}{(\lambda\gamma+\delta)^2}(p_l - p_h)$
And it is easy to prove that $k_{bm} < k_m$ if $\gamma > 0$.
In a graph with γ in the vertival axis and k in the horizontal axis, for a given c, $\gamma_{bm} < \gamma_m$. For any k, the equilibrium proportion of HPs is lower under the bm-case than under m-case, except for those k values that imply a corner solution in both cases.

(iv) $\underline{k}_{bm} < \underline{k}_m < \bar{k}_{bm} = \bar{k}_m < k$. In the $bm-case$, $\gamma^*_{bm} = 0$ since $\bar{k}_{bm} < k$. In the $m-case$, $\gamma^*_m = 0$ since $\bar{k}_m < k$. Hence $\gamma^*_{bm} = \gamma^*$.

Proposition 2.7 $\gamma^*_c \leq \gamma^*_m$. *The equilibrium proportion of HP firms in the* $m-case$ *is never lower than the equilibrium proportion of HP firms in the* $c-case$

This Proposition is a consequence of Propositions 2.4 and 2.5. It is hardly surprising since the only difference between the $c-case$ and the $m-case$ is that in the $c-case$ HP firms see part of their rents extracted by LP firms through the strategic bidding commitment. This reduces the profits of HP firms and increases the profits of HP firms. As noted in Proposition 2.1, $\pi^l_c > \pi^l_m$, and the previous intuition implies an easy to prove relation $\pi^h_c < \pi^h_m$.The formal proof is by noting that:

$$\begin{aligned} \bar{k}_c &< \bar{k}_m \text{ and } \underline{k}_c < \underline{k}_m \\ \underline{k}_m &< \bar{k}_m \text{ and } \underline{k}_c < \bar{k}_c \end{aligned}$$

To be rigorous, one must allow for two possible cases. Assume first that $\underline{k}_m < \bar{k}_c$[11]. If this is true, then

[11] This occurs if

$$\lambda < \frac{-3\delta^2 + 1 + \sqrt{-3\delta^4 + 6\delta^2 + 1}}{6\delta}$$

$$\underline{k}_c < \underline{k}_m < \bar{k}_c < \bar{k}_m$$

Consider the following values of k :

(i) $k \leq \underline{k}_c < \underline{k}_m < \bar{k}_c < \bar{k}_m$. Then $\gamma_m^* = \gamma_c^* = 1$ since $k \leq \underline{k}_c$ and $k < \underline{k}_m$

(ii) $\underline{k}_c < k \leq \underline{k}_m < \bar{k}_c < \bar{k}_m$. In the $m - case$, $\gamma_m^* = 1$ since $k \leq \underline{k}_m$. In the $c - case$, $0 < \gamma_c^* < 1$ since $\underline{k}_c < k < \bar{k}_c$. Hence, $\gamma_c^* < \gamma_m^*$

(iii) As k increases higher than $\underline{k}_m$, both γ^* and γ_c^* decrease. It can be shown that $\frac{\partial \gamma_c^*}{\partial c} < \frac{\partial \gamma_m^*}{\partial c}$. Because $\gamma_c^* < \gamma_m^*$ to start with, this ensures it remains lower as long as both continue decreasing.

(iv) $\underline{k}_c < \underline{k}_m < \bar{k}_c \leq k < \bar{k}_m$. In the $m - case$, $0 < \gamma_m^* < 1$ since $\underline{k}_m < k < \bar{k}_m$. In the $c - case$, $\gamma_c^* = 0$ since $\bar{k}_c \leq k$. Hence $\gamma_c^* < \gamma_m^*$.

(v) $\underline{k}_c < \underline{k}_m < \bar{k}_c < \bar{k}_m \leq k$, then $\gamma_c^* = \gamma_m^* = 0$ since $\bar{k}_c < k$ and $\bar{k}_m \leq k$.

Assume now that $\bar{k}_c < \underline{k}_m$[12]. Then

$$\underline{k}_c < \bar{k}_c < \underline{k}_m < \bar{k}_m$$

Consider the following values of k :

(i) $k \leq \underline{k}_c < \bar{k}_c < \underline{k}_m < \bar{k}_m$. Then $\gamma_m^* = \gamma_c^* = 1$ since $k \leq \underline{k}_c$ and $k < \underline{k}_m$

(ii) $\underline{k}_c < k < \bar{k}_c < \underline{k}_m < \bar{k}_m$. In the $m - case$, $\gamma_m^* = 1$ because $k \leq \underline{k}_m$. In the $c - case$, $0 < \gamma_c^* < 1$ since $\underline{k}_c < k < \bar{k}_c$. Hence, $\gamma_c^* < \gamma_m^*$.

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[12] True if $\lambda > \frac{-3\delta^2+1+\sqrt{-3\delta^4+6\delta^2+1}}{6\delta}$

(iii) $\underline{k}_c < \bar{k}_c \leq k < \underline{k}_m < \bar{k}_m$. In the $m-case$, $\gamma_m^* = 1$ since $k < \underline{k}_m$. In the $c-case$, $\gamma_c^* = 0$ since $\bar{k}_c \leq k$. Hence $\gamma_m^* > \gamma_c^*$

(iv) $\underline{k}_c < \bar{k}_c < \underline{k}_m < k < \bar{k}_m$. In the $m-case$, $0 < \gamma_m^* < 1$ since $\underline{k}_m < k < \bar{k}_m$. In the $c-case$ $\gamma_c^* = 0$,since $\bar{k}_c < k$. Hence, $\gamma_c^* < \gamma_m^*$

(v) $\underline{k}_c < \bar{k}_c < \underline{k}_m < \bar{k}_m \leq k$ Then $\gamma_c^* = \gamma_m^* = 0$ since $k \geq \bar{k}_m > \bar{k}_c$.

2.6 Conclusion

This paper has analysed three environments with on-the-job search; the focus has been on the type of contracts signed between firms and workers, and its consequences regarding wage distributions, firms profits and endogenous investment decisions. The first two environments shown are applications of existing contributions to the literature, namely Burdett and Mortensen (1998) and Postel Vinay and Robin (2002a and 2002b). I have shown how the contracts offered by the firms in those two environments are not optimal, in the sense that they do not fully exploit the possibilities offered by the worker's on-the-job search effort. In Burdett-Mortensen, because firms are completely passive to the outside offers their workers may receive; in Postel Vinay and Robin because, although firms are not passive, they do not offer forward looking contracts. When firms do offer forward looking contracts, which is the third case presented, they are able to extract benefits from alternative firms met by their workers, by forcing those firms to attract the workers but pay them their full productivity. Because this is a more attractive contract for unemployed workers, firms can attract them by offering lower wages, therefore making higher profits. This has implications for the incentives to invest in quality. As firms will see rents extracted by any employed worker that they attract, firm's investments in productivity need not be more attractive, since they only imply higher wages for those workers, rather than higher profits for the firm. Hence, although these forward looking contracts are privately efficient for individual firms, they may result in lower investments in productivity in the market.

If lower wages are needed in order to implement the contract approach described in the paper, then any reason why the wage cannot be sufficiently low will means that such a contract cannot be implemented. The existence of minimium wages is one reason. Another can be the imperfection of financial markets, which impede the workers to borrow against a (expected) higher income, which they would need if the wage determined by the contract falls below a given subsistence level

Chapter 3

Bargaining and on-the-job-search

3.1 Introduction

The present paper addresses a search market with ex-ante heterogenous firms and on-the-job search, with two particular characteristics: (a) Wages are determined through Nash Bargaining (b) Workers on-the-job search effort, which is undertaken at a cost, is not contractible.

The aim is to characterise in detail the process of wage formation and search behaviour in an economy where search is costly and non-contractible. The non-contractibility of the worker's search effort seems like a natural assumption, almost in the same category as the worker's freedom to leave their employment without any liability: in modern labour markets, workers cannot be tied legally to their employment. Even though this may be natural assumptions, they still are imperfections that limit the contractual arrangements in any employment relationship. In particular, I look at how does this imperfection affect the wage formation process. Do the conditions of the labour market matter for the wage determination process, other than just for the wage level? How does this affect the worker's search behaviour? The paper therefore endogenises not only the wage level, but also the wage formation process, which will differ in equilibrium according to the parameter values of the model. The reason is that the non-contractibility of search effort imposes a constraint that may or may not be binding in the Nash Bargaining process whenever firm-worker pairs are negotiating over a possible contract.

In any firm-worker relationship, there will always be two variables which are at the discretion of at least one of the agents involved: the wage level and the search behaviour of the worker, who may search for better employment. In the absence of imperfections, and given the adoption of the Nash Bargaining solution, the agents would choose the worker's search behaviour that maximises the joint value of their contract; and subject to this they would choose the wage level that "settles the score" given the Nash Bargaining solution.

With the imperfection introduced by the non-contractibility of on-the-job search, things change. The pair cannot agree on the search behaviour to maximise the value of the contract, since it is not contractible. Rather, in the bargaining process, the firm knows that the worker will always undertake on-the-job search if he individually has incentives to do so. These incentives are of course affected by the wage finally agreed upon. Given search is costly, the worker is less likely to engage in search for better wages if he is already earning a high enough wage. In particular, for high enough current wages, the relative expected gain of on-the-job search is too low compared to the cost of search. For low enough wages, the relative expected gain of on-the-job search is high enough to justify costly search by the worker. The wage that separates those two wage ranges can be though of as an incentive compatible wage level (ICW). The bargaining process therefore must consider the constraint imposed by the imperfection, i.e. the existence of ICW, which says when the worker will or will not search on the job. Focusing on the Nash Bargaining problem, the effect on the frontier of the utility space must be considered: When the wage is higher than the ICW, the contract will be "stable" in the sense that the worker will not search on the job. For wages above the ICW, changes in the wage represent only a pure transfer of utility. When the wage decreases below the ICW, the contract stops being "stable", since now the wage gives the worker incentives to search on the job. In addition to being a pure transfer of utility, this decrease represents a change in the total value of the contract, which results from the change in the search behaviour of the worker. This creates a discontinuity in the frontier at this particular wage level. Further wage decreases represent only a pure transfer of utility from firm to worker, while the worker remains searching on the job.

When on-the-job search is a possibility, one must consider how current employers react to outside offers received by their workers. The paper will abstract from such complication by following an assumption introduced in Pissarides (1994) and Gautier (2002): Workers must leave their current employment in order to negotiate with firms met through search on the job[1]. Formally, this assumption implies that the workers' only threat in any bargaining is unemployment. Pissarides interprets this assumption as the study of short term contracts. With long term contracts, a worker could negotiate with the new employer whilst holding the current job, which would favour him in this bargaining process. The assumption therefore decreases the workers' incentives to search on the job, as they cannot use their employment status to bargain better wages. Gautier defends it on the grounds that the outside option of any new job is unemployment. I believe this latter argument is only valid if one actually assumes that only short term contracts can be signed. I argue that this assumption qualitatively does not affect qualitatively the results obtained; but only quantitatively, since it just decreases the workers incentives to search on the job. Further, it allows to focus on the role of firm heterogeneity as a reason for on-the-job search: because worker's only threat in any bargaining is unemployment, there is no point

[1]If such an assumption was not made, then one would have to consider in more detail how do firm's react to outside offers reeived by their workers. This has been addressed in various ways in the literature.

Burdett and Mortensen (1998) assume that firms do not respond to outside offers received by their workers. Burdett and Coles (2003) acknowledge that, even if firms cannot observe if their workers actually received an outside offers, the porbability that they do is possitive. This results in firms offering wage-tenure contracts, in which wage increases with tenure.

Postel-Vinay and Robin (2003a, 2003b) assume that firms can observe the outside offers received by their employers, and therefore enter a Bertrand competition for the worker with the outside firm when their workers find one. The result is that the worker goes to the more productive firm, but at a wage equal to the productivity at the least productive firm. There is another way of handling the issue. One can, in the Postel-Vinay and Robin setup, imagine a firm-worker pair contracting in a way that anticipates the possibility of the worker meeting a more productive firm. The current, less productive employer, may commit to strategically bid up to the productivity of the poaching, more productive firm (strategic bidding contracting). The worker then moves to the most productive firm, but this time at a wage equal to his productivity at the poaching firm. This would "extract" rents from the poaching fiirm.

in leaving one employer for another of the same productivity; both will pay the same wage.

Rocheteau (2001) also presents a search model, *without* on-the-job search, in which wages are sometimes determined by an interior solution to the Nash Bargaining problem, and sometimes by a corner solution determined by an incentive compatibility constraint. The need to meet an incentive compatibility constraint comes from the introduction of imperfectly observable shirking by the worker. In his model, workers have an incentive to *shirk*, and this incentive comes from an *exogenously* given cost of effort $\bar{e}$. In the present paper, the need for incentive compatible wages comes from the workers' incentives to *search on the job*, the attractiveness of which is *endogenously* given by the expectation of finding a better wage through search, and how better that wage is. While Rocheteau endogenises the wage formation process by introducing efficiency wage considerations, the present paper endogenises both the wage formation process and the worker's search behaviour by introducing (non-contractible) costly search.

The next section introduces and discusses the assumptions of the model, and gives a summary of the results obtained. Section 3 addresses the individual decisions faced by the agents. Section 4 addresses the bargaining process between firms (good and bad) and workers, paving the way for the results in Section 6. Section 5 discusses the steady state conditions and Section 6 closes the model by addressing the types of contracts signed in equilibrium. Section 7 gives a numerical example of the workings of the model. The Propositions and Theorems in the main body present explicit solutions to all relevant values.

3.2 Assumptions and Set-up of the model

A1 There is a unit mass of workers and a unit mass of vacancies. Workers are homogenous, but vacancies may be good or bad. These types will be denoted by subscript $i = g, b$. Half of the vacancies are good (g) and half of them are bad (b). Vacancies can accept at most one worker. The measure of unemployed workers is denoted by u. The measures of empty vacancies, good and bad, are identified respectively as v_g and v_b.

A2 The production of a worker is denoted as Y in a good vacancy and $X < Y$ in a bad vacancy. The wages paid to the worker in a good and in a bad vacancy are respectively identified as w_g and w_b. I will assume that these wages are determined by Nash Bargaining between vacancy and worker. Vacancies can accept at most one worker.

A3 Workers, regardless of their employment status, may search for vacancies at a cost denoted by c. Vacancies are assumed to be in the search market only while empty, and for them this is costless. If a worker does pay the cost c, he contacts good (empty) vacancies at a rate λ_g, and bad (empty) vacancies at rate λ_b. The rate at which vacancies, regardless of the type, contact workers is given by ε. Worker's on-the-job search behaviour is not contractible.

A4 The arrival rates previously described are determined by a quadratic search technology, where the number of contacts is the product of the measure of searching workers and the measure of searching vacancies.

A5 In any given worker-firm relationship, there is a risk of relationship breakdown, that comes at a rate given by δ.

A6 I will assume quit fees are illegal for workers, so they can leave their employment relationship costlessly whenever they have incentives to do so. Further, I rule out bond posting by firms. Other than that, workers and vacancies sign enforceable contracts to rule the employment relationship.

Workers can be either unemployed, employed in a bad vacancy or employed in a good vacancy. In each case, they may be searching or not. Only cases where unemployed workers engage in search will be considered. Their lifetime discounted value will be denoted by W^u. The lifetime discounted value of a worker employed in a vacancy of type $i = g, b$ will be denoted as W^i. The lifetime discounted value of an empty vacancy of type i will be denoted by V^u for $i = b$ and $V^{g,u}$ for $i = g$. The lifetime discounted value of filled vacancies of type i will be denoted V^i.

Because search effort is not contractible, workers will always search on the job if there are incentives to do it. For any given wage, the attractiveness of on-the-job search comes from the prospect of finding a better wage with another vacancy. Workers must compare this benefit with the cost c in order to decide wether to search or not. Intuitively, if a worker is already earning a relatively high wage, the expected benefit of searching on the job is not very high, and therefore he should have weaker incentives to search on the job.

Employers know if a given wage provides incentives to the worker to search on the job. Hence, for any given wage, they know what the search behaviour of the worker will be. This is crucial in understanding the wage formation process. Because the search behaviour affects the total value of the contract, and the wages are determined through Nash bargaining, it affects the wage finally agreed upon because the employers know it. This would not be the case if employers where not able to discern the workers' search behaviour for any given wage level, since then they could not discern the exact value of their contract.

Because of the above, even though the contract finally signed between a firm and a worker will only include the wage level, the bargaining process will determine both the wage level and the search behaviour, and this bargaining process is in turn affected by the possibilities of on-the-job search.

I finally come to an assumption that both simplifies the analysis and allows to focus on the role of vacancy heterogeneity:

A7 Upon meeting a firm, any worker (employed or unemployed) observes its type and decides wether to enter negotiations with her. If the worker is employed, he must become unemployed before starting these negotiations[2]. This means that, should negotiations fail, he would not be able to go back to his previous employment, but if he rejects negotiations completely, then he can stay

[2]The assumption assumes away the potential complication that an employed worker may find another firm and trigger a Bertrand competition for his services, as in Postel-Vinay and Robin (2003a, 2003b).

in his current employment.

The formal implication is that worker's outside option when bargaining with a firm is always unemployment, regardless of the actual employment status at the time of the contact. Being able to use employment status as an outside option would only improve an employed worker's bargaining position vis-a-vis new employers, and therefore increase the attractiveness of on-the-job search. For this reason, I argue that this latter assumption only reinforces the results of the paper, since it only means that workers have less incentives to search on the job.

Because the bargaining position of a worker vis-a-vis any given firm is always unemployment, regardless of employment status, workers will not leave a vacancy of type $i = k$ for another vacancy of the same type $i = k$, since the wage that will be paid in either is the same. Because of Nash Bargaining, better wages are only paid by firms in which workers are more productive. In the context of the paper, this means that workers in bad vacancies will search only for the prospect of finding a good vacancy, which will pay a higher wage since the worker's productivity is higher there. It also means that workers in good vacancies have no incentive to search on the job, since there is no prospect of finding a better wage. The derivations that follows make full use of these two conclusions.

Throughout the paper, when I refer to a given type of match or contract as dominant, or when I say that it dominates, it must be understood that it is jointly efficient in the sense that it maximises the joint value for the firm and the worker. As will be seen, transferable utility through Nash Bargaining will not suffice for dominant contracts to be signed, and this inefficiency is due to the imperfection created by the non-contractibility of on-the-job search

Summary of Results The paper focuses on the wage and search behaviour in worker-bad firm relationships. The result in worker-good firms relationships is not interesting; the assumptions made force them to be "stable" in the sense that the worker never has an incentive to search on the job. This implies also that the wages are always determined by an interior solution to the Nash Bargaining problem.

Further, the paper focuses on the cases where worker-bad firm relationships value is maximised if the worker abstracts from on-the-job search. I call this $ns-contracts$ (ns for no search), as opposed to relationships where the worker searches on the job, which I call $s-contracts$. Notice that this is true, everything else being equal, for high enough values of X as compared to Y. A high enough value of X implies that it is not jointly efficient for the worker to engage in costly search for a new job in which the productivity is not that much higher. Two equilibria are possible:

Case 1

i) $ns - contracts$ are signed, which can only be true if

ii) The wage agreed upon is incentive compatible, higher than or equal to the ICW.

Case 2

i) $s - contracts$ are signed, which can only be true if

ii) The wage agreed upon is lower than the ICW.

Consider *Case* 1 first. For contracts to be of the ns type, it must be the case that the wage is higher than or equal to the ICW, so that the worker decides not to search on the job. There are two possible ways in which this can be the case. The first one can be thought of as the "natural" way. It occurs if, under the assumption that the worker will not search on the job, the wage that splits the surplus in two (i.e. the simple solution to the Nash Bargaining problem) is actually compatible with this assumption, i.e. is higher than or equal to the ICW. Everything else being equal, this is more likely to be the case for X high enough; Nash Bargaining implies the wage increases directly with X. This implies that, if agents behave ignoring the constraint that the wage must be high enough, the constraint is not relevant in equilibrium . Given that we work in the region where $ns - contracts$ dominate, this is a sufficient conditions for $ns - contracts$ to be signed. If there were no imperfections (or if they are irrelevant in equilibrium), then the agents will sign the contract that dominates.

But it is also possible that $ns - contracts$ are signed even if the above is not true. Imagine that the wage that splits the surplus in two is lower than the ICW, so the worker would actually search on the job if that wage was agreed. Then the wage must be increased up to the level of the ICW if the worker is going to give up search. But this implies a "redistribution" of profits, in which firms are losers and workers are winners. For high enough values of X compared to the ICW, the ICW will not "redistribute" too much of these profits, hence both parties will still agree that the $ns-contract$ is their better option. For low enough value of X compared to the ICW, it may be that the firms are so worse-off that they would prefer to sign an $s-contract$; and pay the worker lower wages. In this case, firms and workers interests are in conflict; which contract is signed would depend on the bargaining power of each party. Because the bargaining powers are fixed to 0.5 for each agent, the result would actually depend on how worse-off (relatively) firms are if they pay the ICW. If X is low, but not very low, then the damage done to the firm is not too much; and the worker would "win" in the bargaining and the $ns-contract$ would be signed. If X is very low, then firms are so worse-off that they are winners in the bargaining, which results in $Case$ 2.

Hence, one expects the equilibrium in $Case$ 1 to arise for high enough values of X, and the equilibrium in $Case$ 2 to obtain for low enough values of X. $Case$ 1 and its two possibilities are addressed in Propositions and Theorems 3.1 and 3.2 in Section 6. $Case$ 2 is addressed in Proposition and Theorem 3.3.

Notice that a particularly relevant variable in determining which case occurs is the (endogenous) ICW. Any factor that increases its value makes it more likely that the outcome will be $Case$ 1. A relevant variable that affects the ICW is Y, since the value of Y implies a higher wage in a worker-good firm match, and therefore stronger incentives for workers in bad firms to search on the job. But this is not so interesting because Y is exogenous. More interesting is to note that the ICW is also affected by the conditions of the labour market, in particular the endogenously determined number of good vacancies in the searching in the market[3]. If there are less good vacancies, the attractiveness of on-the-job search is lower, since it is less likely that search will result in contacting a good vacancy. This implies that the ICW level will be lower.

Notice the implications of all of the above for the wage formation process. The wage in an $ns - contract$ will sometimes be determined by the workers productivity in his employment with the bad firm (if the equal split of the surplus results in an incentive compatible wage); and sometimes the wage will be determined by the workers productivity in an alternative employment with a good firm, even though the worker does not search on the job for a good firm (if the firm must increase the wage to meet the ICW level, which in turn depends on Y). This distinction is also addressed in Propositions and Theorems 3.1 and 3.2.

A further, a quite intuitive, finding is the existence of parameter values such that no pure strategy steady state equilibrium exists. This is exemplified in the numerical exercise in Section 7. As is usually the case in matching models, the result has to do with the composition effects caused by the workers' equilibrium search behaviour. Compare the steady state numbers of firms of either type in *Cases* 1 and 2. Since in *Case* 2 workers in bad firms search on the job, they will be leaving bad firms empty as they move to fill good firms met through search. Hence, the steady state number of empty bad firms is higher and the number of empty good firms is lower in *Case* 2 compared to *Case* 1. This means

[3]The number of good vacancies affects the rate at which workers meet good firms. The assumption of a quadratic search technology implies that this arrival rate is independent of the number of workers searching.

that the rate at which workers find good firms is lower in *Case* 2 compared to *Case* 1, if they do search on the job. This makes search less attractive, and therefore reduces the ICW level. For any value of X, the profits of firms that pay the ICW will be higher with the arrival rate consistent with *Case* 2, since the implied ICW is lower. Hence, there will be values of X which (i) Are not consistent with $ns-contracts$ equilibrium arrival rate, since firms' profits are too low if the implied ICW is paid, and hence $s-contracts$ will be signed, but (ii) Are not consistent either with the $s-contracts$ equilibrium arrival rate, since firms' profits are high enough when the implied ICW is paid, and $ns-contracts$ will be signed.

3.3 Individual Decision Problems

Workers Because search is costly when unemployed, the first problem of the workers is whether to participate in the search market while unemployed, i.e. they will search of $W^u > 0$. The conditions for this to be the case will arise as part of the analysis. Hence, I only worry in this section about the decision of employed workers of whether to search on the job or not.

Workers in Good Vacancies Workers in good vacancies have no incentive to search on the job, since there are no better vacancies to be found in the market. Further, vacancies have been assumed to search only while empty. Hence, any relationship between a worker and a good vacancy is broken only for exogenous reasons. The lifetime discounted value of a worker in such a contract is given by:

$$rW^g = w_g + \delta(W^u - W^g)$$

Filled Good Vacancies For the above reasons, the lifetime discounted value of a filled good vacancy is given by:

$$rV^g = Y - w_g + \delta(V^{g,u} - V^g)$$

Workers in Bad Vacancies Consider now the case of a worker in a bad vacancy. Because search effort is not contractible, workers with bad vacancies will always search on the job, provided that the incentives are given. Assuming $w_g > w_b$, (which is intuitive given wage bargaining and $Y > X$. It will be formally confirmed later), then a worker should prefer to be working in a good vacancy than in a bad vacancy; and this implies that he may have incentives to search on the job for a good vacancy. For any given wage level w_b, the lifetime discounted value of a worker in a bad vacancy is therefore given by:

$W^b = \max(W^{ns}, W^s)$, where :

$$rW^{ns} = w_b + \delta(W^u - W^{ns}) \quad (3.1)$$

$$rW^s = w_b + \delta(W^u - W^s) + \lambda_g(W^g - W^s) - c \quad (3.2)$$

Using 1 and 2 and W^g above:

$$W_{ns} \geq W_s \Leftrightarrow w \geq w_{ic} = \frac{\lambda_g w_g - c(r+\delta)}{\lambda_g} \tag{3.3}$$

This implies that the worker will give up on-the-job search only if the wage received in the bad vacancy is high enough. I call this lower limit for w_b the incentive compatibility constraint. If $w_b > w_{ic}$, the worker has no incentive to search on the job. If $w_b < w_{ic,}$ then the worker prefers to pay the cost of search and look for a good vacancy, where a higher wage will be earned. Notice that for a higher cost of search, w_{ic} is lower, implying that the current wage need not be so high to be incentive compatible, since search is already very costly. Likewise, w_{ic} increases with w_g and with λ_g. If either of them is higher, search is more attractive since the expected wage to be found through search is higher. Hence, w_b needs to be higher in order to be incentive compatible. In particular, a higher λ_g implies the prospect of finding a good firm is better, and this makes on-the-job search more attractive.

Filled Bad Vacancies Given a wage level w_b, the firms lifetime discounted value is determined by the worker's search behaviour.

$$rV^b = rV^{ns} = X - w_b + \delta(V^u - V^{ns}) \text{ if } w_b \geq w_{ic} \tag{3.4}$$

$$rV^b = rV^s = X - w_b + (\delta + \lambda_g)(V^u - V^s) \text{ if } w_b < w_{ic} \tag{3.5}$$

3.4 The bargaining process

Workers-Bad Vacancies Strictly speaking, firm and worker bargain over the wage level. Not over the search behaviour, since it is not contractible. Nevertheless, the wage level affects the search behaviour of the worker, and therefore the total value of the contract. Because firms know if the wages give the correct incentives to the worker, they know the total value of the contract for any given wage level, and this affects the bargaining process.

The contract can be of one of two types:

1– Firms pay wages that meet the incentive compatibility level and the worker does not search on the job. Since there is no search in these contracts, I will refer to them as $ns - contracts.$

2–Firms pay wages that do not meet the incentive compatibility level and the worker searches on the job. Since there is search in these contracts, I will refer to them as $s - contracts.$

The bargaining process determines *both* the search behaviour and the wage level. In general, the contract type (ns or s) finally agreed upon depends on which of the two maximises the global Nash Product.

$ns - contracts$ contracts will be signed if:

$$\max_{w_{nb} \geq w_{ic}} \left[V^{ns}(w_{ns}) - V^{u}\right]^{\frac{1}{2}} \left[W^{ns}(w_{ns}) - W^{u}\right]^{\frac{1}{2}} \geq$$

$$0\ ,\ \max_{w_s \leq w_{ic}} \left[V^s(w_s) - V^u\right]^{\frac{1}{2}} \left[W^s(w_s) - W^u\right]^{\frac{1}{2}}$$

$s - contracts$ will be signed if:

$$\max_{w_s \leq w_{ic}} \left[V^s(w_s) - V^u\right]^{\frac{1}{2}} \left[W^s(w_s) - W^u\right]^{\frac{1}{2}} >$$

$$0\ ,\ \max_{w_{nb} \geq w_{ic}} \left[V^{ns}(w_{ns}) - V^u\right]^{\frac{1}{2}} \left[W^{ns}(w_s) - W^u\right]^{\frac{1}{2}}$$

Notice the two constraints above. In the $ns-contracts$, the constraint $w_{nb} \geq w_{ic}$ says that the final wage agreed upon cannot be lower than the w_{ic}. Otherwise the worker would search on the job, contradicting the initial assumption of no search. In the $s - contracts$ case, the constraint says that the final wage agreed upon cannot be higher than the w_{ic}. Otherwise, the worker would not search on the job, contradicting the initial assumption that the worker would search on the job. It will be important, and it is done later, to identify the cases in which either constraint is or is not binding.

A further analysis of each individual case will prove helpful for further sections, Section 6 in particular. Consider first an $ns - contract$. Abusing notation slightly, I will call $w_{ns} = w^*_{nb}$ if the constraint on the maximisation problem is not binding, i.e. the solution to the Nash Bargaining problem is an interior solution; and $w_{ns} = w_{ic}$ if the constraint is binding, i.e. the solution to the Nash Bargaining problem is a corner solution. The first case occurs if:

$$w_{nb}^{*} = \arg\max_{w_{nb}} \left[V^{ns}(w_{ns}) - V^{u}\right]^{\frac{1}{2}} \left[W^{ns}(w_{ns}) - W^{u}\right]^{\frac{1}{2}} > w_{ic}$$

Applying the well known solution to the Nash Bargaining problem:

$$w_{nb}^{*} = \frac{1}{2}\left[X + r(W_{1}^{u} - V_{1}^{u})\right] \tag{3.6}$$

and it is known that this implies an equal split of the contract surplus, hence:

$$V^{ns}(w_{nb}^{*}) - V^{u} = W^{ns}(w_{nb}^{*}) - W^{u} = \frac{1}{2}\left[V^{ns}(w_{nb}^{*}) - V^{u} + W^{ns}(w_{nb}^{*}) - W^{u}\right]$$

In turn, this implies that the (maximum of the) corresponding Nash Product simplifies to :

$$\left[V^{ns}(w_{nb}^{*}) - V^{u}\right]^{\frac{1}{2}} \left[W^{ns}(w_{nb}^{*}) - W^{u}\right]^{\frac{1}{2}} =$$

$$\left[V^{ns}(w_{nb}^{*}) - V^{u} + W^{ns}(w_{nb}^{*}) - W^{u}\right] \tag{3.7}$$

If the constraint is binding, then:

$$\begin{aligned} w_{ns} &= w_{ic} > w_{nb}^{*} = \arg\max_{w_{ns}} \left[V^{ns}(w_{ns}) - V^{u}\right]^{\frac{1}{2}} \left[W^{ns}(w_{ns}) - W^{u}\right]^{\frac{1}{2}} \\ w_{ns} &= w_{ic} = \frac{\lambda_{g} w_{g} - c(r+\delta)}{\lambda_{g}} \text{ as given by equation 3} \end{aligned}$$

Because this constraint binds, then the (maximum of the) corresponding Nash Product cannot be reduced, and it remains:

$$[V^{ns}(w_{ic}) - V^u]^{\frac{1}{2}} [W^{ns}(w_{ic}) - W^u]^{\frac{1}{2}} \tag{3.8}$$

Naturally, the unbounded maximum is higher than the bounded maximum:

$$\frac{1}{2}[V^{ns}(w_{ns}) - V^u + W^{ns}(w_{ns}) - W^u] > [V^{ns}(w_{ic}) - V^u]^{\frac{1}{2}} [W^{ns}(w_{ic}) - W^u]^{\frac{1}{2}}$$

A particularly relevant condition for further sections will be the condition for the constraint not to be binding:

$$w^*_{nb} \geq w_{ic} \tag{3.9}$$

Using equations 6 and 3 above, this reduces to:

$$X \geq f(V^u, W^u) = \frac{r\lambda_g V^u - 2c(r+\delta)}{\lambda_g} + \frac{2Y(r+\delta) + r\phi W^u}{2r+\phi+2\delta}$$

This says that X must be high enough for the constraint in the bargaining over an $ns-contract$ not to be binding. This is very intuitive, since w^*_{nb} increases directly with X.

For $s-contracts$, the case is analogue. Again abusing notation slightly, I will call $w_s = w^*_s$ if the constraint is not binding, and $w_s = w_{ic}$ if the constraint is binding. Because only the first case will be relevant in what follows, I only analyse this one.

If the constraint is not binding, then:

$$w_s = w_s^* = \arg\max_{w_s} [V^s(w_s) - V^u]^{\frac{1}{2}} [W^s(w_s) - W^u]^{\frac{1}{2}} < w_{ic}$$

The application of the Nash Bargaining solution implies:

$$w_s = w_s^* = \frac{1}{2} [X + c + r(W_u - V_u) + \lambda_g(W_u - W_g)] \tag{3.10}$$

and because this is the unconstrained solution, the (maximum of the) corresponding Nash Product can be reduced to:

$$[V^s(w_s) - V^u]^{\frac{1}{2}} [W^s(w_s) - W^u]^{\frac{1}{2}} = \frac{1}{2} V^s(w_s) - V^u + W^s(w_s) - W^{u\frac{1}{2}} \tag{3.11}$$

Notice that the wages, in all cases, are a pure transfer from firm to worker, $V^s(w) + W^s(w)$ and $V^{ns}(w) + W^{ns}(w)$ is independent of w. I use this from now on to ease notation, and write $V^s(.) + W^s(.)$ as $V^s + W^s$, and $V^{ns}(.) + W^{ns}(.)$ as $V^{ns} + W^{ns}$.

Workers-Good Vacancies Because workers never have an incentive to search on the job, there is never a problem of incentives in contracts between workers and good vacancies. Further, these contracts will always be signed since neither the worker nor the firm can expect a better opportunity. Hence, the wage is always given by the Nash bargaining solution:

$$w_g = arg\max(V^g(w_g) - V^{g,u})^{\frac{1}{2}}(W^g(w_g) - W^u)^{\frac{1}{2}} \;\; = \frac{1}{2}\left[Y + r(W^u - V^{g,u})\right]$$

3.5 Steady States and Arrival Rates

The arrival rates and steady state numbers of unemployed and employed workers, and of filled and empty vacancies of either type, depend on the search and matching behaviour of workers and vacancies. The only cases that will concern us are where workers search while unemployed; and always sign a contract of type either ns or s if they meet a vacancy of any type.

Vacancies have been assumed to search only while empty. But workers search behaviour while employed depends on the outcome of the bargaining problem analysed in the previous section.

No search on the job I will analyse first the case when $ns - contracts$ are signed. Given the quadratic meeting technology, the number of contacts is given by $u(v_g + v_b)$. The arrival rate for workers is $\frac{u(v_g+v_b)}{u} = v_g + v_b$. The unconditional arrival rate for firms of either type is $\varepsilon = \frac{u(v_g+v_b)}{v_g+v_b} = u$, since both are searching in the same side of the market (they cause congestion to each other). I will denote with η the effective arrival rate for bad vacancies and ϕ for good vacancies. Because all contacts lead to a contract being signed, regardless of the type of the vacancy, the effective arrival rates are both equal to u.

The measure of unemployed and employed workers is constant when the flow into unemployment equals the flow out of unemployment:

$$\begin{aligned} u(v_g + v_b) &= (1-u)\delta \Rightarrow \\ u &= \frac{\delta}{v_g + v_b + \delta} \end{aligned}$$

The measures of filled and empty vacancies (good and bad) are determined by:

$$\begin{aligned} v_g u &= (0.5 - v_g)\delta \\ v_b u &= (0.5 - v_b)\delta \end{aligned}$$

The simultaneous solution to these equations gives the steady state measures of unemployed workers and filled vacancies:

$$\begin{aligned} u &= \frac{1}{2}\left(-\delta + \sqrt{\delta(4+\delta)}\right) \\ v_g &= v_b = \frac{1}{4}\left(-\delta + \sqrt{\delta(4+\delta)}\right) \\ 0 &< v_g, v_b < 0.5 \text{ and } 0 < u < 1 \text{ if } 0 < \delta < \infty \end{aligned}$$

Search on the job Let us turn now to the case when search-contracts are signed. In this case, the total number of workers searching is the sum of those unemployed (u) and those working in bad vacancies $0.5 - v_b$. The number of contacts is then given by $(u+0.5-v_b)(v_g+v_b)$. The unconditional arrival rate for workers is therefore (v_g+v_b), and for vacancies of either type is $\varepsilon = (u+0.5-v_b)$

The employed and unemployed stocks of workers are constant when:

$$
\begin{aligned}
u(v_g + v_b) &= (1-u)\delta \Rightarrow \\
u &= \frac{\delta}{v_g + v_b + \delta}
\end{aligned}
$$

The unconditional arrival rate for vacancies of either type is $\varepsilon = (u+0.5-v_b)$. Good vacancies are filled every time they meet a worker, since they attract both employed or unemployed workers because they pay the highest wages. So their effective arrival rate is actually $\phi = \varepsilon$. Bad vacancies are only filled if they meet an unemployed worker (employed workers either with good or bad firms reject them) which occurs with probability $\frac{u}{(u+0.5-v_b)}$ conditional on having met a worker. Hence, their effective arrival rate is $\eta = (u+0.5-v_b)\frac{u}{(u+0.5-v_b)} = u$. Further, good vacancies with a worker become unfilled only if the contract is exogenously destroyed. On the other hand, bad vacancies with a worker become unfilled both if the exogenous destruction comes or if their worker meets a good vacancy, which occurs with probability $[v_g + v_b]\frac{v_g}{v_g+v_b} = v_g$. The stocks of empty and filled vacancies are therefore constant when:

$$v_g(u + 0.5 - v_b) = (0.5 - v_g)\delta$$

$$v_b u = (0.5 - v_b)(\delta + v_g)$$

The solution to these three equations yields:

$$\begin{aligned}
u &= \frac{1}{2}\left(-\delta + \sqrt{\delta(4+\delta)}\right) \\
v_g &= k \\
v_b &= u - k
\end{aligned}$$

where:

$$\begin{aligned}
k &= \frac{1}{2}\left(-\delta - \frac{1}{2} + \frac{1}{2}\sqrt{4\delta^2 + 12\delta + 1}\right) \\
0 &< v_g, v_b < 0.5 \text{ if } 0 < \delta < \infty
\end{aligned}$$

To avoid confusion , I will distinguish the arrival rates in the following way:

When $ns-contracts$ are signed:	When $s-contracts$are signed:
$\eta = u, \phi = u$	$\hat{\eta} = \hat{u} = u, \hat{\phi} = (u + 0.5 - \hat{v}_b)$
$\lambda_g = v_g$	$\hat{\lambda}_g = \hat{v}_g$
$\lambda_b = v_b$	$\hat{\lambda}_b = \hat{v}_b$

The solutions above yield the following conclusions regarding steady state stocks and arrival rates:

(i) The number of unemployed workers is independent of on-the-job search behaviour. This is because when workers search on the job, the total number of unfilled vacancies does not change as they move from bad to good firms.

(ii) The rate at which empty good vacancies become filled is higher when workers search on the job ($\hat{\phi} > \phi$). This is because there are more workers searching, and good vacancies attract them either if they are employed or unemployed.

(iii) The rate at which empty bad vacancies become filled is independent of on-the-job search behaviour ($\hat{\eta} = \eta$). This is because they contact workers at a higher rate when there is on-the-job search, but only attract them if the workers are unemployed.

(iv) The number of unfilled good vacancies is lower if workers search on the job. This is a consequence of (ii) and implies that the rate at which workers contact good empty vacancies ($\hat{\lambda}_g < \lambda_g$) is lower.

(v) The number of unfilled bad vacancies is higher if workers search on the job. Hence, the rate at which workers contact bad vacancies ($\hat{\lambda}_b < \lambda_b$) is higher..

Notice that (iv) has important implications for the level of the ICW. When all workers search on the job, the rate at which employed workers meet good vacancies is lower. This implies that the incentives to search on the job are lower, which reduces the level of the *ICW*. This composition effect and its consequences will be addressed further in Section 6

Summing up:

$$\eta = \hat{\eta}, \phi < \hat{\phi}, \lambda_g > \hat{\lambda}_g, \lambda_b < \hat{\lambda}_b$$

3.6 Contracts

In order to get to the stage of the contract decision between vacancies and workers, it must be the case that unemployed workers do find it optimal to search while employed. They will do it if, after paying the search cost c, $W^u > 0$ is still true. Hence, a common condition to all the derivations below is that $W^u > 0$.

Below are several Propositions that address when do workers and vacancies sign each type of contract. As we will see later, the conditions under which these decisions represent equilibrium decisions imply $W^u > 0$.

Workers-Good Firm Contracts The lifetime discounted values in contracts between workers and good firms have been stated above. They can be completed by the lifetime discounted value of an empty good vacancy and solved separately from the rest of the model. Since good firms contact workers at rate

ϕ, the lifetime discounted value of a good empty vacancy is given by

$$rV^{g,u} = \phi(V^g - V^{g,u})$$

Using $V^{g,u}, V^g, W^g$ and w_g, the solutions to the system of equations is reported in Set of Equations 3.7

$$\begin{aligned} V^g &= \frac{(Y - rW^u)(r + \phi)}{r(2r + \phi + 2\delta)} \\ W^g &= \frac{W^u(r + \phi + 2\delta) + Y}{2r + \phi + 2\delta} \\ V^{g,u} &= \frac{(Y - rW^u)\phi}{r(2r + \phi + 2\delta)} \\ w_g &= \frac{rW^u(r + \phi + \delta) + Y(r + \delta)}{2r + \phi + 2\delta} \end{aligned} \tag{3.12}$$

It is helpful to solve for them at this stage since they will be of use later.

Workers-Bad firms contracts This three propositions that follow address the contracts signed by workers and bad firms. Whenever confusion may arise, endogenous variables are sub-indexed with the number of the corresponding proposition.

Proposition 3.1 *Workers and bad firms sign ns − contracts and the wage is determined by an interior solution to the Nash Bargaining problem ($w_b = w^*_{ns} = w^*_{nb}$) if:*

$X \geq f(V_1^u, W_1^u) > g(V_1^u, W_1^u, \phi)$

$X \geq r(V_1^u + W_1^u)$

where

$f(V_1^u, W_1^u) = \frac{r\lambda_g V_1^u - 2c(r+\delta)}{\lambda_g} + \frac{2Y(r+\delta)+r\phi W_1^u}{2r+\phi+2\delta}$

$g(V_1^u, W_1^u, \phi) = \frac{r\lambda_g V_1^u - c(r+\delta)}{\lambda_g} + \frac{(r+\delta)Y+r(r+\delta+\phi)W_1^u}{2r+\phi+2\delta}$

Keeping W_1^u and V_1^u exogenous for the moment, the equations corresponding to configuration are 7, but luckily we already have a closed form solution for 4 of them, which we solved for and reported in Set of Equations 3.12. The full set of equations is given by Set P1:

$W^b = W^{ns}$ and $V^b = V^{ns}$ as given by equations 3.1 and 3.4

$w_b = w_{ns} = w_{nb}^*$ as given by equation 3.6

$V^{g,u}, V^g, W^g$ and w_g as given by Set of Equations 3.12

Proposition 3.1 is the simplest case, and it occurs when the constraint in the bargaining for an $ns - contract$ does not bind, given by equation 3.9:

$$w_{nb}^* \geq w_{ic} \Rightarrow$$

$$X \geq f(V_1^u, W_1^u) = \frac{r\lambda_g V_1^u - 2c(r+\delta)}{\lambda_g} + \frac{2Y(r+\delta) + r\phi W_1^u}{2r+\phi+2\delta}$$

Notice that in this case, the wage is given by $w_{nb}^* = \frac{1}{2}\left[X + r(W_1^u - V_1^u)\right]$; it is mainly determined by X, the workers productivity in the current employment.

This establishes that if $X \geq f(V_1^u, W_1^u)$, then the constraint in the determination of w_{ns}^* is not binding, so $w_b = w_{ns}^* > w_{ic}$

Using V_s and W_s from equations 3.2 and 3.5, $V^{ns} + W^{ns} \geq V^s + W^s \Leftrightarrow$

$$X \geq g(V_1^u, W_1^u, \phi) = \frac{r\lambda_g V_1^u - c(r+\delta)}{\lambda_g} + \frac{(r+\delta)Y + r(r+\delta+\phi)W_1^u}{2r+\phi+2\delta}$$

Hence, $ns-contracts$ dominate $s-contracts$ if the above condition holds. Finally, $ns-contracts$ dominate not signing a contract at all if and $V_{ns} + W_{ns} > V_1^u + W_1^u \Leftrightarrow$

$$r(V_1^u + W_1^u) \leq X$$

The three conditions are sufficient conditions for $ns-contracts$ to be signed. In fact, what they mean is that the constraint is not binding if the agents behave as if the constraint did not exist. Therefore the equilibrium that obtains is the equilibrium that would obtain if there were no imperfections.

For this to be interesting, it must be the case that:

$$f(V_1^u, W_1^u, \phi) > g(V_1^u, W_1^u, \phi)$$

If the above holds, then when X is high enough that $w^*_{nb} > w_{ic}$, it is also true that the $ns-contract$ dominates. But it also implies there exists an interval such that if $g(V_1^u, W_1^u, \phi) < X < f(V_1^u, W_1^u)$, then $ns-contracts$ dominate, but the wage that results from the equal split of the surplus is not incentive compatible. A different contract is signed, addressed in Proposition 3.2[4]. Since the condition $f(V_1^u, W_1^u, \phi) > g(V_1^u, W_1^u, \phi)$ is needed for incentive compatibility problems to arise, I will only consider parameter values such that it is true. Hence, all the Propositions below are stated for cases when $f(V_1^u, W_1^u, \phi) > g(V_1^u, W_1^u, \phi)$

Functions f and g will be recurrent in the Propositions that follow. In order to save space, the explicit functional form will not be written unless particularly relevant for the argument. Keep in mind then that

(*i*) f is the limit such that $X \geq f \Rightarrow w_{ns} = w^*_{nb} \geq w_{ic}$

(*ii*) g is the limit such that $X \geq g \Rightarrow V^{ns} + W^{ns} \geq V^s + W^s$

Proposition 3.2 *Workers and bad firms sign $ns-contracts$ and the wage is determined by a corner solution to the Nash Bargaining problem, $w_b = w_{ns} = w_{ic}$ if:*

$$\begin{aligned} h(V_2^u, W_2^u) &< X < f(V_2^u, W_2^u, \phi) \\ V^{ns} - V_2^u &> 0, W^{ns} - W_2^u > 0 \end{aligned}$$

[4]Notice, if $g(V_1^u, W_1^u) > f(V_1^u, W_1^u)$, then when $X > g(V_1^u, W_1^u, \phi)$(a no search contract dominates) it is already true that $X > f(V_1^u, W_1^u)$. If this is the case, $w^*_{nb} > w_{ic}$ is true always if $ns-contracts$ are dominant. There is no problem of incentives in this case.

where

$$h = \frac{i \pm 2\sqrt{(\lambda_g(\delta + r + \lambda_g)^3)}(c(2r + \phi + 2\delta) + \lambda_g r W_2^u - \lambda_g Y)}{(r + \delta)\lambda_g(2r + \phi + 2\delta)}$$

$$i = \begin{array}{c}[r + \delta]\,[(-(2\delta + 2r + 3\lambda_g + 2\lambda_g^2)(c(2r + \phi + 2\delta) - \lambda_g Y)] + \\ V_2^u\lambda_g r(2r + \phi + 2\delta) - W_2^u\lambda_g r(2\lambda_g^2 + 3\lambda_g - \phi))\end{array}$$

The fact that the wages paid are determined by the corner solution to the Nash Bargaining problem occurs because the equal split of the surplus results in a wage lower than the ICW. Hence, an $ns - contract$ is only possible if the firm increases the wage offer up to the ICW level. This implies a redistribution of profits from the firm to the worker, harming the interests of the firm. If the productivity X falls within the specified range, then decrease in firms' flow profits (X minus the wage paid) is not so high and the agents still sign an $ns - contract$. Notice that the contracts signed are the workers' preferred contract, the $ns - contract$ in which they get high ICW.

Keeping W_2^u andV_2^u exogenous for the moment, the equations that describe this configuration are the same as for Proposition 1, with one exception, which is that now the wage is given by the incentive compatibility condition. Hence, we now must use a slightly different Set of Equations P2:

$W^b = W^{ns}$ and $V^b = V^{ns}$ as given by 3.1 and 3.4

$w_b = w_{ns} = w_{ic} =$ as given by equation 3.3

$V^{g,u}, V^g, W^g$ and w_g as given by Set of Equations 3.12

In this configuration, the wage resulting from the unconstrained Nash Bargaining in an $ns - contract$ is lower than the wage given by the incentive compatibility constraint. Condition 3.9 implies:

$$w_{ic} > w_{nb}^* \Longleftrightarrow X < f(V_2^u, W_2^u, \phi)$$

Notice that in this case, the wage is given by $w_{ic} = \frac{\lambda_g w_g - c(r+\delta)}{\lambda_g}$. Even though the worker is not searching on the job for a good firm, his wage is determined not by X, but by w_g which is the wage he would earn if employed in a good firm.

It can be shown that $X < f(V_2^u, W_2^u, \phi)$ implies $w_s^* < w_{ic}$: The equal split of the surplus of an $s - contract$ actually gives the workers incentives to search. This is also intuitive; if an $ns - contract$ dominates, then one expects the wage determined by equal split of surplus to be higher in an $ns - contract$ than in an $s - contract$, $w_s^* < w_{nb}^*$. Because $X < f(V_2^u, W_2^u, \phi) \Leftrightarrow w_{nb}^* < w_{ic}$, then $w_s^* < w_{nb}^* < w_{ic}$. Hence, the (the maximum of the) corresponding Nash product in a $s - contract$ is given by:

$$\frac{1}{2}\left[V^s - V_2^u + W^s - W_2^u\right]$$

as shown in Section 3.4.

Because the wage paid in the conjectured contract $ns - contract$ is a corner solution to the Nash Bargaining problem ($w_b = w_{ic} > w_{nb}^*$), the surplus of the contract is not split in equal shares. The (maximum of the) corresponding Nash Product is therefore given by:

$$\left[V_{ns}(w_{ic}) - V_2^u\right]^{\frac{1}{2}}\left[W_{ns}(w_{ic}) - W_2^u\right]^{\frac{1}{2}}$$

Hence, to prove that $ns - contracts$ will be signed, one must prove that:

$$\left[V_{ns}(w_{ic}) - V_2^u\right]^{\frac{1}{2}}\left[W_{ns}(w_{ic}) - W_2^u\right]^{\frac{1}{2}} \geq$$

$$0\ ,\ \frac{1}{2}\left[V_s - V_2^u + W_s - W_2^u\right]$$

Provided all the expressions in parenthesis are positive, one can replace the previous inequality with the equivalent:

$$\left[V_{ns}(w_{ic}) - V_2^u\right]\left[W_{ns}(w_{ic}) - W_2^u\right] \geq \left[\frac{1}{2}\left[V_s - V_2^u + W_s - W_2^u\right]\right]^2$$

The two roots are given by $h(V_2^u, W_2^u)$ in Proposition 3.2. For consistency, it must be the case that:

(i) At least one of the roots is lower than $f(V_2^u, W_2^u, \phi)$, and therefore consistent with the behaviour assumed.

(ii) Both $V_{ns}(w_{ic}) - V_2^u$ and $W_{ns}(w_{ic}) - W_2^u$ are positive for the root that satisfies (i). Otherwise no contract would be signed.

In this configuration, $ns - contracts$ dominate a $s - contracts$ if

$V_{ns} + W_{ns} \geq V_s + W_s$, where V_s and W_s are given by 3.2 and 3.5. This condition can be reduced to

$$X > g(V_2^u, W_2^u, \phi)$$

Finally, if $g(V_2^u, W_2^u, \phi) < X < h(V_2^u, W_2^u, \phi)$, then $ns - contracts$ dominate, but are no longer signed. This inefficiency would be caused by the non-contractibility of the search effort. This case is addressed in Proposition 3.

Proposition 3.3 *Workers and bad firms sign $s - contracts$ and the wage is determined by an interior solution to the Nash Bargaining problem ($w_b^* = w_s^*$) if:*

$$\begin{aligned} X &< f(V_3^u, W_3^u, \hat{\phi}) \\ X &< h(V_3^u, W_3^u, \hat{\phi}) \\ X &> l(V_3^u, W_3^u, \hat{\phi}) \end{aligned}$$

Proposition 3.3 considers the case where the wage in an alternative $ns-contract$ would be a corner solution to the Nash Bargaining problem, i.e.,when $w_{ns} = w_{ic} > w^*_{nb}$. As in Proposition 3.2, the firm must increase the wage up to the ICW for the contract to be an $ns-contract$. But now, for very low values of X, as specified above, firms' profits $X - w_{ic}$ fall so low that firms and workers interests are in conflict. Firms have a strong preference for an $s-contract$ that allows them to pay lower wages, and workers prefer an $ns-contract$ that pays them higher wages. Because firms' profits are so low, they are "winners" in the Nash Bargaining, and their preferred contract, not the workers', is signed.

Keeping W^u_3 and V^u_3 exogenous for the moment, the equations corresponding to this configuration are given by Set of Equations P3:

$$W^b = W^s \text{ and } V^b = V^s \text{ as given by 3.2 and 3.5}$$

$$w_b = w_s = w^*_s \text{ as given by equation 3.10}$$

$$V^{g,u}, V^g, W^g \text{ and } w_{ic} \text{as Set of Equations 3.12}$$

Proposition 3.3 considers the case where the wage in an alternative $ns-contract$ would be a corner solution to the Nash Bargaining problem, i.e.,when:

$$w_{ns} = w_{ic} > w^*_{nb}$$

Again, condition 3.9 implies that this is true if:

$$X < f(V_3^u, W_3^u, \hat{\phi})$$

Because the wage in the $ns-contract$ is a corner solution to the Nash Bargaining problem, the corresponding Nash Product cannot be reduced, and in general is given by:

$$[V^{ns}(w_{ic}) - V_3^u]^{\frac{1}{2}} \, [W^{ns}(w_{ic}) - W_3^u]^{\frac{1}{2}} \, .$$

Again, one can show that $X < f(V_3^u, W_3^u, \hat{\phi}) \Rightarrow w_s^* < w_{ic}$[5]. This means that the (maximum of the) corresponding Nash product is given by:

$$\frac{1}{2} \left[W^s + V^s - W_3^u - V_3^u\right]$$

$s-contracts$ will be signed if:

$$\frac{1}{2} \left(W^s + V^s - W_3^u - V_3^u\right) >$$

$$0 \, , \; [V^{ns}(w_{ic}) - V_3^u]^{\frac{1}{2}} \, [W^{ns}(w_{ic}) - W_3^u]^{\frac{1}{2}}$$

[5]This can be explained intuitively with the same reasoning as in Proposiion 2.

Provided that all the expressions in parenthesis are positive, one can replace this inequality with the equivalent:

$$\left[\frac{1}{2}\left(W^s + V^s - W_3^u - V_3^u\right)\right]^2 > \left[V^{ns}(w_{ic}) - V_3^u\right]\left[W^{ns}(w_{ic}) - W_3^u\right]$$

The solutions to this quadratic are given by $h(V_3^u, W_3^u, \hat{\phi})$. As in Proposition 2, consistency requires that:

(i) At least one of the roots is lower than $f(V_3^u, W_3^u, \hat{\phi})$, and therefore consistent with the behaviour assumed.

(ii) $W_s + V_s - W_3^u - V_3^u$ is positive.

Finally, in this equilibrium $s-contracts$ dominate $ns-contracts$ if $W_s+V_s > W_{ns} - V_{ns}$, which can be reduced to

$$X < g(V_3^u, W_3^u, \hat{\phi}) \quad .$$

Notice if $g(V_3^u, W_3^u, \hat{\phi}) < X < h(V_3^u, W_3^u)$, then $s-contracts$ are signed, but $ns-contracts$ dominate. The inefficiency in the contract decision is caused by the non-contractibility of on-the-job search.

A fourth case could be considered, complementing Proposition 3.3. This would be the case where (i) Workers and bad firms sign search contracts and the wage is given by w_s^*as in Proposition 3.3, and (ii) Wages in an alternative $ns-contract$ would be an interior solution to the Nash Bargaining problem: $w_{ns}^* > w_{ic}$. Although interesting, this case would be out of the scope of the paper, since a necessary condition for this configuration to obtain is that $s - contracts$ dominate.

Composition Effects Section 3.5 introduced the composition effects created by the workers equilibrium search behaviour.

Consider the two configurations in Propositions 3.2 and 3.3. They differ in the workers' search behaviour, and this has consequences on the steady state numbers of searching workers and of empty good and bad vacancies. This "composition effect" of workers' equilibrium search behaviour implies that, for some parameter values, no steady state equilibrium will obtain. Compare the steady state numbers of firms of either type implied by worker's search behaviour in Propositions 3.2 and 3.3. Since in Proposition 3.3 workers in bad firms search on the job, they will be leaving bad firms empty as they move to fill good firms met through search. Hence, the steady state number of empty bad firms is higher and the number of empty good firms is lower in Proposition 3.3 compared to Proposition 3.2. This means that the rate at which workers find good firms is lower in Proposition 3 compared to Proposition 3.2, if they do search on the job. This makes search less attractive, and therefore reduces the ICW level. For any value of X, the profits of firms that pay the ICW will be higher with the arrival rate consistent with $Case$ 2, since the implied ICW is lower. Hence, there will be values of X which (i) Are not consistent with $ns - contracts$ equilibrium arrival rate, since firms' profits are too low if the implied ICW is paid, and hence $s - contracts$ will be signed, but (ii) Are not consistent either with the $s - contracts$ equilibrium arrival rate, since firms' profits are high enough when the implied ICW is paid, and $ns - contracts$ will be signed. Ceteris paribus, there exists a range such that if X falls within that range, no steady state equilibrium will obtain. In terms of the limits obtained above, this implies that $h(V_3^u, W_3^u) < h(V_2^u, W_2^u)$, and no steady state equilibrium obtains if $h(V_3^u, W_3^u) < X < h(V_2^u, W_2^u)$. This is exemplified is the numerical example in Section 3.7[6]. It is also true that the number of searching workers differs across the configurations. But the assumption of a quadratic search technology implies that the rate at which workers meet firms is independent of the number of workers searching. Therefore, this has no consequence..

On the contrary, the limits that separate Propositions 3.1 and 3.2 are equal, since the search behaviour across them is the same. Hence, $f(V_1^u, W_1^u, \phi) = f(V_2^u, W_2^u, \phi)$. This will be shown in the following Theorems 3.1 and 3.2 and in the numerical example in Section 3.7.

Theorem 3.1 *Workers and bad firms sign $ns-contracts$ and the wage is determined by an interior solution to the Nash Bargaining problem ($w_b = w^*_{ns} = w^*_{nb}$) if:*

$$Y > Y_1^a = 2c(\delta + r + \phi)\frac{1}{\lambda_g}$$

[6]There is a further reason why $h(V_3^u, W_3^u) < X < h(V_2^u, W_2^u)$. In Proposition 2, workers are "winners" in the bargaining, in the sense that their preferred contract is signed (the $ns - contracts$). In Proposition 3, workers are "loosers" in the Nash Bargaining, as the contracts signed are the firms' preferred contracts (the $s - contracts$). Workers are therefore worse-off in the configuration in Proposition 3. This means that, when they encounter good firms, they can only bargain lower wages from them, as compared to Proposition 2, and this is another reason why the ICW in Proposition 3 is lower than that in Proposition 2, contributing to the existence of the "gap".

$X \geq X_1^a = \frac{[\lambda_g Y - c(2r+2\delta+\phi)][(2\delta+\lambda_b+2r+\eta+\lambda_g]}{\lambda_g(\lambda_b+2r+2\delta+\phi+\lambda_g)}$

Theorem 3.1 determines when does the configuration in Proposition 3.1 obtains as an equilibrium configuration. This is done by endogenising the values of W_1^u and V_1^u.Because firms and workers sign stable contracts, these values are given by the following Bellman Equations:

$$\begin{aligned} rW_1^u &= \lambda_b(W_{ns}(w_{nb}^*) - W_1^u) + \lambda_g(W_g - W_1^u) \\ rV_1^u &= \eta(V_{ns}(w_{nb}^*) - V_1^u) \end{aligned}$$

Here, bad vacancies effective arrival rate $\eta = \phi$,as was detailed in Section 3.5. The solutions to W_1^u and V_1^u are found using $W^{ns}(w_{ns}^*), V^{ns}(w_{ns}^*)$ from the solutions to Set of Equations P1.

Limits $g(V_1^u, W_1^u, \phi)$ and $f(V_1^u, W_1^u)$ become respectively:

$$\begin{aligned} f(V_1^u, W_1^u) &= X_1^a \\ &= \frac{[\lambda_g Y - c(2r+2\delta+\phi)]\,[(2\delta+\lambda_b+2r+\eta+\lambda_g]}{\lambda_g(\lambda_b+2r+2\delta+\phi+\lambda_g)} \end{aligned}$$

$$\begin{aligned} g(V_1^u, W_1^u, \phi) &= X_1^b \\ &= \frac{[\lambda_g Y - 2c(2r+2\delta+\phi)]\,[(2\delta+\lambda_b+2r+\eta+2\lambda_g]}{\lambda_g(\lambda_b+4r+4\delta+2\phi+2\lambda_g)} \end{aligned}$$

And it is easy to show that $X_1^a > X_1^b$, (as required by Proposition 1), if

$$Y > Y_1^a = 2c(\delta + r + \phi)\frac{1}{\lambda_g}.$$

The intuition is given by the incentives to search on the job. When Y is high enough, the expected profit of on-the-job search is high, since w_g will be higher. Hence, the worker gives up search even for very high wage levels, which would be the result of very high levels of X This implies that the limit X_1^a is very high. Hence, if Y is high enough, $(Y > Y_1^a)$ then X_1^a is high enough $(X_1^a > X_1^b)$

Finally:

$$X > r(V_1^u + W_1^u) \Longleftrightarrow X > \frac{\lambda_g Y - c(2\delta + \eta + 2r)}{2r + \eta + \lambda_g + 2\delta}$$

and this is true when $X > X_1^a$ because $Y > Y_1^a \Longleftrightarrow X_1^a > \frac{\lambda_g Y - c(2\delta + \eta + 2r)}{2r + \eta + \lambda_g + 2\delta}$

Endogenising W_1^u and V_1^u allows also to find and explicit solution to:

$$w_{nb}^* = \frac{-(\delta+r+\eta)(2c\delta+c\phi-Y\lambda_g+2cr)+X[(\delta+r)(2\delta+\lambda_g+\phi+2\lambda_b+2r)+\lambda_b\phi]}{\eta\lambda_g+\eta\phi+\lambda_b\phi+2(\delta+r)(2\delta+\lambda_g+\eta+\phi+\lambda_b+2r)}$$

For further reference, notice that using $X = f(V_1^u, W_1^u)$ in w_{nb} yields:

$$w_{ns_{x=f}}^* = \frac{(\delta + r + \lambda_g + \lambda_b)(\lambda_g Y - c(2r + 2\delta + \phi))}{(2\delta + \lambda_g + \phi + \lambda_b + 2r)\lambda_g}$$

Since condition $Y > Y_1^a = 2c(\delta + r + \phi)\frac{1}{\lambda_g}$ is needed for incentive compatibility problems to arise, I will restrict the parameter value of Y in such a way that $Y > Y_a^1$.

Theorem 3.2 *Workers and bad firms sign ns – contracts and the wage is determined by a corner solution to the Nash Bargaining problem,* $w_b = w_{ns} = w_{ic}$ *if:*

$$X < X_2^a = \frac{[\lambda_g Y - c(2r + 2\delta + \phi)]\,[(2\delta + \lambda_b + 2r + \eta + \lambda_g]}{\lambda_g(\lambda_b + 2r + 2\delta + \phi + \lambda_g)}$$

and $X > X_2^c =$

$$\frac{\left[j \pm 2\sqrt{\lambda_g(r+\eta+\delta)^2(\delta + r + \lambda_g)^3}\right][c(2\delta + 2r + \phi) - \lambda_g Y]}{(r+\delta)^2\lambda_g(\lambda_b + \ \lambda_g + \phi + 2r + 2\delta)}$$

$$j = -(r+\delta)\ \left[(r+\delta)\ (2\delta + \lambda_b + 4\lambda_g + 2r + \eta) + 3\eta\lambda_g + 2\lambda_g^2\right] - 2\lambda_g^2\eta$$

Theorem 3.2 states the conditions for the configuration in Proposition 3.2 to obtain as an equilibrium. Again, firms and workers sign stable contracts. Hence, the values of filled vacancies and unemployed workers are as in Theorem 3.1.

$$\begin{aligned} rW_2^u &= \lambda_b(W_{ns}(w_{ic}) - W_2^u) + \lambda_g(W_g - W_2^u) \\ rV_2^u &= \eta(V_{ns}(w_{ic}) - V_2^u) \end{aligned}$$

The solutions to W_2^u and V_2^u are found using W_{ns}, V_{ns} from the solutions to Set of Equations P2. Using them in $f(V_2^u, W_2^u, \phi)$ and $h(V_2^u, W_2^u, \phi)$ in Proposition 3.2 yields $f(V_2^u, W_2^u, \phi) = X_2^a$ and $h(V_2^u, W_2^u, \phi) = X_2^c$

The following conditions, as required by Proposition 3.2, can easily be shown with algebraic manipulations:

(i) The root X_2^c with a $-$ sign is higher than $f(V_2^u, W_2^u)$, and therefore inconsistent with the behaviour assumed. The root with the $+$ sign is always lower than $f(V_2^u, W_2^u)$, hence this is the one that concerns us. Further,

(ii)Plugging this root in $[V^{ns} - V_2^u]$, $[W^{ns} - W_2^u]$ and $[V_s - V_2^u + W_s - W_2^u]$ results in all being positive.

As regards to limit $g(V_2^u, W_2^u)$, in Proposition 3.2, the use of V_2^u, W_2^u allows to reduce it to:

$$\begin{aligned} g(V_2^u, W_2^u) &= X_2^b \\ &= \frac{(\lambda_b + \lambda_g + r + \delta)\,(Y\lambda_g - c(2\delta + 2r + \phi))}{2\delta + \lambda_g + \lambda_b + 2r + \phi)\,\lambda_g} \end{aligned}$$

Hence, if $X > X_2^b$, $ns - contracts$ dominate if everybody else signs no-search contracts. It can also be shown that $X_2^c > X_2^b$. This means that as X falls lower than X_2^c, agents no longer sign no-search contracts, but they still dominate. This inefficiency in the decision of contract is caused by the non-contractibility of on-the-job search behaviour.

Notice that X_1^b (from Theorem 3.1) differs from X_2^b. The limit such that a no-search contract dominates the search contract changes when the wage paid changes from w_{ns} to w_{ic}. At first glance, it seems strange that a differences in wages paid causes X_1^b to differ from X_2^b. This because the wage is only a transfer between the two agents in the contract, and changing it should therefore not cause changes in the total value of the contract. This would be true if no other agents were involved in the determination of the equilibrium. In this case, the presence of good vacancies, with whom workers interact, invalidates the previous reasoning. The reason is that, when workers earn wages determined by the incentive compatibility constraint, their lifetime value both in a bad vacancy and when unemployed, increases. As a consequence of this increase, they are able to bargain better wages from good vacancies whenever they meet them. This "additional" extraction of value from good vacancies increases the total value in search-contracts. Hence, as a result of having a third party with whom workers interact, the wages stop being a pure transfer of value and do affect the total value of a search contract.

The wages paid in this equilibrium are given by the incentive compatibility constraint, and can be reduced to:

$$w_{ic} = \frac{(\delta + r + \lambda_g + \lambda_b)(\lambda_g X_H - c(2r + 2\delta + \phi))}{(2\delta + \lambda_g + \phi + \lambda_b + 2r)\lambda_g} = w^*_{nb_{x=f}}$$

Theorem 3.3 *Workers and bad firms sign $s-$contracts and the wage is determined by an interior solution to the Nash Bargaining problem ($w_b^* = w_s^*$) if:*

$$Y < Y_3^a = 2c(\delta + r + \hat{\phi})\frac{1}{\hat{\lambda}_g}$$

$$X < X_3^c = \frac{\left[k \pm \sqrt{\hat{\lambda}_g(\delta + r + \hat{\lambda}_g)(2r + \hat{\lambda}_g + 2\delta + \hat{\phi})^2}\right](2c\delta + 2cr - \hat{\lambda}_g Y + c\hat{\phi})(2\delta + 2r + \hat{\eta} + 2\hat{\lambda}_g + \hat{\lambda}_b)}{\hat{\lambda}_g\left[(r+\delta)(4(r+\delta)(\delta + \hat{\lambda}_g + \hat{\phi} + r + \hat{\lambda}_b) + (\hat{\lambda}_b + \hat{\phi})^2 + \hat{\lambda}_g(6\hat{\lambda}_b + 2\hat{\phi} + \hat{\lambda}_g)\right] + \hat{\lambda}_g\hat{\lambda}_b(\hat{\lambda}_b + 2\hat{\phi} + 2\hat{\lambda}_g))}$$

$$k = -(r + \delta)(2\delta + \hat{\lambda}_b + 3\hat{\lambda}_g + \hat{\phi} + 2r) - \lambda(\hat{\phi} + \hat{\lambda}_g + \hat{\lambda}_b)$$

Theorem 3.3 shows if and when the configuration in Proposition 3.3 obtains as an equilibrium configuration. The endogenous values of unemployed workers and filled vacancies are given by:

$$\begin{aligned} rW_3^u &= \hat{\lambda}_b(W_s(w_s^*) - W_3^u) + \hat{\lambda}_g(W_g - W_3^u) \\ rV_3^u &= \eta(V_s(w_s^*) - V_4^u) \end{aligned}$$

Their explicit values are obtained by using the solutions to the system P3. Using this, one can show that:

$$\begin{aligned} f(V_3^u, W_3^u, \hat{\phi}) &= X_3^a \\ h(V_3^u, W_3^u, \hat{\phi}) &= X_3^c \end{aligned}$$

is as specified in Theorem 3.3.

As required by Proposition 3.3, it is east to show that:

(i) The root X_3^c with a "+" sign is lower than X_3^a. This is the one that concerns us since it is consistent with the behaviour assumed (The root with a "-" sign is higher than X_3^a)

(ii) Plugging the solution from (i) into $[V^{ns}(w_{ic}) - V_3^u]$ $[W^{ns}(w_{ic}) - W_3^u]$ and $W_s + V_s -\ W_3^u - V_3^u$ results in all being positive.

Finally, using W_3^u and V_3^u in $g(V_3^u, W_3^u, \hat{\phi})$ yields:

$$\begin{aligned} g(V_3^u, W_3^u, \hat{\phi}) &= X_3^b \\ &= \frac{(2r + 2\delta + \hat{\eta} + 2\hat{\lambda}_g + \hat{\lambda}_b)(\hat{\lambda}_g Y - c(2\delta + c\hat{\phi} + 2r))}{\hat{\lambda}_g(\hat{\lambda}_b + 2\hat{\lambda}_g + 4\delta + 4r + 2\hat{\phi})} \end{aligned}$$

And it can be shown that $X_3^b < X_3^c$. This means that there is an interval (X_3^b, X_3^c) such that if X lies within that interval, $ns-contracts$ dominate but $s-contracts$ are signed. This inefficiency is caused by the non-contractibility of on-the-job search behaviour.

The wage paid in this case is given by:

$$\begin{aligned} w_s &= \frac{(\hat{\lambda}_g + \delta + r)(\hat{\lambda}_g Y - c(2r + 2\delta + \hat{\phi}))}{(2r + \hat{\lambda}_g + 2\delta + \hat{\phi})\hat{\lambda}_g} \\ &+ \frac{\hat{\lambda}_b(\delta + r + \hat{\phi})X}{(2r + \hat{\lambda}_g + 2\delta + \hat{\phi})(2\hat{\lambda}_g + \hat{\lambda}_b + 2r + \hat{\eta} + 2\delta)} \end{aligned}$$

3.7 Numerical Example

This section addresses a numerical example in order to provide a better understanding of the workings of the model. I will use the following values for the relevant parameters:

$$r = 0.1, \delta = 0.1, Y = 40, c = 2$$

For this I first report the steady state stocks and arrival rates when there is no on-the-job search. These come from the derivations in section 3.5:

$$u = \tfrac{1}{2}(-\delta + \sqrt{\delta(4+\delta)} = 0.27 \qquad \phi = \eta = u = 0.27$$
$$v_g = \tfrac{1}{4}(-\delta + \sqrt{\delta(4+\delta)} = 0.13 \qquad \lambda_g = v_g = 0.13$$
$$v_b = \tfrac{1}{4}(-\delta + \sqrt{\delta(4+\delta)} = 013 \qquad \lambda_b = 0.13$$

I will investigate first if and when do the configurations in Theorems 3.1 and 3.2 obtain as an equilibrium. The first thing to note is that a problem of incentives exists if $Y = 40 > Y_{a,1}$, where:

$$Y_1^a = c(2\delta + 2r + \phi)\frac{1}{\lambda_g} = 10.30$$

We conclude that a problem of incentives may arise depending on the value of X. The next step is to address the configuration in Theorem 3.1. For this configuration to obtain as an equilibrium, we must have:

$$\begin{aligned} X &\geq X_1^a = \frac{[\lambda_g Y - c(2r + 2\delta + \phi)]\,[(2\delta + \lambda_b + 2r + \eta + \lambda_g]}{\lambda_g(\lambda_b + 2r + 2\delta + \phi + \lambda_g)} \\ X &\geq X_1^a = 30.77 \end{aligned}$$

Hence, the workers and bad firms sign no-search contracts, and the wage is given by the w_{nb} if $X \geq 30.77$. The thing to note is that a no search contract dominates if:

$$\begin{aligned} X &\geq X_1^b = \frac{[\lambda_g Y - 2c(2r + 2\delta + \phi)]\,[(2\delta + \lambda_b + 2r + \eta + 2\lambda_g]}{\lambda_g(\lambda_b + 4r + 4\delta + 2\phi + 2\lambda_g)} \\ X &\geq X_1^b = 18.53. \end{aligned}$$

The wage paid is $w_{nb}^* =$

$$\frac{-(\delta+r+\eta)(2c\delta+c\phi-Y\lambda_g+2cr)+X[(\delta+r)(2\delta+\lambda_g+\phi+2\lambda_b+2r)+\lambda_b\phi]}{\eta\lambda_g+\eta\phi+\lambda_b\phi+2(\delta+r)(2\delta+\lambda_g+\eta+\phi+\lambda_b+2r)}$$

$$3.029571593 + .3992048651X$$

Hence, there is an interval such that if $18.53 < X < 30.77$, $ns - contracts$ dominate, but the wage must be determined by the incentive compatibility constraint to implement them.

This takes us to investigate the configuration in Theorem 3.2. It obtains as an equilibrium strategy if:

$$\begin{aligned} X &< X_2^a = \frac{[\lambda_g Y - 2c(2r + 2\delta + \phi)]}{\lambda_g} \\ X &< X_2^a = 30.77 \end{aligned}$$

and

$$X_2^a \geq X_2^c = 20.66$$

Hence, provided that $20.66 < X < 29.69$, an equilibrium obtains where workers and bad firms sign no-search contracts, but the wage is given by the incentive compatibility constraint. This wage is:

$$w_{ic} = \frac{(\delta + r + \lambda_g + \lambda_b)(\lambda_g Y - c(2r + 2\delta + \phi))}{(2\delta + \lambda_g + \phi + \lambda_b + 2r)\lambda_g} = 15.03$$

Notice that $w_{ic} = w_{nb}$ if $X = 29.69$

Again, one must note that $ns - contracts$ dominate if:

$$\begin{aligned} X &> X_2^b = \frac{(\lambda_b \ + \lambda_g \ + r + \delta)\ (Y\lambda_g - c(2\delta + 2r + \phi))}{(2\delta + \lambda_g \ + \lambda_b \ + 2r + \ \phi)\ \lambda_g} \\ X &> X_2^b = 15.03 \end{aligned}$$

This implies that for $15.03 < X < 20.66$, $ns - contracts$dominate, but $s - contracts$ are signed because they maximise the global Nash Product. The inefficiency in the contract decision is caused by the non-contractibility of on-the-job search behaviour.

We turn now to the configurations in Theorem 3.3. I first report the steady state arrival rates if workers search on the job.

$$\begin{array}{ll} \hat{u} = \frac{1}{2}(-\delta + \sqrt{\delta(4+\delta)} = 0.27 & \hat{\eta} = \hat{u} = 0.27 \\ \hat{v}_b = \frac{1}{2}(\frac{1}{2} - \frac{1}{2}\sqrt{\delta^2 + 12\delta + 1}) + \sqrt{\delta(4+\delta)} = 0.19 & \hat{\lambda}_b = \hat{v}_b = 0.19 \\ \hat{v}_g = \frac{1}{2}(-\delta - \frac{1}{2} + \frac{1}{2}\sqrt{4\delta^2 + 12\delta + 1}) = 0.07 & \hat{\lambda}_g = \hat{v}_g = 0.07 \end{array}$$

$$\hat{\phi} = \hat{u} + 0.5 - \hat{v}_b = 0.5741$$

What we must ask is if and when does the configuration in Theorem 3.3 obtains. We must ask if $Y > Y_{a3}$, where

$$Y_3^a = c(2\delta + 2r + \hat{\phi})\frac{1}{\hat{\lambda}_g} = 26.21$$

Hence, we conclude that it may obtain for suitable values of X. The three conditions are:

$$\begin{aligned} X &< X_3^a = 9.89 \\ X &< X_3^c = 7.78 \end{aligned}$$

Notice that if $X_2^c = 7.78 < X < 20.66 = X_3^c$, then no pure strategy steady state equilibria obtains. This is a consequence of the composition effects referred to in Sections 3.5 and 3.6

But notice that $s - contracts$dominate only if:

$$X < X_3^b = 6.07$$

Hence, for X such that $6.07 < X < 7.78$, $s - contracts$are signed but $ns -$ $contracts$ dominate. This inefficiency is again caused by the non-contractibility of on-the-job search behaviour. The wages are given by

$$\begin{aligned} w_{ns} &= \frac{(\hat{\lambda}_g + \delta + r)(\hat{\lambda}_g Y - c(2r + 2\delta + \hat{\phi}))}{(2r + \hat{\lambda}_g + 2\delta + \hat{\phi})\hat{\lambda}_g} + \\ & \frac{\hat{\lambda}_b(\delta + r + \hat{\phi})X}{(2r + \hat{\lambda}_g + 2\delta + \hat{\phi})(2\hat{\lambda}_g + \hat{\lambda}_b + 2r + \hat{\eta} + 2\delta)} \\ &= 3.584717043 + .1426473611X \end{aligned}$$

3.8 Conclusion

This paper has studied equilibrium contracting when on-the-job search is costly and the terms of contract are decided by bargaining; and the consequences for the search effort of employed workers and for wage formation . With the natural assumption that the search effort is non contractible, the search decision is individually made by the worker, taking as given the wage level agreed upon. I have shown that two search equilibria exist, and that the imperfection created by non-contractible search may imply that the contract signed is privately inefficient, even in the presence of bargaining. Further, I have shown how a worker's wage need not be determined by his productivity in the firm, but may be determined by his productivity in alternative (better) employment, although (or perhaps because) he is not searching for better employment. As a consequence of the matching framework used, the composition effects of the equilibrium search behaviour of employed workers implies that there is a region such that no pure strategy steady state equilibrium exists.

Bibliography

[1] Altenburg, Lutz and Martin Straub, Taxes on Labour and Unemployment in a Shirking Model with Union Bargaining, Labour Economics 8 (2002), 721-744.

[2] Albrecht, James and Bo Axell, An Equilibrium Model of Search Unemployment, The Journal of Political Economy, Volume 92, Issue 5, (October 1984), 824-840.

[3] Becker, Gary, A Theory of Marriage: Part I, Journal of Political Economy (81), 1973 , 813-846

[4] Becker, Gary, A Theory of Marriage: Part II, Journal of Political Economy (82), 1974 , S11

[5] Burdett, Kenneth and Kenneth Judd, Equilibrium Price Dispersion, Econometrica, Vol. 51, No. 4. (July,1983), 955-970.

[6] Burdett, Kenneth and Dale Mortensen, Wage Differentials, Employer Size, and Undemployment, International Economic Review, Volume 39, Issue 2 (May, 1998), 257-273.

[7] Burdett, Kenneth and Melvyn Coles, Marriage and Class, Quarterly Journal of Economics, pp141-165, 1998

[8] Burdett, Kenneth and Melvyn Coles, Separation Cycles, Journal of Economic Dynamics and Control, pp1069-1090, 1998

[9] Burdett, Kenneth and Melvyn Coles, Long Term Partnership Formation: Marriage and Employment, Economic Journal, June 1999.

[10] Burdett, Kenneth, Ryoichi Imai and Randall Wright, Unstable Relationships, Working Paper, 1999.

[11] Burdett, Kenneth, Melvyn Coles, Nobuhiro Kiyotaky and Randall Wright, Economic Evolution with Market Frictions, American Economic Review Papers and Proceedings, 1995, pp. 281-287.

[12] Burdett, Kenneth and Melvyn Coles, A Note on Positive Assortative Matching, Programme on Labour Market Dynamics in a Changing Environment, Discussion Paper Series 99/38, December 1999.

[13] Burdett, Kenneth and Melvyn Coles, Separations and Trust, working paper, February 2000.

[14] Burdett, Kenneth and Melvyn Coles, Equilibrium Wage-Tenure Contracts, Econometrica, forthcoming 2003.

[15] Burdett, Kenneth; Mortensen, Dale T, Wage Differentials, Employer Size, and Unemployment, International Economic Review, 1998, vol. 39, no. 2, pp. 257.

[16] Butters, Gerard R., Equilibrium Distributions of Sales and Advertising Prices, Review of Economic Studies 44(October 1977), 465-491.

[17] Coles, M. G, Equilibrium Wage Dispersion, Firm Size, and Growth, Review of Economic Dynamics, 2001, vol. 4, no. 1, pp. 159-187.

[18] Diamond, P. and Eric Maskin, An Equilibrium Analysis of Search and Breach of Contract I: Steady States, The Bell Journal of Economics, 10, 1979, 282-316

[19] Gautier, Pieter, Unemployment and Search Externalities in a model with Heterogeneous Jobs and Heterogenous Workers, mimeo?, June 1999.

[20] Mortensen Dale L., Specific Capital, and Labor Turnover, Bell Journal of Economics, 9 (1978) 572-586.

[21] Mortensen, Dale T., Equilibrium Unemployment with Wage Posting: Burdett-Mortensen meet Pissarides, mimeo, September 1998

[22] Muthoo, Abhinay, Bargaining Theory with Applications, New York : Cambridge University Press, 1999

[23] Pissarides, Christopher, Search Unemployment with On-the-job Search, Review of Economic Studies (1994) 61, 457-475.

[24] Postel-Vinay and Jean-Marc Robin, To Match or Not To Match? Optimal Wage Policy with Endogenous Worker Search Intensity, Working Paper, January, 2002

[25] Postel-Vinay, Fabien and Jean-Marc Robin, The Distribution of Earning in an Equilibrium Search Model with State-Dependent Offers and Counteroffers., International Economic Review, Volume 43, Issue 4 (November 2002a), 989-1016

[26] Postel-Vinay, Fabien and Jean-Marc Robin, Equilibrium Wage Dispersion with Worker and Employer Heterogeneity, Econometrica, Vol. 70, No. 6 (November 2002b), 2295-2350

[27] Rocheteau, Guillaume, Equilibrium Unemployment and Wage Formation with Matching Frictions and Worker Moral Hazard, Labour Economics 8 (2001), 75-102

[28] Roth, Alvin and Marilda A. Oliveira Sotomayor, Two-sided matching : a study in game-theoretic modeling and analysis , Cambridge : Cambridge University Press, 1990

[29] Shi, 1998, Frictional Assignemnt, UQAM, Working Paper No. 74.

[30] Shimer, Robert and L. Smith, Assortative Matching and Search, Econometrica, Vol 68, 2000, 343-337

[31] Shimer, Robert and L. Smith, Matching, Search and Heterogeneity, Advances in Macroeconomics (B.E. Journals in Macroeconomics), Vol 1: No. 1, Article 5.

[32] Webb, Tracy, Equilibrium Quits and Lay-offs in a Model With On-the-Job Search, University of Bristol , mimeo