*Studies in Nonlinear Dynamics & Econometrics*

*Volume* 12, *Issue* 3 2008 *Article* 7

REGIME-SWITCHING MODELS IN ECONOMICS AND FINANCE

# Markov-Switching GARCH Modelling of Value-at-Risk

Rasoul Sajjad* Jerry Coakley† John C. Nankervis‡

*University of Essex, rsajja@essex.ac.uk
†University of Essex, jcoakley@essex.ac.uk
‡University of Essex, jcnank@essex.ac.uk

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## Abstract

This paper proposes an asymmetric Markov regime-switching (MS) GARCH model to estimate value-at-risk (VaR) for both long and short positions. This model improves on existing VaR methods by taking into account both regime change and skewness or leverage effects. The performance of our MS model and single-regime models is compared through an innovative backtesting procedure using daily data for UK and US market stock indices. The findings from exceptions and regulatory-based tests indicate the MS-GARCH specifications clearly outperform other models in estimating the VaR for both long and short FTSE positions and also do quite well for S&P positions. We conclude that ignoring skewness and regime changes has the effect of imposing larger than necessary conservative capital requirements.

# 1 Introduction

Value-at-Risk (VaR) is one of the most popular approaches to quantifying market risk. It yields an estimate of the likely losses which could rise from price changes over a pre-determined horizon at a given confidence level. It is usual that VaR is separately computed for the left and right tails of the returns distribution depending on the position of the risk managers or traders. Traders with long positions are exposed to the risk of price falls (left tail VaR) while those with short positions stand to lose when prices increase (right tail VaR). Symmetric VaR models of the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) class have difficulties in correctly modeling the tails of the returns distribution (Giot and Laurent, 2003) due to leverage effects.[1]

There are two main approaches that allow for the leverage effect in volatility forecasting. The first is the use of conditional asymmetric models which extend ARCH models by imposing an asymmetry parameter in the conditional variance equation. The second approach is based on the use of asymmetric density functions for the error term or an asymmetric confidence interval around the predicted volatility.[2] Although such approaches provide an improvement in fit compared with symmetric models, the empirical evidence suggest that the persistence in the conditional variance is likely to exhibit substantial upward bias. One potential source of bias is that the means and variances are assumed fixed rather than varying over the entire sample period (Lamoureux and Lastrapes, 1990, Timmermann, 2000, and Mikosch and Starica, 2004).

A Markov Regime-Switching (MS) approach can resolve this by endogenising changes in the data generating process. Gray (1996) extended the Hamilton (1994) MS model to the MS-GARCH framework by allowing within-regime, GARCH type heteroskedasticity. This was subsequently modified by Klaassen (2002). Marcucci (2005) compares a set of GARCH, EGARCH and GJR-GARCH models within an MS-GARCH framework (Gaussian, Student's *t* and Generalized Error Distribution for innovations) in terms of their ability to forecast S&P100 volatilities. Ane and Ureche-Rangau (2006) extend the regime-switching model developed by Gray (1996) to an Asymmetric Power (AP) GARCH model to analyze empirically Asian stock indices returns. Their empirical results indicate that all the generalizations introduced by the MS-APGARCH model are statistically and economically significant.

In this paper we introduce a MS-GARCH framework to take account of both asymmetry and regime changes in returns data in forecasting VaR. Our study

[1] This means that a negative shock leads to a higher conditional variance (volatility) in the subsequent period than a positive shock would.

[2] See Bond (2000) for a survey on early asymmetric conditional density functions.

builds upon the previous literature in several ways. First, we focus on VaR for long and short positions allowing for asymmetries in both conditional variance and distribution of error terms. This is crucially important in taking account of the leverage effect in stock markets. Existing studies such as Marcucci (2005) assume symmetric distributions and focus on long VaR only. Second, while Giot and Laurent (2003) model the long and short VaR using a single-regime APARCH model combined with the skewed Student's *t* distribution, we extend the analysis to the MS context since our results indicate that regime change matters. Third, we evaluate out-of-sample model performance by using a novel combination of exceptions and regulatory-based backtesting procedures. This provides a more robust evaluation of model performance than the in-sample analysis found in existing studies such as that of Ane and Ureche-Rangau (2006).

The rest of the paper is organized in the following way. Section 2 presents the selected volatility models implemented to model VaR for the long and short trading positions. The empirical results for model specification and diagnostic tests are presented in Section 3. In Section 4, the performance of competing models in forecasting VaR is examined. Finally, Section 5 concludes the paper.

# 2 VaR Models

This section presents the VaR models that are used to model the long and short sides of daily trading positions. The GARCH model, originally introduced by Bollerslev (1986), is the most popular model in volatility forecasting and financial risk management. We go beyond the single-regime GARCH framework and consider MS-GARCH models for modeling VaR for long and short trading positions.

## 2.1 MS-GARCH model

The standard GARCH family models are implemented to mimic the volatility clustering exhibited by most financial time series. However, they are not able to capture possible regime change in the variance process since they often entail a high volatility persistence of individual shocks (see Lamoureux and Lastrapes, 1990, and Timmermann, 2000). One possible way of modeling changing volatility persistence is to combine MS models, as introduced in Hamilton (1994), with GARCH type models in which volatility persistence can take different values depending on whether it is in a high or low volatility regime (state).

Let $s_t$ be a random variable that can assume only integer values {1, 2… M}. Then, an M-state Markov chain with transition probabilities $p_{ij}$ is a process in which the probability that unobserved $s_t$ equals some particular value $j$ depends on the past only through the most recent value $s_{t-1}$. That is

$$\Pr\{s_t = i \mid s_{t-1} = j, s_{t-2} = k, \ldots\} = \Pr\{s_t = i \mid s_{t-1} = j\} = p_{ij}, \quad \text{for } i, j = 1,2,\ldots, M. \tag{2.1}$$

A $M \times M$ matrix *P*, known as the transition matrix, contains transition probabilities $p_{ij}$ giving the probability that state *i* will be followed by state *j*. Then the dynamics of returns is given by:

$$\begin{aligned} \phi_{s_t}(L)(r_t - \gamma_{s_t}) &= \theta_{s_t}(L)\varepsilon_t, \\ \varepsilon_t &= h_{t,s_t}^{1/2} Z_t \end{aligned} \tag{2.2}$$

where the innovations $Z_t$ are *i.i.d.* with zero mean, unit variance and marginal density function $f_z(Z)$ and *L* is the lag operator. The conditional variance $h_t$ can be defined either by:

$$\beta_{s_t}(L) h_{t,s_t}^{\eta_{s_t}/2} = \omega_{s_t} + [\beta_{s_t}(L) - \varphi_{s_t}(L)](1 + \lambda_{s_t} S_t) \mid \varepsilon_t \mid^{\eta_{s_t}}, \tag{2.3}$$

giving the ARCH class of models, or by:

$$\beta_{s_t}(L) \log h_{t,s_t} = \omega_{s_t} + [\beta_{s_t}(L) - \varphi_{s_t}(L)](h_{t,s_t}^{-1/2}(\mid \varepsilon_t \mid + \lambda_{s_t} \varepsilon_{tt})), \tag{2.4}$$

giving the EGARCH class, where $\eta > 0$ denotes the power parameter, $s_t = 1$ if $\varepsilon_t < 0$ and 0 otherwise, and $\lambda$ is the *leverage* or *asymmetry* parameter.

The usual ARCH (Engle, 1982) and GARCH (Bollerslev, 1986) models are obtained if in equation (2.3) only the parameters $\omega$, $\beta(L)$ and $\varphi(L)$ are included and the power parameter $\eta$ is fixed at 2. Alternatively, the power parameter $\eta$ can be estimated rather than imposed as 2, yielding the Power ARCH (or PARCH) class proposed by Ding et al. (1993). When $\eta$ is fixed at 2, equation (2.3) gives a variant of the Threshold GARCH or GJR-GARCH as introduced independently by Zakoïan (1994) and Glosten et al. (1993). Finally, equation (2.4) represents the Exponential GARCH model (Nelson, 1991).

Since $s_t$ is an unobserved variable, the conditional variance in (2.3) and (2.4) depends on the entire sequence of regimes up to time *t*. This means that, for a sample of length *T*, the likelihood function requires integrating over $M^T$ sequences of (unobserved) regime paths rendering the model essentially intractable and practically impossible to estimate. This problem is known as path dependence in MS-GARCH models. Gray (1996) and Klaassen (2002) suggest using the conditional expectation of the lagged variance as a proxy for lagged variance. In other words, the conditional variance of lagged $\varepsilon_t$ is composed of all component variances as well as the time-varying conditional regime probabilities.[3]

The probability that the observed regime at time *t* is *j* evolves according to the filtering (updating) equation:

$$\Pr\{s_t = j \mid \psi_t\} = \frac{f(r_t \mid s_t = j, \psi_{t-1}; \Lambda).\Pr\{s_t = j \mid \psi_{t-1}\}}{\sum_{i=1}^{M} f(r_t \mid s_t = i, \psi_{t-1}; \Lambda).\Pr\{s_t = i \mid \psi_{t-1}\}}. \tag{2.5}$$

where $f(r_t \mid s_t = j, \psi_{t-1}; \Lambda)$ denotes the (conditional) probability density of the return at time *t* conditional on $\psi_{t-1}$ and when regime *j* is operating. The vector $\Lambda$ comprises parameters in the conditional mean and variance equations and parameters characterizing the conditional density distribution. Then the maximum likelihood estimate of $\Lambda$ is obtained by maximizing

$$L(\theta) = \sum_{t=1}^{T} f(r_t \mid \psi_{t-1}; \Lambda),$$

where

$$f(r_t \mid \psi_{t-1}; \Lambda) = \sum_{j=1}^{M} f(r_t \mid s_t = j, \psi_{t-1}; \Lambda).\Pr\{s_t = j \mid \psi_{t-1}\}. \tag{2.6}$$

The key probability in (2.5) and (2.6) has a first-order recursive structure which can be written as

$$\Pr\{s_t = j \mid \psi_{t-1}\} = \sum_{i=1}^{M} \Pr\{s_{t-1} = i \mid \psi_{t-1}\} \Pr\{s_t = j \mid s_{t-1} = i, \psi_{t-1}\}. \tag{2.7}$$

## 2.2 Long and short VaR

Suppose that, at the time index *t*, we are interested in the risk of a financial position for the next $\ell$ periods. The $\Delta V(\ell)$, being the change in value of the asset(s) in the financial position from time t to $t+\ell$, is a random variable at *t*. The VaR of a long position (left tail of the distribution function) over the time horizon

[3] Haas et al. (2004) present a new MS-GARCH model to overcome the path dependence problem. In their model the regime variances only depend on past shocks and their own lagged values.

$\ell$ with probability $p$ is defined as

$$p = \Pr[\Delta V(\ell) \leq VaR] = F_{\ell}(VaR), \tag{2.8}$$

where $F(x)$ denotes the cumulative distribution function, CDF, of $\Delta V(\ell)$. Alternatively,

$$VaR_{\ell}^{p} = F_{\ell}^{-1}(p), \tag{2.9}$$

where $F^{-1}$ is the so-called *quantile function* defined as the inverse of the CDF. The VaR for a short position is similarly computed where the same definition is used for the right tail of the distribution function, i.e. *1-p* substitutes for *p.*[4]

Since quantiles are direct functions of the variance in parametric models, the ARCH class models present a dynamic measure of VaR. More precisely, the VaR for time *T+1* based on the ARCH family models can be defined as

$$VaR_{T+1}^{p}(r) = \mu_{T+1} + h_{T+1}^{1/2} F_{p}^{-1}(z), \tag{2.10}$$

where $F_p^{-1}(z)$ denotes the p*th* quantile of the distribution of variance-adjusted residuals in (2.2). $\mu_{t+1}$ and $h_{t+1}$ are one-step forecasts of the conditional mean and conditional variance, respectively. Equation (2.10) shows that the conditional variance at time *T+1* and the distribution chosen for *F*, the innovations in (2.2), directly affect the level of the $VaR_{T+1}^{p}(r)$ measure.

The empirical evidence suggests that it is crucial to consider the leverage effect in forecasting stock market volatilities (see Nelson, 1991, among others). An asymmetric response of VaR to positive and negative shocks can be modeled in two ways: imposing an asymmetric parameter in the conditional variance equation or imposing a skewness parameter in the distribution of error term. While the latter approach leads to an asymmetric quantile, $F_p^{-1}$, the former leads to a differential response of the conditional variance, $\sigma_{T+1}$, to bad and good news. The skewed Student's *t* distribution can be used to obtain an asymmetric quantile in modeling VaR for long and short positions.

Lambert and Laurent (2001) show that the quantile function with such a density is:

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[4] See, for example, Dowd (2005) for a comprehensive survey of VaR methods.

$$SkSt^*_{p,\zeta,\nu} = \begin{cases} \frac{1}{\zeta} St_{p,\nu}[\frac{p}{2}(1+\zeta^2)] & if \quad p < \frac{1}{1+\zeta^2} \\ -\zeta \; St_{p,\nu}[\frac{1-p}{2}(1+\zeta^{-2})] & if \quad p \geq \frac{1}{1+\zeta^2} \end{cases} \tag{2.11}$$

where $\zeta$ is the asymmetry coefficient and $St_{p,\nu}$ is the quantile function of the (unit variance) Student's $t$ density with $\nu$ degrees of freedom. Then the associated quantile function is obtained from

$$SkSt_{p,\zeta,\nu} = \frac{SkSt^*_{p,\zeta,\nu} - m}{s}, \tag{2.12}$$

where parameters $m$ and $s^2$ are the mean and the variance of the non-standardized skewed Student's $t$, respectively:

$$m = \frac{\Gamma(\frac{\nu-1}{2})\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\nu/2)}(\zeta - \frac{1}{\zeta}), \qquad s^2 = (\zeta^2 + \frac{1}{\zeta^2} - 1) - m^2 .$$

For skewed Student's $t$ innovations, $F^{-1}$ in (2.10) for long and short positions is given by $SkSt_{p,\nu,\zeta}$ and $SkSt_{1-p,\nu,\zeta}$ indicating $p$% quantiles, with $\nu$ degrees of freedom and asymmetry coefficient $\zeta$, on the left and right tails, respectively. If $\zeta < 1$ (or $\log(\zeta) < 0$), $|SkSt_{p,\nu,\zeta}| > |SkSt_{1-p,\nu,\zeta}|$ and the VaR for long trading positions will be larger (for the same conditional variance) than the VaR for short trading positions. The opposite result holds when $\zeta > 1$.

# 3 Empirical Findings

The class of model developed in the previous section enables us to consider excess kurtosis and skewness as well as possible structural changes exhibited by most financial time series data. In the following subsections we implement this model to examine stock market behavior.

## 3.1 Data

The data set analyzed in this paper comprises daily observations on two major stock market indices returns, namely, the FTSE100 and the S&P 500 (hereafter the FTSE and S&P). The sample covers the time interval from 1 January 1991 to 31 December 2004, resulting in 3599 daily observations.[5] The first 2599 observations are used for in-sample estimation while the remaining 1000 observations are taken as the out-of- sample for forecast evaluation process in which a sliding window (rolling) method of 1000 days is implemented. The usual descriptive statistics of the data are given in Table 1. The moments of the stock index returns are shown along with the results of an aggregate autocorrelation (Ljung-Box) test for returns and their squares.

Table 1: Moments of the FTSE100 and S&P500 returns along with aggregate autocorrelation test results.

| Moments specifications | | | | Ljung-Box test: Returns | | Ljung-Box test: Squared returns | |
|---|---|---|---|---|---|---|---|
| | FTSE | S&P | | FTSE | S&P | FTSE | S&P |
| Mean | 0.010 | 0.016 | Q-stat(12) | 55.76 | 15.28 | 1273 | 633 |
| Minimum | -4.654 | -3.914 | | {0} | {0.23} | {0} | {0} |
| Maximum | 2.419 | 2.420 | Q-stat(24) | 81.47 | 32.94 | 1768 | 888 |
| Std. Dev. | 0.479 | 0.492 | | {0} | {0.11} | {0} | {0} |
| Skewness | -0.371 | -0.237 | Q-stat(36) | 91.95 | 53.63 | 211 | 1049 |
| Kurtosis | 7.720 | 7.003 | | {0} | {0.03} | {0} | {0} |

*p*-values in curly braces.
Q-stat (*q*) denotes a modified Ljung-Box type statistic, which combines the first *q* squared normalized autocorrelation estimates.

As can be seen, for both indices, the mean return is quite small, the skewness is significant and negative, implying a possible leverage effect in data, and the kurtosis is significantly higher than that of a Gaussian distribution (excess kurtosis) indicating fat-tailed returns. This suggests the need for a fat-tailed or skewed fat-tailed distribution, for example Student's *t* or skewed Student's *t*, to describe the returns' conditional distribution. In addition, the large Q-statistics up to 12, 24 and 36 orders strongly reject the null hypothesis of no serial correlation in both returns and squared returns for the FTSE but only in squared returns for the S&P index.[6]

We also present, in Figure 1, squared returns, $r_t^2$, for the last four years in order to give an indication of high and low volatility periods. At first glance, plots demonstrate substantial volatility clustering as periods of low volatility mix with periods of high volatility and large positive and negative returns. This indicates

[5] All data have been obtained from DataStream.
[6] Other test statistics like: the BDS, McLeod–Li, Engle LM, Tsay and Bicovariance tests can be used to examine whether the residuals are *i.i.d.* (see for example Panagiotidis, 2005).

the potential benefits of allowing for conditional heteroskedasticity.[7] More precisely, the plots demonstrate two crucial points: the pressure relieving effect and volatility persistence (clustering). The latter implies that individual shocks sometimes have a long effect on subsequent volatility, while the former implies that a shock sometimes is followed by a period of low instead of high volatility.

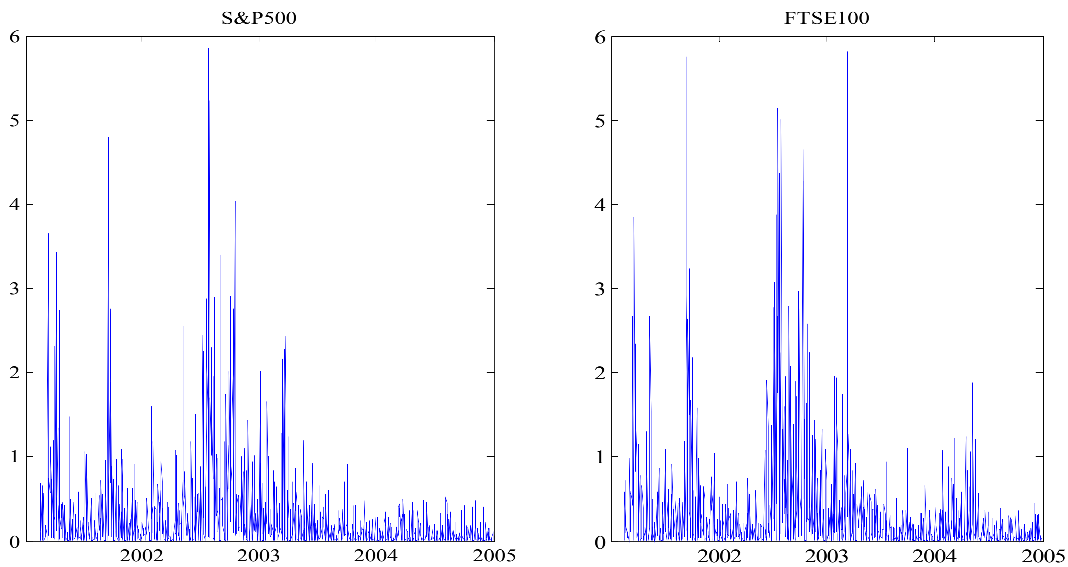


Figure 1: Squared returns of the S&P500 and FTSE100 rates over the sample period January 2001 to December 2004.

It seems that the standard single-regime ARCH models cannot capture the pressure relieving effect, since they typically imply large persistence for individual shocks. However, it is possible that regime-switching models allowing for a switch from a high to a low volatility regime can explain both the large volatility persistence of individual shocks and the pressure relieving effect. Furthermore, imposing an asymmetric parameter in the conditional variance equation and/or distributional form of error terms enables us to capture possible leverage effects in the data. The following subsection examines these issues.

[7] The formal test results for conditional heteroskedasticity are also available upon request.

## 3.2 Specification and diagnostic results

We estimate all models using the Maximum Likelihood (ML) method, assuming normal (N), Student's *t* (t) and skewed Student's *t* (skt) innovations.[8] Table 2 summarizes the estimation results from the single-regime models. The *p*-values are associated with critical values corrected for skewness using a wild bootstrap simulation.[9] Using the Likelihood Ratio (LR) test and the Schwarz criterion,[10] we try different orders of the ARMA process for the conditional mean equations (not shown here) and conclude that no dynamics is preferred to model the conditional mean in our data (only a constant is included). The results also indicate that the asymmetry parameters in both the conditional variance and the skewed Student's *t* innovations of the EGARCH specification are highly significant for both indices. These results resemble those of Marcucci (2005) in modeling the S&P100 returns. However, the asymmetry parameters are insignificant for both the FTSE and the S&P returns with GARCH specifications. In other words, it seems that imposing an asymmetry parameter in either the conditional variance equation or the Student's *t* distribution of the GARCH specification is unnecessary fully to model the dynamics of our return series.

**Regime-switching models**

The ML estimation results assuming normal and Student's *t* innovations for the FTSE and the S&P returns are respectively shown in Tables 3-6. In order to find out the appropriate MS structure for the returns series, three different MS models are analyzed: a partial MS model with no dynamics in the mean equation, a partial MS and a full MS model with appropriate dynamics (preferred dynamics as indicated by the single-regime models) in the mean equation. In the full MS model all parameters in the mean and variance equations are allowed to switch between regimes while in the partial MS model only the variance equation parameters differ across regimes.

According to the information criterion and the LR test, the null of the partial MS structure can be rejected in favor of the full MS structure for the S&P returns with Student's *t* innovations. However, allowing the mean equation's parameters to differ across regimes (full MS model) results in some insignificant parameters in the mean or variance equations. Consequently, it seems the partial MS model in conjunction with the ARMA (1, 1) structure for the mean equation, with either

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[8] All estimations are performed in the TSMod package developed by James Davidson (see http://www.timeseriesmodelling.com/)
[9] See Arghyrou and Gregoriuo (2007) for a very recent application of the wild bootstrap technique.
[10] We use the Schwarz Criterion, which provides consistent order-estimation in the context of linear ARMA models (see Hannan, 1980).

Table 2: ML estimation results from the single-regime models.

| ARCH specifications | FTSE100 | | | S&P500 | | |
|---|---|---|---|---|---|---|
| | *N* | *t* | *skt* | *N* | *t* | *skt* |
| ***Variance parameters:*** | | | | | | |
| GARCH Intercept: $\omega^{1/2}$ | 0.044 | 0.072 | 0.072 | 0.059 | 0.049 | 0.048 |
| | [0.012] | [0.018] | [0.018] | [0.020] | [0.019] | [0.021] |
| ARCH term: $\alpha$ | 0.054 | 0.047 | 0.047 | 0.072 | 0.045 | 0.044 |
| | {0} | {0} | {0.002} | {0} | {0.001} | {0.111} |
| GARCH term: $\beta$ | 0.954 | 0.949 | 0.949 | 0.935 | 0.949 | 0.948 |
| | {0} | {0} | {0.001} | {0} | {0} | {0} |
| GARCH asymmetry: $\lambda$ | -0.198 | 0.129 | 0.132 | 0.133 | 0.576 | 0.655 |
| | {0.283} | {0.515) | {0.193} | {0.205} | {0.194} | {0.076} |
| Power GARCH term: $\eta$ | 1.451 | 1.117 | 1.120 | 1.148 | 1.395 | 1.442 |
| | [0.312] | [0.209] | [0.212] | [0.307] | [0.508] | [0.585] |
| Log skewness term: $Ln(\zeta)$ | | | -0.008 | | | -0.047 |
| | | | {0.504} | | | {0.452} |
| Student's *t* d. f. | | 6.492 | 6.508 | | 5.165 | 5.309 |
| ***Log Likelihood:*** | -1439.11 | -1368.96 | -1368.92 | -1487.23 | -1388.24 | -1386.78 |
| ***Ljung-Box Q(12):*** | 19.1 | 19.2 | 19.2 | 6.8 | 8.1 | 7.9 |
| | {0.085} | {0.083} | {0.083} | {0.872} | {0.778} | {0.793} |

| EGARCH specifications | FTSE100 | | | S&P500 | | |
|---|---|---|---|---|---|---|
| | *N* | *t* | *skt* | *N* | *t* | *skt* |
| ***Variance parameters:*** | | | | | | |
| EGARCH Intercept: $\omega^{1/2}$ | 3.686 | 3.744 | 3.743 | 3.439 | 3.447 | 3.441 |
| | [0.63] | [0.60] | [0.60] | [0.57] | [0.48] | [0.48] |
| ARCH term: $\alpha$ | 0.063 | 0.062 | 0.062 | 0.140 | 0.126 | 0.127 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| GARCH term: $\beta$ | 0.976 | 0.976 | 0.976 | 0.941 | 0.949 | 0.948 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| EGARCH asymmetry: $\lambda$ | -0.293 | -0.593 | -0.607 | -0.196 | -0.314 | -0.354 |
| | {0} | {0} | {0} | {0} | {0} | {0.045} |
| Log skewness term: $Ln(\zeta)$ | | | -0.021 | | | -0.053 |
| | | | {0} | | | {0} |
| Student's t d. f. | | 6.274 | 6.306 | | 4.684 | 4.773 |
| ***Log Likelihood:*** | -1465.53 | -1380.85 | -1380.58 | -1550.04 | -1437.97 | -1436.12 |
| ***Ljung-Box Q(12):*** | 24.987 | 22.63 | 22.48 | 21.346 | 23.60 | 23.12 |
| | {0.015} | {0.031} | {0.032} | {0.046} | {0.023} | {0.027} |

Standard errors are given in square brackets and *p*-values in curly braces.
The ML results assuming Normal, Student's *t* and skewed Student's *t* for the error terms are presented in the columns labeled *N*, *t* and *skt*, respectively. The upper part of the table reports the estimated coefficients for the ARCH specifications, while the lower part reports those for the EGARCH specifications. Q(12) indicates the aggregate autocorrelation test for the squared normalized residuals up to lag 12.

normal or Student's $t$, can be considered as the appropriate dynamics for the S&P returns, while no dynamics is preferred for the FTSE returns. We note that Marcucci (2005) and Ane and Ureche-Rangau (2006) do not compare the partial and full MS and instead use the full MS structure in their studies. By contrast, Haas et al. (2005) implement the partial MS structure in analyzing foreign exchange data.

Table 3: ML estimation results from the two-regime GARCH models with normal innovations for the FTSE100 returns.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***Mean parameters:*** | | | | | | |
| Intercept: $\gamma_s$ | 0.020 | | 0.020 | | -0.035 | 0.027 |
| | {0.006} | | {0.010} | | {0.457} | {0.003} |
| AR1: $\theta_s$ | | | 0.045 | | 0.084 | 0.041 |
| | | | {0.015} | | {0.601} | {0.063} |
| ***Variance parameters:*** | | | | | | |
| GARCH Intercept: $\omega_s^{1/2}$ | 0.048 | 0.000 | 0.049 | 0.000 | 0.048 | 0.000 |
| | [0.009] | [0] | [0.009] | [0] | [0.009] | [0] |
| ARCH term: $\alpha_s$ | 0.029 | 0.083 | 0.029 | 0.096 | 0.030 | 0.100 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| GARCH term: $\beta_s$ | 0.939 | 0.971 | 0.939 | 0.967 | 0.939 | 0.966 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| Stay probabilities: $(p_{11}, p_{22})$ | 0.817 | 0.001 | 0.823 | 0.001 | 0.827 | 0.001 |
| ***Test summary:*** | | | | | | |
| Log Likelihood | -1375.44 | | -1373.25 | | -1372.29 | |
| Schwarz Criterion | **-1406.86** | | -1408.60 | | -1419.42 | |
| Ljung-Box Q(12) | 16.673 | | 15.879 | | 16.986 | |
| | {0.162} | | {0.197} | | {0.15} | |

Standard errors are given in square brackets and $p$-values in curly braces.
The upper and middle parts of the table report the estimated coefficients for the mean and variance equations. In each pair of columns, left and right columns report parameter estimates for regime 1 and 2, respectively. Non-switching parameters are reported under regime 1 (left columns). The lower part reports diagnostic test results, where the second row shows the increase in the Log-likelihood value compared to the one for the corresponding single-regime model. For each of the criteria, boldface entries indicate the best regime switching model for the particular criterion. Q (12) indicates the aggregate autocorrelation test for the squared normalized residuals up to lag 12.

Tables 3-6 also present the results of the Q-test to test for autocorrelation in the standardized residuals. The $p$-values are high for both series, indicating that the models are rich enough to remove all traces of autocorrelation in the normalized residuals up to lag 12. Finally, our results (available upon request) indicate that the skewness parameter in the skewed Student's $t$ distribution is insignificant for both series, similar to the single-regime GARCH models. These results contrast with those of Ane and Ureche-Rangau (2006) who find significant asymmetry in an MS-APARCH specification for different (Asian) stock market indices.

Table 4: ML estimation results from the two-regime GARCH models with Student's *t* innovations for the FTSE100 returns.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***Mean parameters:*** | | | | | | |
| Intercept: $\gamma_s$ | 0.021 | | 0.021 | | -0.015 | 0.022 |
| | {0.004} | | {0.006} | | {0.007} | {0.005} |
| AR1: $\theta_s$ | | | 0.039 | | -0.918 | 0.051 |
| | | | {0.031} | | {0} | {0.006} |
| ***Variance parameters:*** | | | | | | |
| GARCH Intercept: $\omega_s^{1/2}$ | 0.038 | 0.092 | 0.039 | 0.093 | 0.009 | 0.044 |
| | [0.009] | [0.033] | [0.008] | [0.022] | [0.027] | [0.010] |
| ARCH term: $\alpha_s$ | 0.007 | 0.043 | 0.007 | 0.044 | 0.001 | 0.034 |
| | {0} | {0} | {0.006} | {0.013} | {0.444} | {0} |
| GARCH term: $\beta_s$ | 0.980 | 0.927 | 0.979 | 0.925 | 0.911 | 0.956 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| Student's *t* d. f. | 6.896 | | 6.863 | | 5.331 | |
| Stay probabilities: $(p_{11}, p_{22})$ | 0.998 | 0.999 | 0.998 | 0.999 | 0.686 | 0.993 |
| ***Test summary:*** | | | | | | |
| Log Likelihood | -1366.66 | | -1364.77 | | -1363.12 | |
| Schwarz Criterion | **-1402.01** | | -1407.98 | | -1410.25 | |
| Ljung-Box Q(12) | 15.024 | | 15.453 | | 17.791 | |
| | {0.24} | | 0.218} | | {0.122} | |

See the legend of Table 3 for explanations.

## 3.3 Comparing single-regime and MS models

Unlike single-regime models, MS-GARCH models distinguish two sources of volatility persistence to capture the clustering of large changes as well as the pressure relieving effect (see Klaassen, 2002). These are within-regime volatility persistence with different unconditional variances and regime shifts with different periods of persistence (different regime persistence). High (within-regime) volatility persistence manifests itself in a significant large persistence term, as measured by the sum $\hat{\alpha} + \hat{\beta}$, similar to the single-regime model. This implies that the effect of an individual shock takes a long time to dissipate. The persistence of regimes can be illustrated by $p_{11}$ and $p_{22}$ which are usually referred to as the staying probabilities of regimes. The expected duration of regimes is also utilized to get a better idea about regime persistence.

Table 5: ML estimation results from the two-regime GARCH models with normal innovations for the S&P500 returns.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***Mean parameters:*** | | | | | | |
| Intercept: $\gamma_s$ | 0.031 | | 0.032 | | -0.381 | 0.044 |
| | {0} | | {0} | | {0.546} | {0} |
| AR1: $\theta_s$ | | | 0.795 | | 0.752 | 0.757 |
| | | | {0} | | {0.058} | {0} |
| MA1: $\phi_s$ | | | 0.839 | | 0.320 | 0.822 |
| | | | {0} | | {0} | {0} |
| ***Variance parameters:*** | | | | | | |
| GARCH Intercept: $\omega_s^{1/2}$ | 0.019 | 0.078 | 0.016 | 0.096 | 0.000 | 0.021 |
| | [0.011] | [0.070] | [0.012] | [0.065] | [0] | [0.007] |
| ARCH term: $\alpha_s$ | 0.027 | 0.271 | 0.027 | 0.330 | 0.130 | 0.034 |
| | {0.024} | {0} | {0.006} | {0} | {0.506} | {0} |
| GARCH term: $\beta_s$ | 0.953 | 0.896 | 0.954 | 0.870 | 0.942 | 0.946 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| Stay probabilities: $(p_{11}, p_{22})$ | 0.775 | 0.117 | 0.771 | 0.103 | 0.049 | 0.837 |
| ***Test summary:*** | | | | | | |
| Log Likelihood | -1404.84 | | -1396.51 | | -1387.38 | |
| Schwarz Criterion | -1440.23 | | **-1439.76** | | -1442.43 | |
| Ljung-Box Q(12) | 8.96 | | 9.567 | | 10.675 | |
| | {0.706} | | {0.654} | | {0.557} | |

See the legend of Table 3 for explanations.

Table 7 presents the unconditional probabilities, $\pi^j$, the expected durations, $\delta_j$, and the unconditional variances, $E\sigma_{jt}^2$, for the preferred MS-GARCH models. Assuming normal or Student's *t* innovations leads to different regime-switching structures in our data. The first column of Table 7 assumes normal innovations. The unconditional probability, $\pi^1$, of being in the first (lower volatility) regime is 85% and 80% with expected duration of 6 and 4 trading days for the FTSE and S&P returns, respectively. The unconditional probability of being in the second (high-volatility) regime is 15% and 20% for the FTSE and S&P, respectively, with an expected duration of around one day for both series. Thus the low volatility periods are generally longer lasting. This is known as the mean-reverting phenomenon and is first addressed by Dueker (1997) in equity markets.

This result is consistent with Tables 3 and 5 where $\hat{\alpha}_2 + \hat{\beta}_2 > 1$ (1.05 for the FTSE and 1.17 for the S&P returns) indicates that the process is non-stationary in high volatility periods. However, the probability of staying in this non-stable regime ($p_{22}$) is small for both series. Consequently, as noted by Yang (2000), the process endogenously collapses back from its explosive state en route to a stable regime and is stationary in the long run.

Table 6: ML estimation results from the two-regime GARCH models with Student's *t* innovations for the S&P500 returns.

| | | | | | | |
|---|---|---|---|---|---|---|
| ***Mean parameters:*** | | | | | | |
| Intercept: $\gamma_s$ | 0.031 | | 0.032 | | 0.052 | -2.941 |
| | {0} | | {0} | | {0} | {)} |
| AR1: $\theta_s$ | | | 0.795 | | 0.760 | 0.953 |
| | | | {0} | | {0} | {0} |
| MA1: $\phi_s$ | | | 0.837 | | 0.830 | 0.526 |
| | | | {0} | | {0} | {0.004} |
| ***Variance parameters:*** | | | | | | |
| GARCH Intercept: $\omega_s^{1/2}$ | 0.050 | 0.098 | 0.049 | 0.097 | 0.015 | 0.017 |
| | [0.014] | [0.016] | [0.013] | [0.015] | [0.012] | [0.099] |
| ARCH term: $\alpha_s$ | 0.039 | 0.043 | 0.038 | 0.042 | 0.048 | 0.014 |
| | {0.005} | {0} | {0.005} | {0} | {0} | {0.527} |
| GARCH term: $\beta_s$ | 0.926 | 0.926 | 0.927 | 0.929 | 0.946 | 0.972 |
| | {0} | {0} | {0} | {0} | {0} | {0} |
| Student's *t* d. f. | 5.331 | | 5.309 | | 5.565 | |
| Stay probabilities: $(p_{11}, p_{22})$ | 0.998 | 0.999 | 0.998 | 0.999 | 0.859 | 0.072 |
| ***Test summary:*** | | | | | | |
| Log Likelihood | -1388.45 | | -1382.03 | | -1373.20 | |
| Schwarz Criterion | -1427.77 | | -1417.42 | | **-1412.52** | |
| Ljung-Box Q(12) | 10.493 | | 9.970 | | 6.062 | |
| | {0.573} | | {0.619} | | {0.913} | |

See the legend of Table 3 for explanations.

The second column of Table 7 for the Student's *t* case indicates persistence in the two regimes with a staying probability, $p_{11}$ and $p_{22}$, both exceeding 0.99. The regimes are also characterized by different unconditional variances, $E\sigma_{jt}^2$: the unconditional variances in the high-volatility regime are about twice as large as those in the low-volatility regime. Consequently, the degree of persistence due to the Markov effects is close to one, i.e. $p_{11} + p_{22} - 1 \approx 1$. As noted by Timmermann (2000) and Morana (2002), these values of the staying probabilities representing infrequent mixing of regimes may be interpreted as closely resembling structural break models. In this case, estimates of the GARCH parameters from models ignoring the switching may be overwhelmed by substantial upward bias.

Table 7: Regime switching properties in FTSE100 and S&P500 markets.

| | FTSE100 | | | | S&P500 | | | |
|---|---|---|---|---|---|---|---|---|
| | Normal | | Student's *t* | | Normal | | Student's *t* | |
| $P_{11}$ , $P_{22}$ | 0.817 | 0.001 | 0.998 | 0.999 | 0.771 | 0.103 | 0.998 | 0.999 |
| $\pi^1$ , $\pi^2$ | 0.845 | 0.155 | 0.420 | 0.580 | 0.797 | 0.203 | 0.243 | 0.757 |
| $\delta_1$ , $\delta_2$ | 5.471 | 1.001 | 541 | 746 | 4.370 | 1.114 | 617 | 1923 |
| $E\sigma^2_{1t}$ , $E\sigma^2_{2t}$ | 0.115 | 0.503 | 0.176 | 0.262 | 0.301 | 1.371 | 0.156 | 0.288 |

The table shows the unconditional properties of estimated MS-GARCH models with normal and Student's *t* innovations. $P_{jj}$ , j=1, 2, are the staying probabilities and give the probability that state *j* will be followed by state $j$ . $\pi^j$ , *j*=1, 2, are the unconditional probabilities of being in regime *j*, that is $\pi^j = (1-P_{ii})/(2-P_{ii}-P_{jj})$ , *j*=1, 2 and $i \neq j$ . $\delta_j$ , *j*=1 2, are the expected duration times for regime *j*, that is $\delta_j = 1/(1-P_{jj})$ . $E\sigma^2_{jt}$ , *j*=1, 2, denotes the unconditional expectation of the variance.

The above results resemble those in which regime-switching GARCH models are implemented in modeling the dynamics of stock returns. For instance they are similar to those of Marcucci (2005) for the S&P100 and of Ane and Ureche-Rangau (2006) for Asian stock market indices.

Finally we note that the FTSE 100 moved from an auction to an electronic trading system on October 10, 1997. This may affect the regime change and leverage effects in our FTSE100 data. Therefore, the models are also estimated for the pre- and post- October 10, 1997 periods. The results indicate (available upon request) that the data follows similar structures in the sub-periods as well. The asymmetry coefficient for the sub-period before (after) October 10, 1997 is smaller (bigger) than the estimated coefficient for the whole sample (in absolute value). Therefore, it seems that under an electronic trading system the market reacts more strongly to bad news as compared to the previous trading system. Furthermore, the MS results reveal that the FTSE100 market follows a mean reverting feature under the old trading system while it exhibits a structural break feature after October 10, 1997. Using the rolling method in the next subsection, we consider these effects in evaluating the out-of-sample performance of models in forecasting VaR.

# 4 Performance of Models in Forecasting VaR

The diagnostic tests in the previous section show that standard econometric tests for model specification may not be appropriate for choosing the best model among different GARCH models. In particular, as demonstrated by Hansen (1996) and McLachlan and Peel (2000), the standard likelihood test cannot be employed for testing the single-regime versus the MS model.[11] Furthermore, Sarma et al. (2003) show that different methodologies can yield different VaR measures for the same portfolio and can sometimes lead to significant errors in risk measurement.

One can compare the out-of-sample performance in forecasting VaR by the competing models to overcome the above problem. The approach focusing on the past performance of VaR models is referred to as backtesting which checks whether a model's risk estimates are consistent with its assumptions. Furthermore, special attention is devoted to the validation of internal risk assessment models within the Basle Accord (1996) framework.

Having considered alternative methods for backtesting VaR,[12] we utilize a set of exceptions and regulatory-based backtesting methods to evaluate the performance of our competing VaR models. The first set comprises the Christoffersen (2003) LR test and the Hurlin and Tokpavi (2006) Multivariate Portmanteau (MP) test. The second set is based on the traffic light regulations proposed by the Basle Committee on Banking Supervision (1996). While the first set (the LR and MP tests) evaluates the statistical accuracy of the competing VaR models, the regulatory-based backtest measures the loss to the economic agent using the model.

Based on the diagnostic results in the preceding section, we compare three different groups of VaR models: the GARCH, EGARCH and MS-GARCH models. For each group, three types of innovations are considered: the Normal, Student's *t* and skewed Student's *t*, resulting in nine GARCH family VaR models. The models are used to estimate one day ahead VaR of both long and short trading positions (left and right tails of returns distribution) with different probabilities (at different tail quantiles): 0.5%, 1%, 5%, 95%, 99% and 99.5%.

## 4.1 Exceptions-based backtesting results

A rolling window method of 1000 days is used to estimate the daily VaR for each model. Thus the indicator variable $I_t$ contains the last 1000 (2001 t0 2004) hit sequences of the VaR violations. The motivation behind the rolling window technique is to consider dynamic time-varying characteristics of the data in different time periods.

The results of Christoffersen's LR test for the FTSE and the S&P are summarized in Tables 8 and 9, respectively. The number of failures is shown along with *p*-values for an unconditional coverage test, $LR_{uc}$, an independence

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[11] It is also demonstrated by Dacco and Satchell (1999) that the evaluation of forecasts from non-linear models like regime-switching models based on statistical measures might be misleading.
[12] See Dowd (2005) for a survey of backtesting VaR models.

test, $LR_{ind}$, and a conditional coverage test, $LR_{cc}$. It is obvious that there are some big discrepancies between the number of failures for the long and short positions (hereafter $f_l$ and $f_s$, respectively) obtained from the symmetric models (normal and Student's *t* innovations). For the FTSE returns $f_l$ is higher than $f_s$ at all levels of VaR, and vice-versa for the S&P returns. This can be considered as clear evidence of the asymmetry in our returns data.

The specification results showed it was not necessary to impose a skewness parameter in the Student's *t* distribution in modeling the distribution of our returns series. We check whether the skewed Student's *t* may be able to improve the out-of-sample results for both the negative and positive returns. In fact, it is expected that skewed Student's *t* innovations with $\zeta < 1$ can decrease $f_l$ and increase $f_s$, compared to $f_l$ and $f_s$ obtained from a symmetric Student's *t*. The opposite result holds when $\zeta > 1$.

Tables 8 and 9 indicate that, in general, the skewed Student's *t* improves on the out-of-sample performance of the corresponding GARCH and MS-GARCH models with Student's *t* innovations. The overall improvement for both negative and positive returns is 50% and 42% for the FTSE and S&P returns, respectively.[13]

On the other hand, the EGARCH specification with symmetric innovations generally leads to an acceptable performance for out-of-sample VaR prediction. Consequently, the EGARCH model with skewed Student's *t* innovations generally fails to improve on the number of failures compared to those obtained with symmetric Student's *t* innovations.

The results unsurprisingly show that the VaR models based on normal innovations have difficulties in modeling large returns. In particular, the normal MS-GARCH model consistently underestimates the return (risk) of both series at different tails, specifically at the 0.5% and 99.5% tails. In other words, failure numbers are much greater than the expected one at a given quantile, in the case of normal innovations. This leads to low *p*-values for the $LR_{uc}$ test, indicating an insignificant model for volatility forecasting and the VaR estimation. On the other hand, the models with Student's *t* innovations perform very well, irrespective of the model and the tail one takes into account. Thus, the LR test results show that a switch from normal to Student's *t* innovations yields a significant improvement in the VaR performance.

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[13] "Improvement" implies to the situation where the number of failures obtained from an asymmetric model is closer to the expected one, compared with the corresponding symmetric model.

Table 8: LR test results for different confidence level of VaR for the FTSE100 returns.

| VaR models | $N_F$ | p-values $LR_{cc}$ | p-values $LR_{ind}$ | p-values $LR_{cc}$ | $N_F$ | p-values $LR_{cc}$ | p-values $LR_{ind}$ | p-values $LR_{cc}$ |
|---|---|---|---|---|---|---|---|---|
| | VaR 5% | | | | VaR 95% | | | |
| GARCH-N | 54 | 0.562 | 0.960 | 0.844 | 45 | 0.465 | 0.039 | 0.092 |
| GARCH-t | 55 | 0.470 | 0.571 | 0.656 | 48 | 0.776 | 0.028 | 0.085 |
| GARCH-skt | 53 | 0.661 | 0.479 | 0.707 | 50 | 1.000 | 0.260 | 0.531 |
| EGARCH-N | 56 | 0.389 | 0.301 | 0.404 | 56 | 0.389 | 0.469 | 0.530 |
| EGARCH-t | 56 | 0.389 | 0.301 | 0.404 | 55 | 0.470 | 0.508 | 0.619 |
| EGARCH-skt | 56 | 0.389 | 0.620 | 0.610 | 59 | 0.201 | 0.362 | 0.292 |
| MS-GARCH-N | 63 | 0.068 | 0.987 | 0.190 | 53 | 0.661 | 0.015 | 0.047 |
| MS-GARCH-t | 58 | 0.254 | 0.721 | 0.490 | 53 | 0.661 | 0.194 | 0.390 |
| MS-GARCH-skt | 57 | 0.316 | 0.670 | 0.553 | 52 | 0.768 | 0.214 | 0.443 |
| | VaR 1% | | | | VaR 99% | | | |
| GARCH-N | 14 | 0.229 | 0.528 | 0.398 | 9 | 0.749 | 0.686 | 0.875 |
| GARCH-t | 11 | 0.752 | 0.621 | 0.842 | 6 | 0.171 | 0.788 | 0.377 |
| GARCH-skt | 11 | 0.752 | 0.621 | 0.842 | 8 | 0.512 | 0.719 | 0.756 |
| EGARCH-N | 19 | 0.011 | 0.391 | 0.027 | 8 | 0.512 | 0.719 | 0.756 |
| EGARCH-t | 13 | 0.360 | 0.558 | 0.555 | 8 | 0.512 | 0.719 | 0.756 |
| EGARCH-skt | 12 | 0.536 | 0.589 | 0.713 | 8 | 0.512 | 0.719 | 0.756 |
| MS-GARCH-N | 18 | 0.022 | 0.416 | 0.052 | 13 | 0.360 | 0.558 | 0.555 |
| MS-GARCH-t | 11 | 0.752 | 0.621 | 0.842 | 8 | 0.512 | 0.719 | 0.756 |
| MS-GARCH-skt | 12 | 0.536 | 0.589 | 0.713 | 9 | 0.749 | 0.686 | 0.875 |
| | VaR 0.5% | | | | VaR 99.5% | | | |
| GARCH-N | 11 | 0.020 | 0.621 | 0.060 | 3 | 0.334 | 0.893 | 0.621 |
| GARCH-t | 7 | 0.397 | 0.753 | 0.664 | 2 | 0.126 | 0.929 | 0.309 |
| GARCH-skt | 8 | 0.215 | 0.719 | 0.435 | 3 | 0.334 | 0.893 | 0.621 |
| EGARCH-N | 9 | 0.107 | 0.686 | 0.251 | 8 | 0.215 | 0.719 | 0.435 |
| EGARCH-t | 4 | 0.644 | 0.858 | 0.884 | 5 | 1.000 | 0.823 | 0.975 |
| EGARCH-skt | 2 | 0.126 | 0.929 | 0.310 | 6 | 0.662 | 0.788 | 0.877 |
| MS-GARCH-N | 11 | 0.020 | 0.621 | 0.060 | 6 | 0.662 | 0.788 | 0.877 |
| MS-GARCH-t | 10 | 0.048 | 0.653 | 0.129 | 5 | 1.000 | 0.823 | 0.975 |
| MS-GARCH-skt | 10 | 0.048 | 0.653 | 0.129 | 2 | 0.126 | 0.929 | 0.310 |

The table shows the backtesting results from the Likelihood Ratio test of Christoffersen (1998). *P-values* for unconditional coverage, $LR_{uc}$, independence, $LR_{ind}$, and conditional coverage, $LR_{cc}$, tests along with the number of failures, $N_F$, for both long and short positions are reported in the left and right panel of the table, respectively. The models are successively the GARCH, EGARCH and MS-GARCH specifications, where *N, t* and *skt* denote normal, Student's *t* and skewed Student's *t* error terms, respectively.

Table 9: LR test results for different confidence level of VaR for the S&P500 returns.

| VaR models | $N_F$ | p-values $LR_{cc}$ | $LR_{ind}$ | $LR_{cc}$ | $N_F$ | p-values $LR_{cc}$ | $LR_{ind}$ | $LR_{cc}$ |
|---|---|---|---|---|---|---|---|---|
| | VaR 5% | | | | VaR 95% | | | |
| GARCH-N | 43 | 0.302 | 0.042 | 0.074 | 46 | 0.561 | 0.931 | 0.842 |
| GARCH-t | 48 | 0.776 | 0.030 | 0.091 | 51 | 0.879 | 0.800 | 0.957 |
| GARCH-skt | 46 | 0.561 | 0.020 | 0.055 | 55 | 0.470 | 0.508 | 0.619 |
| EGARCH-N | 47 | 0.666 | 0.252 | 0.472 | 57 | 0.316 | 0.126 | 0.188 |
| EGARCH-t | 47 | 0.666 | 0.596 | 0.791 | 56 | 0.389 | 0.141 | 0.233 |
| EGARCH-skt | 45 | 0.465 | 0.502 | 0.611 | 56 | 0.389 | 0.141 | 0.233 |
| MS-GARCH-N | 66 | 0.026 | 0.423 | 0.061 | 64 | 0.050 | 0.224 | 0.070 |
| MS-GARCH-t | 58 | 0.254 | 0.165 | 0.199 | 59 | 0.201 | 0.100 | 0.115 |
| MS-GARCH-skt | 53 | 0.661 | 0.076 | 0.189 | 65 | 0.036 | 0.503 | 0.090 |
| | VaR 1% | | | | VaR 99% | | | |
| GARCH-N | 9 | 0.749 | 0.056 | 0.175 | 11 | 0.752 | 0.621 | 0.842 |
| GARCH-t | 4 | 0.030 | 0.858 | 0.094 | 5 | 0.079 | 0.823 | 0.209 |
| GARCH-skt | 4 | 0.030 | 0.858 | 0.094 | 7 | 0.315 | 0.753 | 0.575 |
| EGARCH-N | 8 | 0.512 | 0.719 | 0.756 | 12 | 0.536 | 0.589 | 0.713 |
| EGARCH-t | 6 | 0.171 | 0.788 | 0.377 | 9 | 0.749 | 0.686 | 0.875 |
| EGARCH-skt | 6 | 0.171 | 0.788 | 0.377 | 11 | 0.752 | 0.621 | 0.842 |
| MS-GARCH-N | 22 | 0.001 | 0.091 | 0.001 | 16 | 0.079 | 0.470 | 0.165 |
| MS-GARCH-t | 12 | 0.536 | 0.130 | 0.263 | 12 | 0.536 | 0.589 | 0.713 |
| MS-GARCH-skt | 11 | 0.752 | 0.106 | 0.258 | 16 | 0.079 | 0.470 | 0.165 |
| | VaR 0.5% | | | | VaR 99.5% | | | |
| GARCH-N | 4 | 0.644 | 0.858 | 0.884 | 5 | 1.000 | 0.823 | 0.975 |
| GARCH-t | 4 | 0.644 | 0.858 | 0.884 | 2 | 0.126 | 0.929 | 0.310 |
| GARCH-skt | 4 | 0.644 | 0.858 | 0.884 | 3 | 0.334 | 0.893 | 0.621 |
| EGARCH-N | 7 | 0.397 | 0.753 | 0.665 | 4 | 0.644 | 0.858 | 0.884 |
| EGARCH-t | 3 | 0.334 | 0.893 | 0.621 | 2 | 0.126 | 0.929 | 0.310 |
| EGARCH-skt | 3 | 0.334 | 0.893 | 0.621 | 3 | 0.334 | 0.893 | 0.621 |
| MS-GARCH-N | 11 | 0.020 | 0.106 | 0.018 | 11 | 0.020 | 0.621 | 0.060 |
| MS-GARCH-t | 6 | 0.662 | 0.788 | 0.877 | 7 | 0.397 | 0.753 | 0.664 |
| MS-GARCH-skt | 4 | 0.644 | 0.858 | 0.884 | 10 | 0.048 | 0.653 | 0.129 |

See the legend of Table 8 for explanations.

We also examine the statistical accuracy of the VaR models using Hurlin and Tokpavi's (2006) MP test. The results for the FTSE and S&P return are reported in Tables 10 and 11, respectively. Following their suggestion concerning the choice of lag order $K$ and number of coverage rates $m$, the following sets are considered for each VaR model, $k \in \{1,3,5\}$, $\Theta = \{5\%,\ 1\%\}$ and $\Theta = \{5\%,\ 0.5\%\}$ for $m = 2$ and $\Theta = \{5\%,\ 1\%,\ 0.5\%\}$ for $m = 3$. The $p$-values corresponding to $Q_m(k)$ for long and short positions are reported in the left and right panel of the tables, respectively. Consider the total number of violations of the no autocorrelation null. The MS-GARCH models with a total of just 6 (out of a total of 54 possible) violations outperform the single-regime GARCH and EGARCH models with 13 and 16 violations, respectively, for the FTSE 100. However, the EGARCH model (8 violations) outperforms the MS-GARCH (14 violations) and single-regime GARCH (19 violations) for the S&P500.

Comparing the MP test results with those of the LR test, we conclude that the MS-GARCH models outperform single-regime models as there is no case in which the $p$-values for all lag orders are less than 5%. The only small exception are the results on the left tail of the S&P returns with $\Theta = \{5\%,\ 1\%,\ 0.5\%\}$ where the MS-GARCH model with skewed Student's $t$ innovations is rejected at all lag orders. On the other hand, the single-regime models are more likely to be rejected by the MP test, especially those with large lag orders and high coverage rates. For instance, the null of no autocorrelation in the VaR violation sequences left by GARCH and EGARCH models is rejected when the MP test with the coverage set $\Theta = \{95\%,\ 99\%,\ 99.5\%\}$ and lags orders k=3,5 is implemented for the FTSE returns. The same result is achieved for the GARCH model in forecasting VaR for the S&P returns with the coverage set $\Theta = \{5\%,\ 1\%,\ 0.5\%\}$ at all lag orders, k=1, 3, 5.

Overall, the MS-GARCH-skt model is favored by our exceptions-based tests in forecasting of both the long and short VaR for FTSE returns. The same results hold for the MS-GARCH-t model in the case of S&P returns. Our findings are an improvement on those in Marcucci (2005) in which no model clearly outperforms the others in forecasting the long VaR of the S&P100 returns.

## 4.2 Regulatory-based backtesting results

We implement the Basle traffic light regulation to compute the capital requirements imposed by the previously introduced VaR methods using our 1000 daily VaR numbers previously estimated for exceptions-based backtesting. Since the multiplication factor is determined based on the number of exceptions over the previous 250 trading days, our regulatory-based backtesting sample contains the last 751 daily capital requirements imposed by each VaR model.

Table 10: MP test results for different multivariate coverage rates of VaR for the FTSE100 returns.

| VaR models | Θ ={5%, 1%} | | | Θ ={95%, 99%} | | |
|---|---|---|---|---|---|---|
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.61 | 0.08 | 0.05 | 0.70 | 0.11 | 0.32 |
| GARCH-t | 0.47 | 0.19 | 0.11 | 0.64 | 0.00 | 0.01 |
| GARCH-skt | 0.38 | 0.13 | 0.06 | 0.88 | 0.10 | 0.35 |
| EGARCH-N | 0.38 | 0.12 | 0.04 | 0.91 | 0.14 | 0.38 |
| EGARCH-t | 0.77 | 0.16 | 0.14 | 0.89 | 0.17 | 0.42 |
| EGARCH-skt | 0.77 | 0.11 | 0.10 | 0.89 | 0.13 | 0.38 |
| MS-GARCH-N | 0.80 | 0.64 | 0.37 | 0.54 | 0.35 | 0.70 |
| MS-GARCH-t | 0.58 | 0.29 | 0.06 | 0.83 | 0.22 | 0.46 |
| MS-GARCH-skt | 0.79 | 0.07 | 0.04 | 0.85 | 0.92 | 0.98 |
| | Θ ={5%, 0.5%} | | | Θ ={95%, 99.5%} | | |
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.75 | 0.28 | 0.13 | 0.70 | 0.51 | 0.90 |
| GARCH-t | 0.71 | 0.03 | 0.09 | 0.63 | 0.27 | 0.74 |
| GARCH-skt | 0.57 | 0.05 | 0.10 | 0.90 | 0.66 | 0.95 |
| EGARCH-N | 0.57 | 0.02 | 0.01 | 0.93 | 0.14 | 0.38 |
| EGARCH-t | 0.61 | 0.01 | 0.04 | 0.95 | 1.00 | 0.99 |
| EGARCH-skt | 0.93 | 0.00 | 0.00 | 0.92 | 0.99 | 0.98 |
| MS-GARCH-N | 0.76 | 0.25 | 0.09 | 0.54 | 0.00 | 0.00 |
| MS-GARCH-t | 0.66 | 0.25 | 0.03 | 0.85 | 0.83 | 0.94 |
| MS-GARCH-skt | 0.82 | 0.02 | 0.07 | 0.87 | 0.39 | 0.81 |
| | Θ ={5%, 1%, 0.5%} | | | Θ ={95%, 99%, 99.5} | | |
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.97 | 0.66 | 0.55 | 0.99 | 0.00 | 0.01 |
| GARCH-t | 0.94 | 0.45 | 0.01 | 0.98 | 0.00 | 0.00 |
| GARCH-skt | 0.90 | 0.55 | 0.00 | 1.00 | 0.00 | 0.00 |
| EGARCH-N | 0.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| EGARCH-t | 0.94 | 0.10 | 0.10 | 1.00 | 0.00 | 0.00 |
| EGARCH-skt | 0.99 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 |
| MS-GARCH-N | 0.93 | 0.17 | 0.07 | 0.96 | 0.05 | 0.55 |
| MS-GARCH-t | 0.95 | 0.92 | 0.77 | 1.00 | 0.79 | 0.99 |
| MS-GARCH-skt | 0.99 | 0.49 | 0.00 | 1.00 | 0.97 | 1.00 |

This table shows the *p*-values for the MP test of Hurlin and Tokpavi (2006). The models are successively the GARCH, EGARCH and MS-GARCH specifications, where *N, t* and *skt* denote normal, Student's *t* and skewed Student's *t* error terms, respectively. For each model, the sliding window (rolling) method with a size of 1000 days is implemented to estimate the one day ahead VaR. $\Theta$ denotes discrete set of coverage rate in testing the null hypothesis corresponding to the joint null by the autocorrelations of order 1 in k = 1,3,5, for the hit sequences of VaR violations.

Table 11: MP test results for different multivariate coverage rates of VaR for the S&P500 returns.

| VaR models | Θ ={5%, 1%} | | | Θ ={95%, 99%} | | |
|---|---|---|---|---|---|---|
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.00 | 0.01 | 0.01 | 0.83 | 0.16 | 0.45 |
| GARCH-t | 0.05 | 0.11 | 0.20 | 0.91 | 0.09 | 0.28 |
| GARCH-skt | 0.03 | 0.06 | 0.13 | 0.93 | 0.38 | 0.75 |
| EGARCH-N | 0.04 | 0.33 | 0.68 | 0.76 | 0.04 | 0.15 |
| EGARCH-t | 0.55 | 0.87 | 0.91 | 0.78 | 0.01 | 0.08 |
| EGARCH-skt | 0.55 | 0.92 | 0.97 | 0.78 | 0.12 | 0.45 |
| MS-GARCH-N | 0.09 | 0.27 | 0.24 | 0.79 | 0.55 | 0.78 |
| MS-GARCH-t | 0.03 | 0.34 | 0.35 | 0.73 | 0.23 | 0.56 |
| MS-GARCH-skt | 0.01 | 0.05 | 0.00 | 0.90 | 0.47 | 0.80 |
| | Θ ={5%, 0.5%} | | | Θ ={95%, 99.5%} | | |
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.05 | 0.16 | 0.26 | 0.97 | 0.80 | 0.95 |
| GARCH-t | 0.04 | 0.10 | 0.18 | 0.98 | 0.59 | 0.81 |
| GARCH-skt | 0.02 | 0.06 | 0.12 | 0.96 | 0.48 | 0.85 |
| EGARCH-N | 0.02 | 0.25 | 0.59 | 0.78 | 0.21 | 0.40 |
| EGARCH-t | 0.92 | 0.98 | 1.00 | 0.80 | 0.00 | 0.00 |
| EGARCH-skt | 0.88 | 0.99 | 1.00 | 0.80 | 0.01 | 0.10 |
| MS-GARCH-N | 0.02 | 0.07 | 0.20 | 0.85 | 0.45 | 0.78 |
| MS-GARCH-t | 0.38 | 0.85 | 0.53 | 0.75 | 0.00 | 0.00 |
| MS-GARCH-skt | 0.14 | 0.22 | 0.02 | 0.90 | 0.14 | 0.40 |
| | Θ ={5%, 1%, 0.5%} | | | Θ ={95%, 99%, 99.5} | | |
| | K=1 | K=3 | K=5 | K=1 | K=3 | K=5 |
| GARCH-N | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.01 |
| GARCH-t | 0.00 | 0.00 | 0.00 | 1.00 | 0.08 | 0.59 |
| GARCH-skt | 0.00 | 0.00 | 0.00 | 1.00 | 0.93 | 1.00 |
| EGARCH-N | 0.20 | 0.95 | 1.00 | 0.99 | 0.43 | 0.88 |
| EGARCH-t | 0.64 | 1.00 | 1.00 | 0.99 | 0.00 | 0.07 |
| EGARCH-skt | 0.58 | 1.00 | 1.00 | 0.99 | 0.16 | 0.75 |
| MS-GARCH-*N* | 0.06 | 0.21 | 0.06 | 1.00 | 0.00 | 0.06 |
| MS-GARCH-t | 0.00 | 0.11 | 0.16 | 0.99 | 0.02 | 0.33 |
| MS-GARCH-skt | 0.00 | 0.01 | 0.00 | 0.95 | 0.01 | 0.13 |

See the legend of Table 10 for explanations.

Figure 2 shows the capital requirement for the FTSE and SP imposed by three selected GARCH, EGARCH-t and MS-GARCH models with Student's *t* innovations. One striking feature is that the capital requirements of these three VaR models display a very similar pattern. However, the MS-GARCH models usually impose lower daily capital requirements, compared to the single-regime models. This helps risk managers avoid over-conservative estimation of VaR and so save on their minimum capital requirements.

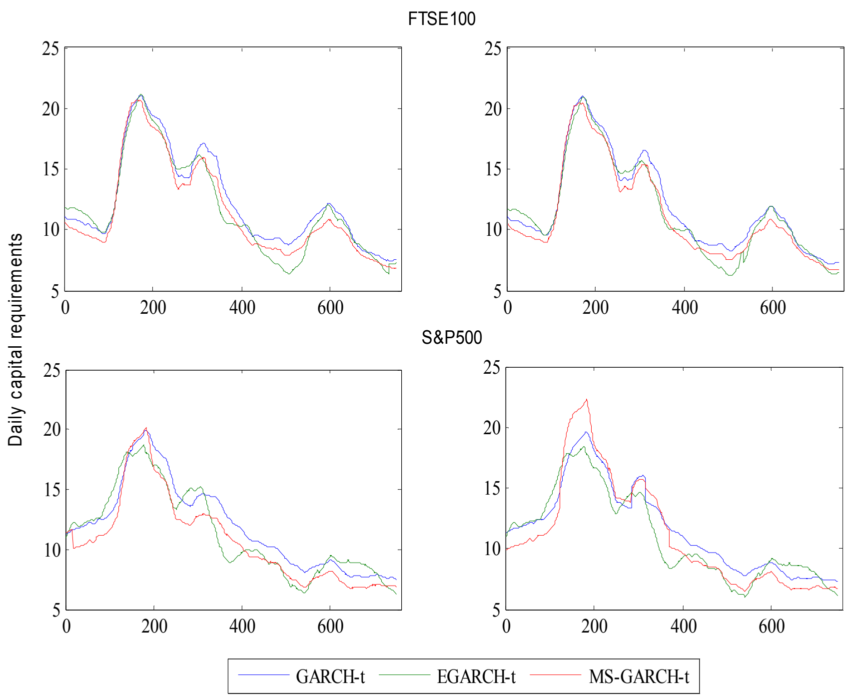


Figure 2: Comparison of capital requirement imposed by different VaR models over 751 days for the left and right tails of the FTSE100 and S&P500 returns.

Tables 12 and 13 summarize the results of regulatory-based backtesting for the FTSE and the S&P data, respectively. The numbers under the green, yellow and red columns indicate how many times during the 751 days each model has been placed in that particular zone by the Basle traffic light regulation. The average daily capital requirement is also reported along with its variance over the sample period. We implement the Hansen (2005) Superior Predictive Ability (SPA) test to compare the performance of the VaR models in terms of regulatory-based backtesting. The last column of the Tables presents the ranking between comparable models based on the SPA test, where the benchmark model is

GARCH-t. The 'best' model is that which has the most significant performance relative to the benchmark model. The other four pair-wise comparisons are those models with a performance that corresponded to the 75% (second best), 50% (median), 25% (second worst) and 0% (worst) quantile of model performance.

Table 12: Results of regulatory-based backtesting over 751 days for the FTSE100.

| VaR Models | Areas | | | Capital Requirements | | |
|---|---|---|---|---|---|---|
| | Green | Yellow | Red | Average | Variance | SPA Rank |
| ***Long position*** | | | | | | |
| GARCH-N | 660 | 91 | 0 | 11.97 | 14.06 | Median |
| GARCH-t | 751 | 0 | 0 | 12.30 | 14.32 | Benchmark |
| GARCH-skt | 751 | 0 | 0 | 12.50 | 14.01 | |
| EGARCH-N | 452 | 299 | 0 | 12.57 | 25.97 | Worst |
| EGARCH-t | 733 | 18 | 0 | 11.69 | 15.34 | Second best |
| EGARCH-skt | 751 | 0 | 0 | 12.01 | 16.08 | Second worst |
| MS-GARCH-N | 480 | 271 | 0 | 11.84 | 19.81 | |
| MS-GARCH-t | 751 | 0 | 0 | 11.49 | 14.59 | Best |
| MS-GARCH-skt | 699 | 52 | 0 | 11.62 | 14.42 | |
| ***Short position*** | | | | | | |
| GARCH-N | 619 | 132 | 0 | 11.72 | 13.02 | Worst |
| GARCH-t | 751 | 0 | 0 | 11.93 | 14.17 | Benchmark |
| GARCH-skt | 751 | 0 | 0 | 11.70 | 14.73 | Second worst |
| EGARCH-N | 741 | 10 | 0 | 11.12 | 12.33 | Best |
| EGARCH-t | 741 | 10 | 0 | 11.47 | 14.99 | |
| EGARCH-skt | 741 | 10 | 0 | 11.17 | 14.27 | |
| MS-GARCH-N | 452 | 299 | 0 | 11.36 | 11.99 | |
| MS-GARCH-t | 751 | 0 | 0 | 11.18 | 14.19 | Second best |
| MS-GARCH-skt | 741 | 10 | 0 | 11.31 | 15.20 | Median |

This table summarizes the capital requirements imposed by different VaR models. For each model, the numbers under the green, yellow and red columns indicate how many times during the 751 days the model has been placed in that particular zone by the Basle traffic light. Average daily capital requirement is also reported along with its variance over the sample period. The last column reports the ranking between comparable models based on the Superior Predictive Ability (SPA) approach, where the benchmark model is GARCH-t. The 'best' model is that model had the most significant performance relative to the benchmark model. The other four pair-wise comparisons are those models with a performance that corresponded to the 75% (second best), 50% (median), 25% (second worst) and 0% (worst) quantile of model performance. There is no ranking for the models which have some red zone record, as a placing in the red zone implies a problem within the VaR model. This is also the case for those models which have weak performance in the exception-based testing.

Table 13: Results of regulatory-based backtesting over 751 days for the S&P500.

| VaR Models | Areas | | | Capital Requirements | | |
|---|---|---|---|---|---|---|
| | Green | Yellow | Red | Average | Variance | SPA Rank |
| ***Long position*** | | | | | | |
| GARCH-N | 749 | 2 | 0 | 11.23 | 10.24 | |
| GARCH-t | 751 | 0 | 0 | 11.92 | 12.38 | Benchmark |
| GARCH-skt | 751 | 0 | 0 | 12.25 | 13.13 | Worst |
| EGARCH-N | 749 | 2 | 0 | 10.81 | 10.34 | Second best |
| EGARCH-t | 751 | 0 | 0 | 11.35 | 12.56 | Median |
| EGARCH-skt | 751 | 0 | 0 | 11.55 | 14.89 | Second worst |
| MS-GARCH-N | 352 | 396 | 3 | 11.73 | 20.83 | |
| MS-GARCH-t | 734 | 17 | 0 | 10.76 | 12.71 | Best |
| MS-GARCH-skt | 746 | 5 | 0 | 10.95 | 11.62 | |
| ***Short position*** | | | | | | |
| GARCH-N | 504 | 247 | 0 | 11.59 | 16.42 | Worst |
| GARCH-t | 720 | 31 | 0 | 11.63 | 12.45 | Benchmark |
| GARCH-skt | 720 | 31 | 0 | 11.26 | 12.00 | Second worst |
| EGARCH-N | 699 | 52 | 0 | 10.57 | 10.81 | Best |
| EGARCH-t | 751 | 0 | 0 | 11.01 | 12.54 | |
| EGARCH-skt | 751 | 0 | 0 | 10.80 | 12.81 | Second best |
| MS-GARCH-N | 435 | 287 | 29 | 11.10 | 19.09 | |
| MS-GARCH-t | 504 | 247 | 0 | 11.17 | 19.63 | Median |
| MS-GARCH-skt | 435 | 287 | 29 | 10.87 | 17.90 | |

See the legend of Table 13 for explanations.

Overall, the MS-GARCH-t model is ranked best for long positions in both indices whereas EGARCH specifications are best for short positions. However, the actual difference in performance between both sets of models is not large in economic terms. This means that the null hypothesis of the SPA test (the benchmark model imposes minimum capital requirements) is strictly rejected when the benchmark is one of the standard GARCH models. This result support those found in earlier studies. Marcucci (2005) obtains similar results with the S&P100 data, finding that MS-GARCH and EGARCH specifications perform better than the GARCH specification.

# 5 Conclusions

It is usually found that the GARCH family of models is a good candidate for estimating conditional VaR over short-term time horizons. In this paper, we extend this analysis to take account of both skewness and regime changes in forecasting VaR for long and short positions. The empirical study shows the MS-GARCH models lead to considerable improvements in correctly forecasting one-day-ahead VaR for long and short positions of the FTSE100 and S&P500 indices.

We use a novel combination of exceptions and regulatory-based backtesting procedures to evaluate out-of-sample model performance. This indicates that the MS-GARCH-t clearly outperforms other models in estimating the VaR for both long and short positions of the FTSE returns data. The exceptions-based (LR and MP) tests indicate that it is an acceptable VaR model and it imposes lower capital requirements according to the regulatory-based test. By contrast, the MS-GARCH-t and EGARCH-t models outperform others in the case of the S&P returns data. Imposing lower capital requirements, on either short or long position, they are acceptable VaR models for both long and short positions. Furthermore, analogous to the findings of Giot and Laurent (2003), the LR test confirms that assuming skewed Student's *t* innovations improves on the out-of-sample performance of the corresponding GARCH and MS-GARCH models with Student's *t* innovations.

Finally, we believe further research could endeavour to replicate these results for different firm sizes (e. g. the FTSE 250, FTSE Small Cap and FTSE All Share indices). This would be to investigate if smaller firms have greater leverage effects due to the additional volatility imposed upon them as a consequence of lower trading volume. This may shed further light on quantifying the effects of skewness and regime change in forecasting VaR for long and short positions.

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