University of Essex

Research Repository

Advance booking and returns contract in agricultural-input supply chains

Accepted for publication in Transportation Research Part E: Logistics and Transportation Review

Research Repository link: https://repository.essex.ac.uk/44285/

Please note:

Changes made as a result of publishing processes such as copy-editing, formatting and page numbers may not be reflected in this version. For the definitive version of this publication, please refer to the published source. You are advised to consult the published version if you wish to cite this paper.
https://doi.org/10.1016/j.tre.2026.105408

www.essex.ac.uk

University of Essex

Advance Booking and Returns Contract in Agricultural-Input Supply Chains

Abstract

Agricultural input supply chains face significant operational challenges arising from seasonal demand, demand uncertainty and the need to coordinate production and distribution decisions. While extant research has largely focussed on farm-level yield uncertainty, less attention has been paid to upstream players like manufacturers and wholesalers of agricultural inputs. A prevalent practice in agricultural input supply chains is advance booking, where manufacturers offer discounts for pre-season orders. This study investigates how advance booking influences supply chain decisions and performance under alternative channel leadership structures. Using a game-theoretic framework, we analyze two supply chain configurations and examine decisions on booking quantities, sales effort, and production quantities. We find that wholesalers exert greater sales effort as supply chain leaders, enabling them to place larger advance orders and reduce reliance on lower-margin regular purchases. Advance booking discounts play a crucial role in incentivizing wholesalers and shaping their ordering decisions. We also find that the manufacturer's production decisions are sensitive to both discount levels and cost structures, with higher costs constraining production output. Further, under intermediate discount and cost conditions, a returns policy can mitigate wholesaler risk and lead to mutually beneficial outcomes. Finally, incorporating weather-induced demand uncertainty, we find that the key insights remain qualitatively unchanged, demonstrating the robustness of the results. The study highlights the importance of advance booking incentives and risk-sharing mechanisms in improving coordination and performance in agricultural input supply chains.

Keywords: Supply chain management, agricultural inputs, advance booking, returns policy, emerging economy

1. Introduction

Agricultural productivity and food security depend critically on the timely availability and efficient distribution of quality agricultural inputs, including seeds, fertilizers and agrochemicals. However, agricultural input supply chains face significant operational challenges arising from seasonal demand, uncertain market conditions and the need to coordinate production and distribution within narrow selling windows. These challenges are particularly pronounced in agriculture-dependent economies, where fragmented distribution networks and reliance on intermediaries complicate supply chain planning. In India, for instance, agriculture supports the livelihoods of over 40% of the population,[1] highlighting the importance of efficient agricultural input supply chains. Consequently,

[1] https://pib.gov.in/PressReleasePage.aspx?PRID=2034943 Accessed on 2nd November 2024

effective coordination between manufacturers and wholesalers is essential to ensure timely product availability, manage demand uncertainty and improve supply chain performance.

One mechanism through which manufacturers address this challenge is *advance booking*, whereby wholesalers commit to purchase quantities before the start of the selling season in exchange for discounted prices or other incentives. Advance booking programs are widely practiced in agricultural input industries, particularly in seeds and agrochemicals. In India, seed producers such as Penna Seeds,[2] Kirtiman Agrogenetics[3] and Kalash Seeds[4] offer advance booking schemes for staple crops like wheat, maize, gram, and paddy while agrochemical firms such as UPL Limited,[5] Dhanuka Agritech,[6] and Tropical Agro[7] offer advance booking discounts on a range of their products. Similar pre-season ordering programs are also operated by multinational firms such as Syngenta[8], Bayer Crop Science, Corteva Agriscience[9] and BASF. Table 1 illustrates representative differences between advance booking and regular period prices for selected agricultural inputs.

Agricultural input	Regular period price (INR Per Kg)	Advance Booking price (INR Per Kg)
Maize seeds	500	475
Gram seeds	300	280
	(INR Per Litre)	(INR Per Litre)
Butachlor herbicide	700	660
2,4-D herbicide	400	370
Fenoxaprop herbicide	2500	2300

Table 1: Advance booking discount, prices in INR, 1 USD $\approx$ 95 INR (representative figures based on interviews with seeds and agrochemical firms)

Advance Booking. The prevalence of advance booking is closely linked to the operational characteristics of agricultural input supply chains. Farmers typically purchase seeds, fertilizers, and crop-protection products within a short period preceding sowing seasons such as the Rabi and Kharif seasons in South Asia.[10] Once planting is completed, demand declines significantly until the next production cycle. Consequently, manufacturers and wholesalers operate within a narrow selling window (Bansal and Dyer, 2020; Schlapp et al., 2022).

At the same time, production lead times are often substantial. Seed production requires at least

[2]Penna Seeds Private Limited is a seed-producing company based in Hyderabad, dealing in hybrid seeds of corn, paddy, vegetables, etc.

[3]Kirtiman Agrogenetics Limited, incorporated on 21 February 2008, supplies seeds, crop solutions, and soil health products.

[4]Kalash Seeds private limited, incorporated in 2011, https://www.kalashseeds.com/ Accessed on 20 May 2026.

[5]UPL Limited is an Indian multinational company engaged in agrochemical and seed production. Website: https://www.upl-ltd.com Accessed on 8th October 2024.

[6]Dhanuka Agritech is a major Indian manufacturer of agricultural inputs. Website: https://www.dhanuka.com Accessed on 5th October 2024.

[7]Tropical Agro is an Indian firm specializing in crop protection and plant nutrition. Website: https://tropicalagro.in Accessed on 7th October 2024.

[8]Syngenta Partner Program: `https://assets.syngentaebiz.com/images/Partner_Program_WCU_Rules.pdf`

[9]Corteva Agriscience Crop Protection Rewards Program: `https://www.corteva.com/content/dam/dpagco/corteva/na/ca/en/files/general-resources/DF-CPW-Flex-Reward-TC-EN.pdf`

[10]Rabi crops are sown in winter, while Kharif crops are planted during the monsoon.

one crop cycle, while the manufacturing and distribution of fertilizers and agrochemicals also require considerable planning. This mismatch between long production lead times and short selling seasons creates significant inventory risk (Cho and Tang, 2013).

If realized demand exceeds production, unmet demand results in lost sales; if it falls short, manufacturers are left with surplus inventory (Bansal and Dyer, 2020). Wholesalers—who supply inputs to retailers and farmers—face similar risks. Wholesalers risk overstocking if advance booking (AB) orders exceed demand, and understocking if demand exceeds AB quantities. In the latter case, they may place replenishment orders, but only while manufacturers' inventories last; once stocks are depleted, unmet demand again leads to lost sales. Figure 1 depicts the agricultural input supply chain structure, highlighting the stage where advance booking is facilitated.

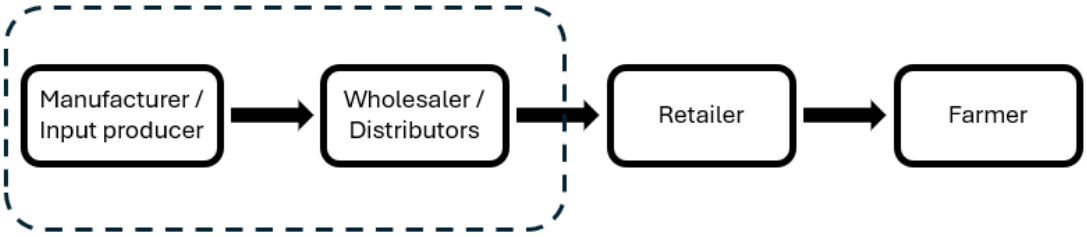


Figure 1: Agricultural input supply chain structure with the focal segment of the paper highlighted

Demand. Demand uncertainty in agricultural input markets is further compounded by the important role played by wholesalers. Wholesalers and field representatives provide technical advice and product recommendations that influence purchasing decisions of downstream buyers. To influence demand, wholesalers frequently invest in field demonstrations, farm visits, exhibitions, technical advisory services and dedicated sales personnel. These activities require substantial resources but can significantly expand product demand. Therefore, demand in agricultural input markets is not merely exogenous; it is actively influenced by the sales effort exerted by channel members.

Agricultural Input Supply Chain Characteristics. The characteristics described above distinguish agricultural input supply chains from traditional advance ordering settings. In many advance ordering models, demand is exogenous and advance orders primarily serve inventory-planning or demand-learning purposes. In contrast, advance booking in agricultural input supply chains serves multiple objectives.

First, advance booking discounts function as pricing and coordination mechanisms (Chintapalli et al., 2017). Manufacturers use discounts, rebates and price-protection guarantees to secure early commitments, improve production planning and attract customers (Li and Zhang, 2013).

Second, demand is strongly influenced by sales effort (Taylor, 2006). Wholesalers play an active role in creating and expanding demand through relationship-building, technical support, and promotional activities (Dar et al., 2024). As a result, demand generation becomes an endogenous decision rather than an exogenous market condition.

Third, agricultural input markets exhibit heterogeneous channel power structures. Large multinational firms such as Bayer Crop Science, Syngenta, Corteva Agriscience and BASF collectively control a substantial share of the global seed and crop-protection markets[11] and routinely operate pre-season booking programs. In such settings, manufacturers possess considerable bargaining power and use advance booking as a coordination and market-management tool.

In contrast, many smaller manufacturers in developing-country markets depend heavily on a limited number of influential wholesalers to access retailers and farming communities. Through our interactions with industry stakeholders, we noted that such wholesalers often possess strong relationships with downstream buyers and play a crucial role in product promotion and market development. Consequently, wholesalers can exert substantial influence over booking decisions and demand-generation activities. These observations motivate the consideration of both Manufacturer-Stackelberg (MS) and Wholesaler-Stackelberg (WS) supply chain configurations.

Empirical evidence highlights the existence of such configurations in agricultural input supply chains. OECD (2018) document the high concentration of global seed and agrochemical manufacturers, where a small number of multinational firms control a substantial share of the market and possess significant channel power. Other studies highlight the important role played by wholesalers and agro-dealers in influencing market demand and product adoption. Dar et al. (2024) show that wholesalers and agro-dealers serve as key intermediaries in transferring information from manufacturers to farmers and facilitating technology adoption. They further note that these channel partners invest effort in providing reliable recommendations to strengthen long-term customer relationships. Similarly, Rutsaert et al. (2021) find that maize seed companies in Tanzania rely heavily on wholesalers and dealers for product distribution, technical support and the dissemination of yield-enhancing technologies. In Ethiopia, the agricultural input market is characterized by a limited number of wholesalers and distributors due to regulatory constraints (Minten et al., 2013), further increasing their influence within the supply chain. Collectively, these studies suggest that both manufacturer- and wholesaler-dominant supply chain structures exist, motivating our analysis of both MS and WS configurations. The two configurations considered in the paper differ in the timing of decision making and, consequently, in the sequence of events, consistent with the modeling approach adopted in the existing advance booking literature (Taylor, 2006; Li and Zhang, 2013; Wei and Zhang, 2018).

Research Questions. Despite the prevalence of advance booking programs in agricultural input industries, it remains less explored in literature. While extant studies have focussed on yield uncertainty (Borodin et al., 2016), contract farming (Anderson and Monjardino, 2019; Bellemare and Novak, 2017; Wang et al., 2014; Nematollahi et al., 2021), agri-food logistics (Yu and Xiao, 2021; Han et al., 2021; Verdouw et al., 2018), and technology adoption (Cao et al., 2022; Xu et al., 2025), limited attention has been given to upstream entities such as manufacturers and wholesalers.

This paper, therefore, addresses the following research questions: How do advance booking dis-

[11] https://www.etcgroup.org/content/top-10-agribusiness-giants, Accessed on 15th May 2026

counts influence ordering, sales effort and profitability decisions in agricultural input supply chains? How do manufacturer-led and wholesaler-led channel structures affect equilibrium outcomes? Under which configuration do advance booking programs create value for both manufacturers and wholesalers? Can returns contracts improve supply chain performance, and if so, under what conditions?

To answer these questions, we model a dyadic agricultural input supply chain consisting of a manufacturer and a wholesaler. Demand is influenced by the wholesaler's sales effort, while advance booking discounts act as the primary instrument influencing order quantity decisions. We focus on advance booking schemes in which the wholesaler pays the entire purchase amount upfront in exchange for a discounted price. We analyze two channel structures: a Manufacturer-Stackelberg (MS) configuration and a Wholesaler-Stackelberg (WS) configuration.

We further extend the analysis by incorporating a returns contract under which the wholesaler can return unsold inventory for partial reimbursement. While uncommon, such contracts exist (e.g., in firms like Penna Seeds, Kalash Seeds and Kirtiman Agrogenetics) and can reduce wholesaler risk while enabling manufacturers to redirect some products to alternate markets or regions with staggered seasons. Finally, because agricultural activity is inherently exposed to weather-related uncertainty, we evaluate the robustness of our results by incorporating a random weather shock affecting demand (refer to Appendix A).

Key Findings. Our results reveal several key insights. First, contrary to the common assumption that upstream leadership can benefit the manufacturer by increasing control over pricing and quantities, our findings indicate that wholesalers may exert equal or greater sales effort when they act as Stackelberg leaders. This higher effort enables them to place larger advance booking orders, reducing their reliance on regular period purchases and ultimately increasing demand for the manufacturer.

Second, advance booking discounts do not merely advance demand across time but can actually increase total demand by enabling wholesalers to invest more in sales effort and market development.

Third, while high discounts are expected to increase production through higher advance orders, our analysis shows that when discounts become large and production costs increase, a manufacturer acting as a Stackelberg leader may optimally reduce production to protect margins.

Fourth, we obtain a particularly counterintuitive result regarding returns contracts: although such contracts are often perceived as exposing manufacturers to greater risk due to buyback obligations (for instance, if in a particular crop cycle, demand remains low, then the manufacturer will be liable to buyback the advance booked products and pay the returns price to the wholesaler), our findings show that properly designed return policies can improve profitability of both parties. By reducing the wholesaler's risk, returns contracts encourage larger advance orders which can increase realized demand and expand total supply chain profits.

Finally, our weather-shock analysis demonstrates that the principal insights remain qualitatively unchanged under stochastic environmental conditions, demonstrating the robustness of our results. Overall Our results suggest that in agricultural input supply chains where downstream effort plays a crucial role in shaping demand, risk-sharing mechanisms and incentive structures can create value for all supply chain players.

The rest of the paper is organized as follows: In the next section, we discuss relevant literature related to this paper. In Section 3, we present the model preliminaries. In Section 4, we present the model results, followed by a comparative analysis in Section 5. Numerical analysis is presented in Section 6 and returns contract analysis in Section 7. Managerial insights and discussions are presented in the concluding Section 8.

2. Literature review

This paper is related to three streams of literature: agricultural supply chains, advance booking, and returns contract. In this section, we review these streams. We first provide an overview of the agricultural supply chain literature. Next, we discuss the literature on advance booking, with particular emphasis on the role of sales effort and its implications for different supply chain players. Finally, we review the returns contract literature. Across these streams, we identify key research gaps and highlight how our work contributes to the existing literature.

Agricultural supply chain. The agricultural supply chain comprises a wide range of interconnected activities, including input procurement, cultivation, post-harvest handling, processing, storage and distribution. One of the key decisions in this regard is crop production planning, which has received significant attention from researchers (Miller et al., 1997; Sharma and Jana, 2009; Cholette, 2009).

More recently, Boyabatlı et al. (2019) study the crop rotation benefits for a farmer; they develop a dynamic farmland allocation policy and find that crop rotation benefits the farmers in comparison to industrial farming. Borodin et al. (2014) study the impact of weather uncertainty on harvest scheduling while Huh and Lall (2013) study its impact on crop selection under contract farming, they also characterize the optimal policies for the farmers. Benini et al. (2025) study crop rotation and planning problem, when farmers receive incentives to comply to sustainability policies. They employ metaheuristics to solve the crop rotation decision problem of the farmers. Glen (1987), Hayashi (2000) and Borodin et al. (2016) review the literature on the application of operations research tools in agricultural supply chains over different time periods. In their review, Borodin et al. (2016) pointed out that exogenous uncertainties — such as volatile weather patterns and market fluctuations - has received much attention from the researchers, however, the role of endogenous uncertainty, where the stochastic nature of the environment is shaped by managerial decisions, remains relatively under-explored. This paper addresses this gap by conceptualizing sales effort as a strategic lever through which wholesalers endogenize demand uncertainty. Furthermore they also call for research on collaborative working between different stakeholders. In this regard, we frame advance booking as a collaborative approach because by aligning the incentives of participating members, it serves as a mutually beneficial instrument that facilitates joint value creation within the supply chain.

Advance Booking in Supply Chains. Advance booking (AB) has been extensively studied in service industries such as travel and hospitality. Early work by Gale and Holmes (1993) and Dana (1998) showed that AB enables better capacity allocation and customer segmentation through price

discrimination—even in the absence of market power. Subsequent studies extended these results under various assumptions, including heterogeneous consumer behavior (Desiraju and Shugan, 1999; Shugan and Xie, 2000), cancellation policies (Xie and Gerstner, 2007; Noori-daryan et al., 2023; Mir et al., 2026), and competitive settings (McCardle et al., 2004). While these studies focus on services with varying customer valuations and flexible consumption timing, our context differs from the extant literature.

We investigate AB in the context of agricultural input supply chains, where the manufacturer offers an AB discount to wholesalers. Unlike services, product valuation in our setting (seeds, fertilizers, and agrochemicals) is consistent across buyers and tied to short, seasonally constrained selling periods. Furthermore, our work focuses not on consumer segmentation but on how AB affects wholesaler's order quantities and sales effort decisions.

In supply chain literature, AB is shown to aid demand forecasting (Moe and Fader, 2002; Chu and Zhang, 2011; Li et al., 2025), substitute for capacity (Özer and Wei, 2004; Kuthambalayan et al., 2015), and coordinate production and sourcing decisions (Tang et al., 2004; Tang and Girotra, 2017; Zhao and Li, 2023). While these models often emphasize inventory or information asymmetry, our study assumes a fixed AB scheme and derives prescriptive decisions for order quantities and sales efforts under stochastic demand. Additionally, our model includes a returns policy that encourages higher AB commitment and contrasts the outcomes under two vertical configurations: wholesaler-led and manufacturer-led supply chains.

Lastly, unlike prior work on advance booking for new or perishable goods (Li and Zhang, 2013), we examine AB in agricultural input markets, where product characteristics are known and the manufacturer offers AB to improve his production decision.

Sales Effort and Advance Booking. The interaction between sales effort and advance booking (AB) has received limited attention in the literature. Khan et al. (2020) examine a perishable inventory model with advertisement-dependent demand and advance payment. Cheng et al. (2018) consider a newsvendor setting with dual marketing decisions—AB discount and advertising—and use a backward search algorithm to optimize pricing and promotion. Wu et al. (2021) explore whether sellers should advertise AB schemes for new products, finding that advertising is not optimal when AB discounts are high or few customers are strategic. Peng and Tian (2022) analyze ordering with outsourced demand information acquisition via a sales agent, showing incentive alignment through contract design. Bai et al. (2024) study advance selling of new products when its consumer valuation is stochastic and the seller puts an advertisement effort.

While these studies focus on pricing or advertising by the AB-offering firm, our model differs in two key aspects. First, we do not consider inventory costs due to the short selling season typical of agricultural inputs. Second, the wholesaler, not the manufacturer, exerts sales effort and is the beneficiary of the AB discount, aligning with practices observed in agricultural input supply chains.

Numerous studies have examined advance booking under demand uncertainty. Zhao and Stecke (2010) analyze pre-ordering for a yet-to-be-released product under demand and valuation uncertainty and characterize the seller's optimal advance selling strategy assuming normal distribution.

Prasad et al. (2011) examine pricing and inventory decisions for advance booking in a newsvendor framework with uncertain demand. Maiti et al. (2009) study advance payment decisions under stochastic lead times and price-dependent demand. Chen et al. (2025) investigate the effect of retailer fairness concerns on the manufacturer in a manufacturer–Stackelberg advance booking model with uniformly distributed demand. Other studies also consider uncertain demand in advance booking settings (Li and Zhang, 2013; Jia et al., 2022; Fan et al., 2024). While we consider demand uncertainty similar to the aforementioned studies, unlike these studies, where demand uncertainty is exogenous to the decision maker, we assume that the wholesaler can influence demand through sales effort.

Returns Contract. Extant literature has examined returns contracts or refund policies. In the context of the service industry, Xie and Gerstner (2007) study cancellation of a pre-purchased service contract of a capacity-constrained service from the perspective of the seller. The study shows that offering refunds for service cancellation can be profitable for the seller in comparison to the no refund scenario. Zhang et al. (2021) show that the partial refund policy can effectively act as a price commitment mechanism in the service industry. Guo (2009) analyze service cancellation with partial or no refund in a competitive environment. The study derives a set of equilibrium outcomes for the two competing firms and shows that even two symmetric firms may end up in asymmetric equilibrium. Some other studies consider returns policy in different contexts like buy online, pickup from store (Shi et al., 2018), omnichannel selling (Jin et al., 2020; He et al., 2020), product review (Fan et al., 2025), information disclosure (Wang et al., 2024) and e-commerce with social influence (Li et al., 2024). In this paper, we study the returns policy combined with advance booking discounts on agricultural input products. In the above-cited studies, multiple selling of the same product or service is considered the primary reason for making refunds profitable for the seller. In contrast, we do not consider multiple selling, rather, our objective is to study returns contract from the perspective of demand impact and whether or not a returns policy with advance booking discount can benefit the manufacturer and the wholesaler.

Furthermore, there exist studies that examine returns or buyback contract as a supply chain coordination mechanism. He and Zhao (2012) study a multi-echelon supply chain with demand and supply uncertainties and show that a returns policy in combination with wholesale price can coordinate the supply chain. Xie et al. (2021) study a supply chain with demand and yield uncertainties and show that a buyback contract can coordinate the supply chain if the seller has greater bargaining power. Additional studies on buyback contracts by Hu et al. (2013) discuss flexible ordering policy, and Wang et al. (2021) discuss the signaling effect of buyback contracts. These papers show the potential supply chain coordination capabilities of a returns contract, however, the studies consider a one-time ordering decision, where once the order is placed, the buyer can not place another order, whereas in our paper, the wholesaler can place multiple orders until the manufacturer's stock lasts, representing a more generic practical scenario. Moreover, we particularly discuss the returns contract decisions in the agricultural input supply chain, which is different from the contexts of the above papers.

Our review of extant literature shows that in the context of agricultural input supply chains, there is limited work done on AB discount schemes. Although, multiple studies explore AB in a generic newsvendor configuration (in other industrial contexts) (Zhao and Stecke, 2010; Prasad et al., 2011; Zhao et al., 2016; Su and Zhang, 2008, 2009; Li and Li, 2023), a majority consider the seller as a Stackelberg leader, given the role in offering AB schemes and deciding the AB terms. In contrast, our study considers both manufacturer-led and wholesaler-led Stackelberg configurations. We derive the equilibrium values for manufacturing quantity, AB quantity, and sales effort under each configuration. We evaluate the two configurations to determine which is preferable for the supply chain and the players. This distinction is particularly relevant in our context, as the wholesaler plays a critical role in exerting sales effort which influences the total demand. Next, we discuss the model preliminaries.

3. Model

We model a dyadic supply chain consisting of a manufacturer and a wholesaler where both are assumed to be risk-neutral and profit-maximizing. We assume that the manufacturer produces one agricultural input such as a particular type of seed and the wholesaler, after buying it from the manufacturer, sells it to her customers. The wholesaler faces a sales effort (y) dependent stochastic demand given as

$$D(y, \epsilon) = \phi(y) + \epsilon \tag{1}$$

Here, $\phi(y) = m + ny$, a linear function of y, $n \neq 0$ (Petruzzi and Dada, 1999; Ma et al., 2013; He et al., 2009). ϵ is the random component distributed in the range $[a, b]$ (Ding and Song, 2020; Petruzzi and Dada, 1999). The PDF and CDF of ϵ are $f(\cdot)$ and $F(\cdot)$ respectively with mean μ and standard deviation σ.

We denote the manufacturer's per unit production cost by c_m and his per unit selling price in the regular period by p_m. In the advance booking period, he offers a discounted price of αp_m, where $0 \leq \alpha \leq 1$ is considered exogenous in our problem (We show in section 6 the range of α acceptable to the manufacturer). Note that the discount received by the wholesaler is, therefore, $(p_m - \alpha p_m) = (1 - \alpha)p_m$.

The manufacturer offers advance booking before the regular selling period; the wholesaler chooses to place an AB order which is fulfilled in the regular selling period. If the wholesaler's realized demand exceeds the AB order quantity, she may place an additional regular period order (RPO). However, fulfillment of the RPO is not guaranteed as it is contingent on the production quantity of the manufacturer. The manufacturer can fulfill the RPO if he has more than the RPO quantity available with him, otherwise, the order may be only partially fulfilled, or not at all. Given the multi-period decisions, we assume a present value (PV) factor denoted by δ to take into account the time value of money and calculate the value of all the transactions at the point of advance booking. It follows $0 \leq \alpha \leq \delta \leq 1$, to ensure that advance booking is feasible for the wholesaler.

We represent the wholesaler's selling price by p_w and her cost coefficient of sales effort by c_w. In line with the literature, we consider an increasing cost of sales effort that follows a non-linear, convex form, $c_w y^2$ (Sun and Liu, 2023; Heese and Swaminathan, 2010). In addition to the sales effort, the wholesaler decides her advance booking quantity q_a, while the manufacturer decides his manufacturing quantity q_m. Table 2 lists all the notations. To ensure that the business is profitable for both the players, we assume $p_m \leq p_w$ (*i.e*, the selling price of the wholesaler is greater than her purchase price) and $c_m \leq \alpha p_m$ (*i.e* the manufacturing cost is less than the discounted price).

Notation	Definition
Decisions	
q_a	Advance booking quantity
q_m	Manufacturing quantity
y	Sales effort of the wholesaler
Parameters	
p_w	Wholesaler's per unit selling price
p_m	Manufacturer's per unit selling price in regular period
δ	Present value (PV) factor
c_w	Cost coefficient of the sales effort for the wholesaler
c_m	Per unit manufacturing cost for the manufacturer
α	Advance booking discount parameter
$\phi(y) = m + ny$	Effect of sales effort on the wholesaler's demand
ϵ	Random component of the wholesaler's demand
a, b	Lower and upper limits of the random component of the wholesaler's demand
$f(\cdot), F(\cdot)$	PDF and CDF of the random component of the wholesaler's demand
Outcomes	
Π_m, Π_w	Manufacturer's and wholesaler's profit

Table 2: Model Notations

From the above notations, we represent the wholesaler's and manufacturer's profit functions as,

$$\Pi_w\left(y, q_a; q_m\right) = \delta\left[p_w\Big(min\big(D(y,\epsilon), q_m\big)\Big) - p_m\Big(min\big(D(y,\epsilon), q_m\big) - q_a\Big)^+\right] - \alpha p_m q_a - c_w y^2 \quad (2)$$

$$\Pi_m\left(q_m; y, q_a\right) = \delta\left[p_m\Big(min\big(D(y,\epsilon), q_m\big) - q_a\Big)^+\right] + \alpha p_m q_a - c_m q_m \quad (3)$$

The first term in equation (2) refers to the net present value of the total revenue while the second term refers to the purchase cost of the wholesaler in the regular period *i.e*. if the advance booking quantity q_a is less than the realized demand, the wholesaler places a regular period order purchased at price p_m. The third term denotes her purchase cost in the advance booking period and the fourth term refers to her non-linear quadratic cost of sales effort. Note that this effort, and the corresponding cost, are incurred *prior* to the regular selling period, consistent with the common practice of wholesalers investing in sales activities much in advance of the selling season.

The first term in equation (3) refers to the net present value of the revenue for the manufacturer

in the regular period while the second term refers to the AB revenue. The third term denotes his manufacturing cost. Substituting the demand expression in equations (2) and (3) we derive the expected profit functions as shown in equations (4) and (5).

$$\begin{aligned} E\big(\Pi_w\left(y, q_a; q_m\right)\big) = \delta p_w\big(\phi(y)+\mu\big) - \delta p_w \int_{q_m-\phi(y)}^{b} \Big(\phi(y)+u-q_m\Big) f(u)du - \alpha p_m q_a - c_w y^2 \\ -\delta p_m\big(q_m-q_a\big)\Big(1-F\big(q_m-\phi(y)\big)\Big) - \delta p_m \int_{q_a-\phi(y)}^{q_m-\phi(y)} \Big(u-q_a+\phi(y)\Big) f(u)du \end{aligned} \tag{4}$$

$$\begin{aligned} E\big(\Pi_m\left(q_m; y, q_a\right)\big) =& \delta p_m \left[\int_{q_a-\phi(y)}^{q_m-\phi(y)} \Big(u-q_a+\phi(y)\Big) f(u)du + \big(q_m-q_a\big)\Big(1-F\big(q_m-\phi(y)\big)\Big) \right] \\ &+ \alpha p_m q_a - c_m q_m \end{aligned} \tag{5}$$

In equation 4, the first term is the expected revenue when the entire demand is satisfied, while the second term represents the loss of revenue due to lost sales. The third and fourth terms are the costs of AB order and sales effort, while the last two terms collectively refer to the expected purchase cost for the regular period order.

In equation 5, the first term represents the manufacturer's expected revenue when he fully satisfies the wholesaler's demand. The second term represents the expected revenue when a portion of the demand is lost due to underproduction. The third and fourth terms are the revenues from AB order and manufacturing cost respectively.

We next describe the two supply chain configurations – the wholesaler Stackelberg and manufacturer Stackelberg.

3.1. Wholesaler Stackelberg (WS) model

The decision sequence is the following – In the advance booking stage (stage 1), the wholesaler decides the quantity to reserve in the advance booking period and the sales effort to put in. In stage 2, the manufacturer determines the production quantity after observing the wholesaler's ordering decisions. The scenario reflects the case of small and medium-scale manufacturers as discussed in the Introduction section. The manufacturer supplies the AB order quantity at the beginning of the selling season, and the wholesaler realizes her season's demand. If the demand exceeds the AB quantity, the wholesaler places a regular period order with the manufacturer, who fulfills the order, provided sufficient quantity is available. Otherwise, he fulfills partially, and the excess demand is lost. Figure 2 illustrates the sequence of events of the WS model.

This is a two-stage game, where the wholesaler's problem is represented by Equation (6). The constraint indicates that the advance booking quantity q_a is non-negative and atleast satisfies the minimum market demand. Equation (7) represents the manufacturer's problem where he decides his manufacturing quantity; the two constraints of this problem are derived from the assumptions

that the manufacturing quantity is greater than the AB quantity and lesser than the maximum possible demand of the selling season.

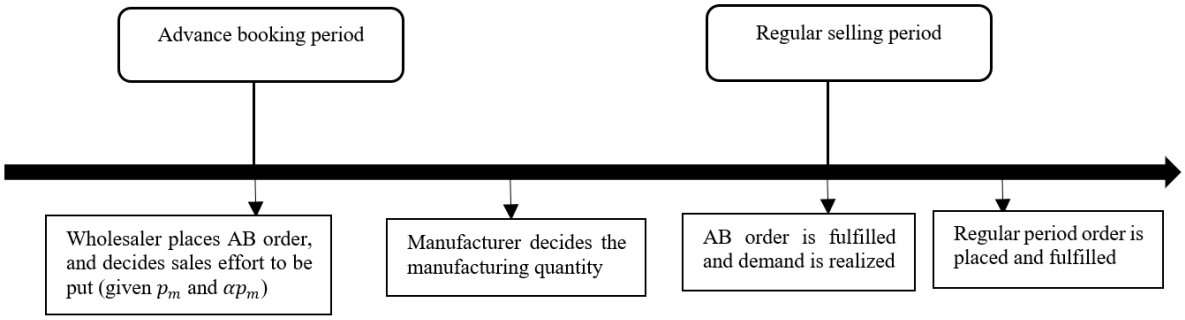


Figure 2: Sequence of events- Wholesaler Stackelberg (WS) model.

WS model, the wholesaler's optimization problem:

$$\begin{aligned} \max_{y,q_a} \quad & E\big(\Pi_w\left(y, q_a; \hat{q_m}\right)\big) \\ \text{s.t.} \quad & 0 \leq D_{min} \leq q_a \end{aligned} \tag{6}$$

WS model, the manufacturer's optimization problem:

$$\begin{aligned} \max_{q_m} \quad & E\big(\Pi_m\left(q_m; y, q_a\right)\big) \\ \text{s.t.} \quad & q_m \geq q_a \\ & q_m \leq \phi(y) + b \end{aligned} \tag{7}$$

We solve the model using backward induction. We first solve the manufacturer's problem to find the expressions of optimal manufacturing quantity, represented by $\hat{q_m}$. Substituting it into the wholesaler's problem, we solve for q_a and y. We present the results in section 4.

3.2. Manufacturer Stackelberg (MS) model

Figure 3 illustrates the sequence of events in the MS model. Given the advance booking price and regular period price, first, the manufacturer decides his manufacturing quantity q_m, using the wholesaler's response function; following which the wholesaler decides her AB quantity q_a and sales effort y in the second stage. **[*Case Instance:*]** Instances of MS configuration can be found in Rwanda's fertilizer supply chain with a few large producers and numerous small-scale wholesalers.[12] Advance booking offered by the seed producing companies in India is also another example of Manufacturer Stackelberg. The seed producers typically offer advance booking schemes two to three months before the selling season, whereas the production of seeds must be completed at least one season in advance (usually more than six months). As a result, the manufacturers need to do their production planning and decide the quantity to be produced before the advance booking information

[12]https://www.rwandatenders.com/tender/rwanda-fertilizerpesticide-value-chain-and-regulatory-assessment-324e943.php. Accessed December 8, 2024.

is available to them, this inevitably makes it a manufacturer Stackelberg configuration. Here, it is important to note that, under the MS configuration, advance booking cannot directly affect the current season's production quantity, as production decisions are made before the advance booking information becomes available. Rather, in this configuration advance booking helps manufacturers improve inventory allocation, secure early demand commitments, and manage cash flows during the current season. In addition, the demand information generated through advance booking can support production planning and capacity decisions for subsequent production cycles.

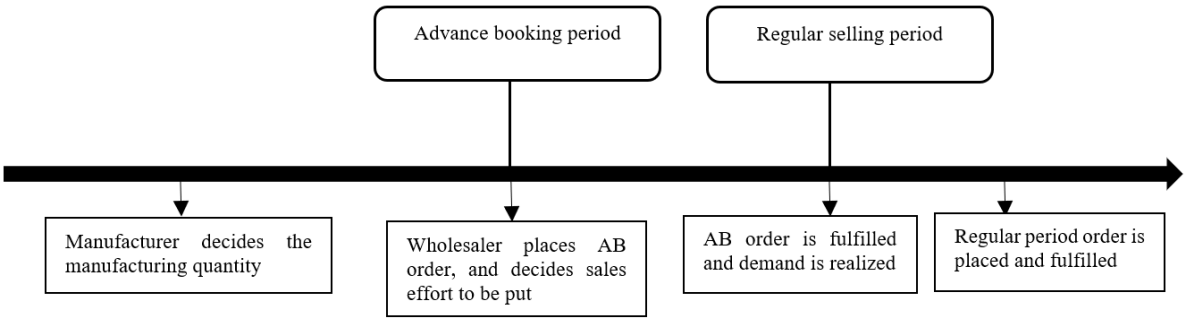


Figure 3: Sequence of events- Manufacturer Stackelberg (MS) model

The manufacturer's decision problem is shown in Equation (8), while Equation (9) represents the wholesaler's problem. The first constraint in the wholesaler's problem indicates that the advance booking quantity q_a is non-negative and atleast satisfies the minimum market demand.; the second constraint shows that the AB quantity is less than the total manufacturing quantity, while the third constraint shows that the manufacturing quantity is less than the maximum possible demand of the selling season.

MS model, the manufacturer's optimization problem:

$$\max_{q_m} \quad E\big(\Pi_m\left(q_m; \hat{y}, \hat{q_a}\right)\big) \tag{8}$$

MS model, the wholesaler's optimization problem:

$$\begin{aligned} \max_{y, q_a} \quad & E\big(\Pi_w\left(y, q_a; q_m\right)\big) \\ \text{s.t.} \quad & 0 \leq D_{min} \leq q_a \\ & q_a \leq q_m \\ & \phi(y) + b \geq q_m \end{aligned} \tag{9}$$

We solve the wholesaler's problem first and find the expressions for optimal values of AB quantity and sales effort represented by $\hat{q_a}$ and $\hat{y}$ respectively. We substitute these expressions in the manufacturer's problem and solve for his decision.

4. Model results in the WS and MS Configurations

In this section, we present the equilibrium results for the WS and MS configurations. To simplify model results and present meaningful analysis, we subsequently derive outcomes considering demand to follow a uniform distribution. Later, through a numerical analysis in Section 6, we show that the results also hold when demand follows a truncated normal distribution.

Lemma 1. *a) The equilibrium value of sales effort, y^* in the WS model is given as $\frac{n}{2c_w}(\delta p_w - \alpha p_m)$.*

b) The equilibrium value of advance booking quantity, q_a^* is given as $\phi(y^*) + F^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$ when $\frac{c_m}{p_m} \leq \alpha < \alpha_1$, while $\phi(y^*) + F^{-1}\big(1 - \frac{\alpha}{\delta}\big)$ when $\alpha_1 \leq \alpha \leq \delta$.

c) The equilibrium value of manufacturing quantity, q_m^* is given as $\phi(y^*) + F^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$ when $\frac{c_m}{p_m} \leq \alpha < \alpha_1$, while $\phi(y^*) + F^{-1}\big(1 - \frac{c_m}{\delta p_m}\big)$ when $\alpha_1 \leq \alpha \leq \delta$.

Proof: To solve the manufacturer's and wholesaler's problems, we introduce two transformation variables namely, $z = q_a - \phi(y)$ and $w = q_m - \phi(y)$ and two intermediate functions $\theta(z, w) = \int_z^w (u - z) f(u) du$ and $\gamma(w) = \int_w^b (w - u) f(u) du$. Substituting them in the manufacturer's and wholesaler's profit functions we get:

$$\begin{aligned} \max_{w} \quad E\big(\Pi_m(w; y, z)\big) =& \delta p_m \Big(\theta(z, w) + (w - z)\big(1 - F(w)\big)\Big) + \alpha p_m \big(z + \phi(y)\big) \\ & - c_m \big(w + \phi(y)\big) \\ \text{s.t.} \quad w \geq& z \\ w \leq& b \end{aligned} \tag{10}$$

$$\begin{aligned} \max_{y,z} \quad E\big(\Pi_w(y, z; \hat{w})\big) =& \delta p_w \big(\mu + \phi(y)\big) - \alpha p_m \big(z + \phi(y)\big) - c_w y^2 - \delta p_m \theta(z, \hat{w}) + \delta p_w \gamma(\hat{w}) \\ & - \delta p_m \big(\hat{w} - z\big)\big(1 - F(\hat{w})\big) \\ \text{s.t.} \quad \phi(y) + a \geq& 0 \\ z \geq& a \end{aligned} \tag{11}$$

We solve the manufacturer's optimization problem using Karush Kuhn Tucker (KKT) conditions of optimality and substitute the values in the wholesaler's optimization problem to derive the equilibrium solutions. Detailed derivations are provided in the Appendix. The equilibrium solutions are presented in Table 3.

The equilibrium expressions of the advance booking quantity and manufacturing quantity indicate that they depend on the sales effort (y) and its effect on the wholesaler's demand ($\phi(\cdot)$). Therefore, higher the sales effort, higher is the AB quantity and manufacturing quantity. Moreover, higher the effect of sales effort on the demand (influenced by the parameters m, n), higher the equilibrium values of q_a and q_m.

Lemma 2. *In the WS configuration, the sales effort by the wholesaler increases with the advance booking discount ($1 - \alpha$) and the wholesaler's selling price (p_w), while it decreases with the cost of sales effort (c_w).*

Solution case	Case A	Case B
Range of α	$\frac{c_m}{p_m} \leq \alpha < \alpha_1$	$\alpha_1 \leq \alpha \leq \delta$
y^*	$\frac{n}{2c_w}(\delta p_w - \alpha p_m)$	$\frac{n}{2c_w}(\delta p_w - \alpha p_m)$
q_a^*	$\phi(y^*) + F^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$	$\phi(y^*) + F^{-1}\big(1 - \frac{\alpha}{\delta}\big)$
q_m^*	$\phi(y^*) + F^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$	$\phi(y^*) + F^{-1}\big(1 - \frac{c_m}{\delta p_m}\big)$

where α_1 is the solution of the following equation:

$$c_m\Big(1 - \frac{p_w}{p_m}\Big)F^{-1}\Big(1 - \frac{c_m}{\delta p_m}\Big) = \delta p_w\left(\int_{F^{-1}\left(1-\frac{\alpha p_m}{\delta p_w}\right)}^{F^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} uf(u)du\right) - \delta p_m\left(\int_{F^{-1}\left(1-\frac{\alpha}{\delta}\right)}^{F^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} uf(u)du\right)$$

Table 3: Equilibrium solution of the WS model under general distribution

This result is in line with the intuition that as the wholesaler's selling price increases or the advance booking discount offered by the manufacturer increases, the wholesaler's profit margin increases. Therefore, she puts higher sales effort to influence demand. In contrast, as the cost of sales effort increases, it becomes costly for her to influence demand leading to a reduction in her sales effort.

We further observe that the equilibrium decisions depend on the AB discount parameter, α. In Case A, when the value of α is less than the threshold α_1, *i.e*, when the AB discount $(1-\alpha)$ is high, it is optimal for the manufacturer to produce only the AB quantity to reduce his risk of unsold quantity (note, $q_a^A = q_m^A$). However, in Case B, when the AB discount is lower, the manufacturer produces more than the AB quantity, expecting a regular season order from the wholesaler.

Lemma 3. *a) The equilibrium value of sales effort, y^* in the MS model is given as $\frac{n\delta}{2c_w}\Big(F\big(q_m^* - \phi(y^*)\big)\big(p_w - p_m\big) + p_m\big(1 - \frac{\alpha}{\delta}\big)\Big)$*

b) The equilibrium value of advance booking quantity, q_a^* is given as $\phi(y^*) + F^{-1}\big(\frac{\delta-\alpha}{\delta}\big)$

c) The equilibrium value of manufacturing quantity, q_m^* is given by the solution to the following equation, $\frac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m)f\big(q_m^* - \phi(y^*)\big) = c_m - \delta p_m\big(1 - F\big(q_m^* - \phi(y^*)\big)\big)$, when $\frac{c_m}{p_m} \leq \alpha < \alpha_2$ and $\phi(y^*) + b$, when $\alpha \geq \alpha_2$.

Where, $\alpha_2 = \frac{c_m}{p_m}\Big(1 + \frac{2c_w}{n^2\delta(p_w - p_m)f(b)}\Big)$

Proof: We replace $z = q_a - \phi(y)$, $w = q_m - \phi(y)$, $\theta(z,w) = \int_z^w (u-z)f(u)du$ and $\gamma(w) = \int_w^b (w-u)f(u)du$ in the MS model to get the wholesaler's and manufacturer's problems as follows:

$$\begin{aligned}
\max_{y,z} \quad E\big(\Pi_w(y,z;w)\big) =& \delta p_w\big(\mu + \phi(y)\big) - \alpha p_m\big(z + \phi(y)\big) - c_w y^2 - \delta p_m\theta(z,w) + \delta p_w\gamma(w) \\
& - \delta p_m\big(w - z\big)\big(1 - F(w)\big) \\
\text{s.t.} \quad \phi(y) + a \geq& 0 \\
z \geq& a \\
w - z \geq& 0 \\
b - w \geq& 0
\end{aligned} \tag{12}$$

$$\max_{w} \quad E\big(\Pi_m\left(w;\hat{y},\hat{z}\right)\big) = \delta p_m\Big(\theta(\hat{z},w) + (w-\hat{z})\big(1-F(w)\big)\Big) + \alpha p_m\big(\hat{z}+\phi(\hat{y})\big) - c_m\big(w+\phi(y)\big) \tag{13}$$

Using backward induction method, we first solve the wholesaler's optimization problem and substitute the values into the manufacturer's problem to derive equilibrium outcomes shown in Table 4. To ensure concavity of the problem, we make additional assumption of $\frac{f'(w)}{f(w)} \leq \frac{2p_m c_w}{n^2(p_w-p_m)(\alpha p_m - c_m)}$. Proofs are detailed in the Appendix.

Solution case	Case C	Case D
Range of α	$\frac{c_m}{p_m} \leq \alpha < \alpha_2$	$\alpha_2 \leq \alpha \leq \delta$
q_m^*	Solution to $c_m - \delta p_m\big(1-F\big(q_m^* - \phi(y^*)\big)\big)$ $= \dfrac{n^2\delta}{2c_w}(p_w-p_m)(\alpha p_m - c_m) f\big(q_m^* - \phi(y^*)\big)$	$\phi(y^*) + b$
y^*	$\frac{n\delta}{2c_w}\Big(F\big(q_m^* - \phi(y^*)\big)(p_w-p_m) + p_m\big(1-\frac{\alpha}{\delta}\big)\Big)$	$\frac{n\delta}{2c_w}\Big(F\big(q_m^* - \phi(y^*)\big)(p_w-p_m) + p_m\big(1-\frac{\alpha}{\delta}\big)\Big)$
q_a^*	$\phi(y^*) + F^{-1}\Big(\frac{\delta-\alpha}{\delta}\Big)$	$\phi(y^*) + F^{-1}\Big(\frac{\delta-\alpha}{\delta}\Big)$
where $\alpha_2 = \frac{c_m}{p_m}\Big(1 + \frac{2c_w}{n^2\delta(p_w-p_m)f(b)}\Big)$		

Table 4: Equilibrium solution of the MS model under general distribution

Here as well, we observe that the equilibrium values of AB quantity depends on the sales effort and its effect on the wholesaler's demand.

4.1. WS and MS models with Uniform distribution

To develop richer model insights, we simplify our analysis by assuming hereon that the random component of the wholesaler's demand is uniformly distributed between zero to b, *i.e*, $\epsilon \sim U(0,b)$, while its sales effort dependent component is $\phi(y) = y$. We substitute these parameters in the previous results for the WS and MS models and find the equilibrium decision outcomes under uniform distribution, as shown in Tables 5 and 6. In the numerical analysis section (Section 6), we later show that our results hold for general distributions using both uniform and truncated normal distributions, demonstrating the robustness of our findings.

Solution case	Case A	Case B
Range of α	$\frac{c_m}{p_m} \leq \alpha < \alpha_1$	$\alpha_1 \leq \alpha \leq \delta$
y^*	$\frac{\delta p_w - \alpha p_m}{2c_w}$	$\frac{\delta p_w - \alpha p_m}{2c_w}$
q_a^*	$\frac{(\delta p_w - \alpha p_m)(\delta p_w + 2c_w b)}{2\delta c_w p_w}$	$\frac{b(\delta-\alpha)}{\delta} + \frac{\delta p_w - \alpha p_m}{2c_w}$
q_m^*	$\frac{(\delta p_w - \alpha p_m)(\delta p_w + 2c_w b)}{2\delta c_w p_w}$	$\frac{\delta p_m(\delta p_w - \alpha p_m) + 2c_w b(\delta p_m - c_m)}{2\delta c_w p_m}$
where $\alpha_1 = \frac{c_m}{p_m}\sqrt{\frac{p_w}{p_m}}$		

Table 5: Equilibrium solution of the WS model under uniform distribution

Solution case	Case C	Case D
Range of α	$\frac{c_m}{p_m} \leq \alpha < \alpha_2$	$\alpha_2 \leq \alpha \leq \delta$
y^*	$\frac{\delta p_w - \alpha p_m + c_m}{2c_w} - \frac{c_m p_w}{2c_w p_m} + \frac{\delta(\alpha p_m - c_m)(p_w - p_m)^2}{4c_w^2 p_m b}$	$\frac{\delta p_w - \alpha p_m}{2c_w}$
q_a^*	$\frac{b(\delta-\alpha)}{\delta} + \frac{\delta p_w - \alpha p_m + c_m}{2c_w} - \frac{c_m p_w}{2c_w p_m} + \frac{\delta(\alpha p_m - c_m)(p_w - p_m)^2}{4c_w^2 p_m b}$	$b - \frac{\alpha b}{\delta} + \frac{\delta p_w - \alpha p_m}{2c_w}$
q_m^*	$\frac{(\alpha p_m - c_m)\left(\delta(p_w - p_m) + 2c_w b\right)}{2\delta c_w p_m} + \frac{b(\delta-\alpha)}{\delta} + \frac{\delta p_w - \alpha p_m + c_m}{2c_w} - \frac{c_m p_w}{2c_w p_m} + \frac{\delta(\alpha p_m - c_m)(p_w - p_m)^2}{4c_w^2 p_m b}$	$b + \frac{\delta p_w - \alpha p_m}{2c_w}$

where $\alpha_2 = \frac{c_m}{p_m}\left(1 + \frac{2c_w b}{\delta(p_w - p_m)}\right)$

Table 6: Equilibrium solution of the MS model under uniform distribution

Next, we present a comparative analysis of the outcomes of the two models.

5. Comparative analysis

In this section, we discuss the results obtained above and their implications. We compare the expressions of sales effort, advance booking quantity and manufacturing quantity for various ranges of AB discounts and derive several insights.

For ease of comparison, we divide the results into four regions based on the value of the discount parameter α. Refer to Table 7. The first region, or Region 1, is where the AB discount is high and thus the value of α is less than both α_1 and α_2, i.e. $\alpha \leq min(\alpha_1, \alpha_2)$. Region 3 is the region with the lowest AB discount, thus α is greater than both α_1 and α_2, i.e. $\alpha \geq max(\alpha_1, \alpha_2)$. For intermediate levels of AB discount, that is when the value of α is between α_1 and α_2, there are two possible regions: Region 2L, when $\alpha_1 \leq \alpha \leq \alpha_2$, and Region 2H, when $\alpha_2 < \alpha \leq \alpha_1$. We compare the decision outcomes, namely, sales effort (y), advance booking quantity (q_a) and manufacturing quantity (q_m) in these regions. For representation, we write each case as a superscript; for example, advance booking quantity in Case A as q_a^A. Recall that cases A and B are the solutions for the WS model and cases C and D are the solutions for the MS model.

Regions	Range of α	y^*	q_a^*	q_m^*
Region 1	$\alpha \leq min(\alpha_1, \alpha_2)$	$y^A \geq y^C$	$q_a^A \geq q_a^C$, if $c_m \geq c_{m1}$ $q_a^A < q_a^C$, otherwise	$q_m^A \geq q_m^C$, if $c_m \geq c_{m2}$ $q_m^A < q_m^C$, otherwise
Region 2L	$\alpha_1 \leq \alpha \leq \alpha_2$	$y^B \geq y^C$	$q_a^B \geq q_a^C$	$q_m^B \geq q_m^C$, if $c_m \geq c_{m2}$ $q_m^B < q_m^C$, otherwise
Region 2H	$\alpha_2 < \alpha \leq \alpha_1$	$y^A = y^D$	$q_a^A \geq q_a^D$	$q_m^A \leq q_m^D$
Region 3	$\alpha \geq max(\alpha_1, \alpha_2)$	$y^B = y^D$	$q_a^B = q_a^D$	$q_m^B \leq q_m^D$

Where $c_{m1} = \frac{\alpha p_m}{\delta p_w} \frac{\delta^2 p_w (p_w - p_m) - 4c_w^2 b^2}{\delta(p_w) - p_m + 2c_w b}$ and $c_{m2} = \frac{\alpha p_m}{p_w} \frac{\delta p_w (p_w - p_m)(\delta(p_w - p_m) + 2c_w b) + 4p_m c_w^2 b^2}{(\delta(p_w - p_m) + 2c_w b)^2}$

Table 7: Region-wise comparison of decision variables

5.1. Analysis of sales effort and advance booking quantity

We observe that in region 1, $y^A \geq y^C$ and in region 2L, $y^B \geq y^C$, i.e. the sales effort of the wholesaler is higher in the WS model than the MS model. However, in regions 2H and 3, $y^A = y^D$ and $y^B = y^D$, that is the wholesaler puts the same sales effort in both settings. We, therefore, state:

Proposition 1. ***(Sales effort)****: (a) When the discount is relatively high (regions 1 and 2L), the wholesaler puts higher sales effort in the WS configuration than the MS configuration.*

(b) When the AB discount is relatively low (regions 2H and 3), the wholesaler puts the same sales effort irrespective of WS or MS configuration.

Result (a) suggests that under a higher advance booking discount and as a Stackelberg leader, the wholesaler puts higher sales effort to increase the market demand. Result (b) suggests when the advance booking discount is relatively low, her sales effort is the same irrespective of her supply chain leadership. This is because, in regions 1 and 2L, where the AB discount is high, the wholesaler purchases higher quantities in the AB period, and therefore, puts in higher sales effort. Therefore, sales effort and advance booking quantity are complementary decisions. A larger advance booking quantity increases the wholesaler's exposure to inventory risk, while greater sales effort helps mitigate this risk by expanding demand. Therefore, while discounts encourage larger advance commitments, they simultaneously encourage higher sales effort. This effect is particularly significant in the WS configuration.

However, in the MS configuration, the manufacturer decides his manufacturing quantity beforehand, restricting the wholesaler's decision. As the AB discount decreases in regions 2H and 3, the incentive to book higher quantities in the AB period also reduces and consequently, the wholesaler reduces her sales effort.

The result has important implications for agricultural input supply chains. Our findings suggest that advance booking discounts can encourage wholesalers to invest in promotional and advisory activities that expand market demand. For manufacturers, the result highlights the importance of designing advance booking programs that adequately reward wholesalers for demand-generation efforts. In markets where wholesalers possess strong relationships with retailers and farmers, providing attractive advance booking incentives can increase sales effort, stimulate demand and improve overall supply chain performance.

[*Case Instance:*] The agricultural input sector in India is replete with several micro, small and medium-scale enterprises that don't have free access to institutional finance or banks. Most of them depend on informal finance. Our results show that in supply chains with wholesalers as leaders, access to advance booking schemes can benefit wholesalers (and supply chains) as such schemes incentivize the wholesalers to put in higher sales effort. The role of wholesalers as agricultural input channel partners for manufacturers is critical. It is their sales effort and information know-how sharing with the farming community that drives farm yields and agricultural input demand. **[*Case Instance:*]** In 2023, the Joint Secretary (Ministry of Agriculture and Farmers Welfare, Government of India) acknowledged that agricultural input dealers and channel partners play an important role in communicating with farmers about the advancements in use of agricultural inputs.[13] Summarily, a robust relationship between manufacturers and agricultural input wholesalers with advance booking

[13] https://krishijagran.com/news/focus-on-technology-adoption-and-creating-infrastructure-to-build-resilient-agri-value-chain-joint-secretary-agriculture-farmers-welfare-ministry/ Accessed on 15th October 2024

discounts can drive higher sales efforts in supply chains.

A comparison of the AB quantity shows that the order of the relationship in the WS or MS configuration depends on the manufacturer's cost of production.

Proposition 2. ***(Advance booking quantity)****: (a) When the AB discount is high (region 1) and the cost of manufacturing is above a threshold, the AB order quantity is higher in the WS model. Otherwise, the AB order quantity is higher in the MS model.*

(b) For intermediate values of AB discount (regions 2L and 2H), the wholesaler places a higher AB order in the WS model than in the MS model.

(c) When the AB discount is relatively low (region 3), the wholesaler places the same AB order irrespective of the WS or MS configuration.

In region 1, due to high AB discount, the wholesaler aims to purchase more in the AB period and the order quantity is higher when she is the first-mover compared to the MS configuration. However, when the manufacturing cost is lower than a threshold ($c_m \leq c_{m1}$), the manufacturer benefits due to a lower cost of production that allows him to increase his manufacturing quantity (we discuss this further in Proposition 3). This allows the wholesaler to increase the AB quantity (in the MS configuration). The result highlights that manufacturer's leadership is not necessarily restrictive; it can support larger advance bookings when production is sufficiently economical.

In part (b), we find that in regions 2L and 2H, the AB order in the WS configuration is always greater. We infer that the AB discount is still quite lucrative for the wholesaler to increase her AB quantity. In the WS configuration, the wholesaler benefits from advance booking and can coordinate her booking quantity with sales effort. This makes her more willing to reserve a larger quantity in advance.

In part (c), we find that as the AB discount becomes lower (region 3), the wholesaler's expected losses due to overbooking surpasses the expected gain, therefore even when she is the Stackelberg leader, she chooses to order the same quantity as of MS. This occurs because the discount is no longer large enough to compensate the wholesaler for the risk of overbooking. Since the benefit of advance booking becomes weak, the wholesaler has little incentive to increase her AB order even when she has first-mover advantage.

We note that the booking discount drives advance booking quantity and results in a higher order quantity in the WS model. The role of advance booking cannot be overstated therefore. For agricultural input supply chain players, the result has several implications. For manufacturers, offering large AB discounts may not generate higher advance orders if production costs are high and the manufacturer retains leadership. For wholesalers, attractive AB discounts create an opportunity to secure agricultural inputs at lower cost, but larger bookings should be supported by sales effort. For farmer cooperatives and rural intermediaries, advance booking can therefore improve access to affordable inputs, especially when wholesalers are able to aggregate demand and commit early.

[*Case Instance:*] Several agricultural input companies in Tanzania work closely with village based advisors (VBAs) to provide pricing incentives for advertising and booking their products.

The VBAs provide further know-how to rural farmers downstream to support farming practices and aggregate their demand for agricultural inputs.[14] We find that such wholesaler-led supply chains lead to higher reservation quantities.

5.2. Analysis of manufacturing quantity

A comparison of the manufacturing quantity (q_m) shows,

Proposition 3. ***(Manufacturing quantity)****: (a) When the advance booking discount is high and cost of manufacturing is above a threshold, the manufacturer produces lesser quantity when he is the Stackelberg leader. Otherwise, the manufacturer produces a higher quantity when he is the Stackelberg leader.*

(b) When the advance booking discount is lower, the manufacturer produces more when he is the Stackelberg leader.

When the AB discount is relatively high, the wholesaler is incentivized to place larger orders during the AB period. However, if the manufacturer's production cost exceeds a certain threshold c_{m2}, the manufacturer, as a Stackelberg leader, reduces his production quantity to lower total costs. In contrast, when the production cost is below this threshold ($c_m \leq c_{m2}$), the manufacturer increases production as the Stackelberg leader to better capitalize on the AB orders.

For agricultural input manufacturers, this implies that discount design should be closely linked to production cost conditions. When production costs are low, manufacturers can use high AB discounts to encourage early commitments and expand output. However, when costs are high, large discounts may lead manufacturers to adopt a more conservative production plan. For wholesalers, the result suggests that early booking incentives are most effective when manufacturers have sufficient cost flexibility to fulfill larger seasonal demand.

This strategic interplay between pricing discounts and production planning is observable in practice. **[*Case Instance:*]** For example, PT Pupuk Indonesia, a major fertilizer producer, offers discounted fertilizer prices to support farmers in preparation for the rice planting season.[15] Similarly, Nirman Agri Genetics Limited in India secured advance bookings worth USD 13.8 million ahead of the rabi season, aligning production planning with anticipated demand.[16] In Pakistan, Engro Fertilizers offers price discounts to farmers, leading to increased early purchases and higher production volumes.[17]

In contrast, in regions 2H and 3, where AB discounts are relatively low, the wholesaler exerts similar sales effort across both strategic configurations (as observed in Proposition 1(b)) and relies

[14] https://dailynews.co.tz/private-extension-systems-missing-link-in-sustaining-africas-food-system-momentum/ Accessed on 17th September 2024

[15] https://www.argusmedia.com/en/news-and-insights/latest-market-news/2699211-pupuk-indonesia-distributes-subsidised-fertilizers Accessed on 18th October 2024

[16] https://tradebrains.in/features/agrochemical-stock-jumps-4-after-board-to-consider-bonus-issue-dividend-and-expansion/ Accessed on 28th October 2024

[17] https://profit.pakistantoday.com.pk/2018/11/18/fertiliser-industry-withholds-gidc-payment-worth-billions/ accessed on 7th November 2024.

more on orders during the regular selling period. Anticipating this, the manufacturer increases production as the Stackelberg leader to capitalize on regular period demand and to mitigate the risks associated with potential underproduction. The comparative outcomes across these settings are summarized in Table 8.

For the manufacturing quantity decision, it is important to note that, under the MS configuration, the manufacturer determines the production quantity before the advance booking information becomes available. Therefore, the manufacturer cannot adjust the production quantity after observing the AB information. Nevertheless, the advance booking information can still help him with inventory allocation and cash-flow management, while also providing valuable information for planning production in future cycles. In contrast, under the WS configuration, the manufacturer determines the production quantity after observing the advance booking information and therefore has the flexibility to adjust the production volume accordingly, providing the manufacturer with greater operational flexibility in the supply chain.

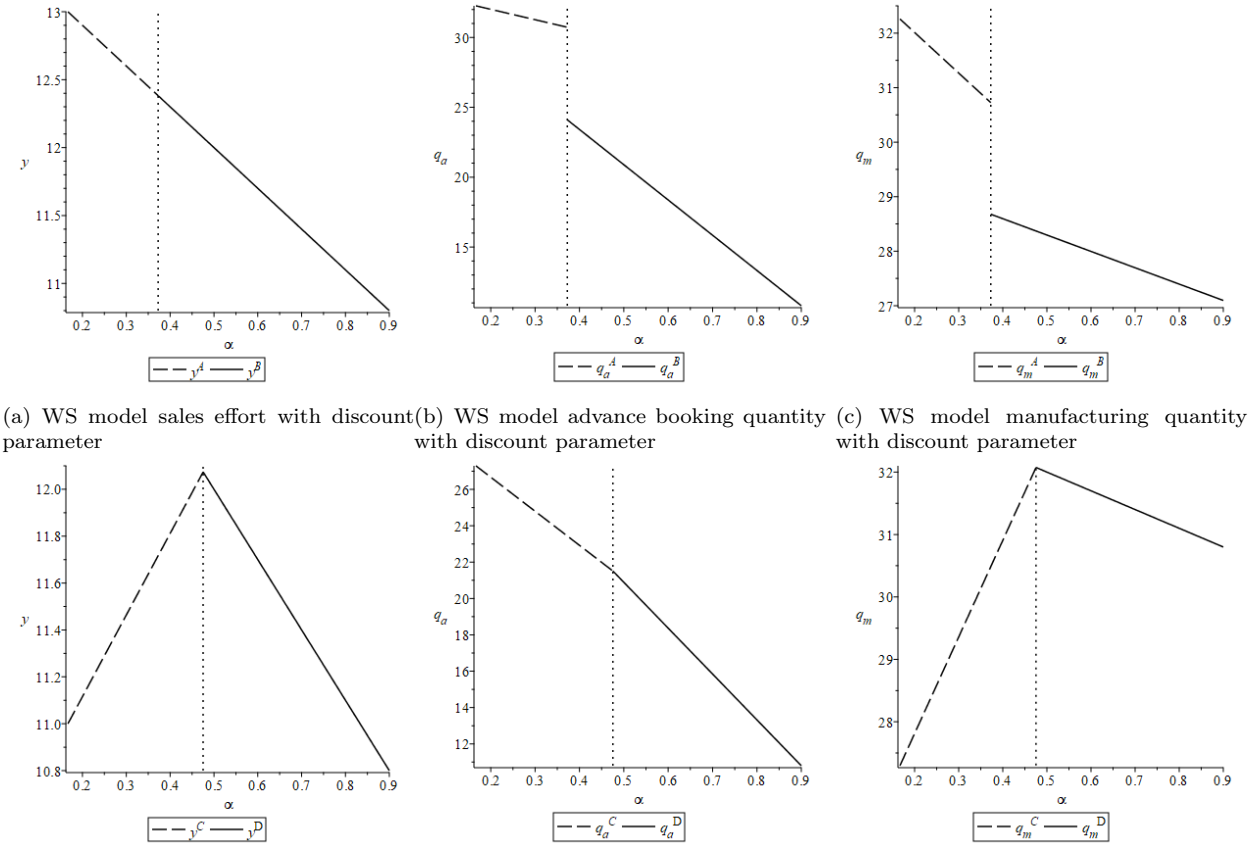
Regions	Range of α	Sales effort of wholesaler	AB order quantity of wholesaler	Manufacturing quantity of manufacturer
Region 1	$\alpha \leq min(\alpha_1, \alpha_2)$	Higher in WS configuration	Higher in WS configuration for high manufacturing cost ($c_m \geq c_{m1}$); otherwise higher in MS configuration	Higher in WS configuration for high manufacturing cost ($c_m \geq c_{m2}$), otherwise higher in MS configuration
Region 2L	$\alpha_1 \leq \alpha \leq \alpha_2$	Higher in WS configuration	Higher in WS configuration	Higher in WS configuration for high manufacturing cost $c_m \geq c_{m2}$, otherwise higher in MS configuration
Region 2H	$\alpha_2 < \alpha \leq \alpha_1$	Equal in WS and MS configuration	Higher in WS configuration	Higher in MS configuration
Region 3	$\alpha \geq max(\alpha_1, \alpha_2)$	Equal in WS and MS configuration	Equal in WS and MS configuration	Higher in MS configuration

Table 8: Summary of comparison of the decision variables

6. Numerical analysis

In this section, we study the behavior of decision variables and profit outcomes with respect to various model parameters using numerical analysis. We begin by analyzing the effect of AB discount parameter, α. We generate a grid of plots of AB quantity, manufacturing quantity and sales effort by setting parameter values as $p_m = 300, \delta = 0.9, c_w = 50, b = 20, c_m = 50$ and $p_w = 1500$ in Tables 5 and 6 where α is varied between 0.167 to 0.9 within the bounds of the problem parameters. Note that the manufacturer will participate only when $\alpha \geq 0.167$, else, the transaction becomes loss making for him and he will not participate. The WS and MS plots are presented in Figures 4 and 5. Additional plots of decision variables and profits with respect to the wholesaler's selling price are also provided in the Appendix.

Impact of advance booking discount. We observe in the WS configuration (Figures 4b and 4c) that the AB quantity (q_a) and manufacturing quantity (q_m) decrease as α increases (or AB discount



(a) WS model sales effort with discount parameter

(b) WS model advance booking quantity with discount parameter

(c) WS model manufacturing quantity with discount parameter

(d) MS model sales effort with discount parameter

(e) MS model advance booking quantity with discount parameter

(f) MS model manufacturing quantity with discount parameter

Figure 4: Plot of decision variables (AB quantity, manufacturing quantity and sales effort) with discount parameter under uniform distribution; cases A, B – WS model; cases C, D – MS model

decreases). Similarly, sales effort (y) decreases with α, irrespective of the solution case (Figure 4a). This is because when the AB discount is high (low α), the wholesaler has to pay less in the AB period, making AB profitable; therefore, as a Stackelberg leader (WS model), she puts in higher sales effort and places a higher AB order. To fulfil higher expected demand and AB order, the manufacturer also produces more, leading to a higher production quantity. Case A in particular shows that the manufacturer produces exactly what is ordered in the AB period ($q_m^A = q_a^A$) as the order quantity is high.

In the MS configuration (Figures 4d – 4f), we find that although the AB quantity decreases with increasing α, the manufacturing quantity and sales effort first increase and then decrease. Particularly in case C, we interestingly find that as the AB discount decreases, the wholesaler increases her sales effort and the manufacturer increases his manufacturing quantity (until a threshold). The intuition is as follows: In the MS configuration, the manufacturer decides his manufacturing quantity first and as the AB discount is high, the wholesaler aims to convert the same into sales through her sales effort. Therefore, as the discount decreases, she increases her sales effort upto a given threshold to convert the manufactured volume into sales. In case D, the AB discount is low; therefore, the

cost of increasing sales effort outweighs the benefit from advance booking, therefore, she reduces her sales effort to reduce her costs.

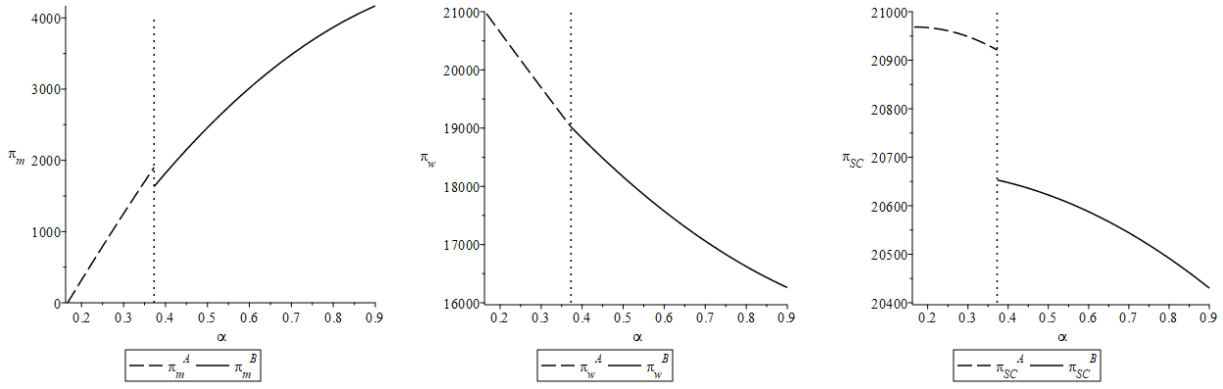
Impact of advance booking discount on profits. Next, we study the impact of the discount parameter (α) on the profits of both the players and the supply chain. We use the same parameter values as in Figure 4 and plot the profits with α ranging between 0.167 to 0.9. The plots are presented in Figure 5.

We observe that in the WS model, irrespective of case A or B, the wholesaler's profit decreases as the AB discount decreases (or α increases), while the manufacturer's profit increases. Observing the supply chain, we find that supply chain profit also decreases in the WS model with increasing α. This suggests that the supply chain profit is the highest when the AB discount is maximum. As the discount reduces, the manufacturer gains at the expense of the wholesaler. However, the loss in the wholesaler's profit offsets the gain for the manufacturer, resulting in a net loss to the supply chain. This is explained by our observation in Figure 4a, where the sales effort of the wholesaler decreases in the WS configuration and as the final demand is dependent on it, demand also reduces. This shrinks the total supply chain profit.

Contrary to the WS model, in the MS model, we observe that although the wholesaler and manufacturer profits follow a similar pattern, the total supply chain profit first increases and then decreases. Specifically in case C, the supply chain profit increases and in case D it decreases as the value of discount parameter increases. In case C, with an increase in α, the wholesaler increases her sales effort (Figure 4d), which leads to an increase in demand that drives the supply chain profit. In case D, however, the converse occurs as the wholesaler reduces her sales effort which reduces the total supply chain profit. Note here that the manufacturer benefits at the expense of the wholesaler, but the reduced profit of the wholesaler more than offsets the gain for the manufacturer to drive down the overall supply chain profit.

Decision outcomes and profits under Normal distribution (truncated). In the following section, we test the robustness of our results. We examine whether the comparative analysis and the directionality of our results hold if demand is normally distributed (truncated). To test this, we plot a grid of decision outcomes for the WS and MS models considering demand to be normally distributed (truncated) with $\mu = 20, \sigma = 5, m = 0, n = 1, a = 5, b = 25$. The other parametric values are consistent with those used in the previous numerical analyses. The plots are presented in Figure 6.

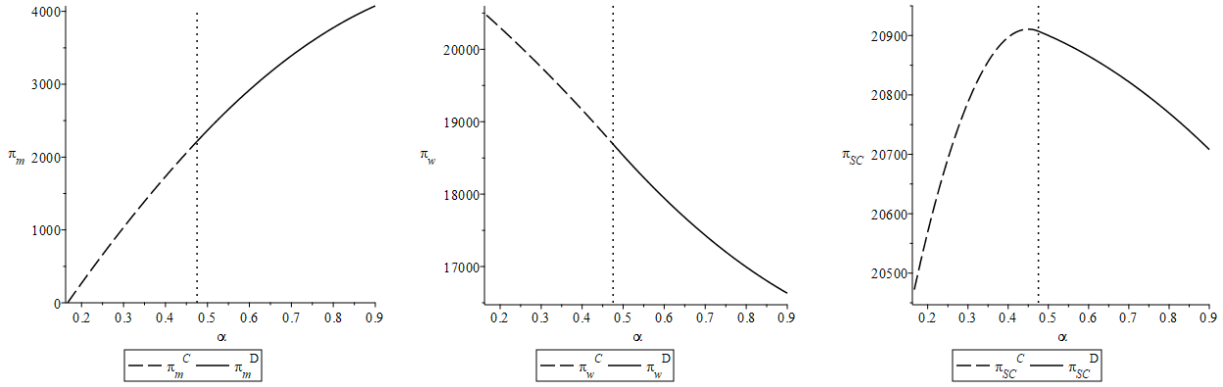
We observe similar patterns in sales effort, advance booking, and manufacturing quantities across the WS and MS models. In the WS model, we find that the sales effort, advance booking and manufacturing quantities reduce with increasing discount parameter (or equivalently lower discount) consistent with the rationale discussed in Figures 4a-4c. The MS model exhibits analogous behaviour. Furthermore, Figure 7 presents the corresponding profits of each player and the supply chain in both models, and the profit trends reflect consistency with our earlier findings. Additionally, we also generate the plots with a separate set of parameter values for uniform distribution and find that the behavior of plots are similar, these plots are included in the appendix.



(a) WS model manufacturer's expected profit with discount parameter

(b) WS model wholesaler's expected profit with discount parameter

(c) WS model supply chain expected profit with discount parameter



(d) MS model manufacturer's expected profit with discount parameter

(e) MS model wholesaler's expected profit with discount parameter

(f) MS model supply chain expected profit with discount parameter

Figure 5: Plot of expected profits of the manufacturer, wholesaler and supply chain with discount parameter under uniform distribution; cases A, B – WS model; cases C, D – MS model

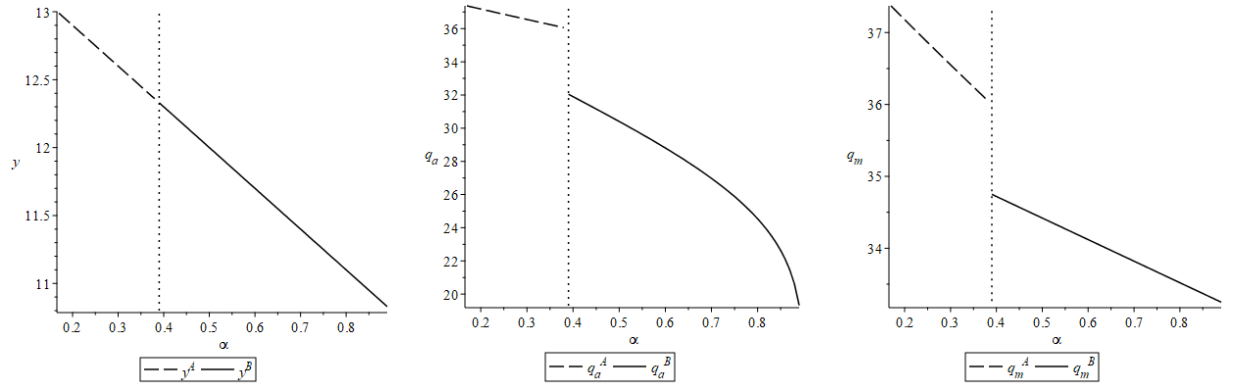
We conclude, therefore, that our results for the general distribution (outlined in Tables 3 and 4) hold for both uniform and (truncated) normal distributions and demonstrate robustness to variations in parametric values.

6.1. Profit analysis

We ask if a supply chain configuration benefits the players and under what conditions? Do players prefer a specific supply chain configuration? To answer this we compare each player's profit in both configurations to understand which is more profitable for them and under what conditions. We generate region plots of wholesaler and manufacturer profits with the discount parameter varying between 0 to 0.9 and the selling price between 300 to 3000. The values of other parameters are set as: $p_m = 300, \delta = 0.9, c_w = 50, b = 20, c_m = 50$ in Tables 5 and 6. The plots are shown in Figures 8 and 9. Note that in the region plots we demonstrate both the participation and non-participation regions.

We make the following observations:

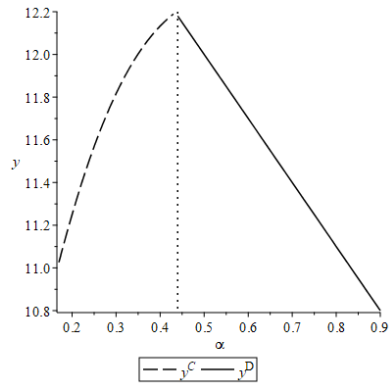
Observation 1: *When the AB discount is high, the manufacturer will opt out*: In both Figures 8a and 8b, we find that when the AB discount is high ($\alpha \leq 0.167$), the manufacturer's revenue



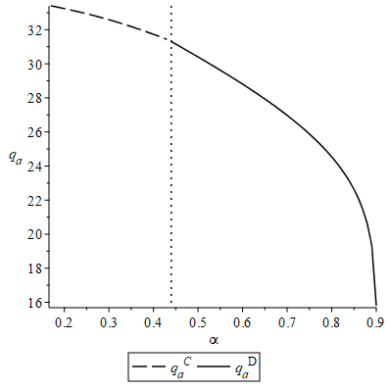
(a) WS model sales effort with discount parameter

(b) WS model advance booking quantity with discount parameter

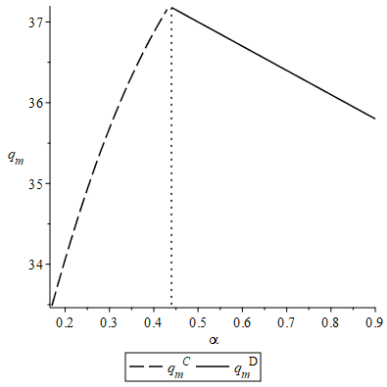
(c) WS model manufacturing quantity with discount parameter



(d) MS model sales effort with discount parameter



(e) MS model advance booking quantity with discount parameter



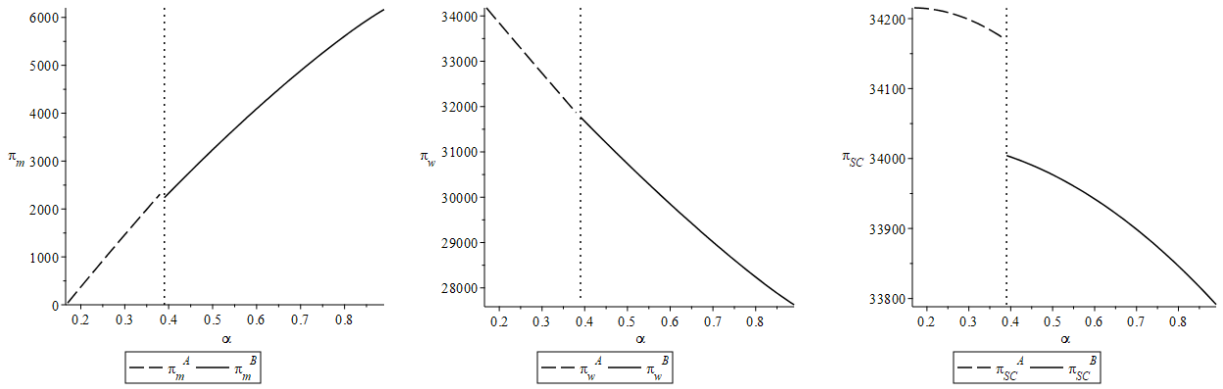
(f) MS model manufacturing quantity with discount parameter

Figure 6: Plot of decision variables (AB quantity, manufacturing quantity and sales effort) with discount parameter under normal distribution (truncated); cases A, B – WS model; cases C, D – MS model (ND parameters: $\mu = 20, \sigma = 5, m = 0, n = 1, a = 5, b = 25$)

in the AB period drops below his manufacturing cost, causing loss with every unit sold in this period, therefore, in this region, the manufacturer will choose not to sell to avoid losses (Refer to the unshaded area termed as *no selling region*).

Observation 2: *When the AB discount is moderately high, being the Stackelberg leader is more profitable for the wholesaler*: The light shaded portion of Figure 8a shows the area under which the wholesaler's profit is higher in the WS model in comparison to the MS model. In this area, the AB discount is moderately high which enables the wholesaler to take the benefit of Stackelberg leadership and influence the manufacturer to produce a higher quantity than in the MS configuration. The increased production quantity and moderately high AB discount together increase her profit in the WS configuration. The result is also supported from our observations in Figure 4.

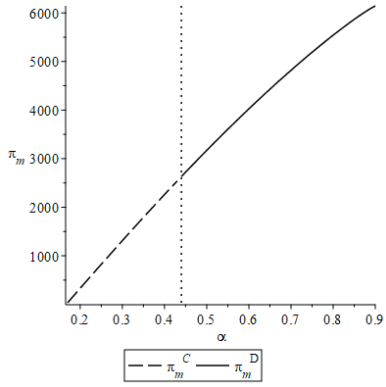
Observation 3: *When the AB discount is low, being a Stackelberg follower is more profitable for the wholesaler*: The dark shaded portion of Figure 8a shows the area where the wholesaler's profit is higher in the MS configuration. In this area, it is more profitable for the wholesaler to first observe the manufacturing quantity and based on that make her decisions. Here, increasing the AB quantity in the WS configuration is no more beneficial for her because she may face losses due to



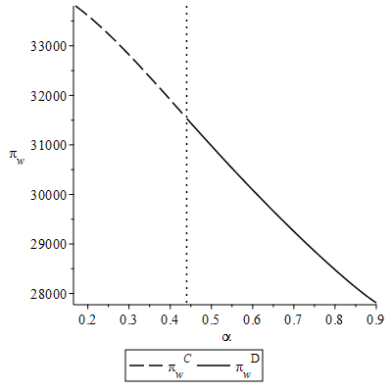
(a) WS model manufacturer's expected profit with discount parameter

(b) WS model wholesaler's expected profit with discount parameter

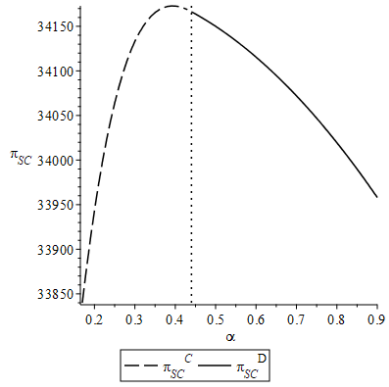
(c) WS model supply chain expected profit with discount parameter



(d) MS model manufacturer's expected profit with discount parameter

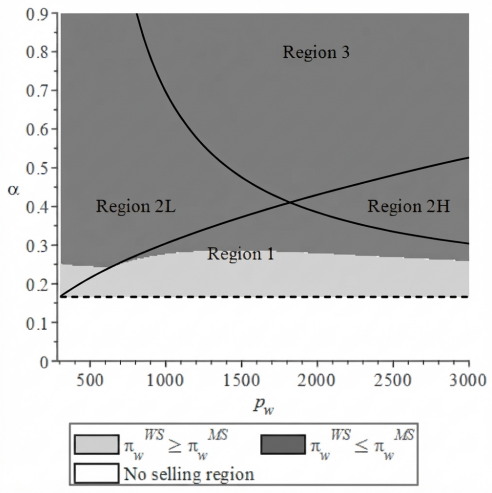


(e) MS model wholesaler's expected profit with discount parameter

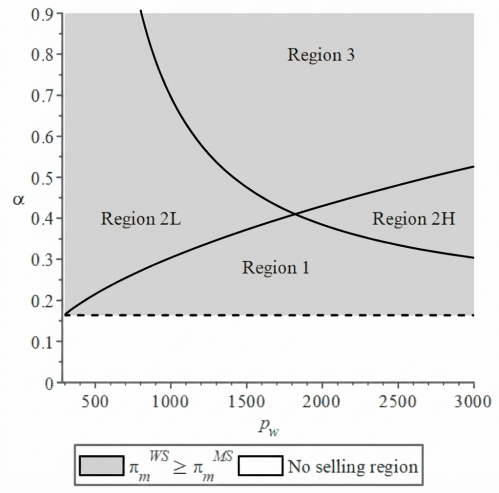


(f) MS model supply chain expected profit with discount parameter

Figure 7: Plot of expected profits of the manufacturer, wholesaler and supply chain with discount parameter under normal distribution (truncated); cases A, B – WS model; cases C, D – MS model (ND parameters: $\mu = 20, \sigma = 5, m = 0, n = 1, a = 5, b = 25$)



(a) Wholesaler profit region plot

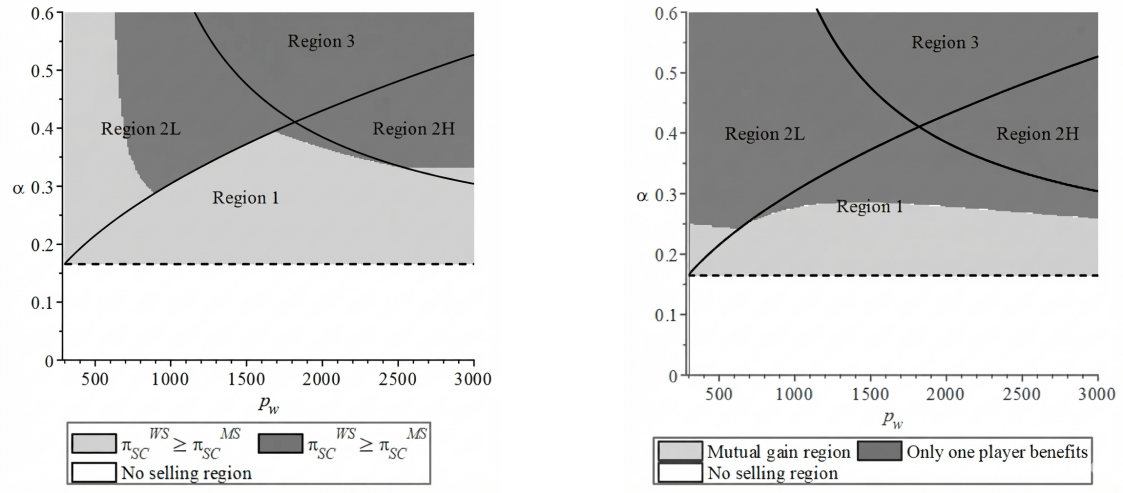


(b) Manufacturer profit region plot

Figure 8: Profit plots (The dashed horizontal line separates the participation and non-participation regions.)

overstocking and the expected value of that loss is higher than the expected gain by increasing the AB order.

Observation 4: *For the manufacturer it is beneficial to be the Stackelberg follower*: In Figure 8b, we observe that the manufacturer's profit is always higher in the WS configuration than in the MS configuration, therefore it is always beneficial for him to be the Stackelberg follower. As we observe in Proposition 1 the wholesaler puts in higher (or equal) sales effort in the WS configuration compared to the MS configuration, which increases the total demand that ultimately benefits the manufacturer.



(a) Supply chain profit region plot

(b) Win-win region for both the players under WS configuration

Figure 9: Supply chain profit plots (The dashed horizontal line separates the participation and non-participation regions.)

Observation 5: *For moderate to high levels of AB discount, the supply chain profit is higher in the WS configuration*: Figure 9a shows the region plot of the supply chain profit. The light shaded area in the plot shows the region where the supply chain profit is higher in the WS configuration, whereas in the dark shaded region, MS configuration yields higher supply chain profit. A key insight is that in agricultural input supply chains, specifically, in least developing economies, it would be preferable for manufacturers to provide discounts to farmer collectives or wholesaler-led supply chains so that supply chains benefit. **[*Case Instance:*]** The result also finds support in YARA[18] South Asia Managing Director's acknowledgment that agricultural input channel partners play an important role in serving farmers. In particular, we find that even standard schemes like advance booking provide incentives to drive higher benefits for agricultural input supply chains.

Observation 6: *For moderately high AB discount, WS configuration is a win-win scenario for both the players*: Observation 4 suggests that the WS configuration is more beneficial for the manufacturer, and observation 2 suggests that for moderately high values of AB discount the WS

[18]YARA International is a Norwegian manufacturer of fertilizers, crop nutrition and ammonia products, founded in 1905 and present in 150 countries; www.yara.com Accessed on 17th January 2025

configuration is more beneficial for the wholesaler. Combining the two, we infer that the WS configuration is more beneficial for the players, creating a win-win situation for them. Figure 9b shows the region, in which the WS configuration yields a higher profit for the players over the MS configuration. Particularly, we note that in the light shaded regions, WS configuration offers a mutual gain for both the players. We infer that farmer collectives or wholesaler-led supply chains benefit agricultural input supply chains more compared to manufacturer-led supply chains.

7. Returns Contract analysis

In the previous section, we observed that when the discount parameter, α, is low (high AB discount), the manufacturer's profit is low, and as α increases, his profit increases but the supply chain profit reduces (specifically in the WS Model). This is primarily because lower discount benefits the manufacturer but not the wholesaler and the supply chain. We therefore ask if there are other mechanisms in agricultural input supply chains that can benefit the players. It is imperative that wholesalers and farmer cooperatives have access to agricultural inputs at competitive rates and schemes that allow them to improve farm yield and tackle production issues. We examine such a contract. In particular, we model a returns contract and study its implications on both the players and the supply chain.

In our problem, the manufacturer offers a returns contract to encourage the wholesaler to place a higher order. Under this contract, the wholesaler books an AB quantity q_a in the AB period and if the realized demand in the regular period falls short of the quantity booked, she can return the unsold items to the manufacturer at a return price p_r. Equations 14 and 15 show the profit functions of the wholesaler and manufacturer under the returns contract.

$$\Pi_w^R(y, q_a; q_m) = \delta\left[p_w\Big(min\big(D(y,\epsilon), q_m\big)\Big) + p_r\Big(q_a - D(y,\epsilon)\Big)^+ - p_m\Big(min\big(D(y,\epsilon), q_m\big) - q_a\Big)^+\right] - \alpha p_m q_a - c_w y^2 \tag{14}$$

$$\Pi_m^R(q_m; y, q_a) = \delta\left[p_m\Big(min\big(D(y,\epsilon), q_m\big) - q_a\Big)^+ - p_r\Big(q_a - D(y,\epsilon)\Big)^+\right] + \alpha p_m q_a - c_m q_m \tag{15}$$

The profit functions with returns contract are similar to the original profit functions in equations (2) and (3), except that these contain terms with return price p_r, particularly, return revenue for the wholesaler and return cost for the manufacturer. The expected profit functions of each player are therefore given as,

$$E\big(\Pi_w^R(y,q_a;q_m)\big) = \frac{\delta}{b}\left[p_w\left(q_m b - \frac{(q_m-y)^2}{2}\right) + p_r\frac{(q_a-y)^2}{2} - p_m\big(q_m-q_a\big)\Big(y+b-\frac{q_m+q_a}{2}\Big)\right] \\ -\alpha p_m q_a - c_w y^2 \tag{16}$$

$$E\big(\Pi_m^R(q_m;y,q_a)\big) = \frac{\delta}{b}\left[p_m\big(q_m-q_a\big)\Big(y+b-\frac{q_m+q_a}{2}\Big) - p_r\frac{(q_a-y)^2}{2}\right] + \alpha p_m q_a - c_m q_m \tag{17}$$

To solve the returns contract corresponding to the WS model, we formulate a two-stage problem similar to the WS model and solve it using backward induction. We also assume that the present value of the return price offered by the manufacturer is no more than the AB price, that is, $p_r \leq \alpha p_m/\delta$. This assumption ensures that the wholesaler does not make abnormal profits through returns only. The equilibrium results obtained after solving the optimization problems are shown in Table 9.

Solution case	Case A'	Case B'
Range of α	$\frac{c_m}{p_m} \leq \alpha < \alpha_3$	$\alpha_3 \leq \alpha \leq \delta$
y^*	$\frac{\delta p_w - \alpha p_m}{2c_w}$	$\frac{\delta p_w - \alpha p_m}{2c_w}$
q_a^*	$\frac{(\delta p_w - \alpha p_m)(\delta(p_w-p_r)+2c_w b)}{2\delta c_w(p_w-p_r)}$	$\frac{b(\delta-\alpha)}{\delta} + \frac{\delta p_w - \alpha p_m}{2c_w} + \frac{p_r b(\delta-\alpha)}{\delta(p_m-p_r)}$
q_m^*	$\frac{(\delta p_w - \alpha p_m)(\delta(p_w-p_r)+2c_w b)}{2\delta c_w(p_w-p_r)}$	$b - \frac{c_m b}{\delta p_m} + \frac{\delta p_w - \alpha p_m}{2c_w}$

where $\alpha_3 = \frac{\delta p_m p_r + c_m\sqrt{(p_w-p_r)(p_m-p_r)}}{p_m^2}$

Table 9: Equilibrium solution of the WS model with returns contract

An immediate observation is that the sales effort put in by the wholesaler (y) is equal to the sales effort in the WS model without return. This is because although the returns contract reduces the wholesaler's risk of over booking, it does not directly incentivize her to increase her sales effort, therefore it remains unchanged. Our main objective is to find if the contract benefits both the players and the supply chain. We analyze this below.

7.1. Profit comparison

We compare the wholesaler and manufacturer profits with and without returns contract to understand if the contract benefits them. We find that the contract benefits the players conditionally. We state,

Proposition 4. *A returns contract benefits the supply chain if the discount parameter is at intermediate levels, that is, $\alpha_1 \leq \alpha \leq \alpha_3$ and the manufacturing cost lies between the thresholds* $\max(c_{m3}, c_{m5}) \leq c_m \leq c_{m4}$

where, $c_{m3}, c_{m4} = \frac{p_m(\alpha p_m - \delta p_r)}{p_w - p_r} \mp \frac{\sqrt{p_m\left(\alpha^2 p_m\left(p_w-p_m\right)^2 - \delta^2 p_w^2 p_r + p_m p_r\left(p_r(\delta-\alpha)^2 + \alpha(2\delta-\alpha)(2p_w-p_m)\right)\right)}}{p_w-p_r}$

and $c_{m5} = p_m\sqrt{\frac{\alpha^2 p_m(p_w-p_m) + \alpha p_m p_r(2\delta-\alpha) - \delta^2 p_w p_r}{(p_w-p_r)(p_w-p_m)}}$

The contract also benefits each player, that is, $E(\Pi_m^R) \geq E(\Pi_m)$ *and* $E(\Pi_w^R) \geq E(\Pi_w)$

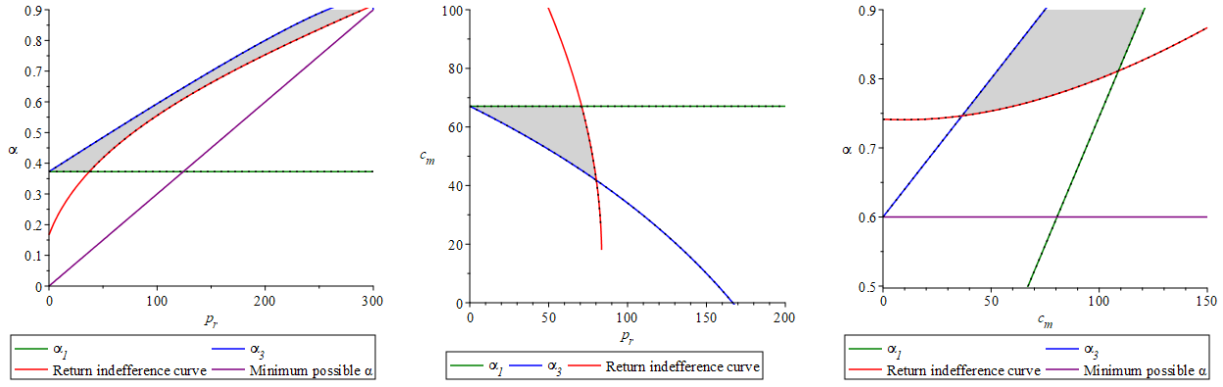
Index R denotes the returns contract. The result suggests that the returns contract benefits the supply chain for intermediate levels of AB discount ($\alpha_1 \leq \alpha \leq \alpha_3$) and manufacturing cost. When the AB discount is low, the manufacturer may not be willing to offer a returns contract as his revenues are higher in the absence of the contract. Similarly, when the AB discount is high, at equilibrium, the manufacturer produces only how much is ordered in the AB period; therefore, again, he is better off without offering the contract. For intermediate values of AB discount when the manufacturing cost is within the thresholds as above, the manufacturer benefits the most from a returns contract. We also find that under these conditions, the wholesaler's profits are higher than her reservation profits without the contract. The returns contract, therefore, creates a win-win scenario for both the players.

We assume in the above analysis that returned products have zero residual value for the manufacturer. This allows us to demonstrate that a returns policy can create a win–win outcome for the players even when the manufacturer cannot recover any value from returned products. In practice, however, returned products often retain some salvage value. For example, grain seeds may be sold as food grains or redirected to regions with later sowing seasons, allowing manufacturers to recover part of their value through alternative channels. Consequently, incorporating salvage revenue would likely further expand the win–win region identified in our analysis.

The results demonstrate the importance of wholesaler-manufacturer coordination in agricultural input supply chains. **[*Case Instance:*]** In India, small landholdings and the low purchasing power of farmers often deter manufacturers from providing inputs at affordable prices. Even when farmers form cooperatives for the purchase of agricultural inputs, prices remain a critical issue. In such cases, incentives through a combination of advance booking and returns contract can benefit farmer cooperatives or wholesalers. We find that in equilibrium the decisions under such incentive schemes benefit the manufacturers of agricultural inputs as well. Such partnerships can greatly aid in the financial support of farmer cooperatives and wholesalers.

We conduct a numerical analysis to depict the regions where the returns contract benefits the supply chain. We generate different plots setting parameter values similar to our previous analysis. First, we generate a region plot of return price (p_r) and discount parameter (α) (Figure 10a). We use the following parameter values: $b = 20, c_m = 50, c_w = 50, \delta = 0.9, p_m = 300, p_w = 1500$ with α and p_r varying between zero to 0.9 and zero to 300 respectively. Figure 10b depicts a region plot of return price (p_r) and manufacturing cost (c_m) while Figure 10c depicts a region plot of manufacturing cost (c_m) and AB discount parameter (α).

Figure 10a shows that in the region between α_1 and α_3, for every unique value of AB discount parameter (α), there exists a set of return prices that benefit the players and the supply chain. Figure 10b shows that in the region, α between α_1 and α_3, the returns contract benefits the supply chain if the return price is below a threshold. Figure 10c shows that in the region (α between α_1 and α_3), for a given manufacturing cost, the returns contract can benefit the supply chain.



(a) Plot of discount parameter and return price

(b) Plot of manufacturing cost and return price

(c) Plot of discount parameter and manufacturing cost

Figure 10: Region plots of returns contract; shaded regions depict combinations of discount parameter, returns price and manufacturing cost where each player and the supply chain has higher profit under the returns contract.

8. Discussion and conclusion

Motivated by the practice of advance booking in agricultural input supply chains, in this paper, we analyze two models differentiated by their Stackelberg leadership. We compare the decision outcomes and draw several insights from the analysis.

8.1. Study Insights

The findings of our study provide answers to the research questions posed in Section 1 and offer several managerial insights for agricultural input supply chains.

Our first research question concerns the impact of advance booking discounts on ordering, sales effort, production, and profitability decisions. The analysis reveals that advance booking discounts do not merely shift purchases from the regular selling period to the advance booking period. Rather, by reducing procurement costs, they encourage wholesalers to invest more in sales effort and market-development activities. This effect is particularly important in agricultural input markets, where demand is strongly influenced by product promotion and field demonstrations. Consequently, advance booking discounts can increase overall demand rather than simply redistribute it over time.

The second research question examines how alternative channel leadership structures influence equilibrium outcomes. A comparison of the MS and WS configurations shows that the wholesaler always exerts higher (or equal) sales effort when acting as the Stackelberg leader. This strategic behavior enables her to place larger advance booking orders and reduce reliance on less profitable regular period purchases. Since sales effort directly influences demand, the manufacturer may benefit from observing the wholesaler's advance booking decision before determining production quantities. This finding highlights the importance of channel leadership in shaping both demand-generation activities and ordering behavior.

Our third research question focuses on the configurations under which advance booking programs create value for supply chain members. The analysis shows that the answer depends critically on the interaction between booking discounts and production costs. At relatively low discount levels, the

manufacturer produces more when acting as the Stackelberg leader. However, as discounts increase, production decisions become conditional on manufacturing costs. When production costs are sufficiently high, the manufacturer may optimally reduce production under the MS configuration despite receiving larger advance commitments. These findings demonstrate that advance booking does not universally increase production and that cost considerations play a critical role in determining the manufacturer's optimal strategy.

The fourth research question investigates whether returns contracts can improve supply chain performance. Our results indicate that, for intermediate levels of advance booking discount and production cost, a returns policy can benefit both the manufacturer and the wholesaler, thereby improving overall supply chain performance. Although returns contracts are often viewed as increasing the manufacturer's exposure to demand uncertainty, the analysis reveals a more nuanced outcome. By reducing the wholesaler's risk of overstocking, returns policies encourage larger advance booking quantities which can increase realized demand and profitability for both parties. Therefore, manufacturers may benefit from incorporating appropriately designed returns contracts into their advance booking programs under suitable market conditions.

Taken together, these findings suggest that advance booking serves not only as an inventory-planning mechanism but also as a demand-generation and coordination tool in agricultural input supply chains. The results further demonstrate that channel leadership, pricing incentives through advance booking discounts, and risk-sharing mechanisms jointly determine the effectiveness of advance booking programs.

Tables 10 and 11 summarize the key results and corresponding managerial recommendations for manufacturers and wholesalers under different operating conditions. These recommendations provide practical guidance for selecting suitable advance booking, pricing, and coordination strategies across alternative supply chain configurations.

Scenario	Manufacturer's recommended measures	Reference results
High discount in the WS configuration	Produce as per the reservation quantity	Lemma 1(b), (c); Figures 4(b) and (c); where, q_a^A equals q_m^A
High discount in the MS configuration	Produce more when the manufacturing cost is low	Proposition 3(a); Figures 4(e) and (f); where, q_m^C is greater than q_a^C
Low discount	Produce more as a Stackelberg leader	Proposition 3(b); Figures 4(c) and (f); where q_m^D is greater than q_m^B
An intermediate range of discount	Offer an appropriate returns policy (when the manufacturing cost lies in the intermediate range)	Proposition 4; Figure 10 shaded region

Table 10: Recommended measures for the manufacturer

8.2. Future Research and Concluding Remarks

Our analysis assumes that the wholesaler has sufficient financial resources to pay for the advance booking order and to exert sales effort. In practice, however, this assumption may not hold, particularly for smaller wholesalers or farmer collectives who face financial constraints. Manufacturers

Scenario	Wholesaler's recommended measures	Reference results
High discount	Put higher sales effort and place higher booking order as a Stackelberg leader (when the manufacturer's production cost is above a threshold)	Propositions 1, 2; Figures 4(a), (b), (d) and (e); where, y^A is greater than y^C and q_a^A is greater than q_a^C
Low discount	Put lower sales effort and place lower booking order (and rely more on regular period order)	Propositions 1, 2; Figures 4(a), (b), (d) and (e); where, y^B, y^D, q_a^B and q_a^D reduces with α

Table 11: Recommended measures for the wholesaler

often address this through partial-prepayment schemes, whose impact on supply chain decisions presents an interesting avenue for future research.

Second, regulatory interventions may influence the implementation of advance booking programs. In many countries, governments subsidize agricultural inputs and regulate selling prices. Incorporating regulatory considerations into advance booking models also represents a promising direction for future research.

Beyond advance booking and returns contracts, studies could examine the role of digital agricultural marketplaces in improving input accessibility and distribution efficiency. Platforms such as Freshokartz (www.freshokartz.com), DeHaat(www.agrevolution.in), and AgroStar (www.agrostar.in) leverage local intermediaries to aggregate demand, improve price transparency and streamline distribution. It would be interesting to examine their impact on farmers and other supply chain stakeholders.

Our model could be extended to examine discount sharing arrangements in which wholesalers or farmer collectives pass a portion of the advance booking discount to farmers and evaluate their impact on the decisions and profitability of different supply chain stakeholders. Another extension could be to endogenize pricing decisions alongside sales effort to better understand their joint influence on supply chain outcomes.

References

Anderson, E. and Monjardino, M. (2019). Contract design in agriculture supply chains with random yield. *European Journal of Operational Research*, 277(3):1072–1082.

Bai, Y., Li, H., and Gu, M. (2024). Advance selling policies with stochastic consumer valuations and advertising effects: dynamic pricing and inventory decisions. *International Journal of Production Economics*, 277:109401.

Bansal, S. and Dyer, J. S. (2020). Planning for end-user substitution in agribusiness. *Operations Research*, 68(4):1000–1019.

Bellemare, M. F. and Novak, L. (2017). Contract farming and food security. *American Journal of Agricultural Economics*, 99(2):357–378.

Benini, M., Detti, P., and Nerozzi, L. (2025). Optimization models and algorithms for sustainable crop planning and rotation: An arc flow formulation and a column generation approach. *Omega*, 135:103320.

Borodin, V., Bourtembourg, J., Hnaien, F., and Labadie, N. (2014). A quality risk management problem: case of annual crop harvest scheduling. *International Journal of Production Research*, 52(9):2682–2695.

Borodin, V., Bourtembourg, J., Hnaien, F., and Labadie, N. (2016). Handling uncertainty in agricultural supply chain management: A state of the art. *European Journal of Operational Research*, 254(2):348–359.

Boyabatlı, O., Nasiry, J., and Zhou, Y. (2019). Crop planning in sustainable agriculture: Dynamic farmland allocation in the presence of crop rotation benefits. *Management Science*, 65(5):2060–2076.

Cao, Y., Yi, C., Wan, G., Hu, H., Li, Q., and Wang, S. (2022). An analysis on the role of blockchain-based platforms in agricultural supply chains. *Transportation Research Part E: Logistics and Transportation Review*, 163:102731.

Chen, Y., Nie, T., Du, S., and Liu, M. (2025). Optimal decision making considering retailer fairness concerns in a supply chain under advance selling. *International Journal of Production Economics*, page 109835.

Cheng, Y., Li, H., and Thorstenson, A. (2018). Advance selling with double marketing efforts in a newsvendor framework. *Computers & Industrial Engineering*, 118:352–365.

Chintapalli, P., Disney, S. M., and Tang, C. S. (2017). Coordinating supply chains via advance-order discounts, minimum order quantities, and delegations. *Production and Operations Management*, 26(12):2175–2186.

Cho, S.-H. and Tang, C. S. (2013). Advance selling in a supply chain under uncertain supply and demand. *Manufacturing & Service Operations Management*, 15(2):305–319.

Cholette, S. (2009). Mitigating demand uncertainty across a winery's sales channels through postponement. *International Journal of Production Research*, 47(13):3587–3609.

Chu, L. Y. and Zhang, H. (2011). Optimal preorder strategy with endogenous information control. *Management Science*, 57(6):1055–1077.

Dana, Jr, J. D. (1998). Advance-purchase discounts and price discrimination in competitive markets. *Journal of Political Economy*, 106(2):395–422.

Dar, M. H., De Janvry, A., Emerick, K., Sadoulet, E., and Wiseman, E. (2024). Private input suppliers as information agents for technology adoption in agriculture. *American Economic Journal: Applied Economics*, 16(2):219–248.

Desiraju, R. and Shugan, S. M. (1999). Strategic service pricing and yield management. *Journal of marketing*, 63(1):44–56.

Ding, W. and Song, H. (2020). Financing the price-setting newsvendor with sales effort. *Journal of Management Analytics*, 7(4):564–590.

Fan, H., Khouja, M., and Zhou, J. (2025). Optimal price, quantity, and return policy decisions of a two-period newsvendor with product reviews. *European Journal of Operational Research*.

Fan, J., Zhang, X., Dan, B., Tian, Y., and Chen, Z. (2024). Advance selling of fresh produce e-tailer with uncertain demand and risk-averse consumers. *Managerial and Decision Economics*, 45(3):1702–1717.

Gale, I. L. and Holmes, T. J. (1993). Advance-purchase discounts and monopoly allocation of capacity. *The American Economic Review*, pages 135–146.

Glen, J. J. (1987). Mathematical models in farm planning: A survey. *Operations Research*, 35(5):641–666.

Guo, L. (2009). Service cancellation and competitive refund policy. *Marketing Science*, 28(5):901–917.

Han, J.-W., Zuo, M., Zhu, W.-Y., Zuo, J.-H., Lü, E.-L., and Yang, X.-T. (2021). A comprehensive review of cold chain logistics for fresh agricultural products: Current status, challenges, and future trends. *Trends in Food Science & Technology*, 109:536–551.

Hayashi, K. (2000). Multicriteria analysis for agricultural resource management: a critical survey and future perspectives. *European journal of operational research*, 122(2):486–500.

He, Y., Xu, Q., and Wu, P. (2020). Omnichannel retail operations with refurbished consumer returns. *International Journal of Production Research*, 58(1):271–290.

He, Y. and Zhao, X. (2012). Coordination in multi-echelon supply chain under supply and demand uncertainty. *International Journal of Production Economics*, 139(1):106–115.

He, Y., Zhao, X., Zhao, L., and He, J. (2009). Coordinating a supply chain with effort and price dependent stochastic demand. *Applied Mathematical Modelling*, 33(6):2777–2790.

Heese, H. S. and Swaminathan, J. M. (2010). Inventory and sales effort management under unobservable lost sales. *European Journal of Operational Research*, 207(3):1263–1268.

Hu, F., Lim, C.-C., and Lu, Z. (2013). Coordination of supply chains with a flexible ordering policy under yield and demand uncertainty. *International Journal of Production Economics*, 146(2):686–693.

Huh, W. T. and Lall, U. (2013). Optimal crop choice, irrigation allocation, and the impact of contract farming. *Production and Operations Management*, 22(5):1126–1143.

Jia, H., Li, J., Liang, L., Peng, W., Xie, J., and Xie, J. (2022). Advance selling of uncertain demand in low-carbon supply chain. *Industrial Management & Data Systems*, 122(12):2680–2709.

Jin, D., Caliskan-Demirag, O., Chen, F. Y., and Huang, M. (2020). Omnichannel retailers' return policy strategies in the presence of competition. *International Journal of Production Economics*, 225:107595.

Khan, M. A.-A., Shaikh, A. A., Konstantaras, I., Bhunia, A. K., and Cárdenas-Barrón, L. E. (2020). Inventory models for perishable items with advanced payment, linearly time-dependent holding cost and demand dependent on advertisement and selling price. *International Journal of Production Economics*, 230:107804.

Kuthambalayan, T. S., Mehta, P., and Shanker, K. (2015). Managing product variety with advance selling and capacity restrictions. *International Journal of Production Economics*, 170:287–296.

Li, C. and Zhang, F. (2013). Advance demand information, price discrimination, and preorder strategies. *Manufacturing & service operations management*, 15(1):57–71.

Li, H., Yang, K., Yao, Y., and Zhang, G. (2024). Selling mode choice and return policy for advance selling in an e-commerce platform with social influence. *Computers & Industrial Engineering*, 197:110620.

Li, J., Hu, C., Dai, M., Shen, W., and Chen, Z. (2025). Advance selling in the presence of valuation uncertainty and demand correlation. *Omega*, page 103407.

Li, K. J. and Li, X. (2023). Advance selling in marketing channels. *Journal of Marketing Research*, 60(2):371–387.

Ma, P., Wang, H., and Shang, J. (2013). Supply chain channel strategies with quality and marketing effort-dependent demand. *International journal of production economics*, 144(2):572–581.

Maiti, A., Maiti, M., and Maiti, M. (2009). Inventory model with stochastic lead-time and price dependent demand incorporating advance payment. *Applied Mathematical Modelling*, 33(5):2433–2443.

McCardle, K., Rajaram, K., and Tang, C. S. (2004). Advance booking discount programs under retail competition. *Management Science*, 50(5):701–708.

Miller, W., Leung, L., Azhar, T., and Sargent, S. (1997). Fuzzy production planning model for fresh tomato packing. *International Journal of Production Economics*, 53(3):227–238.

Minten, B., Koru, B., and Stifel, D. (2013). The last mile (s) in modern input distribution: Pricing, profitability, and adoption. *Agricultural economics*, 44(6):629–646.

Mir, V. R., Tortella, B. D., Leoni, V., and Nicolau, J. L. (2026). Decoding booking cancellations: Quantitative insights and theoretical advances in tourism behavior. *Tourism Management*, 114:105384.

Moe, W. W. and Fader, P. S. (2002). Fast-track: Article using advance purchase orders to forecast new product sales. *Marketing science*, 21(3):347–364.

Nematollahi, M., Tajbakhsh, A., and Sedghy, B. M. (2021). The reflection of competition and coordination on organic agribusiness supply chains. *Transportation Research Part E: Logistics and Transportation Review*, 154:102462.

Noori-daryan, M., Taleizadeh, A. A., and Aickelin, U. (2023). Time-sensitive cancellation refund in advance booking: Effect of online-to-offline marketing policy. *Computers & Industrial Engineering*, 183:109445.

OECD (2018). *Concentration in Seed Markets: Potential Effects and Policy Responses*. OECD Publishing, Paris.

Özer, Ö. and Wei, W. (2004). Inventory control with limited capacity and advance demand information. *Operations Research*, 52(6):988–1000.

Peng, W. and Tian, Z. (2022). Information acquisition, selling effort and pre-order strategy. *International Journal of Production Economics*, 249:108538.

Petruzzi, N. C. and Dada, M. (1999). Pricing and the newsvendor problem: A review with extensions. *Operations research*, 47(2):183–194.

Prasad, A., Stecke, K. E., and Zhao, X. (2011). Advance selling by a newsvendor retailer. *Production and Operations Management*, 20(1):129–142.

Rutsaert, P., Chamberlin, J., Oluoch, K. O., Kitoto, V. O., and Donovan, J. (2021). The geography of agricultural input markets in rural tanzania. *Food Security*, 13(6):1379.

Schlapp, J., Fleischmann, M., and Sonntag, D. (2022). Inventory timing: How to serve a stochastic season. *Production and Operations Management*, 31(7):2891–2906.

Sharma, D. K. and Jana, R. (2009). Fuzzy goal programming based genetic algorithm approach to nutrient management for rice crop planning. *International Journal of Production Economics*, 121(1):224–232.

Shi, X., Dong, C., and Cheng, T. (2018). Does the buy-online-and-pick-up-in-store strategy with pre-orders benefit a retailer with the consideration of returns? *International Journal of Production Economics*, 206:134–145.

Shugan, S. M. and Xie, J. (2000). Advance pricing of services and other implications of separating purchase and consumption. *Journal of Service Research*, 2(3):227–239.

Su, X. and Zhang, F. (2008). Strategic customer behavior, commitment, and supply chain performance. *Management Science*, 54(10):1759–1773.

Su, X. and Zhang, F. (2009). On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science*, 55(5):713–726.

Sun, S. and Liu, T. (2023). Pricing and sales-effort coordination facing free riding behaviors between a brick-and-mortar retailer and a platform store owned by the manufacturer. *Transportation Research Part E: Logistics and Transportation Review*, 179:103285.

Tang, C. S., Rajaram, K., Alptekinoğlu, A., and Ou, J. (2004). The benefits of advance booking discount programs: Model and analysis. *Management science*, 50(4):465–478.

Tang, W. and Girotra, K. (2017). Using advance purchase discount contracts under uncertain information acquisition cost. *Production and Operations Management*, 26(8):1553–1567.

Taylor, T. A. (2006). Sale timing in a supply chain: When to sell to the retailer. *Manufacturing & service operations management*, 8(1):23–42.

Verdouw, C., Robbemond, R. M., Verwaart, T., Wolfert, J., and Beulens, A. J. (2018). A reference architecture for iot-based logistic information systems in agri-food supply chains. *Enterprise information systems*, 12(7):755–779.

Wang, H. H., Wang, Y., and Delgado, M. S. (2014). The transition to modern agriculture: Contract farming in developing economies. *American Journal of Agricultural Economics*, 96(5):1257–1271.

Wang, S., Gurnani, H., and Subramanian, U. (2021). The informational role of buyback contracts. *Management Science*, 67(1):279–296.

Wang, Y., Li, J., Zhu, S., and Quan, P. (2024). Advance selling strategy with return service: The impacts of information disclosure and disclosure methods. *Asia-Pacific Journal of Operational Research*, 41(04):2440009.

Wei, M. M. and Zhang, F. (2018). Advance selling to strategic consumers: preorder contingent production strategy with advance selling target. *Production and Operations Management*, 27(7):1221–1235.

Wu, M., Zhu, S. X., and Teunter, R. H. (2021). Advance selling and advertising: A newsvendor framework. *Decision Sciences*, 52(1):182–215.

Xie, J. and Gerstner, E. (2007). Service escape: Profiting from customer cancellations. *Marketing Science*, 26(1):18–30.

Xie, L., Ma, J., and Goh, M. (2021). Supply chain coordination in the presence of uncertain yield and demand. *International Journal of Production Research*, 59(14):4342–4358.

Xu, X., Chen, X., Hou, J., Cheng, T., Yu, Y., and Zhou, L. (2025). Should live streaming be adopted for agricultural supply chain considering platform's quality improvement and blockchain support? *Transportation Research Part E: Logistics and Transportation Review*, 195:103950.

Yu, Y. and Xiao, T. (2021). Analysis of cold-chain service outsourcing modes in a fresh agri-product supply chain. *Transportation Research Part E: Logistics and Transportation Review*, 148:102264.

Zhang, Z., Lim, W., Cui, H., and Wang, Z. (2021). Partial refunds as a strategic price commitment device in advance selling in a service industry. *European Journal of Operational Research*, 291(3):1062–1074.

Zhao, S. and Li, W. (2023). Game-theoretic analysis of a two-stage dual-channel supply chain coordination in the presence of market segmentation and price discounts. *Electronic Commerce Research and Applications*, 57:101222.

Zhao, X., Pang, Z., and Stecke, K. E. (2016). When does a retailer's advance selling capability benefit manufacturer, retailer, or both? *Production and Operations Management*, 25(6):1073–1087.

Zhao, X. and Stecke, K. E. (2010). Pre-orders for new to-be-released products considering consumer loss aversion. *Production and Operations Management*, 19(2):198–215.

Appendix A – Advance Booking with weather shock

We extend our model to include a random weather shock on the wholesaler's demand to examine its impact on the model outcomes. Weather shock could be due to untimely heavy rainfall, floods, cyclones, hailstorm, delayed monsoon onset etc, which impact the farmers' crop preferences and hence lead to demand shocks for the manufacturers and wholesalers.

We consider ν to be a random weather shock on the wholesaler's demand. We assume its PDF and CDF to be $g(\cdot)$ and $G(\cdot)$ respectively.

The wholesaler's new demand function is as follows:

$$D(y, \epsilon, \nu) = \phi(y) + \epsilon + \nu \tag{A1}$$

Where, $\phi(y) = m + ny$, a linear function of y, $n \neq 0$; ϵ and ν are random variables representing random component of demand and weather shock defined on the support of $[a, b]$ and $[a', b']$ respectively.

The new expressions of profit function, represented by $\bar{\Pi}_w$ and $\bar{\Pi}_m$ are:

$$\bar{\Pi}_w\left(y, q_a; q_m\right) = \delta\left[p_w\Big(min\big(D(y,\epsilon,\nu), q_m\big)\Big) - p_m\Big(min\big(D(y,\epsilon,\nu), q_m\big) - q_a\Big)^+\right] - \alpha p_m q_a - c_w y^2 \tag{A2}$$

$$\bar{\Pi}_m\left(q_m; y, q_a\right) = \delta\left[p_m\Big(min\big(D(y,\epsilon,\nu), q_m\big) - q_a\Big)^+\right] + \alpha p_m q_a - c_m q_m \tag{A3}$$

As ϵ and ν are two random variables, their convolution is another random variable, we call it $\bar{\epsilon}$ and denote its mean, standard deviation, PDF and CDF by $\bar{\mu}$, $\bar{\sigma}$, $\bar{f}(\cdot)$ and $\bar{F}(\cdot)$, respectively, its support is $[a + a', b + b']$. We calculate the expected profits of both players given as:

$$\begin{aligned} E\big(\bar{\Pi}_w\left(y, q_a; q_m\right)\big) = {} & \delta p_w\big(\phi(y) + \bar{\mu}\big) - \delta p_w \int_{q_m - \phi(y)}^{b+b'} \Big(\phi(y) + u - q_m\Big)\bar{f}(u)du - \alpha p_m q_a - c_w y^2 \\ & - \delta p_m\big(q_m - q_a\big)\Big(1 - \bar{F}\big(q_m - \phi(y)\big)\Big) - \delta p_m \int_{q_a - \phi(y)}^{q_m - \phi(y)} \Big(u - q_a + \phi(y)\Big)\bar{f}(u)du \end{aligned} \tag{A4}$$

$$\begin{aligned} E\big(\bar{\Pi}_m\left(q_m; y, q_a\right)\big) = {} & \delta p_m\left[\int_{q_a - \phi(y)}^{q_m - \phi(y)} \Big(u - q_a + \phi(y)\Big)\bar{f}(u)du + \big(q_m - q_a\big)\Big(1 - \bar{F}\big(q_m - \phi(y)\big)\Big)\right] \\ & + \alpha p_m q_a - c_m q_m \end{aligned} \tag{A5}$$

We note that the expected profits of both players are similar to the expected profits in the base model represented by equations 4 and 5. Consequently, we write the wholesaler and manufacturer's optimization problems in the WS and MS configuration as follows:

WS model, the manufacturer's optimization problem:

$$\begin{aligned} \max_{q_m} \quad & E\big(\bar{\Pi}_m\left(q_m; y, q_a\right)\big) \\ \text{s.t.} \quad & q_m \geq q_a \\ & q_m \leq \phi(y) + b + b' \end{aligned} \tag{A6}$$

WS model, the wholesaler's optimization problem:

$$\begin{aligned} \max_{y,q_a} \quad & E\big(\bar{\Pi}_w\left(y, q_a; \hat{q_m}\right)\big) \\ \text{s.t.} \quad & 0 \leq D_{min} \leq q_a \end{aligned} \tag{A7}$$

MS model, the manufacturer's optimization problem:

$$\max_{q_m} \quad E\big(\bar{\Pi}_m\left(q_m; \hat{y}, \hat{q_a}\right)\big) \tag{A8}$$

MS model, the wholesaler's optimization problem:

$$\begin{aligned} \max_{y,q_a} \quad & E\big(\bar{\Pi}_w\left(y, q_a; q_m\right)\big) \\ \text{s.t.} \quad & 0 \leq D_{min} \leq q_a \\ & q_a \leq q_m \\ & \phi(y) + b + b' \geq q_m \end{aligned} \tag{A9}$$

We solve both the models and the equilibrium outcomes are shown in the following tables. We particularly note that the equilibrium decision outcomes of both the players are similar to the base case, and therefore the principal insights remain qualitatively unchanged under stochastic environmental conditions.

Solution case	Case $\bar{A}$	Case $\bar{B}$
Range of α	$\frac{c_m}{p_m} \leq \alpha < \bar{\alpha}_1$	$\bar{\alpha}_1 \leq \alpha \leq \delta$
y^*	$\frac{n}{2c_w}(\delta p_w - \alpha p_m)$	$\frac{n}{2c_w}(\delta p_w - \alpha p_m)$
q_a^*	$\phi(y^*) + \bar{F}^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$	$\phi(y^*) + \bar{F}^{-1}\big(1 - \frac{\alpha}{\delta}\big)$
q_m^*	$\phi(y^*) + \bar{F}^{-1}\big(1 - \frac{\alpha p_m}{\delta p_w}\big)$	$\phi(y^*) + \bar{F}^{-1}\big(1 - \frac{c_m}{\delta p_m}\big)$

where $\bar{\alpha}_1$ is the solution of the following equation:

$$c_m\Big(1 - \tfrac{p_w}{p_m}\Big)\bar{F}^{-1}\Big(1 - \tfrac{c_m}{\delta p_m}\Big) = \delta p_w\left(\int_{\bar{F}^{-1}\left(1-\frac{\alpha p_m}{\delta p_w}\right)}^{\bar{F}^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} u f(u)du\right) - \delta p_m\left(\int_{\bar{F}^{-1}\left(1-\frac{\alpha}{\delta}\right)}^{\bar{F}^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} u f(u)du\right)$$

Table A12: Equilibrium solution of the WS model with random weather shock

Solution case	Case $\bar{C}$	Case $\bar{D}$
Range of α	$\frac{c_m}{p_m} \leq \alpha < \bar{\alpha}_2$	$\bar{\alpha}_2 \leq \alpha \leq \delta$
q_m^*	Solution to $c_m - \delta p_m\big(1 - \bar{F}\big(q_m^* - \phi(y^*)\big)\big)$ $= \dfrac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m)\bar{f}\big(q_m^* - \phi(y^*)\big)$	$\phi(y^*) + b + b'$
y^*	$\frac{n\delta}{2c_w}\Big(\bar{F}\big(q_m^* - \phi(y^*)\big)(p_w - p_m) + p_m\big(1 - \frac{\alpha}{\delta}\big)\Big)$	$\frac{n\delta}{2c_w}\Big(\bar{F}\big(q_m^* - \phi(y^*)\big)(p_w - p_m) + p_m\big(1 - \frac{\alpha}{\delta}\big)\Big)$
q_a^*	$\phi(y^*) + \bar{F}^{-1}\Big(\frac{\delta - \alpha}{\delta}\Big)$	$\phi(y^*) + \bar{F}^{-1}\Big(\frac{\delta - \alpha}{\delta}\Big)$

where $\bar{\alpha}_2 = \frac{c_m}{p_m}\Big(1 + \frac{2c_w}{n^2\delta(p_w - p_m)\bar{f}(b+b')}\Big)$

Table A13: Equilibrium solution of the MS model with random weather shock

With this model extension of Advance booking in the presence of weather shock, we derive the following economic insights:

Insight 1: *Weather uncertainty changes the source of risk without altering the equilibrium structure:* We find that the equilibrium expressions under the weather uncertainty model retains the structure of the base model. This suggests that the equilibrium structure of the advance booking decision problem is robust to the source of demand uncertainty. In other words, whether the demand uncertainty arises from the stakeholder preferences and market conditions or from exogenous natural factors such as weather conditions, the model structure remains consistent with the base model.

Insight 2: *Weather uncertainty can change the optimal AB order quantity, sales effort and production quantity, even when the equilibrium structure remains unchanged:* As the weather uncertainty adds an additional source of variability to the existing market demand uncertainty, it changes the overall distribution of demand faced by the supply chain members. Thereby changing the optimal value of advance booking quantity, sales effort, and production quantity, even though the underlying two-case equilibrium structure remains unchanged. These changes are captured by the inverse CDF term $\bar{F}^{-1}(\cdot)$ in the results table. Furthermore, it also changes the threshold conditions and the parameter regions associated with each equilibrium case, denoted by $\bar{\alpha}_1$ and $\bar{\alpha}_2$.

Insight 3: *Weather uncertainty changes the risk borne by the wholesaler and manufacturer:* Weather uncertainty changes the overall risk borne by the supply chain members. This is because weather uncertainty increases the probability of mismatch between the ordered quantity and the actual demand realization.

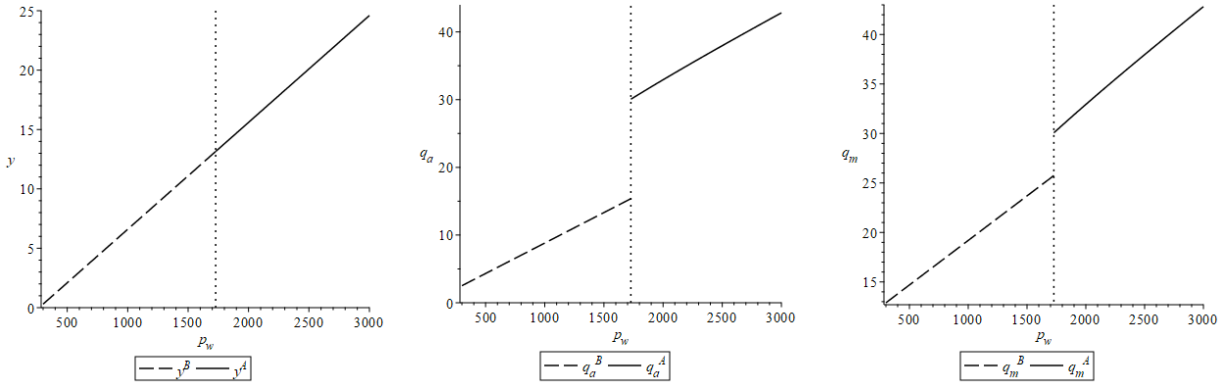
Overall we find that the weather uncertainty model not only makes our results robust but also gives us additional insights into the advance booking decision problem in agricultural input supply chains.

Appendix B – Proofs

Additional Numerical Analysis

We perform additional numerical analysis with wholesaler's selling price (p_w). We generate the grid of plots of AB quantity, manufacturing quantity, sales effort and profits by setting parameter values as $p_m = 300, \delta = 0.9, c_w = 50, b = 20, c_m = 100$ and $\alpha = 0.8$. We vary p_w between 300 to 3000 within the bounds of the problem parameters. The WS and MS plots of decision variables and profits are presented in Figures .11 and .12.

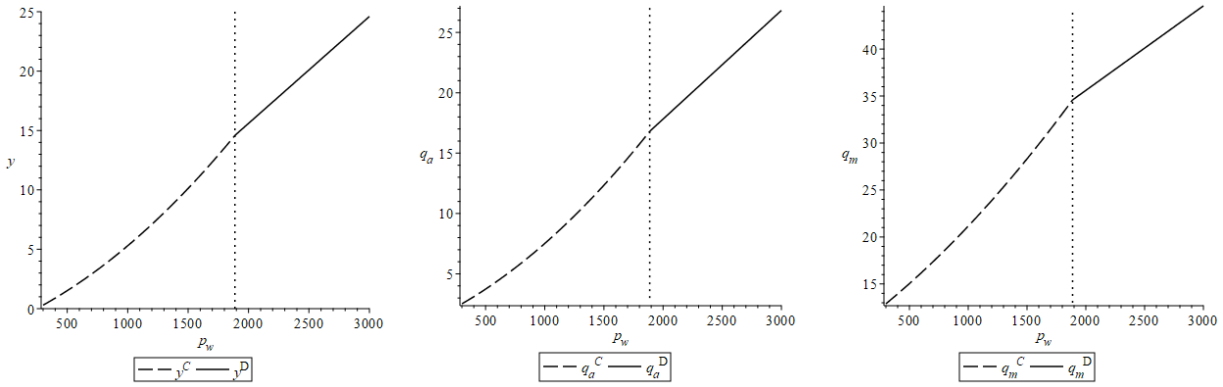
We observe that the equilibrium values of the sales effort, AB quantity and the production quantity increases with the wholesaler's selling price in both the configurations. We also find that profits of both the players and the overall supply chain increase with the wholesaler's selling price.



(a) WS model sales effort with wholesaler's selling price

(b) WS model advance booking quantity with wholesaler's selling price

(c) WS model manufacturing quantity with wholesaler's selling price

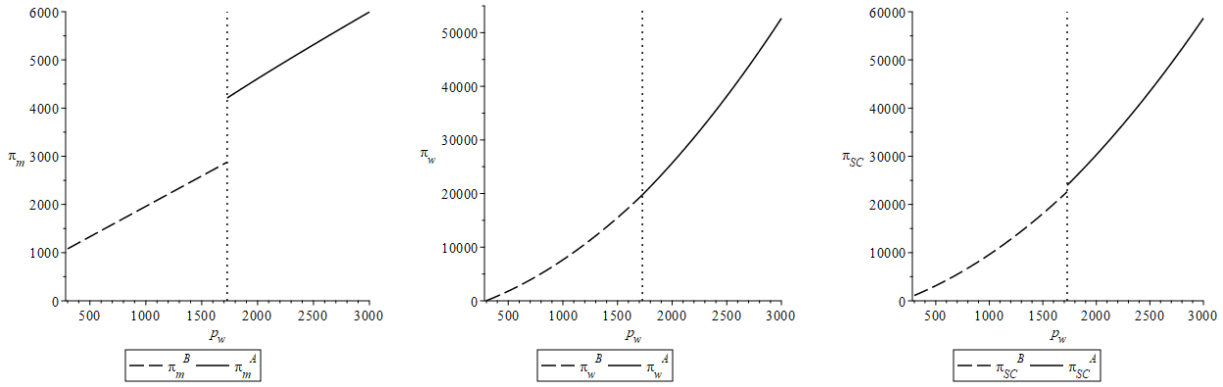


(d) MS model sales effort with wholesaler's selling price

(e) MS model advance booking quantity with wholesaler's selling price

(f) MS model manufacturing quantity with wholesaler's selling price

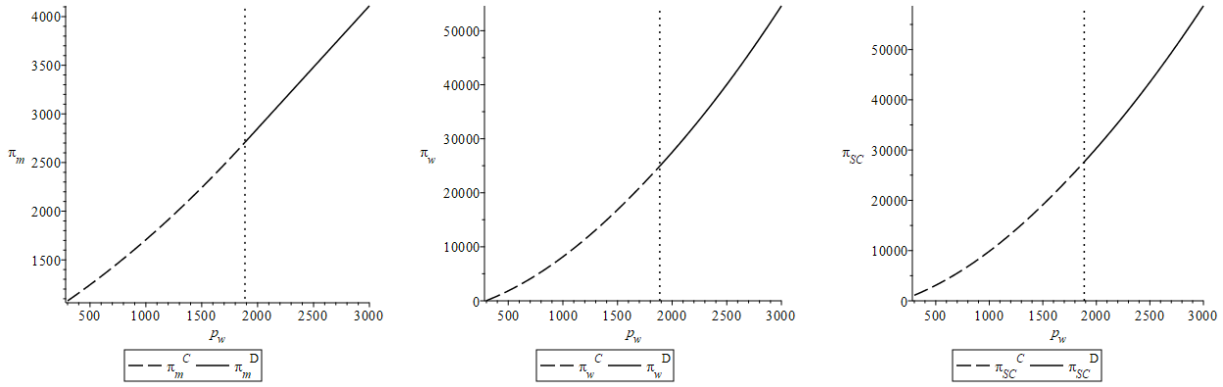
Figure .11: Plot of decision variables (AB quantity, manufacturing quantity and sales effort) with wholesaler's selling price under uniform distribution; cases A, B – WS model; cases C, D – MS model



(a) WS model manufacturer's expected profit with wholesaler's selling price

(b) WS model wholesaler's expected profit with wholesaler's selling price

(c) WS model supply chain expected profit with wholesaler's selling price



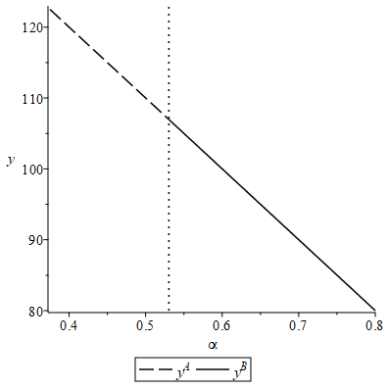
(d) MS model manufacturer's expected profit with wholesaler's selling price

(e) MS model wholesaler's expected profit with wholesaler's selling price

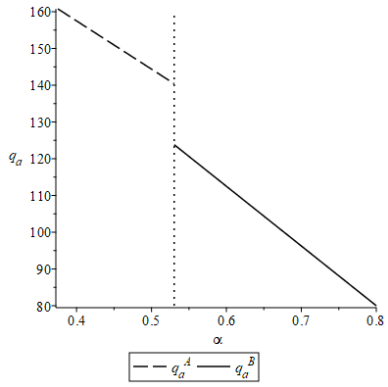
(f) MS model supply chain expected profit with wholesaler's selling price

Figure .12: Plot of expected profits of the manufacturer, wholesaler and supply chain with wholesaler's selling price under uniform distribution; cases A, B – WS model; cases C, D – MS model

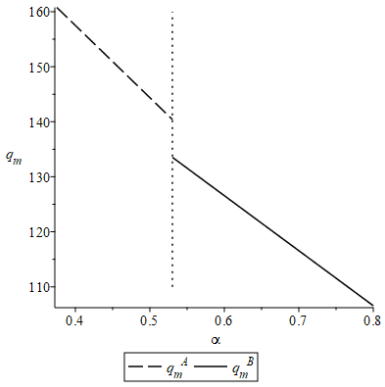
Furthermore, to check the robustness of the results, we generate plots with a separate set of parameter values under uniform distribution. We set the parameter values as: $p_m = 400$, $\delta = 0.8$, $c_w = 2$, $b = 50$, $c_m = 150$ and $p_w = 800$. We vary α in its feasible range of 0.35 to 0.8. The WS and MS plots of decision variables and profits are presented in Figures .13 and .14.



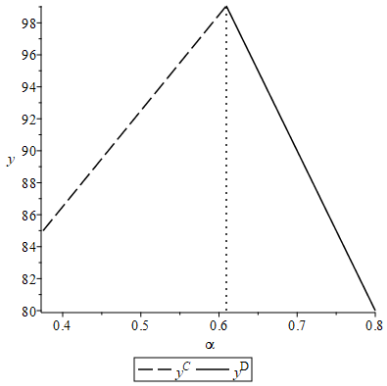
(a) WS model sales effort with discount parameter



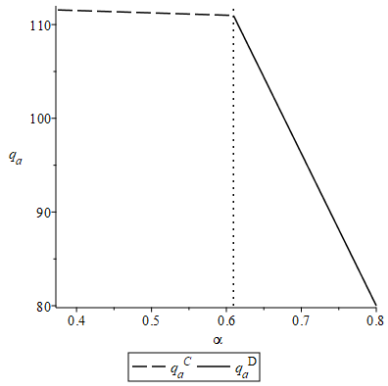
(b) WS model advance booking quantity with discount parameter



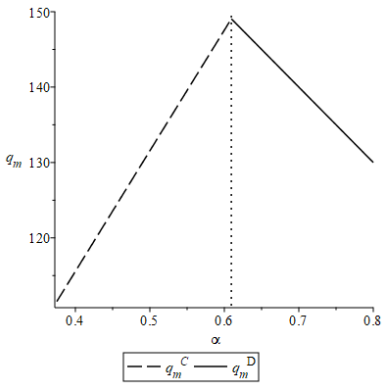
(c) WS model manufacturing quantity with discount parameter



(d) MS model sales effort with discount parameter

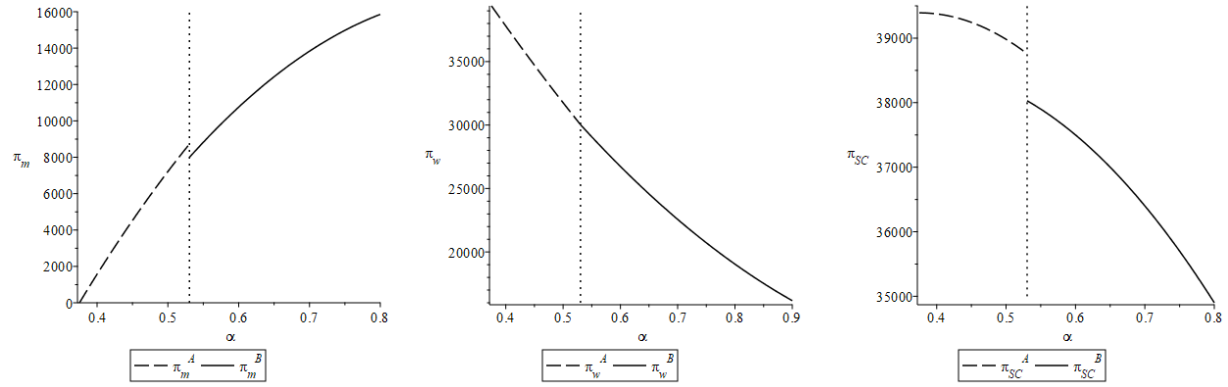


(e) MS model advance booking quantity with discount parameter



(f) MS model manufacturing quantity with discount parameter

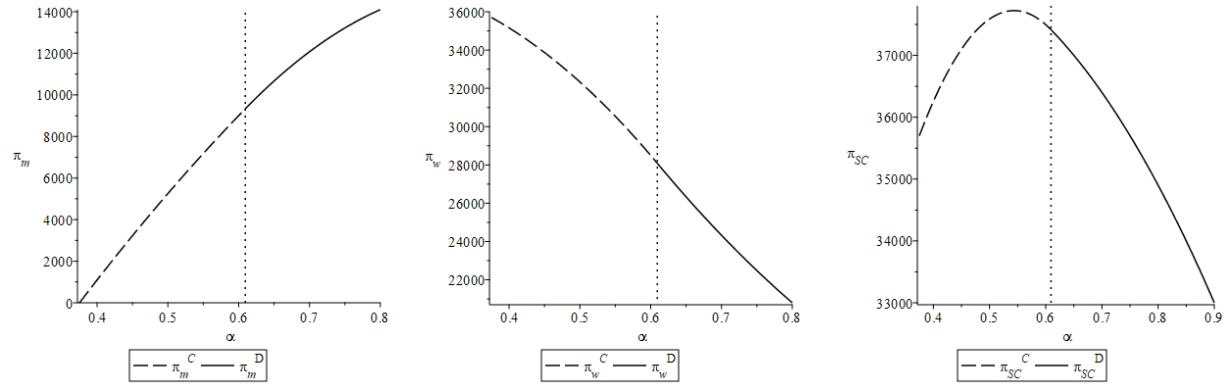
Figure .13: Additional plots of decision variables (AB quantity, manufacturing quantity and sales effort) with discount factor under uniform distribution; cases A, B – WS model; cases C, D – MS model



(a) WS model manufacturer's expected profit with discount parameter

(b) WS model wholesaler's expected profit with discount parameter

(c) WS model supply chain expected profit with discount parameter



(d) MS model manufacturer's expected profit with discount parameter

(e) MS model wholesaler's expected profit with discount parameter

(f) MS model supply chain expected profit with discount parameter

Figure .14: Plot of expected profits of the manufacturer, wholesaler and supply chain with discount factor under uniform distribution; cases A, B – WS model; cases C, D – MS model

Proof of Lemma 1.

Before solving the manufacturer and wholesaler's problems, we first consider two transformation variables namely, $z = q_a - \phi(y)$ and $w = q_m - \phi(y)$ and to simplify them further we also consider two intermediate functions namely, $\theta(z,w) = \int_z^w (u-z)f(u)du$ and $\gamma(w) = \int_w^b (w-u)f(u)du$. Substituting them above, we get the transformed manufacturer's and wholesaler's problems as follows:

$$\begin{aligned} \max_{w} \quad E\big(\Pi_m(w;y,z)\big) =& \delta p_m\Big(\theta(z,w) + (w-z)\big(1-F(w)\big)\Big) + \alpha p_m\big(z+\phi(y)\big) \\ & - c_m\big(w+\phi(y)\big) \\ \text{s.t.} \quad & w \geq z \\ & w \leq b \end{aligned} \tag{A10}$$

$$\begin{aligned}\max_{y,z} \quad E\big(\Pi_w\,(y,z;\hat{w})\big) =& \delta p_w\big(\mu+\phi(y)\big) - \alpha p_m\big(z+\phi(y)\big) - c_w y^2 - \delta p_m \theta(z,\hat{w}) + \delta p_w \gamma(\hat{w}) \\ & - \delta p_m\big(\hat{w}-z\big)\big(1-F(\hat{w})\big) \\ \text{s.t.} \quad \phi(y)+a \geq& 0 \\ z \geq& a\end{aligned} \tag{A11}$$

Employing the backward induction method, we first solve the manufacturer's optimization problem (A10) using Karush Kuhn Tucker (KKT) conditions of optimality. Before that, we check the concavity of the objective function using second derivative test and find that $\frac{\partial^2(E(\Pi_m))}{\partial w^2} = -\delta p_m f(w)$. Here the objective function is concave in w and all the constraints are convex functions (linear function) and satisfy the regularity conditions, therefore the KKT constraints are necessary and sufficient for optimality.

We obtain the following KKT conditions:

$$\begin{aligned}\delta p_m\big(1-F(w)\big) - c_m + \lambda_1 - \lambda_2 &= 0 \\ \lambda_1(w-z) &= 0 \\ \lambda_2\big(b-w\big) &= 0 \\ \lambda_1 \geq 0, \quad \lambda_2 &\geq 0\end{aligned} \tag{A12}$$

Solving these optimality conditions we find the following equilibrium values of w:

$$\hat{w} = \begin{cases} F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big), & \text{if } z \leq F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big) \leq b. \qquad CaseW1 \\ z, & \text{if } F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big) < z \leq b. \qquad CaseW2 \end{cases} \tag{A13}$$

We next solve the wholesaler's problem (A11) for cases W1 and W2 separately. For case W1, we replace the value of $\hat{w}$ by $F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big)$ and include the additional constraints of case W1, that is $z \leq F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big)$ and $b \geq F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big)$ in the wholesaler's problem. We then check the concavity of the objective function using Hessian matrix, which we find as:

$$H_{W1} = \begin{bmatrix} -\delta p_m f(z) & n\delta p_m f(z) \\ n\delta p_m f(z) & -2c_w - n^2 \delta p_m f(z) \end{bmatrix}$$

Here we find that the determinant of the Hessian matrix, $|H| = 2c_w \delta p_m f(z)$, which is non negative, also the first principal minor, $H_{11} = -\delta p_m f(z)$, which is negative. Therefore, the objective function is concave. In addition, as all the constraints are linear, they qualify the regularity conditions and therefore the following KKT conditions provide necessary and sufficient conditions for optimality:

$$
\begin{aligned}
n(\delta p_w - c_m) - 2c_w y - n\delta p_m\Big(1 - \frac{c_m}{\delta p_m} - F(z)\Big) + n(\lambda_1 - \lambda_2 + \lambda_3) &= 0 \\
c_m - \alpha p_m + \delta p_m\Big(1 - \frac{c_m}{\delta p_m} - F(z)\Big) + \lambda_2 - \lambda_3 &= 0 \\
\lambda_1\big(\phi(y) + a\big) &= 0 \\
\lambda_2(z - a) &= 0 \\
\lambda_3\bigg(F^{-1}\Big(1 - \frac{c_m}{\delta p_m}\Big) - z\bigg) &= 0 \\
\lambda_4\bigg(b - F^{-1}\Big(1 - \frac{c_m}{\delta p_m}\Big)\bigg) &= 0 \\
\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq 0, \quad \lambda_4 &\geq 0
\end{aligned}
\tag{A14}
$$

We solve the above conditions to find the following equilibrium solutions for Case W1:
$z^* = F^{-1}\Big(\frac{\delta-\alpha}{\delta}\Big)$, $y^* = \frac{n}{2c_w}(\delta p_w - \alpha p_m)$, when $\frac{c_m}{p_m} \leq \alpha \leq \delta$

Substituting the values of z and w, we get the following equilibrium solutions for case W1, which represents case B in the solution table:

$$
\begin{aligned}
q_m^B &= \phi(y) + F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big) \\
y^B &= \frac{n}{2c_w}(\delta p_w - \alpha p_m) \\
q_a^B &= \phi(y) + F^{-1}\Big(\frac{\delta - \alpha}{\delta}\Big)
\end{aligned}
\tag{A15}
$$

Similarly we construct wholesaler's problem in case W2 by replacing the value of $\hat{w} = z$ in (A11) and adding its constraints $z > F^{-1}\Big(\frac{\delta p_m - c_m}{\delta p_m}\Big)$ and $z \leq b$.

We again check the concavity of the objective function with respect to z and y, we find the following Hessian matrix:

$$
H_{W2} = \begin{bmatrix} -\delta p_w f(z) & n\delta p_w f(z) \\ n\delta p_w f(z) & -2c_w - n^2 \delta p_w f(z) \end{bmatrix}
$$

We calculate the determinant, $|H_{W2}| = 2c_w \delta p_w f(z)$, which is positive and the first principal minor is negative, therefore the objective function is concave and the constraints are linear therefore, the KKT conditions provide necessary and sufficient conditions for optimality.

For case W2, we get the following KKT conditions for optimality:

$$
\begin{aligned}
\delta p_w n\big(F(z)\big) - 2c_w y + n(\lambda_1 - \lambda_2) &= 0 \\
\delta p_w\big(1 - F(z)\big) - \alpha p_m + \lambda_2 &= 0 \\
\lambda_1\big(\phi(y) + a\big) &= 0 \\
\lambda_2(z - a) &= 0 \\
\lambda_3\big(b - z\big) &= 0 \\
\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 &\geq 0
\end{aligned}
\tag{A16}
$$

Solving the KKT conditions for case W2, we find the following equilibrium solutions:
$z^* = F^{-1}\Big(\frac{\delta p_w - \alpha p_m}{\delta p_w}\Big)$, $y^* = \frac{n}{2c_w}(\delta p_w - \alpha p_m)$, when $\alpha < \frac{c_m p_w}{p_m^2}$

Substituting the values of z and w, we get the following equilibrium solutions for case W2, which represents the case A in the solution table:

$$\begin{aligned} q_m^A = q_a^A =& \phi(y) + F^{-1}\Big(\frac{\delta p_w - \alpha p_m}{\delta p_w}\Big) \\ y^A =& \frac{n}{2c_w}(\delta p_w - \alpha p_m) \end{aligned} \tag{A17}$$

Observing the range of α where case A and case B is applicable, we find that for $\frac{c_m}{p_m} \leq \alpha < \frac{c_m p_w}{p_m^2}$, both case A and case B are applicable. Therefore to find out which among the two gives the optimal solution, we compare the wholesaler's (Stackelberg leader) expected profit in the two cases, that is, $E\big(\Pi_w(y^A, q_a^A; q_m^A)\big) = E\big(\Pi_w(y^B, q_a^B; q_m^B)\big)$. By solving this equation, we find that,

$$c_m\Big(1 - \frac{p_w}{p_m}\Big)F^{-1}\Big(1 - \frac{c_m}{\delta p_m}\Big) = \delta p_w\left(\int_{F^{-1}\left(1-\frac{\alpha p_m}{\delta p_w}\right)}^{F^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} uf(u)du\right) - \delta p_m\left(\int_{F^{-1}\left(1-\frac{\alpha}{\delta}\right)}^{F^{-1}\left(1-\frac{c_m}{\delta p_m}\right)} uf(u)du\right) \tag{A18}$$

We call the solution of the above equation to be α_1, which gives the boundary of the range of α where case A and B are applicable. ■

Proof of Lemma 3.

Similar to Lemma 1, we again use the transformation and replace $z = q_a - \phi(y)$, $w = q_m - \phi(y)$, $\theta(z,w) = \int_z^w (u-z)f(u)du$ and $\gamma(w) = \int_w^b (w-u)f(u)du$ in the expected profits. By substituting, we get the wholesaler's and manufacturer's problems as follows:

$$\begin{aligned} \max_{y,z} \quad E\big(\Pi_w(y,z;w)\big) =& \delta p_w\big(\mu + \phi(y)\big) - \alpha p_m\big(z + \phi(y)\big) - c_w y^2 - \delta p_m \theta(z,w) + \delta p_w \gamma(w) \\ & - \delta p_m\big(w - z\big)\big(1 - F(w)\big) \\ \text{s.t.} \quad \phi(y) + a \geq& 0 \\ z \geq& a \\ w - z \geq& 0 \\ b - w \geq& 0 \end{aligned} \tag{A19}$$

$$\max_{w} \quad E\big(\Pi_m(w;\hat{y},\hat{z})\big) = \delta p_m\Big(\theta(\hat{z},w) + (w-\hat{z})\big(1-F(w)\big)\Big) + \alpha p_m\big(\hat{z} + \phi(\hat{y})\big) - c_m\big(w + \phi(\hat{y})\big) \tag{A20}$$

Employing the backward induction method, we first solve the wholesaler's optimization problem, A19. We check the concavity of the optimization problem using Hessian Matrix, which we find as follows:

$$H_M = \begin{bmatrix} -\delta p_m f(z) & n\delta p_m f(z) \\ n\delta p_m f(z) & n^2\delta\big(p_m\big(f(w) - f(z)\big) - p_w f(w)\big) - 2c_w \end{bmatrix}$$

We calculate the determinant of the Hessian matrix and find that, $|H_M| = 2\delta p_m c_w f(z) + n^2\delta^2 f(z)f(w)p_m(p_w - p_m)$, which is positive and the first principal minor is negative, therefore the wholesaler's objective function is concave. Also, all the constraints are linear and satisfy regularity conditions therefore, the KKT conditions are necessary and sufficient for optimality. We write down the KKT conditions as follows:

$$
\begin{aligned}
n\delta\big(p_w F(w) - p_m\big(F(w) - F(z)\big)\big) - 2c_w y + n(\lambda_1 - \lambda_2 + \lambda_4) &= 0 \\
\delta p_m\big(1 - F(z)\big) - \alpha p_m + \lambda_2 - \lambda_3 &= 0 \\
\lambda_1\big(\phi(y) + a\big) &= 0 \\
\lambda_2(z - a) &= 0 \\
\lambda_3(w - z) &= 0 \\
\lambda_4(b - w) &= 0 \\
\lambda_1 \geq 0, \quad \lambda_2 \geq 0, \quad \lambda_3 \geq 0, \quad \lambda_4 &\geq 0
\end{aligned}
\tag{A21}
$$

Solving the above KKT conditions we find the following equilibrium values for the interior solution: $\hat{z} = F^{-1}\big(\frac{\delta-\alpha}{\delta}\big)$, $\hat{y} = \frac{n\delta}{2c_w}\big[F(w)(p_w - p_m) + p_m\big(1 - \frac{\alpha}{\delta}\big)\big]$, when $F^{-1}\big(1 - \frac{\alpha}{\delta}\big) \leq w \leq b$.

Substituting $\hat{z}$ and $\hat{y}$, in the manufacturer's problem A20, we get the manufacturer's optimization problem as follows:

$$
\begin{aligned}
\max_{w} \quad E\big(\Pi_m\left(w; \hat{y}, \hat{z}\right)\big) =& \delta p_m\Big(\theta(\hat{z}, w) + (w - \hat{z})\big(1 - F(w)\big)\Big) + \alpha p_m\big(\hat{z} + \phi(\hat{y})\big) - c_m\big(w + \phi(\hat{y})\big) \\
\text{s.t.} \quad & w \geq F^{-1}\big(1 - \frac{\alpha}{\delta}\big) \\
& w \leq b
\end{aligned}
\tag{A22}
$$

We check the concavity of the objective function using the second derivative test and find that $\frac{\partial^2(E(\Pi_m(w;\hat{z},\hat{y})))}{\partial w^2} = -\delta p_m f(w) + \frac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m)f'(w)$. Therefore for concavity, it should satisfy, $\frac{f'(w)}{f(w)} \leq \frac{2p_m c_w}{n^2(p_w - p_m)(\alpha p_m - c_m)}$. Assuming this, we write down the KKT conditions,

$$
\begin{aligned}
\delta p_m\big(1 - F(w)\big) - c_m + \frac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m) + \lambda_1 - \lambda_2 &= 0 \\
\lambda_1\Big(w - F^{-1}\big(1 - \frac{\alpha}{\delta}\big)\Big) &= 0 \\
\lambda_2\big(b - w\big) &= 0 \\
\lambda_1 \geq 0, \quad \lambda_2 &\geq 0
\end{aligned}
\tag{A23}
$$

Here also all the constraints are linear and satisfy regularity conditions, therefore KKT conditions are necessary and sufficient for optimality. We find a two case solution, however we were unable to reduce it to close form. Following is the two case solution we find:

When $\alpha \leq \frac{c_m}{p_m}\big(1 + \frac{2c_w}{n^2\delta(p_w - p_m)f(b)}\big)$, w^* can be obtained by solving $\delta p_m F(w) - \frac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m)f(w) = \delta p_m - c_m$, and when $\alpha > \frac{c_m}{p_m}\big(1 + \frac{2c_w}{n^2\delta(p_w - p_m)f(b)}\big)$, $w^* = b$.

Substituting the values of w and z, we get the following two case equilibrium solutions:

$$
\begin{aligned}
q_m^* =& \begin{cases} \text{Solution of } \left[\dfrac{n^2\delta}{2c_w}(p_w - p_m)(\alpha p_m - c_m) f\big(q_m^* - \phi(y^*)\big) \right. \\ \qquad\qquad \left. = c_m - \delta p_m\big(1 - F\big(q_m^* - \phi(y^*)\big)\big)\right] & \text{, for } 0 \le \alpha \le \alpha_2 \\ \phi(y^*) + b & \text{, for } \alpha_2 \le \alpha \le \delta \end{cases} \\
y^* =& \frac{n\delta}{2c_w}\Big(F\big(q_m^* - \phi(y^*)\big)(p_w - p_m) + p_m\big(1 - \frac{\alpha}{\delta}\big)\Big) \qquad \text{(A24)} \\
q_a^* =& \phi(y^*) + F^{-1}\Big(\frac{\delta - \alpha}{\delta}\Big) \\
\text{where, } \alpha_2 =& \frac{c_m}{p_m}\Big(1 + \frac{2c_w}{n^2\delta(p_w - p_m) f(b)}\Big)
\end{aligned}
$$

The same solution is presented in the results table. ■

Proof of Proposition 1.

Propositions 1, 2 and 3 are derived from the comparison of WS and MS model results. We divide the results into four regions (Region 1, 2L, 2H and 3) based on the range of α. We compare the results in all four regions to draw the propositions. In region 1 ($\alpha \le min(\alpha_1, \alpha_2)$), case A provides equilibrium results for WS model and case C for the MS model. In region 2L ($\alpha_1 \le \alpha \le \alpha_2$), Case B for the WS model and case C for the MS model are applicable. Similarly in region 2H ($\alpha_2 \le \alpha \le \alpha_1$), case A and case D offer equilibrium results and in region 3 ($\alpha \ge max(\alpha_1, \alpha_2)$, case B and case D provides the equilibrium results for WS and MS models respectively.

We draw this proposition by comparing the equilibrium values of sales effort in all four regions.

In region 1, we get: $y^A - y^C = \frac{(p_w - p_m)\big(2c_m c_w b - \delta(p_w - p_m)(\alpha p_m - c_m)\big)}{4p_m c_w^2 b}$

Solving it for α, we get: $y^A \ge y^C \quad if \quad \alpha \le \frac{c_m}{p_m}\Big(1 + \frac{2c_w b}{\delta(p_w - p_m)}\Big)$

However, the above threshold is same as α_2, therefore in region 1, $y^A \ge y^C, always$

In region 2L, $y^B - y^C$ needs to be evaluated, however as $y^B = y^A$, the comparison result is the same as region 1, therefore, in region 2L, $y^B \ge y^C, always$

In region 2H, we find that $y^A = y^D = \frac{\delta p_w - \alpha p_m}{2c_w}$

While in region 3, $y^B = y^D = \frac{\delta p_w - \alpha p_m}{2c_w}$

Combining the results of all four regions we conclude that, when α is relatively low (regions 1 and 2L), the wholesaler puts higher sales effort in the WS configuration while when α is relatively higher (regions 2H and 3), she puts same sales effort in both the configurations. ■

Proof of Proposition 2.

Under this proposition, we present the comparison results of the advance booking quantities under WS and MS models in different regions.

In region 1, we get:

$q_a^A - q_a^C = \frac{(p_w - p_m)\big(2\delta p_w c_m c_w b + 4\alpha p_m c_w^2 b^2 - \delta^2 p_w(\alpha p_m - c_m)(p_w - p_m)\big)}{4\delta p_w p_m c_w^2 b}$

Solving it for c_m we get:

$q_a^A \ge q_a^C \quad if \quad c_m \ge \frac{\alpha p_m}{\delta p_w}\frac{\delta^2 p_w(p_w - p_m) - 4c_w^2 b^2}{\delta(p_w - p_m) + 2c_w b}$

In region 2L,

$q_a^B - q_a^C = \frac{p_w - p_m}{4p_m c_w^2 b}\big(2c_m c_w b - \delta(p_w - p_m)(\alpha p_m - c_m)\big)$

Solving it for α we get: $q_a^B \ge q_a^C \quad if \quad \alpha \le \frac{c_m}{p_m}\Big(1 + \frac{2c_w b}{\delta(p_w - p_m)}\Big)$,

which is the same as α_2, and in this region the condition is always satisfied.

Therefore $q_a^B \geq q_a^C, always$

In region 2H, the comparison gives: $q_a^A - q_a^D = \frac{\alpha b(p_w - p_m)}{\delta p_w}$

Which is always positive, therefore, $q_a^A \geq q_a^D, always$

In region 3, we observe that $q_a^B = q_a^D = \frac{b(\delta-\alpha)}{\delta} + \frac{\delta p_w - \alpha p_m}{2c_w}$

Therefore in region 3, $q_a^B = q_a^D, always$

Combining the comparison results of all four regions, we conclude that for small values of α (region 1), the manufacturing cost decides whether q_a is higher or lower; for intermediate levels of α (regions 2L and 2H), equilibrium value of q_a is higher in WS configuration while for higher values of α (region 3), AB order quantity is same in both the configurations. ■

Proof of Proposition 3.

This proposition presents the comparison of equilibrium values of manufacturing quantity (q_m) in WS and MS configuration.

By comparing q_m in region 1, we get:

$q_m^A - q_m^C = \frac{1}{4p_m p_w \delta c_w^2 b}\Big(\delta^2(-\alpha p_m + c_m)p_w^3 + 4\delta[-\frac{1}{2}p_m(-\alpha p_m + c_m)\delta + bc_w(-\frac{1}{2}\alpha p_m + c_m)]p_w^2 + [4(-\frac{1}{2}\delta p_m + bc_w)(-\frac{1}{2}p_m(-\alpha p_m + c_m)\delta + bc_m c_w)]p_w - 4\alpha b^2 p_m^2 c_w^2\Big)$

Solving it for c_m we get:

$q_m^A \geq q_m^C \quad if \quad c_m \geq \frac{\alpha p_m}{p_w\Big(\delta(p_w - p_m) + 2c_w b\Big)^2}\Big(\delta^2 p_w(p_w - p_m)^2 + 2\delta p_w c_w b(p_w - p_m) + 4p_m c_w^2 b^2\Big)$

Comparison in region 2L gives:

$q_m^B - q_m^C = \frac{p_w - p_m}{4p_m c_w^2 b}\Big(2c_m c_w b - (\alpha p_m - c_m)\big(\delta(p_w - p_m) + 2c_w b\big)\Big)$

By solving the above, we get:

$q_m^B \geq q_m^C \quad if \quad c_m \geq \frac{\alpha p - m}{\delta(p_w - p_m) + 4c_w b}\Big(\delta(p_w - p_m) + 2c_w b\Big)$

Further comparison of q_m^* in region 2H gives:

$q_m^A - q_m^D = \frac{-\alpha b p_m}{\delta p_w}$

which is always negative, therefore, $q_m^A \leq q_m^D, always$

Finally, comparison in region 3, gives: $q_m^B - q_m^D = \frac{-bc_m}{\delta p_m}$

Which is again always negative, therefore, $q_m^B \leq q_m^D, always$

Combining the comparison results of q_m^* of all the four regions, we conclude that for relatively small values of α (regions 1 and 2L), manufacturing cost decides, whether q_m^* is higher or lower while for relatively higher values of α (regions 2H and 3) q_m^* is higher in the MS configuration. ■

Contract Analysis

We use the same solution method as outlined in the WS configuration to solve the optimization problems with returns policy considering Uniform distribution, we first solve the manufacturer's problem using the following KKT conditions:

$$\begin{aligned} \frac{\delta p_m}{b}(y + b - q_m) - c_m + \lambda_1 - \lambda_2 &= 0 \\ \lambda_1(q_m - q_a) &= 0 \\ \lambda_2(y + b - q_m) &= 0 \\ \lambda_1 \geq 0, \quad \lambda_2 &\geq 0 \end{aligned} \tag{A25}$$

Here also, we obtain a two case solution. We solve them separately after substituting the expression of equilibrium q_m. The KKT conditions obtained for the first case are as follows:

$$\begin{aligned}
\frac{\delta p_m}{b}(y+b-q_a)+\frac{\delta p_r}{b}(q_a-y)-\alpha p_m+\lambda_1-\lambda_3&=0\\
\frac{\delta(p_m-p_r)(q_a-y)}{b}+\delta(p_w-p_m)-2c_w y-\lambda_1+\lambda_2+\lambda_3&=0\\
\lambda_1(q_a-y)&=0\\
\lambda_2(y)&=0\\
\lambda_3(y+b-\frac{c_m b}{\delta p_m}-q_a)&=0\\
\lambda_1\geq 0,\quad \lambda_2\geq 0,\quad \lambda_3&\geq 0
\end{aligned} \tag{A26}$$

Similarly, we solve the second case and combine the results to get the equilibrium solutions, presented in the result table. ■

Proof of Proposition 4.

We compare the manufacturer and wholesaler profits of AB with and without the returns policy (WS configuration). As both the models have two case solutions, we compare each of cases A and B with cases A' and B' to find the scenarios when the returns policy benefits both players. We find that when $\alpha_1 \leq \alpha \leq \alpha_3$ (solution case B without returns and A' with returns), the profits of both the players are higher with returns policy when the manufacturing cost is in the given range. We also calculate the gain in expected profits of the players and find the following expressions:

$$\pi_w^{A'}-\pi_w^B=\frac{b\left(p_m^4\alpha^2+[(\alpha-2\delta)p_r-\alpha p_w]\,\alpha p_m^3+\delta^2 p_m^2 p_r p_w+c_m^2(p_w-p_r)(p_w-p_m)\right)}{2\delta p_m^2(p_w-p_r)} \tag{A27}$$

$$\pi_m^{A'}-\pi_m^B=b\Big(\frac{\alpha^2 p_m}{2\delta}-\frac{c_m^2}{2\delta p_m}+\frac{c_m(\alpha p_m-\delta p_r)}{\delta(p_w-p_r)}+\frac{\alpha p_m(2p_w-p_r)(2\delta p_r-\alpha p_m)-\delta^2 p_w^2 p_r}{2\delta(p_w-p_r)^2}\Big) \tag{A28}$$

By solving the wholesaler's profits equation A27, we find that returns policy benefits her when $c_m \geq p_m\sqrt{\frac{\alpha^2 p_m(p_w-p_m)+\alpha p_m p_r(2\delta-\alpha)-\delta^2 p_w p_r}{(p_w-p_r)(p_w-p_m)}}$. We further solve the manufacturer's profit equation A28 and find that for returns to be profitable, the manufacturing cost must lie in the range of $c_{m3} \leq c_m \leq c_{m4}$.

Combining the above intervals, we conclude that returns policy offers a win-win situation for both the players when $\alpha_1 \leq \alpha \leq \alpha_3$ and $\max(c_{m3}, c_{m5}) \leq c_m \leq c_{m4}$. Where, $c_{m3}, c_{m4} = \frac{p_m(\alpha p_m-\delta p_r)}{p_w-p_r} \mp \frac{\sqrt{p_m\left(\alpha^2 p_m\left(p_w-p_m\right)^2-\delta^2 p_w^2 p_r+p_m p_r\left(p_r(\delta-\alpha)^2+\alpha(2\delta-\alpha)(2p_w-p_m)\right)\right)}}{p_w-p_r}$ and
$c_{m5}=p_m\sqrt{\frac{\alpha^2 p_m(p_w-p_m)+\alpha p_m p_r(2\delta-\alpha)-\delta^2 p_w p_r}{(p_w-p_r)(p_w-p_m)}}$ ■